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Thermal Engineering

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Preface

It gives me immense pleasure to present this book on 'Thermal Engineering'. This text is intended for undergraduate students of Mechanical, Automobile and Aeronautical engineering as well as AMIE courses and competitive examinations. It integrates thermodynamics, applied thermodynamics and thermal engineering and hence covers the syllabi of almost all subjects pertaining to thermal engineering taught from the first year to the final year of engineering curriculum.

Aim

During my teaching span of more than two decades, I felt that the subjects based on thermal engineering are often perceived as difficult by students. I observed that customarily, major problems are faced by students in understanding the text and illustrations. They need a text written in a simple and interesting way which exposes the subject systematically along with a variety of illustrative examples supporting the theoretical concepts.

Through this book I am making an attempt to overcome the problems of students as well as to impart sound knowledge. The presentation is simple, lucid and easy to understand. The topics are explained right from the fundamentals with the help of illustrative figures, enabling even a beginner to understand the subject very easily. Solutions for the problems also are explained with the help of illustrative figures so that the logic behind them is easily understood.

This book discusses the basic concepts first and then supports the theory with applications and solved numerical problems. This approach will help the students in developing an analytical mind. An engineer with an analytical mind and approach would be able to face any problems encountered in the actual engineering field. Moreover, it is my earnest hope that this book will provide a unique combination of features that will make it inviting and effective for both faculty and students.

Salient Features

The salient features of the book are the following:

- Complete coverage of both courses (a) Engineering Thermodynamics (b) Applied Thermodynamics
- Tutorial Approach of problem solving
- Solved Examples based on questions from numerous universities all across India as well as competitive examinations like GATE, IES etc
- Diverse and useful pedagogical features like Summary, Glossary, Review Questions, Problems and Objective Questions
- Well-labeled and apt schematic diagrams supporting theoretical and mathematical explanations

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Organisation

Principally, the book is divided into three parts.

The **First Part** of the book deals with the subject of thermodynamics, which is a core course taught to the first-year students of all disciplines in almost all the engineering colleges and universities. It includes the first eight chapters—chapters 1 to 8.

The **Second Part** of the book is designed for an applied thermodynamics course. This part includes the next eight chapters—chapters 9 to 16.

The **Third Part** provides the foundation learning material on thermal engineering. This part covers steam engineering, internal combustion engines, air compressors, gas turbines, jet and rocket propulsions, air conditioning and an introduction to heat transfer it includes remaining chapters—chapters 17 to 30.

Chapter 1 provides an overview of the basic concepts of thermodynamics. Concepts are an essential part of any science and in the case of thermodynamics, experience has shown that this is an area students find difficult. Several illustrations and day to day examples are provided in support of definitions and concepts to help the students have a thorough understanding of the topics.

Energy is a basic requirement for work transfer. **Chapter 2** gives a detailed treatment of energy and its forms, work and heat transfer. **Chapter 3** deals with the properties of pure substances. The properties of common working substances, steam and ideal gases are worked out in this chapter.

The first and second laws of thermodynamics are regarded as pillars of thermodynamics. The first law speaks of energy and its conservation (quantity), while the second law deals with the quality aspect of energy. Applications of the first law of thermodynamics to non-flow processes (for closed systems) and flow processes (open systems or control volume) are explained with support of several examples in **Chapters 4 and 5**, respectively. The second law of thermodynamics is treated in a comprehensive manner in **Chapter 6** and prolonged in **Chapter 7** with the concept of *entropy*. Entropy is an abstract property of the second law and can be thought of as a measure of disorder in the system responsible for energy degradation taking place in real processes. Other second-law concepts, *availability* and *irreversibility*, are introduced in **Chapter 8** along with the development of a procedure for performance evaluation of a system.

Chapter 9 deals with thermodynamic relations. Maxwell's relations, volumetric expansivity, isothermal and isentropic compressibility, Joule—Thomson coefficient and Clausius—Clapeyron equations are explained in this chapter. **Chapter 10** deals with compressible fluid flow. It uses the concept of the first and second laws for a moving fluid. The comprehensive thermodynamic analysis of gas power cycles (**Chapter 11**), vapour power cycles (**Chapter 12**), refrigeration cycles (**Chapter 13**), ideal gas mixtures (**Chapter 14**) and psychrometry (**Chapter 15**) are carefully considered. **Chapter 16** deals with fuels and combustion. Various types of fuels and determination of their calorific value are explained in this chapter.

Emphasis is given to steam engineering from Chapter 17 to Chapter 23. **Chapter 17** gives an overview of different types of boilers. **Chapter 18** incorporates various important boiler mountings and accessories. The boiler draught and performance are discussed in **Chapter 19**. Even though steam engines are obsolete nowadays, still the concept and preliminary analysis of steam engines are taken up in **Chapter 20**. The relevance of compressible fluid flow to steam nozzles are explained in **Chapter 21**. The concept and analysis of impulse and reaction steam turbines are incorporated in **Chapter 22**. **Chapter 23** gives an elementary treatment to steam condensers. The internal combustion engines are discussed in **Chapter 24**. **Chapters 25** and **26** take up the analytical treatment to reciprocating and rotary air compressors. **Chapter 27** gives an elementary treatment to gas turbines. **Chapter 28** provides an outlook to jet and rocket propulsions. **Chapter 29** takes up theoretical and analytical treatment to air conditioning and lastly, **Chapter 30** provides an elementary introduction of heat transfer.

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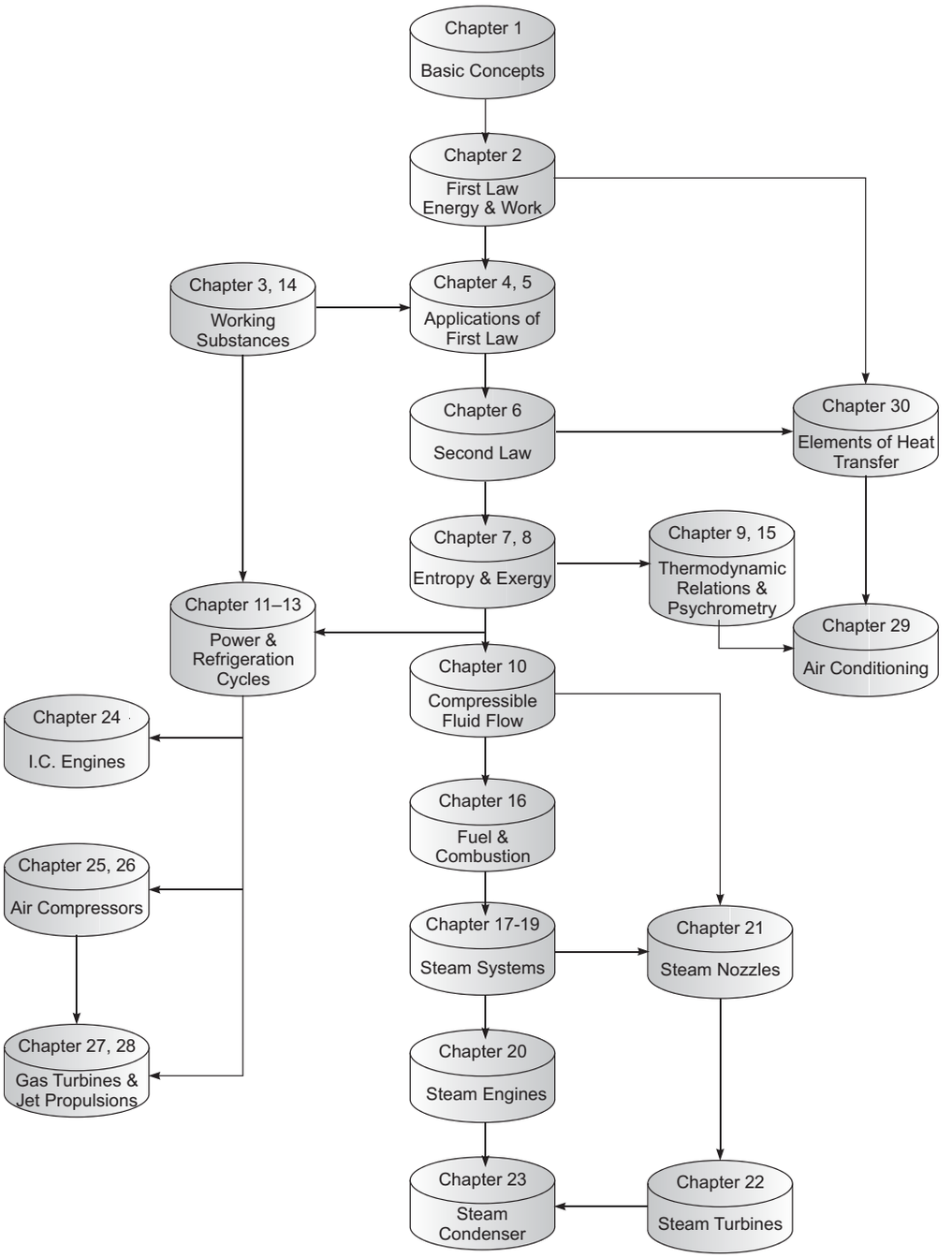
Dependency Chart *The chart given below depicts the preferred sequence of chapters which would help the readers plan their study. However readers are free to follow the sequence of their choice.*



Dependency Chart

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Web Supplements

The web supplements for this book can be accessed at http://www.mhhe.com/rathore/te/1e and contains the following material:

- ***For Instructors***
  - Solution Manual
  - PowerPoint Slides
- ***For Students***
  - Links to Reference Material

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A human creation can never be perfect. Some mistakes might have crept in the text. My efforts in writing this book will be rewarded if readers send their constructive suggestions and objective criticism with a view to improve the usefulness of the book. For any suggestion, query or difficulty, you are most welcome to write to me at mmrathore@gmail.com.

M M Rathore

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Nomenclature

A	area,
A_c	cross-sectional area
a	linear acceleration, specific Helmhotz function, accoustic velocity
b	constant
BP	brake power
$Bsfc$	brake specific fuel consumption
bwr	back work ratio
C	specific heat, constant
CV	calorific value
C_p	specific heat at constant pressure
C_T	constant temperature coefficient
C_v	specific heat at constant volume
c	clearance ratio
D	dimeter, constant
d	bore; diameter
E	total energy
e	energy per unit mass
F	force
FP	friction power
G	Gibbs function, H-TS
g	gravitational acceleration, specific gibbs function, h-Ts,
H	enthalpy, chimney height
HCV	higher calorific value
h	specific enthalpy, heat transfer coefficient, draught in water column
I	irreversibility
i	specific irreversibility
IP	indicated power
k	thermal conductivity, spring constant, number of cylinders, blade velocity coefficient
KE	total kinetic energy
ke	kinetic energy per unit mass
L	stroke, length, thickness, length dimension
LCV	lower calorific value
M	mass dimension, Mach number

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$\mathcal{M}$	molecular weight
m	mass
$\dot{m}$	rate of mass flow
N	rotational speed
n	polytropic index, number of moles
P	power
p	absolute pressure
PE	total potential energy
pe	potential energy per unit mass
p_m	mean effective pressure
Q	heat transfer
$\dot{Q}$	rate of heat transfer
q	heat transfer per unit mass, heat flux
R	gas constant, radius
R_u	universal gas constant
r	radius, compression ratio
r_w	work ratio
S	entropy
s	specific entropy, displacement
T	absolute temperature; torque, temperature dimension
t	time, time dimension
U	internal energy
u	specific internal energy
V	volume
V_c	clearance volume
V_s	swept volume
$\mathcal{V}$	velocity
v	specific volume
$\bar{v}$	molar volume
W	weight, work transfer
$\dot{W}$	rate of work transfer, power
W_{sh}	shaft power
w	specific weight, workdone per kg
w_{sh}	shaft work/kg
x	dryness fraction; length
x_i	mass fraction
y_i	mole fraction
Z	compressibilty factor
z	elevation.

Greek Symbols

α	absorptivity
β	coefficient of volumetric expansion
Δ	finite change in quantity

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ε	emissivity, effectiveness
γ	ratio of specific heats, C_p/C_v,
Φ	avaialability of closed system
ϕ	specific avaialability of closed system, relative humidity
ψ	specific availability of flow system
v	kinematic viscosity
μ	Joule–Thomption coefficient
η	efficiency
θ	temperature difference, angle, total energy of flow system
ρ	density, cutoff ratio, reflectivity
σ	Stefan's–Boltzmann constant
τ	transmissivity,
ω	specific humidity, angular speed.

Suffix

act	actual
atm	atmosphere
c	cross section, clearance
cr	critical point
g, *gauge*	gauge
d	diameter
db	dry bulb
dp	dew point
f	liquid state, of formation
fg	liquid vapour mixture
i	initial state, ice point, intermediate state in multistage, ith component in a mixture
g	gaseous state
H	high (temperature, T_H)
HP	heat pump
L	low (temperature, T_L)
O	exit state
p	constant pressure
prop	propulsive
R	reaction
r	reduced coordinates
REF	refrigerator
ref	referenece temperatures
s, *sat*	saturated state
sup	superheated state
t	total
th	throat, theoretical, thrust
t_p	triple point
v	constant volume

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w	water
wb	wet bulb
wet	wet state
0	dead state, stagnation state
1	initial state
2	final state

UNITS AND DIMENSIONS

Base Units

Quantity	*Units*	*Dimensions*
Length	metre	m
Mass	kilogram	kg
Time	second	*s*
Electric current	ampere	A
Temperature	kelvin	K

Derived Units

Acceleration	a	metre per second squared	m/s^2
Angular acceleration	ω	radiation per second squared	rad/s^2
Area	A	square metre	m^2
Electric current	I	ampere	A
Electric potential difference	V	volt	W/A
Electric resistance	R_e	ohm	Ω
Energy	*E*	joule	J or N.m
Entropy	*s*	joule per kelvin	J/K
Force	*F*	newton	$kg.m/s^2$
Frequency	*v*	hertz	Hz or 1/s
Heat energy	*Q*	joule	J or N.m
Power	*P*	watt	W or J/s
Radiation Intensity	*I*	watt per steradian	W/sr
Specific heat	C_p	joule per kilogram kelvin	J/kg-K
Stress	σ	pascal	N/m^2
Thermal conductivity	*k*	watt per metre-kelvin	W/m-k
Velocity	*U*	metre per second	m/s
Volume	V	cubic metre	m^3
Work	*W*	Joul	J or N./m

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VISUAL WALKTHROUGH

*Each chapter begins with an **Introduction** that describes the learning objectives and chapter importance.*

1

Basic Concepts

Introduction

The definition of the basic concepts forms a sound foundation for understanding of any subject. We start this chapter with an overview of thermodynamics, and a discussion of some basic concepts such as closed and open systems, isolated and adiabatic systems, working substance, continuum, property, state, path, process, cycle, and equilibrium. Then we discuss the pressure, pressure measurement, temperature and its measurement in this chapter. A careful study of these topics is necessary for understanding the following chapters.

There is frequent use of analytical treatment as well as graphical proof for conceptual explanation.

5.6 RELATION BETWEEN NON -FLOW WORK $\int pdv$ AND FLOW WORK $-\int vdp$

The steady-flow energy equation for unit mass-flow rate

$$q - w = h_2 - h_1 + \frac{\mathcal{V}_2^2}{2} - \frac{\mathcal{V}_1^2}{2} + (z_2 - z_1)g$$

It can be written in differential form as

$$\delta q - \delta w = dh + \mathcal{V}d\mathcal{V} + gdz$$

Using $h = u + pv$ and

$$\delta q = du + pdv$$

Inserting it above, we get

$$du + pdv - \delta w = d(u + pv) + \mathcal{V}d\mathcal{V} + gdz$$

or $$du + pdv - \delta w = du + d(pv) + \mathcal{V}d\mathcal{V} + gdz$$

$\therefore$ $$pdv - \delta w = d(pv) + \mathcal{V}d\mathcal{V} + gdz$$

or $$pdv = \delta w + d(pv) + \mathcal{V}d\mathcal{V} + gdz$$

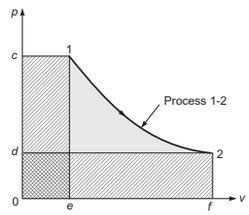

Fig. 5.18 *Relation between* $\int pdv$ *and* $-\int vdp$

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For easy understanding of the topics, a ***strong graphical and schematic support*** *is given in the book.*

Complicated figures are explained by simple, easily redrawable schematics.

In some chapters, the whole concept is picturised with the help of a simple figure.

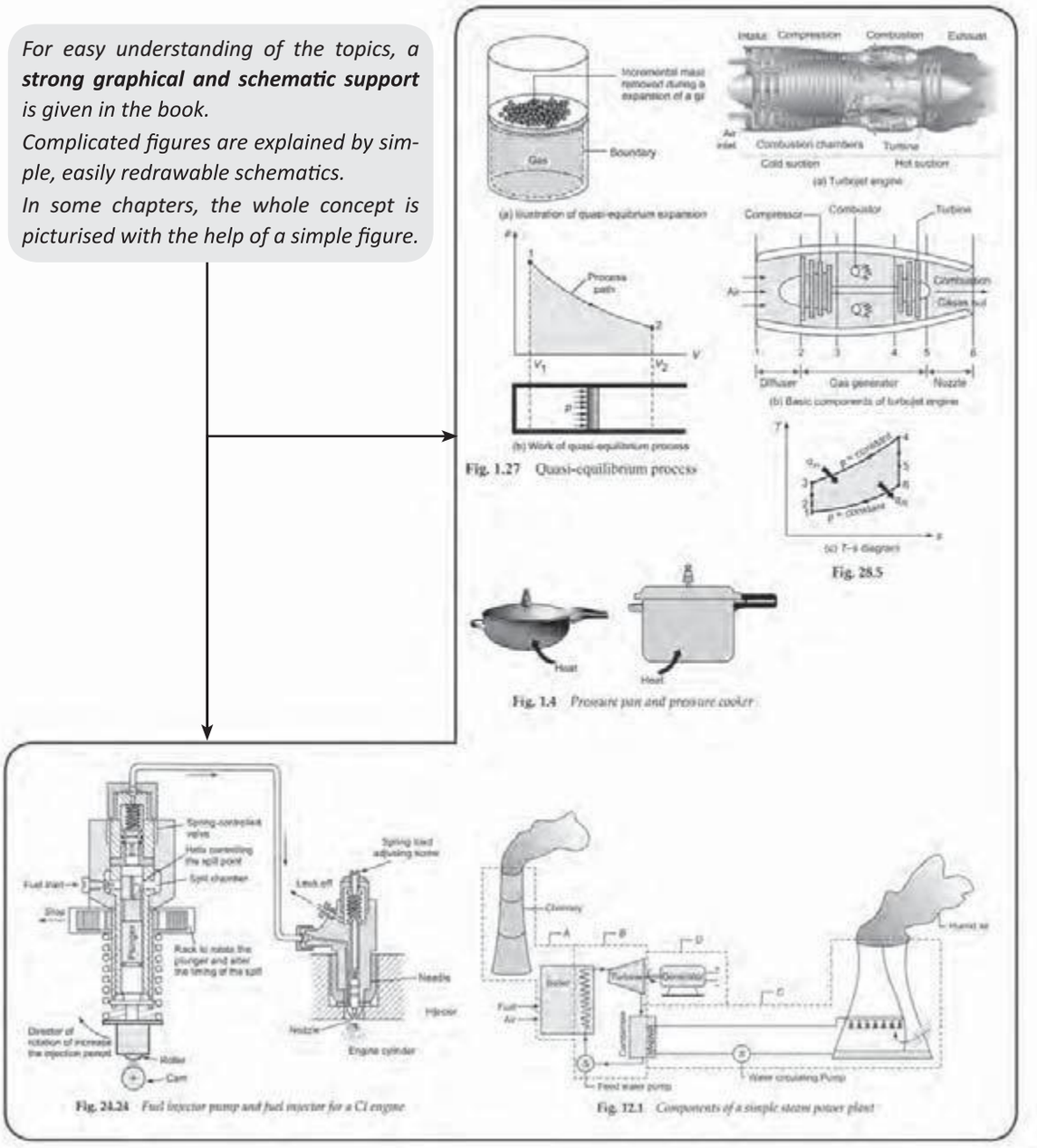


Fig. 1.27 Quasi-equilibrium process

Fig. 28.5

Fig. 1.4 *Pressure pan and pressure cooker*

Fig. 24.24 *Fuel injector pump and fuel injector for a CI engine*

Fig. 12.1 *Components of a simple steam power plant*

Example 12.10 *The boiler produces dry and saturated steam at 30 bar. The steam expands in the turbine to a condenser pressure of 20 kPa. Compare the cyclic work done and thermal efficiency of Carnot and Rankine cycles for these conditions.*

Solution

Given Dry saturated steam pressure

Boiler pressure p_1 = 30 bar

Condenser pressure; p_2 = 20 kPa

To find

(i) Work done by Carnot cycle,
(ii) Work done by Rankine cycle,
(iii) Thermal efficiency of Carnot cycle,
(iv) Thermal efficiency of Rankine cycle.

Properties of steam

At p_1 = 30 bar;

h_f = 1008.42 kJ/kg
h_g = 2804.2 kJ/kg
T_H = 233.9°C
s_g = 6.1869 kJ/kg
At p_2 = 20 kPa;
v_f = 0.001017 m^3/kg
T_L = 60.06°C
h_f = 251.38 kJ/kg
h_{fg} = 2358.33 kJ/kg
s_f = 0.8319 kJ/kg·K
s_{fg} = 7.0766 kJ/kg·K

The specific enthalpies

h_1 = 2804.2 kJ/kg
h_4 = 1008.42 kJ/kg

Carnot Cycle

Analysis

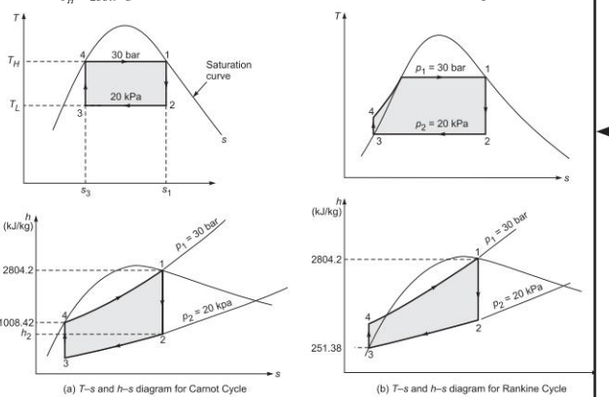
The efficiency of Carnol cycle is given by

$$\eta_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{(60.06 + 273)}{(233.9 + 273)}$$

= 0.343 or **34.3%**

The heat supplied per kg of steam

$q_{in} = h_1 - h_4 = 2804.2 - 1008.42$
= 1795.78 kJ/kg



(a) *T–s* and *h–s* diagram for Carnot Cycle

(b) *T–s* and *h–s* diagram for Rankine Cycle

Fig. 12.13

Numerous Worked examples *are provided in each chapter and at appropriate locations, to aid in understanding of the text material.*

Each solved example in the book is made student-friendly by adopting a step by step methodology.

For quick review, a bulleted ***summary*** *gives the essence of each chapter in brief.*

Summary

- An internal combustion engine is a heat engine that converts chemical energy of fuel into mechanical energy.
- The two-stroke engine completes its working cycle in two strokes of the piston. The three ports; inlet, transfer, and exhaust ports are used for suction, transfer and discharge of charge, respectively. A deflector-shapped piston is used to direct the charge inside the cylinder.
- The four-stroke engine completes its working cycle in four strokes of the piston as suction, compression, expansion and exhaust stroke. They are widely used on motor cycles, cars, buses, trucks and aeroplanes. Due to good thermal efficiency of four-stroke engines, the specific fuel consumption is less.
- Petrol engines use low compression ratio in the range of 4 to 10, while Diesel engines use high compression ratio usually in the range of 14 to 21. The petrol engines induct carburetted homogeneous air–fuel mixture as charge into the cylinder while Diesel engines induct only air during suction, and diesel is injected at the end of the compression stroke. The fuel burns in the presence of hot air. Therefore, the Diesel engines are also called compression ignition (CI) engines.
- The petrol engines are quantity-governing engines, while Diesel engines are quality-governing engines. The hit-and-miss governing is used for high speed gas engines.
- The internal combustion engines are subjected to very high temperature during combustion of charge. Due to overheating of engine, there may be uneven expansion in some parts, burning of lubricant, valve seats, etc. Therefore, the engine should be provided with adequate cooling arrangement.
- The air-cooling system is simple, cheaper and maintenance-free and is widely used on two wheelers and light-duty engines.
- The heavy-duty engines release a large amount of heat during combustion, and therefore, they are water cooled. The water-cooling arrangement consists of a pump, a fan, a water jacket around the engine and a radiator.
- The lubrication minimizes the friction between moving parts. The mist lubrication system is used in two-stroke engines, while all four-stroke engines use wet or dry sump lubrication system.
- The supercharging refers to a process which supplies the charge to the cylinder above atmospheric pressure. Thus, the volumetric efficiency of the engine improves and the engine produces more power.
- Scavenging is a gas exchange process into the cylinder of the engine where the admitted charge pushes the combustion products out of the cylinder. Scavenging takes place in two-stroke

Glossary

Triple point The point where the solid, liquid and vapour phases coexist in thermal equilibrium

Vaporization The transformation of liquid into vapour by supplying heat

Evaporation The transformation of liquid into vapour when liquid itself exacts its latent heat

Condensation The transformation of a vapour into a liquid

Melting The transformation of a solid into a liquid, also known as fusion

Solidification The transformation of a liquid into a solid, also called freezing

Sublimation The transformation of solid into vapour

Saturation The condition that exists when two or more phases coexist in equilibrium

Critical state The peak of the saturation curve

Quality (x) The ratio of mass of vapour present to the total mass of steam

Wetness fraction The ratio of mass of liquid present to the total mass of steam (= 1 – quality = 1 – *x*)

Wet steam A mixture of moisture and vapour, it is a substances that exists under the saturation curve.

Phase The physical state of matter

Dry steam Steam at saturation temperature, which is free from moisture

Superheated steam Dry steam at a temperature greater than the saturation temperature

Ideal gas An ideal working substance for thermodynamic cyles

*A list of commonly used terms in the chapter with their meanings are given in the form of a **glossary**. The glossary will serve as a dictionary for students.*

*Each chapter contains a set of **Review Questions** to recall the essence of the concepts discussed in the chapter.*

Review Questions

1. What are the limitations of the first law of thermodynamics?
2. State the importance of the second law of thermodynamics.
3. Define thermal reservoir, heat engine, refrigerator and heat pump.
4. What is a thermal energy reservoir? Define in terms of heat source and heat sink.
5. What are the characteristic features of a heat engine?
6. What is a heat pump. How does it differ from a refrigerator?
7. What do you mean by coefficient of performance? Show that $(COP)_{HP} = (COP)_R + 1$.
8. What do mean by thermal efficiency? Derive an expression for thermal efficiency of a heat engine.
9. State the Kelvin–Planck and Clausius statements of the second law of thermodynamics.
15. List suitable examples of reversible and irreversible processes.
16. What are the conditions to be a reversible process?
17. What is a thermodynamic temperature scale?
18. Explain the establishment of a thermodynamic temperature scale. Why is the thermodynamic temperature scale called absolute temperature scale?
19. Sketch the Carnot cycle on a p–v diagram.
20. Explain Carnot cycle and prove that

$$\eta_{carnot} = 1 - \frac{T_L}{T_H}.$$

21. Is it possible to operate a heat engine on a Carnot cycle? What purpose does a Carnot cycle serve?
22. State and prove the Carnot theorem.
23. Prove that a reversible engine is more efficient than an irreversible engine operating between the same temperature limits.

Problems

1. A rigid tank of volume 2 m^3 is filled with saturated steam at 2 bar. It contains 0.2 m^3 of moisture and the rest is vapour. Calculate the mass of moisture, mass of vapour and the quality of steam.
[(i)188.5 kg (ii) 2.032 kg (iii) 0.01066]

2. Steam at 0.75 bar and 150°C is condensed reversibly. Find the heat removed and change in entropy. Sketch the process on a T–s diagram.
[2393.8 kJ/kg, 6.536 kJ/kg·K]

3. Determine the increase in entropy of 1 kg of water when it is heated at atmospheric pressure from –23°C to 152°C. Take sp. heat of fusion and superheating as 2.093 kJ/kg·K and for water as 4.187 kJ/kg·K. *[9.0403 kJ/kg]*

4. Obtain specific volume, specific enthalpy, entropy of 2 kg of water at 1 bar and 60°C.

5. A perfect gas of 0.01 –kg mass occupies a volume of 0.02 m^3 at a pressure of 286.4 kPa and 30°C respectively. The gas is allowed to expand until the reduced by using helium instead of air in the tyres? What other considerations are involved in deciding whether to use helium?

10. A tank with a volume of 50 m^3 is being filled with air. At a particular instant, the air in the tank has a temperature of 400 K and a pressure of 1380 kPa. For this system, the pressure is increasing at the rate of 138 kPa/s and temperature is increasing at a rate of 25 K/s. Calculate the air flow rate into the tank in kg/s. *[21.25 kg/s]*

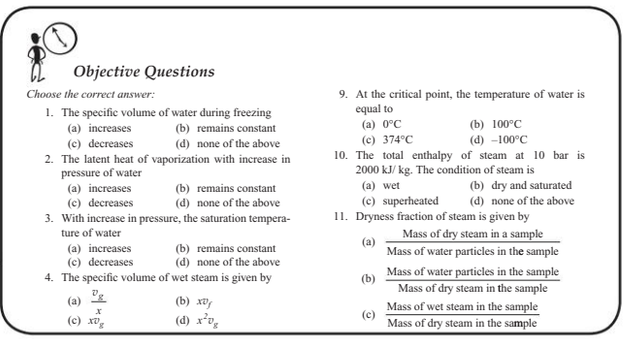
11. A spherical vessel of 3-m diameter contains steam at a pressure of 7 bar (gauge) and a temperature of 200° C, Find

(a) Total enthalpy of steam
(b) Total internal energy of steam
(c) Total mass of steam

Take atmospheric pressure as 1 bar.
[(a) 156.16 MJ (b) 144.85 MJ (c) 55.16 kg]

*A wide variety of **problems with answer(s)** are given as exercise to the students. These are very helpful to teachers in setting class work, assignments, quizzes and examinations.*

Objective questions *enable the user to have a clear comprehension of the subject matter. Answers to all the objective questions are provided.*

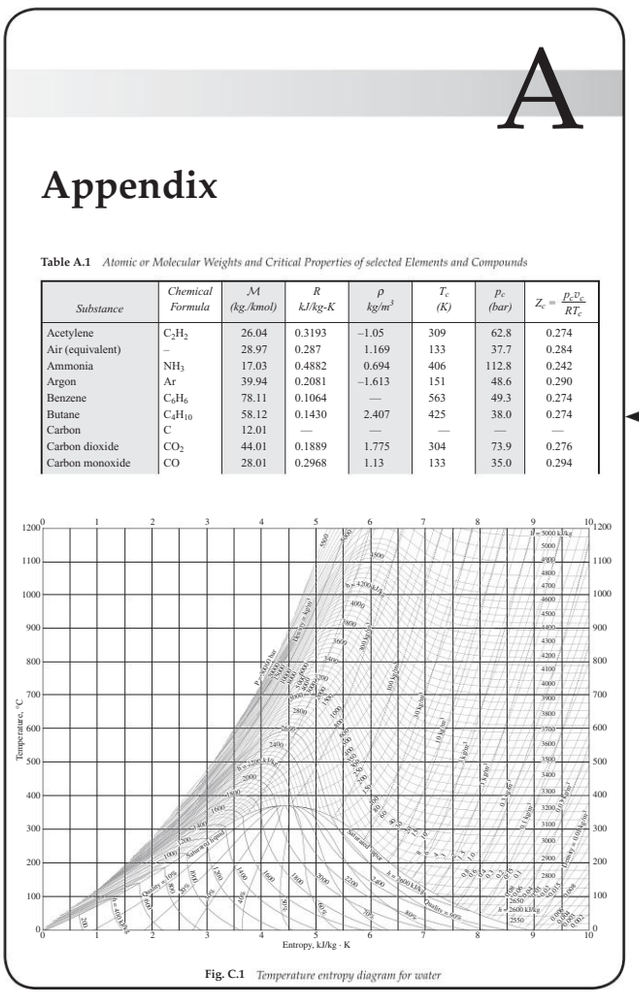


Objective Questions

Choose the correct answer:

1. The specific volume of water during freezing
   (a) increases (b) remains constant
   (c) decreases (d) none of the above
2. The latent heat of vaporization with increase in pressure of water
   (a) increases (b) remains constant
   (c) decreases (d) none of the above
3. With increase in pressure, the saturation temperature of water
   (a) increases (b) remains constant
   (c) decreases (d) none of the above
4. The specific volume of wet steam is given by
   (a) $\frac{v_g}{x}$ (b) xv_f
   (c) xv_g (d) x^2v_g
9. At the critical point, the temperature of water is equal to
   (a) 0°C (b) 100°C
   (c) 374°C (d) –100°C
10. The total enthalpy of steam at 10 bar is 2000 kJ/ kg. The condition of steam is
   (a) wet (b) dry and saturated
   (c) superheated (d) none of the above
11. Dryness fraction of steam is given by
   (a) $\frac{\text{Mass of dry steam in a sample}}{\text{Mass of water particles in the sample}}$
   (b) $\frac{\text{Mass of water particles in the sample}}{\text{Mass of dry steam in the sample}}$
   (c) $\frac{\text{Mass of wet steam in the sample}}{\text{Mass of dry steam in the sample}}$

For easy understanding the properties of elements, a large variety of tables and graphs are provided in ***Appendices—A, B,*** *and* ***C****.*



A

Appendix

Table A.1 *Atomic or Molecular Weights and Critical Properties of selected Elements and Compounds*

Substance	*Chemical Formula*	$\mathcal{M}$ *(kg./kmol)*	R *kJ/kg-K*	ρ *kg/m³*	T_c *(K)*	p_c *(bar)*	$Z_c = \frac{p_c v_c}{RT_c}$
Acetylene	C_2H_2	26.04	0.3193	–1.05	309	62.8	0.274
Air (equivalent)	–	28.97	0.287	1.169	133	37.7	0.284
Ammonia	NH_3	17.03	0.4882	0.694	406	112.8	0.242
Argon	Ar	39.94	0.2081	–1.613	151	48.6	0.290
Benzene	C_6H_6	78.11	0.1064	—	563	49.3	0.274
Butane	C_4H_{10}	58.12	0.1430	2.407	425	38.0	0.274
Carbon	C	12.01	—	—	—	—	—
Carbon dioxide	CO_2	44.01	0.1889	1.775	304	73.9	0.276
Carbon monoxide	CO	28.01	0.2968	1.13	133	35.0	0.294

Fig. C.1 *Temperature entropy diagram for water*

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1

Basic Concepts

Introduction

The definition of the basic concepts forms a sound foundation for understanding of any subject. We start this chapter with an overview of thermodynamics, and a discussion of some basic concepts such as closed and open systems, isolated and adiabatic systems, working substance, continuum, property, state, path, process, cycle, and equilibrium. Then we discuss the pressure, temperature and its measurement in this chapter. A careful study of these topics is necessary for understanding the following chapters.

1.1 THERMODYNAMICS

Thermodynamics can be defined as the 'science of energy'. In fact, the name 'thermodynamics' originates from two Greek words, *threme* (heat energy) and *dynamics* (motion or power). Thus, the subject of thermodynamics deals with energy and its transformation, including heat, work and physical properties of substances. It also deals with thermodynamic equilibrium and feasibility of processes.

Every engineering activity involves an interaction between energy and matter, and it is hard to find an area which does not relate to thermodynamics in some respect. An ordinary house has a gas stove, an electric iron, fans, a cooler, a refrigerator, pressure cooker and televisions. The design of each item requires the knowledge of thermodynamics.

In the engineering field, thermodynamics plays an important role in the design of automobile engines, compressors, turbines, refrigerators, rockets, jet engines, solar collectors, conventional and nuclear power plants. An energy-efficient home is designed for minimum heat loss in winter and minimum heat gain in summer. The size, location and power input to an air-conditioner is decided after thermodynamic analysis of a room.

1.2 THERMODYNAMIC SYSTEM

A thermodynamic system, or simply a ***system***, is defined as *a certain quantity of matter* or *a prescribed region in space considered for thermodynamic study.*

The region outside the system is called the ***surroundings*** *or* ***environment***. The real or imaginary surface that separates the system from its surroundings is called the ***boundary***. The boundary of the system may be fixed or movable. The system and its surroundings constitute the ***universe***. These terms are illustrated in Fig 1.1.

Types of Systems The thermodynamic systems can be classified as

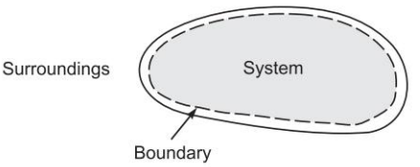

Fig. 1.1 *System, surroundings and boundary*

1. Closed and open systems
2. Homogeneous and heterogeneous systems

1.2.1 Closed System

A closed system (also known as ***control mass***) has the following characteristics:

(i) It consists of a fixed amount of mass, and no mass can cross its boundary, i.e., no mass can enter or leave a closed system.
(ii) The volume of a closed system may vary and hence its boundary is movable.
(iii) The energy in the form of heat or work can cross the boundary.

Figure 1.2 shows the representation of a close a system. Some examples of closed system are discussed below.

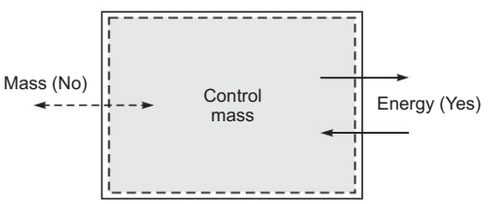

Fig. 1.2 *Representaion of a closed system*

1. Gas Trapped within a Piston–cylinder Device, Fig. 1.3

(i) The inside surface of the cylinder and piston forms the boundary.
(ii) With the movement of the piston, a part of the boundary can move.
(iii) The movement of the piston is restricted by a stopper, so no mass of gas can leave the system.
(iv) The energy, as heat and work, can cross the boundary.

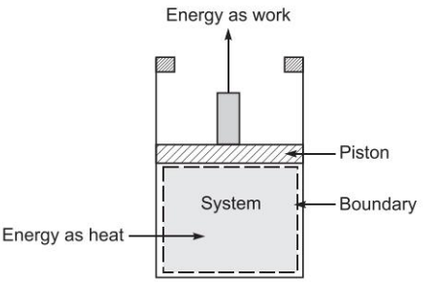

Fig. 1.3 *Gas trapped in a cylinder–piston device*

2. Food Items in a Pressure Cooker, Fig. 1.4

(i) The inside surface of the pressure cooker and its cover forms the boundary.
(ii) The boundary of the system is fixed.
(iii) No mass of the food can cross the boundary, unless by the process of whistling.
(iv) Energy as a heat can leave or enter the boundary of the system.

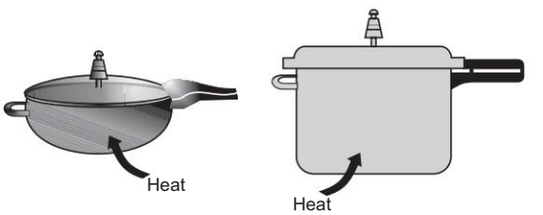

Fig. 1.4 *Pressure pan and pressure cooker*

3. Refrigerator and Ice-cream Freezer Figure 1.5 (b) shows the basic components of a refrigerator as a system along with its boundary. The working substance is the *refrigerant*.

(i) The compressor, condenser, capillary tube and condenser together constitute a system.
(ii) No mass of the working substance can leave the system.
(iii) The boundary of the system is fixed.
(iv) The energy as electrical work enters the compressor, and energy as heat leaves the condenser and enters the evaporator. Thus, energy crosses the boundary.

Hence, the refrigerator is a *closed system*.

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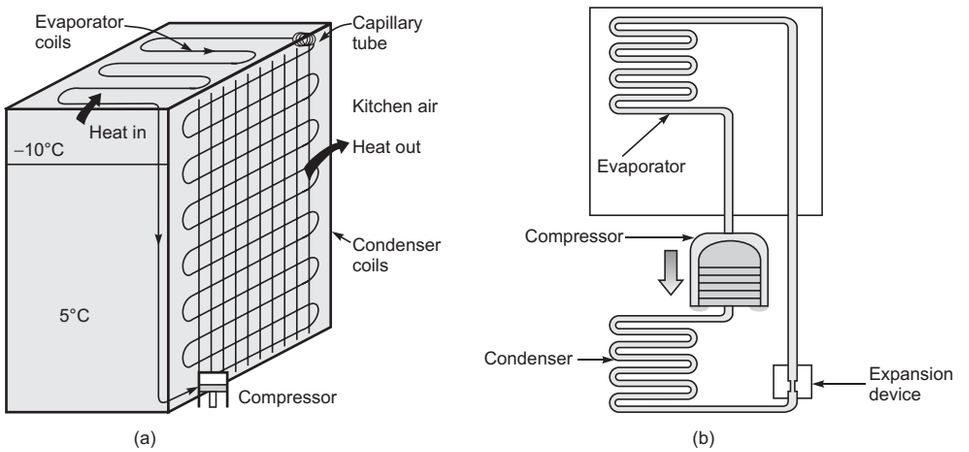

Fig. 1.5 *Schematic and basic components of a refrigerator*

4. Steam (Thermal) Power Plant Figure 1.6 shows the basic components of a steam power plant as a closed system along with its boundary. The working substance is water vapour.

(i) The boiler, turbine, condenser, and feed pump together constitute a system.
(ii) No mass of the working substance can leave the system.
(iii) The boundary of the system is fixed.
(iv) The energy as heat and work can cross the boundary of the system.

Thus, the steam power plant is a closed system.

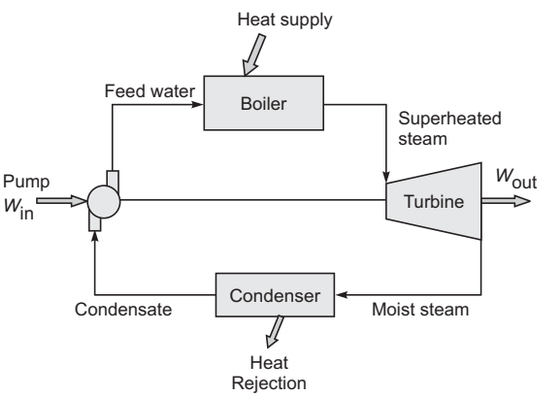

Fig. 1.6 *Steam power plant*

5. Electrolytic Battery, Fig. 1.7

(i) The acid and lead plates in a plastic box make up the system.
(ii) The boundary of the system is fixed.
(iii) No mass of the acid can leave the boundary.
(iv) Energy in electrical form can leave or enter the boundary of the system.

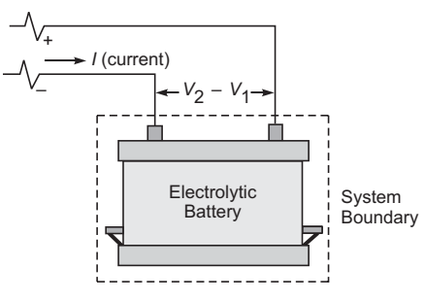

Fig. 1.7 *Electrolytic battery*

6. Bulbs and Lamps, Fig. 1.8

(i) The mass of the inert gas remains fixed inside bulbs and lamps.
(ii) The boundary of the system is fixed.
(iii) Energy as electricity, heat and light can cross the boundary of the system.

Isolated System An *isolated system* is a special case of a closed system, in which energy can also

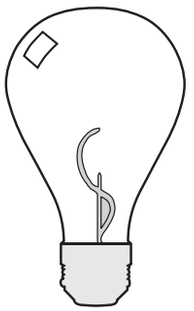

Fig. 1.8 *Incandescent bulb*

not cross the boundary of the system. This system is not in communication with its surroundings in any way.

Thus, an **isolated system** neither exchanges energy in any form nor any mass with the surroundings. Hence, by definition, the *universe* can be considered as an isolated system. The representation of an isolated system is shown in Fig. 1.9. Other examples are thermos flask (Fig. 1.10) and ice box.

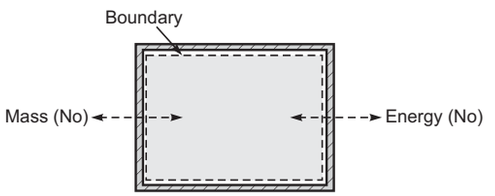

Fig. 1.9 *Representation of an isolated system*

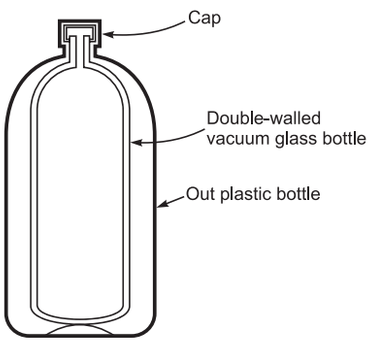

Fig. 1.10 *Thermos flask*

1.2.2 Open System

An *open system* (or a *control volume*) is a properly selected region in space. It usually encloses a device which involves mass flow, such as a compressor, turbine or nozzle. Flow through these devices is best studied by selecting a region within the devices as control volume.

The boundary of the control volume is called the *control surface*. Both mass and energy can cross the control surface. Thus, for an open system:

(i) The system has a selected region (fixed volume), called control volume.
(ii) The boundary of an open system is fixed.
(iii) Mass can cross the control surface.
(iv) The energy, in the form of heat and work, can cross the control surface.

An open system can be represented by Fig. 1.11. Some examples of open systems are discussed below.

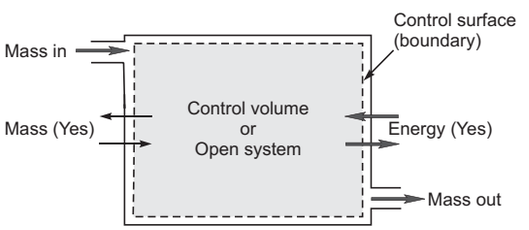

Fig. 1.11 *Representation of open system*

1. Flow through Tubes and Nozzles, Fig. 1.12

(i) The interior surface of the tube or nozzle forms the real boundary, and left and right openings form the imaginary boundary.
(ii) The mass can enter and leave the imaginary boundary of the control volume.
(iii) Energy as well as mass across the boundary can enter or leave the system.

2. Water Boiler, Fig. 1.13

(i) The interior surface of the boiler shell forms the real boundary, and the left and right openings form the imaginary boundary.

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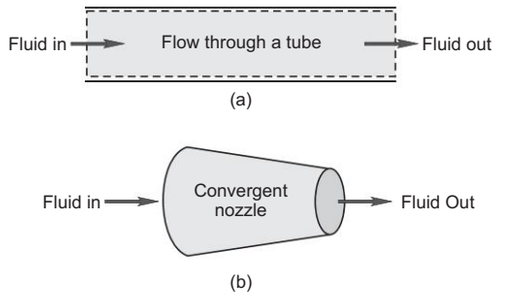

Fig. 1.12 *Flow through (a) Tube, and (b) Nozzle*

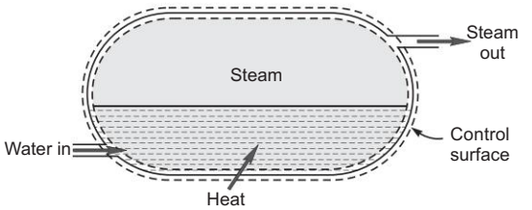

Fig. 1.13 *Water boiler*

(ii) The mass of the water enters, and the mass of steam comes out the control surface.
(iii) Energy as heat enters the control surface.
(iv) The energy in the form of heat or work can cross the boundary.

3. Reciprocating Air Compressor Figure 1.14 shows the reciprocating air compressor as an open system.

(i) The interior surface of cylinder and piston forms the control surface.

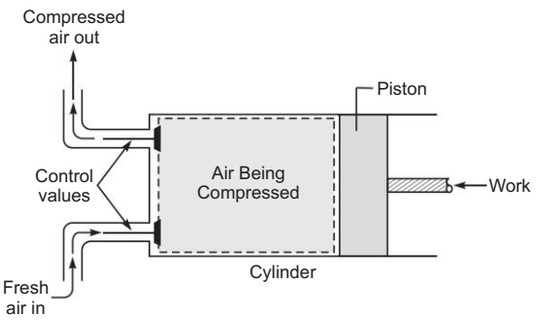

Fig. 1.14 *Reciprocating air compressor*

(ii) Mass of air can enter and leave the control volume through the valves.
(iii) Energy as work and heat can cross the control surface.

4. Internal combustion engines, gas turbine, Fig. 1.15

(i) The interior surface and openings form the control surface.
(ii) Mass as air and fuel mixture enters and leaves as combustion products.
(iii) Some part of the combustion heat is converted into work and the remaining is discharged as waste heat from the system.

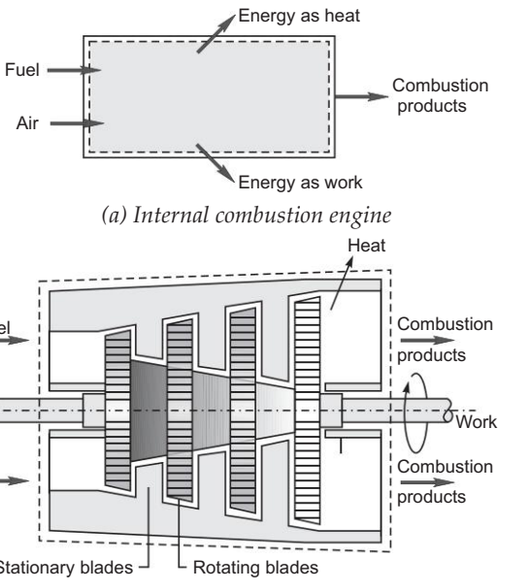

Fig. 1.15

Adiabatic System An *adiabatic system* is a special case of an open system, in which mass can cross the control surface, but energy in the form of heat is not allowed to cross the control surface of the system. However, energy in other forms can enter and leave the system. Insulated turbines, throttle valves, water pumps, water turbines, insulated heat exchangers, etc., are some examples of adiabatic systems.

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Some common features of these devices are the following:

(i) The interior surface of the device forms the control surface.
(ii) Fluid enters and leaves the control surface.
(iii) The energy other than heat energy can enter or leave the control surface.
(iv) Heat transfer is negligible at its outer control surface.

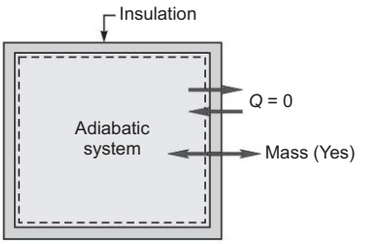

Fig. 1.16 *Representation of an adiabatic system*

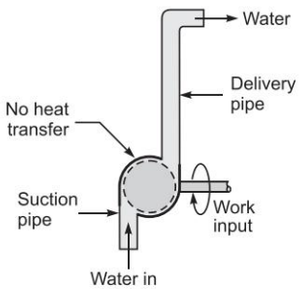

Fig. 1.17 *Water pump does not involve heat transfer*

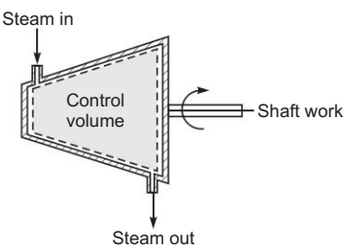

Fig. 1.18 *Insulated turbine*

If the heat exchanger considered above in Fig. 1.19 is not insulated at its outer boundary then the heat transfer will take place across its boundary

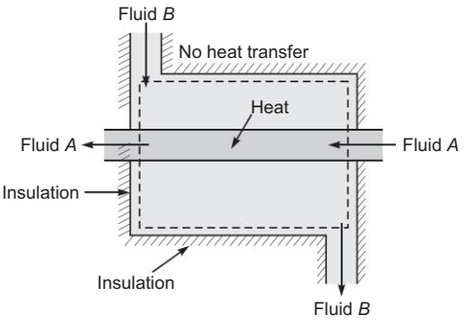

Fig. 1.19 *Heat exchanger insulated at its outer surface*

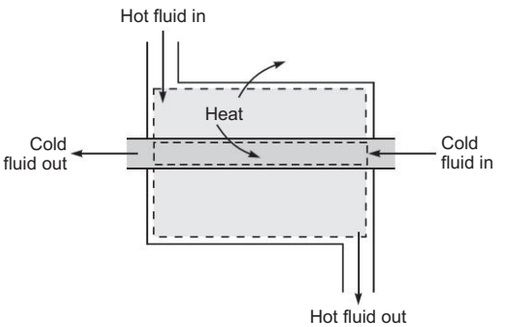

Fig. 1.20 *Heat exchanger which involves heat transfer at the outer boundary*

as shown in Fig.1.20 and it will not be treated as an adiabatic system.

The thermodynamic relations that are applicable to closed and open systems are different. Therefore, it is important that we must recognise the type of system before analysing it.

1.2.3 Difference Between Closed and Open Systems

	Closed system	*Open system*
1.	It is also called a *non-flow system*.	It is also called a *flow system*.
2.	A certain quantity of matter is considered for study. Thus, a closed system has a control mass.	A certain region is considered for study. This region is called control volume.

Contd.

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3.	The system is surrounded by a real boundary, which may be fixed or movable.	An open system is surrounded by a control surface, which is a combination of real and imaginary boundaries.
4.	No mass can cross the boundary, while energy can enter or leave the boundary of the system.	Mass as well as energy can enter or leave the control surface of the system.
5.	If energy transfer does not take place across the boundary then the closed system is called an *isolated system*.	If heat transfer does not take place across the control surface then an open system is called an *adiabatic system*.
6.	Examples of a closed system are pressure cooker and refrigerator.	Examples of an open system are scooter engine, air compressor and gas turbine.

1.2.4 Difference Between Isolated and Adiabatic Systems

	Isolated system	*Adiabatic system*
1.	It is a special type of a closed system.	It is a special type of an open system.
2.	Mass and energy do not cross the boundary of the system.	Mass and energy, except heat energy, can cross the boundary of the system.
3.	It is a closed system, which is insulated at its boundaries, thus it becomes isolated from its surroundings.	It is an open system, which is insulated at its real boundaries, thus heat cannot cross it.
4.	Examples of an isolated system are ice box and thermos flask, etc.	Examples of an adiabatic system are water pump, throttle valve and insulated steam turbine etc.

1.2.5 Homogeneous System

A system is called a **homogeneous system**, if it consists of a single *physical phase*, either solid, liquid or gas phase only. It is treated as one constituent for its analysis. Thus, analysis becomes simple, for example, ice, water and steam (three distinct phases of water), sugar or salt dissolved in water, air, oxygen gas and nitrogen gas.

1.2.6 Heterogeneous System

When a system is a mixture of two or more than two phases of matter, it is called a **heterogeneous system**.

Since each constituent present in the system has its own properties independent of each other, the system cannot be analysed as a single constituent, for examples, mixture of ice and water; mixture of water and steam; dal, rice and water in a pressure cooker, etc.

1.3 MACROSCOPIC V/S MICROSCOPIC VIEWS

It is well known that every substance is composed of a large number of molecules. The properties of the substance depend on the behavior of these molecules. In the *macroscopic approach*, a certain quantity of matter is considered without the events occurring at the molecular level. The macroscopic approach in the study of thermodynamics is also called *classical thermodynamics*. It provides a direct and easy way to the solution of engineering problems. In the macroscopic approach,

1. The structure of the matter is not considered,
2. Only a few variables are needed to describe the state of the system,
3. The values of these variables can be measured.

The *microscopic approach* is more elaborate. We know that every system is composed of a large number of molecules. All have the same mass but each moves with a velocity independent of others. Similarly, each molecule has its own position, temperature, etc. The microscopic approach of such a system will involve a large number of equations, specifying three location coordinates and three velocity components for each molecule. It is

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very difficult to adopt in practice even with high-speed computers. The properties of the system are based on the average behavior of a large group of molecules under consideration. Thus, this approach is also called *statistical thermodynamics*. In the microscopic approach,

1. The knowledge of the structure of the matter is necessary
2. A large number of variables are needed to describe the state of the system
3. The values of the variables cannot be measured easily.

1.4 WORKING FLUID

The matter contained within the system boundaries is called *working fluid*. It is used in thermodynamic devices as a medium for energy transport between the system and surroundings, while undergoing a thermodynamic process or cycle. A working fluid may be gas, vapour, liquid or any non-reactive mixture of these constituents. The working fluids frequently absorb, store or release energy. For examples, water vapour is the working fluid in a steam power plant, and a refrigerant is the working fluid in a refrigerator.

1.5 CONTINUUM

Matter is made up of discrete particles, called *atoms*. These atoms are widely spaced (free path) in a gaseous phase and matter may also have some voids. The macroscopic approach is applicable when the smallest unit of the matter is large enough compared to the mean free path of the atoms. Under such circumstances, the matter in a system is considered as continuous and homogeneous without any hole. This is called the concept of *continuum*.

Let us consider the mass Δm in a volume ΔV surrounded the point P as shown in Fig. 1.21. The ratio of $\Delta m/\Delta V$ is the average mass density of the system within the volume ΔV. The volume $\Delta V'$ is the smallest volume about the point P, for which the mass can be considered continuous. Any volume

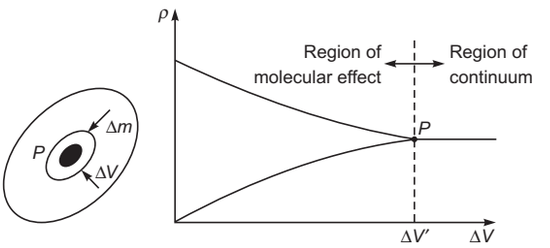

Fig. 1.21 *Concept of continuum*

smaller than this volume will lead to discontinuity in the particles, atoms and electrons, etc., and the density becomes unpredictable.

The variation in density is tentatively shown in Fig. 1.21. When the volume approaches zero, the density becomes uncertain.

$$\rho = \lim_{\Delta V \to \Delta V'} \left(\frac{\Delta m}{\Delta V} \right) \qquad ...(1.1)$$

Similarly, the definition of pressure is also required for a minimum area $\Delta A'$ at the which force ΔF_n acts.

$$\rho = \lim_{\Delta A \to \Delta A'} \left(\frac{\Delta F_n}{\Delta A} \right) \qquad ...(1.2)$$

The continuum is not applicable when the number of molecules in a system becomes negligible, e.g., a system under high vacuum.

1.6 THERMODYNAMIC PROPERTIES OF A SYSTEM

Any characteristic of a system is called a ***property***. Every system has certain characteristics by which its physical condition may be described, e.g., volume, temperature, pressure, etc. The list can be extended to include velocity, viscosity, thermal conductivity, modulus of elasticity, coefficient of thermal expansion, resistivity and elevation, etc.

The salient features of a thermodynamic property are the following:

(a) A property is a measurable characteristic, describing the state of a system.

(b) It has a definite value when the system is in a particular state.

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(c) It also helps to distinguish one system from another.

(d) The magnitude of a property depends on the state of the system, and it is independent of the path or route followed by a system during a process.

(e) A property is an exact differential.

The differential quantity of a property P is designated as dP, and its integral between states 1 and 2 of the system is

$$\int_1^2 dP = P_2 - P_1 \qquad \ldots(1.3)$$

For a given expression of a property,

$$dP = Mdx + Ndy$$

A simple check can be a useful tool to recognize whether a quantity is a property or not.

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y \qquad \ldots.(1.4)$$

Types of Properties Properties may be classified as

1. Intrinsic and extrinsic properties according to their origin, and
2. Intensive and extensive properties according to their dependability.

1.6.1 Intrinsic and Extrinsic Properties

Intrinsic Properties These are the basic properties, and cannot be defined in terms of other properties. Their values can be assigned independently for example, length, mass, time, area volume, pressure, temperature, electric current, etc.

Extrinsic Properties These are those properties whose values cannot be assigned independently. These are characteristic of the motion or position of a system and are measured in reference to certain datum such as velocity, acceleration, potential energy, kinetic energy, enthalpy, entropy, etc.

1.6.2 Intensive and Extensive Properties

Intensive Properties These the properties that do not depend on the extent of the system. These properties are independent of the mass also. Properties such as pressure, temperature, density, velocity, etc., are examples of intensive properties.

Extensive Properties These are properties that vary directly with the extent of the system. These properties depend on the mass of the system. The properties such as mass, area, volume, total energy, etc., are examples of extensive properties.

An easy way to distinguish whether a property is intensive or extensive is to divide the system into two equal parts with a partition as shown in Fig. 1.22.

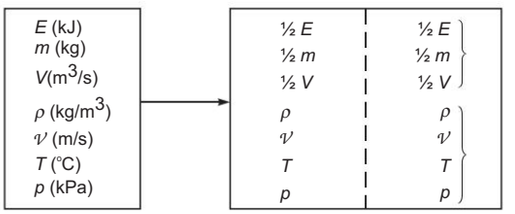

Fig. 1.22 *Recognition of intensive and extensive properties*

After partition, each part will have the same value for the intensive properties as of the original system, but half the value for extensive properties.

1.6.3 Density and Specific Gravity

Specific Property An extensive property expressed per unit mass of the system is called a specific property. Examples include specific volume, specific energy, specific enthalpy, specific internal energy, etc. and thus are intensive properties.

Mass density or simply *density* is a measure of the amount of working substance contained in a given volume and is defined as mass per unit volume.

$$\rho = \frac{\text{Mass of substance}}{\text{Volume occupied}} = \frac{m}{V} \text{ (kg/m}^3\text{)} \quad \ldots(1.5)$$

Similarly, *specific gravity,* or relative density, is defined as the ratio of density of a substance (ρ_s) to the density of water at 4 °C (ρ_{water} = 1000 kg/m^3). It is designated by SG.

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$$SG = \frac{\rho_s}{\rho_{water}} \quad (A\ dimensionless\ quantity)$$

or $\rho_s = SG \times \rho_{water} = SG \times 1000\ (kg/m^3)$...(1.6)

The *specific volume* (v) is the reciprocal of mass-density and is defined as the volume per unit mass of a system.

$$v = \frac{V}{m} = \frac{1}{\rho}\ (m^3/kg) \qquad ...(1.7)$$

The *specific weight* (γ) is defined as the weight of a substance per unit volume, or

$$\gamma = \frac{mg}{V} \qquad ...(1.8)$$

The relation between specific weight and mass density is given as

$$\gamma = \rho g \qquad ...(1.9)$$

Example 1.1 *Recognise whether the following quantities are properties or non-properties:*

(a) $pdv + vdp$ (b) pdv

(c) vdp

where p is the pressure and v is the specific volume.

Solution

(a) The quantity is $pdv + vdp$.

Its differential is $d(pv)$.

It is an exact differential, and thus the quantity $pdv + vdp$, is a property.

(b) The quantity is pdv.

Here, p is functionally related with v and its intergration cannot be evaluated unless the relationship between p and v is known. Thus, the quantity pdv is not a property.

(c) The quantity is vdp.

Here, v is functionally related with p, and thus by the same reasoning as given in (ii), the quantity vdp is not a property.

Example 1.2 *If the pressure p, specific volume v and absolute temperature T are funtionally related as*

$\frac{v}{T}dp + \frac{p}{T}dv$, *examine whether the quantity is a property.*

Solution

Given The quantity $\frac{v}{T}dp + \frac{p}{T}dv$.

To find Whether the given quantity is a property or not.

Analysis Comparing the given quantity with $M\,dx + N\,dy$, we get

$$M = \frac{v}{T} \quad \text{and} \quad x = p$$

and $$N = \frac{p}{T} \quad \text{and} \quad y = v$$

Obtaining the partial differentials, we get

$$\left(\frac{\partial M}{\partial v}\right)_p = \frac{\partial}{\partial v}\left(\frac{v}{T}\right) = \frac{1}{T}$$

and $$\left(\frac{\partial N}{\partial p}\right)_v = \frac{\partial}{\partial p}\left(\frac{p}{T}\right) = \frac{1}{T}$$

Since $\left(\frac{\partial M}{\partial v}\right)_p = \left(\frac{\partial N}{\partial p}\right)_v$, thus the given quantity is a property.

1.7 STATE, PATH, PROCESS AND CYCLE

1.7.1 State

The thermodynamic *state* is the *condition of the system* as characterised by certain thermodynamic properties like pressure, temperature, specific volume, etc.

If any system is not undergoing any change then all of its properties can be measured or calculated, which gives us a set of properties that completely describe the condition or state of the system. At this state, all thermodynamic properties of the system have the same value throughout the system. If the value of even one property changes, the system will change its state to a different one.

Consider a certain quantity of a gas as a system in a piston–cylinder device as shown in Fig.1.23 (a). At the position 1 of the piston at any instant, the condition of the system can be described by pressure p_1, temperature T_1 and volume V_1. The

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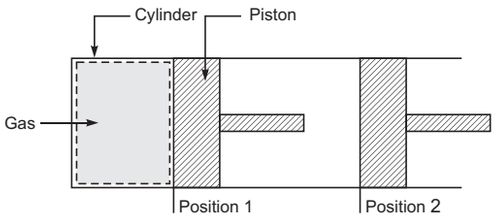
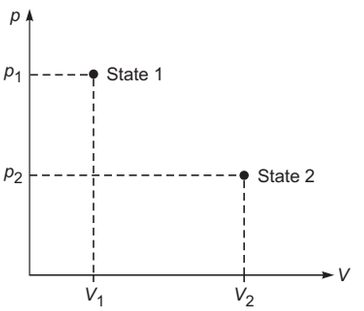
system is said to exist at the state 1. After expansion of the gas, the system will reach a new position 2 (state 2) as shown in Fig.1.23 (b).



(a) Piston at two positions within a cylinder



(b) Representation of states on a plot

Fig. 1.23

1.7.2 Path

When a gas expands in the cylinder, the piston moves outward, the properties of a system change and the system reaches to the new state 2. It is called a *change of state*. *A locus of series of states through which a system passes between initial and final states* is called a *path* as shown in Fig.1.24.

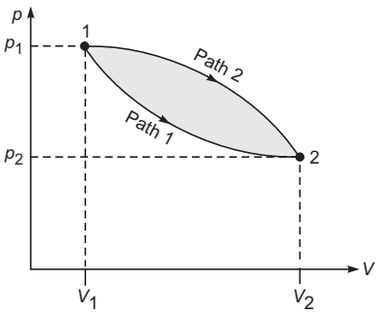


Fig. 1.24 *Two paths between initial and final states*

The system may reach from state 1 to state 2 by a number of paths depending on the type of expansion.

1.7.3 Process

The transformation of a thermodynamic system from one thermodynamic state to another is called a process.

A series of states through which a system passes during a process is called the *path of the process*. The processes are classified as follows:

1. A process undergone by a fluid in a closed system, is referred as a *non-flow process*.
2. A process undergone by a fluid in an open system is referred as a *flow process*.
3. When a process proceeds in such a manner that the system remains almost infinitesimally close to equilibrium, such a process is called a ***quasi-static process***.
4. A process is called a *reversible process* if it once has been taken place between two states, can be reversed to restore the system to initial conditions without leaving any effect on the surroundings. It passes through a series of equilibrium states.
5. A process which cannot be reversed by the same path, and follows in one direction only is called an *irreversible process*. It passes through a series of non equilibrium states.
6. When a system undergoes a process, while enclosed by an adiabatic wall (ideal insulator), the system does not experience any heat exchange between the system and its surroundings. Such a process is called an *adiabatic process*.

The prefix *iso-* is often used to designate a type of process for which a particular property remains constant.

(*i*) *An Isothermal Process* The *temperature* remains constant during the process.

(*ii*) *An Isobaric Process* The *pressure* remains constant during the process.

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(*iii*) *An Isochoric (Isometric) Process* The *volume* remains constant during the process.

(*iv*) *An Isentropic Process* The *entropy* remains constant during the process.

(*v*) *An Isenthalpic Process* The *enthalpy* remains constant during the process.

1.7.4 Cycle

If a system undergoes a series of processes in such a way that its *initial* and *final* states are identical then the system is said to have undergone a *cyclic process* or simply a *cycle*. A thermodynamic cycle is a sequence of processes that begins and ends at the same state as shown in Fig. 1.25. At the conclusion of a cycle, all properties of the fluid have the same values as they had at the initial state.

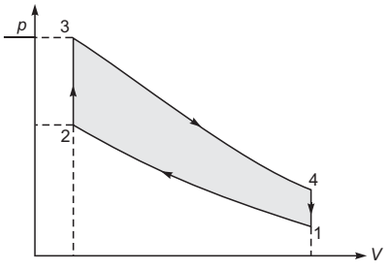

Fig. 1.25 *Cyclic process*

1.8 POINT FUNCTION AND PATH FUNCTION

When a system undergoes a change from one state to another, the properties of the system also change, which depend only on end states and not on the path followed between these two states. Therefore, these properties are called state functions or point functions. *Point functions* can be represented by a point on any plot, e.g., temperature, pressure, volume, etc. These properties have exact differentials designated by the symbol *d*. Therefore, change in volume or pressure is represented by *dV* or *dp,* respectively.

A quantity, whose value depends on the particular path followed during the process is called a *path function*. It requires a particular path and direction to represent the quantity on any plot, e.g., heat, work, etc. The path functions have inexact differentials represented by the symbol δ. Therefore, a differential amount of work or heat is written as δW or δQ. A typical representation of expansion work (a path function quantity) is shown in Fig. 1.26.

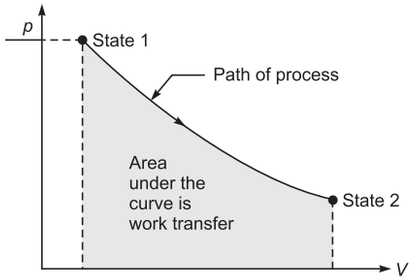

Fig. 1.26 *Reprsentation of path function quantity*

1.9 QUASI-STATIC PROCESS

When a process proceeds in such a manner that the system remains infinitesimally close to an equilibrium state at all times, it is called a *quasi-static*, or *quasi-equilibrium* process. A quasi-static process is viewed as a sufficiently slow process in which system changes its state very slowly under the influence of an infinitesimally small driving force. The system adjusts itself internally, so that the properties in one part of the system do not change any faster than those in the other part.

Figure 1.27 illustrates a quasi-static process. The system consists of a gas initially at equilibrium under the piston in a cylinder. The piston is loaded by a number of small masses. As one mass is removed, the gas expands slightly, allowing the piston to move slowly upward. During such expansion, the molecules get sufficient time to redistribute, and thus the gas would depart only slightly from equilibrium and as the pressure and other intensive properties become uniform, it will attain a new equilibrium state. Moreover, if the same mass is put on the piston, the gas would be restored to its initial state.

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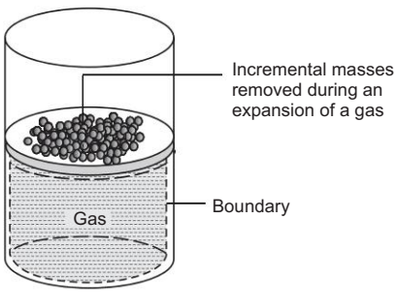
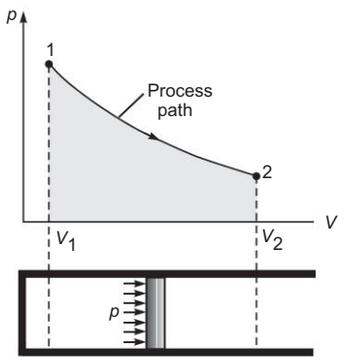


(a) Illustration of quasi-equibrium expansion



(b) Work of quasi-equilibrium process

Fig. 1.27 *Quasi-equilibrium process*

If several masses were removed from piston one after an other, the gas would pass through a series of equilibrium states without ever being far away from equilibrium. If the increments of mass are made negligibly small, the gas would undergo a quasi-equilibrium expansion process. A quasi-equilibrium compression can also be visualized in the similar way.

However, if in actual process, a gas in a piston–cylinder device is compressed suddenly, the molecules near the face of the piston do not have enough time to escape and they pile up in a small region in front of the piston, thus creating a high pressure region. Because of this pressure variation, the system can no longer be said to be in equilibrium and this makes the entire process go in non quasi-static equilibrium. For an actual process, the system does not pass through a series of equilibrium intermediate states, and thus its path cannot be recognized.

A quasi-static process is an ideal process and it is not a true representation of actual processes. But many actual processes closely approximate it and they can be modeled as quasi-equilibrium-process with negligible error. Engineers are interested in quasi-equilibrium processes for two reasons. First, these are easy to analyse; and second, work-producing devices deliver the maximum work, and work-absorbing devices require minimum work when they operate on quasi-equilibrium processes. Therefore, quasi-static processes serve as standard for other processes.

1.10 EQUILIBRIUM

Thermodynamics deals with equilibrium states or state of balance. A system is said to be in *equilibrium state* when there is no unbalance potential (driving force) within the system. It means that the intensive properties are same throughout the entire system and there is no tendency for a change of state. If two systems are in the same intensive state, they are in equilibrium with each other. If the system and its surroundings are in the same intensive state, they are also in equilibrium state.

Thermodynamic equilibrium consists of many types of equilibrium, like thermal, mechanical, chemical, etc.

A system is said to be in ***thermal equilibrium***, if its temperature is the same throughout the entire system, because temperature difference is potential for heat flow.

A system is said to be in ***mechanical equilibrium***, in the absence of any unbalance force. The force is directly related with pressure. Thus, if there is no pressure difference (driving force for mechanical work) throughout the system then the system would be in mechanical equilibrium.

If a system involves two phases, it is in ***phase equilibrium***, when the mass of each phase reaches an equilibrium level and stays there.

Finally, a system is in ***chemical equilibrium*** when a system does not undergo any chemical reaction or its chemical composition does not change with time.

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An isolated system has no interaction with its surroundings and is always in internal equilibrium. When any one of the above conditions of equilibrium are not satisfied, the system is not considered to be in thermodynamic equilibrium.

1.11 DIMENSIONS AND UNITS

Any physical quantity may be characterized by *dimensions*. The reference standard used to measure the dimensions of a physical quantity is called a *unit*. In the *tenth* and *eleventh General Conference of Weight and Measures*, it was decided to use the single universally accepted system of units throughout the world and a system of measurement called the *International System of Units* was introduced. This system of units is called *Système International d 'Unités* and is abbreviated as *SI units*.

The seven units used for the seven fundamental quantities, considered as basic units, are given in Table 1.1.

Table 1.1 *Basic or primary dimensions, units and symbols in SI units*

Qunatity	*Unit*	*Symbol*
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Amount of light	candela	cd
Amount of matter	mole	mol

A list of some derived physical quantities, their symbols, units and dimensions are given in Table 1.2.

Table 1.3 shows a list of some secondary units and their expressions in terms of basic units.

Guidelines for Writing Units and their Symbols
The following guidelines should strictly be followed for writing correct units and their dimensions:

1. All unit names derived from scientist's names are not to be written with an initial capital letter. For example, the unit of force named after Sir Isaac Newton is written as 'newton' (not Newton). Similarly, the unit of temperature named after Lord Kelvin is written as 'kelvin' (not Kelvin).
2. Symbols of unit names derived after scientist's names are always written with an initial capital letter. For example, the symbol 'N' is used for newton and 'K' for kelvin.
3. If the symbol of a unit is not derived from a scientist's name, it is written with a small letter. For example, 'm' for *metre* and 'kg' for *kilogram*.
4. The symbols of units do not take a plural form. However, the full name of the unit

Table 1.2 *Derived quantities, their units and symbols*

Qunatity	*Unit*	*Symbol*
Acceleration	metre per second	m/s^2
Angular acceleration	radian per second square	rad/s^2
Angular velocity	radian per second	rad/s
Area	square metre	m^2
Electric resistance	ohm	Ω
Frequency	hertz	Hz or 1/s
Specific volume	kilogramme per cubic metres	kg/m^3
Velocity	metre per second	m/s
Volume	cubic metre	m^3

Table 1.3 *Secondary units, their symbols and expressions in terms of basic units*

Qunatity	*Unit*	*Symbol*	*Expression in terms of basic units*
Force	newton	N	$kg \cdot m/s^2$
Pressure	pascal	Pa	$kg/m \cdot s^2$
Energy, work, heat	joule	J	$kg \cdot m^2/s^2$
Power	watt	W	$kg \cdot m^2/s^3$

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may be pluralized. For example, the length of an object can be written as 5 m or 5 metres but not 5 ms. Similarly, the mass of an object can be written as '5 kg' or '5 kilograms' but not '5 kgs' .

5. No full stop or other punctuation mark is placed after symbols, unless they appear at the end of a sentence. For example, metre is written as 'm' not 'm.'.
6. For better appearance, a single space must always be provided between a numerical value and a symbol of a unit.

1.12 PRESSURE

Pressure is defined as the *normal force exerted by a fluid per unit area*. We speak of pressure only when we deal with a gas or liquid. The counterpart of pressure in a solid is *stress*. If F is the force normal to the area A, then

$$p = \frac{F}{A} \text{ (N/m}^2\text{)} \qquad ...(1.10)$$

where force, F = mass (m) × acceleration due to gravity (g)

mass, m = volume × density = area (A) × depth (h) × density (ρ)

or $$F = Ah\rho g$$

and $$\text{pressure, } p = \frac{Ah\rho g}{A} = \rho \text{ g h (N/m}^2\text{)} \qquad ...(1.11)$$

1.12.1 Units of Pressure

Pressure is measured in newtons per square metre, which is called ***pascal* (Pa).** The pressure unit pascal being too small, very often kilopascal (kPa), megapascal (MPa), bar and standard atmosphere (atm) are used. These are related as

1 kPa = 10^3 Pa
1 MPa = 10^6 Pa = 10^3 kPa
1 bar = 100 kPa = 10^5 Pa
1 atm = 101, 325 Pa = 101.325 kPa
1 Torr = 133.32 Pa

1.12.2 Atmospheric, Absolute and Gauge Pressures

(*i*) *Atmospheric Pressure* It is the pressure exerted by the envelope of air surrounding the earth's surface. The standard atmospheric pressure is equal to the pressure produced by a 760 mm high column of mercury, the density of mercury being 13,595 kg/m^3 and the acceleration due to gravity being 9.80665 m/s^2 at sea level.

$$p_{atm} = 101325 \text{ N/m}^2 = 1.01325 \text{ bar}$$

(*ii*) *Absolute Pressure* The actual pressure at a given position is called *absolute pressure*. It is designated as p_{abs}, or simply p, and it is measured by a barometer above the absolute zero pressure.

(*iii*) *Gauge Pressure* All pressure-measuring instruments and gauges are calibrated to read zero at atmospheric pressure, and so they indicate the difference between the actual (absolute) pressure and local atmospheric pressure (p_{atm}). This difference is called *gauge pressure.* It is denoted by p_{gauge} and is expressed as

$$p_{gauge} = p_{abs} - p_{atm} \qquad ...(1.12)$$

The pressure measured below atmospheric pressure is called **vacuum (gauge) pressure** and

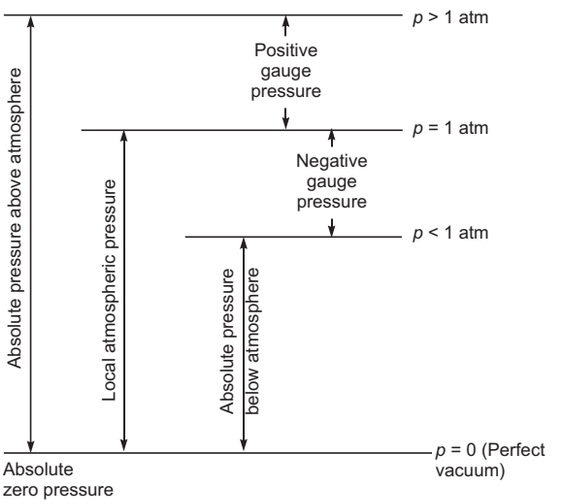

Fig. 1.28 *Relation between atmospheric, absolute and gauge pressures*

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is measured by *vacuum gauges*, which indicate the difference between the atmospheric pressure and actual (absolute) pressure. It is expressed as

$$p_{vacuum} = p_{atm} - p_{abs} = - p_{gauge} \quad ...(1.13)$$

1.13 TEMPERATURE AND ZEROTH LAW OF THERMODYNAMICS

1.13.1 Temperature

Temperature can be defined as a measure of hotness or coldness, but it is not the exact definition. Temperature is a basic property, such as mass, length and time thus cannot be defined precisely.Based on our sensation, we express the level of temperature qualitatively with words like 'cold', 'freezing cold', 'warm', 'hot' and 'red hot'. However, a numerical value cannot be assigned to these feelings, because our sensations may be misleading. For example, a metal chair will feel much colder than a wooden one, even when both are at the same temperature.

1.13.2 Equality of Temperature

If a hot system and cold system are brought into contact with each other, isolated from their surroundings, the hot system gives its heat energy to the cold system till they reach a common temperature, which is the requirement for thermal equilibrium. Two systems attain equal temperature if no changes occur in any property when they remain in contact.

1.13.3 Zeroth Law of Thermodynamics

It states that *when two systems are in thermal equilibrium with a third system, they in turn have thermal equilibrium with each other.*

Consider two systems S_1 and S_2 which are separated by an adiabatic wall, and a third system S_3 is in communication with both the systems as shown in Fig. 1.29.

If systems S_1 and S_2 are individually in thermal equilibrium with a third system S_3, then the systems

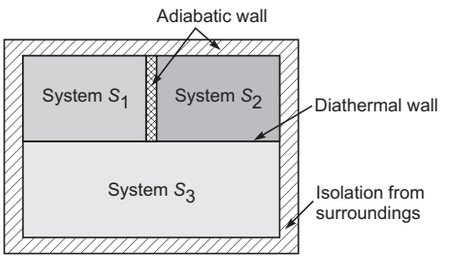

Fig. 1.29 *Three systems in thermal eqilibrium*

S_1 and S_2 will also be in thermal equilibrium with each other, even though they are not in contact.

The zeroth law serves a basis for the validity of temperature measurement, by replacing the third system by a thermometer. The zeroth law can be restated as *two systems are in thermal equilibrium if both have the same temperature reading even if they are not in contact*.

1.14 MEASUREMENT OF TEMPERATURE

The measurement of temperature depends upon the establishment of thermal equilibrium between a system and the device used to measure the temperature. The sensing device should have at least one measurable property that changes with change in temperature. Such a property is called a *thermometric property*. The substance which shows the changes in the thermometric property is called *thermometric substance*.

Fortunately, several properties of materials change with temperature and this forms the basis for temperature measurement. A list of some properties is given below.

(i) *Change in dimension* Expansion or contraction of material, such as mercury in glass *thermometer*.

(ii) Change in electrical resistance of metals and semiconductors, such as resistance thermometers and *thermistors*.

(iii) Thermoelectric *emf* between cold and hot junctions, such as *thermocouples*.

(iv) Change in intensity and colour of emitted radiations, such as *pyrometers*

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The above properties are calibrated into corresponding temperatures through comparison with established standards. The commonly used thermometers are

(a) Liquid-in-glass thermometer, and
(b) Constant volume gas thermometer.

1.14.1 Liquid in Glass Thermometer

The most commonly used thermometer is the liquid-in-glass type as shown in Fig. 1.30. It works on the expansion or contraction of a thermometric substance with temperature. It consists of a uniform-diameter glass capillary tube connected to a bulb filled with a liquid at one end. The assembly is sealed to preserve a partial vacuum in the capillary. As the temperature increases, the liquid expands in volume and rises in the capillary.

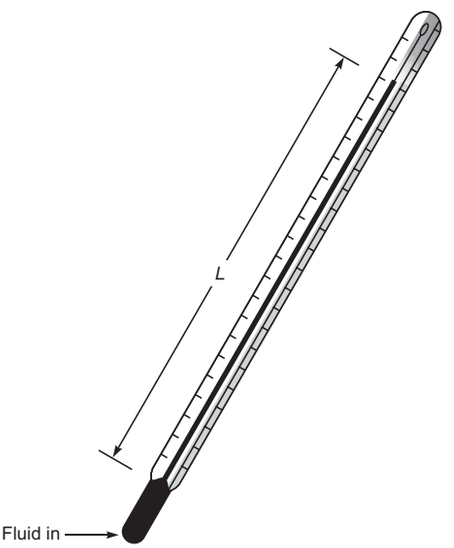

Fig. 1.30 *Liquid-in-glass thermometer*

The height of the liquid column is calibrated into a temperature scale, which may then be read. Mercury is widely used for measuring ordinary temperatures; alcohol, ether, and other liquids are also employed for this purpose.

1.14.2 Constant-Volume Gas Thermometer

The schematic diagram of a constant-volume gas thermometer is shown in Fig. 1.31. The volume of an ideal gas in the sensing bulb *D* is kept constant by adjusting the level of mercury in the arm *B* of the manometer. The arm *B* and the arm *A* are connected by a flexible tube to form a U-tube manometer. The arm *B* is also connected to the gas bulb *D* via a capillary tube *C*, while the other arm *A* of the manometer is open to atmosphere and can be moved vertically to adjust the mercury level, so the mercury just touches the mark *L* of the capillary. The pressure in the bulb *B* is used as a thermometric property and can be given by

$$p = p_{atm} + \rho g h \qquad ...(1.14)$$

where, p_{atm} = atmospheric pressure
ρ = density of the mercury
h = mercury column in manometer

The gas bulb *D* is first placed in a constant-temperature bath at the triple point temeprature T_{tp} of water and the level of mercury is adjusted to touch the mark *L* by moving the manometer arm *A* up and down. As the volume of the bulb becomes

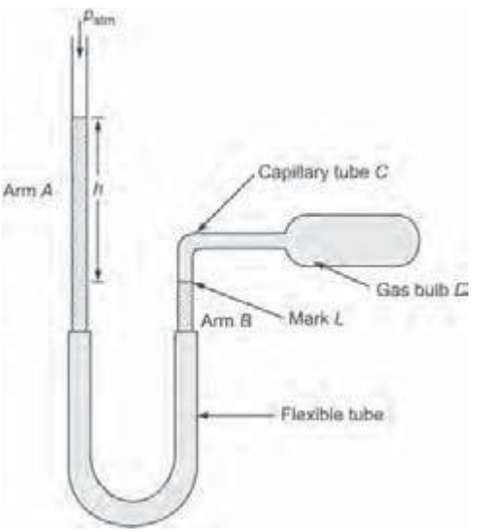

Fig. 1.31 *Constant-volume gas thermometer*

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constant and the height difference of the mercury in the two arms is recorded as h_{tp}, the pressure, p_{tp} corresponding to the mercury column at the triple point is calculated by Eq. (1.14).

Now the bulb is brought in contact with a system whose temperature T, is to be measured. Again, in a similar manner, by keeping the volume of gas in the bulb constant, the height difference of the mercury in the two arms is recorded and the corresponding new pressure p is calculated by Eq.(1.14).

From the ideal gas equation, the new temperature is given by

$$T = 273.15 \times \frac{p}{p_{tp}} \qquad ...(1.15)$$

where 273.15 K is the triple point temperature of water.

1.14.3 Thermocouple

The thermocouple works on the principles of *Seebeck effect*, *Peltier effect*, and *Thomson effect*. When two dissimilar metals are joined together as shown in Fig. 1.32, an *emf* will exist between the two points *A* and *B*. The *Seebeck emf* is caused by the junction of dissimilar metals, the *Peltier emf* is caused by flow of current in the circuit and the *Thomson emf* is caused by the temperature gradient existing in the material. The emf generated at the junction is the function of temperature of two dissimilar metals. This emf generated is calibrated in temperature scale for temperatre measurement.

The temperature is measured by a thermocouple by inserting the probe into a stream of hot gas, hot liquid or attaching it to a hot solid. The thermocouples are widely used in industrial applications to monitor temperature of liquids and gases in storages and flowing pipes and ducts.

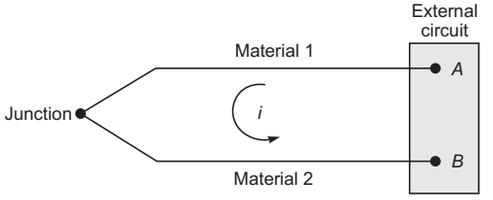

Fig. 1.32 *Principle of temperature measurement by a thermocouple*

The commonly used thermocouple materials are tabulated below.

Table 1.4 *Commonly used thermocouple materials and their temperature range*

	Materials	*Type*	*Range*	*Range*
1	Cooper constantan	T	−220°C to 371°C	± 0.75%
2	Iron constantan	J	−190°C to 760°C	± 1.0% ± 0.75%
3	Chromel–Alumel	K	−190°C to 1260°C	± 0.75%
4	Chromel–constantan	E	− 100°C to 1260°C	
5	Platinum (90%) + rhodium (10%,)	S	0°C to 1482°C	± 0.5%

Copper–constantan Inexpensive and has high output.

Iron–constantan Inexpensive; iron oxidizes rapidly above 760°C.

Chromel–Alumel Good resistant to oxidation within specified temperature limit. It should not be used in reducing atmosphere.

Chromel–constantan Highest output with good stability.

Platinum–Rhodium Thermocouple is expensive, low output, but most accurate. It is very stable and resistant to oxidation. It is used only for high temperature application.

1.15 TEMPERATURE SCALE

Several temperature scales have been introduced so far. All temperature scales are based on easily reproducible states, such as freezing and boiling point temperatures of water, which are also called *ice point* and *steam point*, respectively.

The temperature scale used in SI units is *Celsius scale* (formerly, Centigrade scale). On the Celsius

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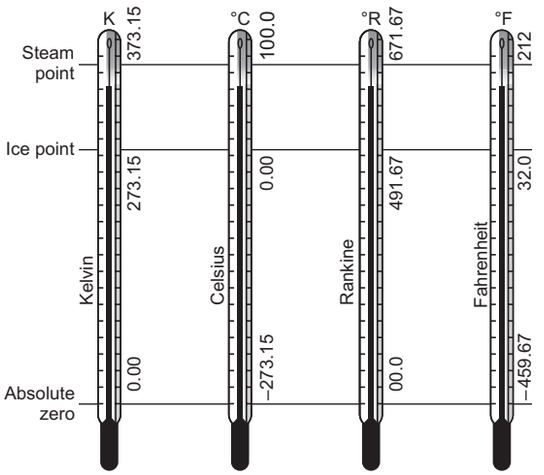

Fig. 1.33 *Comparison of temperature scales*

scale, ice point and steam point are assigned the numerical values of 0 and 100°C, respectively. The English system today uses the *Fahrenheit scale* which assigns 32 and 212°F as the ice point and steam point, respectively.

A more useful temperature scale in thermodynamics is the *absolute temperature scale* (no negative temperature is possible on this scale). This scale is also called the **Kelvin scale**. The temperature unit on this scale is kelvin designated as K, without the degree symbol.

The Kelvin scale is related to the Celsius scale by

$$T\,(\text{K}) = T\,(°\text{C}) + 273.15 \qquad ...(1.16)$$

In the English system, the absolute temperature scale is the *Rankine scale* and it is related to Farhenheit scale by

$$T\,(\text{R}) = T\,(°\text{F}) + 459.67 \qquad ...(1.17)$$

The two temperature scales and corresponding absolute temperature scales are shown in Fig. 1.33. The values 273.15 and 459.67 are often replaced by approximate values of 273 and 460, respectively. The temperature scales on two unit systems are related by

$$T\,(\text{R}) = 1.8\,T\,(\text{K}) \qquad ...(1.18)$$

$$T\,(°\text{F}) = 1.8\,T\,(°\text{C}) + 32 \qquad ...(1.19)$$

Further, it should be noted that the temperature magnitude of each division of 1 K and 1°C are identical. Therefore, when we are dealing with temperature difference, the temperature scale on both the scales is same.

$$\Delta T\,(\text{K}) = \Delta T\,(°\text{C}) \qquad ...(1.20)$$

Similarly, $\Delta T\,(\text{R}) = \Delta T\,(°\text{F})$ ...(1.21)

If a relation involves temperature difference (such as $q = C_p \Delta T$), it makes no difference and either scale can be used. But when the relation is in temperature only (as $pv = RT$), then the temperature in kelvin scale (K) must be used.

1.16 THE INTERNATIONAL PRACTICAL TEMPERATURES SCALE

In the seventh *General Conference on Weights and Measures* held in 1927, a more convenient scale, known as International Practical Temperature Scale was formulated to be used for calibration of temperature-measuring instruments. It was revised in the *Thirteenth General Conference* in 1968. It consists of reproducible reference temperatures, defined by *triple* point, *boiling* point and *melting* point of pure substances.

Triple Point It is a state of equilibrium, where all three phases (solid, liquid and gas) of a substance coexist simultaneously. The triple point of water is 0.01°C (271.16 K).

Boiling Point It is a state of equilibrium, where liquid and gaseous phases of a substance coexist simultaneously. For water, it is 100°C (373.15) at 1 standard atm.

Melting Point It is a state of equilibrium, where solid and liquid phases of a substance coexist simultaneously. More often, all metals have their specific melting point.

The whole temperature scale is divided into four ranges:

(i) From −259.34°C (triple point of hydrogen) to 0°C

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Table 1.2 *Fixed points of the international practical temperature scale of 1968*

	Equilibrium state	*Assigned value of temperature*	
		K	°C
1.	Triple point of hydrogen	13.81	–259.34
2.	Normal boiling point of hydrogen	20.28	–252.87
3.	Normal boiling point of neon	27.10	–246.05
4.	Triple point of oxygen	54.36	–218.79
5.	Normal boiling point of oxygen	90.19	–182.96
6.	Triple point of water	273.16	0.01
7.	Normal boiling point of water	373.15	100.00
8.	Normal freezing point of antimony	630.74	357.59
9.	Normal freezing point of zinc	692.73	419.58
10.	Normal freezing point of silver	1235.08	961.93
11.	Normal freezing point of gold	1337.58	1064.43

(ii) From 0°C to 630.74°C (antimony point),
(iii) From 630.74°C to 1064.43°C (gold point)
(iv) Above 1064.43°C

1.17 ENTROPY

It is an abstract property of the second law of thermodynamics. It is considered as a measure of degree of the molecular disorder in the matter. The change in value is calculated as

$$\Delta S = \int_1^2 \frac{\delta Q}{T} = \int_1^2 m C \frac{dT}{T} \qquad ...(1.22)$$

Example 1.3 *The pressure in a constant gas thermometer is measured as 32 mm of Hg above atmospheric pressure at triple point. Determine the temperature in °C, when the pressure is 76 mm of Hg above atmospheric pressure. The barometer reads 752 mm of Hg.*

Solution

Given Absolute pressure at triple point

$$p_{tp} = 752 + 32 = 784 \text{ mm of Hg}$$

Absolute pressure at given pressure,

$$p = 752 + 76 = 828 \text{ mm of Hg}$$

Triple point temperature ,

$$T_{tp} = 273.15 \text{ K}$$

To find Temperature T at pressure p.

Analysis For constant volume gas thermometer:

$$\frac{p_{tp}}{T_{tp}} = \frac{p}{T}$$

or
$$T = \frac{p}{p_{tp}} \times T_{tp} = \frac{828}{784} \times 273.15$$
$$= 288.48 \text{ K}$$

Unknown temperature,

$$T = 288.48 - 273.15 = \mathbf{15.33°C}$$

Example 1.4 *Calculate the temperature at which the Celsius and Fahrenheit scales agree.*

Solution

Given Condition for two temperature as T (°F) = T (°C)

To find Temperature at which the Celsius and Fahrenhiet scales becomes equal.

Analysis The temperature at the two scales is related as

$$T \text{ (°F)} = 1.8 \text{ T (°C)} + 32$$

Using the given condition T (°F) = 1.8 T (°C) = T say

$$T = 1.8\ T + 32$$

or
$$0.8\ T = -32$$

or
$$T = \mathbf{-40°C}$$

Example 1.5 *The resistance of the windings in a certain motor is found to be 100 ohms at a room temperature of 27°C. When operating at full load under steady conditions, the motor is switched off and the resistance of the winding is found to be 120 ohms. The windings are made of copper wire, whose resistance at t°C is given by*

$$R_t = R_0 (1 + 0.004\ t)$$

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where R_0 is the resistance at 0°C. Find the temperature attained by the coil during full load.

Solution

Given Resistance, $R_1 = 100\ \Omega$,
Temperature, $t_1 = 27°C$
Resistance, $R_2 = 120\ \Omega$

To find The temperature at full load

Analysis The relation between resistance at any temperature is given by

$$R_t = R_0(1 + 0.004t)$$

Using values at the state 1

$$100 = R_0(1 + 0.004 \times 27)$$

or $$R_0 = 90.25\ \text{W}$$

The temperature corresponds to 120 Ω;

$$120 = 90.25\ (1 + 0.004t_2)$$

or $$t_2 = \mathbf{82.4°C}$$

Example 1.6 *The temperature scale of a certain thermometer is given by the relation $t = A \ln p + B$, where A and B are constants and p is the thermometric property of the fluid in thermometer. At ice point and steam point, if the thermometric property is found to be 1.5 and 7.5 respectively, what will be the temperature corresponding to the thermometric property of 3.5 on celsius scale?*

Solution

Given Initial property, $p_1 = 1.5$,
Final property, $p_2 = 7.5$,
Ice-point temperature, $T_1 = 0°C$,
Steam-point temperature, $T_2 = 100°C$,
Thermometric property, $p_3 = 3.5$

To find The temperature corresponding to the thermometric property of 3.5

Analysis The relation for temperature is given as

$$t = A \ln p + B,$$

At ice point, $0 = A \ln (1.5) + B$
or $0 = 0.405A + B$ ...(i)
At steam point, $100 = A \ln (7.5) + B$
or $100 = 2.015A + B$ ...(ii)

Solving these equations, we get,

$$A = 62.13 \quad \text{and} \quad B = -25.2$$

Using A and B for thermometric property at 3.5,

$$t = 62.13 \times \ln (3.5) - 25.2 = \mathbf{54.64°C}$$

Example 1.7 *Two Celsius thermometers A and B with temperature readings T_A and T_B agree at ice point and steam point, but else where they are related by*

$T_A = p + qT_B + rT_B^2$, where p, q and r are constants.

When the thermometers are immersed in an oil bath, A shows a temperature of 51°C, while B shows 50°C. Determine the temperature T_A, when T_B is 25°C.

Solution

Analysis The relation for temperature is given as

$$T_A = p + qT_B + rT_B^2$$

At ice point, $0 = p + q \times 0 + r \times 0$
or $p = 0$
At steam point, $100 = p + 100q + (100)^2 r$
or $100 = 100q + 10000r$ ...(i)
Further, $51 = p + 50q + (50)^2 r$
or $51 = 50q + 2500r$ ...(ii)

Solving these equations, we get,

$$q = 1.04 \quad \text{and} \quad r = -4 \times 10^{-4}$$

Using p, q and r at $T_B = 25°C$,

$$T_A = 0 + 1.04 \times 25 - 4 \times 10^{-4} \times (25)^2 = \mathbf{25.75°C}$$

Example 1.8 *A thermocouple with a test junction at t°C on a gas thermometer scale gives the emf as*

$$e = 0.22t - 5.5 \times 10^{-4}\ t^2\ mV$$

The milivoltmeter is calibrated at ice point and steam points. What will be the reading on this thermometer, when the gas thermometer reads 60°C?

Solution

Given The relation for emf at t°C

$$e = 0.22t - 5.5 \times 10^{-4}\ t^2\ \text{mV}$$

with ice point, $t_1 = 0°C$
Steam point, $t_2 = 100°C$

To find The reading of temperature on the scale corresponding to 60°C

Analysis The *emf* is generated according to

$$e = 0.22t - 5.5 \times 10^{-4}\ t^2\ \text{mV}$$

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at ice point $e_1 = 0.22 \times 0 - 5.5 \times 10^{-4} \times (0)^2 = 0$ mV
at steam point $e_2 = 0.22 \times 100 - 5.5 \times 10^{-4} \times (100)^2$
$= 16.5$ mV
at 60°C $e_3 = 0.22 \times 60 - 5.5 \times 10^{-4} \times (60)^2$
$= 11.22$ mV

The reading of the temperature on the gas thermometer scale corresponds to 60°C.

$$t = \frac{(100°C)}{(16.5\text{ mV})} \times (11.2\text{ mV}) = \mathbf{68°C}$$

Example 1.9 *It is proposed to construct a new scale with the value 5°N assigned to ice point and 20°N to steam point. The pressure of an ideal gas at constant volume is considered as a thermometric property.*

(a) Set up a linear relationship between pressure and temperature in °N on a new scale. What is the kelvin absolute zero on this scale?

(b) Derive an expression between °N and K.

Solution

Given The construction of a new scale with °N.
Ice-point temperature, $T_i = 5$°N and 0°C,
Steam-point temperature, $T_s = 20$°N and 100°C,

To find (a) (i) Linear relationship between pressure and temperature on a new scale, °N,
(ii) Absolute zero on new scale.
(b) Relationship between new scale in °N and kelvin scale.

Analysis

(a) (i) Linear relationship between pressure and temperature with two constants a and b as

$$T = ap + b \qquad ...(i)$$

Assuming pressure p_s at steam point and p_i at ice point, then on Celsius scale;

$$100 = ap_s + b \qquad ...(ii)$$
$$0 = ap_i + b \qquad ....(iii)$$

Subtracting Eq. (iii) from Eq. (ii), we get

$$100 = a(p_s - p_i)$$

or $$a = \frac{100}{p_s - p_i}$$

and constant $$b = -\frac{100}{p_s - p_i} p_i$$

Substituting the value of constants a and b in Eq. (i), we get

$$T(°C) = \frac{100}{p_s - p_i} p - \frac{100}{p_s - p_i} p_i$$

$$= 100\left(\frac{p - p_i}{p_s - p_i}\right)$$

or $$\frac{T(°C)}{100} = \left(\frac{p - p_i}{p_s - p_i}\right) \qquad ...(iv)$$

Similarly, on the new scale °N at steam and ice points, with constants c and d;

$$20 = cp_s + d \qquad ...(v)$$
$$5 = cp_i + d \qquad ....(vi)$$

Substracting Eq. (vi) from Eq. (v), we get

$$c = \frac{15}{p_s - p_i}$$

and constant $$d = 5 - \frac{15}{p_s - p_i} \times p_i$$

Substituting the value of constants c and d in Eq. (i), we get

$$T(°N) = \frac{15}{p_s - p_i} p + 5 - \frac{15}{p_s - p_i} p_i$$

$$= 5 + 15\left(\frac{p - p_i}{p_s - p_i}\right)$$

or $$\frac{T(°N) - 5}{15} = \frac{p - p_i}{p_s - p_i} \qquad ...(vii)$$

Equating Eq. (iv) and (vii), we get

$$\frac{T(°N) - 5}{15} = \frac{T(°C)}{100}$$

or $$\mathbf{T(°N) = \frac{15}{100} T(°C) + 5} \qquad ...(viii)$$

It is the linear relationship between celsius and the new temperature scale on °N.

(ii) At absolute zero on Celsius scale

$$0\text{ K} = -273°C$$

Therefore, $$T(°N) = \frac{15}{100}(-273) + 5$$
$$= \mathbf{-35.95°N}$$

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(b) Relationship between new scale and Kelvin scale

$$T(\text{K}) = T(°\text{C}) + 273$$

or $$T(°\text{C}) = T(\text{K}) - 273$$

Using in Eq. (viii);

$$T(°\text{N}) = \frac{15}{100}\left[T(\text{K}) - 273\right] + 5$$

or $$T(\text{K}) = \frac{100}{15}\left[T(°\text{N}) - 5\right] + 273$$

or $$T(\text{K}) = \mathbf{273} + \frac{\mathbf{20}}{\mathbf{3}}\left[\mathbf{T(°N) - 5}\right]$$

Example 1.10 *Consider a particular Celsius scale assigned the value of 0°C to steam point and 100°C to ice point.*

(a) Using ideal gas as the thermometer medium, set up a relationship between 0°C and pressure for a constant volume thermometer. Proceed to derive the correlation between the two celsius scales. At what temperature are the two scales are numerically equal?

(b) What is the numerical value of obsolute zero for the particular scale? What is 200 K in °C?

Solution

Given Two Celsius scales.

To find (a) (i) Correlation between the two celsius scales,
(ii) Temperature at which two scales are numerically equal,
(b) (iii) Numerical value of absolute zero,
(iv) Temperature in °C corresponding to 200 K.

Analysis

(a) (i) Let the temperature scale relate the pressure linearly:

$$t' = ap + b$$

where p is thermometric property, and a and b are two constants.

Then for a constant-volume gas thermometer

$$100 = ap_i + b \quad \text{...(i)}$$
$$0 = ap_s + b \quad \text{...(ii)}$$

Subtracting (ii) from (i), we get

$$a = \frac{100}{p_i - p_s}$$

and $$b = 100 - \frac{100}{p_i - p_s} p_i$$

Using this, we get

$$t' = \frac{100}{p_i - p_s} p + 100 - \frac{100}{p_i - p_s} p_i$$

$$= 100 \times \left(\frac{p - p_i}{p_i - p_s} \right) + 100$$

or $$\frac{p - p_i}{p_i - p_s} = \frac{t' - 100}{100} \quad \text{...(A)}$$

Similarly, using 0°C for t_i and 100°C for t_s on normal temperature scale with constants c and d;

$$0 = cp_i + d \quad \text{...(iii)}$$
$$100 = cp_s + d \quad \text{...(iv)}$$

Subtracting (iii) from (iv),

$$c = \frac{100}{p_s - p_i} \quad \text{and} \quad d = -\frac{100}{p_s - p_i} p_i$$

Then normal temperature scale

$$t = \frac{100}{p_s - p_i} \times p - \frac{100}{p_s - p_i} \times p_i$$

$$= 100 \times \left(\frac{p - p_i}{p_s - p_i} \right)$$

or $$\frac{t}{100} = \frac{p - p_i}{p_s - p_i} \quad \text{...(B)}$$

Equating (A) and (B), we get

$$\frac{t}{100} = \frac{t' - 100}{100}$$

or two Celsius scales are related by

$$\boldsymbol{t = t' - 100}$$

(ii) If $t = t'$

then $t = 100 - t$

or $2t = 100$

or $\boldsymbol{t = 50°C}$

(b) (iii) At absolute zero

$t = -273°\text{C}$

$-273 = 100 - t'$

or $\boldsymbol{t' = 373°C}$

(iv) At 200 K, $t = 200 - 273 = -73°\text{C}$

Therefore $t' = 100 - t = 100 - (-73) = \mathbf{173°C}$

Summary

- Thermodynamics is the basic science which deals with energy and its transformation.
- A system is a certain quantity of matter or a certain fixed region in space. A system of fixed mass is called a *closed system* or *control mass*, and a system that involves the mass transfer across its boundary is called an *open system*.
- The mass-dependent property is called an *extensive property*, while a property independent of mass is called an *intensive property*.
- The sum of all forms of energy of a system is called *total energy*, which consists of internal, kinetic and potential energy. Internal energy represents the molecular energy of the system and may exist in sensible, latent, chemical and nuclear form.
- A system is said to be in thermodynamic equilibrium, if it maintains thermal, mechanical, phase and chemical equilibrium. Any change from one state to another is called a process. A process with identical states at the end and start is called a *cycle*. During a quasi-equilibrium, the system remains practically in equilibrium at all times.
- Pressure is defined as force per unit normal area. It is measured in Pascal, kPa, bar. The absolute, gauge and vacuum pressure are related as

$$p_{gauge} = p_{abs} - p_{atm}$$

and $$p_{vacuum} = p_{atm} - p_{abs} = -p_{gauge}$$

- The zeroth law of thermodynamics states that the two bodies are in thermal equilibrium, if both have a common temperature even is they are not in contact.
- The temperature scales used in SI unit system are Celsius and Kelvin scales. These are related as

$$T(\text{K}) = T(^\circ\text{C}) + 273.15$$
$$\Delta T(\text{K}) = \Delta T(^\circ\text{C})$$

Glossary

Thermodynamics Science of energy that deals with heat energy and power

System Certain quantity of matter or prescribed region in space considered for analysis

Surroundings Every thing external to a system

Boundary Real or imaginary surface that separates a system from its surroundings

Universe Combination of a system and its surroundings

Closed system Particular quantity of matter under study

Isolated system A closed system that does not interact with its surroundings

Open system Prescribed region in space under study

Adiabatic system An open system in which heat is not allowed to cross the boundary

Physical phase A molecular configuration of matter categorized as either solid, liquid or gas

Working substance A medium for energy transport between a system and its surroundings, while undergoing a thermodynamic process within a device

Homogeneous system A system containing only a single physical phase of a substance

Heterogeneous system A system of mixture of two or more than two phases of matter

Macroscopic Gross or overall behavior

Microscopic Average behavior of molecules making up a system

Thermodynamic State Condition of system described by independent thermodynamic properties

Properties A characteristic of a system

Intensive property Any property, which is independent of mass and size of the system

Extensive property Any property, which depends on mass and size of the system

Equilibrium State of balance

Thermal equilibrium A situation in which the system does not have variation in temperature throughout

Mechanical equilibrium A situation in which all mechanical forces within the system are balanced

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Phase equilibrium A system without phase change

Chemical equilibrium A situation in which a system does not interact chemically

Process Transformation of a system from one equilibrium state to another

Cycle Sequences of processes that begins and ends at the same state

Quasi-static process A process which is always close to thermodynamic equilibrium

Pressure Force acting per unit area of a fluid

Internal energy Sum of all microscopic forms of energies of a system

Review Questions

1. Define thermodynamics and write its importance and applications.
2. How does classical thermodynamics differ from statistical thermodynamics?
3. Explain the concept of macroscopic and microscopic view points applied to the study of thermodynamics.
4. What is meant by classical and statistical thermodynamics ? Explain.
5. Define thermodynamic systems. Classify them.
6. Differentiate between closed system and open system.
7. Define isolated and adiabatic systems and differentiate them.
8. Recognize whether the system is open or closed:
   - (i) A tube of a bicycle filled with air,
   - (ii) A jet engine in flight,
   - (iii) A household refrigerator in operation,
   - (iv) Water pump,
   - (v) Pressure cooker without whistling,
   - (vi) Steam turbine,
   - (vii) Car battery,
   - (viii) An electric geyser.
9. Define control volume and control surface.
10. Differentiate between homogeneous and heterogeneous systems.
11. Define thermodynamic property, state, path, process and cycle.
12. State and differentiate between extensive, intensive and specific properties.
13. What do you mean by thermodynamic equilibrium? How does it differ from thermal equilibrium?
14. Define thermodynamic equilibrium. What are the conditions necessary to establish thermodynamic equilibrium to a system?
15. Define quasi-static process. State its salient features.
16. State different types of processes according to their natures.
17. Define state function and path function.
18. What is the concept of continuum? How are density and pressure defined using this concept?
19. Define zeroth law of thermodynamics. Write its importance in thermodynamics.
20. Define temperature. How is it measured?
21. State and explain zeroth law of thermodynamics used for temperature measurement.
22. What do you mean by thermodynamic property and thermodynamic substance?
23. Establish a relationship between Celsius scale and Fahrenheit scale.
24. What do you mean by absolute scale of temperature? How does the Celsius scale differ from the absolute Kelvin scale?
25. State the operating principle of gas thermometer. Explain its working.

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Problems

1. Convert the following temperatures from °C to °F:

(a) 21°C (b) − 17.78°C
(c) − 50°C (d) 300°C
(e) 100°C

2. Convert the following temperatures from °F to °C:

(a) 212°F (b) 68°F
(c) 32°C (d) − 40°F
(e) − 460°F

3. The resistance of a platinum wire is found to be 11.00 Ω at ice point, 15.247 Ω at steam point and 27.949 Ω at the zinc point (692.73 K). Find the constants in the equation

$R = R_0(1 + AT + BT^2)$ where T is in °C

Find the resistance at a temperature of 250°C.

[$A = 3.915 \times 10^{-3}$, $B = 5.959 \times 10^{-7}$, 21.36 Ω]

4. It is proposed to develop a temperature scale with values 4°N and 16°N assigned as ice and steam points, respectively. The pressure of an ideal gas at constant volume is used as a thermometric property.

(a) Obtain a linear relationship between the pressure and temperature.

(b) What is Kelvin absolute zero on this scale?

(c) Find a relation between the new scale and Kelvin scale.

(d) If the pressure at the steam point is 10 atm, what is the pressuire at 0 degree at new ?

[(a) $\frac{T\,(°C)}{100} = \frac{T(°N) - 4}{12} = \frac{p - p_i}{p_s - p_i}$

(b) – 28.8°N (c) $T(K) = 273 + \frac{25}{3}\left(T\,(°C) - 4\right)$ *(d) – 24.3°N]*

5. A new absolute temperature scale is proposed with ice point as 150°S and steam point as 300°S. Determine the temperature in °C that corresponds to 100°S and 400°S, respectively.

6. A tank which is 4 m long, 3 m wide and 2 m deep is half full of water. How much work is required to raise all the water over the top edge of the tank? *[117.72 kJ]*

7. A rectangular tank measuring 0.6 m ×1 m at the base is filled half to a depth of 15 cm with water. Calculate the following:

(a) Total gravitational force exerted on the base of the tank

(b) The pressure exerted by water at the base of the tank

[(a) 882 N, (b) 1.47 kN/m2]

Objective Questions

1. The unit of force in SI units is
   (a) joule (b) newton
   (c) watt (d) calorie
2. The unit as kN/m^2 is called
   (a) kW (b) kJ
   (c) kPa (d) kcal
3. 1 bar in SI units is
   (a) 1×10^5 Pa (b) 100 kPa
   (c) 0.987 atm (d) All of above.
4. 1 pascal in SI units is
   (a) 1 N/m^2 (b) 100 kPa
   (c) 1×10^5 N/m^2 (d) 1×10^3 N/m^2
5. A closed system is one in which
   (a) both energy and mass cross the boundary of the system
   (b) the mass does not cross the boundary, but energy interaction takes place
   (c) neither mass nor energy cross the boundary of the system

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(d) the mass crosses the boundary but energy does not

6. An isolated system is one in which
   (a) both energy and mass cross the boundary of the system
   (b) the mass does not cross the boundary, but energy interaction takes place
   (c) neither mass nor energy cross the boundary of the system
   (d) the mass crosses the boundary but energy does not
7. An open system is one in which
   (a) both energy and mass cross the boundary of the system
   (b) the mass does not cross the boundary, but energy interaction takes place
   (c) neither mass nor energy cross the boundary of the system
   (d) the mass crosses the boundary but energy does not
8. An adiabatic system is one in which
   (a) both energy and mass cross the boundary of the system
   (b) the mass does not cross the boundary, but energy interaction takes place
   (c) neither mass nor energy cross the boundary of the system
   (d) mass crosses the boundary, heat energy does not cross the boundary of the system
9. Which of the following is an intensive property?
   (a) Volume (b) Temperature
   (c) Energy (d) Work transfer
10. Which of the following is not a property?
   (a) Volume (b) Temperature
   (c) Energy (d) Work transfer
11. Which of the following is an extensive property?
   (a) Volume (b) Temperature
   (c) Pressure (d) Density
12. Mercury in glass thermometer works on the principle of
   (a) fusion
   (b) thermo-electric effect
   (c) expansion of fluid
   (d) change in radiation intensity.
13. A 70-kg woman walks on snow with a total foot imprint area of 500 cm². What pressure does she exert on the snow?
   (a) 0.5 kPa (b) 12.5 kPa
   (c) 13.73 kN/m² (d) 25.46 kN/m²
14. When two bodies are in thermal equilibrium with a third body, then they are in thermal equilibrium with each other. This statement is called
   (a) first law of thermodynamics
   (b) second law of thermodynamics
   (c) third law of thermodynamics
   (d) zeroth law of thermodynamics.
15. A sequence of processes, in which initial and final states of a system are identical, is called a
   (a) path function (b) point function
   (c) cycle (d) none of the above

Answers

1. (b) 2. (c) 3. (d) 4. (a) 5. (b) 6. (c) 7. (a) 8. (d)
9. (b) 10. (d) 11. (a) 12. (c) 13. (c) 14. (d) 15. (c)

2

Energy and Work Transfer

Introduction

Energy is one of the major inputs for the economic development of any country. In the case of developing countries, the energy sector assumes a critical importance in view of the ever-increasing energy needs requiring huge investments to meet them.

Work is a high-grade energy and of prime interest as it is the output from a system, when energy is the input. The rate of work transfer is referred as *power*. High power generation is the need of any developing country.

Sources and forms of energy, enthalpy, forms of work transfer, concept of thermodynamic work transfer, heat, specific heat, sign convention for work and heat transfer are discussed in this chapter as a foundation to the following chapters. The first law of thermodynamics is explained with the help of Joule's experiment and other examples.

2.1 ENERGY

Energy is defined as the *capacity to do work*. It is a scalar quantity. It is measured in kJ in SI units, and kcal in MKS units. Energy can have many forms as shown in Fig. 2.1.

2.2 SOURCES OF ENERGY

The sources of energy can be divided into four categories according to their availabilty:

(*i*) *Transitional Energy* The energy in motion, i.e., wind energy, hydel energy, etc.

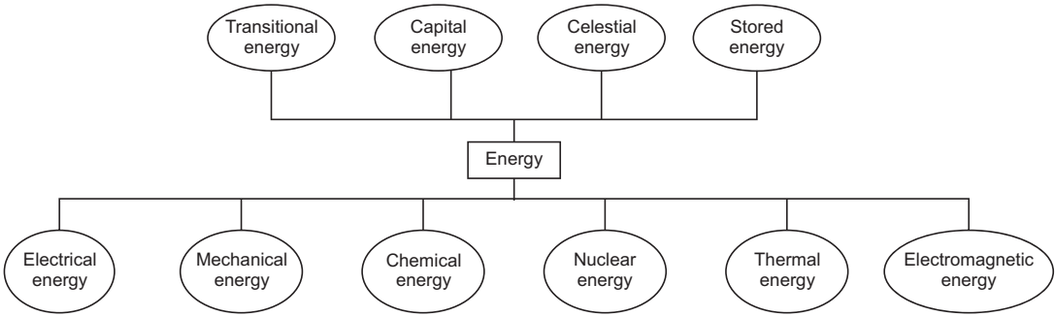

Fig. 2.1 *Sources and forms of energy*

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(*ii*) *Capital Energy* The energy derived from fuels existing in the earth, i.e., fossile fuels, nuclear fuels, etc.

(*iii*) *Celestial Energy* The energy coming from outer atmosphere, i.e., sun, moon, etc.

(*iv*) *Stored Energy* The energy existing in various masses, i.e., flywheel, tides, geothermal, hydraulic energy, etc.

2.3 CLASSIFICATION OF ENERGY SOURCES

The sources and forms of energy can be classified into several types based on the following criteria:

- Primary and secondary energy
- Commercial and non-commercial energy
- Renewable and non-renewable energy

2.3.1 Primary and Secondary Energy

Primary forms of energy are those that are either found or stored in nature. Common primary energy sources are coal, oil, natural gas, and biomass (such as wood). Other primary energy sources available include nuclear energy from radioactive substances, thermal energy stored in the earth's interior (geothermal energy), and potential energy due to the earth's gravity.

Secondary forms of energy are those forms which are derived from the primary forms of energy; for example, coke, oil or gas converted into steam and electricity.

2.3.2 Commercial Energy and Non-Commercial Energy

Commercial forms of energy are available in the market for a definite price. The most important forms of commercial energy are electricity, coal and refined petroleum products. Commercial energy serves the basis of industrial, agricultural, transport and commercial development in the modern world.

Non-commercial forms of energy are not available in the commercial market for a definite price. Non-commercial energy sources include fuels such as firewood, cattle dung and agricultural wastes, which are traditionally gathered, and not bought at a price, and used especially in rural households. These are also called *traditional fuels*. Non-commercial energy is often ignored in energy accounting.

2.3.3 Renewable and Non-Renewable Energy

Renewable energy is obtained from sources that are not exhaustible. These are freely available in nature and can be continuously used.

Examples of renewable resources include wind power, solar power, geothermal energy, tidal power, ocean thermal energy, fuel cells, energy from biomass and hydraulic energy. The most important feature of renewable energy is that it can be harnessed without the release of harmful pollutants.

Non-renewable energy is obtained from conventional fossil fuels such as coal, oil and gas, nuclear fuels, and heat traps which are accumulated in the earth crust. These have been in use for several decades. The sources of non-renewable energy are depleting at a fast rate and may not be sufficient to meet the inceasing energy demand in future. Therefore, these sources are also called *exhaustible sources of energy* and they cannot be replenished immediately.

2.3.4 Comparison between Renewable and Non-Renewable Energy Sources

Renewable sorces of energy

These are inexhaustible.

These are freely available.

These are environment-friendly.

Energy concentration varies from region to region.

Cost of equipments for harnessing energy is high, but maintenance and operational costs are minimum.

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Perfect energy utilization equipments are yet to be designed.

Non-renewable sources of energy

These are exhaustible.

These are available at definite prices.

These are hazardous to the environment.

Energy concentration does not vary.

Cost of equipments, maintenance and operation are high.

Competitive designs are progressing satisfactorily.

2.4 FORMS OF ENERGY

Energy can exist in numerous forms, such as internal, thermal, electrical, mechanical, kinetic, potential, wind, and nuclear energy,

In thermodynamic analysis, all forms of energy can be put into two groups:

(a) Stored energy, and
(b) Transit energy.

(a) Stored Energy The stored form of energy can further be classified as

(i) Macroscopic forms of energy: *potential energy* and *kinetic energy*, and
(ii) Microscopic forms of energy: *internal energy.*

The macroscopic forms of energy are defined as energy with respect to some outside reference.

The microscopic forms of energy are those which are related to the molecular structure of a system and degree of molecular activities and are independent of the outside reference.

(b) Transit Energy Transit energy means energy in transition. It is the energy possessed by a system, which is capable of crossing the boundaries. Heat energy and work transfer are transit forms of energy.

2.4.1 Potential Energy

The energy that a system possesses as a result of its elevation in a gravitational field is called *potential energy* (PE) and is expressed as

$$PE = mgz \text{ (joules)} \qquad ...(2.1)$$

on unit mass basis $pe = gz$ (J/kg) ...(2.2)

where g is the acceleration due to gravity and z is the elevation of the system relative to some outside reference.

2.4.2 Kinetic Energy

The energy that a system possesses, as a result of motion relative to some reference is called *kinetic energy* (KE). When all parts of a system move with the same velocity, the kinetic energy is expressed as

$$KE = \frac{1}{2} m\mathcal{V}^2 \text{ (joule)} \qquad ...(2.3)$$

on unit mass basis $ke = \frac{1}{2}\mathcal{V}^2$ (J/kg) ...(2.4)

where $\mathcal{V}$ is the velocity of the system with respect to some reference.

2.4.3 Internal Energy

The sum of all the microscopic forms of energy is called *internal energy*. The internal energy of a system is the energy stored within the body resulting from the kinetic and potential energy of its molecules. Thus, it is related to molecular structure and degree of molecular activities. The molecules of any system may possess both kinetic and potential energy. Thus, the internal energy of any system may be viewed as the sum of kinetic and potential energy of molecules. It is denoted by U and is measured in joules.

$$U = K + P \text{ (joules)} \qquad ...(2.5)$$

where K = internal kinetic energy of molecules, and
P = internal potential energy of molecules.

(a) Internal Kinetic Energy All molecules in a system move around with some velocity, vibrate about each other, and rotate about an axis during their random motion as shown in Fig. 2.2. The *internal kinetic energy* is the sum of all these motions of molecules. When energy passes into a

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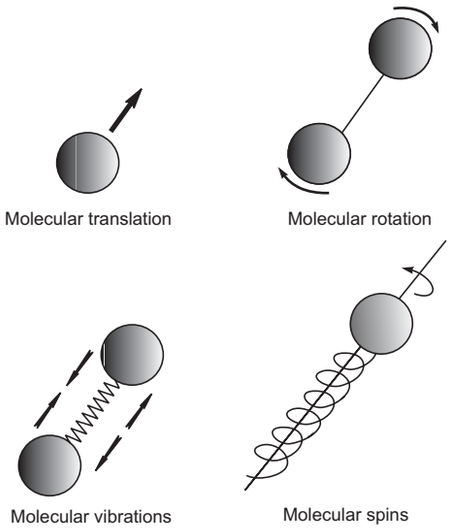

Fig. 2.2 *Molecular motions*

system, it increases the motion of molecules, thus the internal kinetic energy of system is increased, and this change is reflected by an increase in temperature of the system. Sometimes, kinetic internal energy of molecules is referred as *sensible energy*. The average velocity and the degree of activities of molecules are proportional to the temperature of a gas. Thus, at higher temperature, a system will possess higher internal energy.

(*b*) *Internal Potential Energy* Internal potential energy of a system is the energy of molecular separation. It is the energy that the molecules have as a result of their position in relation to one another. The greater the degree of molecular separation, the greater is the internal potential energy.

When a system expands or changes its physical state with addition of energy, a rearrangement of molecules takes place that increases the mean distance between them. An internal work is required to pull the molecules against the forces of attraction between them. An amount of internal potential energy equal to the amount of internal work done for rearrangement of molecules is called *latent energy* or *latent heat*.

2.4.4 Mechanical Energy

Mechanical energy can be defined as a form of energy that can be converted directly and completely into mechnical work by an ideal mechanical device such as an ideal turbine or pump. The kinetic and potential energies are the common forms of mechanical energy. Thermal (heat) energy is not a form of mechanical energy since it cannot be converted to work directly and completely.

An ideal turbine extracts mechanical energy from a flowing fluid by reducing its pressure, while a pump transfers mechanical energy to a fluid by raising its pressure. The pressure force acting on a fluid through a distance produces flow work (pv per unit mass). It is the energy of a flowing fluid, and is thus called *flow energy*. Therefore, the mechanical energy of a flowing fluid on a unit mass basis is viewed as

$$e_{mech} = pv + \frac{v^2}{2} + gz \qquad ...(2.6)$$

where pv is the flow energy, $\frac{v^2}{2}$ is the kinetic energy and gz is the potential energy of unit mass of a fluid. It can also be expressed in the rate form as

$$\dot{E}_{mech} = \dot{m}\left(pv + \frac{v^2}{2} + gz \right) \qquad ...(2.7)$$

where $\dot{m}$ is the mass flow rate of fluid. The mechanical energy change of a fluid is

$$\Delta e_{mech} = (p_2 v_2 - p_1 v_1) + \left(\frac{v_2^2 - v_1^2}{2} \right) + g(z_2 - z_1) \qquad ...(2.8)$$

and $\Delta \dot{E}_{mech}$

$$= \dot{m}\left[(p_2 v_2 - p_1 v_1) + \left(\frac{v_2^2 - v_1^2}{2} \right) + g(z_2 - z_1) \right] \qquad ...(2.9)$$

In the absence of any losses, the mechanical energy change represents the mechanical work supplied to the fluid (if $\Delta e_{mech} > 0$); or extracted from the fluid (if $\Delta e_{mech} < 0$). The shaft work, spring work, acceleration work, gravitational work, work

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done on a solid elastic bar are also some mechanical forms of work.

In *electrical work*, the force is voltage (potential difference) and displacement is electrical current; in *magnetic work*, the force is the magnetic field strength and displacement is magnetic dipole moment and are not mechanical forms of work.

2.5 ENTHALPY

The sum of the internal energy U and the product of pressure p and volume V appears frequently in many thermodynamic analyses. Therefore, it is convenient to give a name to this combination, *enthalpy*. It is also called *total enthalpy* and is designated by H. By definition,

$$H = U + pV \qquad ...(2.10)$$

Since U, p and V all are properties, the enthalpy is also a property of the system. It is measured in units of internal energy, i.e., kJ in SI units. The enthalpy for unit mass system is referred as *specific enthalpy* and is denoted by h (kJ/kg),

$$h = u + pv \qquad ...(2.11)$$

The enthalpy per mole basis is expressed as

$$\bar{h} = \bar{u} + p\bar{v} \qquad ...(2.12)$$

It should be noted that the enthalpy is a combination of other properties and it is not a form of energy.

2.6 HEAT

It is a *transfer form* of energy that flows between two systems (or a system and its surroundings) by virtue of the temperature difference between them. The temperature difference is the potential for heat transfer. There would be no heat transfer between two systems if they are at the same temperature.

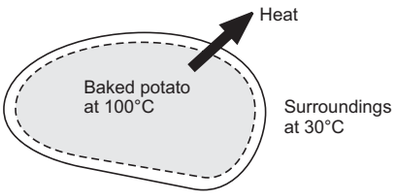
The amount of heat transferred from the state 1 to the state 2 is designated Q_{1-2} or Q and it is measured in joules (J) or kilojoules (kJ) in SI units and calories (cal) or kilocalories (kcal) in MKS units.

Heat transfer per unit mass of a system is denoted by q and is expressed as

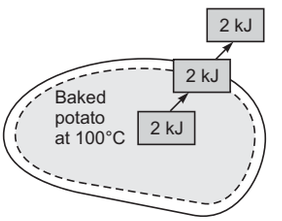
$$q = \frac{Q}{m} \text{ (kJ/kg)} \qquad ...(2.13)$$

2.6.1 Heat Supply and Heat Rejection

The heat or heat energy is generally referred as *heat transfer*. The transfer of heat into a system is called *heat addition* or *heat supply* and the transfer of heat from the system is called *heat rejection*. It is recognized as it crosses the system boundary. For example, a hot potato rejects its internal energy as heat at its boundary to its surroundings as shown in Fig. 2.3(b).



(a) Heat transfer by virtue of temperature difference



(b) Energy as heat crossing the boundary

Fig. 2.3

A process during which there is no heat transfer is called an *adiabatic process* as shown in Fig. 2.4. In an adiabatic process, energy content and the temperature of a system can be changed by other processes, such as work.

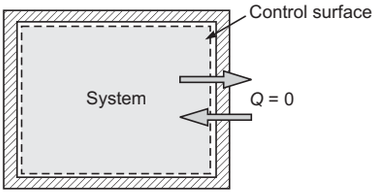


Fig. 2.4 *A system exchanging no heat with surroundings is an adiabatic system*

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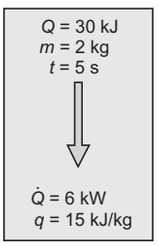

Fig. 2.5 *Relation between Q and q*

2.6.2 Heat-Transfer Rate

The quantity of heat transferred in unit time is called *heat-transfer rate*. It is designate as $\dot{Q}$ and is measured in kJ/s or kW. It is given as

$$\dot{Q} = \frac{Q}{\Delta t} \qquad \text{... (2.14)}$$

2.6.3 Heat vs Internal Energy

In thermodynamics, heat and internal energy are two different forms of energy. Internal energy is a property, while heat is not. A body may contain energy (in stored form) but not heat. The internal energy is associated with a state, while heat is associated with a process. Therefore, *heat or heat energy is defined as a form of energy in transit*. Heat is a path function. It requires a specific direction in its representation on a plot.

2.6.4 Sign Convention for Heat

Heat is a directional quantity, and its specification requires magnitude and direction. Universally accepted sign conventions for heat energy are shown in Fig. 2.6:

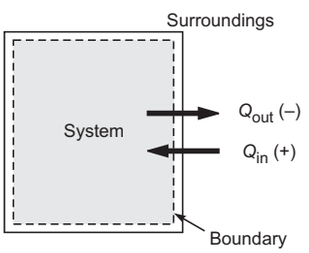

Fig. 2.6 *Sign convention for heat transfer*

1. Heat transferred to a system (heat supply) is considered *positive*.
2. Heat transferred from a system (heat rejection) is considered *negative*.

In other words, the quantity of heat which increases the energy content of a system is positive and any heat transfer that decreases the energy content of a system is negative.

2.7 SPECIFIC HEAT

It is defined as *heat energy required to change the temperature of the unit mass of a substance by one degree*. It is designated as C and is measured in kJ/kg·K or kJ/kg·°C in SI units and kcal/kg·K in MKS units. In general, the specific heat can be calculated as

$$C = \frac{1}{m}\left(\frac{\delta Q}{dT}\right) = \frac{\delta q}{dT} \qquad \text{...(2.15)}$$

Since the heat transfer is a path function, the specific heat also becomes path function and it depends on how the process is executed. The value of energy storage capacity of the substance depends upon specifc heat. The value of specific heat depends upon

(i) molecular arrangement of the system,
(ii) Condition under which change of state occurs,
(iii) How the system executes the process

2.7.1 Specific Heats of Solids and Liquids

Essentially, gases have two specific heats, C_p and C_v. But for liquids and solids, the specific volume is very small and its change with pressure and temperature is negligible. Consequently, the term $d(pv)$ is neglected from the differential form of Eq. (2.11), and thus

$$dh \approx du$$

It indicates that for solids and liquids, the enthalpy is equal to internal energy. Thus they have only one specific heat designated as C. Therefore, for any process of solids and liquids,

$$dq = dh = du \equiv C\,dT \qquad \text{...(2.16)}$$

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2.7.2 Heat Capacity

The product of mass and specific heat is defined as *heat capacity* of the system. It is measured in kJ/K or kJ/°C.

Example 2.1 *4 kg of solid material is heated from 15°C to 115°C with addition of 750 kJ of heat in a furnace. Calculate its specific heat.*

Solution

Given Mass of system, $m = 5$ kg
Initial temperature, $T_1 = 15°C$
Final temperature, $T_2 = 115°C$,
Heat added $Q = 750$ kJ

To find The specific heat of the solid system

Analysis The heat supplied to a system is expressed as

$$Q = mC(T_2 - T_1)$$

$$\therefore \quad C = \frac{Q}{m(T_2 - T_1)}$$

$$= \frac{750 \text{ kJ}}{(1 \text{ kg}) \times (115°C - 15°C)}$$

$$= \mathbf{7.5 \text{ kJ/kg} \cdot °C}$$

Example 2.2 *Estimate the rise in temperature of water when it falls through a height of 50 m. Assume that all the heat generated stays in water. The specific heat of water may be taken as 4.2 kJ/kg · °C.*

Solution

Given A running automobile vehicle is brought to rest
Height of water $h = 100$ m,
Specific heat of water $C = 4.2$ kJ/kg · °C
$= 4200$ J/kg · °C.

To find The temperature rise of water

Assumption Acceleration due to gravity is 9.81 m/s^2.

Analysis The potential energy change of water when it falls through 100 m

$\Delta PE = mgh = m \times (9.81 \text{ m/s}^2) \times (100 \text{ m}) = 981m$ J

Decrease in potential energy will convert into heat energy, thus

$$Q = \Delta PE = 981m \text{ J}$$

and $$Q = mC\Delta T$$

or $$\Delta T = \frac{Q}{mC} = \frac{981m}{m \times 4200} = \mathbf{0.233°C}$$

Example 2.3 *An automobile vehicle of 1500 kg is running at a speed of 60 km/h. The brakes are suddenly applied and the vehicle is brought to rest. Calculate the rise in temperature of brake shoes, if their mass is 15 kg. Take the specific heat of brake shoe material as 0.46 kJ/kg · K.*

Solution

Given A running automobile vehicle is brought to rest.
Initial vehicle speed, $\mathcal{V}_1 = 60$ km/h = 16.67 m/s,
Final vehicle speed $\mathcal{V}_2 = 0$
Mass of vehicle, $m_{vehicle} = 1500$ kg,
Mass of brake shoes, $m_{brake} = 5$ kg,
Specific heat $C = 0.46$ kJ/kg · K

To find The temperature rise of brake shoes

Assumption All the heat generated is absorbed by brake shoes.

Analysis The deceleration work of vehicle

$$W_{deceleration} = \Delta KE = \frac{1}{2} m_{\text{vehicle}} (\mathcal{V}_2^2 - \mathcal{V}_1^2)$$

$$= \frac{1}{2} \times (1500 \text{ kg}) \times ((0 \text{ m})^2 - (16.67 \text{ m})^2)$$

$$= -208417 \text{ J} = -208.417 \text{ kJ}$$

When brakes are applied, this deceleration work converts into heat energy and is transferred to brake shoes. That is,

$$Q = 208.417 \text{ kJ}$$

and $$Q = m_{brake} C\Delta T$$

$$\therefore \quad \Delta T = \frac{(208.417 \text{ kJ})}{(15 \text{ kg}) \times (0.46 \text{ kJ/kg} \cdot \text{K})}$$

$$= \mathbf{30.2°C}$$

Example 2.4 *During a certain process, the specific heat capacity of a system is given by C = (0.4 + 0.004 T) kJ/kg°C. Find the heat transferred and mean specific heat of gas, when the temperature changes from 25°C to 125°C. The mass of the system is 5 kg.*

Solution

Given Initial temperature, $T_1 = 25°C$
Final temperature, $T_2 = 125°C$

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Relation, $C = (0.4 + 0.004T)$ kJ/kg · °C

Mass of system, $m = 5$ kg

To find The heat-transfer rate

Analysis The relation for specific heat is given as

$$C = (0.4 + 0.004T) \text{ kJ/kg°C}$$

The heat transfer can be calculated as

$$Q = m\int_{T_1}^{T_2} C\,dT$$

$$= 5 \times \int_{25}^{125} (0.4 + 0.004T)\,dT$$

or $$Q = 5 \times \left[0.4T + 0.004 \times \frac{T^2}{2}\right]_{25}^{125}$$

$$= 5 \times \left[0.4 \times (125 - 25) + 0.004 \times \frac{(125)^2 - (25)^2}{2}\right]_{25}^{125}$$

$$= 5 \times (40 + 30) = \mathbf{350\ kJ}$$

The heat transfer can also be expressed in terms of mean specific heat as

$$Q = mC(T_2 - T_1)$$

Thus $$C = \frac{350}{5 \times (125 - 25)} = \mathbf{0.7\ kJ/kg°C}$$

2.8 WORK

Work like heat is also a form of *energy in transit*. It is defined as the energy transfer associated with force acting through a distance. It is an interaction between a system and its surroundings.

Energy can cross the boundary of the system as heat or work. Therefore, if the energy crossing the boundary is not heat, then it must be work. A moving piston, a rotating shaft, a rising weight are all associated with work interaction.

Work is also measured in kJ. Work done during a process from the state 1 to the state 2 is denoted by W_{1-2} or W. The work done per unit mass (w) is defined as

$$w = \frac{W}{m} \text{ (kJ/kg)} \qquad ...(2.17)$$

An insulated chamber containing a system of gas, and exchanging energy as work transfer only is shown in Fig. 2.7. The heat energy does not cross the system boundary as the insulated wall and the moving blades add internal energy to the system.

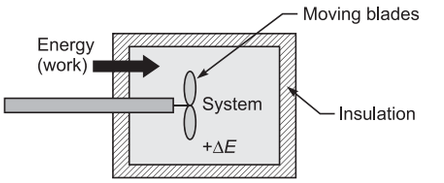

Fig. 2.7 *An energy interaction as work transfer to a insulated system*

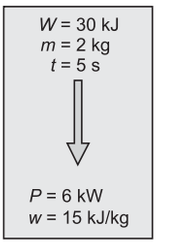

Fig. 2.8 *Relation between W, P and w*

2.8.1 Thermodynamic Definition of Work

In thermodynamics, force and distance are not easily recognized in some energy interactions. According to the thermodynamic definition of work, *an energy interaction between a system and its surroundings during a process can be considered as work transfer, if its sole effect on every thing external to* the *system could have been to raise a weight.*

Thermodynamic work refers to transfer of energy due to potential difference other than temperature difference, without transfer of mass across the system boundary. It is an extension of the concept of work in mechanics.

Consider a system consisting of an electrical battery, switch, and a resistance coil outside the system as shown in Fig. 2.9(a). When the switch is closed, the current flows through the resistance coil. Thus, electrical energy crossing the boundary of the system is converted into heat energy. At the boundary of the system, force and motion are not evident; and thus according to mechanics, this energy interaction cannot be regarded as work transfer.

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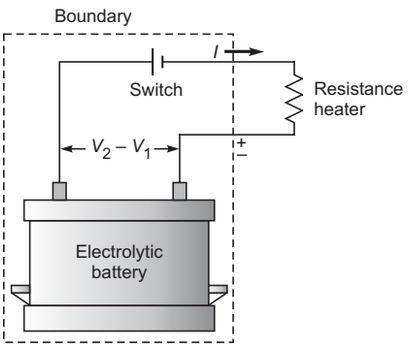
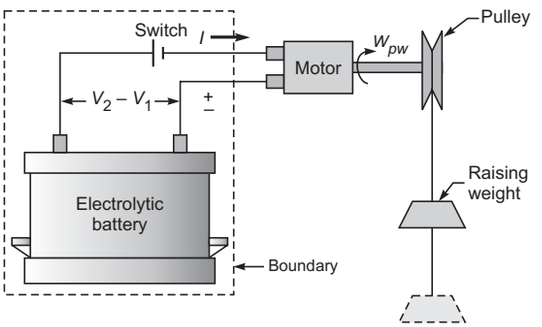


(a) Battery work through a resistance



(b) Battery works through an electric motor-pulley and weight

Fig. 2.9 *Concept of thermodynamic work*

However, an electric current is driven by an electrical potential difference external to the system. Thus, the work is done by the battery according to thermodynamic definition. This concept can be illustrated by replacing the external resistance by an *imaginary* motor-pulley and weight as shown in Fig. 2.9(b). When the switch is closed, the battery drives the motor. The pulley rotates and in turn raises the suspended weight. Thus, the sole effect external to the system becomes to raise a weight.

2.8.2 Power

The quantity of work transfer per unit time is called *power*. It is actually the rate of work transfer. It is denoted by P and measured in kW (= kJ/s).

$$P = \frac{\delta W}{dt} \text{ (kW)} \qquad \ldots(2.18)$$

Sometimes the power is also expressed in horse power (hp). The relation between hp and kW is

$$1 \text{ hp} = 0.746 \text{ kW}$$

2.8.3 Sign Convention for Work Transfer

The complete specification of work transfer also requires magnitude and direction. The universally accepted sign conventions for work transfer are shown in Fig. 2.10.

1. The production of work is desirable, therefore, the work done by a system is considered *positive*.
2. The consumption of work is always undesirable, therefore, work done on a system is considered as *negative*.

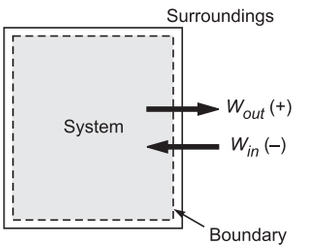


Fig. 2.10 *Sign convention for work transfer*

By the sign convention, work produced by a car engine, hydraulic motor, steam, and gas turbines is positive and work consumed in operation of a compressor, a pump, a refrigerator, a fan, etc., is negative.

2.8.4 Salient Features of Work Transfer

The features of work transfer are given below:

(i) Work is recognized at the system boundary; thus it is a *boundary phenomenon*.

(ii) Work is transferred in specific direction in a process; thus it is *a path function*.

(iii) If the *nature of the process* changes between two given states, the magnitude of work may change.

(iv) Magnitude of work transfer can also be obtained by calculating the area under the path of the process.

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2.8.5 Similarities between Heat and Work Transfer

Heat and work both are the interaction of energy between a system and its surroundings and they have some similarities between them.

1. Both are recognized at the boundary of the system as they cross it, thus, both heat and work are boundary phenomena.
2. A system may have energy, but not heat or work, because, heat and work are transient phenomena.
3. Both are associated with a process, not a state. Therefore, unlike properties heat or work has no meaning at a state.
4. Both are path functions. They are represented by a path followed during the process.
5. The equations for heat and work transfer cannot be differentiated exactly. The differential quantities of heat and work are represented as δQ and δW, respectively.

2.8.6 Dissimilarities between Heat and Work Transfer

1. Heat is a *low-grade energy*, whereas the work is a *high-grade energy*.
2. Heat transfer takes place due to *temperature difference* only, while work transfer may take place due to any *potential difference* in pressure, voltage, height, velocity, temperature, etc.
3. A stationary system cannot do work, while such a restriction is not imposed on heat transfer.
4. The entire quantity of work can be converted into heat or any other form of energy, while conversion of the entire quantity of heat into work is not possible.
5. Conversion of work into heat or another form of energy is possible with a single process, while conversion of heat into work requires a complete cyclic process, like a steam power plant.

2.9 FORMS OF WORK TRANSFER

1. Electrical work
2. Mechanical work
3. Moving boundary work
4. Flow work
5. Gravitational work
6. Acceleration work
7. Shaft work
8. Spring work

2.9.1 Electrical Work

Electrical work is the energy interaction due to crossing of electrons at the system boundary. In an electric field, the electrons in a wire move under the effect of electromotive forces for doing work (driving a motor, fan, etc.). The resistance heating as an electrical work is shown in Fig. 2.11.

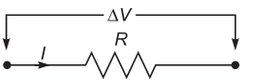

Fig. 2.11 *Resistance heating as electrical work*

Rate of electrical work transfer can be expressed as

$$\dot{W}_E = VI \text{ (watts)} \qquad ...(2.19)$$

The work done W_E in time Δt is

$$W_E = VI\Delta t \text{ (joules)} \qquad ...(2.20)$$

2.9.2 Mechanical Work

In mechanics, the work done by a system is expressed as a product of force (F) and displacement (s)

$$W = Fs \qquad ...(2.21)$$

If the force is not constant, the work done is obtained by adding the differential amounts of work,

$$W = \int_1^2 F\, ds \qquad ...(2.22)$$

2.9.3 Moving Boundary Work

In many thermodynamic problems, mechanical work is the form of moving boundary work. The moving boundary work is associated with real engines and compressors. The pressure difference is the driving force for mechanical work.

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Consider the gas enclosed in a frictionless piston cylinder arrangement as shown in Fig. 2.12. Let the gas pressure is p, volume V and piston cross-sectional area is A. If the piston is allowed to move through a distance ds in a quasi-equilibrium manner, the force applied on piston is

F = pressure × cross-sectional area of piston
$= pA$

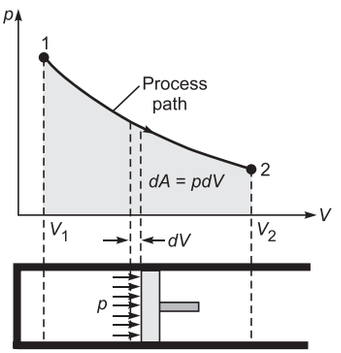

Fig. 2.12 *Schematic and p-V diagram of mechanical work*

Then differential work transfer through a displacement of ds during this process

$$\delta W = pA\,ds = pdV \qquad ...(2.23)$$

Thus, the moving boundary work of a system in differential form is equal to the product of absolute pressure and differential change in its volume dV ($= A\,ds$).

The total boundary work can be obtained by adding all differential works from the initial state 1 to the final state 2 as

$$W = \int_1^2 pdV \text{ (kJ)} \qquad ...(2.24)$$

This work transfer during a process is equal to the area under the curve on a p–V diagram and the work done by each kg of system is

$$w = \int_1^2 pdv \text{ (kJ/kg)} \qquad ...(2.25)$$

2.9.4 Gravitational Work

The work done against the gravitational force is called *gravitational work* or change in potential energy. The gravitation force F_g acting on a body of mass is

$$F_g = mg \qquad ...(2.26)$$

where g is the acceleration due to gravity. The gravitational work done to raise the body from the elevation z_1 to z_2 is

$$W_g = \int_1^2 F_g\,dz = mg\int_{z_1}^{z_2} dz$$
$$= mg(z_2 - z_1) \text{ (joules)} \qquad ...(2.27)$$

It is an increase in potential energy of the body due to gravitation work done on it.

2.9.5 Acceleration Work

The work associated with change in velocity of a system is defined as the *acceleration work*. Acceleration force, F_a according to Newton's second law of motion is

$$F_a = ma \qquad ...(2.28)$$

where a is the acceleration, which is defined in terms of the velocity $\mathcal{V}$ as

$$a = \frac{d\mathcal{V}}{dt} \qquad ...(2.29)$$

and the velocity $\mathcal{V}$ in terms of the displacement s is defined as

$$\mathcal{V} = \frac{ds}{dt} \qquad ...(2.30)$$

the $ds = \mathcal{V}dt$

Thus, the acceleration work

$$W_a = \int_1^2 F_a\,ds = \int_1^2 m\left(\frac{d\mathcal{V}}{dt}\right)(\mathcal{V}dt) = m\int_{\mathcal{V}_1}^{\mathcal{V}_2} \mathcal{V}d\mathcal{V}$$
$$= (1/2)\,m(\mathcal{V}_2^2 - \mathcal{V}_1^2) \text{ (joules)} \qquad ...(2.31)$$

where $\mathcal{V}_1$ and $\mathcal{V}_2$ are the initial and final velocities of the moving mass m. The acceleration work is recognised as the *change in kinetic energy*.

2.9.6 Shaft Work

The shaft work is the work associated with energy transmission with a rotating shaft. It is the product of torque (product of force and radius of shaft) and angular displacement.

Consider a shaft of radius r, rotating with N revolutions per minute as shown in Fig. 2.13. If the

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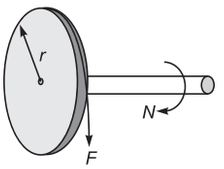

Fig. 2.13 *Work of a moving shaft*

force F is acting through an arm radius r then the *torque* is

$$T = Fr \quad \text{or} \quad F = \frac{T}{r}$$

This force acts through a displacement per unit time,

$$s = (2\pi r) \times \frac{N}{60}$$

Then the shaft work per unit time (shaft power) can be expressed as

$$W_{sh} = Fs = 2\pi r \times \left(\frac{N}{60}\right) \times \frac{T}{r}$$

$$= \frac{2\pi NT}{60} \text{ (watts)} \qquad ...(2.32)$$

2.9.7 Spring Work

When the force is applied on a spring, its length changes as shown in Fig. 2.14. If dx is the change in the length of a spring under the influence of a force F then the work done by the spring is

$$\delta W_{spring} = F dx$$

where the force F exerted can be defined in terms of the spring constant k (N/m) as

$$F = kx \text{ (N)}$$

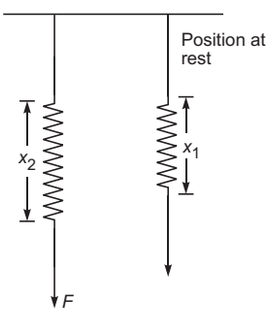

Fig. 2.14 *Elongation of spring under the effect of force*

Then the spring work

$$\delta W_{spring} = kx\, dx$$

If the spring length changes from x_1 (undisturbed position) to x_2 under the action of force, the spring work is

$$W_{spring} = k \int_{x_1}^{x_2} x\, dx$$

$$= (1/2)\, k\, (x_2^2 - x_1^2) \qquad ...(2.33)$$

Example 2.5 *A gas is compressed from an initial volume of 0.38 m^3 to a final volume of 0.1 m^3. During the quasi-equilibrium process, the pressure changes with volume according to the relation, $p = aV + b$, where, $a = -1200$ kPa/m^3 and $b = 600$ kPa. Calculate the work done during this process.*

Solution

Given Compression of gas in quasi-staitic manner

$V_1 = 0.38$ m^3, $V_2 = 0.1$ m^3,

$a = -1200$ kPa/m^3, $b = 600$ kPa

and relation $p = aV + b$

To find The work done by the system.

Analysis The work done by a system can be calculated as

$$W = \int_1^2 p dV = \int_{0.38}^{0.1} (aV + b)\, dV$$

$$= \left[\frac{1}{2} aV^2 + bV\right]_{0.38}^{0.1}$$

Using the numerical values

$$W = -1200 \times \frac{0.1^2 - 0.38^2}{2} + 600 \times (0.1 - 0.38)$$

$$W = 80.64 - 168 = \mathbf{-87.36\ kJ}$$

Example 2.6 *In a reversible non-flow process, the work is done by a substance in accordance with $V = \frac{2.80}{p}$ m^3, where p is the pressure in bar. Find the work done on or by system as pressure increases from 0.7 bar to 7 bar.*

Solution

Given A reversible non-flow process with

$p_1 = 0.7$ bar $= 70$ kPa, $p_2 = 7$ bar $= 700$ kPa

and relation $V = \frac{2.80}{p}$ m^3

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To find The work interaction by the system

Analysis The initial and final volumes of the working substance

$$V_1 = \frac{2.80}{p_1} = \frac{2.8}{0.7} = 4 \text{ m}^3$$

$$V_2 = \frac{2.80}{p_2} = \frac{2.8}{7} = 0.4 \text{ m}^3$$

From the given relation, the pressure p can be expressed as

$$p = \frac{2.80}{V} \text{ bar} = 100 \times \frac{2.80}{V} \text{ kPa}$$

The work done by a system can be calculated as

$$W = \int_1^2 pdV = 100 \times 2.8 \times \int_4^{0.4} \frac{1}{V}\, dV$$

$$= 280 \times \left[\ln(V)\right]_4^{0.4} = 280 \times \ln\left(\frac{0.4}{4}\right)$$

$$= \mathbf{-644.72 \text{ kJ}}$$

Example 2.7 *In a piston–cylinder arrangement, the pressure is inversely proportional to the square of the volume. The initial pressure is 10 bar in the cylinder and the initial volume is 0.1 m^3. The volume is now changed so that the final pressure is 2 bar. Find the work done in kJ.*

Solution

Given The relation $p \propto \frac{1}{V^2}$,

$p_1 = 10 \text{ bar} = 1000 \text{ kPa}$

$V_1 = 0.1 \text{ m}^3$,

$p_2 = 2 \text{ bar} = 200 \text{ kPa}$

To find Work done during the process

Analysis The given relation

$$p \propto \frac{1}{V^2} \quad \text{or} \quad p = \frac{K}{V^2}$$

At the state 1, $p_1 \propto \frac{1}{V_1^2}$ or $p_1 = \frac{K}{V_1^2}$

where K is the constant of proportionality and it is calculated by initial condition. Therefore,

$$K = p_1 V_1^2 = (1000 \text{ kPa}) \times (0.1 \text{ m}^3)^2$$

$$= 10 \text{ kPa.m}^6$$

Now at the state 2,

$$p_2 = \frac{K}{V_2^2}$$

or $$V_2 = \sqrt{\frac{K}{p_2}} = \sqrt{\frac{10}{200}} = 0.223 \text{ m}^3$$

Now, the work done during the process

$$W = \int_1^2 pdV = \int_1^2 \frac{K}{V^2}\, dV = K\left[\frac{1}{V_1} - \frac{1}{V_2}\right]$$

or $$W = 10 \times \left[\frac{1}{0.1} - \frac{1}{0.233}\right]$$

$$= 10 \times (10 - 4.29) = \mathbf{57.1 \text{ kJ}}$$

Example 2.8 *A spherical balloon contains 5 kg of air at 200 kPa and 500 K. The balloon material is such that the pressure inside is always proportional to the square of the diameter. Determine the work done when the volume of the balloon doubles as a result of heat transfer.*

Solution

Given $m = 5$ kg, $p_1 = 200$ kPa,

$T_1 = 500$ K, $V_2 = 2V_1$.

To find The work done by gas, when volume of balloon doubles.

Assumption The gas constant for air is 0.287 kJ/kg·K

Analysis Consider D is the diameter of the balloon. According to the given condition,

$$p \propto D^2 \quad \text{or} \quad p = KD^2$$

It is the equation of state with a constant of proportionality K.

Further, from the relation for a perfect gas

$$p_1 v_1 = RT_1$$

$$v_1 = \frac{R\,T_1}{p_1} = \frac{0.287 \times 500}{200}$$

$$= 0.7175 \text{ m}^3/\text{kg}$$

Thus, the volume of the balloon at initial state;

$$V_1 = m v_1 = 5 \times 0.7175 = 3.5875 \text{ m}^3$$

Thus, the diameter of the balloon can be calculated as

$$V_1 = (1/6) \times \pi \times D_1^3$$

or $$D_1^3 = 6.851$$

or $$D_1 = 1.899 \text{ m}$$

When the volume of the balloon doubles, the diameter of balloon

$$V_2 = (1/6) \times \pi \times D_2^3$$

but $$V_2 = 2V_1 = 7.175$$

or $$D_2^3 = 13.703$$

or $$D_2 = 2.393 \text{ m}$$

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Now from the given relation at the state 1

$$p_1 = KD_1^2$$

or $$K = \frac{p_1}{D_1^2} = \frac{200}{(1.8993)^2} = 55.44$$

The work done by a system;

$$W = \int_1^2 pdV = \int_1^2 KD^2\,dV = K\int_1^2 D^2\,d\left(\frac{\pi}{6}D^3\right)$$

$$= \frac{K}{6}\pi \times \int_1^2 D^2 \times 3D^2\,dD$$

$$= \frac{1}{2}\pi \times 55.44 \int_1^2 D^4 dD$$

$$= 87.08 \times \left[\frac{D_2^5}{5} - \frac{D_1^5}{5}\right]$$

$$= 87.08 \times \left[\frac{(2.393)^5}{5} - \frac{(1.8993)^5}{5}\right]$$

$$= \mathbf{936.22\ kJ}$$

Example 2.9 *The van der Waals equation is given by*

$$\left(p + \frac{a}{v}\right)(v - b) = RT$$

where a and b are constants and other terms have usual meanings. Determine the work done in a reversible isothermal expansion.

Solution The van der Waals equation is

$$\left(p + \frac{a}{v}\right)(v - b) = RT$$

It can be rearranged as

$$p = \frac{RT}{v - b} - \frac{a}{v}$$

The work done by the gas can be calculated as

$$w = \int_{v_1}^{v_2} p\,dv$$

or $$w = RT\int_{v_1}^{v_2} \frac{1}{v - b}\,dv - \int_{v_1}^{v_2} \frac{a}{v}\,dv$$

or $$w = \boldsymbol{RT\ln\left(\frac{v_2 - b}{v_1 - b}\right) + a\left[\frac{1}{v_2} - \frac{1}{v_1}\right]}\ \textbf{(kJ/kg)}$$

Example 2.10 *A quantity of gas is compressed in a piston–cylinder from a volume of 0.8611 m^3 to a final volume of 0.17212 m^3. The pressure (in bar) as a function of volume (m^3) is given by*

$$p = \frac{0.86110}{V} - \frac{8.60673 \times 10^{-5}}{V^2}$$

(a) Find the amount of work done in kJ.

(b) If the atmospheric pressure, i.e., 1 bar acting on the other side of piston is considered, find the net work done in kJ.

Solution

Given $V_1 = 0.8611\ m^3$, $V_2 = 0.17212\ m^3$,

$p_{atm} = 1\ bar = 100\ kPa$

and $$p = \frac{0.86110}{V} - \frac{8.60673 \times 10^{-5}}{V^2}\ (bar)$$

To find

(i) The work done by the gas,

(ii) The work done by the gas when atmospheric pressure on other side of piston is considered.

Analysis

(i) Work done by the gas without considering atmospheric pressure;

$$W = \int_{V_1}^{V_2} pdV$$

$$= (100\ kPa/bar)\int_{0.8611}^{0.17212}\left(\frac{0.86110}{V} - \frac{8.60673 \times 10^{-5}}{V^2}\right)dV$$

$$= 100 \times \left[0.86110\ln(V) - 8.60673 \times 10^{-5} \times \left(-\frac{1}{V}\right)\right]_{0.8611}^{0.17212}$$

$$= 100 \times \left[0.86110 \times \ln\left(\frac{0.17212}{0.8611}\right) - 8.60673 \times 10^{-5} \times \left(\frac{1}{0.8611} - \frac{1}{0.17212}\right)\right]$$

$$= \mathbf{-138.6\ kJ}$$

(ii) Work done on the atmosphere by the piston for its volume change from V_1 to V_2

$$W_{atm} = \int_{V_1}^{V_2} pdV = p(V_2 - V_1)$$

$$= 100 \times (0.17212 - 0.8611)$$

$$= -68.9\ kJ$$

Net work done by the gas

$$W_{net} = W - W_{atm} = -138.6 + 68.9$$

$$= \mathbf{-69.7\ kJ}$$

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Example 2.11 *A system of 1 kg of gas expands from an initial state at pressure of p_1 bar and a volume of v_1 m^3/kg to a volume of v_2 m^3/kg. Calculate the work done by the gas, when expansion is (a) isobaric, (b) isothermal, and (c) polytropic with the law pv^n = constant.*

Solution

Analysis

(i) When the process of expansion is isobaric (constant pressure):

$$p_1 \text{ (bar)} = 100 p_1 \text{ (kPa)}$$

$$w = \int_{v_1}^{v_2} p\,dv = 100\, p_1 \int_{v_1}^{v_2} dv$$

$$= \mathbf{100 p_1 (v_2 - v_1)}$$

(ii) When the process of expansion is isothermal (constant temperature):

The law of process,

$$pv = C \quad \text{or} \quad p = \frac{100\,C}{v}$$

The work done $w = \int_{v_1}^{v_2} p\,dv = 100\,C \int_{v_1}^{v_2} \frac{1}{v}\,dv$

$$= 100\,C \ln\left(\frac{v_2}{v_1}\right)$$

or $$w = \mathbf{100 p_1 v_1 \ln\left(\frac{v_2}{v_1}\right) (kJ/kg)}$$

(iii) When the process of expansion is polytropic:

The law of process,

$$pv^n = C \quad \text{or} \quad p = \frac{100\,C}{v^n}$$

The work done, $w = \int_1^2 p\,dv = 100\,C \int_{v_1}^{v_2} \frac{1}{v^n}\,dv$

$$= 100\,C \left[\frac{v_2^{1-n} - v_1^{1-n}}{n-1}\right]$$

Now using $C = p_1 v_1^n = p_2 v_2^n$

We get, $$w = \mathbf{100} \times \left(\frac{\boldsymbol{p_2 v_2 - p_1 v_1}}{\boldsymbol{n-1}}\right) \text{ kJ/kg}$$

Example 2.12 *A cylinder with a frictionless piston contains 0.1 m^3 of gas at 200 kPa. The piston is connected to a coil spring which exerts a force proportional to the displacement from its equilibrium position. The gas is heated until the volume is doubled, at this state; the pressure is found to be 500 kPa. Determine the work done by the gas. Take the atmospheric pressure as 1 bar.*

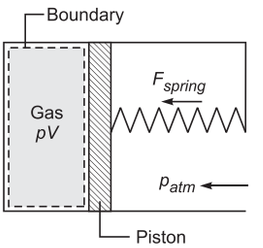

Fig. 2.15 *Gas under spring force*

Solution

Given A cylinder with a piston connected to coil spring.

$V_1 = 0.1$ m^3, $p_1 = 200$ kPa,

$V_2 = 2V_1$, $p_2 = 500$ kPa.

and $F_{spring} \propto x$ or $F_{spring} = kx$

To find Work done by the gas.

Analysis The force balance at any position of the piston

$$pA = p_{atm} A + kx$$

using displacement $x = \frac{V}{A}$

Then $$pA = p_{atm} A + \frac{k\,V}{A}$$

or $$p = p_{atm} + \frac{k\,V}{A^2} \qquad \text{...(i)}$$

The work done

$$W = \int_1^2 p\,dV = \int_1^2 \left(p_{atm} + \frac{kV}{A^2}\right) dV$$

$$= p_{atm}(V_2 - V_1) + \frac{k}{2A^2}(V_2^2 - V_1^2)$$

$$= p_{atm}(V_2 - V_1) + \frac{k}{2A^2}(V_2 - V_1)(V_2 + V_1)$$

$$= (V_2 - V_1)\left[p_{atm} + \frac{k}{2A^2}(V_2 + V_1)\right] \qquad \text{...(ii)}$$

From Eq. (i), we get

$$kV = A^2(p - p_{atm})$$

$$\therefore \quad W = (V_2 - V_1)\left[p_{atm} + \frac{p_2 - p_{atm}}{2} + \frac{p_1 + p_{atm}}{2}\right]$$

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On solving $W = (V_2 - V_1)\left[\frac{p_1 + p_2}{2}\right]$

Using numerical values;

$$W = (0.2 - 0.1) \times \left(\frac{200 + 500}{2}\right) = \mathbf{35\ kJ}$$

2.10 FIRST LAW OF THERMODYNAMICS

The first law of thermodynamics, also known as the *conservation of energy principle.* It states that during any process, if the energy disappears in one form, it appears in other form, but its total quantity remains always constant. That is, *the energy can be neither created nor destroyed, it can only change its form*. For example, for the energy interaction between a system and its surroundings, the energy lost by a system must be exactly equal to the amount of energy gained by the surroundings. The first law can be proved mathematically, but no process in nature is known to have violated the first law of thermodynamics.

It is the relation of *energy balance* and is applicable to any kind of system (open or closed) undergoing any kind of process.

Let us consider a process that involves only heat transfer but no work interaction. A hot potato taken from an oven is exposed to room air as shown in Fig. 2.16. As a result of heat transfer from the hot potato, its energy will decrease. In absence of other effects, the decrease of total energy of the potato becomes equal to the amount of heat transferred to its surroundings. Therefore, the principle of conservation of energy can be expressed as

$$-\Delta E = -Q$$

where $\Delta E = E_2 - E_1$

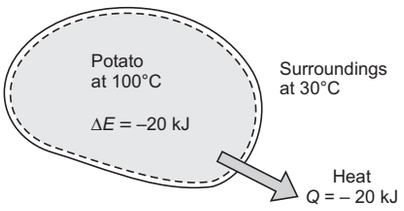

Fig. 2.16 *Hot potato losing its energy as heat at its boundary*

In the absence of any work interaction between a system and its surroundings, the amount of net heat transfer is equal to the change in the energy of a system.

$$Q = \Delta E \quad \text{when} \quad W = 0 \qquad ...(2.34)$$

Now consider a well-insulated room, heated by an electric heater as shown in Fig. 2.17. As a result of electrical work done, the energy of the room will increase. Since the room is adiabatic and cannot have any heat interaction with its surroundings, the conservation of energy principle dictates that electrical work done on the room must be equal to increase in energy of the room. That is,

$$-W = \Delta E$$

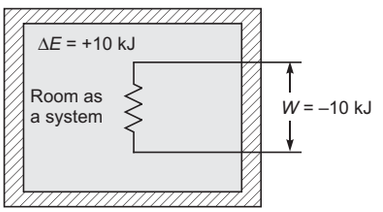

Fig. 2.17 *Electrical work increases the energy in a room*

For an adiabatic processes, the amount of work done is equal to the change in energy of the system. That is,

$$W = -\Delta E \quad \text{when} \quad Q = 0 \qquad ...(2.35)$$

Now consider work and heat transfer simultaneously as shown in Fig. 2.18. Consider an amount of gas in a piston cylinder device. The gas is heated inside the cylinder as a result of heat supply. As the energy of the gas increases, its pressure and temperature also increase. Then the gas will expand and work will be done at the boundary of the system. The conservation of energy principle reveals that

$$Q = W + \Delta E$$

or $$Q - W = \Delta E \text{ (kJ)} \qquad ...(2.36)$$

where Q = net heat transfer across system boundaries

W = net work transfer in all forms

ΔE = net change in total energy of the system $(E_2 - E_1)$

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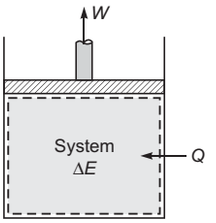

Fig. 2.18 *Heat supplied to a system is equal to work transfer and energy increase*

The total energy of the system is the sum of internal energy U, potential energy PE and kinetic energy KE, i.e.,

$$\Delta E = \Delta U + \Delta PE + \Delta KE \text{ (kJ)} \quad ...(2.37)$$

Most closed systems in practice are stationary, i.e., they do not involve kinetic energy and potential energy during a process. Thus, the stationary systems are called *non-flow systems* and the first law of thermodynamics is reduced to

$$Q - W = \Delta U \text{ (kJ)} \quad ...(2.38)$$

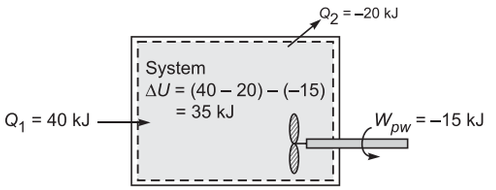

Fig. 2.19 *The energy change of a closed system during a process*

For unit mass,

$$q - w = \Delta u \quad ...(2.39)$$

In rate form of energy, dividing Eq. (2.36), on both sides by the time interval dt

$$\dot{Q} - \dot{W} = \frac{dE}{dt} \quad ...(2.40)$$

In differential form,

$$\delta Q - \delta W = dE \quad ...(2.41)$$

2.10.1 Total Energy

The sum of all the forms of energy is called *total energy* or energy E of a system. Thus

$$E = PE + KE + IE + \text{other forms of energy} \quad ...(2.42)$$

The energy of a system on unit mass basis is denoted as e and is defined as

$$e = \frac{E}{m} \text{ (J/kg)} \quad ...(2.43)$$

The magnetic, wind, thermal, electrical and mechanical energy are transit forms of energy. Most often, in thermodynamic analysis, the *total energy* of a system is considered as the sum of kinetic energy, potential energy and internal energy and is expressed as

$$E = U + KE + PE \quad ...(2.44)$$

On unit mass basis $e = u + ke + pe$ ...(2.45)

2.10.2 Mechanism of Energy Transfer

The energy can be transferred to or from a system in three forms: *heat*, *work* and *mass flow*. The energy interactions are recognised at the system boundary as they cross it, and direction of energy transfer represents the energy gain or loss by a system during a process.

1. Heat Transfer, Q Heat transfer to a system increases the energy of molecules and thus an increase in the internal energy of the system, and heat transfer from a system decreases the energy of molecules and thus results in a decrease in the internal energy of the system.

2. Work Transfer, W The work refers to transfer of energy due to potential difference other than temperature difference, between a system and its boundary. A rising piston, rotating shaft, and an electric wire carrying current crossing the system boundary—all these energy transfers are associated with work interactions. The work transfer to a system (called work done on a system) increases the energy of the system and work transfer from a system (called work done by the system) decreases it.

3. Mass flow, m When mass enters a system, the energy of the system increases, because mass carries energy with it. Similarly, when some mass leaves the system, the energy of the system decreases, because the leaving mass takes some energy with it.

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The net change in energy of a system is the difference between the amounts of heat, work and mass transferred in and out and the energy balance can be written more clearly as

$$\Delta E_{system} = (Q_{in} - Q_{out}) + (W_{in} - W_{out}) + (E_{mass,\,in} - E_{mass,\,out}) \quad ...(2.46)$$

where quantity 'in' and 'out' denote qunatities that enter and leave the system, respectively.

The heat transfer Q is zero for an adiabatic system, the work transfer W is zero for a system that does not involve work interaction and the energy transport with mass E_{mass} is zero for closed systems.

The net change in total energy of a system during a process is equal to the difference between the total energy entering and the total energy leaving the system during that process.

$$\begin{pmatrix}\text{Total energy}\\ \text{entering the system}\end{pmatrix} - \begin{pmatrix}\text{Total energy}\\ \text{leaving the system}\end{pmatrix} = \begin{pmatrix}\text{Net change in total}\\ \text{energy of the system}\end{pmatrix}$$

or $$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work and mass}}} = \underbrace{\Delta E_{system}\,(\text{kJ})}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}} \quad \ldots(2.47)$$

In rate form

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work and mass}}} = \underbrace{\frac{dE_{system}}{dt}\,(\text{KW})}_{\substack{\text{Rate of change in internal,}\\ \text{kinetic, potential, etc., energies}}} \quad \ldots(2.48)$$

For constant rates, the total quantities during a time interval Δt are related to the quanties per unit time as

$$Q = \dot{Q}\Delta t \text{ , and}$$
$$W = \dot{W}\Delta t \text{ ,} \quad ...(2.49)$$

2.11 FIRST LAW OF THERMODYNAMICS FOR A CYCLIC PROCESS—JOULE'S EXPERIMENT

Joule conducted several experiments which led to the formulation of the *first law of thermodynamics*.

In his experimental arrangement as shown in Fig. 2.20, he took a known quantity of water in a rigid insulated tank fitted with a paddle wheel. Water was agitated by a stirrer connected with the paddle wheel (process A). The amount of work done on the water by the stirrer was accurately measured as a product of weight and its displacement. The temperature rise of water during the process was also recorded.

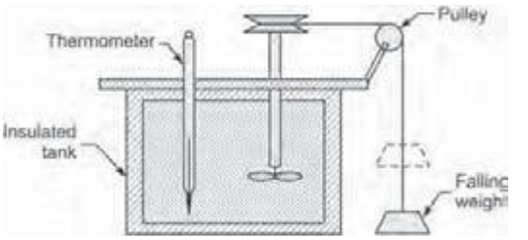

Fig. 2.20 *Joule's experimental arrangement, demonstrating net work transfer is equal to net heat transfer in a cyclic process*

Further, the insulation from the tank was removed and the whole system was placed in a water bath. The heat was transferred from the system (process B) in order to restore the system to the same initial state. Thus process A and process B form a cycle. During process A, there was work done on the system but no heat transfer; during the process B, there was heat transferred but no work done. During a complete cycle, there was net work input and net heat output from the system.

Joule repeated the experiment for different systems, and for different amounts of work interactions, and measured the corresponding amount of heat interaction in each case for restoring the system back to its initial state. Joule found in his experimental observations that,

"*Whenever a closed system undergoes a cycle, the work input to the system is proportional to the net heat output*". It is expressed as

$$\oint \delta W \propto \oint \delta Q$$

or $$\oint \delta W = J \oint \delta Q \quad ...(2.50)$$

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where J is the constant of proportionality, called *mechanical equivalent of heat*, whose value depends on the units selected for W and Q. In SI units its value is unity, therefore;

$$\oint \delta W = \oint \delta Q \qquad ...(2.51)$$

When a system undergoing a cyclic process comprises of more than one process, then

$$\Sigma W = \Sigma Q$$

or $$\Sigma W - \Sigma Q = 0 \qquad ...(2.52)$$

For a cyclic process, the net heat transfer is always equal to the net work transfer.

2.12 ENERGY—A PROPERTY OF THE SYSTEM

For any cyclic process, the first law of thermodynamics is given by Eq. (2.51). Many times we are concerned with heat and work transfer during a single thermodynamic process. Thus, we formulate the first law of thermodynamics for a process.

Consider a system undergoing a change of state from 1 to 2, along a path A and returning from state 2 to 1, along the path B, thus completing a cycle 1–*A*–2–*B*–1 as shown in Fig. 2.21.

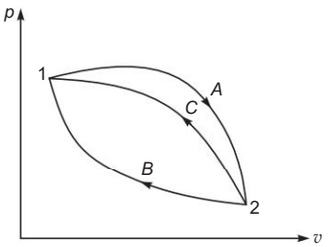

Fig. 2.21 *Cyclic process between two fixed states*

Applying the first law for the cyclic process 1–*A*–2–*B*–1;

$$\int_1^2 \delta Q_A + \int_2^1 \delta Q_B = \int_1^2 \delta W_A + \int_2^1 \delta W_B$$

or $$\int_1^2 \delta Q_A - \int_1^2 \delta Q_B = \int_1^2 \delta W_A - \int_1^2 \delta W_B$$

Rearranging, we get;

$$\int_1^2 \delta Q_A - \int_1^2 \delta W_A = \int_1^2 \delta Q_B - \int_1^2 \delta W_B \qquad ...(2.53)$$

The quantity $\delta Q - \delta W$ is dE according to Eq. (2.36). The integration of each side of Eq. (2.53) leads to the energy transfer during the paths (processes) A and B, thus

$$\Delta E_A = \Delta E_B \qquad ...(2.54)$$

Now consider another cycle 1–*A*–2–*C*–1. The processes A and C together constitute a cycle, for which,

$$\int_1^2 \delta Q_A + \int_2^1 \delta Q_C = \int_1^2 \delta W_A + \int_2^1 \delta W_C$$

or $$\int_1^2 \delta Q_A - \int_1^2 \delta W_A = \int_1^2 \delta Q_C - \int_1^2 \delta W_C$$

or $$\Delta E_A = \Delta E_C \qquad ...(2.55)$$

From Eq. (2.54) and (2.55), we get

$$\Delta E_B = \Delta E_C$$

It indicates that *the change in energy between two states of the system does not depend upon the path followed by the process, but depends on the states between which the process proceeds. Thus, the energy is a state function and property*.

2.13 PERPETUAL MOTION MACHINE OF THE FIRST KIND (PMM1)

A device that violates the first law of thermodynamics by producing work from nothing is called a perpetual motion machine of the first kind (PMM 1).

Consider the steam power plant as shown in Fig. 2.22. It is proposed to produce steam by a resistance heater placed inside the boiler, instead of

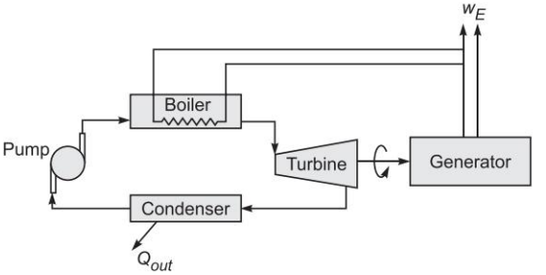

Fig. 2.22 *A perpetual motion machine that violates the first law of thermodynamics*

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by combustion of fuel. The part of electricity generated by the plant is to be used for the resistance heating and pump work. The rest of the electricity is the net output of the plant. The inventor claims, once the system is started, the power plant produces the electricity for an infinite time without requiring any energy input from the outside.

Here, the inventor tries to solve the world energy crisis, if it works. Unfortunately, such a proposal is not possible in practice, because the device only produces energy output as electrical work, W_E and the heat rejection Q_{out} from the condenser without any energy input, which is a violation of the first law of thermodynamics. Therefore, such a device is called the perpetual motion machine of the first kind (PMM1), which is impossible.

Example 2.13 *A system is composed of a gas contained in a cylinder fitted with a piston. The gas expands from the state 1 for which E_1 = 75 kJ to a state 2 for which E_2 = –25 kJ. During the expansion, the gas does 60 kJ of work on the surroundings. Determine the heat transferred to or from the system during the process.*

Solution

Given $E_1 = 75$ kJ, $E_2 = -25$ kJ and $W = 60$ kJ

To find The amount of heat transferred.

Analysis According to the first law of thermodynamics for a process

$$Q - W = \Delta E$$

or $$Q = W + \Delta E$$

where $$\Delta E = E_2 - E_1 = -25 - 75 = -100 \text{ kJ},$$

Using the above $Q = (60 \text{ kJ}) + (-100 \text{ kJ})$

$= \mathbf{-40\ kJ}$ (heat is rejected)

Example 2.14 *A system undergoes a cyclic process composed of four processes 1–2, 2–3, 3–4, and 4–1. The energy transfer is tabulated below:*

Process	*Q kJ/min*	*W kJ/min*	*ΔU kJ/min*
1–2	*400.0*	*150.0*	—
2–3	*200.0*	—	*300.0*
3–4	*–200*	—	—
4–1	*0*	*75*	—

Complete the table and determine the power output.

Solution For any process;

$$Q = W + \Delta U$$

Process 1–2; $Q_{1-2} = W_{1-2} + \Delta U_{1-2}$

Using the values;

$$400.0 \text{ kJ/min} = 150.0 \text{ kJ/min} + \Delta U_{1-2}$$

or $$\Delta U_{1-2} = \mathbf{250.0\ kJ/min}$$

Process 2–3; $Q_{2-3} = W_{2-3} + \Delta U_{2-3}$

Using the values;

$$200.0 \text{ kJ/min} = W_{2-3} + 300.0 \text{ kJ/min}$$

or $$W_{2-3} = \mathbf{-100.0\ kJ/min}$$

Since there are two unknowns in the process 3–4, and they cannot be determined with one equation, hence we consider process 4–1;

Process 4–1; $Q_{4-1} = W_{4-1} + \Delta U_{4-1}$

Using values;

$$0.0 \text{ kJ/min} = 75 \text{ kJ/min} + \Delta U_{4-1}$$

or $$\Delta U_{4-1} = \mathbf{-75.0\ kJ/min}$$

All processes together constitute a cyclic process:

$$\Sigma Q = \Sigma W$$

$$Q_{1-2} + Q_{2-3} + Q_{3-4} + Q_{4-1} = W_{1-2} + W_{2-3} + W_{3-4} + W_{4-1}$$

Using the values

$$400.0 + 200.0 + (-200.0) + 0 = 150.0 + (-100.0) + W_{3-4} + 75.0$$

or $$W_{3-4} = \mathbf{275.0\ kJ/min}$$

Now considering the process 3–4

$$Q_{3-4} = W_{3-4} + \Delta U_{3-4}$$

Using values;

$$-200.0 \text{ kJ/min} = 275.0 \text{ kJ/min} + \Delta U_{3-4}$$

or $$\Delta U_{3-4} = \mathbf{-475.0\ kJ/min}$$

For a cyclic process:

Net heat transfer = net work transfer = 400 kJ/min

The power output

$$P = \frac{\text{Net work transfer}}{\text{time in seconds}} = \frac{400}{60}$$

$$= \mathbf{6.67\ kW}$$

The complete table is

Process	*Q kJ/min*	*W kJ/min*	*ΔU kJ/min*
1–2	400.0	150.0	**250.0**
2–3	200.0	**–100.0**	300.0
3–4	–200	**275.0**	**–475.0**
4–1	0	75.0	**–75.0**

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Example 2.15 *A non-flow system undergoes a frictionless process according to a law $p = (4.5/v) + 2$, where p is in bar and the volume v is in m^3/kg. During the process, the volume changes from 0.12 m^3/kg to 0.04 m^3/kg and the temperature increases by 133°C. The change in internal energy of the fluid is given as $du = C_v dT$, where $C_v = 0.71$ kJ/kg·K and dT is temperature change. Find out (a) heat transferred, and (b) change in enthalpy.*

Assume a fluid quantity of 10 kg.

Solution

Given The process law $p = \left[\frac{4.5}{v} + 2\right]$ bar

$$= 100 \times \left[\frac{4.5}{v} + 2\right] \text{kPa}$$

Initial volume, $v_1 = 0.12$ m^3/kg,
Final volume, $v_2 = 0.04$ m^3/kg
Temperature rise, $\Delta T = 133$°C,
Specific heat, $C_v = 0.71$ kJ/kg·K
$du = C_v dT$,
Mass of fluid, $m = 10$ kg

To find

(i) Heat transferred, and
(ii) change in enthalpy.

Analysis

(i) The change in specific internal energy of the fluid system can be calculated by integrating the given expression

$$\Delta u = \int_1^2 C_v dT = C_v \Delta T$$
$$= (0.71 \text{ kJ/kg·K}) \times (133°\text{C or K})$$
$$= 94.43 \text{ kJ/kg}$$

The work done by each kg of fluid

$$p = 100 \int_1^2 \left[\frac{4.5}{v} + 2\right] dv$$
$$= 100 \times \left[4.5 \times \ln\left(\frac{v_2}{v_1}\right) + 2(v_2 - v_1)\right]$$
$$= 100 \times \left[4.5 \times \ln\left(\frac{0.04}{0.12}\right) + 2 \times (0.04 - 0.12)\right]$$
$$= 100 \times (-4.943 - 0.16) = -510.37 \text{ kJ/kg}$$

The heat transfer for one kg of fluid system:

$$q = w + \Delta u = -510.37 + 94.43$$
$$= -415.94 \text{ kJ/kg}$$

For a 10-kg fluid system,

$$Q = (10 \times 415.94) = \mathbf{41594 \text{ kJ}}$$

(ii) The change in specific enthalpy can be calculated as

$$\Delta h = \Delta u + \Delta(pv) = \Delta u + (p_2 v_2 - p_1 v_1)$$

The pressure p_1 and p_2 can be calculated from the given expression:

$$p_1 = 100 \times \left[\frac{4.5}{v_1} + 2\right] \text{kPa}$$
$$= 100 \times \left[\frac{4.5}{0.12} + 2\right]$$
$$= 3950 \text{ kPa}$$

and $$p_2 = 100 \times \left[\frac{4.5}{v_2} + 2\right] \text{kPa}$$
$$= 100 \times \left[\frac{4.5}{0.04} + 2\right]$$
$$= 11450 \text{ kPa}$$

Thus flow energy,

$$\Delta(pv) = 11450 \times 0.04 - 3950 \times 0.12$$
$$= -16.0 \text{ kJ/kg}$$

Change in enthalpy, $\Delta h = 94.43 + (-16)$
$= 78.43$ kJ/kg

For 10 kg fluid, $\Delta H = m\Delta h = 10 \times 78.43$
$= \mathbf{784.3 \text{ kJ}}$

Example 2.16 *The power developed by a turbine in a certain steam power plant is 1200 kW. The heat supplied to the boiler is 3360 kJ/kg. The heat rejected by the system to the cooling water is 2520 kJ/kg and feed pump work required to pump the condensate back into the boiler is 6 kW. Calculate the steam flow through the cycle in kg/s.*

Solution

Given A steam power plant (Fig. 2.23)

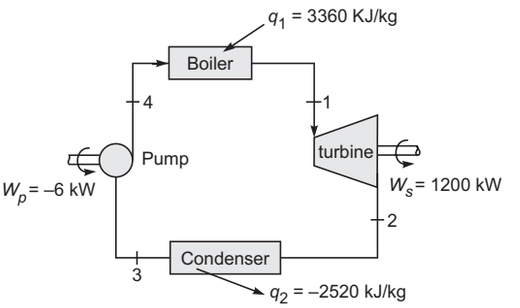

Fig. 2.23

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To find Mass flow of steam

Analysis Since the working substance (steam/water) undergoes a cyclic process, thus

$$\oint \delta Q = \oint \delta W$$

or $\dot{Q}_1 + \dot{Q}_2 = \dot{W}_T + \dot{W}_p$

or $\dot{m}\,(q_1 + q_2) = \dot{W}_T + \dot{W}_p$

or $\dot{m} = \dfrac{1200 - 6}{3360 - 2520} = \mathbf{1.421\ kg/s}$

Example 2.17 *A slow chemical reaction takes place in a fluid at a constant pressure of 0.1 MPa. The fluid is surrounded by a perfect heat insulator during reaction, which begins at the state 1 and ends at the state 2. The insulation is then removed and 105 kJ of heat flows to surroundings as the fluid goes to the state 3. The following data are observed for fluid at states 1, 2 and 3 respectively:*

State	*V (m^3)*	*T (°C)*
1	*0.003*	*20*
2	*0.3*	*370*
3	*0.06*	*20*

For the fluid, calculate E_2 and E_3, if $E_1 = 0$.

Solution

Given A closed system at constant process:

State 1: $V_1 = 0.003\ m^3$,
$T_1 = 20°C = 293\ K$,
$p_1 = 0.1\ MPa = 100\ kPa$,

State 2: $V_2 = 0.3\ m^3$,
$T_2 = 370°C = 643\ K$,
$Q_{1-2} = 0$

State 3: $V_3 = 0.06\ m^3$,
$T_3 = 20°C = 293\ K$,
$Q_{2-3} = -105\ kJ$

To find Magnitude of energy at states 2 and 3

Analysis For constant pressure process 1–2;

$$W_{1-2} = p_1(V_2 - V_1) = 100 \times (0.3 - 0.003) = 29.7\ kJ$$

Applying the first law of thermodynamics

$$Q_{1-2} = W_{1-2} + E_2 - E_1$$

Using the values; $0 = 29.7 + E_2 - 0$

or $E_2 = \mathbf{-29.7\ kJ}$

Process 2–3 also takes place at constant pressure, therefore, work transfer;

$$W_{2-3} = p_1(V_3 - V_2) = 100 \times (0.06 - 0.3) = -24\ kJ$$

Applying the first law of thermodynamics

$$Q_{2-3} = W_{2-3} + E_3 - E_2$$

Using the values;

$-105 = -24 + E_3 - (-29.7)$

or $E_3 = \mathbf{-110.7\ kJ}$

Example 2.18 *The energy (J) of a closed system can be expressed as $E = 100 + 50T + 0.04T^2$. The heat (J) absorbed is given by $Q = 5000 + 20\ T$. The temperature in these relations is expressed in K (Kelvin). Calculate the work done during the process, when the temperature rises from 500 K to 1000 K.*

Solution

Given A closed system:

State 1: $T_1 = 500\ K$,

State 2: $T_2 = 1000\ K$

To find Work transfer during the process

Analysis For any process the work transfer is expressed as

$$W = Q - \Delta E = \int \delta Q - \int dE$$

$$= \int_{T_1}^{T_2} (5000 + 20\ T)\, dT - \int_{T_1}^{T_2} (100 + 50\ T + 0.04\ T^2)\, dT$$

$$= 5000(T_2 - T_1) + 20 \times \frac{(T_2^2 - T_1^2)}{2} - \left[100\,(T_2 - T_1) + 50\,\frac{(T_2^2 - T_1^2)}{2} + 0.04\,\frac{(T_2^3 - T_1^2)}{3}\right]$$

Using the values of T_1 and T_2

$$W = 5000 \times (1000 - 500) + 20 \times \left(\frac{1000^2 - 500^2}{2}\right) - \left[100 \times (1000 - 500) + 50 \times \left(\frac{1000^2 - 500^2}{2}\right) + 0.04 \times \left(\frac{1000^3 - 500^3}{3}\right)\right]$$

$$= 2.5 \times 10^6 + 7.5 \times 10^6 - [50 \times 10^3 + 18.75 \times 10^6 + 11.667 \times 10^6]$$

$$= -20.467 \times 10^6\ J = \mathbf{-20462\ kJ}$$

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Example 2.19 *On a warm summer day, a housewife decides to beat the heat by closing the windows and doors in the kitchen and opening the refrigerator door. At first she feels cool and refreshed but after a while, the effect begins to wear off.*

Evaluate the situation as it relates to the first law of thermodynamics, considering the room including the refrigerator as a system.

Solution Initially, the refrigerator has cold air and when its door is opened, the cold air mixes with room air and makes the room air colder for a while. This makes the lady feel cool and refreshed.

We consider the room with the refrigerator as an isolated system. The refrigerator is operated with electrical work input and then applying first law of thermodynamics for an isolated system

$$Q = \Delta U + W$$

$$O = \Delta U + (-W_e)$$

or $$\Delta U = W_e$$

The internal energy (temperature) of the room will increase on the work input. Hence the cooling effect will wear off.

Summary

- *Energy* is defined as the *capacity to do work. Primary forms of energy* are those that are either found or stored in nature. *Secondary forms of energy* are those forms which are derived from the primary form of energy. *Commercial forms of energy* are available in the market for a definite price. *Non-commercial forms of energy* are not available in the commercial market for a definite price. *Renewable energy* is obtained from sources that are not exhaustible. *Non-renewable energy* is obtained from conventional fossil fuels such as coal, oil and gas, nuclear fuels, and heat traps which are accumulated in the earth crust.
- The energy that a system possesses as a result of its elevation in a gravitational field is called *potential energy PE*, expressed as
$$PE = mgz \text{ (joules)}$$
- The energy that a system possesses as a result of motion relative to some reference is called *kinetic energy KE*.
$$KE = \frac{1}{2} m\mathcal{V}^2 \text{ (joule)}$$
- The sum of all the microscopic forms of energy is called the *internal energy* of the system. The *enthalpy* or *total enthalpy* (H) is the sum of the internal energy U and the product of pressure and volume pV and is expressed as
$$H = U + pV$$
- *Mechanical energy* can be defined as a form of energy that can be converted directly and completely into mechnical work by an ideal mechanical device such as an ideal turbine or pump.
- *Heat energy* is a *transfer form* of energy that flows between two systems (or system and its surroundings) by virtue of the temperature difference between them. It is recognized as it crosses the system boundary.
- *Work transfer* like heat energy is also a form of *energy in transit*. Work is also recognized at the system boundary. The quantity of work transfer per unit time is called *power.*

  The various forms of work are expressed as

  1. Electrical work, $W_E = V I \Delta t$
  2. Mechanical work, $W = \int p dv$
  3. Gravitation work, $\Delta PE = mg(z_2 - z_1)$
  4. Acceleration work,
  $$\Delta KE = (1/2)\, m(\mathcal{V}_2^2 - \mathcal{V}_1^2)$$
  5. Shaft work, $W_s = 2\pi nT$
  6. Spring work,
  $$W_{spring} = (1/2)\, k(x_2^2 - x_1^2)$$
- The first law of thermodynamics states '*energy can be neither created nor destroyed, it can change only its form'*.
- In absence of any work interaction between a system and its surroundings, the amount of net heat transfer is equal to the change in the energy of a system.
$$Q = \Delta E \quad \text{when} \quad W = 0$$

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- For adiabatic processes, the amount of work done is equal to the change in energy of the system.

$$W = -\Delta E \quad \text{when} \quad Q = 0$$

- The conservation of energy principle reveals that

$$Q - W = \Delta E \text{ (kJ)}$$

where the term E is called *total energy* and it is the sum of all the forms of energy of a system. Thus

$$E = PE + KE + IE + \textit{other forms of energy}$$

Glossary

Internal energy Sum of all microscopic forms of energies of a system

Enthalpy Sum of internal energy and product of pressure and volume

Heat A form of energy in transit due to temperature difference

Work Energy transfer associated with force acting through a distance

Thermodynamic work A form of energy in transit due to potential difference other than temperature difference

Power Rate of work transfer per unit time

Specific heat The quantity of heat required to change the temperature of unit mass of substance by one degree Celsius

Latent energy Heat energy associated with phase change of a substance

Review Questions

1. Define energy and its various forms.
2. Define internal energy.
3. Define sensible heat and latent heat with reference to molecular activities.
4. Why is it incorrect to say that a system *contains* heat?
5. Explain the concept of thermodynamic work.
6. Differentiate between mechanical and thermodynamic work.
7. Prove that the work is a function of a process.
8. What are the salient features of work transfer?
9. Define moving boundary work, gravitational work and acceleration work.
10. Define heat energy and specific heat.
11. Write the universally accepted sign convention for heat and work transfer and state the direction of heat transfer and work transfer for each of the following processes:
    (a) Gas in an insulated cylinder expands as the piston is slowly moved outward.
    (b) A closed rigid vessel containing steam at a temperature of 200°C is left in an atmosphere at 20°C.
    (c) The air in a tyre and connecting pump; the pump plunger is pushed down forcing air into the tyre. The tyre, pump and connecting tube are non-conducting.
    (d) The water and steam in a rigid metallic container is heated.
    (e) A viscous fluid is stirred by a paddle wheel in an insulated closed tank.
12. What are the similarities and dissimilarities between heat and work interactions?

Problems

1. A tank which is 4 m long, 3 m wide and 2 m deep is half full of water. How much work is required to raise all the water over the top edge of the tank? *[117.72 kJ]*

2. A rectangular tank measuring 0.6 m × 1 m at the base is filled half to a depth of 15 cm with water. Calculate the following:
    (a) Total gravitational force exerted on the base of the tank

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(b) The pressure exerted by water at the base of the tank *[(a) 882 N, (b) 1.47 kN/m^2]*

3. An engine cylinder has a piston area of 0.1 m^2, and contains gas at a pressure of 1.5 MPa. The gas expands according to a process, which is represented by a straight line on a p–v plane. The final pressure is 0.15 MPa. Calculate the work done on the gas, if the stroke is 0.3 m. *[24.75 kJ]*

4. A fluid system undergoes a non-flow frictionless process. The pressure and volume are related as

$$p = \frac{5}{V} + 1.5$$

where p is in bar and V is in m^3. During the process, the volume changes from 0.15 m^3 to 0.05 m^3. Calculate the work done by the fluid.

[– 564 kJ]

5. A fluid at a pressure of 3 bar and with a specific volume of 0.18 m^3/kg is contained in a cylinder behind a piston. It expands reversibly to a pressure of 0.6 bar according to a law

$$p = \frac{A}{v}$$

where A is a constant. Calculate the work done on the fluid. *[29.84 kJ/kg]*

6. A gas is trapped by a frictionless piston in a vertical cylinder having an inner diameter of 80 cm. The piston is solely supported by gas pressure. The initial gas pressure is 250 kPa. The gas is then heated so that the piston is raised by a distance of 10 cm. Calculate the work done by the gas. How much of this work is done to lift the piston and how much to push back the atmospheric air ? What is the mass of the piston ? Take atmospheric pressure as 100 kPa.

[12.5 kJ, 5kJ, 7.5 kJ, 7706 kg]

7. In a closed system, the gas is compressed frictionlessly from a volume of 0.01 m^3 and a pressure of 0.70 kPa to a volume of 0.025 m^3 in such a manner that $p(v + 0.030)$ = constant, where v is in m^3. Calculate the work done by the gas.

[8.91 J]

8. A gas trapped in a cylinder behind a piston has an initial volume of 0.40 m^3 and its pressure is 159 kPa. It is made to undergo a process, which follows the relationship $(V - 0.2) \times 10^5 = (p - 300)^2$, until the pressure is 400 kPa. Sketch the p–V diagram of this process and calculate the work done. *[9.5 kJ]*

9. In a quasi-static process for a closed system, the gas expands from a volume of 0.15 m^3 and a pressure of 120 kPa to a volume of 0.25 m^3 in such a manner that $p(V + 0.03)$ = constant, where V is in m^3. Calculate the work done. *[9.5 kJ]*

10. Air expands in a cylinder according to the law $pV^{1.4}$ = constant, from an initial volume of 3 m^3 and a pressure of 450 kPa to a final volume of 4 m^3. Compute the work done by the gas.

[368 kJ]

11. A spherical balloon has a diameter D_1, when the pressure of the gas inside it is p_1 and atmospheric pressure p_0. The pressure of the gas inside the balloon is proportional to the diameter. The gas is heated till the volume of the balloon doubles. Calculate the work done by the gas. How much work is done on the atmosphere?

[1.14 p_1V_1, 2 p_oV_1]

12. An electric potential of 100 V is impressed on a certain resistor such that a current of 12 A is drawn. Calculate the energy dissipated in 3 min.

[216 kJ]

13. A certain fluid at 10 bar is contained in a cylinder behind a piston. The initial volume is 0.05 m^3. Calculate the work done by the fluid, when it expands reversibly

(a) At constant pressure to a final volume of 0.2 m^3

(b) According to a linear law to a final volume of 0.2 m^3 and a final pressure of 2 bar

(c) According to the law pV = const. to a final volume of 0.1 m^3

(d) According to the law pV^3 = const. to a final volume of 0.56 m^3

(e) According to the law $p = \frac{A}{V^2} - \frac{B}{V}$ to a final volume of 0.1 m^3 and a final pressure of 1 bar, where A and B are constants. Sketch all processes on p–V diagram.

[(a) 150 kJ, (b) 90 kJ (c) 34.7 kJ (d) 24.8 kJ (e) 19.2 kJ]

14. 0.05 m^3 of a gas at 6.9 bar expands reversibly according to the law pV^2 = const. until the volume is 0.08 m^3. Calculate the work done by the gas and sketch the p–V diagram. *[15.48]*

15. A non-flow reversible process occurs for which pressure and volume are correlated by an expression

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$$p = V^2 + \frac{6}{V}$$

where p is in bar and V is in m^3. What amount of work will be done when the volume changes from 2 to 4 m^3? *[22.83 × 10² kJ]*

16. The properties of a system during a reversible constant pressure non-flow process at 1.8 bar changes from

$v_1 = 0.35$ m^3/kg, $T_1 = 30°C$ to $v_2 = 0.6$ m^3/kg, $T_2 = 270°C$

If $C_p = \left[1.5 + \frac{75}{T + 45}\right]$ kJ/kg·°C, where T is in °C, calculate Q, W, Δu and Δh per kg of mass of system.

[467.63 kJ/kg, 45 kJ/kg, 422.67 kJ/kg, 467.63 kJ/kg]

17. A domestic refrigerator is loaded with fresh food and the door is closed. During a certain period, the machine consumes 1.25 kWh of electrical energy and the internal energy of the food items decreases by 4500 kJ. Calculate the magnitude and direction of heat transfer for the system.

[–9000 kJ]

18. Water is being heated in a closed pan, while being stirred by a paddle wheel. During the process, 40 kJ of heat is added to the water and 8 kJ of heat is lost to the surrounding air. The paddle wheel work amounts to be 500 Nm. Determine the final energy of the system, if its initial energy is 10 kJ.

[42.5 kJ]

19. A closed system undergoes a cycle consisting of two processes. During the first process, 40 kJ of heat is transferred to the system, while the system does 60 kJ of work. During the second process, 45 kJ of work is done on the system.

(a) Calculate the heat transfer during the second process.

(b) Determine the net work and net heat transfer for the cycle.

[(a) –25 kJ, (b) 15 kJ, 15 kJ]

20. A classroom that normally contains 60 students is to be air-conditioned with window air conditioner units of 5 kW rating. A person at rest may be assumed to dissipate heat at a rate of 360 kJ/h. There are 10 tube lights in the room, each of rating 40 W. The rate of heat transferred to the classroom through the walls and windows is estimated to be 12000 kJ/h. If the room air is to be maintained at a constant temperature of 21°C, determine the number of window air-conditioner units required for the classroom. *[2 units]*

21. Fill in the missing data for each of the following processes of a closed system between states 1 and 2.

(Each quantity is expressed in kJ.)

Process	Q	W	E_1	E_2	ΔE
(a)	18	–6		35	
(b)		–10		4	–15
(c)		12	3	32	
(d)	25		14		10

22. A gas in a cylinder fitted with a frictionless piston undergoes a cycle comprising of three processes. First, the gas expands at constant pressure with a heat addition of 400.0 kJ and a work output of 100.0 kJ. Then, it is cooled at a constant volume by removal of 500 kJ of heat. Finally, an adiabatic process brings the system to its initial state. Determine (a) the work of adiabatic process, and (b) the internal energy of the gas at each of two other states, if initial internal energy of the system is 150.0 kJ. *[(a) –200 kJ, (b) 450 kJ, –50 kJ]*

23. A stone of 25-kg mass is dropped in a tank containing 225 kg of water. Initially, the stone is 20 m above the water level and stone and water are in thermal equilibrium. Determine change in internal energy, change in potential energy, change in kinetic energy, heat transfer and work transfer; when

(a) the stone is about to enter the water,

(b) the stone has come to rest in the tank, and

(c) the heat is transferred to surroundings in order to restore the stone and water to their initial temperature.

[(a) $\Delta U = 0$, $W = 0$, $Q = 0$, $\Delta PE = -4905$ J, $\Delta KE = 4905$ J, (b) $\Delta KE = 0$, $W = 0$, $Q = 0$, $\Delta PE = -4905$ J, $\Delta U = 4905$ J, (c) $\Delta KE = 0$, $W = 0$, $Q = -4905$ J, $\Delta PE = 0$, $\Delta U = 4905$ J]

24. The internal energy of system is given by $U = 100 + 50T + 0.04T^2$ joules and the heat transfer $Q = 4000 + 16T$ joules, where T is the temperature in kelvin. If the temperature of the system changes

from 300 K to 500 K, calculate the work transfer and its direction.

[–13.2 kJ, Work is done on the system]

25. 1.5 kg of a gas at 10 bar pressure undergoes a quasi-static expansion according to $p = a + bv$, where a and b are constants, the final pressure being 2 bar. The initial and final volumes are 0.2 m^3 and 1.2 m^3, respectively. The specific internal energy varies according to $u = (1.5pv - 85)$ kJ/kg, where p is in kPa and v in m^3/kg. Calculate the net work and heat transfer.

[900 kJ, 90 J]

Objective Questions

1. The unit of work in SI units is
   (a) joule (b) newton
   (c) watt (d) calorie
2. The unit of heat in SI units is
   (a) joule (b) newton
   (c) watt (d) calorie
3. The unit of power in SI units is
   (a) joule (b) newton
   (c) watt (d) calorie
4. The unit of energy in SI unit is
   (a) joule (b) newton
   (c) watt (d) calorie
5. During a heating process, the temperature of an object rises by 20°C. This temperature rise is equivalent to
   (a) 20°F (b) 52°F
   (c) 36°F (d) 68°F
6. During a cooling process, the decrease of temperature of an object is 15°C. This temperature change on the Kelvin scale is
   (a) 15 K (b) 298 K
   (c) 258 K (d) 59 K
7. The first law of thermodynamics was given by
   (a) Joule (b) Charles
   (c) Wilson (d) James Watt
8. The first law of thermodynamics deals with
   (a) heat and work
   (b) quality of energy
   (c) balance of qunatity of energy
   (d) measurement of energy transfer
9. Joule's experiment states that for a cycle
   (a) change of pressure is proportional to temperature change
   (b) change of volume is proportional to temperature change
   (c) change of internal energy is proportional to temperature change
   (d) sum of all heat transfers is proportional to sum of all work transfers
10. The law of thermodynamics, which states that heat and work are mutually convertible is known as
    (a) zeroth law of thermodynamics
    (b) first law of thermodynamics
    (c) second law of thermodynamics
    (d) third law of thermodynamics
11. The perpetual motion machine of the first kind is impossible according to the
    (a) zeroth law of thermodynamics
    (b) first law of thermodynamics
    (c) second law of thermodynamic
    (d) third law of thermodynamics
12. The heat supplied to a system is considered
    (a) negative (b) positive
    (c) zero, (d) none of the above

Answers

1. (a) 2. (a) 3. (c) 4. (a) 5. (b) 6. (a) 7. (a) 8. (c)
9. (d) 10. (b) 11. (b) 12. (b)

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3

Working Substances

Introduction

Thermodynamic analyses require knowledge of properties of the working substance. Steam and ideal gas are two important working substances discussed in this chapter. In the first part of the chapter, the concept of pure substance and physics of phase-change phenomenon are discussed with the help of various property diagrams and *p-v-T* surface of pure substance. The formats and use of steam tables are demonstrated. In the second part of the chapter, ideal gas, ideal gas laws, equation of state, characteristic gas equation and specific gas constant are incorporated. Finally, specific heats are defined and relations are obtained between two specific heats and for a change in internal energy and enthalpy in terms of specific heat and temperature. Real gases, compressibility factor and other equations of state are discussed at the end of the chapter.

3.1 PURE SUBSTANCE

A pure substance may be defined as a substance, which is chemically homogeneous and has a fixed chemical composition. In general, any substance that appears with invariable chemical composition in either phase or a combination of phases may be treated as a pure substance. Water, nitrogen, helium, carbon dioxide, etc., are pure substances. A system composed of liquid and vapour phases of water (H_2O) is a pure substance, because in either state, the system will always be chemically homogeneous without any change in its chemical composition.

A mixture may also be a pure substance. For example, dry air is mainly a mixture of oxygen and nitrogen. In its gaseous phase, it may be heated, cooled, compressed or expanded with no change in its chemical composition. However, if it is liquefied, it is no longer a pure substance, because liquid air will contain a higher fraction of nitrogen than in the original mixture in the gaseous phase. Similarly, if air contains moisture, and it is evaporated on heating or condensed on cooling, its chemical composition will change, it will be no longer a pure substance. A mixture of carbon monoxide and oxygen is a pure substance as long as they do not react to form CO_2.

3.2 PHASES OF A PURE SUBSTANCE

A *phase* is the homogenous part of a system that is physically distinct. There are three principle phases of a substance: *solid*, *liquid* and *gaseous*. A substance may have several phases. Within the principle phase, each phase has a different molecular structure. For example, carbon may exist as graphite or diamond in solid phase. Pure sulphur and iron, each has three solid phases. Helium has two liquid phases.

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A substance in each phase has a definite molecular structure different from an other phase. It will be helpful to have some understanding of the molecular arrangement involved in each phase.

In a *solid phase*, the molecules are arranged in a three-dimensional lattice,which is repeated throughout its volume. The attraction forces of molecules on each other are very strong and thus, keep the molecules in a fixed position within the solid as shown in Fig. 3.1.

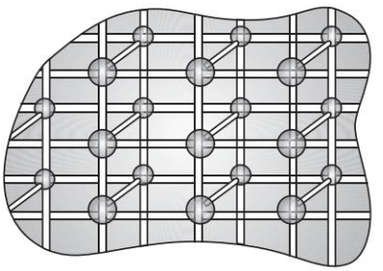
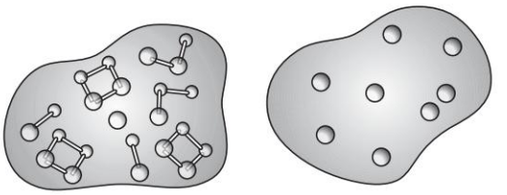


Fig. 3.1 *Molecular pattern in solids*

The *molecules in a solid phase* cannot move relative to each other, they can only oscillate about their equilibrium position. The velocity of molecules during these oscillations depends on temperature. When a solid is subjected to a very high temperature, the velocity of molecules may reach a critical point, where the intermolecular forces become partially weak and a group of molecules break away. It is the beginning of the melting process.

In the *liquid phase*, the molecular spacing is similar to that of the solid phase with relatively weaker molecular forces of attraction. In liquid phase, the molecules do not confine a fixed position relating to each other, therefore, groups of molecules can float easily.

The distances between molecules in the liquid phase are slightly greater than the distances between molecules in the solid phase of the same substance. Water being the only exceptional case, which expands on freezing, and its molecular spacing becomes slightly greater in the solid phase than in the liquid phase.



(a) Molecular pattern in a liquids *(b) Molecular pattern in a gases*

Fig. 3.2 *Molecular pattern in liquids and gases*

In the *gaseous phase,* the molecules are far apart from each other without any molecular pattern and force of attraction. The molecules of a gas are in continuous motion, colliding with each other and rebounding to move in a new direction.

Molecules in the gaseous phase possess a considerable higher energy than they do in solid or liquid phase. Therefore, a gas releases a large amount of heat before it condenses or freezes.

3.3 PHASE-CHANGE PHENOMENON OF A PURE SUBSTANCE

Consider a cylinder fitted with a frictionless piston, which may be loaded to any desired pressure. The cylinder contains 1 kg of ice at –10°C under 1 atm pressure at the state A as shown in Fig. 3.3. The stages of heat addition are illustrated in Fig. 3.4.

Process A–B When any amount of heat is added to ice, it gets warmer and its temperature rises till it approaches 0.01°C (generally referred as 0°C) as shown by the line *AB* in Fig. 3.4.

Process B–C The following facts are observed during this process:

(i) Ice begins to melt at 0°C and a two-phase mixture is formed.

(ii) The temperature of the two-phase mixture (M_1) of ice and water does not change on heat addition as it is shown by the line *BC*.

(iii) There is slight decrease in volume because the liquid water at 0°C is heavier than ice.

(iv) At the point *C,* all ice melts to water without change in pressure (1 atm) and temperature (0°C).

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This phase transformation from solid to liquid is called the *melting* or *fusion of ice*.

Process C–D When heat is added to liquid water, the following facts are observed:

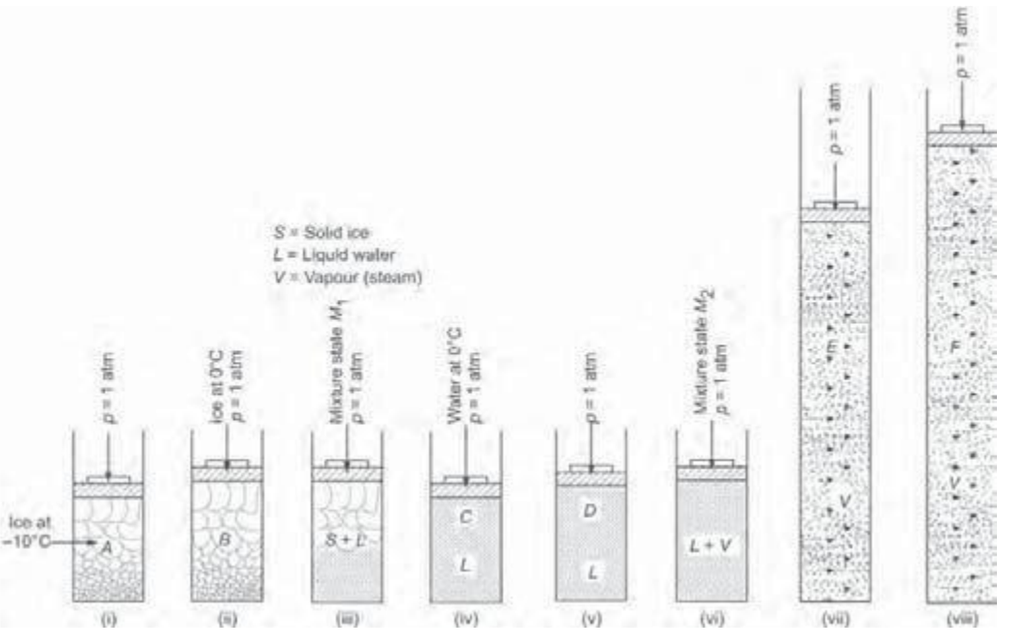

Fig. 3.3 *Water in different thermodynamic states*

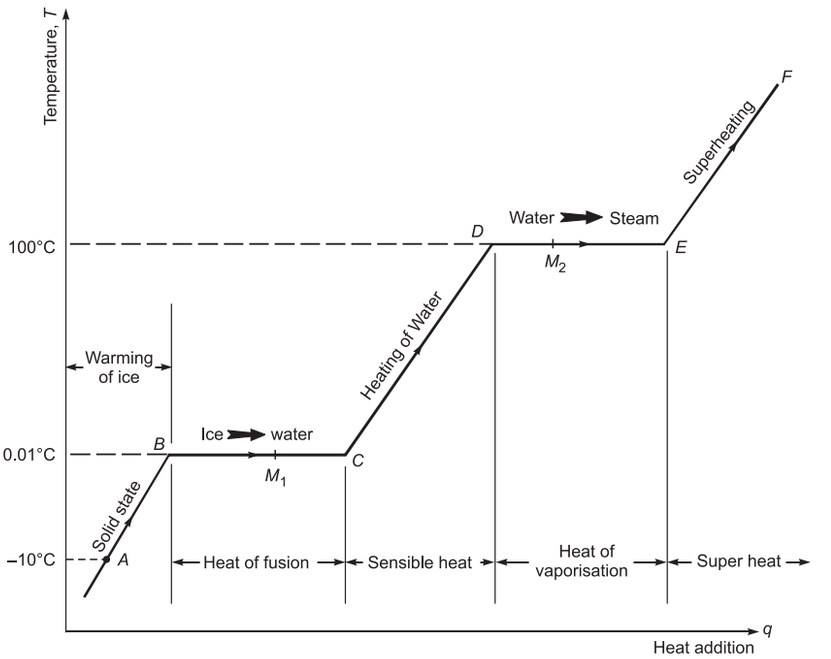

Fig. 3.4 *Phase-change process of water*

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(i) The temperature of water rises with heat supply and keeps on rising until it reaches *boiling point temperature,* the point *D*.

(ii) There is decrease in the specific volume of water, when its temperature rises from 0°C to 4°C and thereafter, the specific volume increases with temperature rise till it reaches the saturation temperature. The piston moves up slightly during this process as shown in Fig. 3.3. (*v*).

(iii) The pressure in the cylinder remains constant at 1 atm.

Process D–E After water reaches saturation temperature (i.e., 100°C at 1 atm), any addition of heat will cause some liquid to vaporise at the same temperature. This is again a phase-change process from saturated water to saturated vapour. During this phase-change process, the following facts are observed:

(i) There exists a two-phase mixture M_2 of water and vapour, called the *wet steam*.

(ii) The temperature of the mixture remains constant until all water does not convert into vapour (steam).

(iii) The process of phase change takes place at constant temperature and constant pressure.

(iv) The specific volume of vapour is considerable larger than that of saturated water as shown in Fig. 3.3 (*vii*).

At the state *E*, all the water has been vaporised and this state of steam is called *dry and saturated steam*. The phase change from liquid to vapour is called *vaporisation.*

Process E–F Once the steam becomes dry and saturated, it bahaves as an ideal gas and its temperature and volume start increasing with further supply of heat. This steam is called *superheated steam.*

3.3.1 Saturation Pressure and Saturation Temperature

At normal atmospheric pressure, the boiling of water and condensation of steam begin at 100°C. Similarly, at higher pressures, the boiling and condensation will take place at higher temperatures; and at lower pressures, these will take place at lower temperatures. The temperature at which the boiling and condensation of a fluid take place is known as *saturation temperature*, denoted as T_{sat}. The corresponding pressure of fluid is called the *saturation pressure*, denoted as p_{sat}. For a given pressure, there is a fixed value of saturation temperature. Figure 3.5 shows the variation of saturation temperature with pressure of fluid.

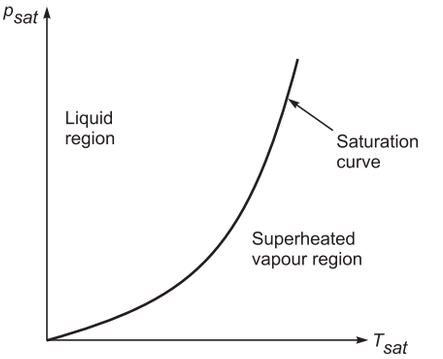

Fig. 3.5 *Liquid–vapour saturation curve for water*

3.3.2 Compressed Liquid and Saturated Liquid

Liquid existing at a temperature lower than saturation temperature is called *compressed liquid* or sub cooled liquid, because pressure of liquid is higher than its saturation pressure and this liquid is not about to vaporise. The temperature by which it is lower than the saturation temperature is known as *degree of subcooling*, that is

$$\text{Degree of subcooling} = T_{sat} - T_i \qquad ...(3.1)$$

where T_i is the initial temperature of fluid. The liquid at saturation temperature at a given pressure is about to vaporise, and is thus called *saturated liquid*.

3.4 TERMINOLOGY OF PURE SUBSTANCES

1. Vaporisation It is the process that involves change of phase from liquid to vapour, when the

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latent heat of phase change is supplied to saturated water.

2. Evaporation It is the process of vapour generation only at the free surface of the liquid. The molecules at the free surface extract their latent heat of phase transformation from the surrounding medium and break away as vapour from the liquid surface and escape to the surrounding atmosphere.

3. Boiling It is the phenomenon of vapour formation in the whole mass of liquid, when heat is supplied.

4. Saturation Temperature T_{sat} It is the temperature at which a pure substance starts to evaporate at a given pressure.

5. Saturation Pressure p_{sat} It is the pressure at which a pure substance starts to evaporate at a given temperature.

6. Steam It is the gases phase of water.

7. Saturated Steam The steam is about to condense.

8. Wet Steam It is the mixture of dry steam and water particles as moisture.

9. Dryness Fraction It is a measure of quality of wet steam.

10. Dry Saturated Steam Saturated vapour, which is free from moisture.

11. Superheated Steam The steam existing at higher temperature than its saturation temperature.

12. Degree of Superheat It is the temperature rise of superheated steam above its saturation temperature.

13. Critical Point A locus on the saturation curve, where saturation liquid line and saturated vapour line meet.

14. Triple Point A locus on the *p–T* diagram, where all three phases of water coexist.

15. Sublimation Transformation of solid phase directly into vapour phase is called sublimation.

16. Sublimation Line It is the solid vapour saturation line on the *p–T* diagram.

3.5 PROPERTY DIAGRAMS

The property diagrams are useful in the study of variation of properties during phase change process. We will discuss *T–v*, *p–v* and *p–T* diagrams for pure substances.

3.5.1 The *T–v* Diagram

The phase-change phenomenon of water (a pure substance) at 1 atm pressure is discussed in detail in the last section and plotted on the *T–q* diagram in Fig. 3.4. The *T–v* diagram is very similar to this diagram from the state *C* to *F* for liquid and vapour states as shown below in Fig. 3.6.

We repeat the process of phase transformation from liquid water to vapour at different pressures by adding different weights on the top of the piston. At new pressures, the paths followed by liquid to vapour phase change, (shown in Fig. 3.7), are very similar to the path at 1 atm pressure with following observations:

(i) as pressure increases, the saturation temperature T_{sat} for boiling and condensation also increases.

(ii) As pressure increases, the specific volume of saturated liquid increases slightly, while the specific volume for saturated vapour decreases considerably, thus the saturation line on the *T–v* diagram will continue to get shorter.

(iii) As pressure increases, the temperature for melting and freezing is slightly decreased.

The *T–v* diagram of Fig. 3.8 is constructed from Fig. 3.7 with following considerations:

(i) All saturated liquid states are connected by a solid line, called the *saturated liquid line.*

(ii) All saturated vapour states are connected by another solid line, called the *saturated vapour line*.

(iii) The saturated liquid line and saturated vapour line meet at the **critical point** and form a dome as shown in Fig. 3.8.

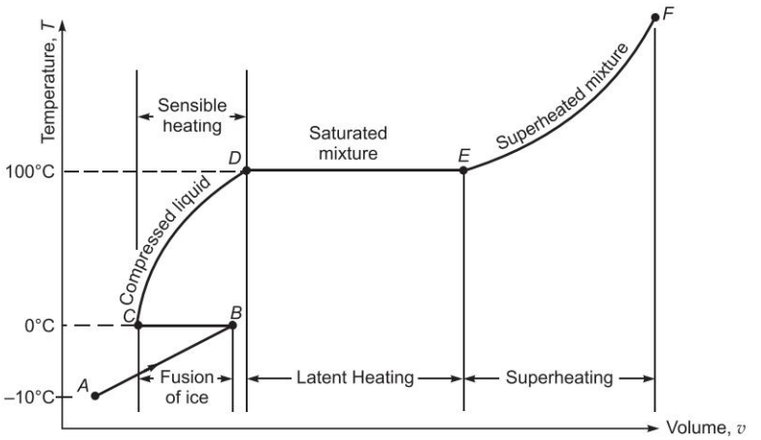

Fig. 3.6 *T–v diagram for water at 1 atm*

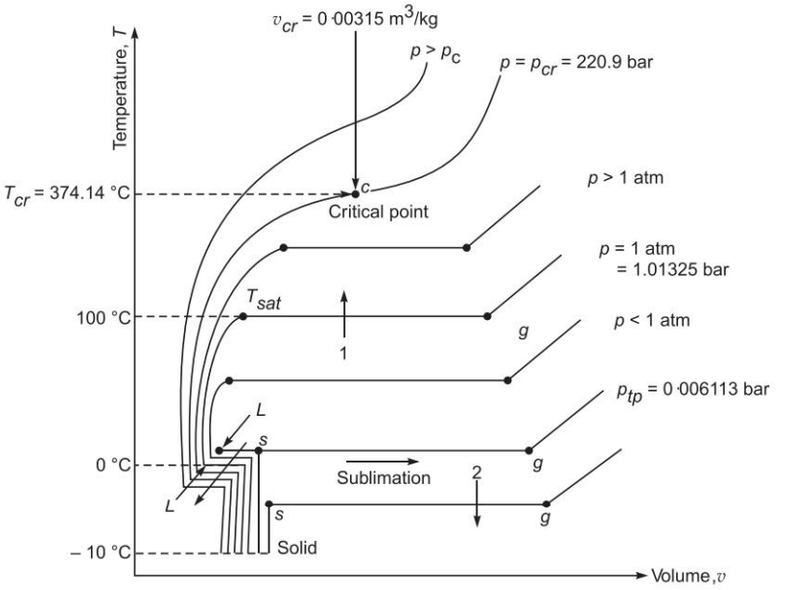

Fig. 3.7 *Changes of state of water at different pressures on the T–v diagram (in Fig. L = water, s and s′ = ice, g = vapour)*

(iv) The region located left to the saturated liquid line is called the *compressed liquid region*.

(v) The region located right to the saturated vapour line is called the *superheated vapour region*.

(vi) The substance can exist in the single phase only in the compressed liquid or superheated vapour region as a liquid or a vapour.

(vii) The region under the dome involves equilibrium between saturated liquid and saturated vapour, and is called the *wet vapour region*.

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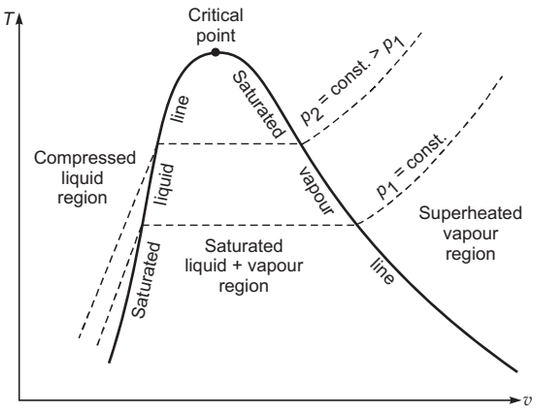

Fig. 3.8 *T–v diagram of a pure substance*

3.5.2 The *p–v* Diagram

The overall shape of a *p–v* diagram of a pure substance is very similar to a *T–v* diagram, except that the constant temperature lines on this diagram have a downward trend. A pressure-specific volume (*p–v*) diagram for water is shown in Fig. 3.9.

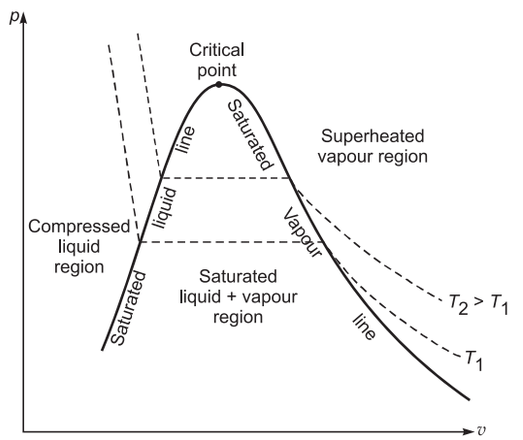

Fig. 3.9 *p–v diagram for water*

It is evident from a *p–v* diagram, as pressure of a pure substance decreases at constant temperature, the specific volume of liquid increases marginally, but the specific volume of vapour increases considerably.

The *T–v* and *p–v* diagrams discussed above represent the equilibrium states involving the liquid and vapour phases only. These diagrams can also be extended to include the solid phase as well as solid–liquid and solid–vapour saturation regions as shown in Fig.3.10. All the three phases of a pure substance exist along a line, called the *triple line.* Along the triple line a substance has the same pressure and temperature, but different specific volumes. Since the water expands on freezing, therefore, a portion of the triple line is extended towards the left to the saturated liquid line.

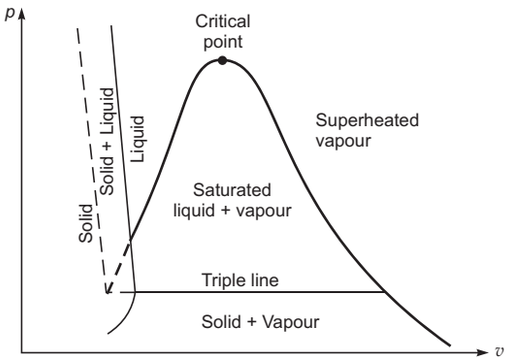

Fig. 3.10 *p–v diagram for water including triple line*

3.5.3 The *p–T* Diagram

The *p–T* diagram of a pure substance is generally called the *phase diagram*, since it shows solid, liquid and vapour regions of a pure substance simultaneously. Its salient features are the following:

(i) Each single phase of a pure substance is separated by saturation lines. The *sublimation line* separates the solid and vapour regions, the *vaporisation line* separates the liquid and vapour regions, and the *fusion line* separates the solid and liquid regions.

(ii) The slope of the fusion line is negative. It indicates that the melting point of ice decreases with increasing pressure.

(iii) The sublimation, fusion and vaporisation lines meet at point, called *triple point* . Thus the triple point can be defined as a locus where all three phases of a substance coexist.

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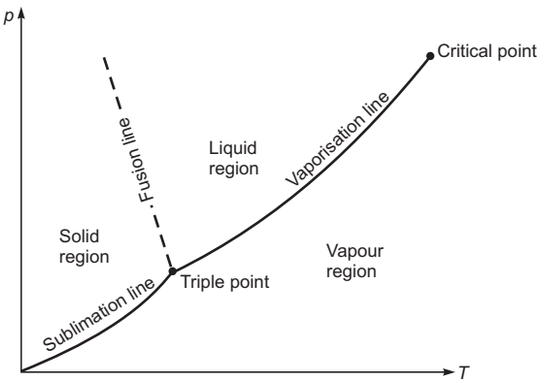

Fig. 3.11 *Phase (p–T) diagram for water*

The phase (p–T) diagram of water for some typical pressure and temperature values is shown in Fig. 3.12.

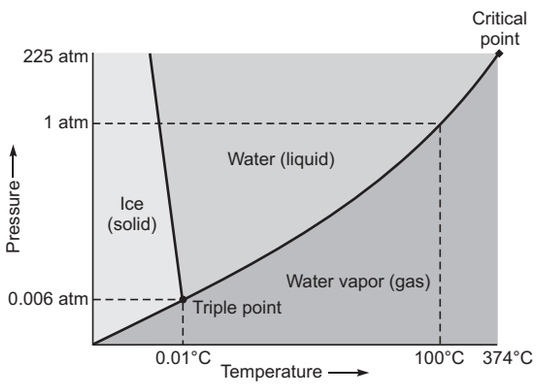

Fig. 3.12 *Phase (p–T) diagram for typical pressure and temperature of water*

3.6 THE *p–v–T* SURFACE

The relationship among pressure, specific volume and temperature of a pure substance can be better understood by the three-dimensional p–v–T diagram.

Figure 3.13 shows a surface p–v–T plot for water, which expands on freezing. It shows p–v, p–T and T–v diagrams simultaneously on a three-dimensional plot. A constant temperature line drawn in the figure passing through the critical point is called **critical isotherm.**

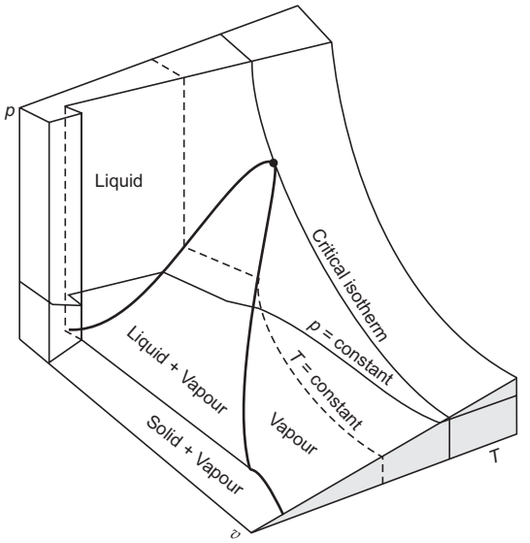

Fig. 3.13 *p–v–T surface for water*

3.7 CRITICAL POINT AND TRIPLE POINT

3.7.1 Critical Point

If the pressure of liquid water is continuously increased to such a state where the saturated liquid state and the saturated vapour state become identical then the saturation line takes the form of a point as shown in Figs. 3.7–3.10. This point is called the *critical point*. Thus, the critical point represents the maximum boiling point temperature and maximum saturation pressure for a single-phase system and above this temperature, the pure substance cannot be liquefied. At the critical point, the *latent heat* of vaporisation of the liquid becomes zero and the liquid directly changes to vapour.

The pressure, temperature and specific volume of a pure substance at the critical point are called *critical pressures* p_{cr}, *critical temperature* T_{cr} and *critical specific volume* v_{cr}, respectively. The critical parameters for water are

$$\text{pressure, } p_{cr} = 22.09 \text{ MPa}$$

$$\text{temperature, } T_{cr} = 374.14 \text{ °C}$$

$$\text{specific volume, } v_{cr} = 0.003155 \text{ m}^3/\text{kg}$$

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3.7.2 Triple Point

The intersection point of the sublimation line, fusion line and vaporisation line on the phase diagram (p–T) represents a point, where all three phases coexist in equilibrium as shown in Fig. 3.11. This point is called *triple point*. On other property diagrams (p–v and T–v diagrams), such a condition is represented by a line called **triple line** as shown in Fig. 3.10.

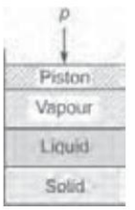

Fig. 3.14 *At triple point, all three phases of water coexist*

The triple point and critical point temperatures and pressures for various substances are given in Table 3.1 below:

Table 3.1 *Triple-point and critical-point data*

	Triple point		*Critical point*	
Substance	*Pressure* p_{tp}, *kPa*	*Temperature* T_{tp}, *K*	*Pressure* p_{cr}, *MPa*	*Temperature* T_{cr}, *K*
Hydrogen	7.000	14.00	5.30	60
Nitrogen	12.500	63.00	3.40	126
Oxygen	0.150	54.00	5.00	154
Water	0.611	273.16 (0.01°C)	22.09	647.29 (374.14°C)

3.8 *T–s* AND *h–s* DIAGRAMS

3.8.1 Temperature–Entropy Diagram

The temperature–entropy (T–s) diagram of a pure substance is shown in Fig. 3.15 with the following observations:

1. The absolute temperature data is plotted along the ordinate, and the specific entropy data is plotted along the abscissa.
2. The value of specific entropy at triple point is zero, and thus the saturated liquid line originates at a temperature of 273.16 K.

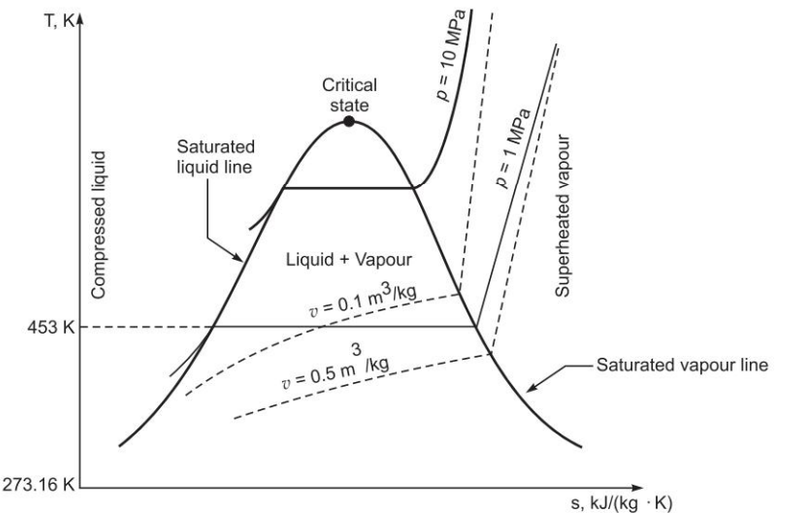

Fig. 3.15 *Schematic of a T–s diagram for water*

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3. The saturated liquid line and saturated vapour line divide the diagram into three regions, i.e., compressed liquid region left to the saturated liquid line, superheated vapour region right to the saturated vapour line and the wet vapour region between these two lines. The two saturation lines meet at the *critical point*.
4. In the compressed liquid region, the constant-pressure lines almost coincide with the saturated liquid line.
5. In the saturated liquid–vapour mixture region, the constant pressure lines and constant temperature lines are horizontal and parallel to each other.
6. In the superheated vapour region, the constant volume lines are steeper than the constant pressure lines.

3.8.2 Enthalpy–Entropy (Mollier) diagram

The enthalpy–entropy diagram is referred as *Mollier diagram*. It is most commonly used to obtain the properties of steam with reasonable accuracy, while analysing the steady flow devices such as a steam turbines, nozzles, etc. The use of a Mollier chart eliminates the complex calculation work and it is also convenient to use.

In the enthalpy–entropy chart, the enthalpy is plotted against entropy. The *h–s* chart covers a pressure range from 0.01 bar to 1000 bar and a temperature up to 800°C. The lines of constant dryness fraction are drawn in the wet region to the value less than 0.5. The lines of constant temperature are drawn in the superheated region. The *h–s* chart does not show the value of specific enthalpy, specific entropy and specific volume for saturated water at pressures which are generally associated with a steam condenser. Hence, this chart is only useful during the expansion process of a steam cycle.

A schematic for *h–s* diagram is shown in Fig. 3.16. The constant pressure lines are indicated by p_1, p_2, p_3 …, etc., the constant temperature lines by T_1, T_2, T_3 …, etc., the constant dryness fraction

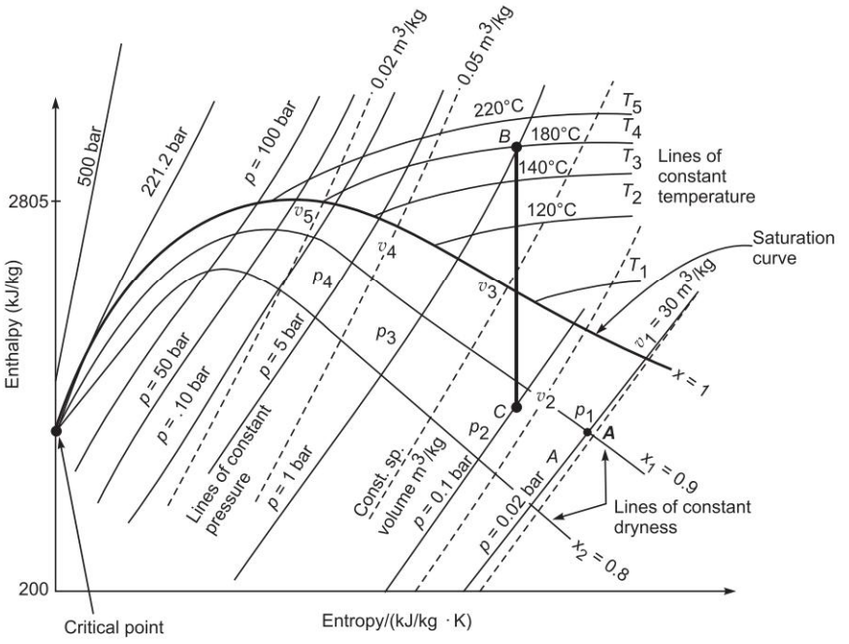

Fig. 3.16 *Schematic of an h–s diagram for water (Mollier diagram)*

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lines by x_1, x_2, x_3 …, etc., and the constant volume lines are drawn dotted lines as indicated by v_1, v_2, v_3 …, etc.

Any two independent properties which appear on the chart are sufficient to define the state of steam. In the wet region, the pressure p_1 and the dryness fraction x_1 define the state A. In the superheated region, the pressure p_3 and the temperature T_4 define the state B. A vertcal line BC of constant entropy can easily be drawn between the pressure p_3 and the pressure p_2 to obtain state C. It consists of the following features:

1. The constant temperature lines are straight and almost horizontal in the superheated vapour region specially at low pressures.
2. The lines of constant dryness fraction (x) are also shown in the diagram, which are parallel to the saturation line.
3. The constant-pressure lines do not change their shape in either region because for a constant-pressure process; $\frac{dh}{ds} = \frac{\delta q}{ds} = T$.
4. The constant-volume lines are steeper than constant-pressure lines.
5. The compressed-liquid region is not shown in the diagram.

3.9 ENTHALPY CHANGES DURING FORMATION OF STEAM

Figure 3.4 illustrates the formation of steam from –10°C ice to superheated steam. The heat is supplied at constant pressure; thus it can be treated as change in enthalpy. During the three stages of phase change, the enthalpy changes are as follows:

1. Change of enthalpy from −10°C ice to 0°C ice (*process A–B*)

$$\Delta h_{A-B} = h_{ice} = C_{p,ice}\,(0 - T_{ice}) \text{ (kJ/kg)} \qquad ...(3.2)$$

where

$C_{p,\,ice}$ = the specific heat of ice taken as 2.1 kJ/kg·K,

T_{ice} = initial temperature of ice.

$\therefore \qquad h_{ice} = 2.1 \times [0 - (-10)] = 21$ kJ/kg

2. Change in enthalpy during fusion of ice from and at 0°C (*process B–C*)

h_{fusion} = Latent heat of fusion of ice = 334.5 kJ/kg.

3. Change in enthalpy during heating of water from 0°C to saturation temperature (*process C–D*)

$$\Delta h_{C-D} = h_f = C_{pw}(T_{sat} - 0) \text{ (kJ/kg)} \qquad ...(3.3)$$

where C_{pw} = 4.187 kJ/kg·K, as the specific heat of water. This enthalpy h_f for heating of water from 0°C to its saturation temperature at a given pressure or temperature can directly be obtained from steam tables.

If water is at the initial temperature of T_i other than 0°C, then the enthalpy of water is calculated as

$$h_f = C_{pw}(T_{sat} - T_i) \text{ (kJ/kg)} \qquad ...(3.4)$$

4. Change in enthalpy during vaporisation of water from and at saturation temperature (*process D – E*)

$\Delta h_{D-E} = h_{fg}$

= Latent heat of vaporisation of water in kJ/kg can be obtained from steam tables.

At normal atmopsheric pressure,

T_{sat} = 100°C; h_{fg} = 2257 kJ/kg

5. Change in enthalpy during superheating of vapour from saturation temperature to some higher temperature (*process E–F*)

$$\Delta h_{E-F} = C_{ps}(T_{sup} - T_{sat}) \text{ (kJ/kg)} \qquad ...(3.5)$$

where C_{ps} = the specific heat of superheated steam, taken as 2.0 to 2.3 kJ/kg·K,

$T_{sup} - T_{sat}$ = *degree of superheat,* i.e., the temperature rise above saturation temperature in K (or °C).

It is observed that the phase change of water from ice to superheated steam involves two types of pattern of changes, when heat is supplied at constant pressure.

1. Temperature changes when phase does not change.
2. Temperature remains constant, when phase changes.

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Further, as a result of heat input, the molecular arrangement of the working substance (water) also changes from strong forces of attraction between molecules in solid phase to free molecules in gaseous phase. These inputs can also be defined as follows:

3.9.1 Latent Heat of Fusion

The *latent heat of fusion* is defined as *the quantity of heat, required to convert one kg of ice into water at constant temperature (0°C).* Its value is taken as 334.5 kJ/kg.

The amount of heat added during the fusion process is used to break up strong molecular bonds in the solid phase into relatively weaker bonds in the liquid phase and giving them a considerable amount of energy for their movement. No temperature rise is recorded during this process. The specific heat during fusion is treated as infinity.

3.9.2 Enthalpy of Saturated Water or Sensible Heat (h_f)

Sensible heat or *enthalpy of saturated water* is defined as an *amount of heat energy absorbed by 1 kg of water during its heating from 0°C to the saturation temperature* (T_{sat}) at a given pressure. It is designated as h_f in the steam tables and can directly be obtained from steam tables.

3.9.3 Enthalpy or Latent Heat of Vaporisation (h_{fg})

It is defined as an *amount of heat energy requied to convert 1 kg of saturated water into dry and saturated steam keeping the temperature and pressure constant.* The magnitude of enthalpy of vaporisation or latent heat decreases as the pressure of water increases and becomes zero at the critical point. It is denoted by h_{fg} and is directly obtained from steam tables.

The heat added during vaporisation is also used to break up molecular bonds in liquid phase, make the molecules free and give them a considerable amount of energy. No temperature rise is recorded during this process. Therefore, the specific heat of the water during vapourisation is also assumed infinity.

3.9.4 Enthalpy of Steam or Total heat (h_g)

It is the sum of *enthalpy of saturated water* and *enthalpy of vaporisation.* It may be defined as *the amount of heat required to convert 1 kg of water at 0°C into dry and saturated steam at a given pressure*. It is designated as h_g and

$$h_g = h_f + h_{fg} \text{ (kJ/kg)} \qquad ...(3.6)$$

3.9.5 Enthalpy of Superheated Steam (h_{sup})

It can be defined as the *amount of heat required to convert 1 kg of water at 0°C into superheated steam at constant pressure.* Normally, it is the sum of enthalpy of dry and saturated steam and heat supplied during superheating of steam. Thus

$$h_{sup} = h_g + C_{ps}(T_{sup} - T_{sat}) \text{ (kJ/kg)} \qquad ...(3.7)$$

3.10 WET STEAM

When saturated steam contains saturated water particles evenly distributed in saturated vapour, it is called the *wet steam*. The wet steam is charecterised by its *dryness fraction*.

3.10.1 Dryness Fraction

The vapour fraction in the wet steam is considered dryness fraction and designated as x. The *dryness fraction* of steam is defined as *ratio of mass of actual dry and saturated vapour to total mass of steam considered*.

If m_g is the mass of dry and saturated steam and m_w is the mass of saturated water in suspension in steam considered then dryness fraction x is expressed as

$$\text{dryness fraction, } x = \frac{m_g}{m_g + m_w} \qquad ...(3.8)$$

Wetness fraction is another term associated with wet steam. It is defined as the *ratio of mass*

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of moisture in suspension with total mass of steam, which contains it. It is denoted by y and expressed as

$$\text{Wetness fraction, } y = \frac{m_w}{m_g + m_w} = 1 - \frac{m_g}{m_g + m_w}$$
$$= 1 - x \qquad ...(3.9a)$$

3.10.2 Priming

It is the representation of wetness fraction in percentage.

$$\text{Priming} = (1 - x) \times 100 \qquad ...(3.9b)$$

3.10.3 Enthalpy of Wet Steam

When steam is not completely dry then its enthalpy of vaporisation is less than that of dry and saturated steam and it cannot be obtained from steam tables. Enthalpy of wet steam is calculated as

Enthalpy of wet steam = Enthalpy of saturated water + dryness fraction × Enthalpy of vaporisation

or $$h_{wet} = h_f + x\, h_{fg} \text{ (kJ/kg)} \qquad ...(3.10)$$

3.11 SUPERHEATED STEAM

3.11.1 Superheating

When water is completely converted into steam, the resulting steam is called *dry and saturated steam*. When the dry and saturated steam is further heated, its temperature increases with corresponding increase in its specific volume. This steam is called *superheated steam* and this process is called *superheating*. The superheating is carried out at constant pressure. The additional amount of heat supplied to steam during superheating is called *superheat,* and the temperature rise of steam above saturation temperature is called *degree of superheat*.

Heat added during superheating

$$q_{superheating} = C_{ps}\,(T_{sup} - T_{sat}) \text{ (kJ/kg)} \qquad ...(3.11)$$

where

C_{ps} = specific heat of superheated steam, which varies from 2.0 to 2.3 kJ/kg · K.

$(T_{sup} - T_{sat})$ = temperature rise is called *degree of superheat*.

Then total heat of superheated steam

$$h_{sup} = h_f + h_{fg} + C_{ps}(T_{sup} - T_{sat}) \text{ kJ/kg}$$
$$= h_g + C_{ps}(T_{sup} - T_{sat}) \text{ (kJ/kg)} \qquad ...(3.12)$$

The superheated steam behaves like a perfect gas and therefore, it follows the gas law pv^n = constant. The value of n (index) for superheated steam is generally assumed as 1.3.

3.11.2 Advantages of Superheating the Steam

All power plants as well as industries use superheated steam. The advantages of the use of superheated steam are the following:

1. The heat content of superheated steam is large and it does more work without increasing pressure drop.
2. Due to large heat content of superheated steam, the steam consumption for a given output is less. It results into reduction in size and weight of the system.
3. The high temperature of superheated steam always results in improved thermal efficiency.
4. Since superheated steam is free from moisture, its use in an apparatus for curing, drying, etc., avoids corrosion and erosion of the surfaces.
5. When superheated steam is used in a steam engine or steam turbine, it can be expanded considerably to do work before it becomes wet.

3.12 SPECIFIC VOLUME OF STEAM

The volume occupied by unit mass of a fluid is called **specific volume of fluid**. It is expressed in cubic metre per kg of the fluid and it is denoted by v.

For dry and saturated steam, the specific volume is designated as v_g and the specific volume during

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evaporation is designated as v_{fg}. The specific volume of steam decreases with increase in pressure and it can directly be obtained from steam tables corresponding to saturation pressure or saturation temperature.

3.12.1 Specific Volume of Wet Steam

The wet steam is a mixture of dry vapour and moisture. Its specific volume is the sum of moisture volume and change of volume during evaporation. If 1 kg of wet steam has a dryness fraction x then

$$v_{wet} = v_f + x\, v_{fg}$$
$$= v_f + x(v_g - v_f)$$

where

v_g = specific volume of dry and saturated steam

v_f = specific volume of moisture (water)

v_{fg} = specific volume change of steam during evaporation = $v_g - v_f$

$$\therefore \quad v_{wet} = x\, v_g + (1 - x)\, v_f \qquad ...(3.13)$$

It is noticed that the volume of moisture at low pressures is very small, and is generally dropped from the expression. Thus

$$v_{wet} \approx x\, v_g \qquad ...(3.14)$$

3.12.2 Specific Volume of Superheated Steam

Since superheated steam behaves like a perfect gas and superheating is carried at constant pressure, thus it follows the property relation

$$\frac{v_g}{T_{sat}} = \frac{v_{sup}}{T_{sup}}$$

or
$$v_{sup} = v_g \times \frac{T_{sup}}{T_{sat}}\ (\text{m}^3/\text{kg}) \qquad ...(3.15)$$

where

T_{sup} = Temperature of superheated steam in K,

T_{sat} = Temperature of dry and saturated steam in K.

3.13 ENTROPY OF A PURE SUBSTANCE

It is calculated as $\int \frac{\delta q}{T}$ between two states. The temperature–entropy diagram in Fig. 3.17 shows the changes in entropy of a pure substance.

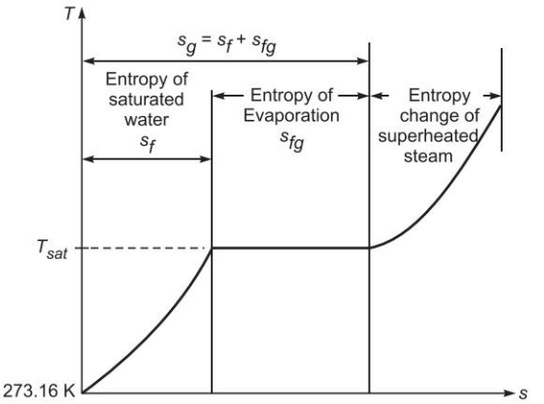

Fig. 3.17 *Temperature–entropy diagram for pure substance*

3.13.1 Entropy Change During Heating of Water

Let 1 kg of water be heated from temperature T_1 to T_2 at constant pressure. The change in entropy is given by

$$\Delta s = \int_1^2 \frac{dq}{T} = C_{pw} \ln\left(\frac{T_2}{T_1}\right) (\text{kJ/kg} \cdot \text{K}) \qquad ...(3.16)$$

The entropy of saturated water, s_f from 273 K to T_{sat} can be obtained directly from the steam tables or it can be calculated as

$$s_f = C_{pw} \ln\left(\frac{T_{sat}}{273}\right) (\text{kJ/kg} \cdot \text{K}) \qquad ...(3.17)$$

3.13.2 Entropy Change During Evaporation

The specific entropy change during evaporation is denoted by s_{fg} for dry and saturated steam and obtained from steam tables. Mathematically, for dry and saturated steam,

$$s_{fg} = \frac{h_{fg}}{T_{sat}} (\text{kJ/kg} \cdot \text{K}) \qquad ...(3.18)$$

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For wet steam, the heat supplied during evaporation is $x\ h_{fg}$ at saturation temperature T_{sat} (K). Then the change of entropy during evaporation,

$$s_{wet} = x\ s_{fg} = \frac{x\ h_{fg}}{T_{sat}} \qquad ...(3.19)$$

3.13.3 Entropy Change of Steam (s_g)

It is the total entropy of steam.

(*a*) *Entropy Change of Dry and Saturated Steam* It is the sum of entropy of saturated water s_f, and the entropy of evaporation s_{fg}. In other words,

$$s_g = s_f + s_{fg}\ (\text{kJ/kg} \cdot \text{K}) \qquad ...(3.20)$$

It can be directly obtained from steam tables.

(*b*) *Entropy Change of Wet Steam* It is the sum of entropy of saturated water s_f and the entropy of partial evaporation. In other words,

$$s_{wet} = s_f + x\ s_{fg}\ (\text{kJ/kg} \cdot \text{K}) \qquad ...(3.21)$$

(*c*) *Entropy Change of Superheated Steam* The entropy change during superheating of 1 kg of dry steam from T_{sat} to T_{sup} at constant pressure

$$s_{superheating} = \int_{T_{Sat}}^{T_{Sup}} \frac{\delta q}{T} = C_{ps} \int_{T_{sat}}^{T_{sup}} \frac{dT}{T}$$

$$= C_{ps} \ln\left(\frac{T_{sup}}{T_{sat}}\right) \qquad ...(3.22)$$

Then the total entropy of superheated steam above the freezing point temperature of water

$$s_{sup} = s_f + s_{fg} + C_{ps} \ln\left(\frac{T_{sup}}{T_{sat}}\right)$$

$$= s_g + C_{ps} \ln\left(\frac{T_{sup}}{T_{sat}}\right) \qquad ...(3.23)$$

Note. The temperatures must be used in Kelvin (K).

3.14 EXTERNAL WORK DONE DURING EVAPORATION

The phase transformation of saturated water to saturated steam takes place at constant pressure, its specific volume increases from v_f to v_g and the piston is displaced due to change in its volume as shown in Fig. 3.18 (b) . Thus, external work is done by steam due to increase in specific volume. This work produced is called *the external work done during evaporation*

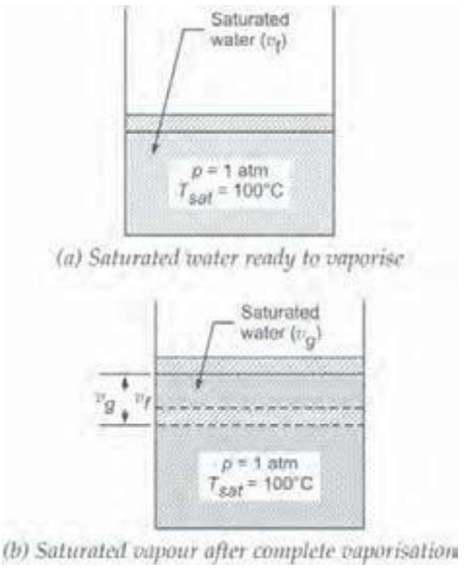

Fig. 3.18

The external work done during evaporation

$$w_{ex} = p\ (v_g - v_f) = p\ v_{fg} \qquad ...(3.24)$$

Since, $v_f << v_g$ at low pressure, it is neglected. Then for dry and saturated steam

$$w_{ex} \approx p\ v_g\ (\text{kJ/kg}) \qquad ...(3.25)$$

For wet steam with a dryness fraction x, the work of evaporation is

$$w_{wet} \approx p\ (x\ v_g)\ (\text{kJ/kg}) \qquad ...(3.26)$$

Note. The pressure p should be expressed in kPa in the above relations.

3.15 INTERNAL LATENT HEAT

Actually, the latent heat or enthalpy of evaporation is always more than the external work done during evaporation. This excess amount of latent heat is called **internal latent heat**. It is required to

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overcome the internal molecular forces to change the phase of substance.

Mathematically, the internal latent heat

= latent heat – external work of evaporation

For dry and saturated steam,

$$\text{internal latent heat} = h_{fg} - pv_g \qquad ...(3.27)$$

For wet steam

$$\text{internal latent heat} = x\, h_{fg} - p\,(xv_g) \qquad ...(3.28)$$

3.16 INTERNAL ENERGY OF STEAM

The internal energy of steam (u) is actually heat energy stored in steam above the freezing point temperature of water, and it is the sum of sensible heat and internal latent heat at a given pressure p in kPa. In other words,

Internal energy of steam

= Sensible heat + Internal latent heat

Total heat of steam

= Sensible heat + Internal Latent heat + External work done

h = Internal energy (u) + pv

Hence $u = h - pv$

For dry steam, $u_g = h_g - pv_g \qquad ...(3.29)$

For wet steam $u_{wet} = (h_f + x\, h_{fg}) - p\,(x\, v_g)$

$= x\, u_{fg} \qquad ...(3.30)$

For superheated steam,

$$u_{sup} = h_g + C_{ps}(T_{sup} - T_{sat}) - pv_g \times \frac{T_{sup}}{T_{sat}} \qquad ...(3.31)$$

The u_f, u_{fg}, u_g and u_{sup} can also be obtained from the steam tables.

3.17 STEAM TABLES

The laws of perfect gases are not applicable to vapours, hence the variation between their properties are obtained from charts and tables. Experimentally determined thermodynamic properties of water are presented in three different forms of tables as follows:

1. Temperature Entry Steam Table In this table, temperature is chosen as an independent variable and properties of coexisting liquid and vapour phases are presented for saturated water/steam. Table 3.2 shows a portion of such a steam table.

2. Pressure Entry Steam Table In this table, the pressure is chosen as an independent variable and other properties are presented against it for saturated water/steam as shown in Table 3.3.

3. Superheated Steam Table In this table, the pressure and temperature are two independent variables and properties of superheated steam can be obtained against these variables as shown in Table 3.4.

Remarks

1. The triple point of water (0.01°C, 0.611 kPa) is chosen as reference state for presenting data in the steam tables. At triple point, the internal energy u_f and entropy s_f for saturated liquid is arbitrarily assigned a zero value.

Table 3.2 *Saturated Steam : Temperature Entry Table*

Temp. T *(°C)*	*Sat. pressure* p_{sat} *(kPa)*	*Specific volume* (m^3/kg)		*Specific enthalpy* (kJ/kg)			*Specific entropy* (kJ/kg·K)	
		v_f	v_g	h_f	h_{fg}	h_g	s_f	s_g
0.01	0.6113	0.001000	206.14	0.01	2505.3	2505.4	0.000	9.1562
100	101.325	0.001044	1.6729	419.02	2257.03	2676.05	1.3068	7.3548
200	1553.8	0.001157	0.12736	852.45	1938.48	2790.93	2.3309	6.4323
374.14	22090	0.003155	0.003155	2099.3	000.00	2099.3	4.4298	4.4298

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Table 3.3 *Saturated Steam : Pressure Entry Table*

Abs. pressure p, kPa	*Sat temp. T_{sat} °C*	*Specific volume m^3/kg*		*Specific enthalpy kJ/kg*			*Specific entropy kJ/kg·K*	
		v_f	v_g	h_f	h_{fg}	h_g	s_f	s_g
0.6113	0.01	0.001000	206.14	0.01	2501.3	2501.4	0.000	9.1562
100	99.63	0.001043	1.6940	417.46	2258.0	2675.46	1.3026	7.3594
1000	179.91	0.001127	0.19444	762.81	2014.29	2777.10	2.1386	6.5864
22090	374.14	0.003155	0.003155	2029.3	0	2029.3	4.4298	4.4298

Table 3.4 *Superheated Steam Table*

Temp. T °C	*Sp. volume v m^3/kg*	*Sp. Enthalpy h kJ/kg*	*Sp. Entropy s kJ/kg·K*
p = 10 kPa (45.81 °C)			
Sat	14.674	2584.63	8.1502
50	14.869	2592.56	8.1749
100	17.196	2687.46	8.4479
150	19.512	2782.99	8.6881
200	25.825	2879.52	8.9037
1300	72.602	5409.70	15.5810

2. The choice of steam table for saturated steam is immaterial, because every table gives the same data of saturated liquid and saturated vapour.

3.18 POINTS TO CONSIDER BEFORE SOLVING THE PROBLEMS

1. The steam table gives the properties on per kg basis. For the different masses they should be multiplied by the given mass.
2. If the value of the independent property of the column 1, i.e., temperature or pressure is not included in steam tables, the properties should be obtained by linear interpolation.
3. A meagre negligible variation in answers is inevitable due to usage of different steam tables.
4. In case the nature of pressure is not specified, it should be assumed absolute pressure.
5. The steam tables give the values of properties above 0°C. Hence if the initial temperature of water is other than 0°C then the initial enthalpy of water can be obtained from the temperature entry steam table at a given temperature. The enthalpy of fluid is then obtained by deducting the initial enthalpy from the total enthalpy of steam.

3.19 ADVANTAGES AND APPLICATIONS OF USE OF STEAM

Steam, as a working substance, offers the following advantages and applications.

Advantages

1. It is capable to supply process heat at constant temperature while condensing.
2. It is cheap, and can be produced everywhere.
3. It is a clean, odourless and tasteless source of heat energy.

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4. It can be used repeatedly again and again as well as first used for power generation and then for process heating.
5. Its flow rate can easily be controlled and readily distributed.

Main Applications

1. The heat content of steam is large and thus it is suitable for process heating (for curing, drying, etc.) in many industries like sugar mills, textile mills, chemical industries, etc.
2. It is also used for power generation in thermal power plants.
3. Due to its high heat content, steam is used for cooking many items like steam rice, *idlies*, etc.
4. Steam can also be used for heating buildings and producing hot water in winter.
5. Steam is also used for creation of vacuum, ejection of gases and sterlisation.

Example 3.1 *Calculate specific enthalpy, specific volume and density of 1 kg of steam at a pressure of 1.9 MPa, having a dryness fraction 0.85.*

Solution

Given 1 kg of wet steam

$p = 1.9$ MPa, $x = 0.85$

To find

(i) Total heat, (ii) Volume, (iii) Density.

Properties of steam At $p = 1.9$ MPa = 19 bar;

$T_{sat} = 209.80°C$ $v_f = 0.001172$ m^3/kg,
$v_g = 0.10467$ m^3/kg $h_f = 896.8$ kJ/kg,
$h_{fg} = 1899.3$ kJ/kg

Analysis

(i) The specific enthalpy (total heat) of steam

$$h = h_f + x\,h_{fg}$$
$$= 896.8 + 0.85 \times 1899.3 = \mathbf{2511.20\ kJ/kg}$$

(ii) The specific volume of 1 kg of steam

$$v = (1 - x)\,v_f + x\,v_g \approx x\,v_g$$
$$= 0.85 \times 0.10467 = \mathbf{0.0889\ m^3}$$

(iii) The density of steam

$$\rho = \frac{1}{v} = \frac{1}{0.0889} = \mathbf{11.248\ kg/m^3}$$

Example 3.2 *Calculate volume, density, enthalpy, and entropy of 2 kg of steam at 80°C and having a dryness fraction of 0.85.*

Solution

Given Wet steam

$m = 2$ kg $T = 80°C$ $x = 0.85$

To find

(i) Volume, (ii) Density,
(iii) Enthalpy, and (iv) Entropy.

Properties of water At 80°C;

$p_{sat} = 47.39$ kPa $v_f = 0.001029$ m^3/kg
$v_g = 3.40715$ m^3/kg $h_f = 334.88$ kJ/kg
$h_{fg} = 2308.77$ kJ/kg·K $s_f = 1.0752$ kJ/kg·K
$s_{fg} = 6.5369$ kJ/kg·K

Analysis

(i) At 80°C, the specific volume of steam;

$$v = (1 - x)\,v_f + x\,v_g \approx x\,v_g$$
$$= 0.85 \times 3.40715 = 2.896\ \text{m}^3/\text{kg},$$

For $m = 2$ kg;

$$V = m\,v = 2 \times 2.896 = \mathbf{5.792\ m^3}$$

(ii) The density of steam;

$$\rho = \frac{1}{v} = \frac{1}{2.896} = \mathbf{0.345\ kg/m^3}$$

(iii) Specific enthalpy of steam;

$$h_{wet} = h_f + x\,h_{fg} = 334.88 + 0.85 \times 2308.77$$
$$= 2297.33\ \text{kJ/kg}$$

Total enthalpy

$$H = m\,h_{wet} = 2 \times 2297.33 = \mathbf{4594.66\ kJ}$$

(iv) Specific entropy of steam;

$$s_{wet} = s_f + x\,s_{fg} = 1.0752 + 0.85 \times 6.5369$$
$$= 6.631\text{kJ/kg·K}$$

Total entropy

$$S = m\,s_{wet} = 2 \times 6.631 = \mathbf{13.26\ kJ/K}$$

Example 3.3 *Calculate volume, density, enthalpy, and entropy of 2 kg of water at 2 bar and 80°C.*

Solution

Given Water

$T = 80°C$ $p = 2$ bar $m = 2$ kg

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To find

(i) Volume, (ii) Density,
(iii) Enthalpy, and (iv) Entropy.

Assumption The specific heat of water as 4.187 kJ/kg·K.

Properties of water At $T = 80°C$

$p_{sat} = 47.39$ kPa

$v_f = 0.001029$ m^3/kg

Analysis At 80°C, water pressure is 2 bar, thus it is compressed liquid.

(i) At 80°C, the specific volume of water

$$v_f = 0.001029 \text{ m}^3/\text{kg}$$

For $m = 2$ kg;

$$V = m\, v_f = 2 \times 0.001029 = \mathbf{0.002058\ m^3}$$

(ii) The density of water

$$\rho = \frac{1}{v_f} = \frac{1}{0.001029} = \mathbf{971.82\ kg/m^3}$$

(iii) The enthalpy of water

$$H = m\, C_{pw}\, (T_{water} - 0)$$
$$= 2 \times 4.187 \times (80°C - 0) = \mathbf{669.92\ kJ}$$

(iv) Entropy of water

$$S = m\, C_{pw} \ln\left(\frac{T_{water} + 273}{0 + 273}\right)$$

$$= 2 \times 4.187 \times \ln\left(\frac{80 + 273}{0 + 273}\right) = \mathbf{2.152\ kJ/K}$$

Example 3.4 *Using steam tables, determine the mean specific heat for superheated steam at 1 bar between temperatures of 150°C to 200°C .*

Solution

Given Superheated steam

$p = 1$ bar $= 100$ kPa, $T_{sup_1} = 150°C$, $T_{sup_2} = 200°C$

To find Mean specific heat of superheated steam

Properties From superheated steam table at 1 bar

At T_{sup_1}: $h_{sup_1} = 2776.38$ kJ/kg

At T_{sup_2}: $h_{sup_2} = 2875.27$ kJ/kg.

Analysis The enthalpy of superheated steam can be expressed as

$$h_{sup_2} = h_{sup_1} + C_{ps}\,(T_{sup_2} - T_{sup_1})$$

or $$2875.27 = 2776.38 + C_{ps}\,(200 - 150)$$

or $$C_{ps} = \frac{2875.27 - 2776.38}{50} = \mathbf{1.978\ kJ/kg \cdot K}$$

Example 3.5 *Identify the type of steam in the following three cases, using the steam tables and giving necessary calculations supporting your claim.*

(a) 2 kg of steam at 8 bar with an enthalpy of 5538.0 kJ at a temperature of 170.4°C

(b) 1 kg of steam at 2550 kPa occupies a volume of 0.2742 m^3. Also, find the steam temperature.

(c) 1 kg of steam at 60 bar with an enthalpy of 2470.73 kJ/kg.

Solution

Case (i): Given The steam

$m = 2$ kg $\qquad p = 8$ bar

$H = 5538$ kJ $\qquad T_{sat} = 170.4°C$

To find Quality of steam.

Analysis The properties of steam at 8 bar;

$T_{sat} = 170.4$ °C $\qquad h_f = 721.1$, kJ/kg

$h_{fg} = 2048.04$ kJ/kg $\qquad h_g = 2769.13$ kJ/kg

The specific enthalpy of a given mass of steam

$$h = \frac{H}{m} = \frac{5538}{2} = 2769.0 \text{ kJ/kg}$$

The specific enthalpy of given steam is approximately equal to the total enthalpy of vapour at saturation temperature, and therefore, the given sample of steam is **dry and saturated**.

Case (ii): Given The steam

$m = 1$ kg $\qquad p = 2550$ kPa $\qquad v = 0.2742$ m^3

To find Quality of steam.

Analysis The properties of steam at 2550 kPa

$T_{sat} = 225$ °C $\qquad v_g = 0.07835$ m^3/kg

The given volume of steam is more than the volume of vapour at saturation temperature. Therefore,

$$v = v_g \times \frac{T_{sup}}{T_{sat}}$$

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or $$0.2742 = 0.07835 \times \frac{T_{sup}}{(225+273)}$$

or $$T_{sup} = 1742.84 \text{ K} = \mathbf{1469°C}$$

The given sample of steam is **superheated**.

Case (iii): Given The steam

$m = 1$ kg $\quad p = 60$ bar $\quad h = 2470.73$ kJ/kg

To find Quality of steam.

Analysis The properties of steam at 60 bar

$T_{sat} = 275.64$ °C $\quad h_f = 1213.32$ kJ/kg
$h_{fg} = 1571.0$ kJ/kg $\quad h_g = 2784.33$ kJ/kg

The enthalpy of the given steam is less than the total enthapy of steam at saturation temperature. Therefore,

$$h = h_f + x\, h_{fg}$$

or $$2470.73 = 1213.32 + x \times 1571.0$$

or $$x = \mathbf{0.8}$$

The given sample of steam is **wet**.

Example 3.6 *Find the internal energy of 1 kg of steam at a pressure of 10 bar, when the condition of steam is*

(a) wet with a dryness fraction of 0.85,
(b) dry and saturated, and
(c) superheated, the degree of superheat being 50°C.

The specific heat of superheated steam at constant pressure is 2.01 kJ/kg·K.

Solution

Given 1 kg of steam at 10 bar with three conditions

(i) $x = 0.85$
(ii) $x = 1$
(iii) $T_{sup} - T_{sat} = 50$°C, and $C_{ps} = 2.01$ kJ/kg·K

To find The internal energy of steam in each case

Analysis Using steam table at 10 bar pressure

$T_{sat} = 179.9$°C $\quad v_g = 0.19444$ m^3/kg
$h_f = 762.79$ kJ/kg $\quad h_{fg} = 2015.3$ kJ/kg
$h_g = 2778.08$ kJ/kg

(i) When steam is wet and the dryness fraction is 0.85
The specific enthalpy of steam

$$h_{wet} = h_f + x\, h_{fg} = 762.79 + 0.85 \times 2015.3$$
$$= 2475.8 \text{ kJ/kg}$$

The specific internal energy of wet steam

$$u_{wet} = h_{wet} - p\,(x\, v_g)$$
$$= 2475.8 - (10 \times 100 \text{ kPa}) \times 0.85 \times (0.19444 \text{ m}^3/\text{kg})$$
$$= \mathbf{2310.52 \text{ kJ/kg}}$$

(ii) For dry steam;

$$u_g = h_g - p\, v_g$$
$$= 2778.08 - (10 \times 100) \times 0.19444$$
$$= \mathbf{2583.64 \text{ kJ/kg}}$$

(iii) When steam is superheated,

$$T_{sup} - T_{sat} = 50°\text{C}$$
$$T_{sup} = 179.9 + 50 = 229.9°\text{C}$$

The specific enthalpy of superheated steam

$$h_{sup} = h_g + C_{ps}\,(T_{sup} - T_{sat})$$
$$= 2778.08 + 2.01 \times 50 = 2878.58 \text{ kJ/kg}$$

Specific volume of superheated steam

$$v_{sup} = 0.19444 \times \frac{229.9+273}{179.9+273} = 0.216 \text{ m}^3/\text{kg}$$

The internal energy of superheated steam

$$u_{sup} = h_{sup} - p\, v_{sup}$$
$$= 2878.58 - (10 \times 100) \times 0.216$$
$$= \mathbf{2662.\ 67 \text{ kJ/kg}}$$

Example 3.7 *Using the data given below, calculate the volume, enthalpy and internal energy of 2 kg of steam at 10.3 bar pressure in each of the following states: (a) Dryness fraction of 0.85, (b) dry and saturated steam, and (c) at a temperature of 220° C.*

Pressure, p, bar	*Sat. Temp T_{sat} °C*	*Sp. volume of dry steam v_g cc/gm*	*Enthalpy of liquid h_f kJ/kg*	*Enthalpy of vapour h_g kJ/kg*
10.02	*180*	*194.10*	*763.22*	*2278.2*
10.50	*182*	*186.00*	*772.20*	*2779.9*

Solution

Given Three types of steam with

$m = 2$ kg $\quad p = 10.3$ bar

Case 1. Wet steam with $x = 0.85$,
2. Dry and saturated steam, and
3. Superheated steam at 220°C.

Tabulated data as

$p_1 = 10.02$ bar $\quad p_2 = 10.5$ bar
$T_{sat_1} = 180°$ C $\quad T_{sat_2} = 182°$ C

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v_{g_1} = 194.10 cc/gm $\quad v_{g_2}$ = 186.00 cc/gm
h_{f_1} = 763.22 kJ/kg $\quad h_{f_2}$ = 772.20 m^3/kg
h_{g_1} = 2278.2 kJ/kg $\quad h_{g_2}$ = 2779.9 kJ/kg

To find In each case of steam

(i) Volume of steam in m^3,
(ii) Enthalpy of steam in kJ, and
(iii) Internal energy of steam in kJ.

Analysis In tabulated data, the data against a pressure of 10.3 bar is not given. Hence, we have to obtain the data against 10.3 bar pressure by interpolating the available data. Using linear interpolation as

$$y_p = y_1 + \frac{y_2 - y_1}{p_2 - p_1}(p - p_1)$$

where y_p is the unknown data to calculate against

p = 10.3 bar

(a) *Saturation temperature*, T_{sat}

$$T_{sat} = T_{sat_1} + \frac{T_{sat_2} - T_{sat_1}}{p_2 - p_1}(p - p_1)$$

$$= 180 + \frac{182 - 180}{10.5 - 10.02} \times (10.3 - 10.02)$$

= 180 + 1.1667 = 181.1667° C

(b) *Specific volume* v_g

$$v_g = v_{g_1} + \frac{v_{g_2} - v_{g_1}}{p_2 - p_1}(p - p_1)$$

$$= 194.10 + \frac{186.00 - 194.10}{10.5 - 10.02} \times (10.3 - 10.02)$$

= 194.10 – 4.725 = 189.375 cc/g

(c) *Specific enthalpy of liquid* h_f

$$h_f = h_{f_1} + \frac{h_{f_2} - h_{f_1}}{p_2 - p_1}(p - p_1)$$

$$= 763.22 + \frac{772.20 - 763.22}{10.5 - 10.02} \times (10.3 - 10.02)$$

= 763.22 + 5.238 = 768.458 kJ/kg

(d) *Specific enthalpy of vapour*, h_g

$$h_g = h_{g_1} + \frac{h_{g_2} - h_{g_1}}{p_2 - p_1}(p - p_1)$$

$$= 2278.2 + \frac{2779.9 - 2278.2}{10.5 - 10.02} \times (10.3 - 10.02)$$

= 2278.2 + 0.9916 = 2570.85 kJ/kg

Now the data in tabulated form:

p bar	T_{sat} °C	v_g cc/gm	h_f kJ/kg	h_g kJ/kg
10.02	180.00	194.10	763.22	2278.2
10.3	**181.1667**	**189.375**	**768.458**	**2570.85**
10.5	182.0	186.00	772.20	2779.9

Case 1: Wet steam with dryness fraction x = 0.85

(i) *Volume of steam in m^3*

$V = m\,(x\,v_g)$

$$= 2 \times (0.85 \times 189.375) \times \frac{10^{-6}}{10^{-3}} = \mathbf{0.322\ m^3}$$

(ii) *Enthalpy of steam*

$H = m\,[h_f + x\,(h_g - h_f)]$

$= 2 \times [768.458 + 0.85 \times (2570.85 - 768.458)]$

$= 2 \times [768.458 + 1709.13] = \mathbf{4601\ kJ}$

(iii) *Internal energy* $U = H - p\,V$

$= 4601 - (10.3 \times 100) \times 0.322$

$= \mathbf{4269.33\ kJ}$

Case 2: Dry and saturated steam

(i) $V = m\,v_g = 2 \times 189.375 \times \dfrac{10^{-6}}{10^{-3}} = \mathbf{0.378\ m^3}$

(ii) $H = m\,h_g = 2 \times 2570.85 = \mathbf{5141.7\ kJ}$

(iii) $U = H - p\,V$

$= \mathbf{5141.7} - (10.3 \times 100) \times 0.378 = \mathbf{4752.36\ kJ}$

Case 3: Superheated steam at 220° C

(i) $V = m\,v_g \times \dfrac{T_{sup}}{T_{sat}}$

$$= 2 \times \left(189.375 \times \frac{10^{-6}}{10^{-3}}\right) \times \left\{\frac{220 + 273}{181.1667 + 273}\right\}$$

$= \mathbf{0.411\ m^3}$

(ii) $H = m\,[h_g + C_{ps}\,(T_{sup} - T_{sat})]$

$= 2 \times [2570.85 + 2.2 \times (220 - 181.1667)]$

$= \mathbf{5312.66\ kJ}$

(iii) $U = H - pV = 5312.66 - (10.3 \times 100) \times 0.411$

$= \mathbf{4889.2\ kJ}$

Example 3.8 *Calculate the amount of heat to be supplied to produce 5 kg of steam at a pressure of 8 bar and a temperature of 320°C from water at 30°C. Take specific heat for superheated steam as 2.2 kJ/kg·K.*

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Solution

Given Generation of steam from water

$m = 5$ kg $\quad p = 8$ bar $\quad T_{sup} = 320°C$

$T_1 = 30°C \quad C_{ps} = 2.2$ kJ/kg·K

To find The amount of heat supplied to make the steam superheated.

Assumptions

(i) The specific heat of water is 4.187 kJ/kg·K.

(ii) The heat is supply at constant pressure.

Analysis At 8 bar, the properties of steam

$T_{sat} = 170.43$ °C $\quad h_g = 2768.4$ kJ/kg

Amount of heat (enthalpy) required by 1 kg of water from 0°C for its superheating to 320 °C;

$$h_{sup} = h_g + C_{ps}(T_{sup} - T_{sat})$$
$$= 2768.4 + 2.2 \times (320 - 170.43)$$
$$= 3097.52 \text{ kJ/kg}$$

Water is initially at 30°C, the amount of heat (enthalpy) already with water at 30°C;

$$h_i = C_{pw}(T_i - 0) = 4.187 \times (30 - 0) = 125.61 \text{ kJ/kg}$$

The net amount of heat/kg (enthalpy) required

$$= h_{sup} - h_i = 3097.52 - 125.61 = 2971.9 \text{ kJ/kg}$$

The total amount of heat required

$$= 5 \times 2971.9 = \mathbf{14859.55\ kJ}$$

Example 3.9 *A certain boiler generates 1000 kg of steam per hour at 10 bar and 0.97 dry. This steam is then taken to a superheater and is heated to a temperature of 280°C keeping the pressure constant. The feed water is available at 30°C. Find*

(a) Heat supplied per hour in the boiler,

(b) Heat supplied per hour in the superheater,

(c) Internal energy of superheated steam,

(d) Change in internal energy of steam.

Solution

Given Generation of steam in a boiler

$m = 1000$ kg/h $\quad p = 10$ bar $\quad x = 0.97$

$T_{sup} = 280$ °C $\quad T_1 = 30°C \quad p =$ constant

To find

(i) Heat supplied per hour in the boiler,

(ii) Heat supplied per hour in superheater,

(iii) Internal energy of superheated steam, and

(iv) Change in internal energy.

Assumptions

(i) Specific heat of water as 4.187 kJ/kg K.

(ii) Specific heat of superheated steam as 2.2 kJ/kg·K.

Analysis The properties of steam at 10 bar;

$T_{sat} = 179.91°C \quad h_f = 762.81$, kJ/kg

$h_{fg} = 2014.3$ kJ/kg $\quad v_g = 0.19444$ m^3/kg

$v_f = 0.001127$ m^3/kg $\quad h_g = 2777.08$ kJ/kg

(i) *Heat supplied per hour in the boiler*

The heat is already available with 1 kg of water at 30°C;

$$h_1 = C_{pw}(T_1 - 0) = 4.187 \times (30 - 0)$$
$$= 125.61 \text{ kJ/kg}$$

Amount of heat required by 1 kg of water to generate steam 0.97 dry from 0°C.

$$h_2 = h_f + x\, h_{fg} = 762.81 + 0.97 \times 2014.3$$
$$= 2716.65 \text{ kJ/kg}$$

Net amount of heat required by 1 kg of water in the boiler;

$$h_3 = h_2 - h_1$$
$$= 2716.65 - 125.61 = 2591.04 \text{ kJ/kg}$$

Total amount of heat required in the boiler per hour

$$= m\, h_3 = 1000 \times 2591.04$$
$$= 2591.04 \times 10^3 \text{ kJ/h} = \mathbf{2591.04\ MJ/h}$$

(ii) *Heat supplied in the superheater per kg of steam*

h_4 = Heat supplied to make steam dry and saturated + Heat supplied to make steam superheated.

Alternatively:

h_4 = Enthalpy of superheated steam/kg – Enthalpy of steam/kg generated in boiler.

Choosing alternate formula:

Enthalpy of superheated steam per kg

$$h_{sup} = h_g + C_{ps}(T_{sup} - T_{sat})$$
$$= 2777.08 + 2.2 \times (280 - 179.91)$$
$$= 2997.30 \text{ kJ/kg}$$

Then $\quad h_4 = h_{sup} - h_3 = 2997.30 - 2591.04$
$$= 406.26 \text{ kJ/kg}$$

Total amount of heat supplied in superheater per hour

$$= m \times h_4 = 1000 \times 406.268$$
$$= 406268 \text{ kJ/hr} = \mathbf{406.26\ MJ/h}$$

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(iii) *Internal energy of superheated steam*

$$u_{sup} = h_{sup} - p v_{sup}$$

where $$v_{sup} = v_g \times \frac{T_{sup}}{T_{sat}}$$

$$= 0.19444 \times \left(\frac{280+273}{179.91+273}\right)$$

$$= 0.2374 \text{ m}^3/\text{kg}$$

Then $$u_{sup} = (2997.30 \text{ kJ/kg}) - (10 \times 100 \text{ kPa}) \times (0.2374 \text{ m}^3/\text{kg})$$

$$= \mathbf{2759.89 \text{ kJ/kg}}$$

(iv) *Change in internal energy*

Initial internal energy of water,

$$u_1 = h_f - p v_f$$

or $$u_1 = 125.61(\text{kJ/kg}) - (10 \times 100 \text{ kPa}) \times 0.001127 \text{ m}^3/\text{kg}$$

$$= 124.483 \text{ kJ/kg}.$$

The change in internal energy

$$u_{sup} - u_1 = 2759.89 - 124.483 = \mathbf{2635.4 \text{ kJ/kg}}$$

Example 3.10 *Prove the following statements:*

(a) Temperature of wet steam equals that of dry and saturated steam at same pressure.

(b) Dryness fraction of steam does not go below zero or above unity.

(c) Specific volume of superheated steam is greater than that of dry and saturated steam at the same pressure.

Solution

(i) Wet steam is produced during the phase change of water and only latent heat of vaporisation is supplied during this process. This latent heat is used to overcome the *molecular forces of attraction* and the *specific heat of water becomes infinity during phase change*. Therefore, the temperature of wet steam equals that of dry and saturated steam at the same pressure.

(ii) The dryness fraction is defined as

$$x = \frac{\text{mass of dry vapour}}{\text{mass of dry vapour + mass of moisture}}$$

(a) As an upper limit, if the steam is completely dry, the mass of moisture will be zero. Then the above relation yields to **unity** dryness.

(b) As a lower limit, if the steam is completely wet, the mass of vapour will be zero. Then the above relation yields to **zero** dryness.

(iii) Superheated steam behaves as a perfect gas, and thus follows the property relation

$$\frac{v_g}{T_{sat}} = \frac{v_{sup}}{T_{sup}}$$

or $$v_{sup} = v_g \times \frac{T_{sup}}{T_{sat}} \text{ (m}^3/\text{kg)}$$

where

T_{sup} = temperature of superheated steam in K,

T_{sat} = temperature of dry and saturated steam in K,

and $T_{sup} > T_{sat}$. Therefore, the specific volume of superheated steam will be greater than the specific volume of saturated steam.

3.20 MEASUREMENT OF DRYNESS FRACTION OF STEAM

There are many calorimeters used to determine the dryness fraction of wet steam. We shall discuss the following calorimeters in this text:

1. Barrel Calorimeter
2. Separating Calorimeter
3. Throttling Calorimeter
4. Combined separating and throttling calorimeter

3.20.1 Barrel Calorimeter

The barrel calorimeter is shown in Fig. 3.19. A known mass of steam at a pressure, p is condensed in the presence of a known quantity of water filled in a barrel calorimeter. As steam condenses, the mass and temperature of water increase. The amount of heat lost by wet steam and amount of heat received by water in the calorimeter are equated to obtain the dryness fraction of wet steam as given below:

Heat lost by wet steam

$$= m_{steam} [(x\, h_{fg}) + C_{pw} (T_{sat} - T_2)] \quad \text{... (i)}$$

Heat gained by water in the calorimeter

$$= (m_c\, C_{pc} + m_w\, C_{pw}) (T_2 - T_1) \quad \text{... (ii)}$$

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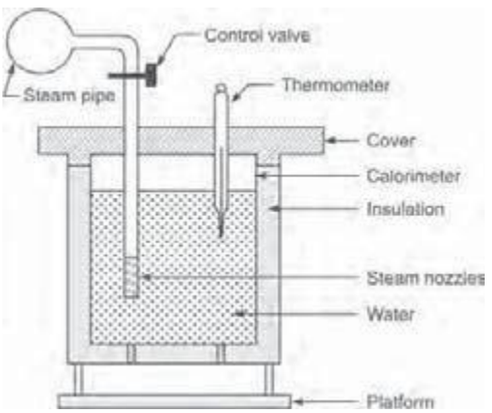

Fig. 3.19 *Barrel calorimeter*

Equating (i) and (ii), we get the dryness fraction.

$$x = \frac{(m_c C_{pc} + m_w C_{pw})(T_2 - T_1) - m_{steam}\, C_{pw}(T_{sat} - T_2)}{m_{steam}\, h_{fg}} \quad ...(3.32)$$

where m_c = mass of the barrel calorimeter, kg

C_{pc} = specific heat of the barrel calorimeter, kJ/kg·K

m_w = mass of water filled in the calorimeter, kg

C_{pw} = specific heat of water, kJ/kg·K

m_{steam} = mass of steam condensed, kg

h_{fg} = latent heat of steam at given pressure p, kJ/kg

T_1 = initial temperature of water, °C

T_2 = final temperature of water, °C

T_{sat} = saturation temperature of steam in °C at given pressure p

The value of the dryness fraction obtained by this method involves considerable error, since the heat losses due to convection and radiation are not taken into account.

Example 3.11 *The steam at a pressure of 5.5 bar is passed into a tank filled with 80 kg of water at 25°C. The tank is made of copper, is 10 kg in mass and has a specific heat of 0.406 kJ/kg·K. After steam has condensed and mixed with water, its temperature reaches 40°C and the mass of the mixture becomes 82.5 kg. Determine the dryness fraction of steam.*

Solution

Given Wet steam with

$p_1 = 5.5$ bar $\quad m_w = 80$ kg

$T_1 = 25$ °C $\quad m_c = 10$ kg

$T_2 = 40$ °C $\quad C_{pc} = 0.406$ kJ/kg·K

$m_w + m_{steam} = 82.5 \quad \therefore \quad m_{steam} = 2.5$ kg

To find Dryness fraction of wet steam.

Assumption The specific heat of water as 4.187 kJ/kg·K

Analysis The enthalpy of vaporisation and saturation temperature at 5.5 bar

$$h_{fg} = 2097 \text{ kJ/kg}, \qquad T_{sat} = 155.48 \text{ °C}$$

The dryness fraction of steam

$$x = \frac{(m_c C_{pc} + m_w C_{pw})(T_2 - T_1) - m_{steam}\, C_{pw}(T_{sat} - T_2)}{m_{steam}\, h_{fg}}$$

$$x = \frac{(10 \times 0.406 + 80 \times 4.187) \times (40 - 25)}{2.5 \times 2097.0} - \frac{2.5 \times 4.187 \times (155.48 - 40)}{2.5 \times 2097.0}$$

$$= \mathbf{0.749}$$

3.20.2 Separating Calorimeter

The separating calorimeter separates water particles from wet steam mechanically by centrifugal action.

The steam passing into the calorimeter is forced to change its direction of motion by means of a perforated cup. The water particles due to their greater moment of energy (being heavier) tend to move away and get separated from the mixture as shown in Fig. 3.20. Some quantity of moisture drains through the perforated cup, some quantity falls as large droplets and some quantity sticks with the wall of the separator. The condensate is collected in the separator and its quantity is measured by a glass tube. The dry steam is passed through a small condenser, where it condenses and its mass is measured.

The dryness fraction x of wet steam is determined as

$$x = \frac{m_g}{m_g + m_w} \quad ...(3.33)$$

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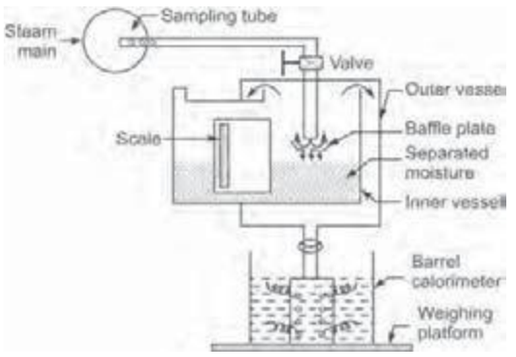

Fig. 3.20 *Separating calorimeter*

where

m_g = mass of dry and saturated steam, kg

m_w = mass of water particles in suspension, kg

It is noticed that the dryness fraction obtained from such experimentation is not accurate, but it is just close approximation.

Example 3.12 *In an experiment, a sample of wet steam is passed through a separating calorimeter. At some instant, the mass of water collected in the separator was 0.2 kg, while the amount of steam condensed was found to be 2.5 kg. Determine the dryness fraction of steam entering the calorimeter.*

Solution

Given $m_w = 0.2$ kg $m_g = 2.5$ kg

To find Dryness fraction of wet steam

Analysis The dryness fraction of wet steam is calculated as

$$x = \frac{m_g}{m_g + m_w}$$

Using numerical values, we get

$$x = \frac{2.5}{2.5 + 0.2} = \mathbf{0.926}$$

3.20.3 Throttling Calorimeter

The schematic of a throttling calorimeter is shown in Fig. 3.21. A sample of wet steam at a pressure p_1 is taken from the steam main through a perforated

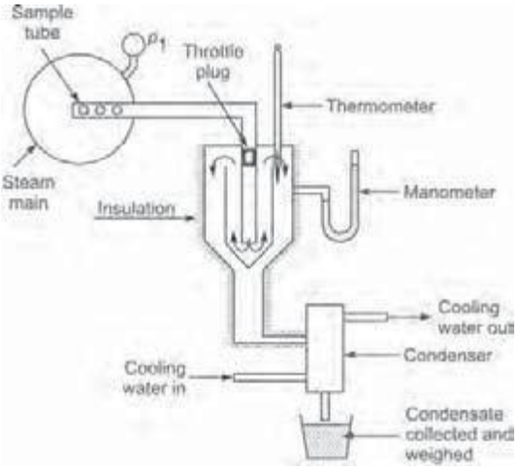

Fig. 3.21 *Throttling calorimeter*

sampling tube. The wet steam is then throttled through a partially opened or restricted valve to a pressure p_2 and temperature T_2 in such a way that the steam reaches the superheated region. During the throttling process, the enthalpy of steam remains constant.

The throttling process is an irreversible process and hence, it is shown by a dotted line on the *h–s* diagram in Fig. 3.22.

The state 1 is defined by pressure p_1 and dryness fraction x_1, while the state 2, in superheated region after throttling can be defined by the pressure p_2 and temperature T_2. Using properties of wet steam at p_1 and superheated steam at p_2 and T_2, we get

$$h_1 = h_2$$

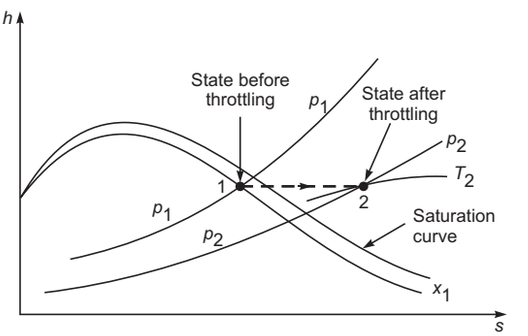

Fig. 3.22 *Throttling process on h–s diagram*

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or $h_{f_1} + x_1 h_{fg_1} = h_2$

or $$x_1 = \frac{h_2 - h_{f_1}}{h_{fg_1}} \quad ...(3.34)$$

where

h_{f_1} = sensible heat of wet steam at p_1, kJ/kg
h_{fg_1} = the enthalpy of vaporisation at p_1, kJ/kg
h_2 = enthalpy of superheated steam at p_2 and T_2
$= h_{g_2} + C_{ps}(T_2 - T_{sat_2})$

To obtain good approximation from a throttling calorimeter, the steam after throttling should be superheated at least by 5°C. If the steam is very wet then the resultant dryness fraction obtained by this technique is not very accurate.

Example 3.13 *In a throttling calorimeter, the steam is admitted at 10 bar, it is throttled to atmospheric pressure and 110°C. Determine the dryness fraction of steam. Assume the specific heat of superheated steam as 2.2 kJ/kg·K.*

Solution

Given The wet steam at state $p_1 = 10$ bar
The steam, after throttling $p_2 = 1.01325$ bar,
$T_2 = 110$ °C
$C_{ps} = 2.2$ kJ/kg·K

To find The dryness fraction of steam.

Analysis From steam table A – 13;

State 1, 10 bar $h_{f_1} = 762.81$ kJ/kg
$h_{fg_1} = 2014.3$ kJ/kg
State 2, 1.01325 bar, $h_{g_2} = 2676$ kJ/kg
$T_{sat_2} = 100$°C

As shown in Fig. 3.22, the enthalpy of steam remains constant during the throttling process. Thus

$$h_1 = h_2$$

or $$h_{f_1} + x\, h_{fg_1} = h_{g_2} + C_{ps}(T_2 - T_{sat_2})$$

or $762.81 + x \times 2014.3 = 2676 + 2.2\,(110 - 100)$

or $$x = \frac{2676 + 22 - 762.81}{2014.3} = \mathbf{0.961}$$

3.20.4 Combined Separating and Throttling Calorimeter

The schematic of combined separating and throttling calorimeter is shown in Fig. 3.23.

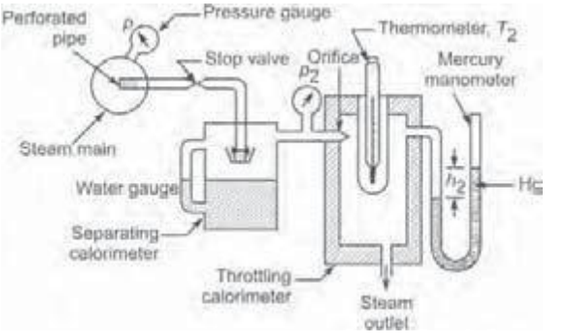

Fig. 3.23 *Combined separating and throttling calorimeter*

When the steam is very wet then the dryness fraction of steam is determined by using a separating and throttling calorimeter. The wet steam taken from a steam main via a steam stop valve is first passed through a separating calorimeter.

Some part of the moisture is removed from steam, due to sudden change in its direction. The resulting semi-dry steam is then throttled into a throttling calorimeter. The steam coming out the throttling calorimeter is superheated at a pressure p_2 and temperature T_2. The throttling calorimeter is well insulated to prevent any heat loss. The steam coming out of the throttling calorimeter is condensed and its mass is measured.

If m_w = mass moisture collected in separator, kg
m_g = mass of steam condensed after throttling, kg

then the dryness fraction of steam from separating calorimeter

$$x_1 = \frac{m_g}{m_g + m_w}$$

Similarly, the dryness fraction x_2, from the throttling calorimeter is obtained as discussed earlier.

$$x_2 = \frac{h_2 - h_{f_1}}{h_{fg_1}}$$

The final dryness fraction of steam in steam main

$$x = x_1\, x_2 \quad ...(3.35)$$

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Example 3.14 *A combined separating and throttling calorimeter was used to determine the dryness fraction of steam flowing through a steam main at a pressure of 9 bar. The pressure and temperature of steam after throttling were 1.25 bar and 115°C, respectively. The mass of steam condensed after throttling was 2.2 kg and the mass of water collected in the separator was 0.20 kg. Estimate the dryness fraction of steam in the main.*

Take specific heat for superheated steam as 2.1 kJ/kg·K.

Solution

Given For separating calorimeter

$m_w = 0.2$ kg $\qquad m_g = 2.2$ kg

For throttling calorimeter

$p_1 = 9$ bar $\qquad p_2 = 1.25$ bar

$T_2 = 115$ °C $\qquad C_{ps} = 2.1$ kJ/kg

To find Final dryness fraction of steam

Analysis The dryness fraction x_1 of wet steam from separating calorimeter

$$x_1 = \frac{m_g}{m_g + m_w} = \frac{2.2}{2.2 + 0.2} = 0.9167$$

For throttling calorimeter, the properties of steam.

Before throttling at $p_1 = 9$ bar,

$h_{f_1} = 742.83$ kJ/kg, $\qquad h_{fg_1} = 2031.1$ kJ/kg

After throttling at $p_2 = 1.25$ bar,

$T_{sat_2} = 106$ °C, $\qquad h_{g_2} = 2685.4$ kJ/kg

Enthalpy before throttling,

$$h_1 = h_{f_1} + x_2\, h_{fg_1} = 742.83 + x_2 \times 2031.1$$

Enthalpy after throttling,

$$h_2 = h_{g_2} + C_{ps}(T_2 - T_{sat_2})$$
$$= 2685.4 + 2.1 \times (115 - 106) = 2704.3 \text{ kJ/kg}$$

Equating two enthalpies, we get

$$x_2 = \frac{2704.3 - 742.83}{2031.1} = 0.9657$$

Final dryness fraction of steam

$$x = x_1 . x_2 = 0.9167 \times 0.9657 = \mathbf{0.885}$$

Example 3.15 *Steam at a pressure of 1 MPa, and a flow rate of 4 kg/min is passed through a separator which extracts moisture at a rate of 0.2 kg/min. The steam is then throttled to a pressure of 3.2 bar and passed through coils which extract 650 kJ/min of heat without loss of pressure. The steam leaves the coil with a dryness fraction of 0.93. Calculate the temperature of steam immediately after throttling and the dryness fraction of steam before and after the separator. Take $C_{ps} = 2.1$ kJ/kg·K.*

Solution

Given A sample of wet steam

$p_1 = 1$ MPa $\qquad m = 4$ kg/min

$m_w = 0.2$ kg/min $\qquad p_2 = 3.2$ bar

$x_3 = 0.93$ $\qquad Q = 650$ kJ/min

$C_{ps} = 2.1$ kJ/kg·K

To find

(i) Temperature of steam after throttling,

(ii) Dryness fraction before and after separator.

Properties of Steam

At $p_1 = 1$ MPa: $\quad h_{f_1} = 762.61$ kJ/kg

$h_{fg_1} = 2013.6$ kJ/kg

At $p_3 = 3.2$ bar: $\quad h_{f_3} = 570.90$ kJ/kg

$h_{fg_3} = 2156.7$ kJ/kg

$T_{sat} = 135.75\ C$

$h_{g_3} = 2727.6$ kJ/kg

Analysis Mass of dry steam,

$$m_g = m - m_w = 4 - 0.2 = 3.8 \text{ kg/min}$$

Then dryness fraction of steam before separator

$$x_1 = \frac{m_g}{m} = \frac{3.8}{4} = 0.95$$

(i) *The heat extracted at the coil*

$$Q = 650 \text{ kJ/min}$$

or $$q = \frac{650}{3.8} = 171.02 \text{ kJ/kg}$$

The heat content in steam at 3.2 bar when it enters the coil

= heat loss on coil

+ enthalpy of steam on leaving the coil

$$h_3 = q + h_{f_2} + x_3\, h_{fg_3}$$
$$= 171.02 + 570.90 + 0.93 \times 2156.7$$
$$= 2747.65 \text{ kJ/kg}$$

Since $h_3 > 2727.6$ kJ/kg (total enthalpy at 3.2 bar), therefore, steam leaving the throttling calorimeter is superheated.

$$h_3 = h_{g_1} + C_{ps}\,(T_{sup} - T_{sat})$$

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$$2747.65 = 2727.6 + 2.1\,(T_{sup} - 135.75)$$

$$\text{or} \quad T_{sup} = \frac{2747.65 - 2727.6}{2.1} + 135.75^{\circ}\text{C} = 145.3^{\circ}\text{C}$$

Degree of superheat = 145.3 – 135.75 = **9.55°C**

(ii) *Dryness fraction of steam after separator : steam pressure, 1 MPa*

$$h_2 = h_f + x_2\, h_{fg_1} = h_3$$

$$2747.65 = 762.61 + x_2\,(2013.6)$$

$$\text{or} \quad x_2 = \frac{2747.65 - 762.61}{2013.6} = 0.986$$

Therefore, the dryness fraction of steam

$$x = x_1\, x_2 = 0.95 \times 0.986 = \mathbf{0.936}$$

3.21 IDEAL GAS MODEL

A gas can be modeled as an ideal gas when it has the following features:

1. It has no intermolecular forces of attraction or repulsion.
2. It does not change its phase during a thermodynamic process.
3. It obeys Boyle's law, Charles's law and the characteristic gas equation.

The internal energy of gases decreases rapidly with decreasing pressure and disappears when absolute pressure approaches zero. At zero pressure, all real gases behave in a similar manner and this state of identical behaviour is referred as *ideal state*. The behaviour of real gases at an ideal state suggests the concept of an ideal gas that will behave in as ideal manner at all pressures.

In actual, no gas is completely ideal gas, but many gases such as air, nitrogen, oxygen, hydrogen helium, argon, neon, krypton, carbon dioxide, etc., can be treated as ideal gases with negligible error. Dense gases, such as water vapour, refrigerant vapour, however, should not be treated as ideal gases.

3.21.1 Boyle's Law

It states that when a gas undergoes a process at constant temperature, its specific volume is inversely proportional to absolute pressure, i.e.,

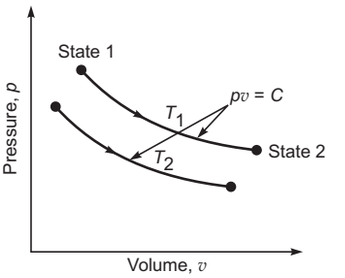

Fig. 3.24 *Boyle's law on a p–v digram*

$$v \propto \frac{1}{p}$$

or $$p\,v = C \qquad \text{...(3.36)}$$

where C is a constant of proportionality.

If a gas changes its state from p_1, v_1 to p_2, v_2 without any change in its temperature, then

$$p_1 v_1 = C$$

and $$p_2 v_2 = C$$

Hence $$p_1 v_1 = p_2 v_2 \qquad \text{...(3.37)}$$

A plot on the p–v diagram is shown in Fig. 3.24. Boyle's law represents a rectangular hyperbola curve. This curve is also called an *isotherm* and the process ocurring at constant temperature is known as an *isothermal process*.

3.21.2 Charles's Law

It states that if a gas undergoes a process at constant pressure, the change in its specific volume is directly proportional to its absolute temperature change, i.e.,

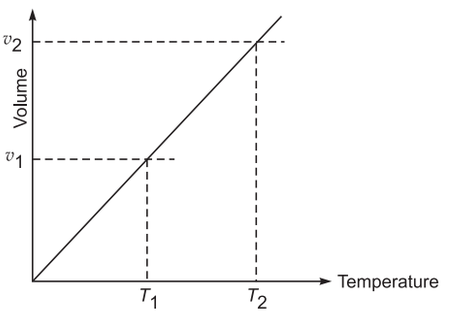

Fig. 3.25 *Charles's law on v–T diagram*

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$$v \propto T$$

or $$\frac{v}{T} = C$$

where C is the constant of proportionality at constant pressure.

For an ideal gas undergoing a constant pressure process from v_1, T_1 to v_2, T_2 ;

$$\frac{v_1}{T_1} = \frac{v_2}{T_2} \qquad ...(3.38)$$

Charles's law has also been stated as the absolute pressure of an ideal gas varies directly with absolute temperature if the volume of a gas is maintained constant during the process. That is,

$$p \propto T$$

or $\frac{p}{T} = C$ (constant of proportionality at constant volume)

For an ideal gas undergoing a constant volume process from p_1, T_1 to p_2, T_2 ;

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \qquad ...(3.39)$$

3.22 EQUATION OF STATE

It is observed that thermodynamic properties are interrelated. Any equation that relates the pressure, temperature and specific volume of a substance is known as an *equation of state*.

For any homogenous system, if the pressure and temperature are specified, its specific volume is automatically fixed by some functional relation. In other words, the thermodynamic properties exist in a certain relationship, such as

$$F(p, v, T) = 0$$

or $$p = f(v, T)$$

and $$v = g(p, T)$$

$$T = h(p, v) \qquad ...(3.40)$$

where F, f, g and h are some functional relationship.

3.23 CHARACTERISTIC GAS EQUATION

In any thermodynamic system of an ideal gas, the pressure, temperature and specific volume vary simultaneously. On the basis of this characteristic, an equation is derived with the help of Boyle's and Charles's laws as

Boyle's law $v \propto \frac{1}{p}$ when T is consant

By Charles's law $v \propto T$ when p is constant

If we combine both relations, we get

$$v \propto \frac{T}{p} \quad \text{when both } p \text{ and } T \text{ vary}$$

or $$v = \frac{RT}{p}$$

or $$pv = RT \qquad ...(3.41)$$

where R is the constant of proportionality and is called *specific gas constant*,
p is the absolute pressure,
T is the absolute temperature, and
v is the specific volume.

This is known as the *characteristic gas equation* for an ideal gas or *ideal-gas equation of state*.

The *molecular* or *molar mass* $\mathcal{M}$ can be defined as the *mass of one kmol* (also known as kg-mol) *of a substance in kilograms.*

The *mass* of a system is the product of its molecular mass $\mathcal{M}$ in kg/kmol and number of kmols, n :

$$m = n\,\mathcal{M}\,(\text{kg}) \qquad ...(3.42)$$

The ideal-gas equation of state can be written in several different forms as follows:

For a given mass system, the total volume

$$V = mv\,(\text{m}^3) \qquad ...(3.43)$$

then $$pV = mRT \qquad ...(3.44)$$

For a given molar specific volume $\overline{v}$ (m^3/kmol), the total volume

$$V = n\overline{v}\ (\text{m}^3) \qquad ....(3.45)$$

then $$pV = n R_u T \qquad ...(3.46)$$

where $R_u = \mathcal{M}R$ is known as the *universal gas constant*. Its value is found to be constant for all gases.

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3.23.1 Universal Gas constant, R_u

When the molecular mass of any gas ($\mathcal{M}$) is multiplied by its specific gas constant (R), it is observed that the product $\mathcal{M}R$ is always same for all gases. This product is called universal gas constant and it is denoted as R_u.

$$R_u = \mathcal{M}R \qquad ...(3.47)$$

For SI system, the value of the universal gas constant is 8.31447 kJ/kmol·K.

3.23.2 Characteristic Gas Constant, R

The constant of proportionality used in Eq. (3.41) is called the *specific gas constant*. Its value is different for different gases.

Unit of R If the absolute pressure is expressed in kN/m^2, specific volume in m^3/kg, and absolute temperature in K (kelvin) then the *specific gas constant R* has the unit of kJ/kg·K.

Calculation of R The value of the gas constant is determined from dividing the universal gas constant R_u by its molecular mass ($\mathcal{M}$) as

$$R = \frac{R_u}{\mathcal{M}} \qquad ...(3.48)$$

Table 3.5 presents the values of molar masses and gas constants for the most commonly used gases.

3.23.3 Change of State of an Ideal Gas

From Eq.(3.44), for an ideal gas undergoing change of state from p_1, V_1, T_1 to p_2, V_2, T_2 we get

at the state 1, $$\frac{p_1V_1}{T_1} = mR$$

at the state 2, $$\frac{p_2V_2}{T_2} = mR$$

Equating the left terms of the two equations, we get

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} = C \qquad ...(3.49)$$

It is a property relation for an ideal gas.

Table 3.5 *Molar masses and specific gas constants for common gases*

Gas	*Molar mass, $\mathcal{M}$, kg/kmol*	*Gas constant, $R = \frac{R_u}{\mathcal{M}}$, kJ/kg·K*
Air	28.97	0.287
Carbon dioxide	44.01	0.189
Carbon monooxide	28.01	0.297
Hydrogen	2.01	4.124
Nitrogen	28.01	0.297
Oxygen	32.00	0.259
Water vapour	18.01	0.462

3.23.4 Avogadro's Law or Avogadro's Hypothesis

It states that the molecular mass of all perfect gases occupies the same volume under identical conditions of pressure and temperature. Avogadro's experiment shows that the average volume for one kmol (kg-mole) for any perfect gas is 22.413 m^3 at standard atmospheric pressure (1.01325 bar) and 0°C (NTP condition).

or
$$V = \frac{\mathcal{M}RT}{p} = \frac{R_uT}{p}$$
$$= \frac{(8.31447 \text{ kJ/kmol·K}) \times (273.15)}{(101.325 \text{ kPa})}$$
$$= 22.413 \text{ m}^3/\text{kmol}$$

Example 3.16 *The gauge pressure of air in an automobile tyre at a temperature of 25°C is 1.75 bar. Due to running condition, the temperature of air in the tyre rises to 70°C. Determine the gauge pressure after running. Assume atmospheric pressure to be 1.0132 bar and volume of air in the tyre to be constant.*

Solution

Given Constant volume of air in an automobile tyre:

State 1: $T_1 = 25°C + 273 = 298$ K and $p_{g_1} = 1.75$ bar
State 2: $T_2 = 70°C + 273 = 343$ K, $p_{atm} = 1.0132$ bar

To find The gauge pressure of air in tyre after running.

Assumptions

(i) Air is an ideal gas.
(ii) Process of heating is reversible.

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Analysis Absolute pressure of air in the tyre at intial state.

p_1 = Atmospheric pressure + gauge pressure

or $p_1 = p_{atm} + p_{g_1} = 1.0132 + 1.75 = 2.7632$ bar

In the tyre, the volume remains constant ($V_1 = V_2$), Eq. (3.49) reduces to

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

Hence after running, i.e., at the state 2, the pressure p_2

$$p_2 = \frac{p_1}{T_1} \times T_2 = \frac{(2.7632 \text{ bar})}{(298 \text{ K})} \times (343 \text{ K}) = 3.18 \text{ bar}$$

The gauge pressure at the state 2;

$$p_{g_2} = p_2 - p_{atm} = 3.18 - 1.0132 = \mathbf{2.167 \text{ bar}}$$

Example 3.17 *The compression ratio of an engine is 15 to 1. The pressure of the gas at the beginning of the stroke is 100 kPa and the temperature is 30°C. Calculate the absolute pressure at the end of compression stroke, if the temperature used to be 980°C.*

Solution

Given Compression of a gas in an engine with

State 1: $V_1 = 15V_2$ $p_1 = 100$ kPa

and $T_1 = 30°\text{C} = 303$ K

State 2: $T_2 = 980°\text{C} + 273 = 1253$ K

To find The pressure of gas after compression

Analysis Using properties relationship, Eq. (3.49)

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

or $$p_2 = \frac{p_1 V_1}{T_1} \times \frac{T_2}{V_2}$$

$$= \frac{(100 \text{ kPa}) \times (15\ V_2) \times (1253 \text{ K})}{(303 \text{ K}) \times V_2}$$

$$= 6203 \text{ kPa} = \mathbf{62.03 \text{ bar}}$$

Example 3.18 *A cylinder of 50-litre capacity contains oxygen at 18°C and at a pressure of 10 MPa. Calculate*

(a) the mass of oxygen in the cylinder,

(b) the molar volume,

(c) the density of oxygen.

The molecular mass of oxygen is 32 kg/kmol.

Solution

Given An oxygen cylinder with constant volume

$V = 50 \text{ lit} = 50 \times 10^{-3} \text{ m}^3$,

$T = 18°\text{C} + 273 = 291$ K

$p = 10 \text{ MPa} = 10 \times 10^3 \text{ kN/m}^2$

$\mathcal{M} = 32$ kg/kmol

To find

(i) Mass of oxygen in the cylinder,

(ii) Molar volume,

(iii) Density of oxygen.

Analysis The universal gas constant $R_u = 8.31447$ kJ/kmol·K and the specific gas constant R for oxygen can be calculated as

$$R = \frac{R_u}{\mathcal{M}} = \frac{8.31447 \text{ kJ/kmol} \cdot \text{K}}{32 \text{ kg/kmol}}$$

$$= 0.259 \text{ kJ/kg} \cdot \text{K}.$$

(i) *Mass of oxygen in cylinder* It can be calculated from the characteristic gas equation.

$$pV = mRT$$

or $$m = \frac{pV}{RT}$$

$$= \frac{(10 \times 10^3 \text{ kN/m}^2) \times (50 \times 10^{-3} \text{ m}^3)}{(0.259 \text{ kJ/kg} \cdot \text{K}) \times (291 \text{ K})}$$

$$= \mathbf{6.63 \text{ kg}}$$

(ii) *Molar volume or molecular volume*

$$pV = R_u T$$

or $$V = \frac{R_u T}{p} = \frac{(8.31447 \text{ kJ/kmol} \cdot \text{K}) \times (291 \text{ K})}{(10 \times 10^3 \text{ kN/m}^2)}$$

$$= \mathbf{0.24 \text{ m}^3/\text{kmol}}$$

(iii) *Density of oxygen* From the characteristic gas equation for unit mass,

$$pv = RT$$

We have $$\rho = \frac{1}{v}$$

Therefore, $$\rho = \frac{p}{RT} = \frac{(10 \times 10^3 \text{ kN/m}^2)}{(0.259 \text{ kJ/kg} \cdot \text{K}) \times (291 \text{ K})}$$

$$= \mathbf{132.68 \text{ kg/m}^3}$$

Example 3.19 *10 kg of carbon monoxide at 40°C occupies 3 m³. Determine the gas pressure in bar. An additional mass of carbon monoxide is then very slowly added to raise the tank pressure to 10 bar. Assuming that the gas temperature remains constant, how much extra mass has been added?*

Assume R = 0.297 kJ/kg·K.

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Solution

Given A constant volume and constant temperature process;

$m_1 = 10$ kg $\quad T_1 = 40°C + 273 = 313$ K $\quad V = 3$ m^3

$T_1 = T_2$, $\quad p_2 = 10$ bar $\quad R = 0.297$ kJ/kg·K

To find

(i) Initial gas pressure in bar,

(ii) Mass of CO added to system.

Assumptions

(i) Carbon monoxide is a perfect gas.

(ii) Process is reversible and nonflow.

Analysis

(i) The characeristic gas equation,

$$p_1 V = m_1 R T_1$$

Initial gas pressure

$$p_1 = \frac{m_1 R T_1}{V}$$

$$= \frac{(10 \text{ kg}) \times (0.297 \text{ kJ/kg} \cdot \text{K}) \times (313 \text{ K})}{(3 \text{ m}^3)}$$

$$= 309.87 \text{ kPa.} = \mathbf{3.098 \text{ bar}}$$

(ii) Since additional CO is added to the tank, as $V = C$ and $T = C$

$$m \propto \text{pressure}$$

Therefore, $\quad \frac{m_2}{m_1} = \frac{p_2}{p_1} = \frac{10}{3.0987}$

and $\quad m_2 = \frac{10 \times 10}{3.0987} = 32.2768$ kg

Additional mass $m = m_2 - m_1$

$= 32.2768 - 10 = \mathbf{22.2768 \text{ kg}}$

Example 3.20 *A spherical balloon of 8 m radius is floating in the atmosphere at 100 kPa pressure and 300 K temperature. Determine the mass and number of moles of air displaced by the balloon.*

If hydrogen gas is filled in the balloon under the same conditions of temperature and pressure, calculate the mass and number of moles of hydrogen. Molecular masses of air and hydrogen are taken as 28.97 and 2, respectively.

Solution

Given A spherical balloon filled with air/hydrogen

$r_o = 8$ m $\quad p = 100$ kPa $\quad T = 300$ K

$\mathcal{M}_{air} = 28.97 \quad \mathcal{M}_{H_2} = 2$

To find In each case,

(i) Mass of gas, and

(ii) Number of moles.

Analysis The specific gas constants:

For air $\quad R_{air} = \frac{R_u}{\mathcal{M}_{air}} = \frac{8.31447}{28.97} = 0.287$ kJ/kg·K

For hydrogen, $\quad R_{H_2} = \frac{R_u}{\mathcal{M}_{H_2}} = \frac{8.31447}{2}$

$= 4.157$ kJ/kg·K

Volume of balloon $V = \frac{4}{3} \pi r_o^3$

$= \frac{4}{3} \times \pi \times (8)^3 = 2144.66$ m^3

(i) *When air is filled in the balloon*

(a) Mass of air filled

$$m_{air} = \frac{pV}{R_{air} T} = \frac{100 \times 2144.66}{0.287 \times 300} = \mathbf{2491 \text{ kg}}$$

(b) Number of moles of air

$$n = \frac{m_{air}}{\mathcal{M}_{air}} = \frac{2491}{28.97} = \mathbf{85.98}$$

(ii) *When hydrogen is filled in the balloon under indentical conditions*

(a) $m_{H_2} = \frac{pV}{R_{H_2} T} = \frac{100 \times 2144.66}{4.157 \times 300} = \mathbf{172 \text{ kg}}$

(b) Number of moles of H_2,

$$n = \frac{172}{2} = \mathbf{86}$$

Example 3.21 *An aerostat balloon is filled with hydrogen, it has a volume of 1000 m^3 at a temperature of 300 K and pressure of 100 kPa. Determine the payload that can be lifted with the aid of the aerostat.*

Solution

Given An aeorstat filled with hydrogen

$V = 1000$ m^3 $\quad p = 100$ kPa $\quad T = 300$ K

To find The payload that can be lifted by the aerostat.

Assumptions

(i) Hydrogen as an ideal gas.

(ii) Molecular weight of H_2 as 2 kg/kmol,

$R_{H_2} = 4.157$ kJ/kg·K.

(iii) Gas constant of air $R_{air} = 0.287$ kJ/kg·K.

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Analysis The mass of hydrogen in the balloon

$$m_{H_2} = \frac{pV}{R_{H_2}T} = \frac{100 \times 1000}{4.157 \times 300} = 80.18 \text{ kg}$$

The balloon displaces the volume of air equivalent to its volume at the same pressure and temperature.

The volume displaced by the balloon

= volume of air equivalent to volume of the balloon (1000 m^3)

Thus, the mass of air displaced

$$m_2 = \frac{pV}{R_{air}T} = \frac{100 \times 1000}{0.287 \times 300} = 1161.44 \text{ kg}$$

The load that can be attached with the hydrogen balloon

$$= m_2 - m_1 = 1161.44 - 80.18 = \mathbf{1081.26\ kg}$$

Example 3.22 *A room of 1000 m^3 volume, contains air at 1 bar and 300 K. In the room, there is a balloon which contains air at 300 K. Suddenly, the balloon bursts. Before any air can escape through the doors or windows, the pressure in the room becomes 1.3 bar, while its temperature is still 300 K. If the volume of balloon was 50 m^3, find:*

(a) mass of air originally in the room (outside the balloon),

(b) initial pressure in bar of air in the balloon, and

(c) final mass of air in the room.

Solution

Given A room with a balloon

$V_{room} = 1000\ m^3$ $V_{balloon} = 50\ m^3$

$T = 300$ K $p_1 = 1$ bar $= 100$ kPa,

$p_2 = 1.3$ bar 130 kPa

To find

(i) Mass of air originally in the room minus balloon contents,

(ii) Final mass of air in the room, and

(iii) Initial pressure of air in the balloon.

Assumption The specific gas constant for air,

$$R = 0.287 \text{ kJ/kg} \cdot \text{K}$$

Analysis The volume of air in the room excluding the balloon's volume

$$V_1 = V_{room} - V_{balloon} = 1000 - 50 = 950\ m^3$$

(i) *Mass of air originally in the room minus volume of balloon*

$$m_{initial} = \frac{p_1 V_1}{RT} = \frac{(100 \text{ kPa}) \times (950\ m^3)}{(0.287 \text{ kJ/kg} \cdot \text{K}) \times (300 \text{ K})}$$

$$= \mathbf{1103.37\ kg}$$

(ii) *Final mass of air after mixing of air of balloon in the room:*

$$m_{final} = \frac{p_2 V_{room}}{RT} = \frac{(130 \text{ kPa}) \times (1000\ m^3)}{(0.287 \text{ kJ/kg} \cdot \text{K}) \times (300 \text{ K})}$$

$$= \mathbf{1509.9\ kg}$$

(iii) *Initial pressure of air in the balloon*

The mass of air in balloon

$$m_{balloon} = m_{final} - m_{initial}$$

$$= 1509.9 \text{ kg} - 1103.37 \text{ kg} = 406.5 \text{ kg}$$

Pressure

$$p_{ballon} = \frac{m_{balloon}\, RT}{V_{balloon}}$$

$$= \frac{(406.5 \text{ kg}) \times (0.287 \text{ kJ/kg} \cdot \text{K}) \times (300 \text{ K})}{(50\ m^3)}$$

$$= \mathbf{700\ kPa\ or\ 7\ bar}$$

3.24 INTERNAL ENERGY AND ENTHALPY OF AN IDEAL GAS

Joule submerged two tanks *A* and *B* connected by a valve in an insulated tank of water as shown in Fig. 3.26.

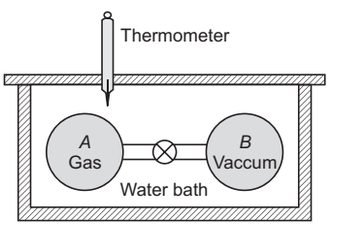

Fig. 3.26 *Joules Experiment demonstrating energy is a function of temperature*

One tank *A* was filled with air and the other tank *B* was evacuated. The air, tanks and surrounding water were allowed to come to thermal equilibrium. The temperature of the water bath was measured.

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Then the valve between the two tanks was opened to let the air pass as a result of free expansion from one tank to other. Of course, no work transfer was involved. Joule observed no change in temperature of the water bath during free expansion of air. From this experiment, he concluded that in the absence of any heat and work interactions to or from the system of air; the internal energy of air did not change. The temperature of air remained constant even though pressure and volume changed. He established that the internal energy of an ideal gas does not depend on pressure and volume, it depends only on temperature.

Hence, the conclusion of Joule's experiment was that the *internal energy of an ideal gas is a function of temperature only.* It is known as *Joule's law* and can be stated as

$$u = u(T) \qquad ...(3.50)$$

In differential form, $du = C_v\, dT$ ...(3.51))

The change in specific internal energy for an ideal gas during a process from the state 1 to state 2 is obtained by integrating Eq. (3.51);

$$\Delta u = \int_1^2 C_v dT$$

or $$\Delta u = u_2 - u_1 = C_v (T_2 - T_1) \qquad ...(3.52)$$

By definition, the enthalpy per unit mass,

$$h = u + pv$$

For an ideal gas, $pv = RT$

Thus $$h = u + RT \qquad ...(3.53)$$

Both terms on the right side of the equation depend on temperature only, so *enthalpy h* is *function of temperature* only. Thus

$$h = h(T) \qquad ...(3.54)$$

In differential form, $dh = C_p\, dT$ ...(3.55)

The change in specific enthalpy for an ideal gas during a process from the state 1 to state 2 is obtained by integrating Eq. (3.55);

$$\Delta h = \int_1^2 C_p dT$$

or $$\Delta h = h_2 - h_1 = C_p (T_2 - T_1) \qquad ...(3.56)$$

3.25 SPECIFIC HEATS OF IDEAL GASES

All the gases have two kinds of specific heats: *specific heat for constant volume* C_v and *specific heat at constant pressure*, C_p.

3.25.1 Specific Heat at Constant Volume, C_v

The *specific heat at constant volume* is defined as the amount of heat energy transferred to change the temperature of the unit mass of a substance by one degree, when the volume of substance is maintained constant.

Consider a closed system of an ideal gas undergoing a change of state at constant volume. According to the first law of thermodynamics for unit mass system,

$$\delta q = \delta w + du = p\, dv + du$$

But for constant volume, $dv = 0$; thus $\delta w = 0$, therefore,

$$\delta q = du = C_v\, dT$$

or $$C_v = \left(\frac{du}{dT}\right)_v \qquad ...(3.57)$$

Further, the specific heat at constant volume C_v can also be defined as *change of specific internal energy of a substance per unit change in temperature at constant volume.* It relates the change in properties, and thus it is also a *property.*

3.25.2 Specific Heat at Constant Pressure C_p

It is defined as the amount of heat energy transferred to change the temperature of unit mass of a substance by one degree when the pressure is maintained constant during the process.

Consider an ideal gas undergoing a process at constant pressure. The enthalpy for unit mass, in differential form can be expressed as

$$dh = du + d(pv)$$

As the pressure is constant, therefore,

$$dh = du + pdv = dq$$

or, $$dh = C_p\, dT$$

or $$C_p = \left(\frac{dh}{dT}\right)_p \qquad ...(3.58)$$

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Further, the specific heat at constant pressure C_p can also be defined as *change in the specific enthalpy of a substance per unit change in temperature at a constant pressure.* It relates the change in properties, and thus it is also a *property.*

These specific heats are properties of the gas and both vary with pressure and temperature. However, for an ideal gas they are assumed constant.

For air $C_p = 1.005$ kJ/kg·K, and
$C_v = 0.718$ kJ/kg·K

3.26 RELATION BETWEEN SPECIFIC HEATS FOR AN IDEAL GAS

A special relationship between C_p and C_v, for an ideal gas can be obtained by differentiating the enthalpy relation Eq. (3.53);

$$dh = du + R\,dT$$

Replacing dh by $C_p\,dT$ and du by $C_v\,dT$, then

$$C_p\,dT = C_v\,dT + R\,dT$$

Dividing both sides by dT, we get

$$C_p = C_v + R \text{ (kJ/kg·K)} \qquad ...(3.59)$$

It is a very important relationship for ideal gases. The ratio of the two specific heats is defined as

$$\gamma = \frac{C_p}{C_v} \qquad ...(3.60)$$

Example 3.23 *10 kg of air is heated in a rigid vessel from 20°C to 100°C. If the ratio of specific heat is 1.4, estimate the values of C_p and C_v, change in internal energy, and enthalpy.*

Solution

Given Air in a rigid vessel

V = Const. m = 10 kg γ = 1.4

State 1: $T_1 = 20°C = 293$ K,

State 2: $T_2 = 100°C = 373$ K,

To find

(i) C_p (ii) C_v
(iii) ΔU and (iv) ΔH

Assumption The specific gas constant for air as R = 0.287 kJ/kg·K.

Analysis

(i) The specific heat at constant volume is given as

$$C_v = \frac{R}{\gamma - 1} = \frac{(0.287 \text{ kJ/kg·K})}{1.4 - 1}$$

$$= \mathbf{0.7175\ kJ/kg·K}$$

(ii) The specific heat at constant pressure is given as

$C_p = C_v + R = 0.7175 + 0.287 =$ **1.0045 kJ/kg·K**

(iii) The change in internal energy

$$\Delta U = m\,C_v\,(T_2 - T_1)$$
$$= 10 \times 0.7175 \times (373 - 293)$$
$$= \mathbf{574\ kJ}$$

(iv) The change in enthalpy

$$\Delta H = m\,C_p\,(T_2 - T_1)$$
$$= 10 \times 1.0045 \times (373 - 293)$$
$$= \mathbf{803.6\ kJ}$$

Example 3.24 *A gas initially at a pressure of 510 kPa and a volume of 142 litres undergoes a process and has a final pressure of 170 kPa and a volume of 275 litres. During the process, the enthalpy decreases by 65 kJ. Take $C_v = 0.718$ kJ/kg·K.*

Determine (a) change in internal energy, (b) specific heat at constant pressure, and (c) specific gas constant.

Solution

Given A gas process

State 1: $p_1 = 510$ kPa $V_1 = 142$ litres $= 142 \times 10^{-3}$ m^3

State 2: $p_2 = 170$ kPa $V_2 = 275$ litres $= 275 \times 10^{-3}$ m^3

$\Delta H = -65$ kJ (since it is decreased)

$C_v = 0.718$ kJ/kg·K

To find

(i) Change in internal energy,
(ii) Specific heat at constant pressure,
(iii) Specific gas constant.

Analysis

(i) The change in enthalpy can be expressed as

$$\Delta H = \Delta U + p_2V_2 - p_1V_1$$

Using numerical values;

$$-65 = \Delta U + (170) \times (275 \times 10^{-3}) - (510) \times (142 \times 10^{-3})$$

or $-65 = \Delta U + (46.75 - 72.42)$

or $\Delta U = -65 + 72.42 - 46.75 = \mathbf{-39.33\ kJ}$

(decrease in internal energy)

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(ii) The change in internal energy can also be expressed as

$$\Delta U = m\,C_v\,(\Delta T)$$

or $\quad -39.33 = 0.718\ (m\Delta T)$

or $\quad m\,\Delta T = -54.777$

Further the change in enthalpy can also be expressed as

$$\Delta H = m\,C_p\,\Delta T = C_p\,(m\,\Delta T)$$

or $\quad -65 = C_p\,(-54.777)$

or $\quad C_p = \dfrac{65}{54.777} = \mathbf{1.1866\ kJ/kg \cdot K}$

(iii) Specific gas constant,

$$R = C_p - C_v$$
$$= 1.1866 - 0.718 = \mathbf{0.468\ kJ/kgK}$$

Example 3.25 *1 kg of a certain gas undergoes a reversible constant pressure process at 1.2 bar, during which its volume changes from 1 m^3 to 1.8 m^3 and the temperature changes from 50°C to 370°C. The specific heat for a substance at constant pressure is given by*

$$C_p = \left[1.1 + \frac{40}{T + 30}\right]\ kJ/kg \cdot K,\ \text{where } T \text{ is in °C.}$$

Find *(a) Heat supplied (b) Work done (c) Change in internal energy, (d) Change in enthalpy*

Solution

Given A constant pressure process with

$m = 1$ kg $\qquad p_1 = 1.2$ bar $= 120$ kPa

State 1: $V_1 = 1\ m^3 \qquad T_1 = 50°C$

State 2: $V_2 = 1.8\ m^3 \qquad T_2 = 370°C$

and $\quad C_p = \left[1.1 + \dfrac{40}{T + 30}\right]$ kJ/kg K

To find

(i) Heat supplied,
(ii) Work done,
(iii) Change in internal energy, and
(iv) Change in enthalpy.

Analysis

(i) Heat supplied at constant pressure

$$Q_{1-2} = \int_1^2 m C_p\,dT$$

$$= m\int_{T_1}^{T_2}\left[1.1 + \frac{40}{T+30}\right]dT$$

$$= m\left[1.1\,T + 40 \times \ln(T+30)\right]_{T_1}^{T_2}$$

$$= m\left[1.1\,(T_2 - T_1) + 40 \times \ln\left(\frac{T_2+30}{T_1+30}\right)\right]$$

Using numerical values, we get

$$Q_{1-2} = 1 \times \left[1.1 \times (370 - 50) + 40 \times \ln\left(\frac{370+30}{50+30}\right)\right]$$

$$= 352 + 64.37 = \mathbf{416.37\ kJ}$$

(ii) Work done at constant pressure

$$W_{1-2} = \int_1^2 pdV = p_1(V_2 - V_1)$$
$$= (120\ \text{kPa}) \times (1.8 - 1)\ (m^3)$$
$$= \mathbf{96\ kPa}$$

(iii) Change in internal energy can be obtained as

$$\Delta U = Q - W = 416.37 - 96$$
$$= \mathbf{320.37\ kJ}$$

(iv) Change in enthalpy

$$\Delta H_{1-2} = Q_{1-2} = \mathbf{416.37\ kJ}$$

3.27 REAL GASES

The ideal gas equation of state, $pv = RT$ can be used with the assumption of very little or no attraction force of molecules within the gas and the volume of molecules is negligibly small in comparison to volume of gas. For many gases, at very low pressure and very high temperature, the forces of intermolecular attraction and volume of molecules compared to the volume of gas are significantly small and the real gases obey very closely the ideal gas equation.

But at higher pressure, the forces of intermolecular attraction and repulsion are significant and the volume of molecules are also appreciable compared to total gas volume. Then the real-gas behaviour deviates from ideal-gas behaviour. The slight modification in the ideal-gas equation of state to fit

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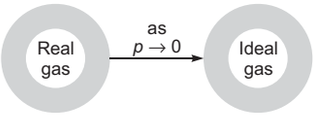

Fig. 3.27 *At very low pressures, all gases approach ideal-gas behaviour (regardless of their temperature)*

real-gas behaviour is to introduce a correction factor called *compressibility factor Z*. It is defined as

$$Z = \frac{pv}{RT} \qquad \ldots(3.61)$$

For an ideal gas, obviously, $Z = 1$

For a real gas, Z is a function of pressure and temperature and it is usually determined empirically.

The real gases behave differently at a given temperature and pressure, but they behave very likely at their reduced pressure and reduced temperature. These are

Reduced pressure $$p_R = \frac{p_{actual}}{p_{critical}} = \frac{p}{p_c} \qquad \ldots(3.62)$$

Reduced temperature $$T_R = \frac{T_{actual}}{T_{critical}} = \frac{T}{T_c} \qquad \ldots(3.63)$$

Reduced sp. volume $$v_R = \frac{v_{actual}}{v_{critical}} = \frac{v}{v_c} \qquad \ldots(3.64)$$

where the subscript c represents critical state.

The disadvantage of the use of compressibility factor chart is that a separate chart is needed for each gas. But at the same reduced coordinates, the compressibility factor Z is approximately same for all gases. It is called *law of corresponding states*, thus

$$v_R = f(p_R, T_R) \qquad \ldots(3.65)$$

A diagram of p_R versus T_R as a parameter plotted from data of any gas would apply to all other gases. The Fig. 3.28 shows a generalised *compressibility chart*, where Z values are plotted against p_R, and T_R for several gases. The use of compressibility chart requires the knowledge of critical parameter given in factor and T_R for several gases. The results obtained by using compressibility chart are accurate within few percentage. The deviation of real-gas behaviour from that of ideal gas is greatest in vicinity of the critical point.

Better results can be obtained for some gases by using pseudo-reduced coordinates in place of reduced properties. These are

$$p'_R = \frac{p}{p_c + A}, \; T'_R = \frac{T}{T_c + B} \qquad \ldots(3.66)$$

where A and B are some constants and presudo-reduced specific volume v'_R is defined as

$$v'_R = \frac{v}{RT_c/p_c} \qquad \ldots(3.67)$$

Thus, v'_R is related to p_c and T_c not to v_c.

Example 3.26 *1 mole of CO_2 at 88.7 bar and 61°C is compressed in reversible isothermal manner till the volume halves. Calculate the work transfer during the process.*

Solution

Given Isothermal compression of CO_2

$p_1 = 88.7$ bar $= 8870$ kPa $\qquad T_1 = 61°C = 334$ K

$v_2 = \frac{1}{2} v_1$, $\qquad n = 1$ mole

To find Work transfer during the process.

Analysis The CO_2 is real gas, the gas compressibility is to include.

The isothermal work

$$W = \int p dv = \int \frac{nR_u T}{v} dv$$

For a real gas

$$W = nR_u T Z \int_{v_1}^{v_2} \frac{dv}{v} = nR_u T Z \ln\left(\frac{v_2}{v_1}\right)$$

where $Z = f(p_R, T_R)$

and $p_R = \frac{p}{p_c}$ and $T_R = \frac{T}{T_c}$

From Table A–1; For CO_2, $p_C = 73.9$ bar, $T_c = 304$ K

Then $p_R = \frac{88.7}{73.9} = 1.2$, $\qquad T_R = \frac{334}{304} = 1.1$

$Z = 0.62$

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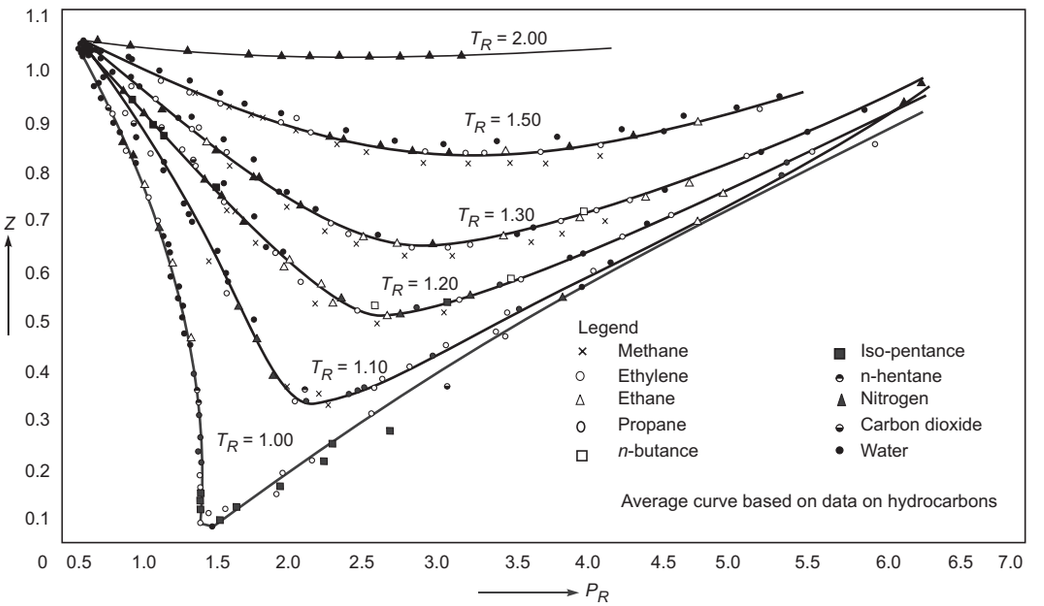

Fig. 3.28 *Compressibility chart for various gases*

The work transfer

$$W = 1 \times 8.314 \times 334 \times 0.62 \ln (1/2)$$
$$= \mathbf{-1229\ kJ}$$

Example 3.27 *Prove that the superheated steam can be modelled as an ideal gas.*

Solution Let v_{sup} is the specific volume of superheated steam and v_g is the volume of an ideal gas.

Consider $p = 10$ kPa and $T_{sup} = 500°C$

$v_{sup} = 35.6789$ m^3/kg (from steam tables)

For superheated steam to be ideal gas

$$v_g = \left(\frac{R_u}{\mathcal{M}}\right)\frac{T}{p} = \left(\frac{8.314}{18}\right) \times \frac{(500 + 373)(\text{K})}{(10\,\text{kPa})}$$
$$= 35.7040 \text{ m}^3/\text{kg}$$

$$\text{Percentage error} = \frac{v_g - v_{sup}}{v_g} \times 100 = 0.07\%$$

Now consider steam at 100 bar and 700°C

$$v_{sup} = 0.04358 \text{ m}^3/\text{kg}$$

For superheated steam as an ideal gas

$$v_g = \left(\frac{8.314}{18}\right)\left(\frac{700 + 273}{100 \times 100\ \text{kPa}}\right)$$
$$= 0.04494 \text{ m}^3/\text{kg}$$

Percentage error = 3%

Steam at 500 bar and 1200°C

$$v_{sup} = 0.013561 \text{ m}^3/\text{kg}$$

For superheated steam as an ideal gas

$$v_g = \left(\frac{8.314}{18}\right) \times \left(\frac{1200 + 273}{500 \times 100}\right)$$
$$= 0.01360 \text{ m}^3/\text{kg}$$

Percentage error = 0.34%

The above calculations show that superheated steam approximates an ideal gas reasonably well at low and high pressures with high degree of superheat.

3.28 OTHER EQUATIONS OF STATE

Many extensive p, v, T data are available, but no equation of state that represents the p, v, T behaviour of the gas could be used accurately over a larger region.

1. Van der Waals Equation In 1873, J.D. van der Waals presented an equation of state which was of interest on physical reasoning, introduced two correction constants in the equation of ideal gas

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equation of state.

$$\left(p+\frac{a}{\overline{v}^2}\right)\left(\overline{v}-b\right)=R_uT \qquad \ldots(3.68)$$

The constant a was introduced to account for the existence of intermolecular attraction, the constant b was introduced to account for volumes of molecules and R_u is the universal gas constant. These constants are evaluated from the behaviour of the gas at critical point. It was assumed that critical temperature line on a p–v diagram is horizontal at critical point, thus

$$\left(\frac{\partial p}{\partial T}\right)_{T_c}=\frac{R_uT_c}{(\overline{v}_c-b)^2}+\frac{2a}{\overline{v}_c^3}=0$$

and $$\left(\frac{\partial^2 p}{\partial \overline{v}^2}\right)_{T_c=Const}=\frac{2R_uT_c}{(\overline{v}_c-b)^3}+\frac{6a}{\overline{v}_c^4}=0$$

Solving these equations, we get

$$a=\frac{27R_u^2T_c^2}{64p_c},\ b=\frac{R_uT_c}{8p_c}$$

and $$\frac{R_uT_c}{p_c\overline{v}_c}=\frac{8}{3} \qquad \ldots(3.69)$$

Except at higher pressures, the real gases do not obey van der Waals equation in all ranges of pressures and temperatures. Despite of its limitations, the van der Waal equation has a historical importance because it was the first attempt to model the behaviour of real gases.

2. Redlich-Kwong-Equation In 1949, Redlich and kwong presented an equation of state with two constants, which is more accurate than the van der Waals equation over a wide range.

$$p=\frac{R_uT}{\overline{v}-b}-\frac{a}{\overline{v}(\overline{v}+b)T^{1/2}} \qquad \ldots(3.70)$$

where R_u = universal gas constant = 8.314 kJ/kg mol.K and $\overline{v}$ is molal volume. The values of these two constants are also determined from critical state parameter and these are

$$a=0.427\frac{R_u^2T_c^{2.5}}{p_c}$$

and $$b=0.0866\frac{R_uT_c}{p_c} \qquad \ldots(3.71)$$

3. Beattie-Bridgeman Equation In 1928, Beattie-Bridgeman proposed an equation of state. The measured values of pressure, temperature and specific volume with good accuracy. It is in the form of

$$p=\frac{R_uT}{\overline{v}^2}\left\{1-\frac{C}{\overline{v}T^3}\right\}\left(\overline{v}+B\right)-\frac{A}{\overline{v}^2} \qquad \ldots(3.72)$$

Where $$A=A_0\left(1-\frac{a}{\overline{v}}\right)$$

and $$B=B_0\left(1-\frac{b}{\overline{v}}\right) \qquad \ldots(3.73)$$

Here $\overline{v}$ = molal volume in m^3/kg mol

R_u = Universal gas constant

= 8.314 kJ/kg mol. K

The constants appearing in above equation are given in Table 3.6 below:

4. Benedict-Webb-Rubin Equation of State In 1940, Benedict-Webb and Rubin extended the work of Beattle Bridgeman by increasing the number of constants to eight. It is expressed as

Table 3.6 *Constants that appear in the Beattie-Bridgeman equations of state*

Gas	A_0	a	B_0	b	$C\times10^{-4}$
Air	131.8441	0.019 31	0.046 11	−0.001 101	4.34
Argon	130.7802	0.023 28	0.039 31	0.0	5.99
Carbon dioxide	507.2836	0.071 32	0.104 76	0.072 35	66.00
Helium	2.1886	0.059 84	0.014 00	0.0	0.0040
Hydorgen	20.0117	−0.005 06	0.020 96	−0.043 59	0.0504
Nitrogen	136.2315	0.026 17	0.050 46	−0.006 91	4.20
Oxygen	151.0857	0.025 62	0.046 24	0.004 208	4.80

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Table 3.7 *The Constants that appear in Benedict-Webb-Rubin equation of state*

Gas	a	A_0	b	B_0	c	C_0	α	γ
n-Butane, C_4H_{10}	190.68	1021.6	0.039 998	0.124 36	3.205×10^7	1.006×10^8	1.101×10^{-3}	0.0340
Carbon dioxide, CO_2	13.86	277.30	0.007 210	0.049 91	1.511×10^6	1.404×10^7	8.470×10^{-5}	0.00539
Carbon monoxide, CO	3.71	135.87	0.002 632	0.054 54	1.054×10^5	8.673×10^5	1.350×10^{-4}	0.0060
Methane, CH_4	5.00	187.91	0.003 380	0.042 60	2.578×10^5	2.286×10^6	1.244×10^{-4}	0.0060
Nitrogen, N_2	2.54	106.73	0.002 328	0.040 74	7.379×10^4	8.164×10^5	1.272×10^{-4}	0.0053

$$p = \frac{R_u T}{\bar{v}} + \left\{ B_0 R_u T - A_0 - \frac{C_0}{T^2} \right\} \frac{1}{\bar{v}^2} + \frac{bR_u T - a}{\bar{v}^3} + \frac{a\alpha}{\bar{v}^6} + \frac{c}{\bar{v} T^3} \left[1 + \frac{\gamma}{\bar{v}^2} \right] e^{-\gamma/\bar{v}^2} \quad \ldots(3.74)$$

where p = pressure in kPa,
$\bar{v}$ = molal volume, m^3/kg mol,
T = Absolute temperature, K
R_u = Universal gas constant
= 8.314 kJ/k mol. K

A_0, B_0, C_0, a, b, c, α, γ are Eight constants.

The values of these constants appearing in the equation are given in Table 3.7 below.

5. Virial Equation of State The equation of state for a real gas can be expressed in the form of power series

$$\frac{pv}{RT} = 1 + \frac{a}{v} + \frac{b}{v^2} + \frac{c}{v^3} + \frac{d}{v^4}$$

where a, b, c, d, etc., are second, third, fourth, etc., virial *coefficients* and these are function of temperature only. These coefficients can be determined experimentally or theoretically from statistical mechanics. The above equation and several other equations of such form are known as *virial equations*.

The accuracy of virial equation of state depends on number of coefficients considered.

The all equations of state discussed above can only be applied to gaseous phase of the substance.

The region in the vicinity of $p = 0$ corresponds to concept of ideal gas with negligible force of attraction between molecules ($pv/RT = 1$). As density increases at constant temperature, the binary collision between molecules becomes more frequent and second virial coefficient exerts with an increasing effect on pv/RT. The effect may be either positive or negative. The positive value means replusion of molecules and negative value indicates attraction between molecules.

It is assumed that when second virial coefficient is negative then third and succeeding virial coefficients must be positive. The number of terms included in virial equation depends upon the region under study.

The virial equation can be converted into power series in the form

$$\frac{pv}{RT} = 1 + A_1 p + A_2 p^2 + A_3 p^3 \quad \ldots(3.75)$$

Where $A_1, A_2, A_3 \ldots$ are *virial coefficients*, function of temperature alone, and virial coefficients are related as

$$A_1 = \frac{a}{RT}, \quad A_2 = \frac{b - a^2}{R^2 T^2} \quad \ldots(3.76)$$

(a) For an ideal gas, an isometric line would be a straight line, starting from origin and with slope proportional to density,

(b) For real gas, the isometrics are displaced from origin and either straight line or a curve.

Note: Eqn (3.68) to (3.76) can also be used for real gases by replacing molar volume $\bar{v}$ by specific volume v and universal gas constant R_u by specific gas constant R.

3.29 ANALYTICAL TEST FOR EQUATION OF STATE

The equation of state and physical picture of an ideal gas can be obtained from property relationships.

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Consider a gas that obeys Joule's law by equation

$$du = C_v\,dT + \left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right]dv$$

and at constants temperature, $dT = 0$, then

$$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - p = 0 \qquad \ldots(3.77)$$

Hence $$\left(\frac{\partial p}{\partial T}\right)_v = \frac{p}{T} = \frac{f(v,T)}{T} \qquad \ldots(3.78)$$

Eq. (3.78) shows that the partial derivative with respect to temperature must be equal to original function of v, T divided by temperature and hence, the solution of Eq. (3.78) is

$$p = f(v, T) \qquad \ldots(3.79)$$

It is the Joule's law of equation of state for a gas with attraction or repulsion forces.

The typical behaviour of internal energy for a real gas is shown in Fig. 3.29.

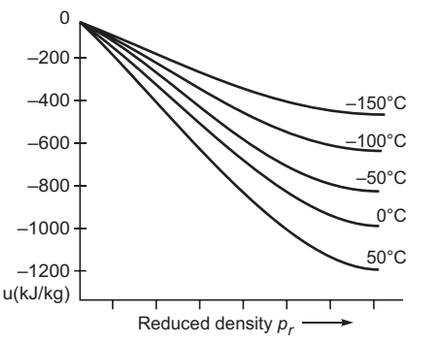

Fig. 3.29 *The internal energy of real gas is function of density and temperature*

There are certain conditions that should be obeyed by an equation of state.

(i) The equation of state should be reduced to an ideal gas equation of state, when its pressure isothermally approaches to zero.

$$\lim_{p\to 0} pv = RT$$

(ii) The critical isotherm of equation of state should have a horizontal tangent and at a point of inflection at critical point

$$\left(\frac{\partial p}{\partial v}\right)_T = 0 \quad \text{and} \quad \left(\frac{\partial^2 p}{\partial v^2}\right) = 0$$

Also at critical point the third derivative should be either zero or negligible, but fourth derivative must be zero.

$$\left(\frac{\partial^3 p}{\partial v^3}\right)_T = (0 \text{ or } -\text{ve}) \quad \text{and} \quad \left(\frac{\partial^4 p}{\partial v^4}\right) = 0$$

(iii) The curvature of isometrics of equation of state on p–T diagram should be negative, positive or zero in the various region. In particular, the critical isometric should be straight, while the curvature of all other isometric should approach zero with either decreasing density or increasing temperature;

$$\left(\frac{\partial^2 p}{\partial T^2}\right)_v \gtreqless 0 \text{ and } \left(\frac{\partial^2 p}{\partial T^2}\right)_v = 0 \Bigg\} \begin{matrix} v = c \\ p \to 0 \\ T \to \infty \end{matrix}$$

(iv) The slope of isotherm of equation of state on the Z-p plot or compressibility factor, Z should be negative, positive or zero in the various regions.

$$\left(\frac{\partial Z}{\partial p}\right)_T = \frac{1}{R_u T}\left[\frac{\partial (pv)}{\partial p}\right]_T$$

$$= \frac{1}{R_u}\left[v + p\left(\frac{\partial v}{\partial p}\right)\right]_T \gtreqless 0$$

$$\lim_{p\to 0}\frac{\partial^2 Z}{\partial p\,\partial T} = 0, \quad \lim_{p\to 0}\frac{\partial^2 Z}{\partial \rho\,\partial T} = 0$$

(v) The equation of state should predict the Joule–Thomson coefficient

$$\mu = \frac{1}{C_p}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = \frac{R_u T^2}{\rho C_p}\left(\frac{\partial Z}{\partial T}\right)_p$$

at inversion state $\mu = 0$

$$T\left(\frac{\partial Z}{\partial T}\right)_p = v \quad \text{or} \quad \left(\frac{\partial Z}{\partial T}\right)_p = 0$$

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Example 3.28 *Consider nitrogen gas at 145 K with specific volume of 0.0022 m^3/kg. Calculate its pressure using Redlich Kwong equation and compare with pressure obtained by ideal gas equation. Use p_c = 3390 kPa, T_c = 126.2 K.*

Solution

Given Nitrogen gas with specific volume

$v = 0.0022\ m^3/kg, \quad T = 145\ K$

$p_c = 3390\ kPa \quad T_c = 126.2\ K$

To find Pressure of gas by Redlich Kwong equation and then by ideal gas equation and compare it.

Analysis Since specific volume of nitrogen is given, thus calculating specific gas constant for nitrogen

$$R_{N_2} = \frac{R_u}{\mathcal{M}_{N_2}} = \frac{8.3144}{28}$$

$$= 0.297\ kJ/kg \cdot K$$

Calculating the constants of Redlich Kwong equation

$$a = 0.427\ \frac{R_{N_2}^2\, T_c^{2.5}}{p_c}$$

$$= 0.427 \times \frac{(0.297)^2 \times (126.2)^{2.5}}{3390}$$

$$= 1.988\ {m^3 K^{0.5}}/{kg}$$

$$b = 0.0866\ \frac{R_{N_2} T_c}{p_c}$$

$$= 0.0866 \times \frac{0.297 \times 126.2}{3390}$$

$$= 9.575 \times 10^{-4}\ m^3/kg$$

using Redlich Kwong equation in terms of v and R_{N_2}

$$p = \frac{R_{N_2} T}{v - b} - \frac{a}{v(v+b)T^{1/2}}$$

$$= \frac{0.297 \times 145}{0.0022 \quad 9.575 \times 10^{\ 4}} - \frac{1.988}{0.0022 \times (0.0022 + 9.575 \times 10^{-4}) \times (145)^{1/2}}$$

$$= 34659.66 - 23766.65$$

$$= 10893\ kPa \text{ or } \mathbf{108.93\ bar}$$

Using ideal gas equation

$$p = \frac{R_{N_2} T}{v} = \frac{0.297 \times 145}{0.0022}$$

$$= 19575\ kPa \text{ or } \mathbf{195.75\ bar}$$

The value of p obtained by Redlich Kwong equation is 44% less than ideal gas equation calculation.

Example 3.29 *Determine the pressure exerted by CO_2 in a container of 1.5 m^3 capacity when it contains 5 kg at 27°C.*

(a) Using ideal gas equation

(b) Using Van der Waal's equation

use $a = 3.6285 \times 10^5\ J^4/kg\ mol^2$, $b = 0.0422\ m^3/kg\ mol$ constants of van der Waal's equation

Solution

Given CO_2 gas in a a container

$m = 5\ kg, \quad V = 1.5\ m^3 \quad T = 27°C = 300\ K$

To find Pressure of gas

Analysis The molecular mass of CO_2 gas

$\mathcal{M}_{CO_2} = 12 + 32 = 44\ kg/kg\ mol.$

Sp. gas constant, $R = \frac{R_u}{\mathcal{M}_{CO_2}} = \frac{8.314}{44}$

$= 0.1889\ kJ/kg \cdot K$

(i) Using perfect gas equation

$$p = \frac{mRT}{V} = \frac{5 \times 0.1889 \times 300}{1.5} = \mathbf{188.95\ kPa}$$

The molar volume $\overline{v}$ of CO_2

$$\overline{v} = \frac{V}{n} = \frac{V\mathcal{M}_{CO_2}}{m} = \frac{1.5 \times 44}{5}$$

$= 13.2\ m^3/kg\ mol.$

(ii) Using Van der Waal's equation

$$\left(p + \frac{a}{\overline{v}^2}\right)(\overline{v} - b) = R_u T$$

or $$p = \frac{R_u T}{\overline{v} - b} - \frac{a}{\overline{v}^2}$$

$$= \frac{(8.314 \times 100) \times 300}{13.2 - 0.0422} - \frac{3.6285 \times 10^5}{(13.2)^2}$$

$= 189560.56 - 2082.477$

$= 187478\ pa \quad \text{or} \quad \mathbf{1.875\ bar}$

Example 3.30 *Determine the critical pressure of oxygen using perfect gas equation and Van der Waal's equation. Take $\overline{v}_c = 0.0745\ m^3/kg\text{-}mol$, $T_c = -118.10°C$.*

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Solution

Given Oxygen gas with

$\overline{v}_c = 0.0745\ \text{m}^3/\text{kg}\cdot\text{mol}$. $T_c = -118.10°\text{C} = 154.9\,\text{K}$

To find Critical pressure of oxygen by using

(i) Perfect gas equation, and
(ii) Van der Waals equation.

Analysis Using perfect gas equation

$$p_c v_c = R_u T_c$$

or $$p_c = \frac{R_u T_c}{v_c} = \frac{8.314 \times 154.9}{0.0745}$$

$$= \mathbf{17{,}286.42\ kPa}$$

using Van der Waals equation at critical point

$$\left(p_c + \frac{a}{\overline{v}_c^2}\right)(\overline{v}_c - b) = R_u T_c$$

with constants $a = \frac{27}{64}\frac{R_u^2 T_c^2}{p_c}$ and $b = \frac{R_u T_c}{8 p_c}$ the Van der Waals equation at critical point yields to

$$\frac{p_c \overline{v}_c}{R_u T_c} = \frac{3}{8}$$

$$\therefore \quad p_c = \frac{3}{8} \times \frac{8.314 \times 154.9}{0.0745} = \mathbf{6482.4\ kPa}$$

Example 3.31 *Determine the density of the steam at 100 bar and 600°C by using (a) steam table, (b) the perfect gas equation, (c) Van der Waals equation with constants*

$$a = 5.5113\ J^4/kg\ mol^2$$
$$b = 0.030\ m^3/kg\ mol$$

and (d) compressibility chart.

Solution

Given Steam at $p = 100\ \text{bar} = 100 \times 10^5\ \text{N/m}^2$
and $T = 600°\text{C} = 873\ \text{K}$

To find Density of steam

Analysis

(i) Using steam table (superheated steam)

at $p = 100$ bar $T_{sup} = 600°\text{C}$
$v = 0.03837\ \text{m}^3/\text{kg}$

Density $\rho = \frac{1}{v} = \frac{1}{0.03837}$

$$= \mathbf{26.06\ kg/m^3}$$

(ii) Using perfect gas equation

$$p_v = RT = \left(\frac{R_u}{\mathcal{M}_{H_2O}}\right) T$$

or $$\rho = \left(\frac{\mathcal{M}_{H_2O}}{R_u T}\right) p = \frac{18 \times (100 \times 10^5)}{8314 \times 873}$$

$$= \mathbf{24.8\ kg/m^3}$$

(iii) Using van der Waals equation with constants

$a = 5.5113 \times 10^5\ \text{J}^4/\text{kg mol}^2$
and $b = 0.03\ \text{m}^3/\text{kg mol}$.

$$\left(p + \frac{a}{\overline{v}^2}\right)(\overline{v} - b) = R_u T$$

or $$\left(100 \times 10^5 + \frac{5.5113 \times 10^5}{\overline{v}^2}\right) \times (\overline{v} - 0.030)$$

$$= 8314 \times 873$$

$$100\,\overline{v} + \frac{5.5113}{\overline{v}} - 3 - \frac{0.16534}{\overline{v}^2} - 72.58 = 0$$

It is a non-linear equation, its numerical solution leads to $\overline{v} = 0.68\ \text{m}^3/\text{kg mol}$.

The specific volume

$$v = \frac{n\overline{v}}{m} = \frac{\overline{v}}{\mathcal{M}_{H_2O}} = \frac{0.68}{18}$$

$$= 0.0378\ \text{m}^3/\text{kg}$$

Density, $\rho = \frac{1}{v} = \frac{1}{0.0378}$

$$= \mathbf{26.455\ kg/m^3}$$

(iv) Using compressibility chart:

For water, $p_c = 221\text{-}2$ bar,
$T_c = 374.15°\text{C}$
$= 647.15\ \text{K}$

Reduced parameter

$$p_r = \frac{p}{p_c} = \frac{100}{221.2} = 0.4522$$

$$T_r = \frac{T}{T_c} = \frac{873}{647.15} = 1.349 \approx 1.35$$

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From generalized compressibility chart compressibility factor, $Z = 0.95$

Then using generalized equation

$$pv = \frac{ZR_uT}{\mathcal{M}_{H_2O}}$$

or $$v = \frac{0.95 \times 8.314 \times 873}{100 \times 100 \times 18}$$

$$= 0.0383 \text{ m}^3/\text{kg}$$

Density $$\rho = \frac{1}{v} = \frac{1}{0.0383}$$

$$= \mathbf{26.10 \text{ kg/m}^3}$$

Summary

- A pure substance has a fixed chemical composition throughout. Water has a constant chemical composition in all of its phases, and therefore, it is considered as a pure substance.
- The heat given to water to raise its temperature from 0°C to saturation temperature is called *sensible heat*. It is denoted by h_f in steam tables.
- The heat given for evaporation of water at its saturation temperature is called *latent heat* or *heat of evaporation or vaporization*. It is denoted by h_{fg} in steam tables.
- The sum of sensible and latent heat is called *total heat* or *total enthalpy of water*. It is denoted by h_g in steam tables.
- The mixture of dry steam and moisture is called *wet steam*. Its quality is defined by *dryness fraction* 'x', which is calculated as

$$x = \frac{\text{mass of dry steam}}{\text{mass of total steam}} = \frac{m_g}{m_w + m_g}$$

where m_w = mass of moisture present in steam sample.

m_g = mass of dry steam in the sample.

Quality of steam = $100\,x\%$

- The properties of wet steam are calculated as

Enthalpy $h_{wet} = h_f + x\,h_{fg}$

Entropy $s_{wet} = s_f + x\,s_{fg}$

Specific Volume $v_{wet} = x\,v_g$

- Steam heated above the saturation temperature is called superheated steam and the temperature rise above the saturation temperature is called degree of superheat. Its properties are calculated as

Degree of superheat = $T_{sup} - T_{sat}$

Enthalpy $h_{sup} = h_g + C_{ps}\,(T_{sup} - T_{sat})$ (kJ/kg)

Entropy $s_{sup} = s_g + C_{ps} \ln\left(\frac{T_{sup}}{T_{sat}}\right)$ (kJ/kg·K)

Specific volume $v_{sup} = v_g \times \frac{T_{sup}}{T_{sat}}$ (m^3/kg)

The density of steam, $\rho = \frac{1}{v}$ (kg/m^3)

- The external work done during phase change is calculated as

$$w_e = pv$$

- Internal energy of steam, $u = h - pv$

Internal latent heat = latent heat – external work of evaporation

- An ideal gas is an ideal working substance for all thermodynamic cycles. The characteristic gas equation for an ideal gas is

$$pV = mRT$$

where R is the specific gas constant and is obtained as

$$R = \frac{R_u}{\mathcal{M}} = \frac{\text{universal gas constant}}{\text{molecular weight}} = \frac{8.31447}{\mathcal{M}}$$

- The thermodynamic properties p, V and T are related as

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

- For all type of processes, the change in internal energy and change in enthalpy can be calculated as

Change in internal energy

$$\Delta U = mC_v(T_2 - T_1)$$

Change in enthalpy

$$\Delta H = mC_p(T_2 - T_1)$$

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Glossary

Triple point The point where the solid, liquid and vapour phases coexist in thermal equilibrium

Vaporization The transformation of liquid into vapour by supplying heat

Evaporation The transformation of liquid into vapour when liquid itself exacts its latent heat

Condensation The transformation of a vapour into a liquid

Melting The transformation of a solid into a liquid, also known as fusion

Solidification The transformation of a liquid into a solid, also called freezing

Sublimation The transformation of solid into vapour

Saturation The condition that exists when two or more phases coexist in equilibrium

Critical state The peak of the saturation curve

Quality (x) The ratio of mass of vapour present to the total mass of steam

Wetness fraction The ratio of mass of liquid present to the total mass of steam (= 1 – quality = 1 – x)

Wet steam A mixture of moisture and vapour, it is a substances that exists under the saturation curve.

Phase The physical state of matter

Dry steam Steam at saturation temperature, which is free from moisture

Superheated steam Dry steam at a temperature greater than the saturation temperature

Ideal gas An ideal working substance for thermodynamic cyles

Review Questions

1. Define pure substance.
2. Explain the phase transformation of water from freezing state to superheated state.
3. Describe the phase-change process of water using a T–v diagram.
4. Define (a) dry and saturated steam (b) wet steam, and (c) superheated steam.
5. Define (a) dryness fraction, and (b) degree of superheat.
6. Explain the following terms:
   (a) Latent heat of fusion
   (b) Latent heat of vaporisation
   (c) Sensible heat of water
   (d) Enthalpy
   (e) Dryness fraction
   (f) Superheated steam
7. What is an internal latent heat and how it relates with enthalpy of the vaporization?
8. Draw a saturation curve on a T–s diagram and mention the different states of water on it.
9. Define (a) triple point, (b) sublimation, and (c) critical point.
10. Explain external work of evaporation.
11. Define (a) sensible heat, (b) latent heat, and (c) super heat.
12. Define dryness fraction and state various methods of determination of dryness fraction.
13. Explain the procedure of obtaining dryness fraction of steam by a throttling calorimeter.
14. Discuss (a) entropy of water, (b) entropy of evaporation, and (c) entropy of steam.
15. Draw a format of pressure entry steam table.
16. Define enthalpy of wet steam and superheated steam with necessary equations.
17. Prove that for a constant pressure process, $dq = dh$.
18. Explain why food cooks faster in a pressure cooker than in conventional cooking?

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Problems

1. A rigid tank of volume 2 m^3 is filled with saturated steam at 2 bar. It contains 0.2 m^3 of moisture and the rest is vapour. Calculate the mass of moisture, mass of vapour and the quality of steam.

[(a)188.5 kg (b) 2.032 kg (c) 0.01066]

2. Steam at 0.75 bar and 150°C is condensed reversibly. Find the heat removed and change in entropy. Sketch the process on a *T–s* diagram.

[2393.8 kJ/kg, 6.536 kJ/kg·K]

3. Determine the increase in entropy of 1 kg of water when it is heated at atmospheric pressure from –23°C to 152°C. Take sp. heat of fusion and superheating as 2.093 kJ/kg·K and for water as 4.187 kJ/kg·K. *[9.0403 kJ/kg]*

4. Obtain specific volume, specific enthalpy, entropy of 2 kg of water at 1 bar and 60°C.

5. A perfect gas of 0.01 –kg mass occupies a volume of 0.02 m^3 at a pressure of 286.4 kPa and 30°C respectively. The gas is allowed to expand until the pressure is 101 kPa and final volume is 4 times the initial volume. Calculate (a) the molal mass of the gas, and (b) final temperature.

[(a) 44 kg/kmol (b) 154.5°C]

6. A quantity of air at a temperature of 80°C is heated at constant pressure until the temperature is 400°C, when the volume was found to be 1.7 m^3. What was initial volume of air ?

[0.892 m^3]

7. The air in a car tyre was at a pressure of 3 bar and at 20°C. After running, the pressure rises to 3.5 bar. Estimate the temperature of air inside the tyre. *[64.8°C]*

8. A vessel having a volume of 0.32 m^3 containing air at 120 kPa, 12°C was sealed 4200 years ago and placed in a tomb of Egypt. If since that time air has leaked from the vessel to cause the pressure to drop to 119.9 kPa with no change in temperature, determine the average rate (molecules per second) at which the air has left the vessel.

9. An airplane has two tyres that are inflated to 1500 kPa. Each tyre has a volume of 0.021 m^3. How much would the weight of the airplane be reduced by using helium instead of air in the tyres? What other considerations are involved in deciding whether to use helium?

10. A tank with a volume of 50 m^3 is being filled with air. At a particular instant, the air in the tank has a temperature of 400 K and a pressure of 1380 kPa. For this system, the pressure is increasing at the rate of 138 kPa/s and temperature is increasing at a rate of 25 K/s. Calculate the air flow rate into the tank in kg/s. *[21.25 kg/s]*

11. A spherical vessel of 3-m diameter contains steam at a pressure of 7 bar (gauge) and a temperature of 200° C, Find

(a) Total enthalpy of steam
(b) Total internal energy of steam
(c) Total mass of steam

Take atmospheric pressure as 1 bar.

[(a) 156.16 MJ (b) 144.85 MJ (c) 55.16 kg]

12. Steam at 15 bar and 0.9 dry is supplied to a super heater at the rate of 5,000 kg/h, the temperature of steam coming out of the super heater is 300°C. The temperature of feed water supplied to the boiler is 45°C. Calculate

(a) Heat supplied in the boiler
(b) Heat supplied in the super heater

[(i) 12.045×10^6 kJ/h, (ii) 2.102×10^6 kJ/h]

13. Wet steam at a pressure of 8.5 bar enters a separator at the rate of 2 kg/s in which it loses 0.05 kg/s of moisture. Then it is throttled to a pressure of 5.4 bar before it is used for process work, during which it loses the heat at the rate of 920 kW at constant pressure until the dryness fraction becomes 0.8. Calculate the temperature of steam immediately after throttling and dryness fraction of steam before and after the separator.

[125.39°C, 0.975, 0.9477]

14. The following data was obtained in a test on a combined separating and throttling calorimeter. The pressure of the steam sample is 15 bar, the pressure of steam at exit is 1 bar, and the temperature is 150°C, discharge from the separating calorimeter is 0.5 kg/min, and the discharge from

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the throttling calorimeter is 10 kg/min. Calculate the dryness fraction of steam.

[x = 0.9477]

15. The following observations were obtained with a separating and throttling calorimeter arranged in series:

Pressure of steam = 12 bar
Water separated = 2 kg
The steam supplied to throttling calorimeter = 20.5 kg
Temperature of steam after throttling = 120°C
Barometer reading 760 mm of Hg
Final pressure = 65 mm of Hg
Estimate the quality of steam. *[0.88]*

Objective Questions

1. The specific volume of water during freezing
   (a) increases (b) remains constant
   (c) decreases (d) none of the above
2. The latent heat of vaporization with increase in pressure of water
   (a) increases (b) remains constant
   (c) decreases (d) none of the above
3. With increase in pressure, the saturation temperature of water
   (a) increases (b) remains constant
   (c) decreases (d) none of the above
4. The specific volume of wet steam is given by
   (a) $\frac{v_g}{x}$ (b) xv_f
   (c) xv_g (d) x^2v_g
5. Internal latent heat of steam is given by
   (a) $h_{fg} - pv_g$ (b) $h_g - pv_g$
   (c) $h_{sup} - pv_g$ (d) $h_{fg} + pv_g$
6. Specific internal energy of steam is given by
   (a) $C_v(T_2 - T_1)$ (b) $h - pv$
   (c) $h + pv$ (d) $h_{fg} + pv_g$
7. During throttling process
   (a) $h_2 = h_1^2$ (b) $h_2 = h_1$
   (c) $h_2 = h_1 + \frac{h_{fg}}{T_{sat}}$ (d) $h_1 = h_2 + \frac{h_{fg}}{T_{sat}}$
8. Critical point is the point at which
   (a) melting point and boiling temperature become equal
   (b) change in volume during evaporation become zero
   (c) latent heat of evapoartion becomes zero.
   (d) none of the above
9. At the critical point, the temperature of water is equal to
   (a) 0°C (b) 100°C
   (c) 374°C (d) –100°C
10. The total enthalpy of steam at 10 bar is 2000 kJ/ kg. The condition of steam is
    (a) wet (b) dry and saturated
    (c) superheated (d) none of the above
11. Dryness fraction of steam is given by
    (a) $\frac{\text{Mass of dry steam in a sample}}{\text{Mass of water particles in the sample}}$
    (b) $\frac{\text{Mass of water particles in the sample}}{\text{Mass of dry steam in the sample}}$
    (c) $\frac{\text{Mass of wet steam in the sample}}{\text{Mass of dry steam in the sample}}$
    (d) $\frac{\text{Mass of dry steam in the sample}}{\text{Total mass of steam in the sample}}$
12. To obtain the dryness fraction of steam from a throttling calorimeter, the steam after throttling should be
    (a) wet
    (b) dry and saturated
    (c) superheated by atleast 5°C
    (d) completely condensed
13. Throttling calorimeter is used to measure the dryness fraction up to
    (a) 0.7 (b) 0.98
    (c) 1.0 (d) none of the above
14. Entropy of wet steam is given by
    (a) $s_{fg} + \frac{h_{fg}}{T_{sat}}$ (b) $s_f + x(s_g - s_f)$
    (c) $s_f + s_{fg}$ (d) $s_g + C_{ps} \ln\left(\frac{T_{sup}}{T_{sat}}\right)$

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15. The perfect gas
    (a) has no intermolecular forces of attraction or repulsion
    (b) does not change its phase during a process
    (c) obeys Boyle's and Charles's laws
    (d) all of the above
16. Universal gas constant of a perfect gas
    (a) increases with increase in temperature
    (b) decrease with temperature
    (c) increases with increase in molar mass
    (d) always remains constant
17. The specific gas constant of any perfect gas
    (a) increases with increase in temperature
    (b) increase with increase in pressure
    (c) is a function of pressure and temperature
    (d) is a constant
18. The ratio of two specific heats of a perfect gas
    (a) always remains constant
    (b) always varies with pressure
    (c) always varies with temperature
    (d) none of the above

Answers

1. (a)	2. (c)	3. (a)	4. (c)	5. (a)	6. (b)	7. (b)	8. (c)
9. (c)	10. (a)	11. (d)	12. (c)	13. (b)	14. (b)	15. (d)	16. (d)
17. (d)	18. (c).						

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4

First Law Applied to Non-Flow Systems

Introduction

In Chapter 2, we have considered various forms of energy and energy transfer, and we have developed a general relation for energy balance. In Chapter 3, we have discussed the properties of working substances: steam, ideal gases and real gases. In this chapter, we will apply the energy balance relation to those systems that do not involve mass flow across their boundary; that is, closed systems.

4.1 IDEAL-GAS PROCESSES

There are an infinite variety of processes. The five important non-flow processes commonly dealt in engineering applications are

1. Constant-volume process
2. Constant-pressure process
3. Constant-temperature process
4. Constant-entropy process
5. Polytropic process

The processes considered in this chapter are reversible non-flow and can be represented by a solid curve on thermodynamic plots. A thermodynamic system can undergo either one or a combination of processes.

4.1.1 Constant-Volume Process

(*a*) *Definition* If any system of gas undergoes a process in such a way that during the process, the volume of the system remains constant, the process is called constant volume process or **isochoric process**, for example, processes taking place in a pressure cooker, gas cylinder, etc.

(*b*) *Properties Relationship* The properties relationship for any system undergoing a process between two states is expressed as

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

For a constant-volume process,

$$V_1 = V_2$$

Then the above relation reduces to

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \qquad ...(4.1)$$

(*c*) *p–V and T–S Diagrams* A constant-volume process is represented by a vertical straight line on a *p–V* diagram and a curved line on a *T–S* diagram as shown in Fig. 4.1.

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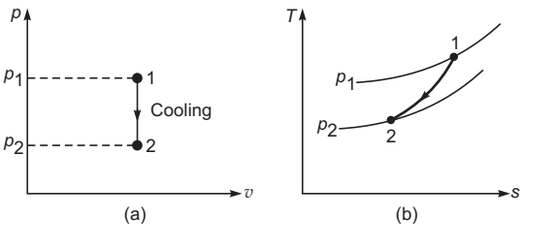

Fig. 4.1 *Constant volume cooling process on p-v and T–s diagrams*

(*d*) *Work Done during the Process* For a non-flow system, the work done can be obtained either from the area under the curve or mathematically as

$$W_{1-2} = \int_1^2 p dV$$

For a constant-volume process,

$$dV = 0$$

Hence $$W_{1-2} = 0 \qquad ...(4.2)$$

It is also evident from the *p–V* diagram, as there is no area under the vertical line.

(*e*) *Change of Internal Energy* For an ideal gas, the change of specific internal energy is calculated as

$$\Delta u = C_v(T_2 - T_1) \text{ (kJ/kg)}$$

where, C_v = specific heat at constant volume in kJ/kg·K and $(T_2 - T_1)$ is temperature difference in °C or K. For a given mass system, the change of internal energy

$$\Delta U = m\Delta u = mC_v(T_2 - T_1) \text{ (kJ)} \qquad ...(4.3)$$

where, m = mass of the system in kg.

(*f*) *Heat Transferred* In general, the heat transferred during a process is expressed as

$$Q = \int_1^2 mC\,dT$$

or Q = mass × specific heat × change of temperature

For a constant-volume process

$$Q = mC_v(T_2 - T_1) = \Delta U \qquad ...(4.4)$$

and for unit mass,

$$q = \int_1^2 C_v dT = C_v(T_2 - T_1) \text{ (kJ/kg)} \qquad ...(4.5)$$

Thus, *for a constant-volume process, the change of internal energy is always equal to heat transferred during the process.*

(*g*) *Change of Enthalpy* The change of specific enthalpy of an ideal gas during a process is calculated as

$$\Delta h = C_p(T_2 - T_1) \text{ (kJ/kg)}$$

where C_p = Specific heat at constant pressure, kJ/kg·K

$(T_2 - T_1)$ = Temperature difference in °C or K.

For a given mass system, the change in enthalpy

$$\Delta H = m\Delta h = mC_p(T_2 - T_1) \text{ (kJ)} \qquad ...(4.6)$$

(*h*) *Change in Entropy* The change in entropy of an ideal gas can be calculated as

$$\Delta S = \int_1^2 \frac{\delta Q}{T} = \int_1^2 mC_v \frac{dT}{T}$$

$$= mC_v \ln\left(\frac{T_2}{T_1}\right) \text{(kJ/K)} \qquad ...(4.7)$$

where T is the absolute temperature in kelvin (K).

Example 4.1 *A certain gas occupies a volume of 0.3 m³ at a pressure of 2 bar. The temperature of the gas at this state is 350 K. The gas undergoes a thermodynamic constant-volume process until the pressure raises to 7 bar. Determine the temperature at the end of the process, change in internal energy, change in enthalpy and change in entropy during this process. Take C_v = 0.712 kJ/kg·K and R = 0.287 kJ/kg·K.*

Solution

Given A constant volume process

State 1: $V = 0.3$ m³ $p_1 = 2$ bar = 200 kPa $T_1 = 350$ K

State 2: $V = 0.3$ m³ $p_2 = 7$ bar = 700 kPa

$C_v = 0.712$ kJ/kg·K, $R = 0.287$ kJ/kg·K

To find

(i) Final temperature of the gas,
(ii) Change in internal energy,
(iii) Change in enthalpy, and
(iv) Change in entropy.

Analysis

We have $R = C_p - C_v$

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It gives $C_p = R + C_v$
$= 0.287 + 0.712 \cong 1.0$ kJ/kg·K

Using characteristic gas equation to calculate mass of the gas in the rigid tank mass

$$m = \frac{p_1 V}{R T_1}$$
$$= \frac{(200 \text{ kPa}) \times (0.3 \text{ m}^3)}{(0.287 \text{ kJ/kg·K}) \times (350 \text{ K})}$$
$$= 0.5973 \text{ kg}$$

(i) *The temperature of gas after constant-volume process*

We have $\frac{p_1}{T_1} = \frac{p_2}{T_2}$

It gives $T_2 = \frac{p_2}{p_1} T_1 = \frac{7}{2} \times (350 \text{ K})$

= 1225 K or 952°C

(ii) *Change in internal energy*

$\Delta U = m\, C_v\, (T_2 - T_1)$
$= (0.5973 \text{ kg}) \times (0.712 \text{ kJ/kg·K})$
$(1225 - 350)(\text{K})$

or $\Delta U =$ **372.12 kJ**

(iii) *Change in enthalpy*

$\Delta H = m\, C_p\, (T_2 - T_1)$
$= (0.5973 \text{ kg}) \times (1.0 \text{ kJ/kg·K})$
$(1225 - 350)(\text{K})$

or $\Delta H =$ **522.63 kJ**

(iv) Change in entropy

$$\Delta S = m\, C_v \ln\left(\frac{T_2}{T_1}\right)$$
$$\Delta S = 0.5973 \times 0.712 \times \ln\left(\frac{1225}{350}\right)$$
$$= \mathbf{0.5328 \text{ kJ/K}}$$

Example 4.2 *A rigid tank of 0.2 m^3 capacity contains nitrogen at 350 kPa and 15°C. The tank is exposed to a constant temperature source at 170°C, which results in 50 kJ of heat being tranferred to nitrogen. Determine*

(a) Final state of the gas,
(b) Change in internal energy,
(c) Change in entropy, Take C_v = 743 J/kg·K for Nitrogen.

Solution

Given A constant temperature and constant volume processes.

State 1: $V = 0.2 \text{ m}^3$
$p_1 = 350$ kPa
$T_1 = 15°\text{C} + 273 = 288$ K

State 2: $T_2 = 170°\text{C} + 273 = 443$ K
$V = 0.2 \text{ m}^3$
$Q = 50$ kJ
$C_v = 743$ J/kg·K = 0.743 kJ/kg·K

To find

(i) Final state of the gas,
(ii) Change in internal energy,
(iii) Change in entropy.

Assumptions

(i) Nitrogen as a perfect gas.
(ii) The ratio of two specific heats, $\gamma = 1.4$ for perfect gas.

Analysis

We have $R = C_p - C_v$ and $\gamma = \frac{C_p}{C_v}$

Hence $R = C_v (\gamma - 1)$
$= 743 \times (1.4 - 1) = 297$ J/kg·K
$= 0.297$ kJ/kg·K.

Using characteristic gas equation to calculate mass of the gas in the rigid tank.

$$m = \frac{p_1 V}{R T}$$
$$= \frac{(350 \text{ KPa}) \times (0.2 \text{ m}^3)}{(0.297 \text{ kJ/kg·K}) \times (288 \text{ K})} = 0.818 \text{ kg}$$

The temperature of gas after 50 kJ of heat supply

$Q = m\, C_v\, (T_2 - T_1)$
50 kJ $= (0.818 \text{ kg}) \times (0.743 \text{ kJ/kg·K})\, (T_2 - 288 \text{ K})$
or $T_2 = 370.26$ K = **97.26°C**

Change in internal energy

$\Delta U = Q =$ **50 kJ** (For constant volume process)

Change in entropy

$$\Delta S = m\, C_v \ln\left(\frac{T_2}{T_1}\right)$$
$$\Delta S = 0.818 \times 0.743 \times \ln\left(\frac{370.28}{288}\right) = \mathbf{0.153 \text{ kJ/K}}$$

Example 4.3 *The usual cooking gas (mostly methane) cylinder is about 25 cm in diameter and 80 cm in height. It is charged to 12 MPa at a room temperature of 27°C.*

(a) Assuming the ideal gas law, find the mass of gas filled in the cylinder.

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(b) If the cylinder is protected against the excessive pressure by means of a fusible plug, at what temperature should the plug melt to limit the maximum pressure of 15 MPa?

Solution

Given A constant-volume process in a cooking cylinder

$d = 25$ cm $= 0.25$ m, $L = 80$ cm $= 0.8$ m

State 1: $p_1 = 12$ MPa $= 12000$ kPa,
$T_1 = 27°C + 273 = 300$ K

State 2: $p_2 = 15$ MPa $= 15000$ kPa

To find

(i) Initial mass of gas in the cylinder, and
(ii) Final temperature.

Analysis The molecular mass of methane

$$\mathcal{M}_{CH_4} = \mathcal{M}_C + 2\,\mathcal{M}_{H_2}$$
$$= 12 + 2 \times 2 = 16 \text{ kg/kmol}$$

Specific gas constant of methane

$$R = \frac{R_u}{\mathcal{M}_{CH_4}} = \frac{8.314}{16} = 0.5196 \text{ kJ/kg·K}$$

The volume of gas cylinder

$$V = \frac{\pi}{4} d^2 L = \frac{\pi}{4} \times (0.25)^2 \times 0.8 = 0.0393 \text{ m}^3$$

Using characteristic gas equation to calculate mass of gas

$$m = \frac{p_1 V}{RT}$$

$$= \frac{(12000 \text{ KPa}) \times (0.0393 \text{ m}^3)}{(0.5196 \text{ kJ/kg·K}) \times (300 \text{ K})} = 3.02 \text{ kg}$$

The final temperature of gas, when the pressure approaches 15000 kPa,

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right) = 300 \times \left(\frac{15000}{12000} \right)$$
$$= \mathbf{375 \text{ K or } 102°C}$$

4.1.2 Constant Pressure, or Isobaric Process

(a) Definition If any system of gas undergoes a process in such a way that during the process, the pressure of the gas remains constant then such process is called a constant pressure or *isobaric process.*

(b) Properties Relationship In general, the properties are related betwen two states as

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

But for a constant pressure process,

$$p_1 = p_2,$$

then

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \qquad ...(4.8)$$

(c) Work Done The $\int pdV$ work done during the process can be evaluated from an area under the curve on a p–V diagram or can be calculated as

$$W = \int_1^2 pdV = p(V_2 - V_1) \text{ (kJ)} \qquad ...(4.9)$$

where p is expressed in kPa (kN/m^2), and V is expressed in m^3.

(d) p-V and T-S Diagrams A constant–pressure process is a horizontal straight line on a p–V plane and an inclined curve on a T–S plane as shown in Fig 4.2.

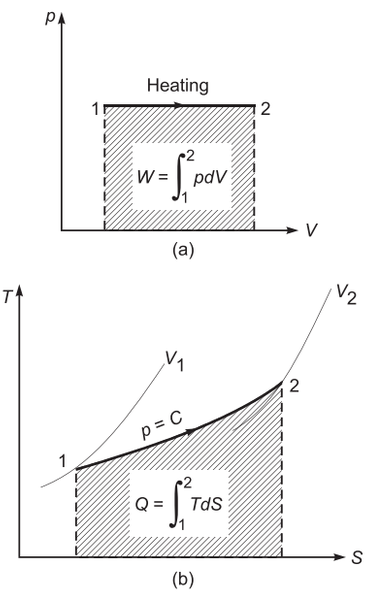

Fig. 4.2 *Constant-pressure process on p–V and T–S diagrams*

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(e) *Change of Internal Energy* For any process, it is calculated as

$$\Delta U = m C_v (T_2 - T_1) \text{ (kJ)}$$

(f) *Heat Transferred* For a constant-pressure process, the heat transferred can be obtained as

$$Q = m\int_1^2 C_p dT = m C_p (T_2 - T_1) \text{ (kJ)} \quad ...(4.10)$$

and for a unit mass system

$$q = C_p (T_2 - T_1) \text{ (kJ/kg)} \quad ...(4.11)$$

where C_p must be expressed in kJ/kg·K or kJ/kg·°C and $(T_2 - T_1)$ as temperature difference in °C or K.

(g) *Change in Enthalpy* For any process, it is calculated as

$$\Delta H = m\, C_p (T_2 - T_1) \text{ (kJ)}$$

and

$$\Delta h = C_p (T_2 - T_1) \text{ (kJ/kg)}$$

Thus *for a non-flow constant pressure process, the change in enthalpy is always equal to the heat transferred during the process.*

(h) *Change in Entropy* It is calculated as

$$\Delta S = \int_1^2 \frac{\delta Q}{T} = \int_1^2 m C_p \frac{dT}{T}$$

$$= m C_p \ln\left(\frac{T_2}{T_1}\right) \text{ (kJ/K)} \quad ...(4.12)$$

The temperature T must be expressed in K (kelvin).

Example 4.4 *In an experimental determination of specific heat of air at constant pressure, 50 g of air at constant pressure was heated from 14°C to 74°C during a period of 300 seconds by a continuous electrical heating method, The power consumed was 10.04 watts. Determine*

(a) Specific heat of air at constant pressure,
(b) Characteristic gas constant R for air,
(c) The density of air at 0°C and 0.1 MPa.

Take Y = 1.4 for air.

Given A constant-pressure process

m = 50 g = 0.05 kg T_1 = 14°C + 273 = 287 K
Δt = 300 *s* T_2 = 74°C + 273 = 347 K
P = 10.04 W γ = 1.4
p = 0.1 MPa = 100 kPa T_o = *O*°C = 273 K

To find

(i) Specific heat at constant pressure C_p,
(ii) Specific gas constant R, and
(ii) Density of air at 0°C and 0.1 MPa.

Assumptions

(i) Air as an ideal gas, and
(ii) Process is reversible.

Analysis

(i) *The amount of heat supplied during the process*

$$Q = P\Delta t = 10.04 \times 300 = 3012 \text{ J}$$

Further the heat transferred during a constant pressure process is also expressed as

$$Q = m C_p (T_2 - T_1)$$

$$C_p = \frac{Q}{m(T_2 - T_1)}$$

$$= \frac{(3012 \text{ J})}{(0.05 \text{ kg}) \times (347 - 287)(\text{K})}$$

$$= \mathbf{1004 \text{ J/kg·K}}$$

(ii) *The specific gas constant R*

We have $\gamma = \dfrac{C_p}{C_v}$

or $C_v = \dfrac{C_p}{\gamma} = \dfrac{1004}{1.4} = 717 \text{ J/kg·K}$

The gas constant

$$R = C_p - C_v$$
$$= 1004 - 717 = 287 \text{ J/kg·K}$$

or $R = \mathbf{0.287 \text{ kJ/kg·K}}$

(iii) *Density of air at 0°C and 100 kPa*

From the characteristic equation for an ideal gas

$$pv = RT$$

or $\dfrac{p}{\rho} = RT$

or $\rho = \dfrac{p}{RT} = \dfrac{(100 \times 10^3 \text{ N/m}^2)}{(287 \text{ J/kg·K}) \times (273 \text{ K})}$

$$= \mathbf{1.276 \text{ kg/m}^3}$$

Example 4.5 *1 kg of nitrogen at a temperature of 150°C occupies a volume of 0.2 m^3. The gas undergoes a fully restricted constant-pressure expansion without friction to a final volume of 0.36 m^3. Calculate the final temperature, work done, heat transferred and change in entropy.Take C_v = 743 J/kg K, R = 0.297 kJ/kg·K.*

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Solution

Given A constant-pressure expansion process of nitrogen is shown in Fig. 4.3.

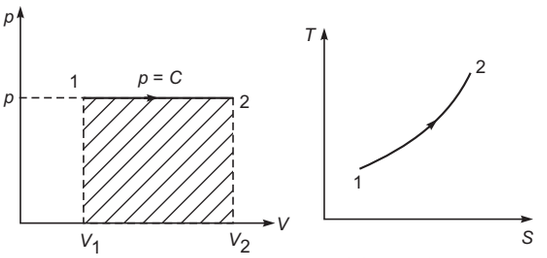
$m = 1$ kg $\quad C_v = 743$ J/kg K $\quad R = 0.297$ kJ/kg K

State 1: $T_1 = 150°C + 273 = 423$ K, $\quad V_1 = 0.2$ m^3

State 2: $V_2 = 0.36$ m^3

To find

(i) Final temperature T_2,
(ii) Work done,
(ii) Heat transferred,
(iv) Change in entropy.



(a) p–V diagram *(b) T–S diagram*

Fig. 4.3

Analysis From the characteristic gas equation

$$p_1 = \frac{mRT_1}{V_1}$$

$$= \frac{(1 \text{ kg}) \times (0.297 \text{ kJ/kg} \cdot \text{K}) \times (423 \text{ K})}{(0.2 \text{ m}^3)}$$

$$= 624.15 \text{ kPa}$$

The specific heat at constant pressure can be calculated as

$$C_p = R + C_v$$
$$= (0.297 \text{ kJ/kg K}) + (0.743 \text{ kJ/kg K})$$
$$= 1.04 \text{ kJ/kg K}$$

(i) *Final temperature T_2*

The properties relationship for constant pressure process,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

or $$T_2 = \frac{V_2}{V_1} \times T_1 = \frac{0.36}{0.2} \times (423 \text{ K})$$

$$= 761.4 \text{ K} = \mathbf{484.4°C}$$

(ii) *Work done*

For a constant-pressure process, the work done can easily be calculated as area under the curve 1–2 and it is

$$W_{1-2} = p(V_2 - V_1)$$

Using numerical values

$$W_{1-2} = (624.15 \text{ kJ}) \times (0.36 - 0.2) \text{ (m}^3\text{)}$$
$$= \mathbf{100.5 \text{ kJ}}$$

(iii) *Heat transferred Q_{1-2}*

It is calculated as

$$Q_{1-2} = m C_p (T_2 - T_1)$$

Then $\quad Q_{1-2} = (1 \text{ kg}) \times (1.04 \text{ kJ/kg K}) \times (761.4 \text{ K} - 423 \text{ K})$

$$= \mathbf{352 \text{ kJ}}$$

(iv) *Change of entropy*

$$\Delta S_{1-2} = mC_p \ln\left(\frac{T_2}{T_1}\right)$$
$$= (1 \text{ kg}) \times (1.04 \text{ kJ/kg K}) \times \ln\left(\frac{761.4}{423}\right)$$
$$= \mathbf{0.611 \text{ kJ/K}}$$

4.1.3 Isothermal Process or Constant Temperature Process

(*a*) *Definition* If a system undergoes a process in such a way that during the process, the temperature of the system remains constant, such a process is called an *isothermal process* or constant temperature process.

(*b*) *Relation between p, V and T* The property relation

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

for a constant temperature process ($T_1 = T_2$) the relation is reduced to

$$p_1V_1 = p_2V_2$$

or $$pV = C \text{ (constant)} \qquad ...(4.13)$$

It is the equation of a *hyperbola*, and hence an isothermal process is also called a ***hyperbolic process***.

(*c*) *p–V Diagram* An isothermal process is represented on a *p–V* and *T–S* diagrams as shown in Fig. 4.4.

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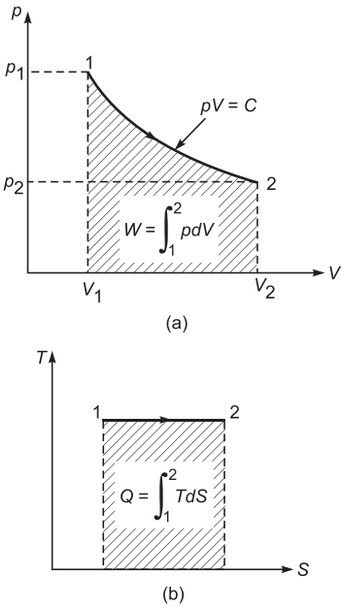

Fig. 4.4 *p-V and T-s diagrams for an isothermal process.*

(*d*) *Work Done* The work done during a process is expressed as

$$W_{1-2} = \int_1^2 p dV$$

For an isothermal process,

$$pV = C$$

$$\therefore \quad p = \frac{C}{V}$$

Then
$$W_{1-2} = \int_1^2 \frac{C}{V} dV = C \int_{V_1}^{V_2} \frac{dV}{V}$$

$$= C \ln\left(\frac{V_2}{V_1}\right)$$

Using $C = p_1 V_1$

we get
$$W_{1-2} = p_1 V_1 \ln\left(\frac{V_2}{V_1}\right) \quad ...(4.14)$$

Further, $p_1 V_1 = m R T_1$

Then
$$W_{1-2} = m R T_1 \ln\left(\frac{V_2}{V_1}\right) \quad ...(4.15)$$

From properties relation, we have

$$\frac{V_2}{V_1} = \frac{v_2}{v_1} = \frac{p_1}{p_2}$$

Then
$$W_{1-2} = m R T_1 \ln\left(\frac{p_1}{p_2}\right) \quad ...(4.16)$$

(*e*) *Change of Internal Energy* For any system,

$$\Delta U_{1-2} = m C_v (T_2 - T_1)$$

But $T_1 = T_2$ (for isothermal process)

Hence $\Delta U = 0$

(*f*) *Heat Transferred* According to the first law of thermodynamics for any process,

$$Q = \int_1^2 TdS = T(S_2 - S_1) \quad ...(4.17)$$

or $Q = \Delta U + W$

Here $\Delta U = 0$

Hence
$$Q_{1-2} = W_{1-2} = p_1 V_1 \ln\left(\frac{V_2}{V_1}\right)$$

$$= m R T \ln\left(\frac{p_1}{p_2}\right) \quad ...(4.18)$$

During an isothermal process, the heat transfer is equal to work transfer.

(*g*) *Change in Enthalpy* For any process, it is calculated as

$$\Delta h = C_p (T_2 - T_1)$$

But $T_1 = T_2$

Thus $\Delta h = \Delta H = 0$

(*h*) *Change in Entropy* It is calculated as

$$\Delta S = \int_1^2 \frac{\delta Q}{T} = \int_1^2 \frac{\delta W}{T} = \int_1^2 \frac{pdV}{T}$$

But
$$\frac{p}{T} = \frac{m R}{V} \text{ from } pV = mRT$$

Then
$$\Delta S = \int_1^2 m R \frac{dV}{V} = m R \ln\left(\frac{V_2}{V_1}\right) \quad ...(4.19)$$

or
$$= m R \ln\left(\frac{p_1}{p_2}\right) \quad ...(4.20)$$

Example 4.6 *A mass of 0.8 kg of air at 1 bar and 25°C is contained in a gas-tight frictionless piston–cylinder device. The air is now compressed to a final pressure of 5 bar. During the process, the heat is transferred from air such that the temperature inside the cylinder remains constant. Calculate the heat transferred and work done during the process and direction of each in the process.*

Solution

Given Constant temperature process

$m = 0.8$ kg,

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State 1: $T_1 = 25°C + 273 = 298$ K,
$p_1 = 1$ bar $= 100$ kPa,
State 2: $p_2 = 5$ bar $= 500$ kPa; $T_2 = T_1$

To find Heat transfer and work transfer

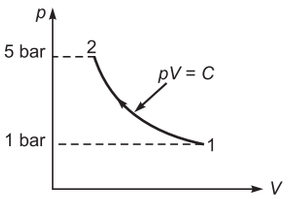

Fig. 4.5 *p–V diagram for constant temperature process*

Assumptions

(i) Air as perfect gas,
(ii) Gas constant for air
$R = 0.287$ kJ/kg·K, and
(iii) The specific heat at constant volume
$C_v = 0.7165$ kJ/kg·K

Analysis For isothermal compression process, the work done is given by Eq.(4.16).

$$W_{1-2} = mRT_1 \ln\left(\frac{p_1}{p_2}\right)$$
$$= 0.8 \times 0.287 \times 298 \times \ln\left(\frac{100}{500}\right)$$
$$= \mathbf{-110.12\ kJ}$$

It is the work supplied to the system during compression process.

For isothermal process, the heat transfer

$$Q_{1-2} = W_{1-2} = \mathbf{-110.12\ kJ}$$

The heat is rejected from the system during compression process.

Example 4.7 *Air initially at 60 kPa pressure, 800 K temperature and 0.1 m^3 volume is compressed isothermally until the volume is halved, and subsequently the air is cooled at contant pressure till the volume is halved again. Sketch the process on a p–V plane and determine*

(a) total work interaction, and
(b) total heat interaction.

Assume ideal-gas behaviour for air and take C_p = 1.005 kJ/kg·K.

Solution

Given Constant temperature and constant pressure processses.

State 1: $p_1 = 60$ kPa $V_1 = 0.1$ m^3, $T_1 = 800$ K,
State 2: $T_2 = T_1$ $V_2 = 0.5\ V_1$, $V_2 = 0.05$ m^3
State 3: $p_2 = p_3$ $V_3 = 0.5\ V_2$, $V_3 = 0.025$ m^3
$C_p = 1.005$ kJ/kg·K.

To find

(i) Total work transfer, and
(ii) Total heat transfer.

Assumption

Gas constant for air $R = 0.287$ kJ/kg·K.

Analysis The isothermal and isobaric processes on *p–V* plane are shown in Fig.4.6.

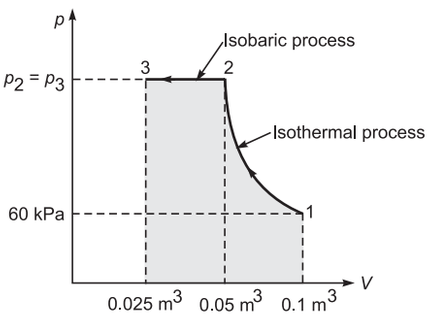

Fig. 4.6 *p–V diagram for isothermal and isobaric processes*

From the characteristic gas equation,

$$m = \frac{p_1V_1}{RT_1} = \frac{(60\text{ kPa}) \times (0.1\text{ m}^3)}{(0.287\text{ kJ/kg·K}) \times (800\text{ K})} = 0.0261\text{ kg}$$

(i) For isothermal compression process 1-2,

$$p_2 = p_1\frac{V_1}{V_2} = (60\text{ kPa}) \times \frac{0.1}{0.05} = 120\text{ kPa}$$

Work transfer is given by Eq. (4.14)

$$W_{1-2} = p_1V_1 \ln\left(\frac{V_2}{V_1}\right)$$
$$= (60\text{ kPa}) \times (0.1\text{ m}^3) \times \ln\left(\frac{0.05}{0.1}\right)$$
$$= \mathbf{-4.158\ kJ}$$

For isothermal process, the heat transfer

$$Q_{1-2} = W_{1-2} = \mathbf{-4.158\ kJ}$$

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(ii) For constant-pressure process 2–3,

$$T_3 = T_2 \frac{V_3}{V_2} = (800\text{ K}) \times \frac{0.025}{0.05} = 400\text{ K}$$

Work transferred during constant-pressure process is given by Eq. (4.9)

$$W_{2-3} = p_2(V_3 - V_2) = (120\text{ kPa}) \times (0.025 - 0.05) = \mathbf{-3\ kJ}$$

Heat transferred during constant-pressure process 2–3,

$$Q_{2-3} = m\ C_p\ (T_3 - T_2)$$
$$= (0.0261\text{kg}) \times (1.005\text{ kJ/kg}\cdot\text{K}) \times (400\text{ K} - 800\text{ K}) = \mathbf{-10.5\ kJ}$$

(a) Total work transfer,

$$W_{Total} = W_{1-2} + W_{2-3} = -4.153\text{ kJ} + (-3\text{ kJ})$$
$$= \mathbf{-7.153\ kJ}$$

(b) Total heat transfer,

$$Q_{Total} = Q_{1-2} + Q_{2-3} = -4.153\text{ kJ} + (-10.5\text{ kJ})$$
$$= \mathbf{-14.643\ kJ}$$

Example 4.8 *An ideal gas is heated at constant volume until its temperature is doubled. Then it is expanded isothermally till it reaches its original pressure. Finally, the gas is cooled at constant pressure till it is restored to the original state. Determine the net work done per kg of gas.*

Solution

Given An ideal-gas cycle

$v_1 = v_2 \quad T_2 = 2T_1 \quad T_2 = T_3 \quad p_3 = p_1$

To find Net work done per kg of gas

Assumption All processes in the cycle are reversible.

Analysis

(i) During constant-volume heating process 1–2, ($v_1 = v_2$)

$$w_{1-2} = 0$$

(ii) During isothermal expansion process 2–3: $T_2 = T_3 = 2T$,

$$w_{2-3} = p_2 v_2 \ln\left(\frac{v_3}{v_2}\right)$$
$$= p_2 v_2 \ln\left(\frac{p_2}{p_3}\right) = RT_2 \ln\left(\frac{p_2}{p_1}\right)$$
$$= 2R\,T_1 \ln\left(\frac{p_2}{p_1}\right)$$

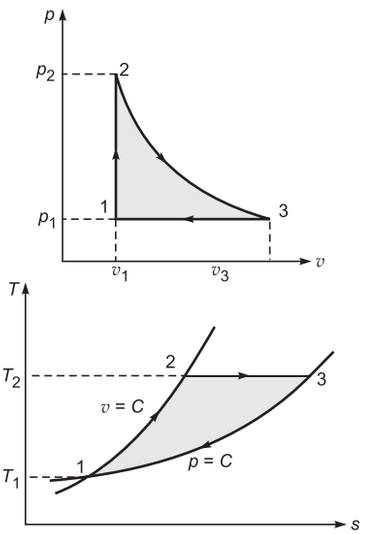

Fig. 4.7

(iii) During constant pressure cooling process, 3–1: $p_3 = p_1$

$$w_{3-1} = p_3(v_1 - v_3) = m\,R\,(T_1 - T_3)$$
$$= R\,(T_1 - 2T_1) = -R\,T_1$$

Net work done $= w_{1-2} + w_{2-3} + w_{3-1}$

$$= 0 + 2RT_1 \ln\left(\frac{p_2}{p_1}\right) - R\,T_1$$

$$= \boldsymbol{R\,T_1}\left[\mathbf{2\ln}\left(\frac{\boldsymbol{p_2}}{\boldsymbol{p_1}}\right) - \mathbf{1}\right]$$

4.1.4 Reversible Adiabatic or Isentropic Process

If any process of expansion or compression is carried out without exchange of heat between system and surroundings, i.e., operation in an insulated cylinder, the process is called ***adiabatic process***. During this process, the system neither receives nor rejects any amount of heat.

When an adiabatic process is reversible, i.e., frictionless, etc., then this process is called ***isentropic*** or *reversible adiabatic* or ***constant entropy process***.

(a) Law of an Isentropic Process For unit mass of a system, the energy equation for a non-flow

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process is expressed as

$$\delta q = du + \delta w$$

For adiabatic process, $\delta q = 0$

$$\therefore \quad du + \delta w = 0$$

or $$C_v\,dT + p\,dv = 0 \qquad ...(4.21)$$

For unit mass of an ideal gas, the characteristic equation

$$p\,v = RT$$

Differentiating both sides, we get

$$p\,dv + v dp = R dT \qquad ...(4.22)$$

or $$dT = \frac{p dv + v dp}{R}$$

Substituting for dT in Eq. (4.21), we have

$$C_v \frac{p dv + v dp}{R} + p dv = 0$$

or $$C_v p\,dv + C_v\,v dp + R p dv = 0$$

Using $R = C_p - C_v$, a relation between two specific heats, we have

$$C_v p\,dv + C_v\ v dp + (C_p - C_v)\,p\,dv = 0$$

or $$C_v v dp + C_p\,p\,dv = 0$$

Dividing both sides by $p\,vC_v$, we get

$$\frac{dp}{p} + \frac{C_p}{C_v}\frac{dv}{v} = 0$$

Assuming ratio of two specific heats $\frac{C_p}{C_v} = \gamma$ and integrating the above equation, we get

$$\log_e p + \gamma \log_e v = \log_e C$$

where $\log_e C$ is the constant of integration

Taking antilog on both sides, we get

$$pv^{\gamma} = C \text{ (a constant)}$$

It is the law of isentropic expansion or compression. For a given mass system, it is

$$pV^{\gamma} = C \text{ (constant)} \qquad ...(4.23)$$

(*b*) *Relation between p, V and T* The thermodynamic properties for a perfect gas are related as

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

or $$\frac{T_2}{T_1} = \frac{p_2 V_2}{p_1 V_1} \qquad ...(4.24)$$

For an isentropic process,

$$pV^{\gamma} = C \quad \text{or} \quad pv^{\gamma} = C$$

or $$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$

or $$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = \left(\frac{V_2}{V_1}\right)^{-\gamma} = \left(\frac{v_2}{v_1}\right)^{-\gamma} \qquad ...(4.25)$$

or $$\frac{v_2}{v_1} = \frac{V_2}{V_1} = \left(\frac{p_2}{p_1}\right)^{-\frac{1}{\gamma}} \qquad ...(4.26)$$

Substituting Eq. (4.25) in Eq. (4.24), we get

$$\frac{T_2}{T_1} = \left(\frac{V_2}{V_1}\right)^{-\gamma}\left(\frac{V_2}{V_1}\right) = \left(\frac{V_2}{V_1}\right)^{1-\gamma} = \left(\frac{v_2}{v_1}\right)^{1-\gamma} \qquad ...(4.27)$$

Using Eq. (4.26) in Eq. (4.24), we get

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)\left(\frac{p_2}{p_1}\right)^{-\frac{1}{\gamma}} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad ...(4.28)$$

(*c*) *p–V Diagram* The *p–V* diagram for an isentropic process is shown in Fig 4.8.

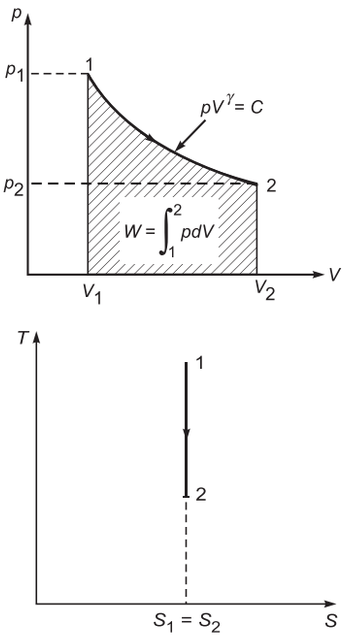

Fig. 4.8 *p–V and T–S diagrams for an isentropic expansion process*

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(*d*) *Work done During the Process* The work done during a non-flow process is expressed as

$$W_{1-2} = \int_1^2 pdV$$

For an isentropic process

$$pV^{\gamma} = C, \text{ or } p = \frac{C}{V^{\gamma}}$$

Then $$W_{1-2} = \int_1^2 \frac{C}{V^{\gamma}} dV = C\int_{V_1}^{V_2} V^{-\gamma} dV$$

or $$W_{1-2} = C\left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_1}^{V_2} = \frac{C\left(V_2^{-\gamma+1} - V_1^{-\gamma+1}\right)}{1-\gamma}$$

Substituting for C as $p_1 V_1^{\gamma}$ and $p_2 V_2^{\gamma}$

$$W_{1-2} = \frac{p_2 V_2 V_2^{-\gamma+1} - p_1 V_1 V_1^{-\gamma+1}}{1-\gamma}$$

We get

$$W_{1-2} = \frac{p_2 V_2 - p_1 V_1}{1-\gamma} = \frac{mR(T_2 - T_1)}{1-\gamma} \quad ...(4.29)$$

If we put $R = C_p - C_v$ and $\gamma = \frac{C_p}{C_v}$, we get

$$C_v = \frac{R}{\gamma - 1}$$

Using in Eq. (4.29), we get

$$W_{1-2} = -m\, C_v\, (T_2 - T_1) = -\Delta U \quad ...(4.30)$$

This expression explains that *during an isentropic expansion process, the work done is always equal to decrease in internal energy during the process*.

(*e*) *The Change of Internal Energy* As explained by Eq. (4.30), the change of internal energy during isentropic process

$$\Delta U_{1-2} = -W_{1-2} \text{ (adiabatic work)}$$

or $$\Delta U_{1-2} = m\, C_v\, (T_2 - T_1)$$

(*f*) *Heat Transferred* During an adiabatic operation, the heat transferred is zero, i.e., no area under the curve on a *T–S* diagram.

$$Q_{1-2} = 0$$

(*g*) *Change in Enthalpy* For any process

$$\Delta H = m\, C_p\, (T_2 - T_1)$$

(*h*) *Change of Entropy* For isentropic process, $S_1 = S_2$;

Thus $$\Delta S = \int \frac{\delta Q}{T} = 0 \quad ...(4.31)$$

Example 4.9 *A quantity of gas occupying 0.14 m^3 at a pressure of 1400 kPa and 300°C is* expanded isentropically to 280 *kPa. Calculate (a) mass of gas, (b) final temperature , (c) work transfer , and (d) change in entropy.*

Take C_p= 1.04 kJ/kg·K and C_v = 0.74 kJ/kg·K

Solution

Given Isentropic expansion of a gas

State 1: $V_1 = 0.14$ m^3
$p_1 = 1400$ kPa
$T_1 = 300°C = 573$ K

State 2: $p_2 = 280$ kPa,
$C_p = 1.04$ kJ/kg·K
$C_v = 0.74$ kJ/kg·K

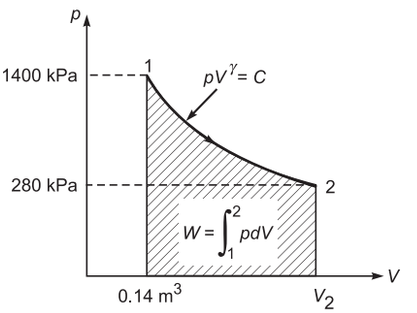

Fig. 4.9

To find

(i) Mass of the gas,
(ii) Final temperature,
(iii) Work transfer, and
(iv) Change in entropy.

Assumption The given gas is a perfect gas.

Analysis

(i) *For an ideal gas*

$$p_1 V_1 = mRT_1$$

where $R = C_p - C_v = 1.04 - 0.74$
$= 0.3$ kJ/kg·K

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Using values, we get

$$m = \frac{p_1 V_1}{R T_1} = \frac{1400 \times 0.14}{0.3 \times 573} = \mathbf{1.14\ kg}$$

(ii) *Final temperature*

For an isentropic process

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

where $\gamma = \frac{C_p}{C_v} = \frac{1.04}{0.74} = 1.405$

then $T_2 = 573 \times \left(\frac{280}{1400}\right)^{\frac{1.405-1}{1.405}}$

$= 360\ \text{K} = \mathbf{87\,°C}$

(iii) *Work transfer*

For an isentropic process

$$W = \frac{p_2 V_2 - p_1 V_1}{1-\gamma} = \frac{mR(T_2 - T_1)}{1-\gamma}$$

$$= \frac{1.14 \times 0.3 \times (360 - 573)}{1 - 1.405} = \mathbf{180\ kJ}$$

(iv) *Change in entropy*

For an isentropic process $S_1 = S_2$

Thus $\Delta S = \mathbf{0}$

Example 4.10 *The following equation, which connects u, p and v for several gases is*

$$u = a + bpv$$

where a and b are constants. Prove that for a reverseble adiabatic process

$$pv^{\gamma} = \text{constant, where } \gamma = \frac{b+1}{b}$$

Solution For a reversible adiabatic process

$$Q = 0$$

For a process of unit mass system

$$dq = du + pdv$$

or $0 = du + p\,dv$ ...(i)

We have $u = a + bpv$

Differentiating both sides,

$$du = b\,vdp + b\,p\,dv$$

Using in Eq. (i);

$$0 = b\,vdp + b\,p\,dv + p\,dv$$

or $p\,(1 + b)\,dv + b\,v\,dp = 0$

Multiplying both sides by $\frac{1}{b\,v\,p}$, we get

$$\frac{1+b}{b}\frac{dv}{v} + \frac{dp}{p} = 0$$

Integrating both sides

$$\left(\frac{1+b}{b}\right) \ln(v) + \ln(p) = \ln(C)$$

Taking antilog

$$p\,v^{\left(\frac{1+b}{b}\right)} = C$$

Using $\gamma = \frac{1+b}{b}$

then $pv^{\gamma} = C$ **Proved**

Example 4.11 *1 kg of gas occupying 0.15 m^3 at a pressure of 12 bar is heated at constant pressure until its volume is 0.28 m^3. The gas is then expanded adiabatically until its volume is 1.5 m^3. Calculate*

(a) Temperature at the end of constant-pressure heating and at the end of adiabatic expansion.

(b) Total work done

Take C_p = 1.068 kJ/kg·K and C_v = 0.775 kJ/kg·K

Solution

Given A gas undergoes constant pressure and adiabatic processes.

$m = 1$ kg, $C_p = 1.068$ kJ/kg·K

$C_v = 0.775$ kJ/kg·K

State 1: $p_1 = 12$ bar $= 1200$ kPa $V_1 = 0.15\ m^3$

State 2: $V_2 = 0.28\ m^3$ $p_2 = p_1$

State 3: $V_3 = 1.5\ m^3$ $s_1 = s_2$

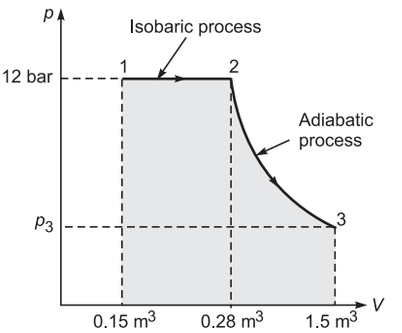

Fig. 4.10 *p-V diagram*

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To find

(i) Temperature after constant pressure heating,
(ii) Temperature afer adiabatic expansion, and
(iii) Total work done.

Analysis For an ideal gas

$$R = C_p - C_v = 1.068 - 0.775 = 0.293 \text{ kJ/kg·K}$$

and $p_1V_1 = m\,R\,T_1$

Then $$T_1 = \frac{p_1V_1}{mR} = \frac{1200 \times 0.15}{1 \times 0.293} = 614 \text{ K} = 341°\text{C}$$

(i) *Temperature after constant pressure heating*

$$T_2 = T_1\frac{V_2}{V_1} = \frac{614 \times 0.28}{0.15}$$

$$= 1146 \text{ K} = \mathbf{873°C}$$

Work done $W_{1-2} = p_1(V_2 - V_1)$

$$= 1200 \times (0.28 - 0.15) = 156 \text{ kJ}$$

(ii) *During adiabatic expansion*

$$\gamma = \frac{C_p}{C_v} = \frac{1.068}{0.775} = 1.38$$

Temperatures and volumes are related as

$$\frac{T_3}{T_2} = \left(\frac{V_3}{V_2}\right)^{1-\gamma} = \left(\frac{1.5}{0.28}\right)^{1-1.38} = 0.525$$

or $T_3 = 1146 \times 0.525 = 601.65 \text{ K} = \mathbf{328.65°C}$

Work done $$W_{2-3} = \frac{mR(T_3 - T_2)}{1-\gamma}$$

$$W = \frac{1 \times 0.293 \times (601.65 - 1146)}{1 - 1.38}$$

$$= 419.72 \text{ kJ}$$

(iii) *Total work transfer*

$$W_{total} = W_{1-2} + W_{2-3} = 156 + 419.72 = \mathbf{575.72 \text{ kJ}}$$

Example 4.12 *0.5 kg of air is compressed reversibly and adiabatically from 80 kPa and 60°C to 0.4 MPa and is then expanded at constant pressure to the original volume. Sketch the process on p-V and T-S diagrams, compute work transfer, heat transfer and change in entropy for whole path.*
Take R = 0.287 kJ/kg·K and γ = 1.4

Solution

Given Reversible adiabatic and constant pressure processes;

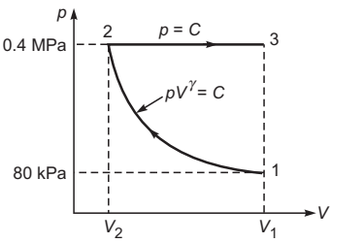
$m = 0.5$ kg $\qquad p_1 = 80$ kPa.
$R = 0.287$ kJ/kg·K $\qquad T_1 = 60°\text{C} + 273 = 333$ K
$\gamma = 1.4$ $\qquad p_2 = 0.4$ MPa $= 400$ kPa
$s_1 = s_2$ $\qquad p_2 = p_3$

To find

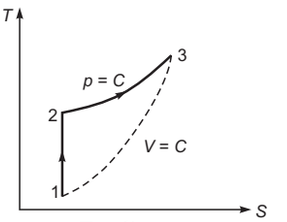
(i) Work done,
(ii) Heat transfer, and
(iii) Change in entropy.

Assumptions

(i) The system is non-flow, and
(ii) Air is a perfect gas.



p–V diagram



T–s diagram

Fig. 4.11 *p–V and T–s diagrams*

Analysis

(i) *Work transfer*

The temperature after isentropic compression1–2;

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 333 \times \left(\frac{400}{80}\right)^{\frac{1.4-1}{1.4}}$$

$$= 527.41 \text{ K}$$

For constant-pressure process 2–3;

$$\frac{T_3}{T_2} = \frac{V_3}{V_2}$$

But $V_3 = V_1$

Hence, $$\frac{T_3}{T_2} = \frac{V_1}{V_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = \left(\frac{400}{80}\right)^{\frac{1}{1.4}} = 3.157$$

or $T_3 = T_2 \times 3.157 = 527.41 \times 3.157$

$= 1665.0$ K

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The work done in reversible adiabatic process

$$W_{1-2} = \frac{(p_2V_2 - p_1V_1)}{(1-\gamma)} = \frac{mR(T_2 - T_1)}{(1-\gamma)}$$

or $$W_{1-2} = \frac{0.5 \times 0.287 \times (527.41 - 333)}{1 - 1.4}$$

$$= \frac{27.89}{-0.4} = -69.744 \text{ kJ}$$

The work done during constant-pressure expansion

$$W_{2-3} = p_2(V_3 - V_2) = m\,R\,(T_3 - T_2)$$
$$= 0.5 \times 0.287 \times (1665.0 - 527.41)$$
$$= 163.24 \text{ kJ}$$

Total work transfer,

$$W_{total} = W_{1-2} + W_{2-3}$$
$$= -69.744 + 163.24 = \mathbf{93.5\ kJ}$$

(ii) *Heat transfer*

(a) During isentropic compression process

$$Q_{1-2} = 0$$

(b) During constant-pressure expansion

$$Q_{2-3} = m\,C_p\,(T_3 - T_2)$$
$$= (0.5 \times 1.005) \times (1665 - 527.41)$$
$$= 571.44 \text{ kJ}$$

Total heat transfer,

$$Q_{Total} = Q_{1-2} + Q_{2-3}$$
$$= 0 + 571.44 = \mathbf{571.44\ kJ}$$

(iii) *Change in entropy*

(a) During isentropic expansion,

$$\Delta S_{1-2} = 0$$

(b) During constant-pressure expansion,

$$\Delta S_{2-3} = \int \frac{\delta Q}{T} = mC_p \ln\left(\frac{T_3}{T_2}\right)$$

$$= 0.5 \times 1.005 \times \ln\left(\frac{1665}{527.41}\right)$$

$$= 0.578 \text{ kJ/K}$$

The total entropy change,

$$\Delta S_{Total} = \Delta S_{1-2} + \Delta S_{2-3} = 0 + 0.578$$
$$= \mathbf{0.578\ kJ/K}$$

4.1.5 Polytropic Process

The most commonly used functional relationship for many processes can be approximated by an equation as

$$p_1V_1^{\,n} = p_2V_2^{\,n} = pV^n = C \qquad ...(4.32)$$

where the exponent n is constant and it is referred as the ***polytropic index***. The process in the form of $pV^n = C$ is designated as *polytropic process*.

(*a*) *Determination of Polytropic Index n* If a polytropic process is taking place from an initial state p_1, V_1 to a final state p_2, V_2, then

$$p_1V_1^{\,n} = p_2V_2^{\,n}$$

Taking logarithms on both sides, we get

$$\ln p_1 + n \ln V_1 = \ln p_2 + n \ln V_2$$

or $$n\,(\ln V_1 - \ln V_2) = \ln p_2 - \ln p_1$$

or $$n = \frac{\ln\left(p_2 / p_1\right)}{\ln\left(V_1 / V_2\right)} \qquad ...(4.33)$$

(*b*) *p, V, T Relationship* If the adiabatic index γ is replaced by polytropic index n in equations (4.25), (4.26), (4.27) and (4.28), we get

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^n = \left(\frac{V_2}{V_1}\right)^{-n} = \left(\frac{v_2}{v_1}\right)^{-n} \qquad ...(4.34)$$

$$\frac{v_2}{v_1} = \frac{V_2}{V_1} = \left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}} \qquad ...(4.35)$$

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{V_2}{V_1}\right)^{-n}\left(\frac{V_2}{V_1}\right)$$

$$= \left(\frac{V_2}{V_1}\right)^{1-n} = \left(\frac{v_2}{v_1}\right)^{1-n} \qquad ...(4.36)$$

and $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)\left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \qquad ...(4.37)$$

The polytropic relation $pV^n = C$ represents the most convenient curve fitting to actual experimental data between p and V with the value of the index

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'n' for the process. Each of the four processes discussed earlier can also be represented by polytropic process as

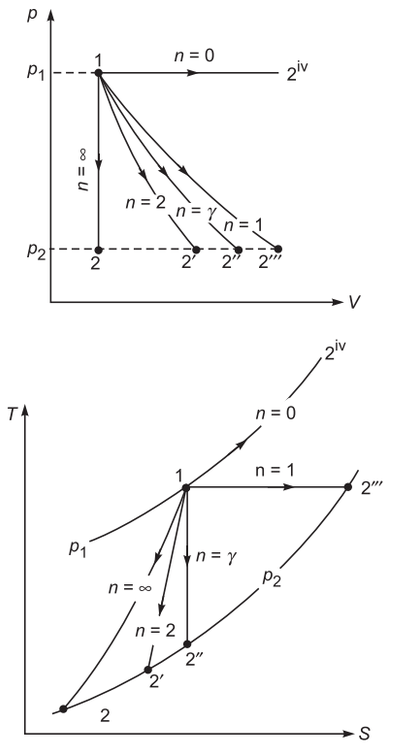

Fig. 4.12 *Polytropic processes on p-V and T-s diagrams*

(i) When $n = 0$, $p_1 V_1^{\,0} = p_2 V_2^{\,0}$
or $p_1 = p_2$

It is a *constant-pressure process.*

(ii) When $n = 1$, the process takes the form of *isothermal* or *hyperbolic process*; $pV = C$

(iii) When $n = \gamma$ (adiabatic index), the law of process takes the form

$$pV^{\gamma} = C$$

It is an *isentropic* or *adiabatic process.*

(iv) When $n = \infty$ then

$$p_1 V_1^{\infty} = p_2 V_2^{\infty}$$

where V^{∞} indicates a very large volume and thus can be treated as *constant,volume process.* It can also be illustrated with the help of Eq. (4.37),

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\infty}}$$

$$= \left(\frac{p_2}{p_1}\right)^{1} \times \left(\frac{p_2}{p_1}\right)^{-\frac{1}{\infty}}$$

$\because \quad \frac{1}{\infty} = 0$ and $\left(\frac{p_2}{p_1}\right)^{-\frac{1}{\infty}} = 1$

Thus $\frac{T_2}{T_1} = \frac{p_2}{p_1}$

It is a relation for the *constant volume process.*

All processes discussed above are illustrated graphically on *p-V* and *T-S* diagrams in Fig. 4.12.

(c) *Work Done* For a non-flow process, the polytropic work transfer is calculated as

$$W_{1-2} = \int_1^2 p\,dV$$

The pressure p can be expressed as

$$p = \frac{C}{V^n} = CV^{-n}$$

Hence $$W_{1-2} = \int_1^2 CV^{-n}dV = C\int_{V_1}^{V_2} V^{-n}dV$$

$$= C\left[\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1}\right]$$

Using $$C = p_1 V_1^{\,n} = p_2 V_2^{\,n}$$

we get $$W_{1-2} = \frac{p_2V_2 - p_1V_1}{1-n}$$

$$= \frac{mR(T_2 - T_1)}{1-n} \qquad ...(4.38)$$

where, $p_2V_2 = mRT_2$ and $p_1V_1 = mRT_1$

Using $$\frac{V_2}{V_1} = \left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}}$$

then $$W_{1-2} = \frac{p_1V_1}{1-n}\left\{\left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}} - 1\right\} \qquad ...(4.39)$$

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(*d*) *Change in Internal Energy* For any process,

$$\Delta U = mC_v(T_2 - T_1)$$

(*e*) *Change in Enthalpy* It is calculated as

$$\Delta H = mC_p(T_2 - T_1)$$

(*f*) *Heat Transferred* According to the first law of thermodynamics,

$$Q = \Delta U + W$$

$$= mC_v(T_2 - T_1) + \frac{mR(T_2 - T_1)}{1-n}$$

$$= m(T_2 - T_1)\left\{C_v + \frac{R}{1-n}\right\}$$

Using $R = C_p - C_v$

we get $$Q = m(T_2 - T_1)\frac{(C_v - nC_v + C_p - C_v)}{1-n}$$

$$= m\left(\frac{C_p - nC_v}{1-n}\right)(T_2 - T_1)$$

or $$Q = mC_n(T_2 - T_1) \quad ...(4.40)$$

where $$C_n = \frac{C_p - nC_v}{1-n} \quad ...(4.41)$$

C_n is called *polytropic specific heat.*

Again rearranging the above equation, we have

$$Q = mC_v\frac{\left(\frac{C_p}{C_v} - n\right)}{1-n} \times (T_2 - T_1)$$

Using $\frac{C_p}{C_v} = \gamma$ and $C_v = \frac{R}{\gamma - 1}$, we get

$$Q = \frac{(\gamma - n)}{(1-n)} \times \frac{mR(T_2 - T_1)}{(\gamma - 1)}$$

$$= \frac{\gamma - n}{\gamma - 1} \times \frac{mR(T_2 - T_1)}{1-n}$$

$$= \frac{\gamma - n}{\gamma - 1} \times \text{Polytropic work transfer} \quad ...(4.42)$$

Rearranging, we get

$$Q = \frac{(\gamma - n)}{(n-1)} \times \frac{mR(T_2 - T_1)}{(1-\gamma)}$$

$$= \frac{\gamma - n}{n-1} \times \text{Adiabatic work transfer} \quad ...(4.43)$$

(*g*) *Change of Entropy* For unit mass, the heat transfer

$$\delta q = C_n dT$$

Then change of entropy between two states

$$\Delta s_{1-2} = \int_1^2 \frac{\delta q}{T} = \int_{T_1}^{T_2} C_n \frac{dT}{T}$$

$$= C_n \ln\left(\frac{T_2}{T_1}\right) \text{ (kJ/kg·K)} \quad ...(4.44)$$

For a given mass system

$$\Delta S = mC_n \ln\left(\frac{T_2}{T_1}\right) \text{ (kJ/K)} \quad ...(4.45)$$

where T is expressed in kelvin (K)

Example 4.13 *The internal energy of a certain substance is given by the following relation:*

$$u = 3.56pv + 84$$

where u is given in kJ/kg, p in kPa, and v in m^3/kg.

A system composed of 3 kg of this substance expands from an initial pressure of 500 kPa and a volume of 0.22 m^3 to a final pressure of 100 kPa in a process in which pressure and volume are related by $pv^{1.2}$ = constant.

(a) If the expansion is quasi-static, find Q, ΔU and W for the process.

(b) In another process, the same system expands according to the same pressure – volume relationship as in part (i), and from the same initial state to the same final state as in part (i), but the heat transfer in this case is 30 kJ. Find the work transfer for this process.

Solution

Given A substance with

m = 3 kg,

$u = 3.56pv + 84$;

State 1: $v_1 = 0.22$ m^3/kg

$p_1 = 500$ kPa

State 2: $p_2 = 100$ kPa;

Case (i) executes a polytropic process with $pv^{1.2}$ = constant,

Case (ii) heat transfer, $Q = 30$ kJ

To find **Case (i)** (a) Total change in internal energy,

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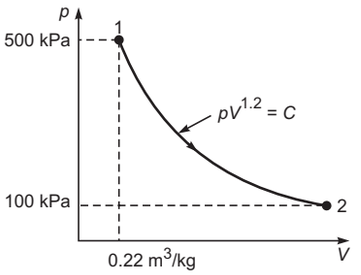

Fig. 4.13

(b) Total work transferred, and
(c) Total heat transferred.

Case (ii) Total work transferred.

Analysis

Case (i) For a polytropic process, between two states

$$p_1 v_1^{1.2} = p_2 v_2^{1.2}$$

Specific volume at the state 2;

$$v_2 = v_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{1.2}}$$

$$= (0.22 \text{ m}^3/\text{kg}) \times \left(\frac{500}{100}\right)^{\frac{1}{1.2}}$$

$$= 0.841 \text{ m}^3/\text{kg}$$

Change in specific internal energy

$$\Delta u = (3.56 p_2 v_2 + 84) - (3.56 p_1 v_1 + 84)$$
$$= 3.56 (p_2 v_2 - p_2 v_2)$$
$$= 3.56 \times (100 \times 0.841 - 500 \times 0.22) = -92.5 \text{ kJ/kg}$$

(a) Total internal energy change

$$\Delta U = m \, \Delta u = 3 \times (-92.5 \text{ kJ/kg}) = \mathbf{-277.5 \text{ kJ}}$$

(b) The work transfer/kg

$$w = \frac{(p_2 v_2 - p_1 v_1)}{1 - n}$$

$$= \frac{(100 \times 0.841 - 500 \times 0.22)}{1 - 1.2} = 129.5 \text{ kJ/kg}$$

Total work transferred;

$$W = m w$$
$$= (3 \text{ kg}) \times (129.5 \text{ kJ/kg}) = \mathbf{388.5 \text{ kJ}}$$

(c) Heat tranfrer $Q = \Delta U + W$
$= -277.5 + 388.5 = \mathbf{111 \text{ kJ}}$

Case (ii) For a system from the same initial state to the same final state, and same pressure–volume relationship, the change in internal energy will **remain same**, thus

$$\Delta U = \mathbf{-277.5 \text{ kJ}}$$

The total work transferred between the two states

$$W = Q - \Delta U$$
$$= 30 \text{ kJ} - (-277.5) = \mathbf{307.5 \text{ kJ}}$$

Example 4.14 *An internal combustion engine has a cylinder diameter of 15 cm and a stroke of 20 cm. The pressure and temperature of gas at the beginning of the compression stroke are 1 bar and 30°C, respectively. If the clearance volume is 1.147 litres and the law of compression is $pV^{1.2}$ = constant, determine pressure and temperature at the end of compression. If the working substance is air, determine the change in internal energy of air during compression. Take C_v = 0.718 kJ/kgK and R = 0.287 kJ/kgK.*

Solution

Given Compression of air

d = 15 cm = 0.15 m, L = 20 cm = 0.2 m

State 1: p_1 = 1 bar = 100 kN/m²;
T_1 = 30°C = 303 K

State 2: $pV^{1.2} = C$
$V_2 = V_c$ = 1.147 lit = 1.147×10^{-3} m³
C_v = 0.718 kJ/kg·K,
R = 0.287 kJ/kg·K

To find

(i) Pressure p_2, and temperature T_2 after compression, and
(ii) Change in internal energy of air after compression.

Assumption The compression process is quasi-static.

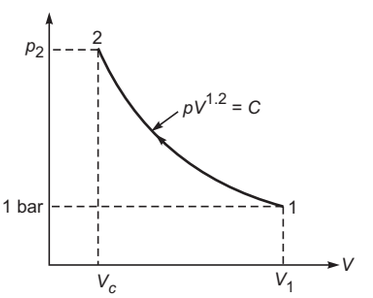

Fig. 4.14 *p-V diagram*

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Analysis The volume of air in the cylinder before compression.

V_1 = clearance volume + swept volume

$= V_c + (\pi/4)\, d^2 L$

$= 1.147 \times 10^{-3} + (\pi/4) \times (0.15)^2 \times 0.2$

$= 4.68 \times 10^{-3}$ m^3

(i) The pressure after compression can be calculated as

$$p_2 = p_1 \left(\frac{V_1}{V_2}\right)^n$$

$$= (100 \text{ kN/m}^2) \times \left(\frac{4.68 \times 10^{-3}}{1.147 \times 10^{-3}}\right)^{1.2}$$

$= 540$ kN/m^2 = **5.4 bar**

And the final temperature

$$T_2 = T_1 \times \left(\frac{V_2}{V_1}\right)^{1-n}$$

$$= 303 \times \left(\frac{4.68 \times 10^{-3}}{1.147 \times 10^{-3}}\right)^{1-1.2}$$

$= 401.4$ K = **124.4°C**

(ii) The change in internal energy

$$\Delta U = m\, C_v\, (T_2 - T_1)$$

where m = mass of air can be calculated from $p_1 V_1 = m R T_1$

or $$m = \frac{p_1 V_1}{R T_1} = \frac{(100 \text{ kPa}) \times (4.68 \times 10^{-3} \text{ m}^3)}{(0.287 \text{ kJ/kg} \cdot \text{K}) \times (303 \text{ K})}$$

$= 5.38 \times 10^{-3}$ kg

Then $\Delta U = 5.38 \times 10^{-3} \times 0.718 \times (401.4 - 303)$

= **0.38 kJ**

Example 4.15 *A cylinder contains 0.12 m^3 of air at 1 bar and 90°C. It is compressed to 0.03 m^3, the final pressure being 6 bar. Find the index of compression, increase in internal energy and heat transfered.*

Take $R = 0.287$ *kJ/kg·K and* $C_v = 0.717$ *kJ/kg·K*

Solution

Given Compression of gas in a cylinder

State 1: $V_1 = 0.12$ m^3

$p_1 = 1$ bar = 100 kPa

$T_1 = 90°C = 363$ K

State 2: $V_2 = 0.03$ m^3

$p_2 = 6$ bar = 600 kPa

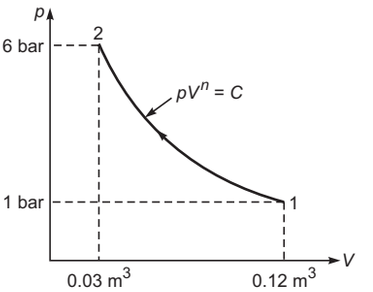

Fig. 4.15

Process: Polytropic process

Constants: $R = 0.287$ kJ/kg,

$C_v = 0.717$ kJ/kg·K

To find

(i) Index of compression,

(ii) Increase in internal energy, and

(iii) Heat transferred.

Analysis

(i) Considering the compression process is a quasi-static polytropic process, thus

$$p_1 V_1^{\,n} = p_2 V_2^{\,n}$$

Index of compression,

$$n = \frac{\ln\left(\frac{p_1}{p_2}\right)}{\ln\left(\frac{V_2}{V_1}\right)} = \frac{\ln\left(\frac{1}{6}\right)}{\ln\left(\frac{0.03}{0.12}\right)} = \frac{-1.7917}{-1.3863}$$

= **1.292**

(ii) Temperature after compression

$$T_2 = T_1 \left(\frac{V_2}{V_1}\right)^{1-n} = 363 \times \left(\frac{0.03}{0.12}\right)^{1-1.292}$$

$= 544.5$ K

Mass of air in the cylinder

$$m = \frac{p_1 V_1}{R T_1} = \frac{(100 \text{ kPa}) \times (0.12 \text{ m}^3)}{(0.287 \text{ kJ/kg} \cdot \text{K}) \times (363 \text{ K})}$$

$= 0.115$ kg

Change of internal energy of air during compression;

$\Delta U = m\, C_v (T_2 - T_1)$

$= 0.115 \times 0.717 \times (544.5 - 363)$

= **14.99 kJ**

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Work transfer

$$W = \frac{p_2V_2 - p_1V_1}{1-n} = \frac{600\times0.03 - 100\times0.12}{1-1.292}$$

$$= -20.548 \text{ kJ}$$

(iii) Heat transfer during compression

$$Q = \Delta U + W$$

Heat transferred $Q = 14.99 - 20.548$

$= -$**5.55 kJ** (Rejected)

Example 4.16 *In a vertical cylinder, air below a frictionless piston occupies a volume of 0.09 m^3 at a pressure of 10 bar. The diameter of the cylinder is 0.6 m. The mass of the piston is 90 kg. A stopper which keeps the piston is suddenly removed. The piston moves upward through a distance of 1.5 m.*

The pressure of air above the piston is 1 bar.

The air expands according to $pV^{1.3} = C$. Neglect the velocity of air and calculate the velocity of the piston when it has risen 1.5 m.

Solution

Given

$V_1 = 0.09$ m^3 $\quad p_1 = 10$ bar $= 1000$ kPa

$d = 0.6$ m $\quad m = 90$ kg

$h = 1.5$ m $\quad p_0 = 100$ kPa

$n = 1.3$

To find Velocity of piston as it releases.

Analysis The change in volume during piston displacement of 1.5 m

$$\Delta V = \frac{\pi}{4}d^2h = \frac{\pi}{4}\times(0.6\text{ m})^2\times(1.5\text{ m}) = 0.4241 \text{ m}^3$$

The volume after expansion

09 + 0.4241 = 0.5141 m^3

pansion

$$\left.\right)^n = 1000\times\left(\frac{0.09}{0.5141}\right)^{1.3}$$

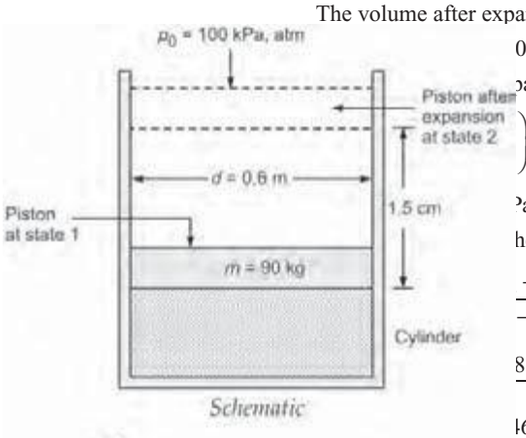
Pa.

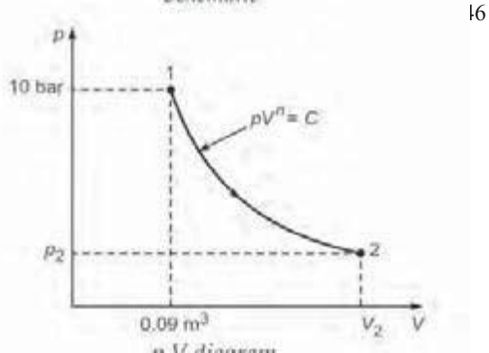
he gas during expansion

$$\frac{-p_1V_1)}{-n)}$$

$$\frac{8\times0.5141 - 1000\times0.09}{1-1.3}$$

46 kJ





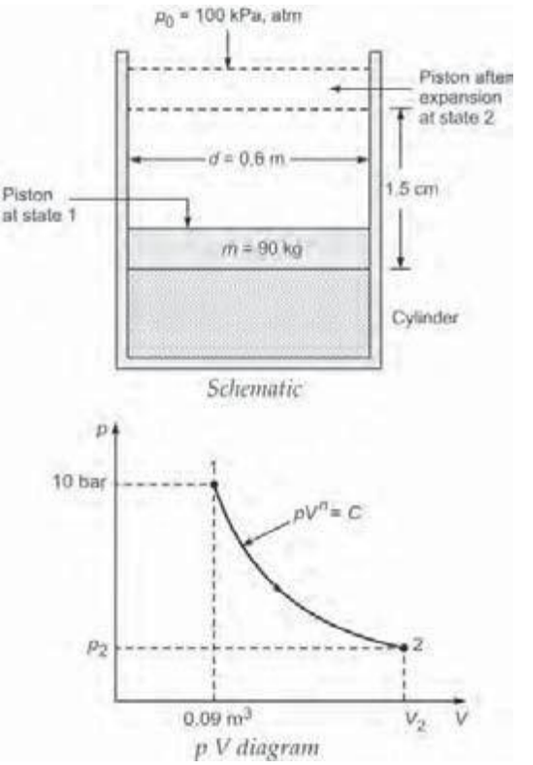


Fig. 4.16

This work is used to increase the potential and kinetic energy of the piston and to displace atmospheric air by 1.5 m.

Thus $W = mgh + (1/2)m\mathcal{V}^2 + p_0(\Delta V)$

or $122.146 = 90\times[(9.81)\times(1.5) + (1/2)\times(\mathcal{V}^2)]$

$+ (1\times10^6 \text{ Pa})\times0.4241 \text{ m}^3$

or $$\mathcal{V}^2 = \frac{(122.146 - 42410 - 1324.35)\times2}{90}$$

$= 1742.5$ m^2

or $\mathcal{V} = \sqrt{(1742.45)}$

$=$ **41.74 m/s**

Example 4.17 *0.2 m^3 of a mixture of fuel and air at 1.2 bar and 60°C is compressed till its pressure becomes 12 bar and temperature 270°C. Then it is ignited suddenly at constant volume and its pressure becomes twice the pressure at the end of compression. Find the maximum temperature reached and change in internal energy. Also, find the heat transfer during the compression process.*

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Consider the mixture as perfect gas.

Take $C_p = 1.05$ *kJ/kg·K and* $\gamma = 1.4$.

Solution

Given A mixture of fuel and air as a perfect gas

State 1: $p_1 = 1.2$ bar $= 120$ kPa $\quad V_1 = 0.2$ m^3

$T_1 = 60°C + 273 = 333$ K

State 2: $p_2 = 12$ bar $= 1200$ kPa $\quad T_2 = 270°C + 273 = 543$ K

$p_3 = 2p_2 \quad V_2 = V_3$

$C_p = 1.05$ kJ/kg·K $\quad \gamma = 1.4$

To find

(i) T_3, the maximum temperature,

(ii) Net change in Internal energy, ΔU,

(iii) Heat transfer during compression Q_{1-2}.

Analysis Since the states before and after compression are known and the type of compression is not mentioned, therefore, calculation of polytropic index:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

Taking logarithms, we get

$$\ln\left(\frac{T_2}{T_1}\right) = \left(1-\frac{1}{n}\right)\ln\left(\frac{p_2}{p_1}\right)$$

$$n = \frac{\ln\left(p_2/p_1\right)}{\ln\left(p_2/p_1\right) - \ln\left(T_2/T_1\right)}$$

Substituting the values in the equation, we get

$$n = \frac{\ln\left(1200/120\right)}{\ln\left(1200/120\right) - \ln\left(543/333\right)} = 1.27$$

The specific heat at constant volume

$$C_v = \frac{C_p}{\gamma} = \frac{1.05}{1.4} = 0.75 \text{ kJ/kg·K}$$

The characteristic gas equation

$$R = C_p - C_v = 1.05 - 0.75 = 0.3 \text{ kJ/kg·K}$$

The mass of the gas

$$m = \frac{p_1 V_1}{RT_1} = \frac{(120 \text{ kPa}) \times (0.2 \text{ m}^3)}{(0.3 \text{ kJ/kg·K}) \times (333 \text{ K})} = 0.24 \text{ kg}$$

The property relationship for constant volume process 2-3;

$$\frac{p_2}{T_2} = \frac{p_3}{T_3}$$

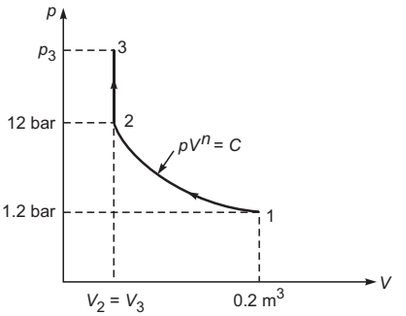

Fig. 4.17

The maximum temperature

$$T_3 = T_2 \frac{p_3}{T_2} = 543 \times 2 = 1086 = \mathbf{813°C}$$

Net change in internal energy,

$$\Delta U_{1-3} = m C_v (T_3 - T_1)$$
$$= 0.24 \times 0.75 \times (1086 - 333)$$
$$= \mathbf{135.67 \text{ kJ/kg·K}}$$

The polytropic specific heat

$$C_n = \frac{C_p - nC_v}{1-n}$$

$$= \frac{(1.05 - 1.27 \times 0.75)}{1-1.27} = -0.36 \text{ kJ/kgK}$$

Heat transfer during compression process

$$Q_{1-2} = m C_n (T_2 - T_1)$$
$$= (0.24 \text{ kg}) \times (-0.36 \text{ kJ/kg·K})\ (543 - 333)(\text{K})$$
$$= \mathbf{-14.2 \text{ kJ}} \text{ (The heat is rejected)}$$

Example 4.18 *A certain mass of air initally at a pressure of 480 kPa and a temperature 190°C is expanded adiabatically to a pressure of 94 kPa. It is then heated at constant volume until it attains its initial temperature when the pressure is found to be 150 kPa. State the type of compression necessary to bring the system back to its original pressure and volume. Determine*

(a) the index of adiabatic expansion,

(b) the work done per kg of air, and

(c) the change in specific entropy of air.

Take $R = 0.29$ *kJ/kg·K.*

Solution

Given Three processes constitute a cycle

State 1: $p_1 = 480$ kPa $\quad T_1 = 190°C + 273 = 463$ K,

$s_1 = s_2 \quad R = 0.29$ kJ/kg·K

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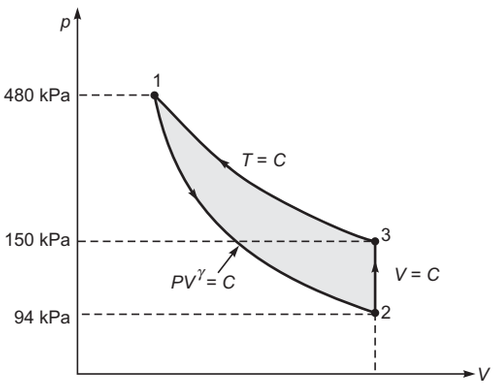
State 2: $p_2 = 94$ kPa $\quad V_2 = V_3$
State 3: $T_3 = T_1$ $\quad p_3 = 150$ kPa

To find

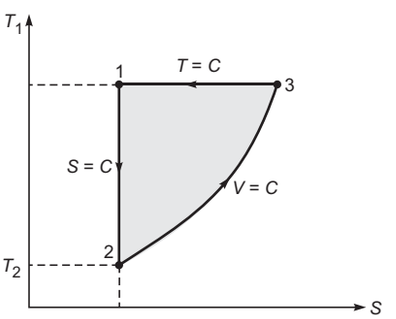
(i) Index of adiabatic expansion,
(ii) The work done per kg of air, and
(iii) Change in specific entropy.

Assumptions

(i) The air is perfect gas, and
(ii) Reversible process.



p–V diagram



T–s diagram

Fig. 4.18

Analysis

(i) To determine adiabatic index, the state before and after expansion must be defined. Therefore, we have to consider constant-volume heating process 2-3, in order to calculate the temperature at state 2.

$$\frac{p_3}{T_3} = \frac{p_2}{T_2}$$

where $T_3 = T_1$

Therefore,

$$T_2 = \left(\frac{p_2}{p_3}\right) T_1 = \left(\frac{94}{150}\right) \times 463 = 290.14 \text{ K}$$

Now the adiabatic index γ can easily be calculated as,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Taking logarithm on both sides, we get

$$\ln\left(\frac{T_2}{T_1}\right) = \left\{\left(\frac{\gamma-1}{\gamma}\right)\right\}. \ln\left(\frac{p_2}{p_1}\right)$$

or $$\gamma\left\{\ln\left(\frac{p_2}{p_1}\right) - \ln\left(\frac{T_2}{T_1}\right)\right\} = \ln\left(\frac{p_2}{p_1}\right)$$

or $$\gamma = \frac{\ln(p_2/p_1)}{\ln(p_2/p_1) - \ln(T_2/T_1)}$$

$$\gamma = \frac{\ln(94/480)}{\ln(94/480) - \ln(290.14/463)}$$

$$= \mathbf{1.401}$$

(ii) (a) Work done per kg of air during adiabatic process 1–2;

$$w_{1\text{-}2} = \frac{R(T_2 - T_1)}{1-\gamma}$$

$$w_{1\text{-}2} = \frac{0.29 \times (290.14 - 463)}{1 - 1.4}$$

$$= 125.32 \text{ kJ/kg}$$

(b) Work done during constant-volume process 2–3;

$$w_{2\text{–}3} = 0$$

(c) Work done during isothermal process 3–1;

$$w_{3\text{-}1} = R\,T_1 \ln\left(\frac{p_3}{p_1}\right)$$

$$= 0.29 \times 463 \times \ln\left(\frac{150}{480}\right)$$

$$= -156.17 \text{ kJ/kg}$$

Net work done per kg of air

w_{net} = 125.32 + 0 – 156.176

= **30.856 kJ**

(iii) (a) Change in specific entropy for isentropic expansion process, $\Delta s_{1\text{–}2} = 0$

(b) Change in specific entropy during constant volume heating process 2–3

$$s_3 - s_2 = C_v \ln\left(\frac{T_3}{T_2}\right)$$

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where $C_v = \dfrac{R}{\gamma - 1} = \dfrac{0.29}{1.401 - 1}$

$= 0.723$ kJ/kg·K

$\therefore \quad \Delta s_{2-3} = 0.723 \times \ln\left(\dfrac{463}{290.14}\right)$

$= 0.337$ kJ/kg·K

(c) Change in entropy during isothermal process 3–1

$$s_3 - s_1 = R \ln\left(\frac{p_3}{p_1}\right)$$

$$= 0.29 \times \ln\left(\frac{150}{480}\right)$$

$$= -0.337 \text{ kJ/kg·K}$$

Net change of entropy during cyclic process 1-2-3-1;

$\Delta s_{cycle} = 0 - 0.337 + 0.337$

$=$ **0** (*since entropy is a property*)

Example 4.19 *3 kg of air at a pressure of 150 kPa and temperature 360 K is compressed polytropically to 750 kPa according to law $pV^{1.2}=C$. The gas is then cooled to initial temperature at constant pressure. The air is then expanded at constant temperature till it reaches original pressure of 150 kPa. Draw the cycle on p–V diagram and determine the net work and heat transfer.*

Solution

Given Three processes constitute a cycle

$m = 3$ kg air

State 1: $p_1 = 150$ kPa, $T_1 = 360$ K

State 2: $p_2 = 750$ kPa $pV^{1.2} = C$

or $p_1 V_1^{1.2} = p_2 V_2^{1.2}$

State 3: $p_2 = p_3 = 750$ kPa, $T_3 = T_1 = 360$ K

To find

(i) Work transfer, and

(ii) Heat transfer.

Assumptions

(i) The specific gas constant $R = 0.287$ kJ/kg·K

(ii) The specific heat $C_p = 1.005$ kJ/kg·K

(iii) The specific heat $C_v = 0.7165$ kJ/kg·K

Analysis The temperature after polytropic compression

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = 360 \times \left(\frac{750}{150}\right)^{\frac{1.2-1}{1.2}}$$

$= 470.75$ K

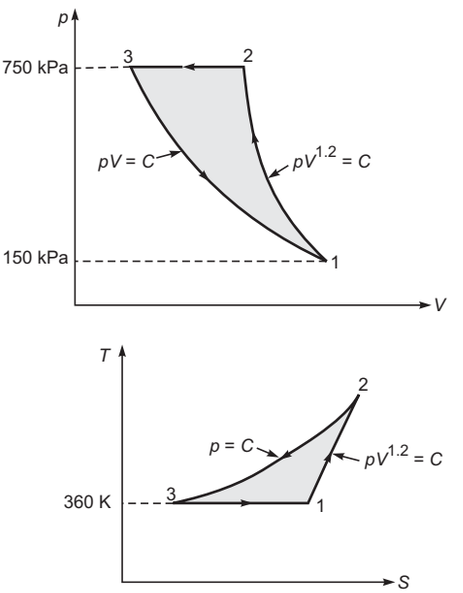

Fig. 4.19 *p-V and T-s diagram*

(i) (a) The work done during polytropic compression process 1–2;

$$W_{1\text{-}2} = \frac{(p_2V_2 - p_1V_1)}{(1-n)} = \frac{mR(T_2 - T_1)}{(1-n)}$$

$$= \frac{3 \times 0.287 \times (470.75 - 360)}{(1 - 1.2)}$$

$= -476.8$ kJ

(b) The work done during constant pressure cooling process 2–3;

$W_{2-3} = mR(T_3 - T_2)$

$= 3 \times 0.287 \times (360 - 470.74)$

$= -95.355$ kJ

(c) The work done during isothermal process 3–1;

$$W_{3-1} = mRT_1 \ln\left(\frac{p_3}{p_1}\right)$$

$$= 3 \times 0.287 \times 360 \times \ln\left(\frac{750}{150}\right)$$

$= 494.86$ kJ

Net work transfer;

$W_{net} = W_{1\text{-}2} + W_{2\text{-}3} + W_{3\text{-}1}$

$= -476.8 + (-95.355) + 494.86$

$=$ **–73.5 kJ**

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(ii) (a) Heat transfer during polytropic compression

$$Q_{1\text{-}2} = \frac{\gamma - n}{\gamma - 1} \times \text{Polytropic work done}$$

$$= \frac{(1.4 - 1.2)}{1.4 - 1} \times (-476.8)$$

$$= -234.4 \text{ kJ}$$

(b) Heat transfer during constant pressure cooling

$$Q_{2\text{–}3} = m\,C_p(T_3 - T_2)$$

$$= 3 \times 1.005 \times (360 - 470.75)$$

$$= -334 \text{ kJ}$$

(c) Heat transfer during isothermal expansion
= work done during the process

$$Q_{3\text{–}1} = W_{3\text{–}1} = 494.86 \text{ kJ}$$

Net heat transfer

$$Q_{net} = Q_{1\text{–}2} + Q_{2\text{–}3} + Q_{3\text{–}1}$$

$$= -234.4 - 334 + 494.86$$

$$= \mathbf{-73.5\ kJ}$$

$\because$ for a cycle $Q_{net} = W_{net} = -$ **73.5 kJ**

Example 4.20 *1 kg of gas expands reversibly according to linear law from 4.2 bar to 1.4 bar. The initial and final volumes are 0.004 m^3 and 0.02 m^3, respectively. The gas is then cooled at constant pressure and finally compressed isothermally back to its initial state of 4.2 bar and 0.004 m^3. Calculate the work done in each process stating its direction. Sketch the cycle on a p–V diagram.*

Solution

Given *Expansion, cooling and compression of gas*

$m = 1$ kg

State 1: $p_1 = 4.2$ bar $= 420$ kPa, $V_1 = 0.004$ m^3

State 2: $p_2 = 1.4$ bar $= 140$ kPa, $V_2 = 0.02$ m^3

State 3: $p_3 = p_2 = 140$ kPa, and p_3V_3 or $p_2V_3 = p_1V_1$

To find

(i) Sketch of processes on a p–V diagram,

(ii) Work transfer during each process.

Assumptions

(i) The gas is an ideal gas, and

(ii) Each process is a reversible process.

Analysis The initial process is proceeded linearly and its law may be

$$p = aV + C$$

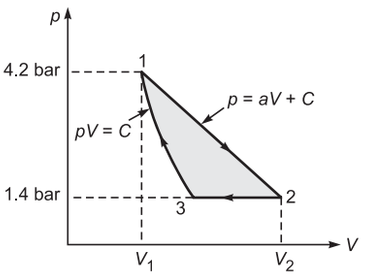

Fig. 4.20 *p–V diagram*

Using p_1, p_2, V_1 and V_2 to obtain constants a and C,

Assuming $p_i = aV_i + C$

then $420 = a \times 0.004 + C$ ...(i)

and $140 = a \times 0.02 + C$ ...(ii)

Solving these equations, we get

$a = -17500$ kPa/m^3

and $C = 490$ kPa

Therefore, for linear law, the pressure is expressed as

$$p = (-17500\,V + 490) \text{ kPa}$$

The work transfer during the process

$$W_{1\text{–}2} = \int_1^2 p\,dV = \int_1^2 (-17500V + 490)\,dV$$

$$= \left[-17500 \times \frac{V^2}{2} + 490V\right]_{0.004}^{0.02}$$

$$= -\frac{17500}{2} \times [0.02^2 - 0.004^2] + 490 \times [0.02 - 0.004]$$

$$= -3.36 + 7.84 = \mathbf{4.48\ kJ}$$

The quantity $W_{1\text{–}2}$ is positive and hence, it is the work done by the system.

Volume after constant-pressure cooling process 3–1

$$V_3 = \frac{p_1V_1}{p_3} = \frac{p_1V_1}{p_2} = \frac{420 \times 0.004}{140}$$

$$= 0.012 \text{ m}^3$$

The work transfer during constant pressure cooling process, $p_2 = p_3$,

$$W_{2\text{–}3} = p_2(V_3 - V_2)$$

Then $W_{2\text{–}3} = (140 \text{ kPa}) \times (0.012 - 0.02) \text{ m}^3$

$$= -\mathbf{1.12\ kJ}$$

It is negative, thus work is done on the system.

Work transfer during an isothermal process,

$$W_{3\text{–}1} = p_1V_1 \ln\left(\frac{V_1}{V_3}\right)$$

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$$= 420 \times 0.004 \times \ln\left(\frac{0.004}{0.012}\right)$$

$$= -\mathbf{1.845\ kJ}$$

It is also negative thus; work done on the system is isothermal compression work.

Example 4.21 *A perfect gas undergoes a cycle comprises of three processes. It is first compressed isothermally from 1 bar and 27°C to one-eighth of its initial volume. The energy is then added at constant pressure, increasing the temperature of gas and the cycle is completed by isentropic expansion to original conditions. Take C_p = 1.25 kJ/kg·K and R = 0.5 kJ/kg·K. Calculate the maximum cycle temperature and pressure. Also, find the net work transfer per kg.*

Solution

Given A perfect gas undergoes a cyclic process ;

State 1: p_1 = 1 bar = 100 kPa, T_1 = 27°C + 273 = 300 K

State 2: $T_2 = T_1$, $v_2 = (1/8)\, v_1$

State 3: $p_3 = p_2$, $s_3 = s_1$

R = 0.5 kJ/kg·K C_p = 1.25 kJ/kg·K

To find

(i) Maximum pressure and temperature in the cycle, and

(ii) Net work transfer in the cycle.

Analysis The specific heat at constant volume

$$C_v = C_p - R$$

$$= 1.25 - 0.5 = 0.75\ \text{kJ/kg·K}$$

Ratio of two specific heats

$$\gamma = \frac{C_p}{C_v} = \frac{1.25}{0.75} = 1.667$$

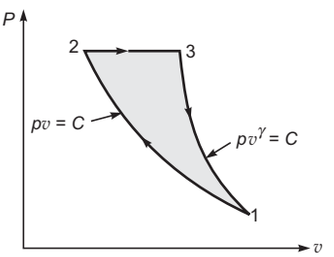

Fig. 4.21

Work done per kg in isothermal compression

$$w_{1\text{-}2} = p_1 v_1 \ln\left(\frac{v_2}{v_1}\right) = RT_1 \ln\left(\frac{v_2}{v_1}\right)$$

$$w_{1\text{-}2} = 0.5 \times 300 \times \ln(1/8) = -312\ \text{kJ/kg}$$

Now $$p_2 = \frac{p_1 v_1}{v_2} = 1 \times 8 = 8\ \text{bar}$$

The maximum pressure in the cycle,

$$p_2 = p_3 = \mathbf{8\ bar}$$

Now considering process 3-1

$$\frac{T_3}{T_1} = \left(\frac{p_3}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

and $$T_3 = T_1\left(\frac{p_3}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

The maximum temperature

$$T_3 = 300 \times \left(\frac{8}{1}\right)^{\frac{1.667-1}{1.667}}$$

$$= 689.4\ \text{K} = \mathbf{416.4°C}$$

Work done per kg during constant pressure process

$$w_{2\text{–}3} = p_2(v_3 - v_2) = R(T_3 - T_2) = R(T_3 - T_1)$$

$$= 0.5 \times (689.4 - 300) = 194.6\ \text{kJ/kg}$$

Work done per kg during isentropic expansion

$$w_{3\text{-}1} = \frac{R(T_1 - T_3)}{1-\gamma} = \frac{0.5 \times (300 - 689.4)}{1 - 1.667}$$

$$= 291.75\ \text{kJ/kg}$$

Net work transfer in the cycle

$$w_{net} = -312 + 194.6 + 291.75 = \mathbf{174.35\ kJ/kg}$$

Example 4.22 *A system expands from the state 1 (p_1, V_1, T_1) to the state 2 (p_2, V_2, T_2) in a reversible non-flow process as defined below:*

(a) $pV^n = C$

(b) A constant-pressure process followed by a constant-volume process.

(c) A constant-volume process followed by a constant-pressure process.

Show these processes on a p–V diagram and find the ratio of work done in these processes, if $p_1 : p_2 = 5 : 2$ and $V_2 : V_1 = 2 : 1$ and $n = 1.4$.

Solution Processes are shown on a *p–V* diagram

Path 1. Process 1–*a* –2 process $pV^n = C$

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Path 2. Process 1– *b* –2 constant-pressure process followed by constant-volume process.

Path 3. Process 1– *c* – 2 constant-volume process followd by constant-pressure process.

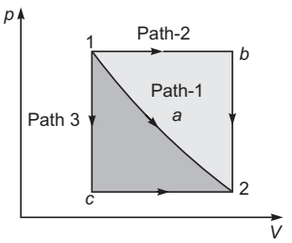

Fig. 4.22

Work done during path 1, polytropic process

$$W_1 = \frac{(p_2V_2 - p_1V_1)}{(1-n)}$$

Work done during path 2,

$$W_2 = p_1(V_2 - V_1) + 0$$

Work done during path 3,

$$W_3 = 0 + p_2(V_2 - V_1)$$

Put $\frac{p_2}{p_1} = \frac{2}{5}$ and $\frac{V_2}{V_1} = \frac{2}{1}$

$$\frac{W_1}{W_2} = \left[\frac{\{p_2V_2 - p_1V_1\}/(1-n)}{[p_1(V_2 - V_1)]}\right]$$

$$= \frac{[p_1V_1\{1 - p_2V_2/p_1V_1\}]}{(n-1)p_1V_1\left\{\left(V_2/V_1\right) - 1\right\}} = \frac{\{1-(2/5)\times 2\}}{0.4\times(2-1)}$$

$$= 0.5 = \frac{5}{10}$$

Now $$\frac{W_2}{W_3} = \frac{\{p_1(V_2 - V_1)\}}{\{p_2(V_2 - V_1)\}} = \frac{5}{2} = \frac{10}{4}$$

Therefore $W_1 : W_2 : W_3 =$ **5 : 10 : 4**

Example 4.23

Process 1: Air initially at 100 kPa and 50°C undergoes reversible adiabatic comprerssion such that its volume is reduced to one-fifth of its initial volume.

Process 2: 940 kJ/kg of heat is added to air to constant volume.

Process 3: Process 2 is followed by a reversible adiabatic expansion up to initial volume.

Process 4: Finally, heat is rejected at constant volume so as to reach the initial condition.

Draw the four processes on a p–V diagram. Determine the maximum temperature, net work done and heat rejected per kg of air. Assume adiabatic index of compression and expansion of 1.4 and constant-volume specific heat as 0.717 kJ/kg·K.

Solution

Given Air undergoes four processes in a cyclic process.

Process 1–2: Reversible adiabatic

$p_1 = 100$ kPa

$T_1 = 50°C = 323$ K

$v_2 = 0.2v_1$

Process 2–3: Constant volume heat addition

$q_{2-3} = 940$ kJ/kg

Process 3–4: Reversible adiabatic expansion and

$v_4 = v_1$

Process 4–1: Constant volume heat rejection to approach initial state:

Constants: $\gamma = 1.4$, $C_v = 0.717$ kJ/kg·K

To find

(i) To draw a *p–V* diagram,

(ii) Maximum temperature T_3 in the cycle,

(iii) Net work done per kg of the cycle, and

(iv) Heat rejected per kg of air.

Analysis

(i) The *p–V* diagram is shown in Fig. 4.23.

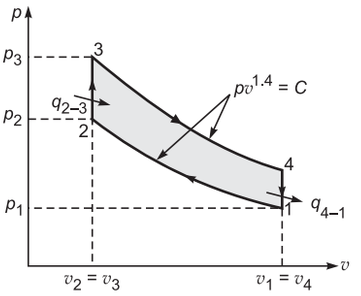

Fig. 4.23 *p–V diagram*

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The specific gas constant for air

$$R = C_v(\gamma - 1) = 0.717 \times (1.4 - 1)$$
$$= 0.287 \text{ kJ/kg . K}$$

(ii) Maximum temperature in the cycle.

(a) Temperature after adiabatic compression

$$T_2 = T_1\left(\frac{v_2}{v_1}\right)^{1-\gamma} = 323 \times \left(\frac{1}{5}\right)^{1-1.4}$$
$$= 614.88 \text{ K}$$

(b) Temperature after constant volume heat addition

$$q_{2-3} = C_v(T_3 - T_2)$$

or $$940 = 0.717 \times (T_3 - 614.88)$$

or $$T_3 = \frac{940}{0.717} + 614.88$$
$$= 1925.9 \text{ K}$$

Max. temperature, T_3 = **1925.9 K or 1652.9°C**

Temperature T_4 can be obtained as

$$\frac{T_4}{T_3} = \left(\frac{v_4}{v_3}\right)^{1-\gamma} = \left(\frac{v_1}{v_2}\right)^{1-\gamma}$$

or $$T_4 = T_3\left(\frac{v_1}{v_2}\right)^{1-\gamma} = 1925.9 \times (5)^{1-1.4}$$
$$= 1011.68 \text{ K}$$

(iii) Net work done per kg of air in the cycle;

$$w_{net} = \Sigma w$$
$$= w_{1-2} + w_{2-3} + w_{3-4} + w_{4-1}$$

But for constant-volume process

$$w_{2-3} = 0 \quad \text{and} \quad w_{4-1} = 0$$

$\therefore$ $$w_{net} = w_{1-2} + w_{3-4}$$

Calculating w_{1-2} and w_{3-4} separately;

(a) Compression work per kg during the process 1–2:

$$w_{1-2} = \frac{R(T_2 - T_1)}{1 - \gamma}$$
$$= \frac{0.287 \times (614.88 - 323)}{1 - 1.4}$$
$$= -209.42 \text{ kJ/kg}$$

(b) Expansion work per kg, duirng the process 3–4:

$$w_{3-4} = \frac{R(T_4 - T_3)}{1 - \gamma}$$
$$= \frac{0.287 \times (1011.68 - 1925.9)}{1 - 1.4}$$
$$= 655.95 \text{ kJ/kg}$$

Net work done per kg in the cycle

$$w_{net} = -209.42 \text{ kJ/kg} + 655.95 \text{ kJ/kg}$$
$$= \mathbf{446.53 \text{ kJ/kg}}$$

(iv) Heat rejection per kg of air during the process 4–1

$$q_{4-1} = C_v(T_1 - T_4)$$
$$= 0.717 \times (323 - 1011.68)$$
$$= \mathbf{-493.78 \text{ kJ/kg}}$$

($\because q_{1-2} = 0$ and $q_{3-4} = 0$ adiabatic processes)

Heat rejection can also be obtained.

$$\Sigma w = \Sigma q$$

$\Rightarrow$ $$q_{4-1} + 940 = 446.53$$

or $$q_{4-1} = -493 \text{ kJ/kg}$$

Example 4.24 *A non-flow system executes four different thermodynamics processes in sequence continuously as followed:*

1–2: Adiabatic compression,

2–3: Isobaric heat addition,

3–4: Adiabatic expansion, and

4–1: Constant volume heat rejection

The temperature at four silient points 1, 2, 3 and 4 are respectivley 300 K, 700 K, 1500 K, and 600 K. The working substance is 1 kg mass of air, for which

$$C_p = 1.005 \text{ kJ/kg·K}, \quad C_v = 0.718 \text{ kJ/kg·K}$$

Determine the following for each process:

(a) Heat transfer,
(b) Work transfer,
(c) Change in internal energy Also, find
(d) Net work and heat transfer for the cycle, and
(e) Change in internal energy for the cycle.

Comment on results (d) and (e).

Solution

Given Four different processes are in sequence, executed by a non-flow system of air.

$m = 1$ kg, $C_p = 1.005$ kJ/kg·K,

$C_v = 0.718$ kJ/kg·K $T_1 = 300$ K,

$T_2 = 700$ K, $T_3 = 1500$ K,

and $T_4 = 600$ K

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To find

(i) For each process,
  (a) Heat transfer,
  (b) Work transfer, and
  (c) Change in internal energy.

(ii) For cyclic process with comments:
  (a) Net work transfer, and
  (b) Net heat transfer.

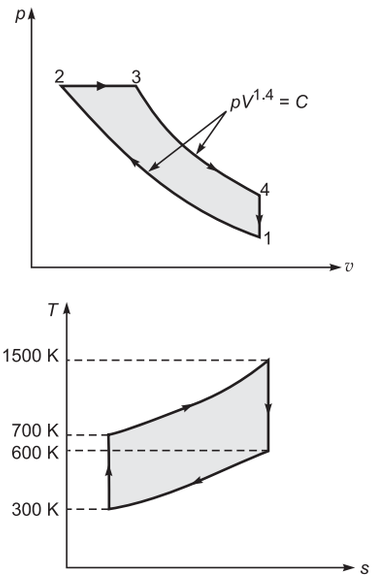

Fig. 4.24 *p–V and T–s diagram*

Assumption All processes are reversible processes.

Analysis The specific gas constant

$$R = C_p - C_v = 1.005 - 0.718 = 0.287 \text{ kJ/kg·K}$$

Ratio of two specific heats

$$\gamma = \frac{C_p}{C_v} = \frac{1.005}{0.718} = 1.4$$

(i) Considering each process separately

1. *Process 1–2: Adiabatic compression process, $pV^{\gamma} = C$*

  (a) Heat transferred:

  $Q_{1-2} = \mathbf{0}$ for adiabatic process

  (b) Work transfer;

$$W_{1-2} = \frac{mR(T_2 - T_1)}{1-\gamma}$$

$$= \frac{1 \times 0.287 \times (700 - 300)}{1 - 1.4}$$

$$= \mathbf{-287 \text{ kJ}}$$

  (c) Change in internal energy:

$$\Delta U = mC_v (T_2 - T_1)$$

$$= 1 \times 0.718 \times (700 - 300) = \mathbf{287 \text{ kJ}}$$

$\because Q_{1-2} = 0 \qquad \therefore \Delta U = -W_{1-2}$

2. *Process 2–3: Isobaric heat addition*

$T_2 = 700$ K, and $T_3 = 1500$ K, p = Const.

  (a) Heat transferred,

$$Q_{2-3} = mC_p (T_3 - T_2)$$

$$= 1 \times 1.005 (1500 - 700) = \mathbf{804 \text{ kJ}}$$

  (b) Work transferred,

$$W_{2-3} = p_2(V_3 - V_2) = mR(T_3 - T_2)$$

$$= 1 \times 0.287 \times (1500 - 700)$$

$$= \mathbf{229.6 \text{ kJ}}$$

  (c) Change in internal energy

$$\Delta U_{2-3} = Q_{2-3} - W_{2-3}$$

$$= 804 - 229.6 = \mathbf{574.4 \text{ kJ}}$$

3. *Process 3–4: Adiabatic expansion process*

$T_3 = 1500$ K, $T_4 = 600$ K

  (a) Heat transferred, $Q_{3-4} = \mathbf{0}$

  (b) Work transferred,

$$W_{3-4} = \frac{mR(T_4 - T_3)}{1-\gamma}$$

$$= \frac{1 \times 0.287 (600 - 1500)}{1 - 1.4}$$

$$= \mathbf{645.75 \text{ kJ}}$$

  (c) Change in internal energy,

$$\Delta U_{3-4} = -W_{3-4} = \mathbf{-645.75 \text{ kJ}}$$

4. *Process 4–1: Constant-volume heat-rejection process*

$T_4 = 600$ K, $T_1 = 300$ K v = Const.

  (a) Heat rejection,

$$Q_{4-1} = mC_v (T_1 - T_4)$$

$$= 1 \times 0.718 \times (300 - 600) = \mathbf{-215.4 \text{ kJ}}$$

  (b) Work transferred,

$$W_{4-1} = \mathbf{0} \qquad (\because V = \text{Const. } dV = 0)$$

  (c) Change in internal energy,

$$\Delta U_{4-1} = Q_{4-1} = \mathbf{-215.4 \text{ kJ}}$$

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(ii) For cyclic process

(a) Net work transfer,

$$W_{net} = \oint \delta W$$
$$= W_{1-2} + W_{2-3} + W_{3-4} + W_{4-1}$$
$$= -287 + 229.6 + 645.75 + 0$$
$$= \mathbf{584.35\ kJ}$$

(b) Net heat transfer

$$Q_{net} = \oint \delta Q = Q_{1-2} + Q_{2-3} + Q_{3-4} + Q_{4-1}$$
$$= 0 + 804 + 0 - 215.4 = \mathbf{584.6\ kJ}$$

Comment: For cyclic process

$$\oint \delta Q = \oint \delta W$$

and

$$\oint dU = 0$$

i.e. $287 + 574.4 - 645.75 - 215.4 = 0$

Example 4.25 *An imaginary engine receives heat and does work on a slowly moving piston at such rates that the cycle of operation of 1 kg of fluid can be represented as a circle of 10 cm diameter on a p–V diagram on which 1 cm = 300 kPa and 1 cm = 0.1 m^3/kg.*

(a) How much work is done by each kg of fluid for each cycle of operation?

(b) Calculate the thermal efficiency of the engine, if heat rejected by the engine is 1000 kJ/kg of working fluid.

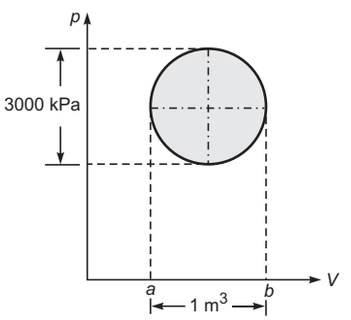

Fig. 4.25

Solution

Given An imaginary engine

$m = 1$ kg

Dia. of circle = 10 cm

1 cm = 300 kPa on p scale

1 cm = 0.1 m^3 on V scale.

$Q_{rej} = 1000$ kJ/kg

To find

(i) Work transfer during a cycle, and

(ii) Thermal efficiency.

Analysis

(i) The net work transfer is the area within the curve = Area of the circle on p–V diagram.

$$\text{Area} = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times (\text{Dia. on } p \text{ scale}) \times (\text{Dia. on } V \text{ scale})$$
$$= \frac{\pi}{4} \times (10 \times 300) \times (10 \times 0.1)$$
$$= \mathbf{2356.2\ kJ/kg.}$$

(ii) The thermal efficiency is expressed as

$$\eta_{th} = \frac{\text{Net work transfer}}{\text{Heat supplied}}$$
$$= \frac{\text{Net work transfer}}{\text{Net work transfer} + \text{Heat rejected}}$$
$$= \frac{2356.2}{2356.2 + 1000} = 0.7020$$
$$= \mathbf{70.20\%}$$

Example 4.26 *The working substance in a Carnot engine is 0.04 kg of air. The maximum cycle temperature is 930 K and the maximum pressure is 4.4×10^3 kPa. The heat added per cycle is 4 kJ. Determine the maximum cylinder volume, if minimum temperature during the cycle is 300 K.*

Solution

Given A Carnot cycle with

$m = 0.04$ kg $\quad T_1 = 930$ K

$T_3 = 300$ K $\quad p_1 = 4.4 \times 10^3$ kPa

$Q_{1-2} = 4$ kJ

To find Maximum cylinder volume, V_3

Assumptions

(i) Specific gas constant for air, $R = 0.287$ kJ/kg·K

(ii) Ratio of two specific heats $\gamma = 1.4$

Analysis Using perfect-gas equation:

$$p_1 V_1 = m R T_1$$

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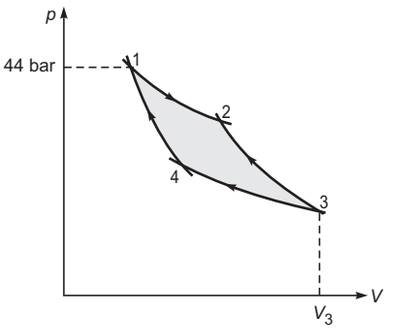

Fig. 4.26 *Carnot cycle on a p–V diagram*

or $$V_1 = \frac{mRT_1}{p_1}$$

$$= \frac{(0.04 \text{ kg}) \times (0.287 \text{ kJ/kg} \cdot \text{K}) \times (930 \text{ K})}{(4.4 \times 10^3 \text{ kPa})}$$

$$= 2.426 \times 10^{-3} \text{ m}^3$$

After heat addition during the process 1–2;

$$Q_{1-2} = p_1 V_1 \ln\left(\frac{V_2}{V_1}\right)$$

or $$4 = 4.4 \times 10^3 \times 2.426 \times 10^{-3} \times \ln\left(\frac{V_2}{2.426 \times 10^{-3}}\right)$$

or $$\ln\left(\frac{V_2}{2.426 \times 10^{-3}}\right) = 0.3746$$

or $$V_2 = 2.426 \times 10^{-3} \times \exp(0.3746)$$

$$= 3.53 \times 10^{-3} \text{ m}^3$$

During isothermal process;

$$T_2 = T_1 = 930 \text{ K}$$

During isentropic expansions 2–3

$$\frac{T_3}{T_2} = \left(\frac{V_3}{V_2}\right)^{1-\gamma}$$

or $$V_3 = V_2 \left(\frac{T_3}{T_2}\right)^{\frac{1}{1-\gamma}} = 3.53 \times 10^{-3} \times \left(\frac{300}{930}\right)^{\frac{1}{1-1.4}}$$

$$= 3.53 \times 10^{-3} \times 16.920 = \mathbf{0.06 \text{ m}^3}$$

Maximum volume in the cycle in 0.06 m^3 or 60 litres

4.2 VAPOUR PROCESS

Equations for ideal gases as $pv = RT$, Joule's law, etc., are not applicable to vapour, though the basic energy equations derived from the first law are applicable. Equations related with work done, heat transfer, enthalpy, internal energy do apply to vapour under specified conditions. The following basic-energy equation can be applied to vapour by taking care of units. For a unit mass system;

Heat transfer; $\delta q = Tds$

$\delta q = du + pdv$

and $\delta q = dh - vdp$

Internal energy; $u = h - pv$

Work transfer; $w = \int_1^2 pdv$

The unknown state property can be obtained by equating constant property between two states of a process, as given:

For constant-volume process; $v_1 = v_2$

For isentropic process; $s_1 = s_2$

For isothermal process; $p_1 v_1 = p_2 v_2$

For polytropic process; $p_1 v_1^n = p_2 v_2^n$

Example 4.27 *Steam is stored in a container of 0.45 m^3 capacity. The pressure of steam is 8 bar and temperature is 200°C. The container is cooled and pressure drops to 2.5 bar. Find (a) quality of steam after cooling, and (b) heat lost to atmosphere due to cooling. Neglect the volume of water.*

Solution

Given The steam is in the container, thus

$V_1 = V_2 = 0.45 \text{ m}^3 = \text{constant}$

$p_1 = 8$ bar, $T_1 = 200°\text{C}$

$p_2 = 2.5$ bar

To find

(i) The quality of steam after cooling, and

(ii) Heat lost to atmosphere during cooling.

Properties of steam

At 8 bar and 200°C (from superheated steam table)

$v_1 = 0.2608 \text{ m}^3/\text{kg}$ $h_1 = 2839.25 \text{ kJ/kg}$

At 2.5 bar (from pressure entry, saturated steam table)

$T_{sat} = 127.43°\text{C}$, $v_{g_2} = 0.71871 \text{ m}^3/\text{kg}$,

$h_{f_2} = 535.34 \text{ kJ/kg}$, $h_{fg_2} = 2185.55 \text{ kJ/kg}$

Analysis

(i) *Quality of steam*

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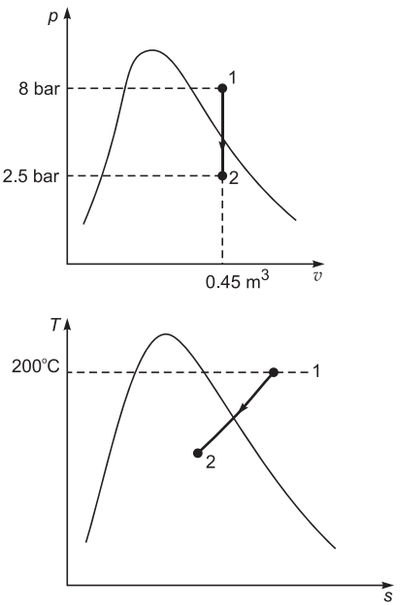

Fig. 4.27 *Constant-volume process on p-V and T-s diagrams*

Since the volume in the container is constant, thus

$$V_1 = V_2$$

or $$v_1 = v_2$$

But specific volume v_{g_2} at 2.5 bar is greater than specific volume v_1, thus steam is wet at 2.5 bar, therefore,

$$v_1 = x v_{g_2}$$

$$0.2608 = x \times 0.7187$$

or $$x = \mathbf{0.3628}$$

(ii) *Heat transferred during cooling*

Mass of steam in container,

$$m = \frac{V}{v_1} = \frac{0.45 \text{ m}^3}{0.2608 \text{ m}^3/\text{kg}} = 1.725 \text{ kg}$$

The heat transferred at constant volume

$$Q = \Delta U = m(\Delta u) = m(u_2 - u_1)$$

where $u_1 = h_1 - p_1 v_1 = 2839.25 - (8 \times 100) \times 0.2608$

$= 2630.61$ kJ/kg

$u_2 = h_2 - p_2 v_2 = (h_{f_2} + x h_{fg_2}) - p_2 (x v_{g_2})$

$= (535.34 + 0.3628 \times 2185.55) - (2.5 \times 100) \times (0.3628 \times 0.7187)$

$= 1263.06$ kJ/kg

Hence $Q = 1.725 \times (1263.06 - 2630.61)$

$= \mathbf{-2359.0}$ kJ (Rejected)

Example 4.28 *A pressure cooker contains 2 kg of dry and saturated steam at 5 bar. Find the quantity of heat that must be rejected so as to reduce quality upto 60% dry. Determine the pressure and temperature at the new state.*

Solution

Given A constant volume cooling process with

$p_1 = 5$ bar $\quad x_1 = 1.0$

$m = 2$ kg $\quad x_2 = 0.6$

To find After cooling,

(i) Pressure and temperature at new state,

(ii) Heat transferred.

Analysis At 5 bar, the properties of steam

$v_g = 0.3749$ m^3/kg, $\quad h_g = 2748.7$ kJ/kg

Since the volume remains constant during cooling process, thus

$$v_{g_1} = x_2 v_{g_2}$$

or $$v_{g_2} = \frac{0.3749}{0.6} = 0.6248 \text{ m}^3/\text{kg}$$

From steam tables, the pressure and saturation temperature can directly be obtained corresponding to vapour specific volume of 0.6248 m^3/kg , if available in the steam table.

But this volume is not available in the steam table. The specific volume of 0.6573 m^3/kg at 2.75 bar and 0.6058 at 3 bar are given.

Approximating the new pressure, by using linear interpolation formula as

$$y = y_1 \left(\frac{y_2 - y_1}{x_2 - x_1} \right) (x - x_1)$$

Here $y_1 = p_1 = 2.75$ bar $\quad y_2 = p_2 = 3.0$ bar

$x_1 = v_{g_1} = 0.6573$ m^3/kg $\quad x_2 = v_{g_2} = 0.6058$ m^3/kg

Using value $x = 0.6248$ m^3/kg for calculation of pressure as

$$p = 2.75 + \left(\frac{3 - 2.75}{0.6058 - 0.6573} \right) \times (0.62483 - 0.657)$$

$= 2.75 + (-4.854) \times (-0.0325) = 2.907$ bar

$= 1.0161 + 1.891 = \mathbf{2.907 \text{ bar}}$

Similarly, other properties can be obtained at this state.

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The saturation temperature corresponding to the new state

$$T_{sat} = \mathbf{133.1°C}$$

The new state is 2.907 bar and 0.6 dry.

$$h_f = 555 \text{ kJ/kg}, \quad h_{fg} = 2166.5 \text{ kJ/kg}$$

(ii) Heat transferred

$$Q = m(u_2 - u_1)$$

where $u_2 = h_2 - p_2 v_2$

$$= (h_{f_2} + x_2 h_{fg_2}) - p_2(x_2 v_{g_2})$$

$$= (555 + 0.6 \times 2166.5) - 290.7 \times 0.6 \times 0.6248$$

$$= 1745.91 \text{ kJ/kg}$$

and $u_1 = h_1 - p_1 v_1$

$$= h_{g_1} - p_1 v_{g_1} = 2748.7 - 500 \times 0.3749$$

$$= 2561.25 \text{ kJ/kg}$$

Hence, heat transferred

$$Q = 2 \text{ kg} \times (1745.91 - 2561.25) \text{ (kJ/kg)}$$

$$= \mathbf{-1630.68 \text{ kJ}} \text{ (rejected)}$$

Example 4.29 *A quantity of steam at 10 bar and 0.85 dry occupies a volume of 0.15 m^3. Determine the heat supplied to raise the temperature of steam to 350°C at constant pressure and percentage of this heat supplied, which appears as external work.*

Solution

Given $p_1 = 10 \text{ bar} = 1000 \text{ kPa}$ $T_{sup} = 350°C$

$x = 0.85$ $V_1 = 0.15 \text{ m}^3$

To find

(i) Heat supplied,

(ii) External work, and

(iii) Percentage of heat supplied as external work done

Assumption Specific heat of superheated steam,

$$C_{ps} = 2.1 \text{ kJ/kg·K}$$

Properties From steam table at 10 bar

$T_{sat} = 179.91°C$ $h_f = 762.81 \text{ kJ/kg}$,

$h_{fg} = 2015.3 \text{ kJ/kg}$, $v_g = 0.19444 \text{ m}^3/\text{kg}$

Analysis Mass of steam

$$m = \frac{V}{x v_g} = \frac{(0.15 \text{ m}^3)}{0.85 \times (0.19444 \text{ m}^3/\text{kg})} = 0.9076 \text{ kg}$$

Heat supplied per kg of steam during superheating

$$h_1 = (1 - x) h_{fg} + C_{ps}(T_{sup} - T_{sat})$$

$$= (1 - 0.85) \times 2015.3 + 2.1 \times (350 - 179.91)$$

$$= 302.3 + 357.2 = 659.5 \text{ kJ/kg}$$

Total heat supplied to steam,

$$Q = m \times h_1 = 0.9076 \times 659.5 = \mathbf{598.5 \text{ kJ}}$$

The work done during superheating,

$$w = p(v_2 - v_1) = p\left[v_g \times \frac{T_{sup}}{T_{sat}} - x v_g\right]$$

$$w = 1000 \times \left[0.19444 \times \left(\frac{350 + 273}{179.91 + 273}\right) - 0.85 \times 0.19444\right]$$

$$= 1000 \times (0.2675 - 0.1653) = 102.18 \text{ kJ/kg}$$

Total external work done

$$W_e = mw = 0.9075 \times 102.18 = \mathbf{92.73 \text{ kJ}}$$

The percentage of total heat supplied as external work

$$= \frac{92.73}{598.5} \times 100 = \mathbf{15.5\%}$$

Example 4.30 *A closed vessel of 0.2 m^3 capacity contains steam at 8 bar and 200°C.*

(a) What is the weight of steam contained in the vessel ?

(b) The vessel is cooled and steam becomes just dry and saturated. What would be the pressure at this state ? Estimate specific entropy.

(c) The vessel is further cooled till the temperature drops to 160.8°C. Find the pressure and condition of steam at this state.

Solution

Given The steam in a closed vessel

$V = 0.2 \text{ m}^3$ $p = 8 \text{ bar}$ $T_{sup} = 200°C$

$x_2 = 1.0$ $T_3 = 160.8°C$

To find

(i) Weight of steam

(ii) (a) Pressure p_2 of steam in saturation state

(b) Specific entropy of saturated steam

(iii) Pressure p_3 and quality x_3 of steam at the state 3.

Assumption Specific heat of superheated steam as 2.1 kJ/kg·K

Analysis

(i) At 8 bar, the properties of steam

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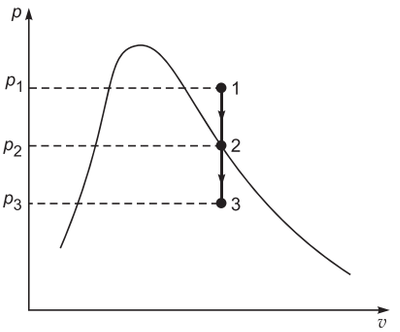

Fig. 4.28

T_{sat} = 170.43°C $\quad v_g = 0.2404$ m^3/kg

$h_g = 2769.13$ kJ/kg

The specific volume of superheated steam

$$v_1 = v_g \times \frac{T_{sup}}{T_{sat}}$$

$$= 0.2404 \times \left(\frac{200 + 273}{170.43 + 273}\right)$$

$$= 0.25643 \text{ m}^3/\text{kg}$$

Note It can also be obtained directly from superheated steam table.

The mass of steam in the vessel

$$m = \frac{V}{v_1}$$

$$= \frac{0.2 \text{ m}^3}{0.25643 \text{ m}^3/\text{kg}} = 0.78 \text{ kg}$$

Weight of steam = $mg = 0.78 \times 9.81 =$ **7.65**

(ii) When steam is cooled to just become saturated (state 2), its specific volume remains constant, but pressure reduces, i.e., $v_{g_2} = 0.25643$ m^3/kg.

(a) From steam tables, saturation pressure corresponding to 0.25643 m^3/kg is **7.5 bar** (approx.)

(b) The specific entropy at this pressure is

$$s_g = \mathbf{6.6847 \text{ kJ/kg} \cdot \text{K}}$$

(iii) When the vessel is cooled to $T_3 = 160.8$°C (state 3), taking this as saturation temperature, the pressure can be calculated by linear interpolation.

At $T_{sat_1} = 160$°C $\quad v_{g_1} = 0.3071$ m^3/kg

$p_1 = 6.178$ bar

At $T_{sat_2} = 165$°C $\quad v_{g_2} = 0.2727$ m^3/kg

$p_2 = 7.005$ bar

Therefore, the pressure at 160.8°C, using linear interpolation formula

$$y = y_1 + \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

or $$p_3 = p_1 + \left(\frac{p_2 - p_1}{T_{sat_2} - T_{sat_1}}\right)(T - T_{sat_1})$$

$$p_3 = 6.178 + \left(\frac{7.005 - 6.178}{165 - 160}\right) \times (160.8 - 160)$$

= 6.31 bar

Similarly, specific volume of steam

$$v_3 = 0.3071 + \left(\frac{0.2727 - 0.3071}{165 - 160}\right) \times (160.8 - 160)$$

= **0.3016 m^3/kg**

The condition of steam can be calculated by equating the volume at two states;

$$v_1 = v_3$$

or $$0.25643 = x_3 \times 0.3016$$

or $$x_3 = \frac{0.255}{0.3016} = \mathbf{0.85}$$

Example 4.31 *Dry saturated steam at 1.4 MPa undergoes a reversible hyperbolic expansion in a non-flow system until its pressure is 0.80 MPa. Calculate the work done during the process.*

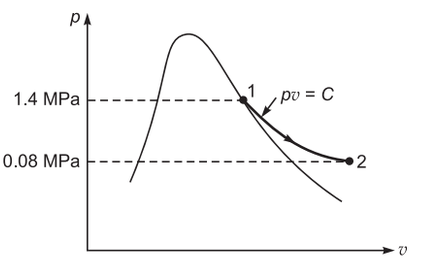

Fig. 4.29

Solution

Given Reversible hyperbolic expansion of steam:

State 1: $p_1 = 1.4$ MPa (Dry)

$x_1 = 1$

State 2: $p_2 = 0.80$ MPa, after hyperbolic expansion.

To find Work done during the process.

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Properties of steam at pressure

$p_1 = 1.4$ MPa $\quad T_{sat_1} = 195.07°C$,

$v_{g_1} = 0.14084$ m^3/kg, $\quad h_{g_1} = 2790.00$ kJ/kg

At a pressure of 0.80 MPa, after hyperbolic expansion

$T_{sat_2} = 170.43°C$, $\quad v_{g_2} = 0.2404$ m^3/kg

$h_{f_2} = 721.1$ kJ/kg $\quad h_{fg_2} = 2048$ kJ/kg

Analysis For hyperbolic process:

$$p_1 v_1 = p_2 v_2$$

$$1.4 \times 0.14084 = 0.8 \times v_2$$

and $$v_2 = 0.2464 \text{ m}^3/\text{kg}$$

The work done per kg of steam during isothermal process,

$$w = p_1 v_1 \ln\left(\frac{v_2}{v_1}\right)$$

$$= (1.4 \times 10^3) \times 0.14084 \times \ln\left(\frac{0.2464}{0.14084}\right)$$

$$= \mathbf{110.34 \text{ kJ/kg}}$$

Example 4.32 *Dry saturated steam at 1550 kPa is isothermally expanded to 100 kPa pressure. Calculate the change in internal energy, work done, and heat transfer during the process.*

Solution

Given Reversible isothermal expansion of dry saturated steam:

State 1: $p_1 = 1550$ kPa (Dry) $\quad x_1 = 1$

State 2: $p_2 = 100$ kPa, after isothermal expansion.

To find

(i) Heat transfer,

(ii) Work done, and

(iii) Change in internal energy during the process.

Assumptions

(i) The steam in superheated region as an ideal gas.

(ii) The specific heat for superheated steam as 2.1 kJ/kg·K.

Properties of steam at pressure $p_1 = 1550$ kPa

$T_{sat_1} = 199.86°C$, $\quad v_{g_1} = 0.1275$ m^3/kg

$h_{g_1} = 2790.8$ kJ/kg, $\quad s_{g_1} = 6.4289$ kJ/kg·K

At pressure 100 kPa, after isothermal expansion

$T_{sat_2} = 99.63°C$, $\quad v_{g_2} = 1.6937$ m^3/kg

$h_{g_2} = 2675.46$ kJ/kg,

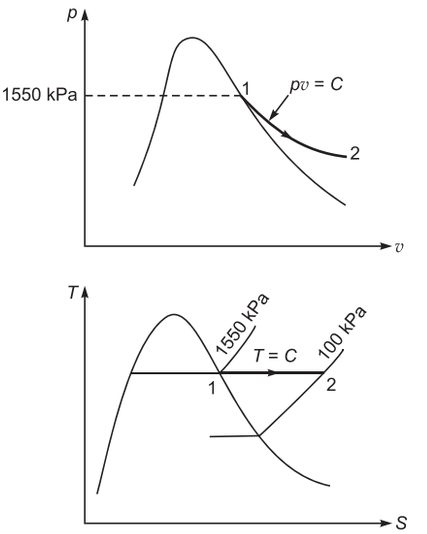

Fig. 4.30

Analysis The steam is superheated during the process, thus it behaves as an ideal gas, i.e.,

$$p_1 v_1 = p_2 v_2$$

$$1550 \times 0.1275 = 100 \times v_2$$

or $$v_2 = 1.976 \text{ m}^3/\text{kg}$$

The temperature at the state 2;

$$T_2 = T_1 = 199.86°C$$

The enthalpy of steam at the state 2

$$h_2 = h_{g_2} + C_{ps}(T_2 - T_{sat_2})$$

$$= 2675.46 + 2.1 \times (199.86 - 99.63)$$

$$= 2885.94 \text{ kJ/kg}$$

(i) The change in specific internal energy:

$$\Delta u = u_2 - u_1 = (h_2 - p_2 v_2) - (h_1 - p_1 v_1)$$

$$= h_2 - h_1 \qquad (\because\ p_2 v_2 = p_1 v_1)$$

$$= 2885.94 - 2790.8 = \mathbf{95.14 \text{ kJ/kg}}$$

(ii) The work done per kg of steam during isothermal process,

$$w = p_1 v_1 \ln\left(\frac{v_2}{v_1}\right)$$

$$= (1550) \times 0.1275 \times \ln\left(\frac{1.976}{0.1275}\right)$$

$$= \mathbf{541.65 \text{ kJ/kg}}$$

(iii) The heat transferred by steam during isothermal process,

$$q = w + \Delta u = 541.65 + 95.14$$

$$= \mathbf{636.79 \text{ kJ/kg}}$$

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Example 4.33 *A piston–cylinder arrangement contains 1 kg of water at 100 kPa. The initial volume is 0.5 m^3. The heat transferred to the water is an amount which is just necessary to cause a slow expansion at constant temperature. The process is terminated when the volume is doubled. Determine the magnitude of heat transfer. Solve the problem if the process of expansion is hyperbolic.*

Solution

Given Water in a piston–cylinder arrangement

$m_1 = 1$ kg, $v_1 = 0.5$ m^3

$v_2 = 1$ m^3 $p_1 = 100$ kPa

To find

(i) Heat supplied to water at constant temperature, and

(ii) Heat supplied during the hyperbolic process.

Properties of water at 100 kPa

$T_{sat} = 99.63°C$ $v_g = 5.694$ m^3/kg

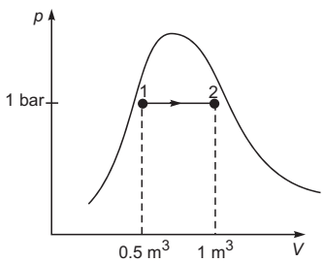
$h_f = 417.46$ kJ/kg $h_{fg} = 2258$ kJ/kg

Analysis

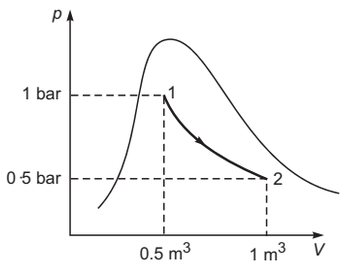
(i) Heat supplied at constant temperature

The initial volume,

$$v_1 = x_1 v_{g_1} = 0.5 \qquad \text{(since } m = 1 \text{ kg)}$$



(a) p–V diagram for constant-temperature heating



(b) p–V diagram for hyperbolic expansion

Fig. 4.31

or $$x_1 = \frac{v_1}{v_{g_1}} = \frac{0.5}{1.694} = 0.295$$

Enthalpy of water $h_1 = h_{f_1} + x_1 h_{fg_1}$

$= 417.46 + 0.295 \times 2258$

$= 1083.57$ kJ/kg

Since the steam is wet, therefore, during further heat addition, its temperature and pressure remain constant, thus,

$$v_2 = x_2 v_g = 1 \text{ m}^3$$

$$x_2 = \frac{1}{1.694} = 0.59$$

Enthalpy $h_2 = h_{f_2} + x_2 h_{fg_2} = 417.46 + 0.59 \times 2258$

$= 1749.68$ kJ/kg

Heat added to water,

$$Q = m(u_2 - u_1)$$

where $u_1 = h_1 - p_1 v_1 = 1083.57 - 100 \times 0.5$

$= 1033.57$ kJ/kg

$u_2 = h_2 - p_2 v_2 = 1749.68 - 100 \times 1$

$= 1649.68$ kJ/kg

$\therefore \quad Q = 1 \times (1649.68 - 1033.57)$

$= \mathbf{616.11\ kJ}$

(ii) If expansion is hyperbolic

$$p_1 v_1 = p_2 v_2$$

or $$p_2 = \frac{p_1 v_1}{v_2} = \frac{100 \times 0.5}{1} = 50 \text{ kPa}$$

From steam table at 50 kPa,

$v_{g_2} = 3.24$ m^3/kg, $h_{f_2} = 340.47$ kJ/kg,

$h_{fg_2} = 2305.4$ kJ/kg,

Specific volume; $v_2 = x_2 v_{g_2} = 1$

$$x_2 = \frac{v_2}{v_{g_2}} = \frac{1}{3.24} = 0.3086$$

$h_2 = h_{f_2} + x_2 h_{fg_2}$

$= 340.47 + 0.3086 \times 2305.4$

$= 1051.92$ kJ/kg

Work done during the hyperbolic process

$$W = m\, p_1 v_1 \ln\left(\frac{v_2}{v_1}\right)$$

$$= 1 \times 100 \times 0.5 \times \ln\left(\frac{1}{0.5}\right) = 34.65 \text{ kJ}$$

Internal energy change for $m = 1$ kg

$\Delta U = m(u_2 - u_1)$

$= m[(h_2 - p_2 v_2) - (h_1 - p_1 v_1)]$

$= \dot{m}\,(h_2 - h_1)$

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$= 1 \times (1051.92 - 1083.57)$

$= 1083.57$ kJ/kg

or $\Delta U = -31.63$ kJ (decrease)

The heat transfer $Q = \Delta U + W$

$= -31.65 + 34.65 =$ **3.00 kJ**

Example 4.34 *Steam at 2 MPa and 250°C is expanded isentropically to 0.35 MN/m^2 and it is then expanded hyperbolically to 0.06 MN/m^2. Using steam tables, determine*

(a) final condition of steam,

(b) change in specific entropy during hyperbolic process.

Solution

Given Isentropic and hyperbolic expansion of steam;

State 1: $p_1 = 2$ MPa, $T_1 = 250$°C

State 2: $p_2 = 0.35$ MPa, $s_1 = s_2$

State 3: $p_3 = 0.06$ MPa, $p_2 v_2 = p_3 v_3$

To find

(i) Final condition of steam, and

(ii) Change in entropy during hyperbolic expansion.

Properties of steam

From steam tables

At 2 MPa (2000 kPa) and 250°C:

$h_1 = 2902.5$ kJ/kg $\quad s_1 = 6.5453$ kJ/kg·K

$v_1 = 0.111$ m^3/kg

At 0.35 MPa or at 350 kPa:

$T_{sat} = 138.9$°C, $\quad h_{f_2} = 584.33$ kJ/kg

$h_{fg_2} = 2148.1$ kJ/kg, $\quad s_{f_2} = 1.727$ kJ/kg·K

$s_{fg_2} = 5.213$ kJ/kg·K $\quad v_{g_2} = 0.5243$ m^3/kg

At 0.06 MPa or 60 kPa:

$T_{sat} = 86$°C $= 359$ K $\quad v_{g_3} = 2.73$ m^3/kg

$h_{f_3} = 359.9$ kJ/kg $\quad h_{fg_3} = 2293.6$ kJ/kg

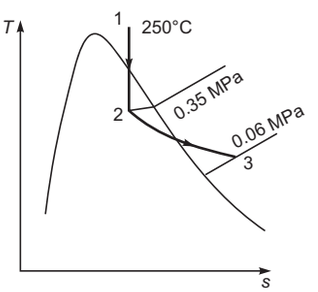

Fig. 4.32

$s_{f_3} = 5.146$ kJ/kg·K $\quad s_{fg_3} = 6.387$ kJ/kg·K,

$s_{g_3} = 7.533$ kJ/kg·K

Analysis

(i) During isentropic expansion 1–2

$$s_1 = s_2 = s_{f_2} + x_2 s_{fg_2}$$

$$6.5453 = 1.727 + x_2 \times 5.213$$

or $$x_2 = \frac{6.5453 - 1.727}{5.213} = 0.924$$

Condition of steam after hyperbolic expansion 1–2;

$$p_2 v_2 = p_3 v_3$$

or $$p_2 (x_2 v_{g_2}) = p_3 v_{g_3} \times \frac{T_{sup_3}}{T_{sat_3}}$$

$$0.35 \times (0.924 \times 0.5243) = 0.06 \times \left[2.73 \times \frac{T_{sup}}{(86 + 273)}\right]$$

or $$T_{sup_3} = \frac{0.16955 \times 359}{0.1638}$$

$= 371.6$ K $= 96.6$°C

Degree of superheat $= 96.6 - 86 =$ **10.6°C**

Hence, steam is superheated by 10.6°C.

(ii) Change in specific entropy during the process 2–3

$$\Delta s = s_3 - s_2$$

where $$s_2 = s_{f_2} + x_2 s_{fg_2}$$

$= 1.727 + 0.924 \times 5.213$

$= 6.5453$ kJ/kg·K

$$s_3 = s_{g_3} + C_{ps} \ln\left(\frac{T_{sup_3}}{T_{sat_3}}\right)$$

$$= 7.533 + 2.1 \times \ln\left(\frac{371.6}{86 + 273}\right)$$

$= 7.605$ kJ/kg·K

Now change in entropy,

$\Delta s = 7.605 - 6.5453 =$ **1.061 kJ/kg·K**

Example 4.35 *1 kg of steam at 8 bar, 250°C expands polytropically to 2 bar according to $pv^{1.3}$ = constant. Determine the quality of steam at final state, heat transfer and change in entropy during expansion process.*

Solution

Given Polytropic expansion of steam:

$m = 1$ kg

$p_1 = 8$ bar $= 800$ kPa

$T_1 = 250$°C $= 523$ K

$p_2 = 2$ bar $= 200$ kPa

$pv^{1.3} =$ constant

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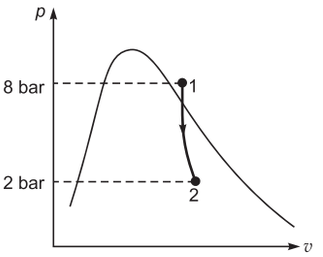

Fig. 4.33

To find

(i) The final quality of steam,
(ii) Heat transfer, and
(iii) Change in entropy.

Assumptions

(i) The reversible process, and
(ii) Superheated steam as perfect gas.

Properties of steam The properties of superheated steam at 8 bar, 250°C:

$$v_1 = 0.2931 \text{ m}^3/\text{kg},$$
$$h_1 = 2948 \text{ kJ/kg}$$
$$s_1 = 7.034 \text{ kJ/kg} \cdot \text{K}$$

The properties of steam at 2 bar:

$v_{g_2} = 0.8857 \text{ m}^3/\text{kg}$ $\quad h_{f_2} = 504.7 \text{ kJ/kg}$
$h_{fg_2} = 2202 \text{ kJ/kg}$ $\quad s_{f_2} = 1.5301 \text{ kJ/kg} \cdot \text{K}$
$s_{g_2} = 7.127 \text{ kJ/kg} \cdot \text{K}$

Analysis

(i) The steam expands polytropically as

$$p_1 v_1^{1.3} = p_2 v_2^{1.3}$$

or
$$v_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{1.3}} v_1 = \left(\frac{8}{2}\right)^{\frac{1}{1.3}} \times 0.2931$$
$$= 0.8514 \text{ m}^3/\text{kg}$$

Specific volume v_2 is less than v_{g_2}, hence the steam is wet after expansion, thus,

$$v_2 = x_2 v_{g_2}$$

or
$$x_2 = \frac{v_2}{v_{g_2}} = \frac{0.8514}{0.8857} = \mathbf{0.961}$$

(ii) The polytropic work done

$$w = \frac{p_2 v_2 - p_1 v_1}{1 - n}$$
$$= \frac{200 \times 0.8514 - 800 \times 0.2931}{1 - 1.3}$$
$$= 214 \text{ kJ/kg}$$

The change in internal energy

$$\Delta u = u_2 - u_1$$

where
$$u_1 = h_1 - p_1 v_1$$
$$= 2948 - 800 \times 0.2931$$
$$= 2713.52 \text{ kJ/kg}$$
$$u_2 = h_2 - p_2 v_2 = (h_{f_2} + x_2 h_{fg_2}) - p_2 v_2$$
$$= (504.7 + 0.961 \times 2202) - 200 \times 0.8514$$
$$= 2450.54 \text{ kJ/kg}$$

Thus
$$\Delta u = 2450.54 - 2713.52$$
$$= -262.98 \text{ kJ/kg}.$$

The heat transfer per kg of steam

$$q = \Delta u + w = -262.98 + 214$$
$$= \mathbf{-48.98 \text{ kJ/kg}}$$

(iii) The entropy change during expansion

$$\Delta s = s_2 - s_1$$

where
$$s_2 = s_{f_2} + x_2 s_{fg_2}$$
$$= s_{f_2} + x_2 (s_{g_2} - s_{f_2})$$
$$= 1.5301 + 0.961 \times (7.1271 - 1.5301)$$
$$= 6.9 \text{ kJ/kg} \cdot \text{K}$$

Thus
$$\Delta s = 6.9 - 7.034 = \mathbf{-0.125 \text{ kJ/kg} \cdot \text{K}}$$

Example 4.36 *A vessel with a partition in it, contains initially 2 kg of dry and saturated steam at 7.0 bar in one compartment and 1 kg of steam with dryness fraction of 0.8 at 3.5 bar in the other compartment. After the partition is removed, the pressure of the mixture is found to be 5 bar. Neglecting the volume of water, find*

(a) Dryness fraction of the mixture
(b) The heat interaction between the vessel and the surroundings

A	*B*
2 kg	1 kg
7 bar	3.5 bar
$x = 1$	$x = 0.8$

Solution

Given $p_1 = 7$ bar $\quad m_1 = 2$ kg $\quad x_1 = 1$
$p_2 = 3.5$ bar $\quad m_2 = 1$ kg $\quad x_2 = 0.8$
$p_3 = 5$ bar of mixture

To find

(i) Dryness fraction of steam after mixing,
(ii) Heat interactions between vessel and surroundings.

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Properties of Steam Properties of steam from steam table:

At 7 bar $v_{g_1} = 0.2729$ m^3/kg $h_{g_1} = 2763.5$ kJ/kg

At 3.5 bar $v_{g_2} = 0.5243$ m^3/kg $h_{f_2} = 584.33$ kJ/kg

$h_{fg_2} = 2148.5$ kJ/kg

At 5.0 bar $v_{g_3} = 0.3749$ m^3/kg $h_{f_3} = 640.23$ kJ/kg

$h_{fg_3} = 2108.5$ kJ/kg

Analysis The volume of the 1st compartment

$$V_1 = m_1 v_{g_1} = 2 \times 0.2729 = 0.5458 \text{ m}^3$$

The volume of the 2nd compartment,

$$V_2 = m_2(x_2 v_{g_2}) = 1 \times (0.8 \times 0.5243) = 0.41944 \text{ m}^3$$

After the removal of partition, the total volume of the compartment

$$V_3 = V_1 + V_2 = 0.5458 + 0.41944 = 0.96524 \text{ m}^3$$

The mass of steam after removal of partition;

$$m = m_1 + m_2 = 2 + 1 = 3 \text{ kg}$$

Specific volume of steam;

$$v_3 = \frac{V}{m} = \frac{0.96524}{3} = 0.32174 \text{ m}^3/\text{kg}$$

But specific volume of dry saturated steam at 5 bar; $v_{g_3} = 0.3749$ m^3/kg, is greater than v_3

Thus $v_3 = x_3 v_{g_3}$

or dryness fraction $x_3 = \dfrac{v_3}{v_{g_3}} = \dfrac{0.32174}{0.3749} = \mathbf{0.858}$

From the first law of thermodynamics,

$$Q = W + \Delta U$$

For a constant–volume process,

$W = 0$

and ΔU = Internal energy after mixing – Internal energy before mixing.

$\therefore \quad Q = U_3 - (U_2 + U_1)$

where $U_3 = m[h_3 - p_3 v_3] = m[(h_{f_3} + x h_{fg_3}) - p_3 v_3]$

$= 3 \times [(640.23 + 0.858 \times 2108.5) - (5 \times 100 \times 0.32174)]$

$= 6865.36$ kJ

$U_1 = m_1[h_1 - p_1 v_1]$

$= 2 \times [(2763.5) - (7 \times 100 \times 0.2729)]$

$= 5144.94$ kJ

$U_2 = m_2 [(h_{f_2} + x h_{fg_2}) - (p_2 \cdot x v_{g_2})]$

$= 1 \times [(584.33 + 0.8 \times 2148.1) - (3.5 \times 100 \times 0.8 \times 0.5243)]$

$= 2156.0$ kJ

Thus $Q = 6865.36 - (5144.94 + 2156.0)$

$= \mathbf{-435.56 \text{ kJ}}$

Heat is rejected from vessel.

Summary

- The characteristic gas equation for an ideal gas is

  $$pV = m\,RT$$

  where R is the specific gas constant and it is obtained as

  $$R = \frac{R_u}{\mathcal{M}} = \frac{\text{Universal gas constant}}{\text{Molecular weight}} = \frac{8.31447}{\mathcal{M}}$$

- The thermodynamic properties p, V and T are related as

  $$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

- For all type of processes, the change in internal energy and change in enthalpy can be calculated as follows:

  Change in internal energy

  $$\Delta U = m\,C_v(T_2 - T_1)$$

  Change in enthalpy $\Delta H = m\,C_p(T_2 - T_1)$

- The summary of equations for ideal processes are given below.

  1. Constant-volume Process

  Law $V = C$, $\dfrac{p_1}{T_1} = \dfrac{p_2}{T_2}$

  Work transfer $W = 0$

  Heat transfer $Q = m\,C_v(T_2 - T_1) = \Delta U$

  Change in entropy $\Delta S = m\,C_v \ln(T_2/T_1)$

  2. Constant-Pressure Process

  Law $p = C$, and $\dfrac{V_1}{T_1} = \dfrac{V_2}{T_2}$

  Work transfer $W = p(V_2 - V_1)$

  Heat transfer $Q = m\,C_p(T_2 - T_1) = \Delta H$

  Change in entropy $\Delta S = m\,C_p \ln(T_2/T_1)$

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3. Isothermal Process

Law $T = C$, or $pV = C$,

$p_1V_1 = p_2V_2$

Work transfer $W = p_1V_1 \ln(V_2/V_1)$

$= mRT_1 \ln(p_1/p_2)$

Heat transfer $Q = W$

Change in entropy $\Delta S = mR \ln(V_2/V_1)$

$= mR \ln(p_1/p_2)$

4. Isentropic Process

Law $pV^{\gamma} = C$,

$$\frac{T_2}{T_1} = \left(\frac{V_2}{V_1}\right)^{1-\gamma} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Work transfer

$$W_{1-2} = \frac{(p_2V_2 - p_1V_1)}{1-\gamma} = \frac{mR(T_2 - T_1)}{1-\gamma}$$

$= -\Delta U$

Heat transfer $Q = 0$

Change in entropy $\Delta S = 0$

5. Polytropic Process

Law $pV^n = C$

Properties relation $\dfrac{T_2}{T_1} = \left(\dfrac{V_2}{V_1}\right)^{1-n}$

$$= \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

Work transfer

$$W = \frac{p_2V_2 - p_1V_1}{1-n} = \frac{mR(T_2 - T_1)}{1-n}$$

Heat transfer $Q = mC_n(T_2 - T_1)$

where polytropic specific heat

$$C_n = \frac{C_p - nC_v}{1-n}$$

Change in entropy $\Delta S = mC_n \ln\left(\dfrac{T_2}{T_1}\right)$

Glossary

Non-flow process A process in which mass of system does not change

Isochoric process Constant-volume process

Isobaric process Constant-pressure process

Isothermal process Constant-temperature process

Isentropic process Constant-entropy process

Polytropic process A general representation of all processes by a relation pV^n = constant.

Isenthalpic process Constant-enthalpy process

Review Questions

1. Define an ideal gas.
2. What is an equation of state?
3. Show that for an ideal gas, $C_p - C_v = R$.
4. State the difference between standard symbols E and U.
5. Define an adiabatic process. Show that a reversible adiabatic process for a given mass of a perfect gas is pV^{γ} = constant.
6. How are the state parameters p, V and T related in a polytropic process? What are values of the polytropic index for isobaric, isochoric, isothermal and adiabatic processes?
7. Show that the work transfer per kg for a perfect gas during an adiabatic expansion process is given by

$$W_{1-2} = \frac{R(T_1 - T_2)}{\gamma - 1}$$

where T_1 and T_2 are initial and final temperatures and R is the charecteristic gas equation.

8. How is polytropic index determined and within what limits can it change?
9. Define polytropic specific heat and prove that

$$C_n = \frac{C_p - C_v}{1-n}$$

What is the significance of its negative sign?

10. For a polytropic process, derive the following relations:

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$$Q_{1-2} = \frac{\gamma - n}{\gamma - 1} \times \text{polytropic work transfer}$$

and $$Q_{1-2} = \frac{\gamma - n}{n - 1} \times \text{adiabatic work transfer}$$

11. Show that when an ideal gas is compressed polytropically with index n, the heat rejection, increase in internal energy and work done is in the ratio

$$(\gamma - n) : (n - 1) : (\gamma - 1)$$

12. The internal energy of a certain closed system is given by $U = M + N\,pV$. Show that if it undergoes a reversible non-flow process with $Q = 0$, the relation between p and V is $pV^{\gamma} = C$, where C is some constant and $\gamma = \dfrac{N+1}{N}$.

Problems

1. 1 kg of gas expands isentropically through a volume ratio of 5. The initial pressure and temperature are 28 bar and 220°C, respectively. Find (a) final pressure and temperature, (b) work done during expansion, and (c) change in entropy. For a gas take

$C_p = 1.024$ kJ/kg·K and $C_v = 0.713$ kJ/kg·K.

[(a) 2.94 bar, − 28.6°C (b) 178 kJ (c) 0]

2. 0.1 kg of gas at a pressure of 1 bar and 15°C is compressed adiabatically to 28 bar. Assuming that $pv = 287\,T$ for one kg of gas and the index of adiabatic compression is 1.404, calculate (a) the final volume and temperature, (b) ratio of compression, and (c) work done during the compression. *[(a) 0.0785 m^3, (b) 487°C (c) 324.7 kJ]*

3. A volume of 2.5 litres of a gas at 14 bar and 1100°C expands to a pressure of 280 kPa according to $pV^{1.28} = C$. Determine (a) final volume, (b) final temperature and (c) work done by gas during expansion.

[(a) 8.8 litre, (b) 692.5°C, (c) 3.7 kJ]

4. The air initially at 75 kPa, 727°C, 0.12 m^3 is compressed isothermally until the volume is one half of the initial volume. Further, it is cooled at a constant pressure till its volume is again halved.

(a) Sketch the process on *p–V* and *T–S* planes.

(b) Determine the total work transfer for the two processes.

(c) Calculate the total heat transferred.

[(b) − 10.74 kJ (c) − 22 kJ]

5. Air initially at 1.00 bar has a specific volume 0.12 m^3/kg. It is compressed polytropically according to the law $pv^{1.3}$ = constant to a pressure of 4.2 bar. It is then allowed to expand isothermally. It is finally cooled at a constant volume to bring the system to initial state.

(a) Show the process on *p–V* and *T–S* diagrams.

(b) Calculate the heat transfer and work transfer in each case.

[W = 16 kJ, 18.46 kJ and Q = − 4.28 kJ, 18.46 kJ, − 11.7 kJ]

6. Air initially at 4 bar, 60°C, expands isothermally to 200 kPa and then it expands isentropically to 100 kPa. Sketch the processes on *p–V* and *T–S* diagrams and calculate for entire expansion process

(a) heat transfer,

(b) change in specific internal energy, and

(c) change in specific entropy.

[(a) 66.24 kJ/kg (b) – 43 kJ/kg (c) 0.198 kJ/kg·K.]

7. 0.4 kg of air at 6 bar receives an amount of heat at constant volume, so that its temperature rises from 383 K to 923 K. It is then expanded polytropically according to $pV^{1.32}$ = constant to initial temperature and finally, it is compressed isothermally to its original volume. Calculate (a) pressure at end states, and (b) work transfer and heat transfer during each process. Take

$C_v = 0.718$ and $R = 0.287$ kJ/kg·K.

[(a) p_2 =14.45 bar, p_3 = 0.3845 bar, (b) W_{1-2} = 0, Q_{1-2} = 155 kJ, W_{2-3} = 193 kJ, W_{2-3} = 38.64, $W_{3-1} = Q_{3-1} = -120.886$ kJ]

8. 0.5 kg air undergoes a Carnot cycle with 50% efficiency. The initial pressure and volume of air are 7 bar and 0.12 m^3, respectively. The heat supplied to air during isothermal expansion is 40 kJ. Calculate:

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(a) maximum and minimum temperature of the cycle,
(b) volume at the end of isothermal expansion,
(c) the work and heat transfer for each of four processes.

[(a) T_{max} = 312.4°C, T_{min} = 19.7°C, (b) V_2 = 0.193 m^3 (c) W_{1-2} = 40 kJ, W_{2-3} = 105 kJ, W_{3-4} = −20 kJ, W_{4-1} = −105 kJ, Q_{1-2} = 40 kJ, Q_{2-3} = Q_{4-1} = 0, Q_{3-4} = 20 kJ].

9. 2 kg of air at 150°C and 3 bar expands according to $pV^{1.2}$ = constant, to a final pressure of 1 bar. Find heat transfer, work transfer, change in internal energy and change in entropy.

Take R = 0.287 kJ/kg·K and γ = 1.4

[101 kJ, 203.12 kJ, –99.1 kJ and 0.26 kJ/K.]

10. One kilogram of oxygen is compressed isothermally in a closed system from 100 kPa and 25°C to 300 kPa. Calculate (a) work transfer, (b) heat transfer, and (c) change in internal energy.

11. 1 kg of air initially at 1 bar and 40°C is compressed adiabatically until its pressure is 50 bar. The air then receives 125.6 kJ of heat at constant pressure. Find for each process, the change in internal energy.

[ΔU_{1-2} = 464.6 kJ, ΔU_{2-3} = 89.73 kJ,]

12. 1 kg of air at 20 bar, 200°C is expanded reversibly to 12 bar, 125°C in such a way that the process is represented by a straight line on a p–V diagram. Determine (a) law of pressure relating volume, (b) work and heat transfer, and (c) change in internal energy.

[(a) p = 40 −294 V (bar); (b) 43.82 kJ, −9.9 kJ; (c) −53.9 kJ]

13. A gas at a pressure of 14 bar and a temperature of 343°C is expanded adiabatically to a pressure of 1.05 bar. The gas is heated at constant volume until it again attains 343°C temperature, when its pressure is found to be 2.222 bar and finally it is compressed isothermally until the original pressure of 14 bar is attained. Sketch the p–V diagram for these operations and for 0.5 kg mass of the gas, calculate the change in internal energy during the adiabatic expansion.

Take C_p = 1.005 kJ/kg·K. *[–115.8 kJ]*

14. A quantity of perfect gas occupies a volume of 0.3 m^3 at a pressure of 1 bar and a temperature of 20°C. This is compressed isothermally to a pressure of 5 bar and then expanded adiabatically to its initial volume. Determine for this quantity of gas: (a) Heat transfer during the compression (b) Change in internal energy during the expansion (c) the mass of the gas.

Take γ = 1.4 and C_p = 1 kJ/kg·K.

[(a) −44.3 kJ (b) −35.5 kJ (c) 0.35 8 kg]

15. A volume of 0.36 m^3 of a gas at 288 K and 1.03 bar is compressed reversibly and adiabatically to 10 bar. It is then cooled at constant pressure to its original temperature after which it expands isothermally. Find the heat transfer at constant pressure, heat transfer during isothermal expansion and work transfer during the cycle.

Take γ = 1.4 and R = 0.287 kJ/kg·K.

[−119.12 kJ, 84.28 kJ, −35.2 kJ]

16. A volume of 115 litres of air at 1.05 bar and 90°C is compressed adiabatically until the volume reaches 11.5 litres. Find the change in internal energy and change in enthalpy.

[45.69 kJ, 63.96 kJ]

17. In a closed system, 0.070 kg of air is compressed polytropically from 100 kPa and a volume of 0.060 m^3 to a volume of 0.030 m^3 with a polytropic exponent of 1.4. Compute (a) final temperature, (b) the work done, (c) the change in internal energy, and (d) the heat transfer.

18. 0.028 m^3 of air initially at 260°C and 700 kPa is expanded at constant pressure to 0.084 m^3. A polytropic process is carried according to law $pV^{1.5}$ = constant. The system is brought to initial state by a constant temperature process.

(a) Sketch the processes on p–V and T–S diagrams,
(b) Determine the net heat transfer and work transfer,
(c) Find the thermal efficiency of the cycle.

[(b) 52.9 kJ, 52.9 kJ, (c) 38.57%]

19. A closed system contains an ideal gas R = 0.27 kJ/kg·K and C_v = 0.5 kJ/kg·K. The system undergoes the following process in a cycle. At the state 1, the temperature is 450 K and pressure is 4.5 bar. The gas is then heated at constant pressure until the temperature becomes 900 K at the state 2. The gas is then compressed at constant temperature unitl the value of entropy equals that at states 1 and 3 and finally, there is an isentropic

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expansion from the state 3 to the state 1. Sketch the p–V and T–S diagrams. Calculate (a) pressure at the state 3, (b) heat transfer from the state 1 to state 3, and (c) cyclic work transfer.

[(a) 32.48 bar, (b) −133.8 kJ/kg, (c) −133.8 kJ/kg]

20. 50 litres of air at 1.013 bar and 100°C temperature is compressed to 28 bar. Volume of air at the end of polytropic compression is found to be 4 litres. Air is now heated at constant volume till pressure rises to 56 bar. Assuming C_p = 1.00 kJ/kg·K and C_v = 0.71 kJ/kg·K over complete range of working. Determine (a) polytropic index of compression, and (b) entropy change in each process.

[(a)1.314 (b) -7.93×10^{-3} kJ/K, 0.023 kJ/K]

21. A cylinder of an engine at the beginning of suction stroke contains 400 cm^3 of gas at a pressure of 100 kPa, and temperature of 110°C. If compression takes place according to law $pV^{1.3}$ = constant, determine the temperature and pressure of the gas when the volume is 80 cm^3. Also calculate the work done on the gas during the compression and change of internal energy of 1 kg of gas if C_v = 0.750 kJ/kg·K.

If the presure is increased to 40 bar during combustion at constant volume, determine the temperature of gas.

[− 0.0826 kJ, 177.5 kJ/kg, 2788°C]

22. A certain quantity of air has a volume of 0.028 m^3 at a pressure of 1.25 bar and 25°C. It is compressed to a volume of 0.0042 m^3 according to the law $pV^{1.3}$ = constant. Calculate the final temperature and work done during compression. Also detemine the reduction in pressure at a constant volume process required to bring the air back to its original temperature.

[T_2 = 253.48°C, W_{1-2} = − 8.945 kJ, Δp_{2-3} = 6.393 bar]

23. If air in a cylinder is kept at constant pressure by a gas tight dead weight piston. The cylinder is 600 mm in diameter, contains a quantity of air at 27°C in such a way that the piston is 150 mm from the base of cylinder. If heat is supplied to the air, increasing the temperature to 55°C, how far the piston will move? *[14 mm]*

24. A volume of 115 litres of air at 1.05 bar and 90°C is compressed adiabatically until the volume reached to 11.5 litres. Find the change in internal energy and change in enthalpy.

[45.69 kJ, 63.96 kJ]

25. Air trapped in a cylinder expands frictionlessly against a piston so that pV = constant. Initially the air is at 400 kN/m^2, 4°C, and occupies a volume of 0.02 m^3. The local value of g is 9.51 m/s^2.

(a) To what pressure must the air expand in order to perform 8100 J of work?

(b) What is the mass of air in the system?

[(a) 145.3kN/m^2 (b) 0.1 kg]

26. 100 kg of air at 300 kPa and 10°C is trapped inside a vertical cylinder, which is fittted at the top with the weighted piston so that the pressure of the air is held constant, there is no heat transfer. A paddle wheel in the cylinder is turned until the volume of air has increased by 20 percent. Determine : (a) the work done on the air and the amount of the work done on the air by paddle wheel. *[(a) –0.409 kJ (b) 0.571 kJ]*

27. 0.05 kg of steam at 15 bar is contained in a rigid vessel of volume 0.0076 m^3, what is the temperature of steam? If the vessel is cooled, at what temperature will the steam be just dry and saturated? The cooling is continued until the pressure in the vessel is 11.0 bar. Calculate the final dryness fraction of steam and heat rejected between initial and final states.

[(a) 72.36°C, (b) 192.5°C, (c) 0.856, (d) 19.48 kJ]

28. A cylinder of volume 0.1 m^3 contains nitrogen gas at 101 kPa and 20°C. If 0.5 kg of nitrogen is pumped into the cylinder, calculate the new pressure, when the cylinder has returned to its initial temperature. Assume that the specific gas constant for nitrogen is 0.297 kJ/kg·K. *[536.1 kPa]*

29. A vessel of 1 m^3 capacity contains steam of dryness fraction of 0.80. The pressure gauge on the vessel shows 9.8 bar and local barometer reads 720 mm of Hg. How many kg of steam is contained in the steam vesslel? *[6.89 kg]*

30. A vessel having a capacity of 0.6 m^3 contains steam at 15 bar and 250°C. The steam is blown off until the pressure drops to 4 bar. Then the valve is closed and vessel is cooled until the pressure falls to 3 bar. Assuming that during the blowing-off period, the entropy remains constant,

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determine

(i) the mass of steam blown off,

(ii) the quality of steam in the vessel after cooling, and

(iii) the heat transferred during the cooling process

[(a) 0.963, (b) 2.601 kg (c) –620 kJ]

31. A rigid vessel of 1 m^3 volume contains steam at 19 bar and 300°C. The vessel is cooled until the steam is just dry and saturated. Calculate the mass of steam in the vessel, the final pressure, and the heat removed during the process.

[7.55 kg, 15 bar and –1361 kJ]

32. 1 kg of steam at 120 bar and 400°C expands reversibly in a perfect insulated cylinder behind a piston until the pressure falls to 38 bar and the steam becomes just dry and saturated. Calculate the work done by the steam. *[196.34 kJ/kg]*

33. A mass of 3 kg of nitrogen occupying 2 m^3 is heated from 30°C to 225°C at a constant volume. Calculate the initial and final pressures of the gas. The universal constant is 8314 J/kgmol.K. The molecular mass of nitrogen is 28.

[1.35 bar, 2.22 bar]

34. A high altitude chamber, whose volume is 30 m^3 is put into operation by reducing the pressure from 1 bar to 0.35 bar and the temperature from 27°C to 5°C.

(i) How many kg of air must be removed from the chamber during the process?

(ii) Express this mass as a volume measured at 1 bar and 27°C.

[21.68 kg, 18.67 m^3]

35. 0.44 kg of gas having a volume 0.28 m^3 and a pressure of 1.4 bar is compressed to a pressure of 14 bar according to $pV^{1.3} = C$. Find the change of internal energy.

Take $C_p = 1.041$ kJ/kg·K

and $C_v = 0.743$ kJ/kg·K.

(68.54 kJ)

Objective Questions

1. The polytropic index n in equation pV^n = constant for a isochoric process is
   (a) 1 (b) 1.4
   (c) 0 (d) ∞
2. A process that does not involve heat transfer is called
   (a) isothermal process (b) isolated process
   (c) Steady process (d) adiabatic process
3. In an isothermal process
   (a) temperature increases gradually
   (b) volume remains constant
   (c) change in internal energy is zero
   (d) enthalpy change is maximum
4. In a reversible adiabatic process, the work transfer is equal to
   (a) decrease in enthalpy
   (b) decrease in internal energy
   (c) heat transfer
   (d) the product of pressure and change in volume

Answers

1. (d) 2. (p) 3. (c) 4. (b)

5

First Law Applied to Flow Processes

Introduction

Most engineering problems involve flow of mass across a system boundary. The mass and energy analyses for an open system are discussed with the help of continuity equation and the first law of thermodynamics in this chapter. For analysis of an open system, the control-volume concept is used. The rate of work and heat interactions with their sign convention is also considered across the control surface. The steady flow work, relation between $\int pdv$ and $-\int vdp$ work are also carried out in the chapter. The steady-flow energy equation is applied to the nozzle, diffuser, turbine compressor, pump, boiler, condenser and throttling processes.

5.1 FLOW PROCESS AND FLOW ENERGY

Most engineering problems involve mass flow in and out of a system, such as turbines, compressors, nozzles and pumps, etc. Such systems are called open systems. Therefore, their analysis is carried out with the concept of control volume, control surface and flow energy.

A *flow process* constitutes an open system as shown in Fig. 5.1, in which the working substance enters and leaves the control surface of a system. Energy interaction in the form of heat and work may also take place within the system.

Flow energy or *flow work* refers to work required to push a certain mass of fluid into and out of the control volume. It is necessary for maintaining continuous flow through a control volume.

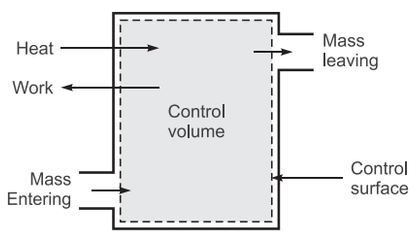

Fig. 5.1 *Flow process and control volume*

Consider a fluid entering the control volume through a cross-sectional area A as shown in Fig. 5.2. If p is the fluid pressure acting uniformly at the imaginary piston at the entrance of the control volume, the force applied on the fluid element by imaginary piston is

$$F = pA \text{ (kN)}$$

If the fluid is pushed by a distance L then the flow work $W_f = pAL = pV$ ...(5.1)

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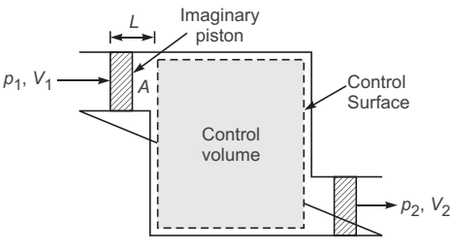

Fig. 5.2 *Illustration for flow work*

the flow work at entrance $W_{f_1} = p_1 V_1$ (kJ)

the flow work at exit $W_{f_2} = p_2 V_2$ (kJ)

5.2 MASS AND ENERGY ANALYSIS OF AN OPEN SYSTEM

5.2.1 Conservation of Mass Principle—Continuity of Mass

The conservation of mass is one of the most fundamental principle for flow systems. It states that the mass of a system can neither be created nor destroyed but its amount remains constant during any process. It only changes its form (phase). For example, when 16 kg oxygen reacts with 2 kg of hydrogen, 18 kg of water is formed. The conservation of mass principle for a control volume (*CV*) can be expressed as

Total mass entering *CV* – Total mass leaving *CV*
= Net change in mass within *CV*

The amount of mass flowing through a cross-section per unit time is called the ***mass flow rate*** and it is calculated as

$$\dot{m} = \frac{A_c \mathcal{V}}{v} \qquad ...(5.2)$$

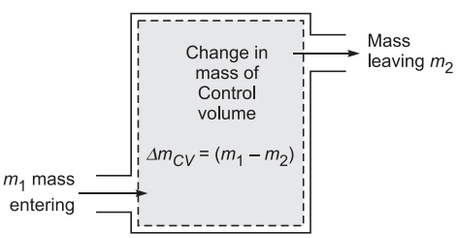

Fig. 5.3 *Conservation of mass principle for a control volume*

where $\dot{m}$ = mass flow rate in kg/s,
A_c = cross sectional area of flow in m^2,
v = sp. volume of fluid in m^3/kg,
$\mathcal{V}$ = fluid velocity in m/s.

Further, $v = \dfrac{1}{\rho}$

Therefore, $\dot{m} = \rho A_c \mathcal{V}$ ...(5.3)

The volume flow rate through a cross-sectional area per unit time is called fluid ***discharge rate*** and it is designated as $\dot{V}$, and is expressed as

$$\dot{V} = A_c \mathcal{V} \text{ (m}^3\text{/s)} \qquad ...(5.4)$$

5.2.2 The Steady-Flow Process

A steady-flow process can be defined as a process during which all properties of fluid at each location within the system remain constant with respect to time. That is, the fluid properties can change from point to point within the control volume but at any fixed location they remain the same during the entire process. *Steady means no change with time.* The steady-flow process requires the following particular conditions:

1. The fluid properties (intensive or extensive) within the control volume remain constant at each location with respect to time.
2. The properties of the fluid crossing the boundary (inlet and outlet) remain constant at each point of the boundary.
3. The mass-flow rate into the system is always equal to the mass flow rate out the system.
4. Heat and work interactions within the surroundings occur at a steady rate.

5.2.3 Total Energy of Closed and Open Systems

As explained earlier, the total energy of a non-flow (closed) system comprises of internal energy, potential energy and kinetic energy as shown in Fig. 5.4(a). For unit mass

$$e = u + ke + pe = u + \frac{\mathcal{V}^2}{2} + gz \text{ (J/kg)}$$

where $\mathcal{V}$ is the velocity and z is the elevation of the system relative to some reference. In an open

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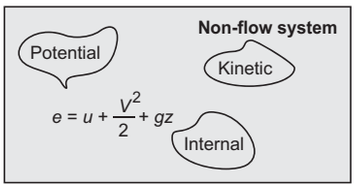
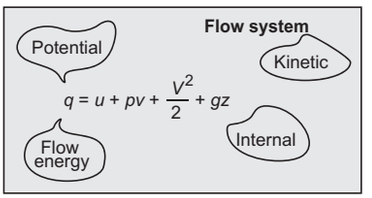


(a) Total energy of a non-flow system



(b) Total energy of a flow system

Fig. 5.4

or flow system as shown in Fig. 5.4(b), the fluid entering and leaving possesses an additional form of energy—*flow energy* (or flow work), pv which pushes the fluid in and out. Then total energy of the flowing fluid on a unit-mass basis (denoted by θ), becomes

$$\theta = pv + e = u + pv + ke + pe$$

$$= u + pv + \frac{\mathcal{V}^2}{2} + gz \qquad ...(5.5)$$

Using $h = u + pv$

Then $$\theta = h + \frac{\mathcal{V}^2}{2} + gz \qquad ...(5.6)$$

5.3 ENERGY BALANCE IN STEADY FLOW

Consider a steady-flow device within a control volume as shown in Fig. 5.5. The working substance enters the device at the section 1 and leaves at the section 2.

Let $\dot{Q}$ = rate of heat transfer to control volume in W

$\dot{W}$ = rate of shaft work done by the fluid in control volume in W

p = pressure in N/m^2

u = specific internal energy in J/kg

h = specific enthalpy in J/kg

$\mathcal{V}$ = velocity of fluid in m/s

z = elevation above datum in m

θ = total energy of the fluid, including flow work per unit mass, J/kg

$\Sigma\dot{m}_1$ = total mass flow rate into the control volume in kg/s

and $\Sigma\dot{m}_2$ = total mass flow rate out of the control volume in kg/s.

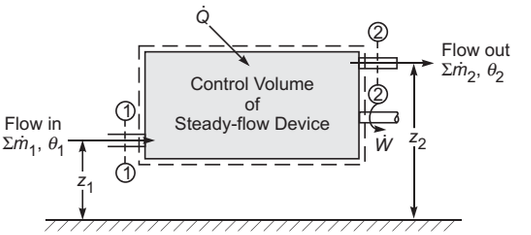


Fig. 5.5 *Schematic diagram flow of steady-flow process*

Energy balance on the control volume reveals:

Total energy rate crossing boundary as heat and work = Total energy rate leaving control surface 2 – Total energy rate entering the control surface 1

or $$\dot{Q} - \dot{W} = \Sigma\dot{m}_2\theta_2 - \Sigma\dot{m}_1\theta_1 \qquad ...(5.7)$$

For a single-stream (one inlet and one exit) system, the mass-flow rate at the inlet and exit can be expressed as $\dot{m}_1$ and $\dot{m}_2$. The inlet and exit states of the fluid for control volume are designated by subscripts 1 and 2, respectively.

$$\dot{Q} - \dot{W} = \dot{m}_2\theta_2 - \dot{m}_1\theta_1 \qquad ...(5.8)$$

Using θ from Eq. (5.5);

$$\dot{Q} - \dot{W} = \dot{m}_2\left[u_2 + p_2v_2 + \frac{\mathcal{V}_2^2}{2} + z_2g\right] - \dot{m}_1\left[u_1 + p_1v_1 + \frac{\mathcal{V}_1^2}{2} + z_1g\right]$$

For steady flow, the mass-flow rate through the entire control volume remains constant, thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$, then

$$\dot{Q} - \dot{W} = \dot{m}\left[(u_2 - u_1) + (p_2v_2 - p_1v_1) + \left(\frac{\mathcal{V}_2^2}{2} - \frac{\mathcal{V}_1^2}{2}\right) + (z_2 - z_1)g\right] \qquad ...(5.9)$$

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Using $h = u + pv$

then $$\dot{Q} - \dot{W} = \dot{m}\left[(h_2 - h_1) + \left(\frac{\mathcal{V}_2^2}{2} - \frac{\mathcal{V}_1^2}{2}\right) + (z_2 - z_1)g\right] \text{ (J/s)} \quad ...(5.10)$$

Dividing both sides by $\dot{m}$, the steady-flow energy equation on a unit-mass basis as

$$q - w = (h_2 - h_1) + \left(\frac{\mathcal{V}_2^2}{2} - \frac{\mathcal{V}_1^2}{2}\right) + (z_2 - z_1)g \quad ...(5.11)$$

or $$q - w = \Delta h + \Delta ke + \Delta pe \text{ (J/kg)}$$

where $q = \dfrac{\dot{Q}}{\dot{m}}$ (heat transfer per unit mass, J/kg)

$w = \dfrac{\dot{W}}{\dot{m}}$ (work transfer per unit mass, J/kg)

$\Delta h = \Delta u + \Delta(pv)$ (specific enthalpy change, J/kg)

The above equation can be arranged as

$$q - w = \Delta u + \Delta(pv) + \Delta ke + \Delta pe \text{ (J/kg)} \quad ...(5.12)$$

If the fluid has negligible change in kinetic energy and potential energy then the steady flow equation reduces to

$$q - w = \Delta h = \Delta u + \Delta(pv) \text{ (J/kg)} \quad ...(5.13)$$

5.4 SOME STEADY-FLOW DEVICES

Some common steady-flow devices are discussed below:

5.4.1 Nozzles and Diffusers

Nozzles and diffusers as shown in Fig. 5.6 are used in a number of engineering devices. A *nozzle* is a device that increases the velocity of a fluid at the expense of pressure drop. A *diffuser* is a device that increases the pressure of a fluid by slowing it down. Nozzles and diffusers are the devices in which cross sectional area changes along the flow. The following point should be noted for flow through these devices.

$\dot{Q}$ = 0, the heat transfer rate is negligible,

$\dot{W}$ = 0, since they do not involve any shaft, thus no work interaction,

Δpe = 0, fluid experiences usually very little or no change in its elevation between inlet and outlet.

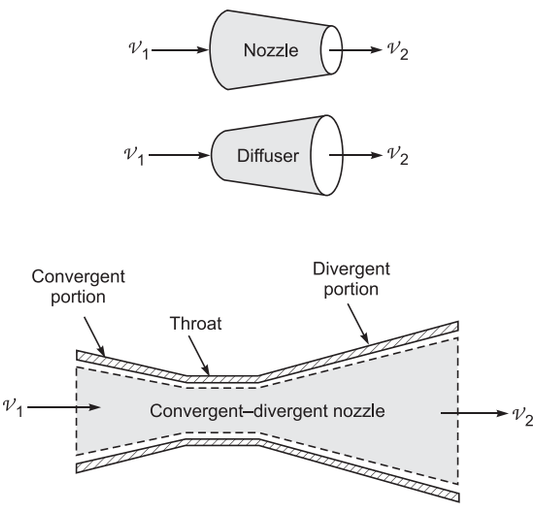

Fig. 5.6 *Nozzle and diffuser*

For nozzles and diffusers the steady-flow energy equation,

$$h_2 - h_1 + \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} = 0 \quad ...(5.14)$$

5.4.2 Water Turbines

Water turbine is a device which takes in water from a height. The water enters into the turbine, a part of

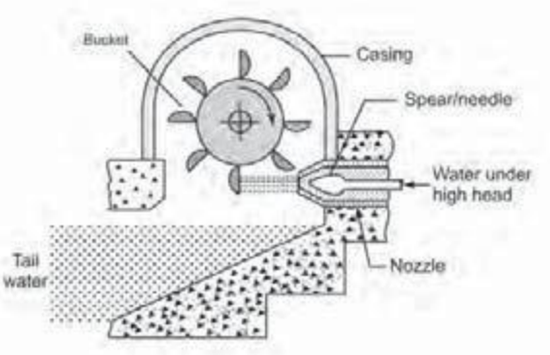

Fig. 5.7 *Water turbine (Pelton wheel)*

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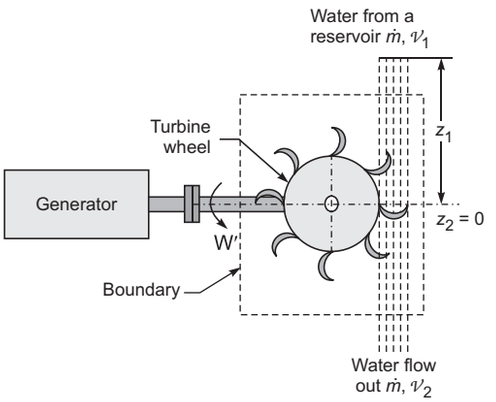

Fig. 5.8 *Schematic of water turbine*

its potential energy is converted into useful work (shaft work), which is used to generate electric power in a generator.

The following facts should be kept in mind, while analysing a water turbine;

Rate of work done by turbine,

$$\dot{W} = +\text{ve}$$

Heat transfer rate from turbine,

$$\dot{Q} = 0,$$

Change in specific internal energy,

$$\Delta u = u_2 - u_1 = 0,$$

(no change in temperature of water)

Specific volume of water, $v_1 = v_2 = v$ (remains constant)

The steady flow energy equation reduces

$$\dot{m}\left[(p_2v_2 - p_1v_1) + \left(\frac{v_2^2}{2} - \frac{v_1^2}{2}\right) + (z_2 - z_1)g\right] + \dot{W} = 0 \quad ...(5.15)$$

5.4.3 Steam and Gas Turbines

These are the devices (Fig. 5.9 and 5.10) in which the fluid (steam or gas) expands, work is produced on the shaft of the turbine. The work produced by a turbine is given to the generator to produce electric power. According to sign convention, the work done by the turbine is considered positive.

Rate of work done $\dot{W} = +\text{ve}$

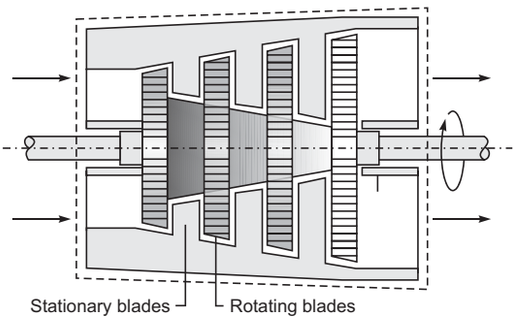

Fig. 5.9 *Axial flow gas turbine*

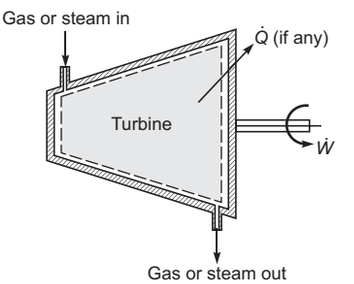

Fig. 5.10 *Schematic of steam or gas turbine*

Change in potential energy,

$$\Delta z = 0 \text{ (if elevations are not mentioned)}$$

Heat-transfer rate,

$$\dot{Q} \approx 0 \text{ (if turbine is insulated)}$$

$$\dot{Q} = -\text{ve (if turbine is not insulated)}$$

The steady-flow energy equation

$$\dot{Q} - \dot{W} = \dot{m}\left[(h_2 - h_1) + \frac{v_2^2 - v_1^2}{2} + (z_2 - z_1)g\right] \quad ...(5.16)$$

5.4.4 Compressor

A *compressor* is a device, which takes in low pressure gas and compresses it to higher pressure.

(i) Reciprocating Compressor The *reciprocating compressor* is shown in Fig. 5.11. It has inlet and outlet gas-flow streams and work and heat interactions at its control volume. For a reciprocating compressor:

Rate of work input $\dot{W} = -\text{ve}$,

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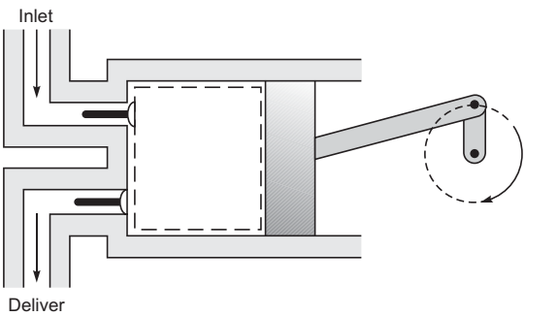


Fig. 5.11 *Reciprocating compressor*

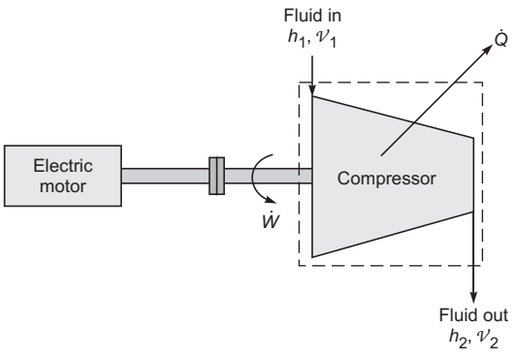


Fig. 5.12 *Schematic of a compressor*

Heat rejection (if any) $\dot{Q}$ = –ve,

Change in potential energy $\Delta pe = 0$,

Change in kinetic energy $\Delta ke = 0$,

The steady-flow energy equation for reciprocating compressor becomes

$$\dot{Q} - \dot{W} = \dot{m}(h_2 - h_1) \text{ (J/s)} \qquad ...(5.17)$$

(ii) Centrifugal Compressor/Root Blower Generally, the change in potential energy is taken negligible, or

$$\Delta pe = 0,$$

Rate of work input is $\dot{W}$ = –ve

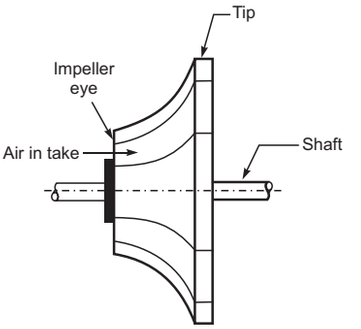
Heat rejection (if any) $\dot{Q}$ = –ve

Then steady-flow energy equation becomes

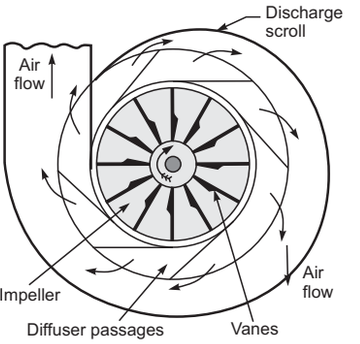
$$\dot{Q} - \dot{W} = \dot{m}\left[(h_2 - h_1) + \left(\frac{v_2^2}{2} - \frac{v_1^2}{2}\right)\right] \text{ (J/s)} \qquad ...(5.18)$$

5.4.5 Centrifugal Water Pump

The water pump (Fig. 5.13) increases the pressure of incompressible fluid like water, therefore, these are power consuming devices. The particular conditions for water pumps;



(a) Impeller of centrifugal compressor



(b) Centrifugal compressor

Fig. 5.13

Rate of work input to pump,

$$\dot{W} = -\text{ve},$$

No heat-transfer rate from the pump,

$$\dot{Q} = 0,$$

Change in specific internal energy,

$$\Delta u = u_2 - u_1 = 0,$$

Specific volume of water,

$$v_1 = v_2 = v = \text{constant}$$

The steady-flow energy equation reduces to

$$-\dot{W} = \dot{m}\left[(p_2 v_2 - p_1 v_1) + \left(\frac{v_2^2}{2} - \frac{v_1^2}{2}\right) + (z_2 - z_1)g\right] \qquad ...(5.19)$$

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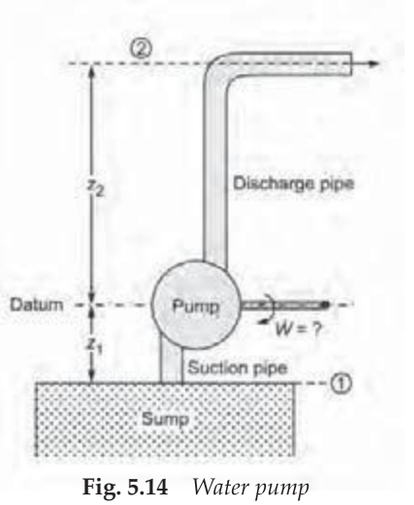
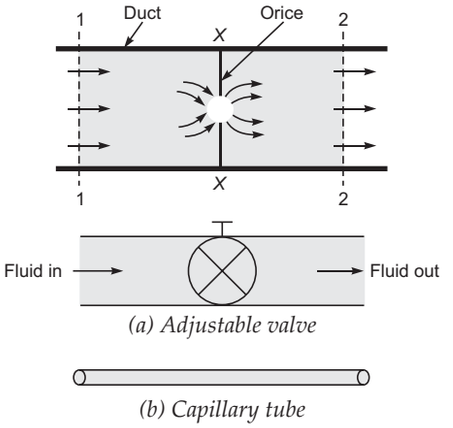


Fig. 5.14 *Water pump*

5.4.6 Throttling Process

It is an irreversible process. Whenever a fluid expands from a region of high pressure to low pressure through a porous plug, partly open valve, an orifice, a capillary tube or through any other obstruction without exchanging any energy as heat and work transfer with surroundings and with negligible change in kinetic and potential energies, the enthalpy of the fluid remains constant and fluid is said to undergo a *throttling process*.

Figure 5.14 shows steady flowing fluid through an insulated duct, passes through an orifice at section $X-X$. Applying steady flow energy equation between sections 1–1 and 2–2,



(a) Adjustable valve

(b) Capillary tube

Fig. 5.15 *Throttling devices*

$$q - w = \Delta h + \Delta ke + pe$$

Here $q = 0, w = 0, \quad \Delta pe = 0$, then

$$\Delta h + \Delta ke = 0$$

or $$h_1 + \frac{\mathcal{V}_1^2}{2} = h_2 + \frac{\mathcal{V}_2^2}{2}$$

When $\mathcal{V}_1 = \mathcal{V}_2$ then $h_1 = h_2$

For a perfect gas $C_p T_1 = C_p T_2$

$$T_1 = T_2$$

Thus for a perfect gas, the temperature before and after throttling is always same.

5.4.7 Boiler, Condenser and Heat Exchanger

These are the devices where the objective is to transfer heat energy between hot and cold fluids, therefore, the heat transfer rate cannot be taken as zero.

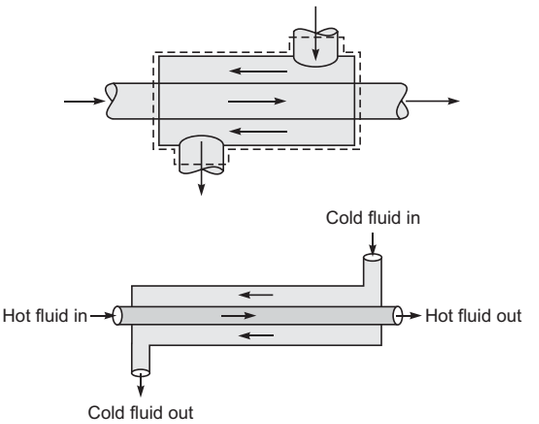


Fig. 5.16 *Heat exchanger*

They do not involve any shaft work or work interaction. Therefore,

$$\dot{W} = 0 \quad \text{and} \quad \dot{Q} \neq 0 \quad \text{(compulsory)}$$

If change in potential energy is negligible then,

$$\Delta pe = 0, \quad \text{(generally)}$$

The steady-flow energy equation for these devices may be expressed as

$$\dot{Q} = \dot{m}\left[h_2 - h_1 + \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2}\right] \text{(J/s)} \qquad ...(5.20)$$

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Note The remaining terms for all devices have usual meanings, if specific conditions are not mentioned.

5.5 SIGNIFICANCE OF $-\int vdp$

The steady-flow energy equation (5.11) for unit mass-flow rate is expressed as

$$q - w = h_2 - h_1 + \frac{\mathcal{V}_2^2}{2} - \frac{\mathcal{V}_1^2}{2} + (z_2 - z_1)g$$

It can be written in differential form as

$$\delta q - \delta w = dh + \mathcal{V}d\mathcal{V} + gdz$$

Using $h = u + pv$

Therefore $dh = du + pdv + vdp$

Inserting it above, we get

$$\delta q - \delta w = du + pdv + vdp + \mathcal{V}d\mathcal{V} + gdz$$

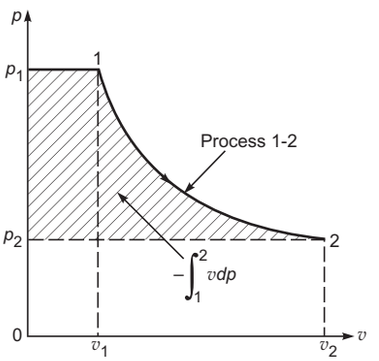

Fig. 5.17 *Area behind the curve represents* $-\int vdp$ *work*

According to the first law of thermodynamics for the moving system, if an observer moving with the fluid,

$$\delta q = du + pdv$$

Then $du + pdv - \delta w = du + pdv + vdp + \mathcal{V}d\mathcal{V} + gdz$

or $-\delta w = vdp + \mathcal{V}d\mathcal{V} + gdz$

Integrating both sides and rearranging, we get

$$w = -\int vdp - \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} - (z_2 - z_1)g \qquad ...(5.21)$$

If the kinetic and potential energy changes are negligible, the above equation is reduced to

$$w = -\int vdp \qquad ...(5.22)$$

It should be noted that the area behind curve is equal to steady flow work $-\int vdp$.

5.6 RELATION BETWEEN NON-FLOW WORK $\int pdv$ AND FLOW WORK $-\int vdp$

The steady-flow energy equation for unit mass-flow rate

$$q - w = h_2 - h_1 + \frac{\mathcal{V}_2^2}{2} - \frac{\mathcal{V}_1^2}{2} + (z_2 - z_1)g$$

It can be written in differential form as

$$\delta q - \delta w = dh + \mathcal{V}d\mathcal{V} + gdz$$

Using $h = u + pv$ and

$$\delta q = du + pdv$$

Inserting it above, we get

$$du + pdv - \delta w = d(u + pv) + \mathcal{V}d\mathcal{V} + gdz$$

or $du + pdv - \delta w = du + d(pv) + \mathcal{V}d\mathcal{V} + gdz$

$\therefore$ $pdv - \delta w = d(pv) + \mathcal{V}d\mathcal{V} + gdz$

or $pdv = \delta w + d(pv) + \mathcal{V}d\mathcal{V} + gdz$

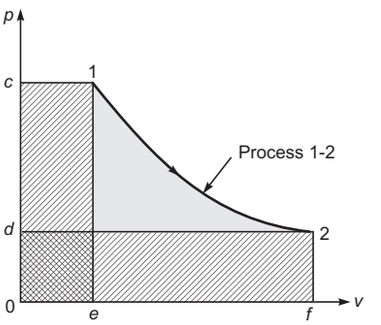

Fig. 5.18 *Relation between* $\int pdv$ *and* $-\int vdp$

Integrating both sides, we get

$$\int pdv = \int \delta w + \Delta(pv) + \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} + (z_2 - z_1)g$$

Inserting w from Eq. (5.21), we get

$$\int pdv = -\int vdp + \Delta(pv)$$

or $-\int vdp = p_1v_1 + \int pdv - p_2v_2 \qquad ...(5.23)$

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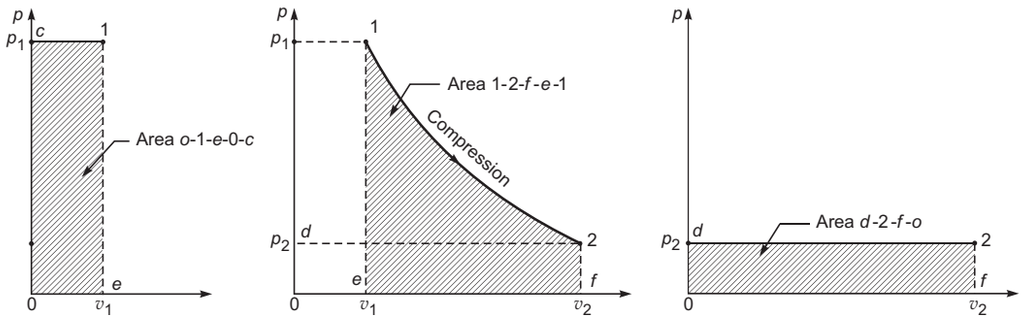

Fig. 5.19 *Three process areas on p - V diagram*

The three areas of Fig. 5.18 are shown separately in Fig. 5.19. Thus the $-\int vdp$ work can also be related with non-flow work $\int pdv$ graphically as

$$\int pdv = \text{area } e\text{-}1\text{-}2\text{-}f$$

$$-\int vdp = \text{area } c\text{-}1\text{-}2\text{-}d$$

$$= \text{area } c\text{-}1\text{-}e\text{-}o + \text{area } e\text{-}1\text{-}2\text{-}f - \text{area } d\text{-}2\text{-}f\text{-}o$$

$$-\int vdp = p_1v_1 + \int pdv - p_2v_2$$

It proves that the steady flow work $-\int vdp$ *is the sum of net flow work and non-flow work* $\int pdv$.

Example 5.1 *A small rigid evacuated cylinder filled from a large high-pressure reservoir. Assuming adiabatic conditions, determine the internal energy of the gas in the cylinder after charging.*

Solution

Given Adiabatic conditions, $q = 0$

To find Internal energy of the gas in the cylinder.

Assumptions

(i) No shaft work done during the process, $w = 0$.
(ii) Since there is no back pressure, therefore, $p_2 v_2 = 0$.
(iii) Change in potential energy is negligible, $z_1 = z_2$.
(iv) Change in kinetic energy is negligible, $\mathcal{V}_1 = \mathcal{V}_2$.

Analysis As the cylinder is very small as compared to reservoir, therefore, the changes in the reservoir are also negligible. Applying energy equation to flow process

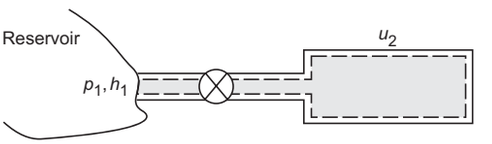

Fig. 5.20 *Schematic*

$$q - w = (u_2 - u_1) + (p_2v_2 - p_1v_1) + \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} + (z_2 - z_1)g$$

The above equation reduces to

$$0 = (u_2 - u_1) - p_1v_1$$

or $$u_2 = u_1 + p_1v_1$$

or $$u_2 = h_1$$

The internal energy of the gas in the cylinder is equal to the enthalpy of air in the reservoir.

Example 5.2 *In a steady-flow system, a substance flows at a rate of 5 kg/s. It enters the system at a pressure of 6 bar, velocity 300 m/s, internal energy 2000 kJ/kg and specific volume of 0.38 m^3/kg. It leaves the system at a pressure of 1.5 bar, velocity 150 m/s, internal energy 1600 kJ/kg and specific volume of 1.26 m^3/kg. During its passage through the system, the substance losses 80 kJ/kg of heat to the surroundings. Determine the power of the system, stating whether it is from or to the system. Neglect any changes in the potential energy.*

Solution

Given A steady-flow system, with negligible potential energy change.

$\dot{m} = 5$ kg/s, $q = -80$ kJ/kg
$\mathcal{V}_1 = 300$ m/s $\mathcal{V}_2 = 150$ m/s
$p_1 = 6$ bar $= 600$ kPa $p_2 = 1.5$ bar $= 150$ kPa

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$u_1 = 2000$ kJ/kg $\quad u_2 = 1600$ kJ/kg
$v_1 = 0.38$ m^3/kg $\quad v_2 = 1.26$ m^3/kg
$\Delta z = 0$

To find Power of the system and its direction with reference to system.

Analysis The steady-flow energy equation for 1 kg of mass of the substance.

$$q - w = \Delta h + \Delta ke + \Delta pe$$
$$= \Delta u + \Delta(pv) + \Delta ke + 0$$

Calculating each quantity separately

$$\Delta u = u_2 - u_1 = 1600 - 2000 = -400 \text{ kJ/kg}$$
$$\Delta(pv) = p_2v_2 - p_1v_1$$
$$= 150 \times 1.26 - 600 \times 0.38 = -39 \text{ kJ/kg}$$
$$\Delta ke = \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} = \frac{150^2 - 300^2}{2}$$
$$= -33.75 \times 10^3 \text{J/kg} = -33.75 \text{ kJ/kg}$$

Now $-80 - w = -400 - 39 - 33.75 = -472.75$
or $w = 472.75 - 80 = 392.75$ kJ/kg

The power output of the system

$$P = \dot{m}w = 5 \times 392.75 = \mathbf{1963.75\ kW}$$

Power developed by system.

Example 5.3 *In a steady-flow apparatus 135 kJ work is done by each kg of fluid. The specific volume of the fluid, pressure and velocity at the inlet are 0.37 m^3/kg, 600 kPa and 16 m/s, respectively. The inlet is 32 m above floor level. The discharge is at floor level. The discharge conditions are 0.62 m^3/kg, 100 kPa and 270 m/s. The total heat loss between inlet and outlet is 9 kJ/kg of the fluid. In flowing apparatus, does the specific internal energy increase or decrease and by how much?*

Solution

Given A steady-flow apparatus as shown in Fig. 5.21.

To find Specific internal energy change device.

Assumptions

(i) Steady-state conditions, and
(ii) Acceleration due to gravity $g = 9.81$ m/s^2.

Analysis The steady-flow energy equation is given by

$$q - w = (u_2 - u_1) + (p_2v_2 - p_1v_1) + \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} + (z_2 - z_1)g$$

Substituting the values of properties with proper care of units

$$(-9000) - 135 \times 10^3 = u_2 - u_1 + (100 \times 10^3 \times 0.62) - (600 \times 10^3 \times 0.37) + \frac{270^2 - 16^2}{2} + (0 - 32) \times 9.81$$
$$-144{,}000 = u_2 - u_1 + 62000 - 222000 + 36322 - 313.92$$
$$u_2 - u_1 = -20008 \text{ J/kg} \approx \mathbf{-20.0\ kJ/kg} \text{ (decrease)}$$

Example 5.4 *75 kg/min air enters the control volume of a steady-flow system at 2 bar and 100°C, at an elevation of 100 m above the datum. The same mass leaves the control volume at 150 m elevation from datum with pressure of 10 bar and at a temperature of 300°C. The entrance velocity is 40 m/s and exit velocity is 20 m/s. During the process 54,000 kJ/h of heat is transferred to control volume and rise in enthalpy is 8 kJ/kg. Calculate the power developed.*

Solution

Given $\dot{m} = 75$ kg/min $= 1.25$ kg/s

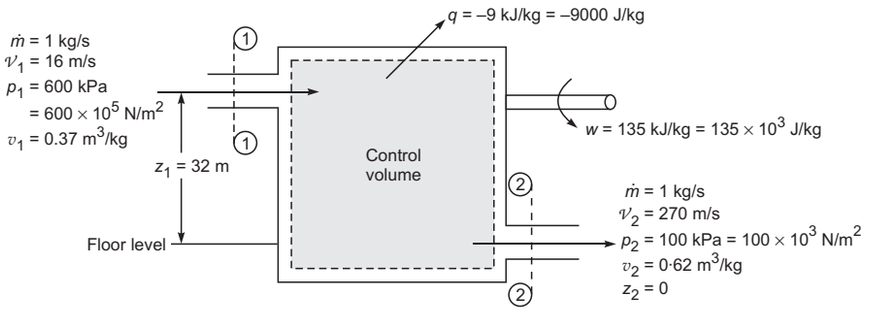

Fig. 5.21 *Schematic diagram*

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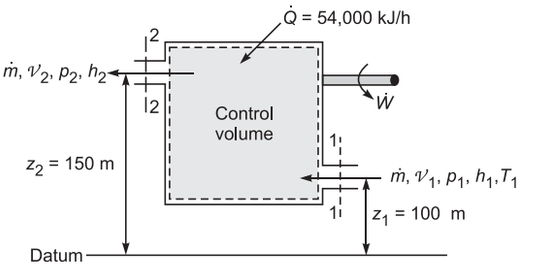

Fig. 5.22 *Schematic diagram*

$\dot{Q}$ = 54,000 kJ/h
= 15×10^3 W (heat supply to system)

p_1 = 2 bar $\quad$ p_2 = 10 bar
$\mathcal{V}_1$ = 40 m/s $\quad$ $\mathcal{V}_2$ = 20 m/s
z_1 = 100 m $\quad$ z_2 = 150 m
T_1 = 100°C $\quad$ T_2 = 300°C
Δh = 8 kJ/kg = 8×10^3 J/kg

To find The power developed by the system.

Analysis The steady-flow energy equation is given by

$$\dot{Q} - \dot{W} = \dot{m}\left[(h_2 - h_1) + \left(\frac{\mathcal{V}_2^2}{2} - \frac{\mathcal{V}_1^2}{2}\right) + (z_2 - z_1)g\right]$$

Substituting the values of properties with proper care of units

$$(15 \times 10^3) - \dot{W} = (1.25) \times [(8 \times 10^3) + \frac{20^2 - 40^2}{2} + (150 - 100) \times 9.81]$$

or
$$\dot{W} = (15 \times 10^3) - (1.25) \times (8 \times 10^3 - 600 + 490.5)$$
$$= (15 \times 10^3) - 9863.125$$
$$= 5136.87 \text{ W} \approx \mathbf{5.13\ kW}$$

Example 5.5 *In a steady-flow device, the inlet and outlet conditions are given below. Determine the heat loss/gain by the system.*

Property	*Inlet*	*Outlet*
Pressure (bar)	*10*	*8.93*
Specific enthalpy (kJ/kg)	*2827*	*2341*
Velocity (m/s)	*20*	*120*
Elevation (m)	*3.2*	*0.5*

Fluid flow rate through the device is 2.1 kg/s. The work output of the device is 750 kW.

Solution

Given Flow through a steady-flow device as shown in Fig. 5.23.

To find Heat-transfer rate from the device.

Assumption Acceleration due to gravity $g = 9.81 \text{m/s}^2$.

Analysis Applying steady-flow energy equation

$$\dot{Q} - \dot{W} = \dot{m}\left\{(h_2 - h_1) + \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} + (z_2 - z_1)g\right\}$$

Calculating each item separately

$$h_2 - h_1 = 2341 - 2827 = -486 \text{ kJ/kg}$$
$$\frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} = \frac{120^2 - 20^2}{2}$$
$$= 7000 \text{ J/kg} = 7 \text{ kJ/kg}$$
$$(z_2 - z_1)g = (0.5 - 3.2) \times 9.81$$
$$= -26.487 = -0.0264 \text{ kJ/kg}$$

Substituting the value in steady-flow energy equation

$$\dot{Q} - 750 \text{ kW} = (2.1 \text{ kg/s}) \times (-486 \text{ kJ/kg} + 7 \text{ kJ/kg} - 0.0264 \text{ kJ/kg})$$

or
$$\dot{Q} = 750 - 1006 = \mathbf{-256\ kW}$$
(Heat is rejected)

Nozzle

Example 5.6 *Air enters a convergent nozzle with a velocity of 40 m/s. The enthalpy of air decreases by*

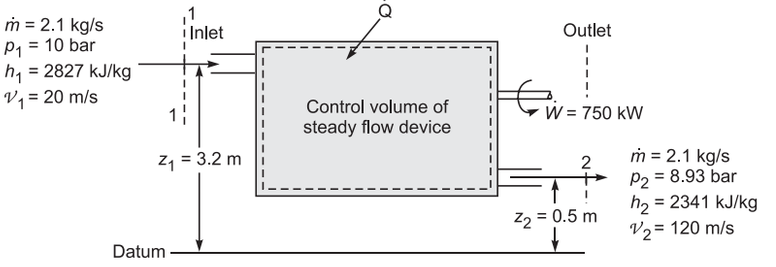

Fig. 5.23

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180 kJ/kg. Determine the exit velocity. Assume adiabatic conditions in the nozzle.

Solution

Given $\mathcal{V}_1 = 40$ m/s,
$\Delta h = -180$ kJ/kg $= -180 \times 10^3$ J/kg,
$\dot{Q} = 0$ (for adiabatic conditions)

To find Exit velocity of air at nozzle exit

Assumptions

(i) No change in potential energy, or $z_1 = z_2$.

(ii) Properties of the system remain constant at their locations.

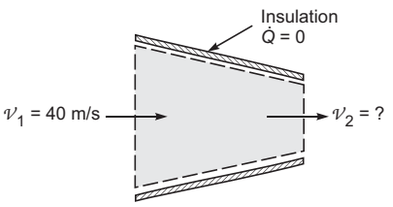

Fig. 5.24 *Convergent nozzle*

Analysis Since the nozzle does not have any work and heat interaction, hence the steady-flow energy equation reduces to

$$0 = h_2 - h_1 + \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2}$$

$$0 = -180 \times 10^3 + \frac{\mathcal{V}_2^2 - (40\text{ m})^2}{2}$$

or $\mathcal{V}_2^2 = 2 \times 180 \times 10^3 + 1600 = 361600$

or $\mathcal{V}_2 = \mathbf{601.33\ m/s}$

Example 5.7 *At the inlet to a convergent–divergent nozzle the enthalpy of the fluid passing is 3000 kJ/kg and the velocity is 60 m/s. At the discharge end, the enthalpy is 2757 kJ/kg. The nozzle is horizontal and the heat loss during flow is negligible. Find (a) velocity of the fluid at the exit of the nozzle, (b) if the inlet area is 0.1 m^2 and the specific volume at the inlet is 0.187 m^3/kg, find the mass flow rate of the fluid, and (c) if the specific volume at the outlet is 0.498 m^3/kg, find the area at the exit of the nozzle.*

Solution

Given A convergent–divergent nozzle shown in Fig. 5.24 with data

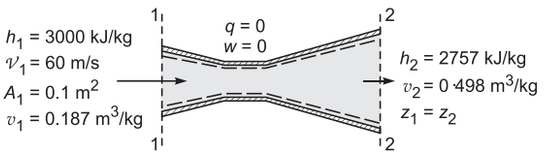

Fig. 5.25 *Schematic diagram of convergent-divergent nozzle*

To find

(i) Exit velocity of the fluid,

(ii) Mass flow rate, and

(iii) Exit area of the nozzle.

Assumption Properties of the system remains constant at their locations.

Analysis

(i) Since the nozzle does not have any work and heat interaction, hence the steady-flow energy equation reduces to

$$0 = h_2 - h_1 + \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2}$$

$$0 = (3000 \times 10^3 \text{ J/kg} - 2757 \times 10^3 \text{ J/kg}) + \frac{\mathcal{V}_2^2 - (60\text{ m})^2}{2}$$

or $\mathcal{V}_2^2 = 2 \times 243 \times 10^3 \text{ J/kg} + 3600 \text{ m}^2$
$= 489600$

or $\mathcal{V}_2 = \mathbf{699.71\ m/s}$

(ii) The mass-flow rate can be calculated by using the continuity equation;

$$\dot{m} = \frac{A_1 \mathcal{V}_1}{v_1} = \frac{A_2 \mathcal{V}_2}{v_2}$$

or $\dot{m} = \dfrac{(0.1\text{ m}^2) \times (60\text{ m/s})}{(0.187\text{ m}^3/\text{s})}$
$= \mathbf{32.08\ kg/s}$

(iii) The exit area of the nozzle can also be calculated by continuity equation;

$$A_2 = \frac{\dot{m} v_2}{\mathcal{V}_2} = \frac{(32.08\text{ kg/s}) \times (0.498\text{ m}^3/\text{kg})}{(699.71\text{ m/s})}$$
$= \mathbf{0.0228\ m^2}$

Example 5.8 *Steam enters the nozzle, operating at steady state at 30 bar and 350°C with negligible velocity and exits at 15 bar and a velocity of 500 m/s. The mass-flow rate is 2.22 kg/s. Determine (a) the exit temperature of steam, and (b) exit area of nozzle.*

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Solution

Given Flow of steam through a nozzle;

$p_1 = 30$ bar $\quad T_1 = 350°C$

$\mathcal{V}_1 = 0 \quad p_2 = 15$ bar

$\mathcal{V}_2 = 500$ m/s $\quad \dot{m} = 2.22$ kg/s

To Find

(i) Exit temperature of steam, and

(ii) Exit area of the nozzle.

Assumptions

(i) No heat loss at the boundary of the nozzle, $q = 0$.

(ii) No change in potential energy, $\Delta pe = 0$

(iii) Specific heat of superheated steam as 2.1 kJ/kg·K

Properties of steam

At 30 bar and 350°C: $h_1 = 3115.3$ kJ/kg

At 15 bar, $T_{sat_2} = 198.32$ C $\quad v_{g_2} = 0.13177$,

$h_{g_2} = 2792.2$ kJ/kg

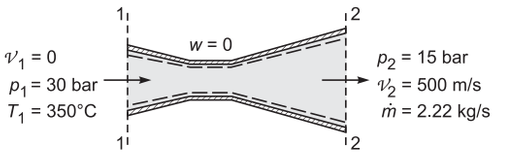

Fig. 5.26 *Schematic for Example 5.8*

Analysis

(i) The steady-flow energy equation for unit mass

$$q - w = \Delta h + \Delta ke + \Delta pe$$

For nozzle; $w = 0$ and $q = 0$

$$\therefore \quad 0 = h_2 - h_1 + \Delta ke$$

$$\Delta ke = \frac{(\mathcal{V}_2^2 - \mathcal{V}_1^2)}{2} = \frac{(500^2 - 0^2)}{2}$$

$$= 125 \times 10^3 \text{ J/kg}$$

And $\quad h_2 = h_1 - \Delta ke = 3115.3 - 125$

$$= 2990.3 \text{ kJ/kg}$$

Further, the h_2 at 15 bar > 2792.2 kJ/kg for dry and saturated steam.

Therefore, steam is superheated at this pressure.

$$h_2 = h_{g_2} + C_{ps}(T_{sup_2} - T_{sat_2})$$

$$2990.3 = 2792.2 + 2.1 \times (T_{sup} - 198.32)$$

$$T_{sup_2} = \frac{2990.3 - 2792.2}{2.1} + 198.32$$

$$= 94.33 + 198.32 = \mathbf{292.65\ C}$$

and $\quad v_2 = v_{g_2} \dfrac{T_{sup_2}}{T_{sat_2}}$

$$= 0.13177 \times \frac{(292.65 + 273)}{(198.32 + 273)}$$

$$= 0.15814 \text{ m}^3\text{/kg}$$

(ii) The exit area can be calculated from the continuity equation

$$\dot{m} = \frac{A_2 \mathcal{V}_2}{v_2}$$

$$A_2 = \frac{\dot{m} v_2}{\mathcal{V}_2} = \frac{2.22 \times 0.15814}{500}$$

$$= 7.02 \times 10^{-4} \text{ m}^2$$

$$= \mathbf{7.02\ cm^2}$$

Example 5.9 *Helium at 300 kPa, 60°C, enters a nozzle with negligible velocity and expands steadily without heat transfer in a quasi-equilibrium manner to 120 kPa. The process is such that $pv^{1.67}$ = constant. Calculate the exit velocity.*

Solution

Given A nozzle with data as shown in Fig. 5.27.

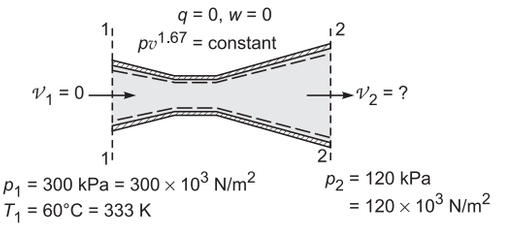

Fig. 5.27 *Schematic*

To find Exit velocity of the fluid.

Assumptions

(i) No change in potential energy, or $z_1 = z_2$

(ii) Properties of the system remain constant at their locations.

(iii) The gas constant for helium as 2078.5 J/kg·K. as

$$R = \frac{8314}{4} \text{ J/kg·K}$$

Analysis The steady-flow energy equation for nozzle

$$q - w = (h_2 - h_1) + \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} + (z_2 - z_1)g$$

where $w = 0$ (no work done in a nozzle), inlet velocity $\mathcal{V}_1$ is negligible and change in potential energy is also

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negligible and the equation is reduced to

$$0 = (h_2 - h_1) + \frac{\mathcal{V}_2^2}{2}$$

This equation cannot be used for calculation of exit velocity $\mathcal{V}_2$, because $h_2 - h_1$ is also unknown. Thus, using the equation

$$w = -\int v dp - \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} - (z_2 - z_1)g$$

Using the assumptions

$$0 = -\int v dp - \frac{\mathcal{V}_2^2}{2}$$

or $$\mathcal{V}_2^2 = -2\int_1^2 v dp$$

but p and v are related as $pv^n = C$

or $$v = \frac{C^{1/n}}{p^{1/n}}$$

Then $$\mathcal{V}_2^2 = -2C^{1/n}\int_1^2 p^{-1/n} dp$$

$$= -2C^{1/n}\left(\frac{p_2^{-1/n+1} - p_1^{-1/n+1}}{(-1/n)+1}\right)$$

using $$C^{1/n} = p_1^{1/n} v_1 = p_2^{1/n} v_2$$

we get $$\mathcal{V}_2^2 = -2n\left(\frac{p_2 v_2 - p_1 v_1}{n-1}\right)$$

$$= -2\left(\frac{n}{n-1}\right) R(T_2 - T_1)$$

where $$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = 333 \times \left(\frac{120}{300}\right)^{\frac{1.67-1}{1.67}}$$

$$= 230.56 \text{ K}$$

Using the values

$$\mathcal{V}_2^2 = -2 \times \left(\frac{1.67}{1.67-1}\right) \times 2078.5 \times [(230.56 - 333)]$$

or $$\mathcal{V}_2^2 = -2 \times 2.4925 \times 2078.5 \times (-102.44)$$

$$= 1061414 \text{ m}^2/\text{s}^2$$

or $$\mathcal{V}_2 = \mathbf{1030.25 \text{ m/s}}$$

Compressor, Pumps

Example 5.10 *Air flows steadily at the rate of 0.5 kg/s through an air compressor, entering at 7 m/s velocity, 100 kPa pressure and 0.95 m^3/kg specific volume and leaving at 5 m/s, 700 kPa and 0.19 m^3/kg, respectively. The internal energy of the air leaving is 90 kJ/kg greater than that of air entering. Cooling water in the compressor jackets absorbs heat from the air at the rate of 58 kW. (a) Compute the power input to the compressor, and (ii) ratio of inlet pipe diameter to outlet pipe diameter.*

Solution

Given An air compressor with $\dot{m} = 0.5$ kg/s of air

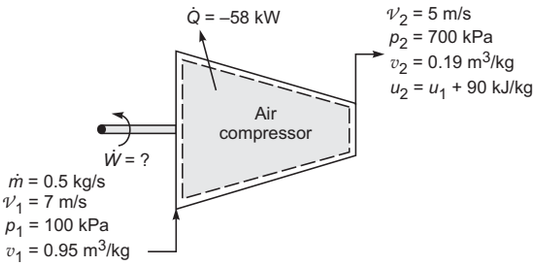

Fig. 5.28 *Schematic*

To find Shaft power input to compressor

Assumptions

(i) No change in potential energy, or $z_1 = z_2$

(ii) Properties of the system remain constant at their locations.

Analysis

(i) The steady-flow energy equation with the above assumptions is given by

$$\dot{Q} - \dot{W} = \dot{m}\left[(u_2 - u_1) + (p_2 v_2 - p_1 v_1) + \left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right)\right]$$

Calculating each term separately;

Change in flow work;

$$p_2 v_2 - p_1 v_1 = (700 \text{ kPa}) \times (0.19 \text{ m}^3/\text{kg}) - (100 \text{ kPa}) \times (0.95 \text{ m}^3/\text{kg}) = 38 \text{ kJ/kg}$$

Change in kinetic energy;

$$\Delta Ke = \frac{\mathcal{V}_2^2}{2} - \frac{\mathcal{V}_1^2}{2} = \frac{5^2 - 7^2}{2} = -12 \text{ J/kg}$$

$$= -0.012 \text{ kJ/kg}$$

Using in the above equation

$$-58 \text{ kW} - \dot{W} = (0.5 \text{ kg/s}) \times [90 \text{ kJ/kg} + 38 \text{ kJ/kg} - 0.012 \text{ kJ/kg}]$$

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or $\dot{W} = -58 - 0.5 \times 127.988$

$= -121.994$ kW

$\approx$ **–122 kW** (input)

(ii) The mass-flow rate can be expressed as

$$\dot{m} = \frac{A_1 \mathcal{V}_1}{v_1} = \frac{A_2 \mathcal{V}_2}{v_2}$$

or $$\frac{A_1}{A_2} = \frac{\pi d_1^2}{\pi d_2^2} = \frac{v_1 \mathcal{V}_2}{v_2 \mathcal{V}_1} = \frac{0.95 \times 5}{0.19 \times 7} = 3.5714$$

$$\frac{d_1}{d_2} = \sqrt{3.5714} = \mathbf{1.89}$$

Example 5.11 *Air at 100 kPa and 280 K is compressed steadily to 600 kPa and 400 K. The mass-flow rate of air is 0.02 kg/s and heat losses of 16 kJ/kg occurs during the process. Assuming the changes in potential and kinetic energies are negligible, determine the necessary power input to the compressor.*

Solution

Given Steady compression of air

$p_1 = 100$ kPa $\quad p_2 = 600$ kPa $\quad T_1 = 280$ K

$T_2 = 400$ K $\quad \dot{m} = 0.02$ kg/s $\quad q = -16$ kJ/kg

$\Delta ke = 0 \quad \Delta pe = 0$

To find Power input to the compressor.

Assumptions

(i) Steady flow process, and

(ii) The specific heat at constant pressure is 1.005 kJ/kg·K.

Analysis Applying steady-flow energy equation for 1 kg of air;

$$q - w = \Delta h + \Delta ke + \Delta pe$$

For air, an ideal gas

$$\Delta h = C_p(T_2 - T_1)$$

$$= 1.005 \times (400 - 280) = 120.6 \text{ kJ/kg}$$

Then $-16 - w = 120.6 + 0 + 0$

or $w = -136.6$ kJ/kg

It is the work input to the compressor.

The power input to compressor

$$p = \dot{m} w = 0.02 \times (-136.6)$$

$$= \mathbf{-2.732\ kW}$$

Example 5.12 *In a test of water-cooled air compressor, it is found that the shaft work required to drive the compressor is 175 kJ/kg of air delivered and the enthalpy of air leaving is 70 kJ/kg greater than that of air entering, and increase in enthalpy of circulating water is 93 kJ/kg. Compute the amount of heat transferred to the atmosphere from the compressor per kg of air.*

Solution

Given Work input, $w = -175$ kJ/kg,

$h_2 - h_1 = 70$ kJ/kg,

$q_{water} = -93$ kJ/kg

(Enthalpy increase of water = heat rejected by system)

To find

(i) The amount of heat transferred to atmosphere

Assumptions

(i) The heat transfer, $q = q_{water} + q_{atmosphere}$,

(ii) $z_1 = z_2$, since no information is provided regarding the potential energy change,

(iii) Neither inlet velocity nor outlet velocity is given, therefore, we assume change in kinetic energy is negligible, i.e., $\mathcal{V}_1 = \mathcal{V}_2$.

Analysis The steady-flow energy equation for unit-mass flow rate

$$q - w = \Delta h + \Delta ke + \Delta pe$$

Using the values

$q - (-175 \text{ kJ/kg}) = (70 \text{ kJ/kg}) + 0 + 0$

or $q = -175 \text{ kJ/kg} + 70 \text{ kJ/kg}$

$= -105$ kJ/kg

Since heat transfer $q = q_{water} + q_{atmosphere}$

$(-105 \text{ kJ/kg}) = (-93 \text{ kJ/kg}) + q_{atmosphere}$

or $q_{atmosphere} = -105 + 93 = \mathbf{-12\ kJ/kg}$

Example 5.13 *A centrifugal air compressor compresses 5.7 m^3/min of air from 85 kPa, 0.35 m^3/kg to 650 kPa, 0.1 m^3/kg. If the suction-line diameter is 10 cm and the discharge line diameter is 6.25 cm, determine (i) the mass flow rate of fluid in kg/min, (ii) the change in*

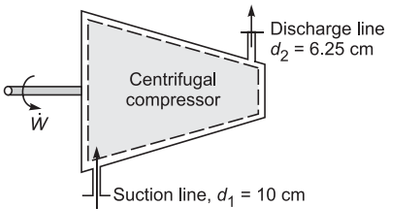

Fig. 5.29 *Schematic*

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flow work between the boundaries, and (iii) the inlet and outlet velocities.

Solution

Given A centrifugal compressor with

$\dot{V} = 5.7 \text{ m}^3/\text{min}$,
$v_1 = 0.35 \text{ m}^3/\text{kg}$,
$p_1 = 85 \text{ kPa} = 85 \times 10^3 \text{ N/m}^2$,
$v_2 = 0.1 \text{ m}^3/\text{kg}$,
$p_2 = 650 \text{ kPa} = 650 \times 10^3 \text{ N/m}^2$,
$d_1 = 10 \text{ cm} = 0.1 \text{ m}$
$d_2 = 6.25 \text{ cm} = 0.0625 \text{ m}$,

To find

(i) Mass-flow rate of fluid in kg/min,
(ii) Change in flow work between the boundaries,
(iii) Inlet and exit velocities.

Assumptions

(i) No change in potential energy, or $z_1 = z_2$,
(ii) No heat transfer at system boundary,
(iii) Properties of the system remains constant at their locations.

Analysis

(i) *The mass-flow rate of air*

$$\dot{m} = \frac{\dot{V}}{v_1} = \frac{5.7 \text{ m}^3/\text{min}}{0.35 \text{ m}^3/\text{kg}}$$
$$= \mathbf{16.3 \text{ kg/min} = 0.271 \text{ kg/s}}$$

(ii) *The change in flow work*

$$\Delta(pV) = \dot{m}(p_2 v_2 - p_1 v_2)$$
$$= 16.3 \times (650 \times 0.1 - 85 \times 0.35)$$
$$= 574 \text{ kJ/min} = \mathbf{9.57 \text{ kW}}$$

(iii) *Inlet and exit velocities*

These velocities can be calculated from the continuity equation as

$$\dot{m} = \frac{A_1 \mathcal{V}_1}{v_1} = \frac{A_2 \mathcal{V}_2}{v_2}$$

or
$$\mathcal{V}_1 = \frac{\dot{m} v_1}{A_1} = \frac{\dot{m} v_1}{(\pi/4) d_1^2}$$
$$= \frac{(0.271 \text{ kg/s}) \times (0.35 \text{ m}^3/\text{kg})}{(\pi/4) \times (0.1 \text{ m})^2} = \mathbf{12 \text{ m/s}}$$

and
$$\mathcal{V}_2 = \frac{\dot{m} v_2}{(\pi/4) d_2^2} = \frac{(0.271 \text{ kg/s}) \times (0.1 \text{ m}^3/\text{kg})}{(\pi/4) \times (0.0625 \text{ m})^2}$$
$$= \mathbf{8.83 \text{ m/s}}$$

Example 5.14 *Air is compressed steadily at a rate of 0.46 kg/s from 100 kPa, 20°C to a final pressure of 320 kPa. The compression is polytropic with a polytropic index of 1.32. The volume of air changes from 3 m^3/kg to 0.8 m^3/kg. The inlet velocity is 25 m/s, while exit velocity is 130 m/s. The delivery connection is 12 m above the inlet. What is the shaft power of the compressor? Is it a power-absorbing or power-producing device?*

Solution

Given An air compressor with mass flow rate, $\dot{m}$ = 0.46 kg/s, of air

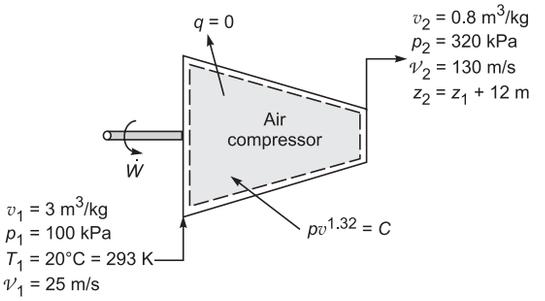

Fig. 5.30 *Schematic diagram*

To find Shaft power input to compressor.

Assumptions

(i) No heat transfer at system boundary.
(ii) Properties of the system remain constant at their locations.
(iii) Acceleration due to gravity is 9.81 m/s².

Analysis The relation for steady work is expressed as

$$w = -\int v\,dp - \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} - (z_2 - z_1) g$$

The p and v are related as $pv^n = C$

or
$$v = \frac{C^{1/n}}{p^{1/n}}$$

we get
$$w = -n\left(\frac{p_2 v_2 - p_1 v_1}{n-1}\right) - \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} - (z_2 - z_1) g$$

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Using the values with proper care of units

$$w = -1.32 \times \left[\frac{320 \times 0.8 - 100 \times 3}{1.32 - 1}\right] - \frac{130^2 - 25^2}{2 \times 1000} - \frac{12 \times 9.81}{1000}$$

The kinetic energy and potential energy changes are divided by 1000 to get each quantity in kJ/kg.

$$w = 181.5 - 8.1375 - 0.117$$
$$= 173.25 \text{ kJ/kg}$$

The power input to the compressor

$$\dot{W} = \dot{m} w$$
$$= (0.46 \text{ kg/s}) \times (173.25 \text{ kJ/kg})$$
$$= \mathbf{79.7 \text{ kW}}$$

Example 5.15 *Water at the rate of 10 kg/s is compressed adiabatically from 5 bar to 50 bar in a steady flow process. Calculate the power required, assuming that the specific volume of water to be 0.001 m^3/kg, which remains almost constant.*

Solution

Given Compression of water

$\dot{m}$ = 10 kg/s $\quad p_1$ = 5 bar = 500 kPa

p_2 = 50 bar = 5000 kPa $\quad Q = 0$

v = 0.001 m^3/kg

To find Power input to pump

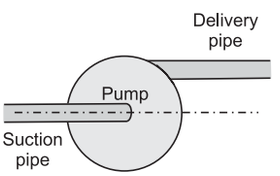

Fig. 5.31

Assumption

(i) Change in potential energy, $\Delta pe = 0$,

(ii) Change in kinetic energy $\Delta ke = 0$,

Analysis In a steady flow process, the work transfer can be obtained as

$$w = -\int v\,dp - \Delta ke - \Delta pe$$

using assumption, then

$$w = -\int v\,dp = -v\int dp$$

During compression of water, the specific volume remains constant, therefore, total power input

$$\dot{W} = \dot{m} v \int_{p_1}^{p_2} dp = -\dot{m} v (p_2 - p_1)$$
$$= -(10 \text{ kg/s}) \times (0.001 \text{ m}^3/\text{kg}) \times (5000 - 500) \text{ (kPa)}$$
$$= -\mathbf{45 \text{ kW}} \text{ (input power)}$$

Example 5.16 *In a water-cooled compressor, 0.6 kg of air is compressed per second. The power required to run the compressor is 40 kW. The heat lost to the cooling water is 30% of input, and 10% of input is lost in bearing and other frictional effects. The air enters the compressor at 1 bar and 30°C. If the changes in potential energy and kinetic energy are neglected, estimate the exit temperature of air. Take C_p for air as 1.005 kJ/kg·K.*

Solution

Given A water-cooled compressor;

$\dot{m}$ = 0.6 kg/s $\quad \dot{W}$ = – 40 kW (work input)

p_1 = 1 bar = 100 kPa $\quad T_1$ = 30°C = 303 K

The total losses of input may be considered as

$$\dot{Q} = (0.3 + 0.1) \text{ of input} = 0.4 \times 40$$
$$= -16 \text{ kW (losses)},$$
$$C_p = 1.005 \text{ kJ/kg·K}$$

To find Exit temperature of air

Assumptions

(i) No change in potential energy, and

(ii) No change in kinetic energy.

Analysis The steady-flow energy equation;

$$\dot{Q} - \dot{W} = \dot{m}(\Delta h + \Delta ke + \Delta pe)$$

Dropping out, Δke and Δpe, and using the values

$$(-16 \text{ kW}) - (-40 \text{ kW}) = (0.6 \text{ kg/s}) \times (\Delta h)$$

we get $\quad \Delta h = 40 \text{ kJ/kg}$

The change in specific enthalpy can be expressed as

$$\Delta h = C_p(T_2 - T_1)$$

or $\quad 40 = 1.005 \times (T_2 - 303)$

or $\quad T_2 = 342.8 \text{ K} = \mathbf{69.8°C}$

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Example 5.17 *A centrifugal pump delivers 50 kg of water per second. The inlet and outlet pressures are 1 bar and 4.2 bar respectively. The suction is 2.2 m below the centre of the pump and delivery is 8.5 m above the centre of the pump. The suction and delivery pipe diameters are 20 cm and 10 cm, respectively. Determine the capacity of electric motor to run the pump.*

Solution

Given A centrifugal water pump with flow rate

$\dot{m} = 50$ kg/s

$d_1 = 20$ cm $= 0.2$ m

$d_2 = 10$ cm $= 0.1$ m

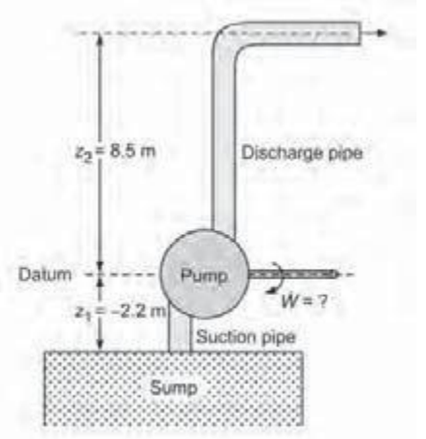
$p_1 = 1$ bar $= 1 \times 10^5$ N/m^2,

$p_2 = 4.2$ bar $= 4.2 \times 10^5$ N/m^2

$z_1 = 8.5$ m

$z_2 = -2.2$ m

To find The power input to pump



Assumptions

(i) No temperature change of water during pumping, thus $\Delta u \approx 0$.

(ii) No heat transfer during the pumping.

(iii) Specific volume of water is constant and is 0.001 m³/kg.

(iv) Acceleration due to gravity, $g = 9.81$ m/s.

Analysis The steady-flow energy equation is given by

$$\dot{Q} - \dot{W} = \dot{m}(\Delta u + \Delta(pv) + \Delta ke + \Delta pe)$$

The flow rate of water can be expressed as

$$\dot{V} = \dot{m}v = (50 \text{ kg/s}) \times (0.001 \text{ m}^3/\text{kg}) = 0.05 \text{ m}^3/\text{s}$$

and $$\dot{V} = \mathcal{V}_1 A_1 = \mathcal{V}_2 A_2$$

$$\mathcal{V}_1 = \frac{\dot{V}}{A_1} = \frac{\dot{V}}{(\pi/4)\, d_1^2} = \frac{(0.05 \text{ m}^3/\text{s}) \times 4}{\pi \times (0.2 \text{ m})^2}$$

$$= 1.59 \text{ m/s}$$

or $$\mathcal{V}_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{(\pi/4) d_2^2} = \frac{(0.05 \text{ m}^3/\text{s}) \times 4}{\pi \times (0.1 \text{ m})^2}$$

$$= 6.37 \text{ m/s}$$

Calculating each term of the steady-flow energy equation separately:

(i) Since the temperature remains constant during pumping, therefore, the specific internal energy change,

$$\Delta u = 0$$

(ii) The change in kinetic energy,

$$\Delta ke = \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} = \frac{6.37^2 - 1.59^2}{2}$$

$$= 19.02 \text{ J/kg} = 0.019 \text{ kJ/kg}$$

(iii) Change in potential energy,

$$\Delta pe = g(z_2 - z_1)$$

$$= (9.81 \text{m/s}^2) \times [8.5 \text{ m} - (-2.2 \text{ m})]$$

$$= 105 \text{ J/kg} = 0.105 \text{ kJ/kg}$$

(iv) Change in flow work,

$$\Delta(pv) = p_2 v_2 - p_1 v_1 = v(p_2 - p_1)$$

$$= (0.001 \text{ m}^3/\text{kg}) \times (420 \text{ kPa} - 100 \text{ kPa})$$

$$= 0.32 \text{ kJ/kg}$$

Substituting the values of each term in the equation we get,

$$0 - \dot{W} = 50 \times (0 + 0.32 + 0.019 + 0.105)$$

or $$\dot{W} = -\mathbf{22.2 \text{ kW}}$$

Turbine

Example 5.18 *Steam enters a turbine with a velocity of 40 m/s and specific enthalpy of 2500 kJ/kg; and leaves with a velocity of 90 m/s and specific enthalpy of 2030 kJ/kg. Heat losses from the turbine to surroundings are 240 kJ/min and the steam-flow rate is 5040 kg/h. Neglect the change of potential energy. Find the power developed by the turbine.*

Solution

Given A steam turbine as shown in Fig. 5.33.

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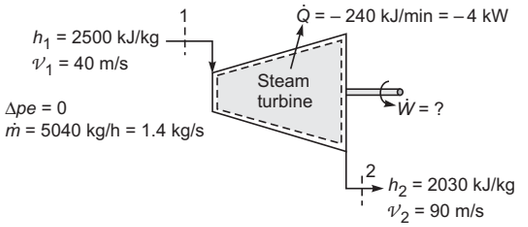

Fig. 5.33 *Schematic for steam turbine*

To find Power developed by the turbine.

Analysis The steady-flow energy equation is

$$\dot{Q} - \dot{W} = \dot{m}(\Delta h + \Delta ke + \Delta pe)$$

Calculating each term of the steady-flow energy equation separately

(i) Change in specific enthalpy,

$$\Delta h = h_2 - h_1 = 2030 - 2500$$
$$= -470 \text{ kJ/kg}$$

(ii) Change in kinetic energy,

$$\Delta ke = \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2}$$
$$= \frac{90^2 - 40^2}{2} = 3250 \text{ J/kg}$$
$$= 3.25 \text{ kJ/kg}$$

Substituting the values of each term in the equation, we get,

$$(-4 \text{ kW}) - \dot{W} = (1.4 \text{ kg/s}) \times (-470 \text{ kJ/kg} + 3.25 \text{ kJ/kg} + 0)$$

or $$\dot{W} = 653.45 - 4 = \mathbf{649.45 \text{ kW}}$$

Example 5.19 *The gas turbine of a turbojet engine receives a steady flow of gases at a pressure of 7.2 bar, a temperature of 850°C and a velocity of 160 m/s. It discharges the gases at a pressure of 1.15 bar, a temperature of 450°C, and a velocity of 250 m/s. Determine the external work output of the turbine in kJ/kg of the gas flow. The process may be assumed to be adiabatic and C_p for combustion gases may be taken as 1.04 kJ/kg·K.*

Solution

Given A gas turbine of a turbojet engine as shown in Fig. 5.34.

To find The work developed by the turbine in kJ/kg.

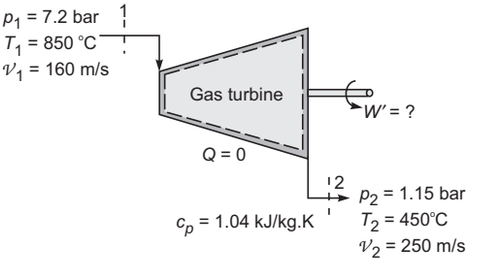

Fig. 5.34 *Schematic*

Assumptions

(i) No change in potential energy, or $\Delta pe = 0$

(ii) Properties of the system remains constant at their locations

Analysis The steady-flow energy equation is

$$q - w = \Delta h + \Delta ke + \Delta pe$$

Calculating each term of the steady-flow energy equation separately

Change in specific enthalpy,

$$\Delta h = C_p(T_2 - T_1)$$
$$= (1.04) \times (450 - 850) = -416 \text{ kJ/kg}$$

Change in kinetic energy,

$$\Delta ke = \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} = \frac{(250)^2 - (160)^2}{2}$$
$$= 18450 \text{ J/kg} = 18.45 \text{ kJ/kg}$$

Substituting the values of these terms in steady-flow energy equation, we get

$$0 - w = (-416 \text{ kJ/kg}) + (18.45 \text{ kJ/kg}) + 0$$

or $$w = \mathbf{397.55 \text{ kJ/kg}}$$

Example 5.20 *A hydraulic turbine is supplied with 25 m^3/s of water. At the turbine inlet, the water is at 5 bar, and 25°C with an elevation above datum of 100 metres and a flow velocity of 1 m/s. At the turbine exit, the water is at 1.2 bar and 25.1°C with zero elevation and a flow velocity of 11 m/s. The turbine loses 5 J of heat per kg of water flowing through it. Assuming that the water is an incompressible fluid with specific heat of 4.178 kJ/kg·K, determine*

(a) the change in potential energy,

(b) the change in internal energy, and

(c) the power output in MW.

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Solution

Given A hydraulic turbine is shown in Fig. 5.35.

$\dot{V} = 25\ m^3/s$ $\quad p_1 = 5$ bar

$p_2 = 1.2$ bar $\quad T_1 = 25°C$

$\mathcal{V}_1 = 1$ m/s $\quad T_2 = 25.1°C$

$z_1 = 100$ m $\quad z_2 = 0$

$\mathcal{V}_2 = 11$ m/s $\quad C_p = 4.178$ kJ/kg·K

$q = -5$ J/kg

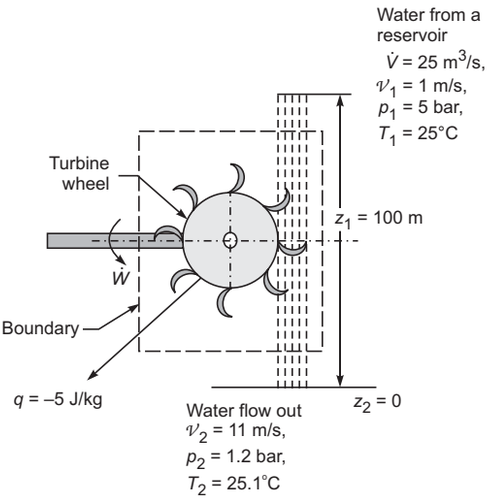

Fig. 5.35 *Schematic of hydraulic turbine*

To find

(i) The change in potential energy,

(ii) The change in internal energy, and

(iii) Power developed by the turbine in MW.

Analysis The mass-flow rate of water

$$\dot{m} = \text{Volume} \times \text{density}$$
$$= 25\ m^3/s \times 1000\ kg/m^3$$
$$= 25000\ kg/s$$

(i) The change in potential energy,

$$\Delta PE = \dot{m} g (z_2 - z_1)$$
$$= (25000) \times (9.81) \times (0 - 100)$$
$$= -24.525 \times 10^6\ J/s$$
$$= \mathbf{-24.525 \times 10^3\ kW}$$

(ii) The change in internal energy

$$\Delta U = \Delta H = \dot{m} C_p \Delta T$$
$$= (25000) \times (4.178) \times (25.1 - 25)$$
$$= \mathbf{10445\ kW}$$

(Water has a single specific heat, which is given as 4.178 kJ/kg·K.

(iii) Change in kinetic energy,

$$\Delta KE = \dot{m} \times \left(\frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} \right)$$
$$= (25000\ kg/s) \times \frac{(11m)^2 - (1\ m)^2}{2}$$
$$= 15 \times 10^5\ J/s = 1500\ kW$$

The total heat-loss rate

$$\dot{Q} = \dot{m} q = (25000\ kg/s) \times (-5\ J/kg)$$
$$= -125 \times 10^3\ J/s = -125\ kW$$

The steady-flow energy equation is given by

$$\dot{Q} - \dot{W} = \dot{m} (\Delta h + \Delta ke + \Delta pe)$$

Substituting the values of each term in the equation, we get,

$$-125\ kW - \dot{W} = 10445\ kW + 1500\ kW - 24.525 \times 10^3\ kW$$

or $\dot{W} = \mathbf{12445\ kW\ or\ 12.445\ MW}$

Example 5.21 *Steam initially at 1.5 MPa and 300°C expands reversibly and adiabatically in a steam turbine to 40°C. Determine the*

(a) condition of steam after expansion,

(b) work done by the turbine per kg of steam.

Solution

Given Isentropic expansion of steam;

$p_1 = 1.5$ MPa $\quad T_1 = 300°C$

$Q = 0$ $\quad T_2 = 40°C$

To find

(i) Condition of steam after expansion,

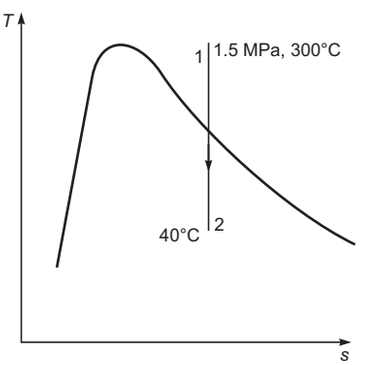

Fig. 5.36

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(ii) Turbine work output.

Assumptions

(i) The kinetic energy change is negligible.
(ii) The potential energy change is negligible.
(iii) No heat transfer occurs during expansion.

Properties of steam At 1.5 MPa (1500 kPa) and 300°C:

$h_1 = 3038.9$ kJ/kg $\quad s_1 = 6.92$ kJ/kg·K

At 40°C $p_2 = 7.3$ kPa $\quad s_{f_2} = 0.572$ kJ/kg·K

$s_{g_2} = 8.257$ kJ/kg·K $\quad h_{f_2} = 167.57$ kJ/kg

$h_{fg_2} = 2406.7$ kJ/kg

Analysis

(i) From observation of the magnitude of entropy values at the two states, we find that the steam should be wet at exhaust. For isentropic expansion,

$$s_1 = s_2 = s_{f_2} + x(s_{g_2} - s_{f_2})$$

$$6.92 = 0.572 + x(8.257 - 0.572)$$

or $$x = \frac{(6.92 - 0.572)}{7.685} = \mathbf{0.826}$$

(ii) Steady-flow work per kg of steam,

$$q - w = \Delta h + \Delta ke + \Delta pe$$

Substituting the values in the above equation, we get

$$w = h_1 - h_2$$

Where $h_2 = h_{f_2} + x h_{fg_2}$

$= 167.57 + 0.826 \times 2406.7$

$= 2155.5$ kJ/kg

and $w = h_1 - h_2 = 3038.9 - 2155.5$

$= \mathbf{883.4\ kJ/kg}$

Example 5.22 *A turbine operating on air has inlet conditions of 10 bar, 750 K, and 200 m/s, while exit conditions are 1.25 bar and 40 m/s. The mass-flow rate of air is 1000 kg/h. The flow of air is assumed to be reversible adiabatic. Calculate*

(a) the temperature of air at exit, and

(b) the power output of the turbine.

Assume C_p = 1.053 kJ/kg·K and adiabatic index = 1.375.

Solution

Given A turbine with operating data as shown in Fig. 5.37.

$\dot{m} = 1000$ kg/h = 0.278 kg/s

$C_p = 1.053$ kJ/kg·K,

$\gamma = 1.375$

To find

(i) The temperature of air at exit,
(ii) The power output of the turbine.

Assumptions

(i) No change in potential energy, or $\Delta pe = 0$,
(ii) Properties of the system remains constant at their locations.

Analysis

(i) The exit temperature can be calculated by the using property relationship for reversible adiabatic process;

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{(\gamma-1)}{\gamma}}$$

or $$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{(\gamma-1)}{\gamma}}$$

$$= 750 \times \left(\frac{1.25}{10}\right)^{\frac{(1.375-1)}{1.375}}$$

$$= 750 \times 0.567 = \mathbf{425.3\ K}$$

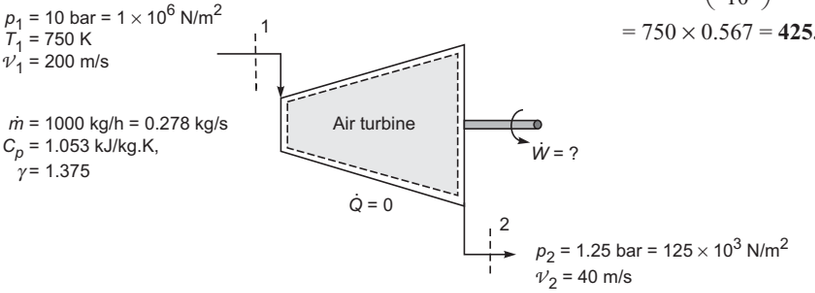

Fig. 5.37 *Schematic*

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(ii) The steady-flow energy equation is

$$\dot{Q} - \dot{W} = \dot{m}(\Delta h + \Delta ke + \Delta pe)$$

Calculating each term of steady-flow energy equation separately:

Change in specific enthalpy,

$$\Delta h = C_p(T_2 - T_1)$$
$$= (1.053) \times (425.3 - 750)$$
$$= -341.83 \text{ kJ/kg}$$

Change in kinetic energy,

$$\Delta ke = \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} = \frac{40^2 - 200^2}{2}$$
$$= -19200 \text{ J/kg}$$
$$= -19.2 \text{ kJ/kg}$$

Substituting the values of these terms in the steady-flow energy equation, we get,

$$0 - \dot{W} = (0.278 \text{ kg/s}) \times [(-341.83 \text{ kJ/kg}) + (-19.2 \text{ kJ/kg}) + 0]$$

or $\dot{W} = \mathbf{100.3\ kW}$

Example 5.23 *A steam turbine receives steam from two boilers One flow is 5 kg/s at 3 MPa, 700°C and the other flow is 15 kg/s at 800 kPa, 500°C. The exit state is 10 kPa, with a quality of 96%. Find the total power output of the adiabatic turbine.*

Solution

Given A steam turbine receiving steam from two boilers:

Boiler 1: $\dot{m}_1 = 5$ kg/s $p_1 = 3$ MPa $T_1 = 700°C$
Boiler 2: $\dot{m}_2 = 15$ kg/s $p_2 = 800$ kPa $T_2 = 500°C$
Exit : $Q = 0$ $p_3 = 10$ kPa $x_3 = 0.96$

To find Power output of the turbine

Assumptions

(i) Steady flow through the turbine,
(ii) Changes in kinetic and potential energies are negligible.

Properties of steam

At 3 MPa or 3000 kPa and 700°C;

$v_1 = 0.14838 \text{ m}^3/\text{kg}$, $h_1 = 3911.72$ kJ/kg

At 800 kPa and 500°C;

$v_2 = 0.44331 \text{ m}^3/\text{kg}$, $h_2 = 3480.6$ kJ/kg

At 10 kPa;

$v_{g_3} = 14.673 \text{ m}^3/\text{kg}$, $h_{f_3} = 191.81$ kJ/kg,
$h_{fg_3} = 2392.82$ kJ/kg, $h_{g_3} = 2584.63$ kJ/kg

Analysis The mass rate of steam leaving the turbine

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 5 + 15 = 20 \text{ kg/s}$$

Total enthalpy of steam coming from the boiler 1;

$$H_1 = \dot{m}_1 h_1 = (5 \text{ kg/s}) \times (3911.72 \text{ kJ/kg})$$
$$= 19558.6 \text{ kJ/s}$$

Total enthalpy of steam coming out of the boiler 2;

$$H_2 = \dot{m}_2 h_2 = (15 \text{ kg/s}) \times (3480.6 \text{ kJ/kg})$$
$$= 52209 \text{ kJ/s}$$

Total enthalpy of steam leaving the turbine

$$H_3 = \dot{m}_3 h_3 = \dot{m}_3 (h_{f_3} + x_3 h_{fg_3})$$
$$= (20 \text{ kg/s}) \times (191.81 + 0.96 \times 2392.82) \text{ (kJ/kg)}$$
$$= 49778.3 \text{ kJ/s}$$

Applying steady-flow energy equation

$$\dot{Q} - \dot{W} = \Delta H + \Delta KE + \Delta PE$$

Calculating each item separately

Given $\dot{Q} = 0$; $\Delta KE = 0$ and $\Delta PE = 0$

Then ΔH = Exit enthalpy – Inlet enthalpy
$= H_3 - (H_1 + H_2)$
$= 49778.3 - (19558.6 + 52209)$
$= -21989.25$ kJ/s

Then $-\dot{W} = \Delta H = -21989.25$ kW

or $\dot{W} = 21989.25$ kW = **21.99 MW**

Example 5.24 *The mass-flow rate of steam into a steam turbine is 1.5 kg/s and the heat transfer from the turbine is 8.5 kW. The steam is entering in the turbine at the pressure of 2 MPa, temperature of 350°C, velocity of 50 m/s, elevation of 6 m and is leaving the turbine at a pressure of 0.1 MPa, quality of 100 %, velocity of 200 m/s, elevation of 3 m. Determine the power output of the turbine.*

Solution

Given Flow through a turbine

$\dot{m} = 1.5$ kg/s, $\dot{Q} = -8.5$ kW
$p_1 = 2$ MPa, $p_2 = 0.1$ MPa
$T_1 = 350°C$ $x_2 = 1.0$
$\mathcal{V}_1 = 50$ m/s $\mathcal{V}_2 = 200$ m/s
$z_1 = 6$ m $z_2 = 3$m

To find Power output of the turbine.

Assumptions

(i) Steady flow condition,
(ii) Acceleration due to gravity as.

$$g = 9.81 \text{ m/s}^2$$

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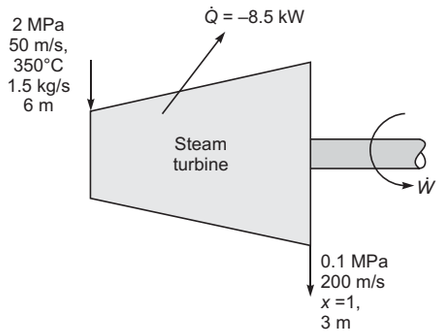

Fig. 5.38

Analysis Properties of steam:

State 1: 2MPa, 350°C, (From superheated steam table)

$$h_1 = 3136.96 \text{ kJ/kg}$$

State 2: 0.1 MPa, (From pressure entry steam table)

$$x = 1.0 \qquad h_2 = 2675.45$$

Using steady-flow energy equation.

$$\dot{Q} - \dot{W} = \dot{m}(\Delta h + \Delta ke + \Delta pe)$$

Calculating each quantity separately

$$\Delta h = h_2 - h_1 = 2675.45 - 3136.96$$
$$= -461.51 \text{ kJ/kg}$$

$$\Delta ke = \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} = \frac{(200)^2 - (50)^2}{2}$$
$$= 18750 \text{ J/kg} = 18.75 \text{ kJ/kg}$$

$$\Delta pe = (z_2 - z_1)\, g = (3 - 6) \times 9.81$$
$$= -29.43 \text{ J/kg} = -0.0294 \text{ kJ/kg}$$

Using these values in the above equation;

$$-8.5 - \dot{W} = 1.5 \times [-461.51 + 18.75 - 0.0294]$$

or $$-\dot{W} = 8.5 + 1.5 \times (-442.78)$$

or $$\dot{W} = \mathbf{655.68 \text{ kW}}$$

Example 5.25 *Steam initially at a pressure of 25 bar and 0.9 dry expands isentropically to 15 bar and it is then throttled until it is just dry. Find*

(a) the quality of steam after isentropic expansion and adiabatic heat drop,

(b) the pressure after throttling, increase in entropy during throttling,

Solution

Given Isentropic expansion and throttling of steam

$p_1 = 25$ bar $\qquad x_1 = 0.9$ dry

$p_2 = 15$ bar, $\qquad x_3 = 1$

To Find

(i) Dryness fraction of steam after isentropic expansion and adiabatic heat drop, and

(ii) Pressure after throttling and increase in entropy during throttling.

Analysis Properties of steam

At pressure 25 bar, $h_{f_1} = 962$ kJ/kg

$h_{fg_1} = 1839$ kJ/kg

$s_{f_1} = 2.5543$ kJ/kg·K

$s_{g_1} = 6.2536$ kJ/kg·K

At pressure 15 bar, $h_{f_2} = 844.75$ kJ/kg

$h_{fg_2} = 1947.2$ kJ/kg

$s_{f_2} = 2.3145$ kJ/kg·K

$s_{g_2} = 6.4406$ kJ/kg·K

(i) *The dryness fraction of steam after isentropic expansion,*

$$s_1 = s_2$$
$$s_{f_1} + x(s_{g_1} - s_{f_1}) = s_{f_2} + x_2(s_{g_2} - s_{f_2})$$
$$2.5543 + 0.9 \times (6.2536 - 2.5543)$$
$$= 2.3145 + x_2\,(6.4406 - 2.3145)$$

or $$x_2 = \frac{5.8848 - 2.3145}{4.1256} = \mathbf{0.865}$$

Enthalpy drop during isentropic expansion

$$\Delta h = h_1 - h_2 = (h_{f_1} + x_1 h_{fg_1}) - (h_{f_2} + x_2 h_{fg_2})$$
$$= (962 + 0.9 \times 1839) - (844.75 + 0.865 \times 1947.2)$$
$$= \mathbf{88.0 \text{ kJ/kg}}$$

(ii) *The pressure after throttling*

Since $h_2 = h_3$

The enthalpy of steam after throttling

$$h_3 = h_2 = 844.75 + 0.865 \times 1947.2$$
$$= 2529.08 \text{ kJ/kg}$$

Pressure corresponds to this enthalpy when steam is just dry.

$$= \mathbf{0.016 \text{ bar}}$$

Entropy at this pressure

$$s_3 = 8.834 \text{ kJ/kg·K}$$

Change in entropy

$$= s_3 - s_2 = s_3 - [s_{f_2} + x_2(s_{g_2} - s_{f_2})]$$
$$= 8.834 - [2.3145 + 0.865\,(6.4406 - 2.3145]$$
$$= \mathbf{2.945 \text{ kJ/kg K}}$$

Example 5.26 *The power output of a steam turbine is 5 MW. The inlet conditions are 2 MPa of pressure, 400°C*

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of temperature, 50 m/s of velocity and 10 m of elevation. The exit conditions are 15 kPa, 0.9 dry quality, 180 m/s, and 6 m elevation.

(a) Compute the magnitude of Δh, Δke, and Δpe.

(b) Determine the work done per kg mass of steam.

(c) Calculate the mass flow rate of steam.

Solution

Given Steam turbine $\dot{W} = 5$ MW

$p_1 = 2$ MPa $= 200$ kPa $\quad \mathcal{V}_1 = 50$ m/s

$T_1 = 400°C + 273 = 673$ K, $\quad z_1 = 10$ m

$p_2 = 15$ kPa $\quad x_2 = 0.9$

$\mathcal{V}_2 = 180$ m/s $\quad z_2 = 6$ m

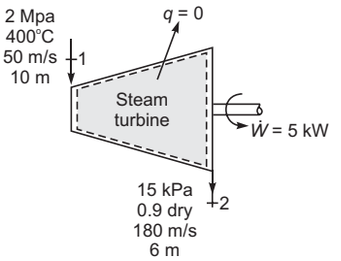

Fig. 5.39

To Find

(i) The magnitude of Δh, Δke and Δpe,

(ii) Work done per kg mass of steam,

(iii) The mass flow rate of steam.

Assumptions

(i) Steady-state conditions through the turbine,

(ii) No heat loss from the boundary of turbine, ($q = 0$).

Analysis

(i) At the inlet conditions, 2 MPa and 400°C:

$$h_1 = 3247.6 \text{ kJ/kg}$$

At the turbine exit, at 15 kPa:

$$h_f = 225.94 \text{ kJ/kg}$$

$$h_{fg} = 2373.1 \text{ kJ/kg}$$

$$h_2 = h_{f_2} + x h_{fg_2}$$

$$= 225.94 + 0.9 \times 2373.1$$

$$= 2361.73 \text{ kJ/kg}$$

Now, $\Delta h = h_2 - h_1$

$$= 2361.73 - 3247.6$$

$$= -\mathbf{885.87 \text{ kJ/kg}}$$

$$\Delta ke = \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2}$$

$$= \frac{(180 \text{ m/s})^2 - (50 \text{ m/s})^2}{2}$$

$$= 14950 \text{ J/kg} = \mathbf{14.95 \text{ kJ/kg}}$$

and $\Delta pe = g(z_2 - z_1)$

$$= (9.81 \text{ m/s}^2) \times (6 \text{ m} - 10 \text{ m})$$

$$= -39.24 \text{ J/kg} = -\mathbf{0.0392 \text{ kJ/kg}}$$

(ii) Work done by the turbine per kg of steam

Applying steady-flow energy equation for unit mass

$$q - w = \Delta h + \Delta ke + \Delta pe$$

$$0 - w = -885.87 + 14.95 - 0.0392$$

or $w = \mathbf{870.95 \text{ kJ/kg}}$

(iii) The required mass-flow rate for a 5-MW power output

$$\dot{m} = \frac{\dot{W}}{w} = \frac{5 \times 1000}{870.95} = \mathbf{5.74 \text{ kg/s}}$$

Example 5.27 *A turbine operating on a steady flow of air is to produce 1 kW of power by expanding air from 300 kPa, 350 K, 0.346 m^3/kg to 120 kPa. For preliminary design, the inlet velocity is assumed to be 30 m/s, the exit velocity is assumed to be 50 m/s and the expansion follows the law $pv^{1.4}$ = constant. Determine the required mass-flow rate of air.*

Solution

Given An air turbine as shown in Fig. 5.40.

To find Mass-flow rate of air.

Assumptions

(i) No change in potential energy, or $z_1 = z_2$,

(ii) No heat transfer from the turbine,

(iii) Properties of the system remains constant at their locations.

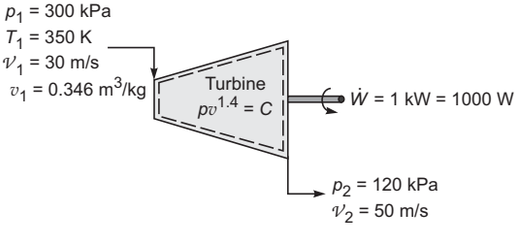

Fig. 5.40 *Schematic diagram of air turbine*

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Analysis The steady-flow energy equation for analysis is given by

$$q - w = h_2 - h_1 + \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} + g(z_2 - z_1)$$

where $q = 0$ and change in potential energy is also negligible and then equation reduces to

$$-w = h_2 - h_1 + \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2}$$

If the work per unit mass is determined, the mass flow rate can be calculated as

$$\dot{W} = \dot{m} w$$

But with this equation also, we cannot proceed for calculation of exit velocity $\mathcal{V}_2$, because $h_2 - h_1$ is also unknown and cannot be directly calculated. Thus using the equation

$$w = -\int v dp - \Delta ke - \Delta pe$$

Dropping potential energy change

$$w = -\int v dp - \Delta ke$$

p and v are related as $pv^n = C$

or $$v = \frac{C^{1/n}}{p^{1/n}}$$

Then $$w = -C^{1/n}\int_1^2 p^{-1/n} dp - \Delta ke$$

$$= -C^{1/n}\left[\frac{p_2^{(-1/n)+1} - p_1^{(-1/n)+1}}{(-1/n)+1}\right] - \Delta ke$$

Using $C^{1/n} = p_1^{1/n} v_1 = p_2^{1/n} v_2$

We get $$w = -n\left(\frac{p_2 v_2 - p_1 v_1}{n-1}\right) - \Delta ke$$

Using $$\frac{v_2}{v_1} = \left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}}$$

We get $$w = -\left(\frac{n}{n-1}\right) p_1 v_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right] - \Delta ke$$

Using the values

$$w = -\left(\frac{1.4}{1.4-1}\right) \times 300 \times 10^3 \times 0.346 \left[\left(\frac{120}{300}\right)^{0.4/1.4} - 1\right] - \frac{50^2 - 30^2}{2}$$

$$w = 82953.18 \text{ J/kg} \cong 83 \text{ kJ/kg}$$

The mass-flow rate can be obtained as

$$\dot{m} = \frac{\dot{W}}{w} = \frac{1 \text{ kW}}{83 \text{ kJ/kg}} = 0.012 \text{ kg/s} = \mathbf{43.3 \text{ kg/h}}$$

Example 5.28 *Consider a gas turbine power plant with air as the working fluid. Air enters at 100 kPa, 20°C, $\rho_1 = 1.19$ kg/m^3 with a velocity of 130 m/s through an opening of 0.112 m^2 cross-sectional area. After being compressed, heated and expanded through the turbine, the air leaves at 180 kPa, 150°C, $\rho_2 = 1.48$ kg/m^3, through an opening of the same size. The power output of the plant is 375 kW. The internal energy and enthalpy of air are given in kJ/kg by u = 0.717 T and h = 1.004 T, where T is the temperature on kelvin scale. Determine the net amount of heat transferred to air in kJ/kg.*

Solution

Given A gas turbine of a power plant

$A = 0.112 \text{ m}^2$

$u = 0.717\ T$ (kJ/kg)

and $h = 1.004\ T$ (kJ/kg)

To find Heat transferred per kg.

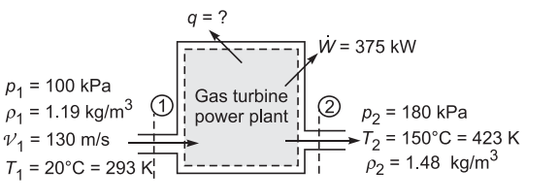

Fig. 5.41

Assumptions

(i) No change in potential energy, or $z_1 = z_2$.

(ii) Properties of the system remains constant at their locations.

Analysis The steady-flow energy equation for unit mass flow rate;

$$q - w = h_2 - h_1 + \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} + g(z_2 - z_1)$$

The mass-flow rate

$$\dot{m} = \rho_1 \mathcal{V}_1 A_1$$

$$= (1.19 \text{ kg/m}^3) \times (130 \text{ m/s}) \times (0.112 \text{ m}^2)$$

$$= 17.326 \text{ kg/s}$$

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The exit velocity of fluid

$$\mathcal{V}_2 = \frac{\dot{m}}{\rho_2 A_2} = \frac{(17.326 \text{ kg/s})}{(1.48 \text{ kg/m}^3) \times (0.112 \text{ m}^2)}$$

$$= 104.5 \text{ m/s}$$

The work done per unit mass;

$$w = \frac{\dot{W}}{\dot{m}} = \frac{375 \text{ kW}}{17.326 \text{ kg/s}} = 21.64 \text{ kJ/kg}$$

The specific change in enthalpy;

$$h_2 - h_1 = C_p (T_2 - T_1)$$
$$= (1.004 \text{ kJ/kg} \cdot \text{K}) \times (423 \text{ K} - 293 \text{ K})$$
$$= 130.52 \text{ kJ/kg}$$

The change in kinetic energy;

$$\Delta ke = \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2}$$
$$= \frac{(104.5^2 - 130^2)\,(\text{m}^2/\text{s}^2)}{2}$$
$$= -2990 \text{ J/kg} = -2.99 \text{ kJ/kg}$$

Substituting these values in the steady-flow energy equation

$$q - 21.64 = 130.52 - 2.99 + 0$$

or $$q = \mathbf{148.55 \text{ kJ/kg}}$$

Example 5.29 *In a steam power plant, the steady flow conditions prevail, 3600 kg of water per hour enters the boiler at a specific enthalpy of 840 kJ/kg and a speed of 300 m per minute at 5-m elevation. The water receives heat at constant pressure in the boiler and increases specific enthalpy to 3140 kJ/kg and then steam formed enters the turbine. The steam leaves the turbine at the speed of 3000 m/min at an elevation of 1 m and a specific enthalpy of 2640 kJ/kg. Heat losses from the turbine and boiler are 72000 kJ/h. Determine the power output of the power plant.*

Solution

Given A steam power plant operates as a steady flow system as shown in Fig. 5.42.

To find Power output from the power plant.

Assumptions

(i) Boiler and turbine together as a system,

(ii) Properties of the system remain constant at their locations.

Analysis The steady-flow energy equation for a combined system is given by

$$\dot{Q} - \dot{W} = \dot{m} [\Delta h + \Delta ke + \Delta pe]$$

Calculating each term separately

(i) The net heat-transfer rate,

$\dot{Q}$ = Heat supplied to the boiler – heat rejected from the system

$$= \dot{m} (h_2 - h_1) - \dot{Q}_{rej}$$
$$= (1 \text{ kg/s}) \times (3140 - 840)(\text{kJ/kg}) - 20 \text{ kW} = 2280 \text{ kJ/kg}$$

(ii) The change in specific enthalpy;

$$\Delta h = h_3 - h_1$$
$$= 2640 \text{ kJ/kg} - 840 \text{ kJ/kg}$$
$$= 1800 \text{ kJ/kg}$$

(iii) The change in kinetic energy;

$$\Delta ke = \frac{\mathcal{V}_3^2 - \mathcal{V}_1^2}{2} = \frac{(50^2 - 5^2)\,(\text{m}^2/\text{s}^2)}{2}$$
$$= 1237.5 \text{ J/kg} = 1.237 \text{ kJ/kg}$$

(iv) The change in potential energy

$$\Delta pe = g (z_3 - z_1)$$
$$= (9.81 \text{ m/s}^2) \times (1 \text{ m} - 5 \text{ m})$$
$$= -39.24 \text{ J/kg} = -0.039 \text{ kJ/kg}$$

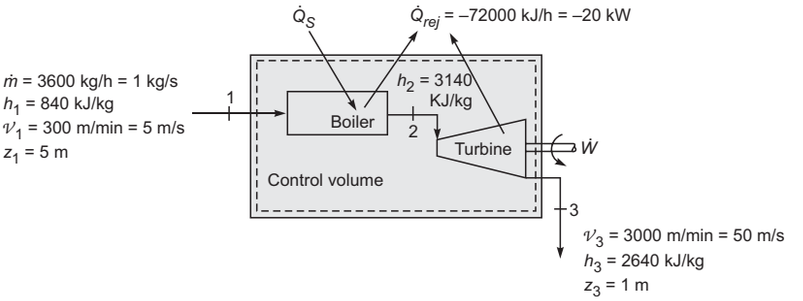

Fig. 5.42 *Schematic*

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Substituting the values of these terms in steady flow energy equation

$$2280 \text{ kW} - \dot{W} = (1 \text{ kg/s}) \times (1800 + 1.237 - 0.039) \text{ (kJ/kg)} = 180.197 \text{ kW}$$

or $$\dot{W} = \mathbf{478.8 \text{ kW}}$$

Example 5.30 *Helium gas is expanded polytropically in a turbine from 4 bar, 300°C to 1 bar such that the final volume is 2.5 times the initial volume. The velocity of gas at the exit is 50 m/s. What is the mass-flow rate of gas required to produce 1 MW turbine output? How much is the heat transfer during the process? Also determine exit area of the turbine. Assume specific heat of helium = 5.193 kJ/kg at constant pressure.*

Solution

Given Polytropic expansion of helium gas through a turbine

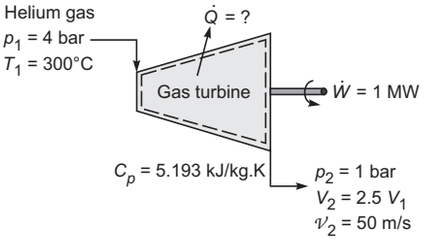

Fig. 5.43 *Schematic*

To find

(i) Mass-flow rate of helium through the turbine,
(ii) Heat transferred during expansion process, and
(iii) Exit area of the turbine.

Assumptions

(i) The molecular weight of the helium = 4 kg/kmol.
(ii) Steady flow conditions through the turbine.
(iii) Universal gas constant $R_u = 8.314$ kJ/kmol·K.
(iv) Inlet velocity of helium gas is negligible *i.e.*, $\mathcal{V}_1 = 0$.
(v) Change in potential energy is negligible.

Analysis

(i) The index of expansion of helium gas

$$p_1 V_1^{\,n} = p_2 V_2^{\,n}$$

or $$n = \frac{\ln(p_1/p_2)}{\ln(V_2/V_1)} = \frac{\ln\left(\frac{4}{1}\right)}{\ln(2.5)} = 1.513$$

The temperature T_2, after polytropic expansion

$$\frac{T_2}{T_1} = \left(\frac{V_2}{V_1}\right)^{1-n}$$

or $$T_2 = T_1\left(\frac{V_2}{V_1}\right)^{1-n}$$

$$= 573 \times (2.5)^{1-1.5135} = 358.1 \text{ K}$$

The gas constant R for helium,

$$R = \frac{R_u}{\mathcal{M}} = \frac{8.314}{4} = 2.0785 \text{ kJ/kg K}$$

The steady flow work/kg during expansion is expressed as

$$w = -\int v\,dp - \Delta ke - \Delta pe$$

where for polytropic expansion process

(a) $$-\int_1^2 v\,dp = -\left(\frac{n}{n-1}\right)(p_2 v_2 - p_1 v_1)$$

$$= -\frac{n}{n-1} R (T_2 - T_1)$$

$$= -\frac{1.513}{(1.513-1)} \times 2.0785 \times (358.1 - 573)$$

$$= 1317.37 \text{ kJ/kg}$$

(b) $$\Delta ke = \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} = \frac{(50 \text{ m})^2 - (0)^2}{2}$$

$$= 1250 \text{ J/kg} = 1.25 \text{ kJ/kg}$$

(c) $\Delta pe = 0$

Therefore, the specific work done during expansion

$$w = 1317.37 + 1.25 + 0$$
$$= 1318.62 \text{ kJ/kg}$$

Further, turbine output

$$\dot{W} = \dot{m}\, w$$

or $$1000 \text{ kW} = \dot{m} \times 1318.62$$

or $$\dot{m} = \mathbf{0.7583 \text{ kg/s}} \text{ or } \mathbf{2730 \text{ kg/h}}$$

(ii) Heat transferred during the steady flow process

$$\dot{Q} - \dot{W} = \dot{m}\,(\Delta h + \Delta ke + \Delta pe)$$

where $$\Delta h = C_p (T_2 - T_1)$$
$$= 5.193 \times (358.1 - 573)$$
$$= -1115.97 \text{ kJ/kg}$$

Therefore, the heat transferred,

$$\dot{Q} - 1000 = 0.7583 \times (-1115.97 + 1.25 + 0)$$

or $$\dot{Q} = 1000 - 845.3 = \mathbf{154.7 \text{ kW}}$$

(iii) Exit area of the turbine can be obtained by using continuity equation;

$$\dot{V}_2 = A_2 \mathcal{V}_2$$

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where $p_1 \dot{V}_1 = \dot{m} R T_1$

$$\dot{V}_1 = \frac{0.7583 \times 2.0785 \times 573}{400}$$

$$= 2.257 \text{ m}^3/\text{s}$$

and $\dot{V}_2 = 2.5 \dot{V}_1 = 5.644 \text{ m}^3/\text{s}$

Therefore, $A_2 = \frac{\dot{V}_2}{\dot{\mathcal{V}}_1} = \frac{5.664}{50} = 0.113 \text{ m}^2$

$$= \mathbf{11.3 \times 10^4 \text{ mm}^2}$$

Example 5.31 *Air-enters at 15°C through a heat exchanger at a velocity of 30 m/s where its temperature is raised to 800°C. It then enters a turbine with the same velocity of 30 m/s and expands until the temperature falls to 650°C. On leaving the turbine, air is taken at a velocity of 60 m/s to a nozzle where it expands until the temperature has fallen to 500°C. If the air-flow rate is 2 kg/s, calculate.*

(a) Rate of heat transfer to the air in the heat exchanger,

(b) Power output of the turbine, assuming no heat loss, and

(c) Velocity of air at the exit from nozzle, assuming no heat loss. Take specific enthalpy of air as h = C_pT, where C_p = 1.005 kJ/kg·K and T is the temperature in °C.

Solution

Given Air undergoes a steady-flow process through a heat exchanger, turbine and nozzle as shown below.

Air $\dot{m} = 2$ kg/s

$h = C_pT$ $C_p = 1.005$ kJ/kg·K

State 1: $T_1 = 15°C$ $\mathcal{V}_1 = 30$ m/s

State 2: $\mathcal{V}_2 = 30$ m/s, $T_2 = 800°C$

State 3: $T_3 = 650°C$, $\mathcal{V}_3 = 60$ m/s

$Q_{2-3} = 0$

State 4: $T_4 = 500°C$, $Q_{3-4} = 0$

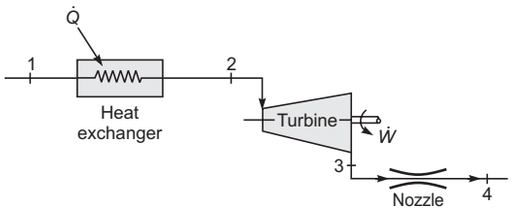

Fig. 5.44

To find

(i) Rate of heat transfer in heat exchanger,

(ii) Power output from turbine, and

(iii) Exit velocity of air from nozzle.

Assumptions

(i) Steady flow condition,

(ii) Since no information regarding the elevation is provided, thus assuming no change in potential energy, i.e., $z_1 = z_2 = z_3 = z_4$

Analysis Considering steady flow process through each device separately.

(i) Control volume: Air heat exchanger

Initial state: $T_1 = 15°C$, $\mathcal{V}_1 = 30$ m/s

Final state: $T_2 = 800°C$, $\mathcal{V}_2 = 30$ m/s

Applying steady flow energy equation to heat exchanger,

$$\dot{Q} - \dot{W} = \dot{m}\left\{(h_2 - h_1) + \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} + (z_2 - z_1)g\right\}$$

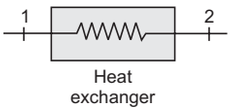

Fig. 5.45

For a heat exchanger

$\dot{W} = 0, \mathcal{V}_1 = \mathcal{V}_2$ and $z_1 = z_2$ (assumed)

Then $\dot{Q} = \dot{m}(h_2 - h_1) = \dot{m} C_p (T_2 - T_1)$

$= (2 \text{ kg/s}) \times (1.005 \text{ kJ/kg·K}) \times (800 - 15)$ (K)

$= \mathbf{1577.85 \text{ kJ/s}}$

(ii) Control volume: Air turbine

Initial state: $T_2 = 800°C$, $\mathcal{V}_2 = 30$ m/s

Final state: $T_3 = 650°C$, $\mathcal{V}_3 = 60$ m/s

and $Q_{2-3} = 0$

Applying steady-flow energy equation to turbine

$$\dot{Q} - \dot{W} = \dot{m}\left\{(h_3 - h_2) + \frac{\mathcal{V}_3^2 - \mathcal{V}_2^2}{2} + (z_3 - z_2)g\right\}$$

Using the data it reduces to

$$-\dot{W} = \dot{m}\left\{C_p(T_3 - T_2) + \frac{\mathcal{V}_3^2 - \mathcal{V}_2^2}{2}\right\}$$

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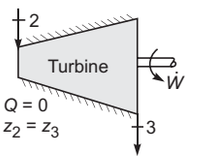

Fig. 5.46

$$\text{or} \quad -\dot{W} = 2 \times \left\{ 1005 \times (650 - 800) + \frac{60^2 - 30^2}{2} \right\}$$

$$= 2 \times (-150750 + 1350)$$

$$= -298800 \text{ W}$$

$$\text{or} \quad \dot{W} = 298800 \text{ W} = \mathbf{298.80 \text{ kW}}$$

(iii) Control volume: Nozzle

Initial state: $T_3 = 650°C$, $\mathcal{V}_3 = 60$ m/s

Final state: $T_4 = 500°C$

For nozzle $\dot{W} = 0$, $\dot{Q} = 0$, $z_3 = z_4$

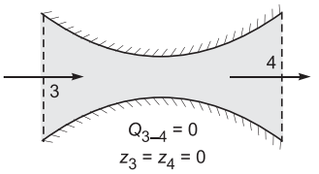

Fig. 5.47

Then steady-flow energy equation reduces to

$$0 = \dot{m} \left\{ (h_4 - h_3) + \frac{\mathcal{V}_4^2 - \mathcal{V}_3^2}{2} \right\}$$

$$2 (h_3 - h_4) + \mathcal{V}_3^2 = \mathcal{V}_4^2$$

$$\text{or} \quad \mathcal{V}_4^2 = 2 C_p (T_3 - T_4) + \mathcal{V}_3^2$$

$$\text{or} \quad \mathcal{V}_4^2 = 2 \times 1005 \times (650 - 500) + (60)^2 = 305100$$

$$\text{or} \quad \mathcal{V}_4 = \mathbf{552.36 \text{ m/s}}$$

Example 5.32 *A heat exchanger receives air at a velocity of 25 m/s and 20°C. The temperature of air increases to 780°C in a heat exchanger.*

The air then passes through the turbine. The velocity of air while entering the turbine is 25 m/s. The air expands in the turbine, where its temperature falls to 630°C. On leaving the turbine, air passes through the nozzle. The velocity of air entering the nozzle is 60 m/s. The air expands in the nozzle till its temperature falls to 500°C. The air-flow rate is 2.5 kg/s. Determine

(a) Rate of heat transfer to air in the heat exchanger

(b) Power output from the turbine, assuming no heat loss

(c) The velocity at the exit from nozzle, assuming no heat loss

Solution

Given Air undergoes a steady flow process through a heat exchanger, turbine and nozzle as shown below:

Air $\dot{m} = 2.5$ kg/s

State 1: $T_1 = 20°C$ $\mathcal{V}_1 = 25$ m/s

State 2: $\mathcal{V}_2 = 25$ m/s, $T_2 = 780°C$

State 3: $T_3 = 630°C$, $\mathcal{V}_3 = 60$ m/s

$Q_{2-3} = 0$

State 4: $T_4 = 500°C$, $Q_{3-4} = 0$

To find

(i) Rate of heat transfer in the heat exchanger,

(ii) Power output from turbine, and

(iii) Exit velocity of air from nozzle.

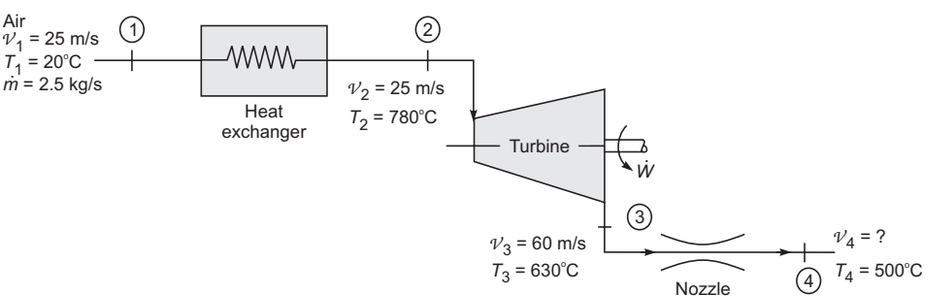

Fig. 5.48

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Assumptions

(i) Steady flow condition,
(ii) Since no information regarding the elevation is provided, thus assuming no change in potential energy, i.e., $z_1 = z_2 = z_3 = z_4$,
(iii) Specific heat of air at constant pressure is 1.005 kJ/kg·K.

Analysis Applying steady-flow process through each device separately.

(i) *Control volume: heat exchanger*

$$\dot{W} = 0, \quad \Delta pe = 0 \quad \Delta ke = 0$$

Initial state: $T_1 = 20°C$, $\mathcal{V}_1 = 25$ m/s
Final state: $T_2 = 780°C$, $\mathcal{V}_2 = 25$ m/s

The steady flow energy equation reduces to

$$\dot{Q} = \dot{m}(h_2 - h_1) = \dot{m} C_p (T_2 - T_1)$$
$$= (2.5) \times (1.005) \times (780 - 20)$$
$$= 1909.5 \text{ kJ/s} = \mathbf{1909.5\ kW}$$

(ii) *Control volume: Air turbine*

Initial state: $T_2 = 780°C$, $\mathcal{V}_2 = 25$ m/s
Final state: $T_3 = 630°C$, $\mathcal{V}_3 = 60$ m/s

$$Q_{2-3} = 0$$

Steady flow energy equation reduces to

$$-\dot{W} = \dot{m}\left\{(h_3 - h_2) + \frac{\mathcal{V}_3^2 - \mathcal{V}_2^2}{2}\right\}$$

or $$-\dot{W} = \dot{m}\left\{C_p(T_3 - T_2) + \frac{\mathcal{V}_3^2 - \mathcal{V}_2^2}{2}\right\}$$

or $$-\dot{W} = (2.5) \times \left\{(1.005) \times (630 - 780) + \frac{(60)^2 - (25)^2}{2} \times (10^{-3})\right\}$$

or $$\dot{W} = \mathbf{373.156\ kW}$$

(iii) *Control volume: Nozzle*

$$\dot{W} = 0, \quad \Delta pe = 0 \quad \dot{Q} = 0$$

Initial state: $T_3 = 630°C$, $\mathcal{V}_3 = 60$ m/s
Final state: $T_4 = 500°C$

Then Steady flow energy equation reduces to

$$0 = \dot{m}\left\{(h_4 - T_3) + \frac{\mathcal{V}_4^2 - \mathcal{V}_3^2}{2}\right\}$$

$$\mathcal{V}_4^2 = 2(h_3 - h_4) + \mathcal{V}_3^2$$

or $$\mathcal{V}_4^2 = 2 C_p (T_3 - T_4) + \mathcal{V}_3^2$$

or $$\mathcal{V}_4^2 = 2 \times 1005 \times (630 - 500) + (60)^2$$
$$= 264900 \text{ m}^2/\text{s}^2$$

or $$\mathcal{V}_4 = \mathbf{514.68\ m/s}$$

Throttle Valve

Example 5.33 *The steam at 2 MPa and 300°C is throttled to an atmospheric pressure of 100 kPa. Determine the final temperature of air. Take the specific heat of superheated steam as 2.2 kJ/kg·°C.*

Solution

Given The throttling of steam

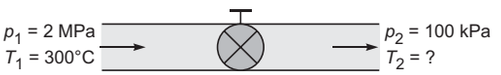

Fig. 5.49

To find The temperature of air after throttling.

Assumptions

(i) Flow through the throttle valve is assumed adiabatic, $\dot{Q} = 0$.
(ii) Throttling valves do not involve any type of work interaction, therefore, $\dot{W} = 0$.
(iii) No information regarding elevations and inlet and exit velocity is provided, thus assuming, $\Delta pe = 0$, $\Delta ke = 0$.

Analysis For a throttling device, the enthalpy of the system before and after throttling remains constant. Thus,

$$h_1 = h_2$$

The enthalpy of steam at 2 MPa and 300°C can be obtained from the steam table for superheated steam as

$$h_1 = 3023.5 \text{ kJ/kg}$$

This enthalpy equals the exit enthalpy, thus

$$h_2 = 3023.5 \text{ kJ/kg}$$

But the exit pressure is 100 kPa, the properties of saturated steam at 100 kPa;

$$h_{g,100\text{ kPa}} = 2675.5 \text{ kJ/kg},$$
$$T_{sat} = 99.63°C$$

Since the enthalpy after throttling is greater than enthalpy of saturated steam at 100 kPa, thus the quality of steam will be superheated. It can be obtained as

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$$h_2 = h_{g,\,100\text{ kPa}} + C_{ps}(T_2 - T_{sat})$$

or $$3023.5\text{ kJ/kg} = 2675.5\text{ kJ/kg} + (2.2\text{ kJ/kg}\cdot\text{°C}) \times (T_2 - 99.63)(\text{°C})$$

or $$T_2 = \frac{3023.5 - 2675.5}{2.2} + 99.63$$

$$= \mathbf{258.81°C}$$

Example 5.34 *Air flows at the rate of 2.3 kg/s in a 15-cm diameter pipe. It has a pressure of 7 bar and a temperature of 95°C before it is throttled by a valve to 3.5 bar. Find the velocity of air demonstration of the restrictions.*

Solution

Given Throttling of air as shown in Fig. 5.50

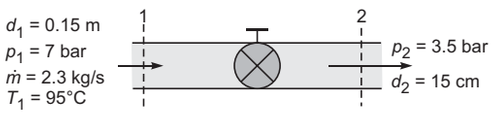

Fig. 5.50

To find The velocity of air after throttling

Assumptions

(i) Adiabatic conditions ($\dot{Q} = 0$),

(ii) Steady flow process,

(iii) Change in potential energy is negligible, i.e., ($z_1 = z_2$)

(iv) Air properties, R = 0.287 kJ/kg·K, C_p = 1.005 kJ/kg·K

Analysis For the given data, initial specific volume

$$v_1 = \frac{RT_1}{p_1} = \frac{0.287 \times 368}{700}$$

$$= 0.151\text{ m}^3/\text{kg}$$

From continuity equation the initial velocity

$$\dot{m} = \frac{A_1 \mathcal{V}_1}{v_1}$$

or $$\mathcal{V}_1 = \frac{\dot{m} v_1}{A_1} = \frac{\dot{m} v_1}{(\pi/4) d_1^2}$$

or $$\mathcal{V}_1 = \frac{2.3 \times 0.151}{\frac{\pi}{4} \times (0.15)^2} = 19.63\text{ m/s}$$

During the throttling process enthalpy remains constant;

$$h_1 = h_2$$

or $$C_p T_1 = C_p T_2 \quad \text{or} \quad T_1 = T_2$$

Then specific volume at exit

$$v_2 = \frac{RT_2}{p_2} = \frac{0.287 \times 368}{3.5 \times 100}$$

$$= 0.301\text{ m}^3/\text{kg}$$

The exit velcoity, from continuity equation

$$\mathcal{V}_2 = \frac{\dot{m} v_2}{A_2} = \frac{2.3 \times 0.301}{(\pi/4)(0.15)^2}$$

$$= \mathbf{39.2\text{ m/s}}$$

Mixing of Streams

Example 5.35 *A certain water heater operates under steady flow conditions, receiving 4.2 kg/s of water at 75°C temperature and an enthalpy of 313.93 kJ/kg. The water is heated by mixing with steam, which is supplied to a heater at a temperature of 100.2°C and an enthalpy of 2676 kJ/kg. The mixture leaves the heater as liquid water at a temperature of 100°C and an enthalpy of 419 kJ/kg. How much steam must be supplied to the heater per hour?*

Solution

Given Heating of water with steam mixing as shown in Fig. 5.51

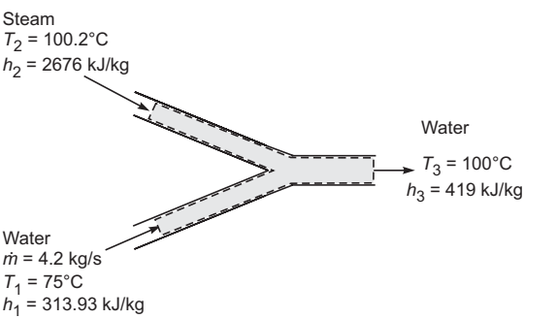

Fig. 5.51 *Schematic of mixer heater*

To find Mass-flow rate of steam to heater.

Assumptions

(i) No heat loss to surroundings,

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(ii) Steady flow process,
(iii) Change in kinetic and potential energies is negligible.

Analysis For adiabatic mixing streams, the steady flow energy equation can be expressed as

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_2) h_3$$

or $4.2 \times 313.93 + \dot{m}_2 \times 2676 = (4.2 + \dot{m}_2) \times 419$

or $1318.5 + 2676\ \dot{m}_2 = 1759.8 + 419\ \dot{m}_2$

or $2257\ \dot{m}_2 = 441.3$

or $\dot{m}_2 = \mathbf{0.1955\ kg/s}$

5.7 TRANSIENT FLOW PROCESSES

Many practical processes involve unsteady flow, which cannot be analysed with the steady-flow energy equation. While charging and discharging a gas cylinder, the properties of gas inside the cylinder vary with time. The mass flow rates into and out of the cylinder are not same and there is accumulation of mass inside the cylinder. Such flow processes are transient and analysis of such processes is made by control mass analysis and then by control volume analysis.

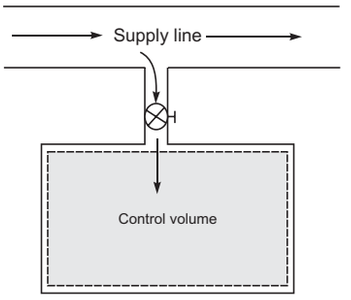

Fig. 5.52 *Charging of cylinder*

Mass of the gas, which enters the cylinder during charging process can be expressed as

$$m_2 - m_1 = \Delta m_{system} \qquad ...(5.24)$$

Energy balance:

$$E_2 - E_1 = \Delta E_{system} \qquad ...(5.25)$$

where suffix 1 = initial state, and 2 = final state of control volume.

The initial energy of fluid in cylinder

$$E_1 = m_1 u_1 + (m_2 - m_1) u_i \qquad ...(5.26)$$

Energy of system at the end of charging

$$E_2 = m_2 u_2 \qquad ...(5.27)$$

When the mass (Δm_{system}) enters the cylinder, the cylinder volume becomes $(m_2 - m_1) v_i$. The work is done on the system to force the gas into the cylinder at pressure p_i acting in the supply mains.

$$W = -(m_2 - m_1)\, p_i v_i.$$

where suffix i indicates the condition in the tank after charging of cylinder.

Applying the first law of thermodynamics to this system

$$\underbrace{Q - W}_{\text{Net energy transfer by heat, work and mass}} = \underbrace{E_2 - E_1}_{\text{Change in energy}}$$

or $Q + (m_2 - m_1)\, p_i v_i = m_2 u_2 - [m_1 u_1 + (m_2 - m_1) u_i]$

or $Q + (m_2 - m_1)(p_i v_i + u_i) = m_2 u_2 - m_1 u_1$

or $Q + (m_2 - m_1) h_i = m_2 u_2 - m_1 u_1$

If the cylinder is insulated ($Q = 0$) and initially evacuated ($m_1 = 0$), then during adiabatic charging, the specific enthalpy of gas in the supply main is equal to the specific internal energy energy of the gas in the cylinder at the end of charging. That is

$$h_i = u_2 \qquad ...(5.28)$$

For an ideal gas $h = C_p T$ and $u = C_v T$, then

$$T_2 = \gamma T_i \qquad ...(5.29)$$

Example 5.36 *A spherical vessel of 1.5-m diameter containing air at 315 K is evacuated until the vaccum inside the vessel is 730 mm of Hg ; the evacuation process is carried out at constant temperature. Determine the mass of air pumped out. Also, calculate the pressure in the tank, if remaining air is cooled to 275 K.*

For air R = 0.287 kJ/kg·K. Take atmospheric pressure as 760 mm of Hg.

Solution

Given Spherical vessel of constant capacity

V = Constant $\quad p_{atm}$ = 760 mm of Hg,

R = 0.287 kJ/ kg·K

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State 1: $D = 1.5$ m or $r_o = 0.75$ m
$T_1 = 315$ K
State2: $p_{vacuum} = 730$ mm of Hg $T_2 = 315$ K
State 3: $T_3 = 275$ K

To find

(i) Mass of air pumped out, and
(ii) Pressure of air in the tank after cooling.

Assumption Density of mercury, $\rho = 13.59 \times 10^3$ kg/m^3.

Analysis

(i) The volume of spherical tank

$$V = \frac{4}{3}\pi r_o^3 = \frac{4}{3}\pi \times (0.75)^3$$
$$= 1.767 \text{ m}^3$$

The initial mass of air ; $m_1 = \dfrac{p_1 V}{RT_1}$

Final mass of air, $m_2 = \dfrac{p_2 V}{RT_1}$ (T_1 and V are constant)

The mass of air pumped out

$$\Rightarrow \quad m_1 - m_2 = \frac{(p_1 - p_2)V}{RT_1} \qquad ...(i)$$

Absolute pressure of air in the tank after evacuation

$$p_2 = p_{am} - p_{vaccum} = 760 - 730$$
$$= 30 \text{ mm of Hg}$$

Then $p_1 - p_2 = 760 - 30 = 730$ mm of Hg

or $p_1 - p_2 = \rho g h$

$$= 13.59 \times 10^3 \times 9.81 \times \left(\frac{730}{1000} \text{m}\right)$$
$$= 97322 \text{ Pa} = 97.32 \text{ kPa}$$

Using in Eq (i) with other numerical values

$$m_1 - m_2 = \frac{97.32 \times 1.767}{0.287 \times 315} = \mathbf{1.90 \text{ kg}}$$

(ii) Pressure of air in the tank after cooling;

$$\frac{p_2}{T_2} = \frac{p_3}{T_3}$$

or $$p_3 = p_2 \times \frac{T_3}{T_2}$$

$$= 30 \text{ mm of Hg} \times \frac{275}{315}$$
$$= 26.2 \text{ mm of Hg}$$

This pressure can be expressed in kPa as

$$p_3 = \rho g h$$
$$= 13.59 \times 10^3 \times 9.81 \times \frac{26.2}{1000} \times 10^{-3}$$
$$= \mathbf{3.49 \text{ kPa}}$$

Example 5.37 *An oxygen cylinder has a capacity of 300 litres and contains oxygen at a pressure of 3.10 MPa and a temperature of 18°C. The stop valve is opened and some gas is used. If the pressure and temperature of the oxygen left in the cylinder falls to 1.7 MPa and 15°C respectively, determine the mass of oxygen used.*

If the stop valve is closed, the oxygen remaining in the cylinder gradually attains its initial temperature of 18°C. Determine the amount of heat transferred through the cylinder wall from the atmosphere. The density of oxygen at 0°C and 0.101325 MPa is 1.429 kg/m^3 and $\gamma = 1.4$ for oxygen.

Solution

Given An oxygen cylinder with

$V = 300$ lit $= 300 \times 10^{-3}$ m^3 $= 0.3$ m^3

State 1: $p_1 = 3.1$ MPa$= 3100$ kPa
$T_1 = 18°\text{C} + 273 = 291$ K
State 2: $p_2 = 1.7$ MPa $= 1700$ kPa
$T_2 = 15°\text{C} + 273 = 288$ K
State 3: $T_3 = 18°\text{C} + 273 = 291$ K
$\rho = 1.429$ kg/m^3
$T_0 = 0°\text{C} + 273 = 273$ K
$p_0 = 0.101\ 325$ MPa $\gamma = 1.4$

To find

(i) Mass of oxygen used,
(ii) Heat transfer $Q_{2\text{–}3}$.

Assumptions

(i) Oxygen as a perfect gas,
(ii) Reversible process.

Analysis

(i) The density of oxygen at 0°C (or 273 K) and 0.101325 MPa (101.325 kPa) is given as 1.428 kg/m^3. Using perfect gas equation for unit mass;

$$\frac{p_0}{\rho} = RT_0$$

or $$R = \frac{(101.325 \text{ KPa})}{(1.429 \text{ kg/m}^3) \times (273 \text{ K})}$$
$$= 0.259 \text{ kJ/kg} \cdot \text{K}$$

From the characteristic gas equation, the initial mass of the oxygen in the cylinder,

$$m_1 = \frac{p_1 V_1}{RT_1} = \frac{(3100 \text{ KPa}) \times (0.3 \text{ m}^3)}{(0.259 \text{ kJ/kg} \cdot \text{K}) \times (291 \text{ K})}$$
$$= 12.3 \text{ kg}$$

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The mass of the oxygen remaining in the cylinder after its use.

$$m_2 = \frac{p_2 V_2}{R T_2}$$

$$= \frac{(1700 \text{ KPa}) \times (0.3 \text{ m}^3)}{(0.259 \text{ kJ/kg} \cdot \text{K}) \times (288 \text{ K})}$$

$$= 6.83 \text{ kg}$$

Therefore, the mass of oxygen used

$$\Delta m = m_1 - m_2 = 12.3 - 6.83 = \mathbf{5.46 \text{ kg}}$$

(ii) The heat transferred during temperature change of remaining oxygen in the cylinder

$$Q_{2-3} = m_2 C_v (T_3 - T_2)$$

where $$C_v = \frac{R}{\gamma - 1} = \frac{0.259}{1.4 - 1}$$

$$= 0.6475 \text{ kJ/kg} \cdot \text{K}$$

Then $$Q_{2-3} = (6.83 \text{ kg}) \times (0.6475 \text{ kJ/kg} \cdot \text{K}) \times (291 \text{ K} - 288 \text{ K})$$

$$= \mathbf{13.26 \text{ kJ}}$$

Example 5.38 *A 2 m^3 tank with perfectly insulated walls contains saturated steam at a pressure of 1.0 MPa. This tank is connected to a steam line through a valve. The valve is opened and the superheated steam at 4 MPa and 400°C flows into the tank until the pressure in the tank is 4 MPa. Calculate the mass and temperature of steam that enters the tank.*

Solution

Given A tank of fixed capacity $V = 2 \text{ m}^3$

State 1: $p_1 = 1$ MPa = 10 bar (Saturated steam)

State 2: $p_2 = 4$ MPa = 40 bar

$T_2 = 400$°C (Superheated steam)

To find

(i) Mass of steam admitted into cylinder,

(ii) Temperature of steam.

Properties of steam

At 10 bar (Saturated)

$v_1 = 0.1944 \text{ m}^3/\text{kg}$

$h_1 = 2778.08 \text{ kJ/kg}$

$u_1 = 2582.64 \text{ kJ/kg}$

At 40 bar and 400°C (Superheated)

$v_2 = 0.07341 \text{ m}^3/\text{kg}$

$h_2 = 3213.51 \text{ kJ/kg}$

Analysis The initial mass of steam in the tank

$$m_1 = \frac{V}{v_1} = \frac{2 \text{ m}^3}{0.1944 \text{ m}^3/\text{kg}} = 10.288 \text{ kg}$$

The mass of steam after steam admission at 4.0 MPa

$$m_2 = \frac{V}{v_2} = \frac{2 \text{ m}^3}{0.07341 \text{ m}^3/\text{kg}} = 27.244 \text{ kg}$$

The mass of steam admitted into tank

$$m_{admitted} = m_2 - m_1 = 27.244 - 10.288$$

$$= \mathbf{16.956 \text{ kg}}$$

For transient filling,

$$Q + (m_2 - m_1) h_i = m_2 u_2 - m_1 u_1$$

For insulated tank $Q = 0$; then

$$16.956 \times 3213.51 = 27.244\, u_2 - 10.288 \times 2582.64$$

It gives $u_2 = 2975.27$ kJ/kg

Interpolating at 40 bar between u(450°C) = 3010.13 kJ/kg and u(400°C) = 2919.88 kJ/kg, the final temperature to be **430°C.**

Summary

- The mass-flow rate through a cross-section is expressed as

$$\dot{m} = \rho A \mathcal{V} \text{ (kg/s)}$$

- The volume-flow rate through a cross-section is given by

$$\dot{V} = A\mathcal{V} = \dot{m} v \text{ (m}^3\text{/s)}$$

- The steady-flow energy equation for a given mass-flow rate of the system is

$$\dot{Q} - \dot{W} = \dot{m} \left[u_2 - u_1 + p_2 v_2 - p_1 v_1 + \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} + (z_2 - z_1) g \right]$$

Using $h = u + pv$

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then $\dot{Q} - \dot{W}$

$$= \dot{m}\left[h_2 - h_1 + \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} + (z_2 - z_1)g\right] \text{ (J/s)}$$

- Dividing both sides by $\dot{m}$, the steady flow energy equation on a unit mass basis as

$$q - w = h_2 - h_1 + \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} + g(z_2 - z_1)$$

- The steady flow process can be modeled for the following devices as follows:

(i) For flow through a nozzle, diffuser, boiler, condenser, heat exchanger, throttle valve and duct, the shaft work w or $\dot{W}$ is always zero.

(ii) For throttle valve $h_1 = h_2$.

(iii) For work consuming devices like water pumps, blowers, and compressors, the shaft work (input) $\dot{W}$ (or w) is always negative.

(iv) For a water pump, $v_1 = v_2$, $u_1 = u_2$ and $q = 0$.

(v) For a turbine engine, the shaft work $\dot{W}$ (or w) is always positive.

(vi) The other terms in the steady-flow energy equation have their usual meaning if not specified in the problem.

Glossary

Internal energy Total energy minus kinetic and potential energy

Enthalpy Sum of internal energy and product of pressure and volume

Steady No change with time

Steady flow process A process of fluid flow with constant properties at each location with respect to time

Flow energy Work associated with fluid push in and out of a control volume

Nozzle Converts pressure energy into kinetic energy

Diffuser Converts kinetic energy into pressure energy

Turbine Rotary engine used to generate power

Compressor A device which increases the pressure of a fluid

Water pump Lifts water through an elevation at the cost of power input

Heat exchanger A device that transfers heat energy between two fluids at different temperatures

Throttling Device A flow-control valve which reduces pressure irreversibly

Review Questions

1. State the continuity equation with its nomenclature.
2. What is the total energy of a flow system? Identify different forms of energy that constitute the total energy of a system.
3. Define and explain how following properties are related with heat energy
   (i) intrenal energy , and (ii) enthalpy.
4. Define and explain the flow work.
5. Distinguish between energy of a non-flow system and a flow system.
6. What is flow energy? Do fluids at rest possess any flow energy?.
7. What is a steady flow process? What are the conditions for a steady flow process?
8. Draw a control volume and write the energy and mass transfer and derive an expression for steady-flow energy equation.
9. Write the steady-flow energy equation and state the significance of various terms involved.
10. State the significance of $-\int v dp$.
11. What is the relation between $-\int v dp$ and $\int p dv$?
12. Differentiate between pv work and $\int p dv$ work.
13. Explain the significance of $\int p dv$ in case of (a) flow process, and (b) non-flow process.
14. Deduce the steady-flow energy equation for (a) reciprocating compressor, and (b) water pump.

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15. State the consequences of the first law of thermodynamics.
16. When air enters a diffuser and decelerates, does its pressure increase or decrease? Deduce the steady-flow energy equation for a diffuser.
17. Define the mass and volume flow rates. How are they related to each other?
18. How is a steady flow system characterised?
19. Why are throttling devices commonly used in refrigeration and air-conditioning appliances?
20. Would you expect the temperature of a liquid to change as it is throttled? Explain.
21. Apply steady-flow energy equation to each of the following :
    (a) Boiler (b) Nozzle (c) Centrifugal pump (d) Steam turbine (e) Water turbine

Problems

1. A stream of gases at 7.5 bar, 750°C and 140 m/s is passed through a turbine of a jet engine. The stream comes out of the turbine at 2.0 bar, 550°C and 280 m/s. The process may be assumed adiabatic. The enthalpies of gas at the entry and exit of the turbine are 950 kJ/kg and 650 kJ/kg of gas respectively.Determine the capacity of the turbine if the gas flow is 5 kg/s. *[1353 kW]*

2. A steam turbime has the following data:

Parameter	*Inlet*	*Outlet*
Steam pressure (bar)	14	0.07
Internal energy (kJ/kg)	2730	2340
Specific volume (m^3/kg)	0.166	18.6
Velocity of steam (m/s)	120	330

The heat rejected from the turbine is 20 kJ/kg and the power output is 20 kW. Neglecting changes in potential energy, determine the mass flow rate of steam. *[0.0471 kg/s]*

3. In one of the section of the heating plant in which there are no pumps, a steady flow of water enters at a temperature of 50°C and a pressure of 3 bar (h = 240 kJ/kg). The water leaves the section at a temperature of 35°C and at a pressure of 2.5 bar (h = 192 kJ/kg). The exit pipe is 20 m above the entry pipe. Assuming change in kinetic energy to be negligible, evaluate the heat transfer from the water per kg of water flowing. *[47.8 kJ/kg]*

4. A petrol engine operates steadily with a work output of 36 kW. The heat transfer rate to cooling water is 125 MJ/h. The sum of all other heat transfers to surroundings and engine foundation is 25 MJ/h. The fuel–air mixture enters the engine at the rate of 150 kg/h. Determine the change in enthalpy of the air–fuel mixture stream in kJ/kg. Neglect changes in kinetic energy and potential energy. *[–1864 kJ/kg]*

5. 12 kg of a fluid per minute goes through a reversible steady flow process. The properties of fluid at the inlet are p_1 = 1.4 bar, ρ_1 = 25 kg/m^3, $\mathcal{V}_1$ = 120 m/s and u_1 = 920 kJ/kg and at the exit are p_2 = 5.6 bar, ρ_2 = 5 kg/m^3, $\mathcal{V}_2$ = 180 m/s and u_2 = 720 kJ/kg. During the passage, the fluid rejects 60 kJ/s and rises through 60 metres. Determine (a) the change in enthalpy (Δh), and (b) work done during the process (W).

[Δh = – 93.6 kJ/kg ; $\dot{W}$ = – 44.2 kW]

6. A gas flows steadily through a rotary compressor. The gas enters the compressor at a temperature of 16°C, a pressure of 100 kPa, and an enthalpy of 391.2 kJ/kg. The gas leaves the compressor at a temperature of 245°C, a pressure of 0.6 MPa and an enthalpy of 534.5 kJ/kg. There is no heat transfer to or from the gas as it flows through the compressor.

(a) Evaluate the external work done per unit mass of gas, assuming the gas velocities at entry and exit to be negligible.

(b) Evaluate the external work done per unit mass of gas when the gas velocity at entry is 80 m/s and that at the exit is 160 m/s.

[143.3 kJ/kg, 152.9 kJ/kg]

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7. In a steady flow process, the working fluid flows at a rate of 240 kg/min. The fluid rejects 120 kJ/s passing through the system. The conditions of fluid at inlet and outlet are given as $\mathcal{V}_1 = 300$ m/s, $p_1 = 6.2$ bar, $u_1 = 2100$ kJ/kg, $v_1 = 0.37$ m^3/kg and $\mathcal{V}_2 = 150$ m/s, $p_2 = 1.3$ bar, $u_2 = 1500$ kJ/kg, $v_2 = 1.2$ m^3/kg. The suffix 1 indicates the conditions at inlet and 2 indicates at outlet of the system. Neglecting the change in potential energy, determine the power capacity of the system in MW. *[2.7086 MW]*

8. 15 kg of air per min. is delivered by a centrifugal compressor. The inlet and outlet conditions of air are $\mathcal{V}_1 = 10$ m/s, $p_1 = 1$ bar, $v_1 = 0.5$ m^3/kg and $\mathcal{V}_2 = 80$ m/s, $p_2 = 7$ bar, $v_2 = 0.15$ m^3/kg. The increase in enthalpy of air passing through the compressor is 160 kJ/kg, and heat loss to the surroundings is 720 kJ/min. Assuming that inlet and discharge lines are at the same level, find

(a) Motor power required to drive the compressor,

(b) Ratio of inlet to outlet pipe diameter.

[(a) 52.78 kW (b) $d_1/d_2 = 5.16$]

9. A compressor takes air at 100 kN/m^2 and delivers the same at 550 kN/m^2. The compressor discharges 16 m^3 of free air per minute. The density of air at inlet and exit are 1.25 kg/m^2 and 5 kg/m^3. The power of the motor driving the compressor is 40 kW. The heat lost to the cooling water circulated around the compressor is 30 kJ/kg of air passing through the compressor. Neglecting changes in PE and KE, determine the change in specific internal energy. *[60 kJ/kg]*

10. 60 kg of water is delivered by a centrifugal pump per second. The inlet and outlet pressures are 1 bar and 4 bar respectively. The suction is 2 m below the centre of the pump and delivery is 8 m above the centre of the pump. Determine the capacity of the electric motor to run the pump. The suction and delivery pipe diameters are 20 cm and 10 cm and respectively. *[27.15 kW]*

11. A gas turbine of a turbojet engine receives a steady flow of gases at a pressure of 7.2 bar, temperature of 850°C and a velocity of 160 m/s. It discharges the gases at a pressure of 1.15 bar, a temperature of 450°C and a velocity of 250 m/s. Determine the work output of the turbine in kJ/kg of the gas flow. The process may be assumed to be adiabatic and C_p of gas as 1.04 kJ/kg·K.

[397.55 kJ/kg]

12. The centrifugal air compressor of a gas turbine receives atmospheric air at 1 bar and 300 K. It discharges the air at 4 bar and 480 K with a velocity of 100 m/s. The mass-flow rate compressor is 15 kg/s. Determine the power input to compressor. *[2810 kW]*

13. A blower handles 1 kg/s of air at 20°C and consumes a power of 15 kW. The inlet and outlet velocities are 100 m/s and 150 m/s respectively. Find the exit air temperature, assuming adiabatic condition. Take C_p of air as 1.005 kJ/kg·K.

[28.38°C]

14. A gas flows steadily through a rotary compressor. The gas enters the compressor at a temperature of 16°C a pressure of 100 kPa and an enthalpy of 391.2 kJ/kg. The gas leaves the compressor at a temperature of 245°C, a pressure of 0.6 MPa and an enthalpy of 534.5 kJ/kg. If the flow through the compressor is adiabatic, calculate

(a) Work input to the compressor per unit mass of gas, assuming no change in kinetic energy of the gas,

(b) Work input per kg of gas, if its inlet velocity is 80 m/s and exit velocity is 160 m/s

[(a) 143.3 kJ/kg, (b) 152.9 kJ/kg]

15. A centrifugal compressor supplies air at 0.5 MPa, 200°C and 250 m/s velocity to a stationary gas turbine plant. The air enters the compressor from an atmosphere at 1 bar, 35°C and with negligible velocity. Determine the work input per unit mass of the air. Assuming adiabatic flow, will an electric motor of 1200 kW rating be adequate for driving this compressor, if the mass-flow rate of the air is 8 kg/s? *[197.07 kJ/kg, No]*

16. Calculate the work input per kg to pump water insentropically from 100 kPa, 30°C to 5 MPa.

[4.92 kJ/kg]

17. Determine the power required to derive a pump which raises the water pressure from 1 bar at entry to 25 bar at exit and delivers 2000 kg/h of water. Neglect the changes in volume, elevation, velocity and take specific volume of water as 0.001045 m³/kg. *[–1.393 kW]*

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18. Exhaust gases from a diesel engine enter a turbine of a turbocharger at 2 bar, 600°C and 50 m/s. The gases leave the turbine at 1 bar, 270°C and 220 m/s. The inlet area is 7.1 cm^2. The heat loss from the turbine may be assumed 15 kJ/kg of gases. Determine

(a) Mass flow rate of exhaust gases,

(b) Power output from the turbine.

Assume properties of gases are same as that of air. *[(a) 0.02833 kg/s (b) 8.32 kW]*

19. Air enters a control volume at 10 bar, 400 K and 20 m/s through a flow area of 20 cm^2. At the exit, the pressure is 6 bar, the temperature is 345.7 K, and the velocity is 330.2 m/s. The air behaves as an ideal gas. For steady-state operation, determine

(a) the mass flow rate, in kg/s

(b) the exit flow area, in cm^2

[(a) 0.3484 kg/s, (b) 1.744 cm^2]

20. Air enters a control volume operating at steady state at 1.2 bar, 300 K, and leaves at 12 bar, 440 K with a volumetric flow rate of 1.3 m^3/min. The work input to the control volume is 240 kJ per kg of air flowing. Neglecting potential and kinetic energy effects, determine the heat transfer rate, in kW. *[–3 kW]*

21. Air expands through a turbine from 10 bar, 900 K to 1 bar, 500 K. The inlet velocity is very small compared to exit velocity of 100 m/s. The turbine operates at steady state and devlops a power of 3200 kW. Heat transfer between the turbine and its surroundings and potential energy effects are negligible. Calculate the mass flow rate of air in kg/s and the exit area in m^2.

[8.06 kg/s, 0.115m^2]

22. In a conference hall, comfort temperature conditions are maintained in winter by circulating hot water through a piping system. The water enters the system at 3-bar pressure and 50°C temperature with an enthalpy of 240 kJ/kg and leaves at 2.5 bar, and 30°C with an enthalpy of 195 kJ/kg. The exit of water is 15 m above its entry. The hot water supplies 30 MJ/h heat to the conference hall. Calculate the quantity of water circulated through the pipe per minute. Assume there is no pump in the system and change in kinetic energy is negligible. *[11.15 kg/mim]*

23. Air at 50 bar and 300 K is flowing through a pipeline. An evacuated and insulated cylinder of 0.1-m^3 volume is connected to the pipeline through a valve. The valve is opened and the cylinder is filled with air till the pressure in the cylinder reaches 50 bar and then the valve is closed. Determine the temperature of air in the cylinder at the end of the filling operation and mass of air that is filled in the cylinder. Take $\gamma = 1.4$. *[420 K and 4.15 kg]*

24. Steam at a pressure of 20 bar and 500°C is flowing in a pipe. An evacuated tank is connected to this pipe through a valve. The valve is opened and the tank is filled with steam until the pressure is 20 bar and then the valve is closed. The process takes place adiabatically and changes in kinetic and potential energy can be assumed negligible. Determine the temperature of steam in the tank at the end of the filling operation. *[698.4°C]*

25. An insulated cylinder with a volume of 0.1 m^3 contains air at 50 bar and 300 K. The valve of the cylinder is opened allowing some of air to escape till the air pressure in the cylinder reduces to 30 bar. Calculate the temperature of air left in the cylinder and mass of air that escaped from the cylinder. *[29.2 K, 1.775 kg]*

26. A rigid and insulated tank of 0.5-m^3 volume contains nitrogen at 20 bar and 400 K. The valve is opened, allowing some of the nitrogen to escape till pressure in the tank reduces to 4 bar. Assuming nitrogen as an ideal gas, calculate the mass of nitrogen discharged from the tank.

[5.75 kg]

27. A rigid vessel of 1-m^3 capacity contains steam at 20 bar and 300°C. A valve on the tank is opened, allowing some steam to escape until the pressure reduces to 2 bar, and the temperature of the tank is maintained constant while steam discharges from the tank. Calculate the energy transferred as heat. *[41.942 MJ]*

28. A container has a capacity of 3-m^3. It contains air at 1.5 bar and 298 K. Additional air is now pumped until its pressure rises to 30 bar and temperature to 333 K. Calculate the mass of air added to the container. If the container is allowed to cool to its initial temperature, determine the pressure of air in the container. Assuming air as an ideal

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gas, calculate the heat transfer and change in entropy during the cooling process.

[88.9 kg, 26.85 bar, –2346.74 kJ, –7.44 kJ/K]

29. A vessel of 3-m^3 capacity contains air at a pressure of 0.5 bar and a temperature of 25°C. Additional air is now pumped into the system until the pressure rises to 30 bar and the temperature rises to 60°C. Determine the mass of air pumped in and express the quantity as a volume at a pressure of 1.02 bar and a temperature of 20°C. The gas is allowed to attain its temperature again at 25°C. Calculate the pressure in the vessel. Determine the heat transferred and change in entropy of the gas during the cooling process.

[Add. Mass = 92.41 kg, Equivalent volume = 76.2 m^3, 26.84 bar, 2363 kJ, − 7.509 kJ/K]

30. Steam at 0.6 MPa, 200°C enters an insulated nozzle with a velocity of 50 m/s. It leaves at a pressure of 5.5 bar and a velocity of 600 m/s. Determine the final temperature, if the steam is superheated in the final state and the quality, if it is saturated. *[x = 0.99]*

31. Steam at 800 kPa, 300°C is throttled to 200 kPa. Change in kinetic energy is negligible. Determine the final temperature of the steam. *[292.4°C]*

32. Steam enters a steam turbine at a pressure of 1 MPa, 300°C and a velocity of 50 m/s. The steam leaves the turbine at a pressure of 150 kPa and a velocity of 200 m/s. Determine the turbine output per kg of steam flowing, assuming the process to be reversible adiabatic. *[377.5 kJ/kg]*

33. Consider the reversible adiabatic flow of steam through a nozzle. Steam enters a nozzle at 1 MPa, 300°C with a velocity of 30 m/s. The pressure of steam at the nozzle exit is 0.3 MPa. Determine the exit velocity of steam leaving the nozzle.

[737 m/s]

34. Steam is supplied to a turbine at 5.4 MPa, 300°C expanded isentropically to an exhaust pressure at 105.3 kPa. To what pressure must the incoming steam be throttled in order to reduce the work done per kilogram to two third of the work obtained without throttling? Assume the flow through the turbine is still reversible adiabatic and exhaust pressure is also same. *[510 kPa]*

35. Dry saturated steam at 800 kPa enters a steady flow system and is expanded reversibly and isothermally to 500 kPa. There is no change of kinetic energy. The flow rate is 5.0 kg/s. Calculate the work done during expansion. *[95.5 kJ/kg]*

36. A turbine in a steam power plant operating under steady state receives 1 kg/s superheated steam at 4 MPa and 300°C. The steam enters the turbine with the velocity of 10 m/s at an elevation of 5 m from the ground level. The turbine discharges the wet steam of 0.85 quality at 50 kPa pressure with a velocity of 50 m/s at an elevation of 10 m above the ground level. The energy losses as heat from the turbine casing are estimated to be 10 kW. Estimate the power output of the turbine. If the changes in kinetic energy and potential energies are neglected, how much error will be introduced? *[682.17 kW, 0.183%]*

37. The power developed by a turbine in a certain steam power plant is 1200 kW. The heat supplied to the steam boiler is 3360 kJ/kg, the heat rejected by the system to cooling water in the condenser is 2520 kJ/kg and the feed pump work required to pump the condensate back into the boiler is 6 kW. Calculate the steam flow through the cycle in kg/s. *[5125.7 kg/h]*

38. Steam at 5 MPa, 500°C enters a nozzle steadily with a velocity of 80 m/s, and it leaves at 2 MPa and 400°C. The inlet area of the nozzle is 38 cm^2, and heat is being lost at a rate of 8 kJ/s. Determine (a) the mass flow rate of the steam, (b) the exit velocity of the steam, and (c) the exit area of the nozzle.

[(a) 4.43 kg/s, (b) 612.5 m/s, (c) 5.09 cm^2]

39. Steam at 3 MPa and 400°C enters an adiabatic nozzle steadily with a velocity of 40 m/s and leaves at 2.5 MPa and 300 m/s. Determine (a) the exit temperature and (b) the ratio of the inlet to exit area A_1/A_2. *[(a) 370°C, (b) 0.165]*

40. Steam at 0.1 MPa and 150°C enters a diffuser with a velocity of 180 m/s and leaves as saturated vapour at 120°C with a velocity of 50 m/s. The exit area of the diffuser is 0.08 m^2. Determine (i) the mass rate of steam (ii) the rate of heat transfer and (iii) the inlet area of the diffuser.

[(a) 4.484 kg/s, (b) 352 kW, (c) 0.054 m^2]

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41. Steam flows steadily through adiabatic turbine where the inlet conditions of the turbines are 12.5 MPa, 500°C and 80 m/s and exit conditions are 10 kPa, 0.92 dry and 40 m/s. The mass-flow rate of the steam is 25 kg/s. Determine (a) the change in kinetic energy; (b) the power output and (c) the turbine inlet area.

[(a) – 2.4 kJ/kg (b) 23.8 MW (c) 0.0080 m^2]

Objective Questions

1. An adiabatic heat exchanger is used to heat cold water from 15°C entering at a rate of 5 kg/s by hot air at 90°C entering also at 5 kg/s. If the exit temperature of air is 20°C, the exit temperature of water is
   (a) 27°C (b) 85°C
   (c) 32°C (d) 52°C
2. A control volume refers to
   (a) a fixed region in space
   (b) a fixed quantity of matter
   (c) an isolated system
   (d) a closed system
3. If all the variables of a flow system are independent of time, the system is said to be
   (a) closed flow (b) steady flow
   (c) constant flow (d) unsteady flow
4. Internal energy of a perfect gas depends on
   (a) temperature, specific heat and enthalpy
   (b) temperature, specific heat and entropy
   (c) temperature, specific heat and pressure
   (d) temperature only
5. Steady flow occurs when
   (a) properties do not change with time
   (b) the system is in equilibrium with its surroundings
   (c) properties change with time
   (d) when $\left(\frac{\partial v}{\partial t}\right)$ is constant
6. A heat exchanger is used to heat cold water from 15°C entering at a rate of 2 kg/s by hot air at 100°C entering at 3 kg/s. The heat exchanger is not insulated and is losing heat at a rate of 40 kJ/s. If the exit temperature of air is 20°C, the exit temperature of water is
   (a) 44°C (b) 72°C
   (c) 49°C (d) 39°C
7. A nozzle
   (a) accelerates the fluid flow
   (b) decelerates the fluid low
   (c) does not affect the velocity of fluid
   (d) none of the above
8. During a throttling process
   (a) internal energy remains constant
   (b) enthalpy of fluid remains constant
   (c) pressure remains constant
   (d) temperature remains constant

Answers

1. (c) 2. (a) 3. (b) 4. (d) 5. (a) 6. (a) 7. (a) 8. (b)

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6

Second Law of Thermodynamics

Introduction

Energy has a quantity as well as quality. The first law of thermodynamics deals with energy as a property and states that the quantity of energy must be conserved during a process. The second law of thermodynamics deals with the quality of energy and asserts that the processes occur in a certain direction only. The second law also recognizes that the mutual conversion of heat into work during a process is impossible, while its opposite is possible. A process cannot take place unless it satisfies both the first and second laws of thermodynamics.

Thermal reservoir, operation of heat engine, refrigerator, and heat pump are discussed first in this chapter to explain the two statements of the second law. Reversible and irreversible processes are explained to give insight to the performance of the devices. Then the Carnot cycle and absolute temperature are introduced.

6.1 LIMITATIONS OF THE FIRST LAW OF THERMODYNAMICS

The first law of thermodynamics states that during any cyclic process, the net work transfer is always directly proportional to heat transfer and therefore, work and heat are mutually convertible one into another. A certain energy balance must be held when a system undergoes a process.

But the first law of thermodynamics does not impose any restriction on the direction in which the process is feasible. As far as the first law is concerned, all heat transferred to a heat engine could be converted to useful work. But we all know it is not possible to convert net heat into work.

Further, it is our experience that if a glass of hot milk is left in the room, it eventually cools off after a certain period. The amount of heat rejected by the milk is gained by the room air and the quantity of heat is balanced. Now let us consider the reverse of the process, i.e., getting cold milk hotter as a result of heat transfer from the room air. But such a process is impossible in nature.

Another common example is that an electrical work can be converted into heat energy through an electric heating element, but its reverse process—supplying an amount of heat to this heating element—will not generate any amount of electric work.

Other common example is when a cycle rider

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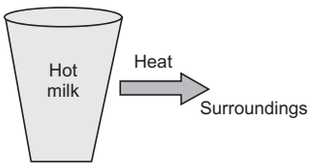
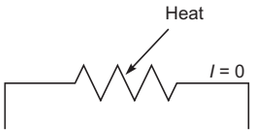


(a) Hot milk losing heat to surroundings



(b) Transferring heat to heating element will not generate electricity

Fig. 6.1

stops his cycle by applying friction brakes. The kinetic energy of the moving wheel is absorbed by brake blocks, whose temperature rises, and hence the kinetic energy lost by the moving wheel is converted into heat energy. The first law should be equally satisfied, if the brake blocks are to cool off and they give their energy back to the cycle wheel, causing it to rotate. However, it is never seen in practice.

Other common experiences are the following:

(i) Heat always flow in the direction of decreasing temperature.

(ii) Water always flows downwards.

(iii) A paddle wheel is operated by falling weight. As it rotates, it stirs a fluid within an insulated container. The decrease in potential energy of weight is equal to increase of internal energy of the fluid mass. However, the reverse of the process—raising the weight by decreasing internal energy of the fluid—does not occur in nature.

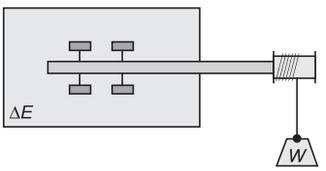


Fig. 6.2 *Supplying heat to a paddle wheel will not cause it to rotate*

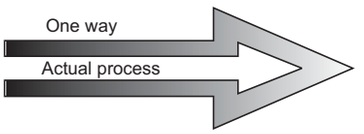


Fig. 6.3 *Process occurs in a certain direction and not in reverse direction*

It is clear from the above discussion that the processes proceed in a certain direction only and not in the reverse direction. The first law of thermodynamics does not place any restriction on the direction of process. Satisfying the first law does not ensure that the process will actually occur, because it only keeps the account of energy in quantity during any process. The second law of thermodynamics is introduced to overcome the remedies of the first law of thermodynamics. The *second law* of thermodynamics takes into account the direction of process as well as quality of energy, and it states that *in any process, high-grade energy can only be converted to low-grade energy*. The energy cannot be upgraded by its own. According to the second law of thermodynamics, heat and work can be classified as low and high-grade energy, respectively. *Therefore, net quantity of work can be converted in any form of energy, but net quantity of heat cannot.*

6.2 THERMAL RESERVOIR

It is a hypothetical body with an infinite heat capacity. A *thermal reservoir* can supply or absorb any amount of heat without affecting its temperature. For example, large bodies of water such as oceans, rivers as well as the atmosphere can be considered as thermal reservoirs.

A two-phase system of fluids can also be treated as a thermal reservoir as long as it does not return to single phase, since the two-phase system can absorb or release any amount of heat while maintaining a constant temperature. Similarly, large industrial furnaces can also be treated as thermal reservoirs. They are maintained almost at constant temperatures and are capable of transferring a large quantity of heat energy in an isothermal manner.

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A reservoir that supplies heat energy is called heat source and one that absorbs heat energy is called heat sink.

6.3 HEAT ENGINE

We know that the net amount of heat cannot be converted into work directly but conversion of heat into work requires some special devices. Such devices are called *heat engines.*

In a broader sense, work-producing devices may be included in heat engines such as internal combustion engines, steam and gas turbines. But these devices are operated in mechanical cycles, not in thermodynamic cycles, since the working fluid does not undergo a complete cycle.

A heat engine is a device which operates in a cycle. It receives the heat energy from a high-temperature reservoir, converts some of the heat energy into work and rejects the remaining heat to a low-temperature reservoir. A steam power plant best fits into the definition of a heat engine. The schematic diagram of a basic steam power plant and a heat engine are shown in Fig. 6.4. The working of the heat engine can be illustrated as follows:

1. It receives heat Q_H from the high-temperature reservoir (source) at T_H.
2. It converts the part of heat supplied into useful work as W_{net}.
3. It rejects the remaining heat Q_L to a low-temperature reservoir (sink) at T_L.

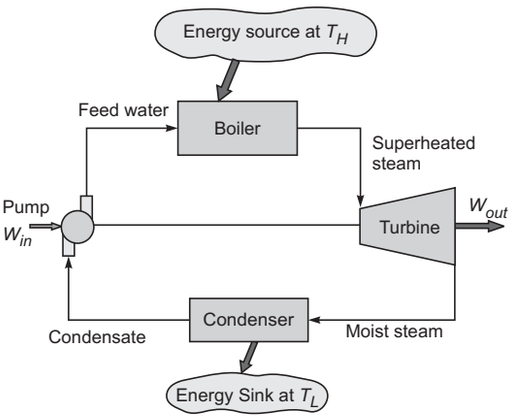
The net work output of a heat engine is the difference between the total work done by the turbine and work input to feed pump, etc.

$$W_{net} = W_{out} - W_{in}$$

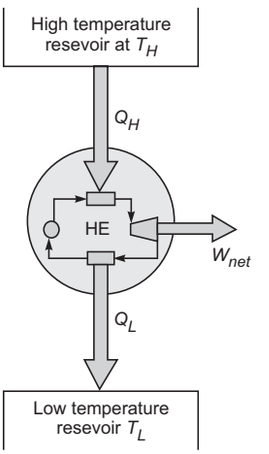
For any cyclic heat engine, the net work can also be obtained from the difference between heat supplied and heat rejection.

$$W_{net} = Q_H - Q_L \qquad ...(6.1)$$

Thermal Efficiency Thermal efficiency is the performance-measuring parameter of a heat engine. In general, it is defined as the ratio of the desired output to the energy input. For a heat engine, the desired output is the net work W_{net}, and the energy input is the heat supplied Q_H. Thermal efficiency is denoted by the symbol 'η_{th}'.



(a) Schematic diagram of a steam power plant



(b) Schematic diagram of a heat engine

Fig. 6.4

Thermal efficiency,

$$\eta_{th} = \frac{\text{Net work output}}{\text{Energy input as heat}} = \frac{W_{net}}{Q_H}$$

or

$$\eta_{th} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H} \qquad ...(6.2)$$

Since Q_H and Q_L both are positive quantities, thermal efficiency of the work producing devices is always less than unity. The ordinary spark ignition internal combustion engine is about 30% efficient,

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while a diesel engine is about 32% efficient. Large gas and steam power plants have a thermal efficiency of about 45%.

6.4 REFRIGERATOR

A refrigerator is a device, operating in a cycle, that maintains a body at lower temperature than its surroundings. A refrigerator extracts heat continuously from a controlled space, and thus it is maintained at a lower temperature than its surroundings. The working fluid in the refrigerator is called the *refrigerant*.

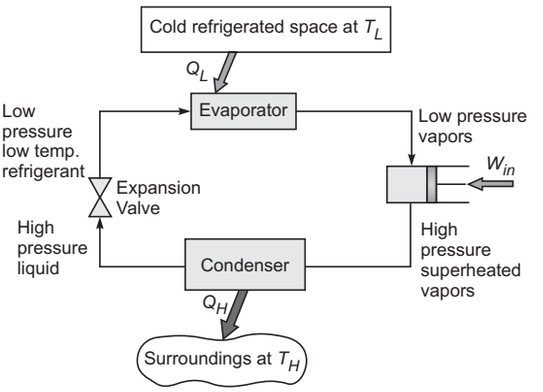
The most frequently used refrigeration cycle is the *vapour compression cycle*. The vapour compression cycle is used in household refrigerators. Its basic components are: a compressor, a condenser, an expansion device and an evaporator as shown in Fig. 6.5(a).

The refrigerant enters the compressor as low pressure vapour and is compressed to high pressure and temperature. In the condenser, the refrigerant vapour passes through the long condenser coils, where it rejects its latent heat to the surroundings. The liquid refrigerant then enters a capillary tube (an expansion device for small refrigerators), where its pressure reduces drastically due to throttling effect and its temperature also reduces. This low-temperature and low pressure refrigerant then passes through the evaporator coils (*freezer*), where it evaporates by absorbing heat from the refrigerated space to keep it at low temperature. The cycle completes as the refrigerant re-enters the compressor.

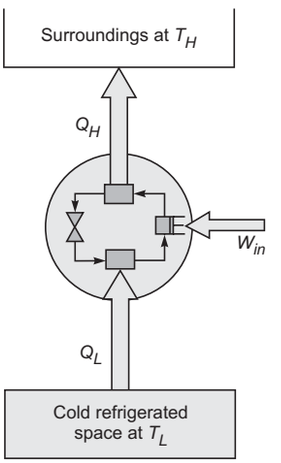
Coefficient of Performance of Refrigerator $(COP)_R$

The performance of a refrigerator is measured in terms of the *coefficient of performance* $(COP)_R$. It is defined as the ratio of the desired output (*refrigerating effect*) to the energy (*work*) input.

Consider an amount of heat Q_L is removed from the refrigerated space at temperature T_L. The work input to the compressor is W_{in} and the heat rejected at the condenser is Q_H.



(a) Basic components of a refrigerator



(b) Schematic diagram of a refrigerator

Fig. 6.5

$$(COP)_R = \frac{\text{Refrigerating Effect}}{\text{Work input}} = \frac{Q_L}{W_{in}} \quad ...(6.3)$$

From the heat energy balance for a cyclic device, the work input W_{in} can be expressed as

$$W_{in} = Q_H - Q_L$$

Hence, $$(COP)_R = \frac{Q_L}{Q_H - Q_L} \quad ...(6.4)$$

The value of $(COP)_R$ of a refrigerator may be greater than unity. That is, the amount of heat removed from the refrigerated space may be greater than the work input.

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6.5 HEAT PUMP

A heat pump is a device, operating in a cycle, that maintains a space at a higher temperature than the surroundings. The heat pump supplies heat continuously to the controlled space, and thus maintained at a higher temperature than its surroundings. The refrigerator and heat pump are operated on the same thermodynamic cycle, but they differ in their objectives. The discharge of heat at higher temperature to a space is the objective for a heat pump. The heat pump absorbs the heat from low-temperature surroundings and supplies it to a higher temperature space at the cost of work input to the compressor.

If an ordinary refrigerator is placed in a window of a house with its evaporator open to the outside, and its condenser located in the room. Then the refrigerator acts as a heat pump and it supplies heat to the room.

Let an amount of heat Q_L be absorbed from the low temperature region and W_{in} be the work input. Then the heat supply, Q_H to the room is the desired effect for a heat pump.

The coefficient of performance of heat pump is expressed as

$$(COP)_{HP} = \frac{\text{Heat supplied}}{\text{Work input}}$$

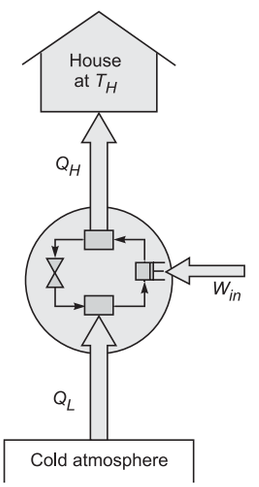

Fig. 6.6 *Schematic of a heat pump*

$$(COP)_{HP} = \frac{Q_H}{W_{in}} \quad ...(6.5)$$

But $$Q_H = Q_L + W_{in}$$

$$(COP)_{HP} = \frac{Q_L + W_{in}}{W_{in}}$$

$$= 1 + \frac{Q_L}{W_{in}} = 1 + (COP)_R \quad ...(6.6)$$

For given values of Q_H and Q_L, the coefficient of performance of a heat pump is always greater than the coefficient of performance of a refrigerator by unity.

6.6 STATEMENTS OF THE SECOND LAW OF THERMODYNAMICS

6.6.1 Kelvin–Planck Statement

It relates the working of heat engines and is stated as

"*It is impossible to construct an engine which while operating in a cycle, produces no other effect except to extract the heat from a single temperature reservoir and do equivalent amount of work*".

According to the Kelvin–Planck statement, a heat engine cannot be 100 per cent efficient. In practice, no heat engine can convert all heat supplied to useful work. The engine receives heat from a high-temperature reservoir and it must reject some amount of heat to a low-temperature reservoir. The work done by a cyclic heat engine is the difference between heat supplied and heat rejection.

6.6.2 Clausius Statement

It relates the working of the refrigerators and heat pumps and it can be stated as,

"*It is impossible for any device that operates in a cycle and produce no effect other than the transfer of heat energy from a lower temperature body to higher temperature body*".

It is well-known tendency of heat energy to flow from a hot body to a cold body, and it cannot flow by itself from a low-temperature body to a high-temperature body without addition of work.

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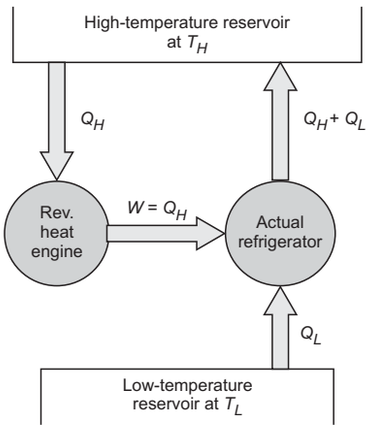
Refrigerators and heat pumps transfer energy from a low-temperature region to higher temperature region at the cost of work input to their compressors.

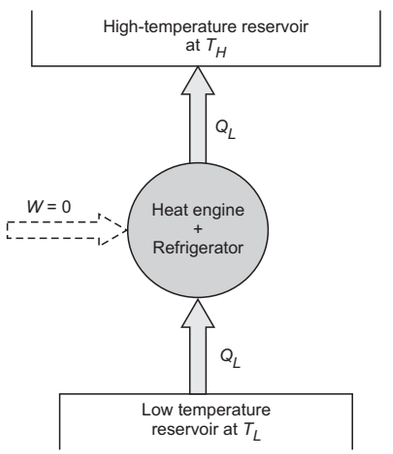
6.6.3 Equivalence of Two Statements

At first sight, the two statements appear to be unrelated but it can easily be proved that these are two parallel statements of the second law. Any device that violates one statement also leads to violation of the other statement.

Consider a heat engine (PMM2) and an actual refrigerator as shown in Fig. 6.7, operating between a high temperature T_H and a low temperature T_L. The heat engine is receiving heat Q_H from the high-temperature reservoir at T_H and it converts all heat into net work W_{net}. It does not *reject any amount of heat to the low-temperature reservoir*, thus violating Kelvin–Planck statement of the second law.



(a) Refrigerator powered by 100% efficient engine



(b) The device (heat engine + refrigerator) which violates Clausius's statement

Fig. 6.7

Let us assume the work produced by the engine is supplied to a cyclic refrigerator, that removes heat Q_L from a low-temperature reservoir at T_L and discharges heat in amount $Q_H + Q_L$ to a high-temperature reservoir at T_H. Now, if the refrigerator and heat engine are grouped together, they constitute a device, whose sole effect is to *transfer heat energy Q_L from the low temperature reservoir to the high-temperature reservoir without any work input from outside*.

This device clearly violates the Clausius statement. Therefore, the violation of Kelvin–Planck statement leads to violation of Clausius statement.

It can also be proved that violation of Clausius statement leads to the violation of the Kelvin–Planck statement.

6.7 PERPETUAL MOTION MACHINE OF THE SECOND KIND

According to the first law of thermodynamics, the heat and work are mutually convertible one into another. If the total heat supplied is converted into net work by any machine then its efficiency would be 100 percent and it is called a *perpetual motion machine of the second kind* (PMM2). A heat engine (of Fig. 6.7a) is PMM2 which absorbs heat from a single temperature reservoir and converts it completely into work, leading to the violation of the second law.

Consider a steam power plant without a condenser as shown in Fig. 6.8. It is thought that the condenser rejects more than one half of heat supplied as waste. Let such a steam power plant be where all the heat transferred into the boiler will be converted to work by turbine, and thus the power plant will theoretically have 100 per cent efficiency. It is a perpetual motion machine of the second kind

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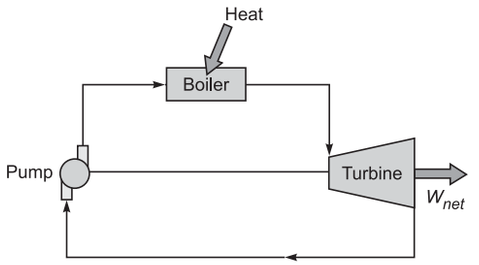

Fig. 6.8 *Perpetual motion machine of the second kind*

which works in a cycle satisfying the first law, but violates the second law. Therefore, it will not work, because heat is a low-grade energy and cannot be completely converted into work.

6.8 REVERSIBLE PROCESS: IDEAL PROCESS

A reversible process is defined as a process that once having taken place in a direction, can be reversed without leaving any trace on either system or surroundings.

A reversible process is one which is proceeded in such a way that at the conclusion of the process, both system and surroundings may be restored to their initial states without producing any change in the universe. This is possible only if the net transfer of heat and work between the system and surroundings for original and reverse process is zero.

Once a cup of hot tea cools off due to heat transfer to its surroundings, it cannot heat up again by recovering the lost heat from the surroundings without any external effect. If it could be possible then the system and surroundings can be restored to their original states and it would be a reversible process.

Actually reversible processes do not occur in nature. They are idealization of actual processes. Reversible processes can be approximated by actual devices, but they can never be achieved. Reversible processes are easy to analyse, since the system passes through a series of equilibrium states. Engineers are always interested in reversible processes, because *work-producing devices such as engines and turbines develop maximum work and work consuming devices, such as compressors, blowers and pumps, etc., consume least work when they are operated reversibly.*

Let the initial state of the system be represented by *A* and let the system be taken to the final state *B* by the path *APB* as shown in Fig. 6.9. If the system and surroundings both are restored to their initial states without any change in the universe, i.e., the system returns by the same path then the process *APB* is called a reversible process.

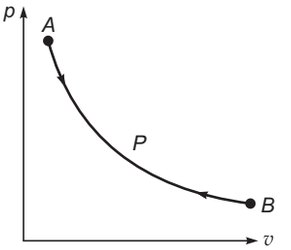

Fig. 6.9 *Reversible process*

6.9 IRREVERSIBLE PROCESSES: ACTUAL PROCESSES

All processes occuring in nature are irreversible. When these processes are reversed, they cannot return to their initial state of the system without changing the surroundings. And when such processes are reversed, they always leave some mark on the surroundings or system. The *irreversibility* of the processes may be caused due to

(i) Mechanical or thermal irreversibility
(ii) Internal and external irreversibility

Mechanical irreversibility is associated with friction. When two bodies have relative movement, a frictional force opposes the motion at the interface of these two bodies and some work is lost to overcome this frictional force. When the direction of motion is reversed, some work is further required to overcome the friction. Friction is also involved between fluid and solid surfaces and between the layers of fluid moving at different velocities during any process.

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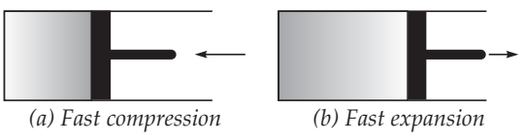
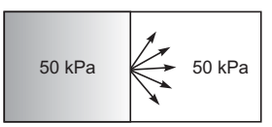
Thermal irreversibility is associated with heat transfer due to finite temperature difference between a system and its surroundings. An amount of heat lost from a system during a compression process cannot be regained during an expansion process, thus causing irreversibility.

All actual processes are internally irreversible. During an *internally irreversible* process, a system does not proceed through a series of equilibrium states. Consider an amount of gas in the piston–cylinder device. If the piston is pushed rapidly, the gas molecules near the piston face will not have sufficient time to escape and they will pile up in front of the piston face as shown in Fig. 6.10(a). This will raise the pressure near the piston face, while the gas in the other part will have lower pressure. Because of this higher value of pressure near the piston face, the compression process will require a large work input than required for a quasi-static process. The non-uniformity of pressure will cause the process irreversible.



(a) Fast compression *(b) Fast expansion*



(c) Unstrained expansion

Fig. 6.10 *Irreversible compression and expansion processes*

When the process is reversed and gas expands rapidly, the gas molecules in the cylinder will not be able to follow the piston as fast, thus creating a low-pressure region before the piston face as shown in Fig. 6.10(b). Because of this low-pressure value at the piston face, the process delivers less work than a corresponding reversible one. Thus, the work done by the gas during expansion is less than the work done by the surroundings on the gas during compression. Thus, surroundings have a net deficit of work.

Internal irreversibility is also caused due to mixing of different layers of fluid at different temperatures. It is also due to free or unrestrained expansion of fluid as shown in Fig. 6.10(c).

External irreversibility is associated with friction at bearings and friction between atmospheric air and rotating members. The system absorbs some work to overcome these frictional forces.

Conditions for a Reversible Process A reversible process should possess the following properties:

1. The process should not involve friction of any kind.
2. Heat transfer should not take place due to finite temperature difference between the system and surroundings.
3. There should not be a mixing of fluid layers at different temperatures.
4. There should not be free and unrestrained expansion.
5. The process must proceed through a series of equilibria.

6.10 CARNOT CYCLE, OR CARNOT ENGINE

A french engineer, Sadi Carnot, in 1824 proposed an engine which works on a reversible cycle. It is theoretically a heat engine that converts the maximum amount of energy into mechanical work. Carnot showed that the efficiency of any engine depends on the difference between the highest and lowest temperatures reached during one cycle. The greater the temperature difference, higher the efficiency.

The Carnot cycle, also called *reversible cycle*, comprises of four reversible processes as given:

1. Reversible isothermal expansion,
2. Reversible adiabatic (isentropic) expansion,
3. Reversible isothermal compression, and
4. Reversible adiabatic (isentropic) compression.

The working of the engine is idealized with the following assumptions:

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1. The working substance for the Carnot engine is a perfect gas.
2. The piston movement in the cylinder is frictionless.
3. The walls of the cylinder and piston are considered perfectly insulated.
4. The cylinder head is so arranged that it is partially a very good conductor of heat and partially a perfect insulator.
5. The heat supply and heat rejection are at constant temperatures.

6.10.1 Working of a Carnot Engine

1. Reversible Isothermal Expansion (Process 1–2)
The cylinder head is a good conductor of heat, and the heat source is brought in its contact. The heat is added to the working medium at constant temperature T_H. The gas expands slowly in an isothermal manner from the state 1 to state 2.

Heat addition during the process,

$$Q_H = p_1 V_1 \ln\left(\frac{V_2}{V_1}\right)$$

The work done during the process,

$$W_{1-2} = Q_H = p_1 V_1 \ln\left(\frac{V_2}{V_1}\right)$$

$$= m R T_H \ln\left(\frac{V_2}{V_1}\right) \qquad ...(6.7)$$

2. Reversible Adiabatic Expansion (Process 2–3)
The cylinder head is now made the insulator. The gas continues to expand slowly doing work on the surroundings until the temperature drops to T_L. The piston movement is assumed frictionless, therefore, the process is adiabatic and reversible (isentropic).

Heat transfer, $Q_{2-3} = 0$

The work done on the gas,

$$W_{2-3} = \frac{p_3 V_3 - p_2 V_2}{\gamma - 1} = \frac{m R\left(T_H - T_L\right)}{\gamma - 1} \qquad ...(6.8)$$

3. Reversible Isothermal Compression (Process 3–4)
The insulation from the cylinder head is removed and it is made a good conductor of heat and brought in contact of the thermal reservoir at low

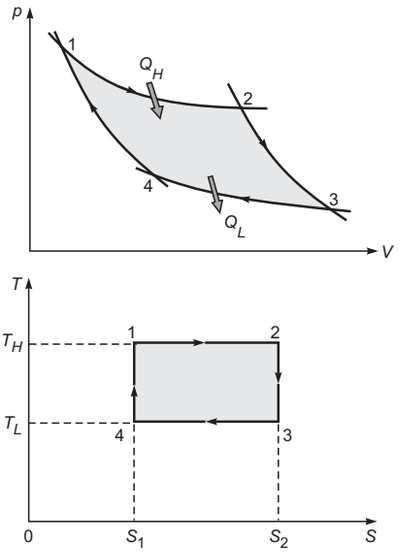

Fig. 6.11 *Carnot Cycle on p–V and T–S diagram*

temperature (heat sink). The piston is pushed inward by an external force and the gas is compressed. As the temperature of gas tends to rise, its heat is absorbed by the heat sink, keeping the temperature of gas constant at T_L. Therefore, heat is rejected in an isothermal manner from the state 3 to 4.

Heat rejection during the process,

$$Q_L = p_3 V_3 \ln\left(\frac{V_3}{V_4}\right) = m R T_L \ln\left(\frac{V_3}{V_4}\right) \qquad ...(6.9)$$

The work done on the gas,

$$W_{3-4} = Q_L = p_3 V_3 \ln\left(\frac{V_3}{V_4}\right)$$

4. Reversible Adiabatic Compression (Process 4–1)
The contact of the heat sink with the cylinder head is removed and the cylinder head is again made an insulator, and the gas is allowed to compress isentropically from the state 4 to 1.

Heat transfer during the process,

$$Q_{4-1} = 0$$

The work done on the gas,

$$W_{4-1} = \frac{p_1 V_1 - p_4 V_4}{\gamma - 1}$$

$$= \frac{mR\left(T_H - T_L\right)}{\gamma - 1} \qquad ...(6.10)$$

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6.10.2 Thermal efficiency of a Carnot Heat Engine

The thermal efficiency of any heat engine can be calculated as

$$\eta_{th} = \frac{\text{Net work done}}{\text{Heat suplied}} = \frac{W_{net}}{Q_H}$$

For a cyclic heat engine,

$$W_{net} = Q_H - Q_L$$

Therefore, $\eta_{th} = \dfrac{Q_H - Q_L}{Q_H} = 1 - \dfrac{Q_L}{Q_H}$

Using Eqs. (6.7) and (6.9), we get

$$\eta_{th} = 1 - \frac{m R T_L \ln(V_3/V_4)}{m R T_H \ln(V_2/V_1)} \quad ...(6.11)$$

Considering the isentropic process 2–3;

$$p_2 V_2^{\gamma} = p_3 V_3^{\gamma}$$

and $$\frac{T_H}{T_L} = \left(\frac{V_3}{V_2}\right)^{\gamma-1} \quad ...(6.12)$$

Similarly, for the isentropic process 4–1,

$$p_1 V_1^{\gamma} = p_4 V_4^{\gamma}$$

and $$\frac{T_H}{T_L} = \left(\frac{V_4}{V_1}\right)^{\gamma-1} \quad ...(6.13)$$

Equating the above two equations , we get

$$\frac{V_3}{V_2} = \frac{V_4}{V_1}$$

or $$\frac{V_3}{V_4} = \frac{V_2}{V_1} \quad ...(6.14)$$

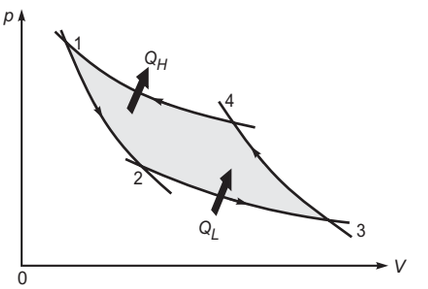
Substituting in Eq. (6.11), we get;

$$\eta_{rev} = 1 - \frac{T_L}{T_H} \quad ...(6.15)$$

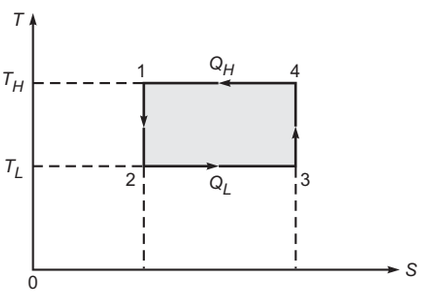
Equation (6.15) reveals that the Carnot cycle efficiency is independent of the working substance. It only depends on source and sink temperatures. The efficiency becomes maximum when the sink temperature T_L approaches zero, but it is not possible because it violates the Kelvin–Planck's statement of the second law. Hence, any value of the efficiency can be obtained when the temperature difference $(T_H - T_L)$ will have a finite value.

6.11 REVERSED CARNOT CYCLE

Since the Carnot cycle is reversible, therefore, its direction can be reversed. In such a case, it can be operated as a *reversible heat pump* or a *reversible refrigerator*. The cycle undergoes the same path and processes, only their directions are reversed. Now the cycle requires the work input W_{net} and it transfers heat Q_L from the low temperature reservoir to the high-temperature reservoir. Its *p–V* and *T–S* diagrams are shown in Fig. 6.12.



(a) p-V diagram for a reversed Carnot cycle



(b) T-S diagram for a reversed Carnot cycle

Fig. 6.12

6.12 CARNOT THEOREM

The *Carnot theorem* is also called *Carnot principle*. It states that

(1) *No engine can be more efficient than a reversible engine operating between the same two temperature reservoirs, or a Carnot* (*reversible*) *engine among all engines is the most efficient.*

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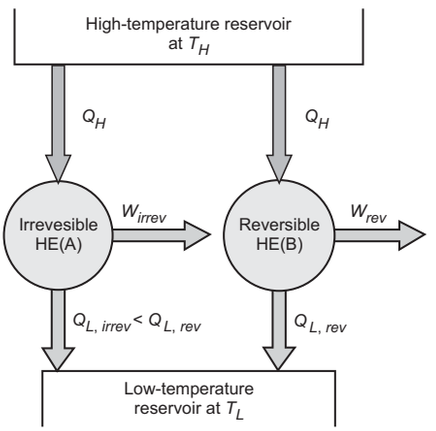
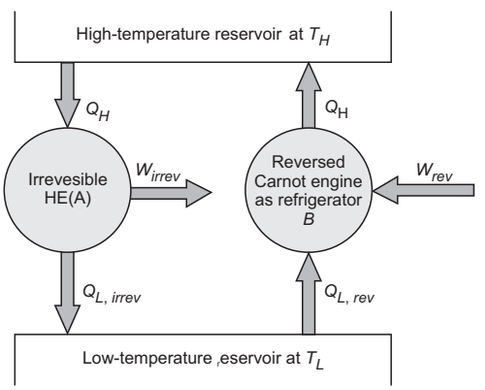
(2) *The efficiency of all reversible heat engines operating between the same two temperature reservoirs are the same.*

To prove the first principle, consider two heat engines operating between the same two thermal reservoirs as shown in Fig. 6.13(a). The engine *A* is an irreversible engine and the engine *B* is a reversible *engine.* Each engine is supplied with an equal amount of heat Q_H. Let the amount of work W_{rev} produced by the reversible engine *B* be less than the work W_{irrev} produced by an irreversible engine *A*, thus irrevesible engine will be more efficient than the reversible engine.

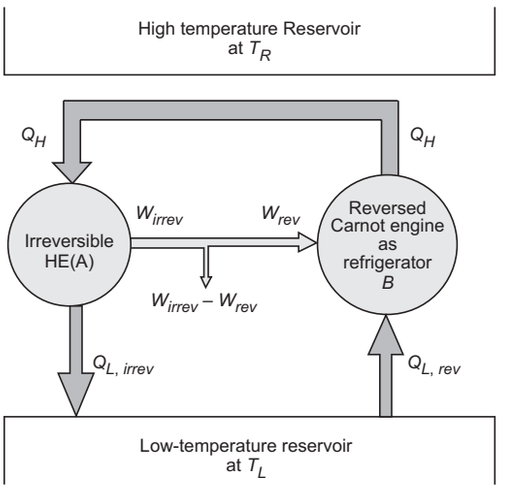
$$\eta_A > \eta_B \quad \text{or} \quad W_{irrev} > W_{rev}$$



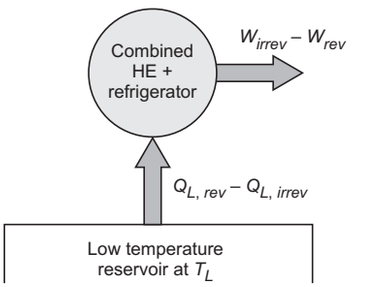
(a) A reversible and irreversible heat engine operating between the same two reservoirs



(b) Arrangement with an irreversible engine and reversed cannot engine operating as a refrigerator



(c) Arrangement with irreversible heat engine drives reversed Carnot engine



(d) Equivalent combined arrangement

Fig. 6.13

Now let the reversible heat engine *B* be reversed as shown in Fig. 6.13(b). The magnitude of heat and work transfer will remain same, but their direction will be reversed and it will operate as a refrigerator. The reversible refrigerator *B* will require the work input of W_{rev}, a part of W_{irrev} and it will supply heat Q_H to a high-temperature reservoir equal to heat input to the irreversible heat engine *A*. Thus, the net heat exchange from high temperature reservoir becomes zero and it can be eliminated as shown in Fig. 6.13(c). The heat discharged by the refrigerator *B* may directly be supplied to the irreversible engine *A*.

The irreversible engine *A* and the reversible refrigerator *B* together constitute a machine as shown

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in Fig. 6.13(d) that produces the net work $W_{irrev} - W_{rev}$, while exchanging the heat $(Q_{L,\,rev} - Q_{L,\,irrev})$ with the low-temperature reservoir only. Thus, this machine *violates the Kelvin Planck's statement* of the second law. Therefore, our initial assumption that $\eta_{irrev} > \eta_{rev}$ is wrong. Thus we can conclude that no heat engine is more efficient than a reversible heat engine operating between the same two thermal reservoirs.

The second Carnot principle can also be proved in a similar manner by replacing the irreversible engine *A* by a reversible engine that is more efficient and thus delivers more work than the reversible engine *B*. By the same reasoning, we can conclude that a machine which operates on a single-temperature reservoir and produces an equivalent amount of work, thus violates the Kelvin Planck's statement of the second law. Therefore, we can state that no reversible heat engine can be more efficient than another reversible engine operating between the same two temperature limits.

6.13 THERMODYNAMIC TEMPERATURE SCALE

A temperature scale that is independent of the properties of the substance and is used to measure the temperature is called a thermodynamic temperature scale.

The thermal efficiency of any heat engine cycle receiving heat Q_H from a source at T_H and rejecting heat Q_L to a sink at T_L is given by

$$\eta_{th} = 1 - \frac{Q_L}{Q_H} = \phi(T_H, T_L) \qquad ...(6.16)$$

It is necessary to maintain a temperature difference in order to obtain the work output from an engine. Further, the efficiency of all reversible heat engines operating between the same temperature limits is same, because the efficiency of any reversible heat engine is only a function of source and sink temperatures. A functional relationship between heat supply and heat rejection can be expressed as

$$\frac{Q_H}{Q_L} = \phi(T_H, T_L) \qquad ...(6.17)$$

If some relationship is assigned between T_H, T_L, Q_H and Q_L, the resulting expression will be the definition of the temperature scale.

Let us consider three Carnot heat engines as shown in Fig. 6.14. The engines *A* and *C* are supplied with the same amount of heat Q_1 from the source at the temperature T_1. The engine *C* rejects heat Q_3 to the sink at the temperature T_3. The engine *A* rejects heat Q_2, which is supplied to the engine *B*. The engine *B* rejects heat Q_3 to the sink at T_3. The amount of heat rejected by engines *B* and *C* is same. Since the engines *A* and *B* can be combined into one reversible heat engine, operating between same temperature limits as the engine *C*, the combined engine will have the same efficiency as the engine *C* has.

Applying Eq. (6.17) to all three engines separately;

$$\frac{Q_1}{Q_2} = \phi(T_1, T_2),$$

$$\frac{Q_2}{Q_3} = \phi(T_2, T_3), \quad \text{and} \quad \frac{Q_1}{Q_3} = \phi(T_1, T_3)$$

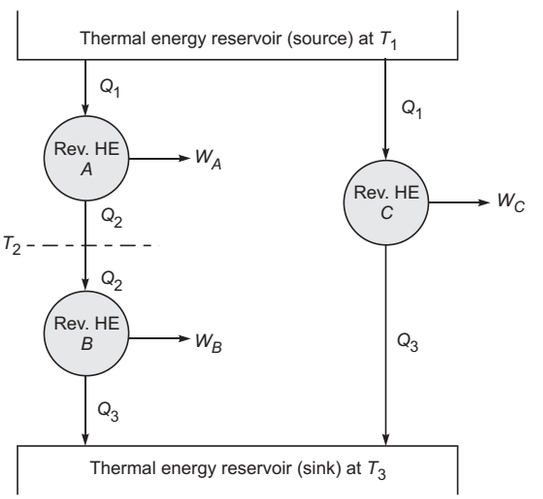

Fig. 6.14 *Arrangement of reversible heat engines to define absolute temperature scale*

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The identity of the reversible heat engines gives

$$\frac{Q_1}{Q_3} = \frac{Q_1}{Q_2} \times \frac{Q_2}{Q_3}$$

$$= \phi(T_1, T_2) \times \phi(T_2, T_3) \quad ...(6.18)$$

But the ratio Q_1/Q_3 depends only on temperatures T_1 and T_3, and is independent of the temperature T_2. This condition will be satisfied only, if the function ϕ has the following form;

$$\phi(T_1, T_2) = \frac{\varphi(T_1)}{\varphi(T_2)} \quad \text{and} \quad \phi(T_2, T_3) = \frac{\varphi(T_2)}{\varphi(T_3)}$$

In this form, $\varphi(T_2)$ will be cancelled from the product of $\phi(T_1, T_2)$ and $\phi(T_2, T_3)$ and the right-hand side functional relation yields to

$$\frac{Q_1}{Q_3} = \phi(T_1, T_3) = \frac{\varphi(T_1)}{\varphi(T_3)}$$

This relation is more specific for functional form of Q_1/Q_3 in terms of T_1 and T_3. Thus, for a reversible heat engine operating between two temperatures T_H and T_L;

$$\frac{Q_H}{Q_L} = \phi(T_H, T_L) = \frac{\varphi(T_H)}{\varphi(T_L)} \quad ...(6.19)$$

Several functions of $\varphi(T)$ will satisfy this equation and the choice is completely arbitrary. Lord Kelvin first proposed by taking $\varphi(T) = T$ to a thermodynamic scale as

$$\frac{\varphi(T_H)}{\varphi(T_L)} = \frac{T_H}{T_L} \quad ...(6.20)$$

This temperature scale is called the *Kelvin scale* and temperature on this scale are called *absolute temperature*. Thus, on the Kelvin scale, for any reversible heat engine, the ratio of temperatures associated with the source and sink is equal to the ratio of energy supplied by the source and energy rejected to the sink.

At the International Conference on Weights and Measures held in 1954, the triple point of water was assigned a value of 273.15 K. The *magnitude of one kelvin* is defined as 1/273.15 of the temperature interval between absolute zero and the triple-point temperature of water. The magnitudes of temperature units on Kelvin and Celsius scales are identical (1 K ≡ 1°C) and the temperature on these two scales is related as

$$T(\text{K}) = T(°\text{C}) + 273.15$$

6.13.1 The Carnot Refrigerator and Heat Pump

A refrigerator that operates on reversed Carnot Cycle is called a *Carnot refrigerator.*

$$(COP)_R = \frac{Q_L}{Q_H - Q_L} = \frac{1}{(Q_H/Q_L) - 1}$$

Since the performance of all reversible devices operating between the same two temperature limits is same, the *COP* of all reversible refrigerators will be equal. The ratio Q_H/Q_L is replaced by T_H/T_L. Thus, for a reversible refrigerator

$$(COP)_{R,\,rev} = \frac{1}{(T_H/T_L) - 1} = \frac{T_L}{T_H - T_L} \quad ...(6.21)$$

Similarly, the *COP* of a heat pump is expressed earlier by Eq. (6.6)

$$(COP)_{HP} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - (Q_L/Q_H)}$$

For a reversible heat engine, replacing Q_L/Q_H by T_L/T_H, we get

$$(COP)_{HP,\,rev} = \frac{1}{1 - (T_L/T_H)} = \frac{T_H}{T_H - T_L} \quad ...(6.22)$$

It is the inverse of efficiency of a reversible heat engine.

Futher, Eq.(6.21) and (6.22) represent the highest coefficient of performance that a refrigerator or a heat pump can attain, when operating between the same two temperature limits of T_H and T_L, because they require least work input for a given output effect.

The coefficient of performance of an actual and a reversible refrigerator can be compared as

$$(COP)_R \begin{cases} < (COP)_{R,rev} & \text{for an irreversible refrigerator} \\ = (COP)_{R,rev} & \text{for a reversible refrigerator} \\ > (COP)_{R,rev} & \text{for an impossible refrigerator.} \end{cases} \quad ...(6.23)$$

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A similar relation can be established between an actual and a reversible heat pump by replacing $(COP)_R$ by $(COP)_{HP}$ in Eq. (6.23).

6.13.2 Effect of Temperature T_H and T_L on Efficiency of Carnot Cycle

The efficiency of the Carnot engine is expressed by Eq. (6.15)

$$\eta_{rev} = 1 - \frac{T_L}{T_H}$$

The efficiency of a reversible engine can be improved by

(i) by increasing the temperature T_H of the high-temperature reservoir, or

(ii) by decreasing the temperature T_L of the low-temperature reservoir.

Both possibilities are analysed below:

1. Keeping the temperature T_L of sink constant, and differentiating above relation with respect to T_H, for maximizing the efficiency,

$$\left(\frac{\partial \eta_{rev}}{\partial T_H}\right)_{T_L = C} = \frac{T_L}{T_H^2} \quad ...(6.24)$$

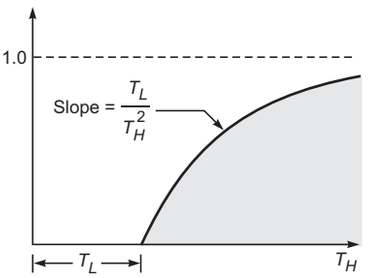
As T_H increases, the efficiency η_{rev} increases and slope $(\partial \eta_{rev}/\partial T_H)_{T_L=C}$ decreases as shown in Fig. 6.15(a).

2. Keeping the temperature T_H of the source constant, and differentiating the above relation with respect to T_L, for maximizing the efficiency

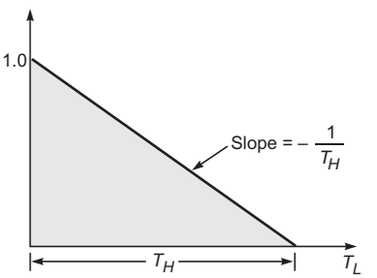
$$\left(\frac{\partial \eta_{rev}}{\partial T_L}\right)_{T_H = C} = -\frac{1}{T_H} \quad ...(6.25)$$

Thus, as T_L decreases, the efficiency increases linearly and $(\partial \eta_{rev}/\partial T_L)_{T_H=C}$ remains constant as shown in Fig. 6.15(b).

Further, the rate of efficiency increase depends on the slope. The slope of the curve of Fig. 6.15(b) remains constant with decrease of T_L. Thus for any reversible engine, comparing two slopes



(a) Effect of source temperature on efficiency



(b) Effect of sink temperature on efficiency

Fig. 6.15

$$\left(\frac{\partial \eta_{rev}}{\partial T_L}\right)_{T_H = C} > \left(\frac{\partial \eta_{rev}}{\partial T_H}\right)_{T_L = C} \quad ...(6.26)$$

Since $T_H > T_L$, therefore, *the efficiency can be increased effectively by decreasing T_L than increasing T_H.*

Example 6.1 *A heat engine operates on a Carnot cycle between source and sink temperatures of 337°C and 57°C, respectively. If the heat engine receives 400 kJ of heat from the source, find the efficiency, net work done and heat rejected to the sink.*

Solution

Given A Carnot heat engine with

$T_H = 337°C + 273 = 610$ K,

$T_L = 57°C + 273 = 330$ K,

$Q_H = 400$ kJ.

To find

(i) The efficiency of reversible heat engine,

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(ii) The net workdone by Carnot engine,
(iii) The heat rejected to sink.

Analysis

(i) The thermal efficiency of a reversible heat engine

$$\eta_{rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{330}{610} = \mathbf{0.459}$$

(ii) Further, the thermal efficiency of a reversible engine can also be given as

$$\eta_{rev} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{W}{Q_H}$$

The work done, $W = \eta_{rev} \times Q_H = 0.459 \times 400$ kJ
$= \mathbf{183.6\ kJ}$

(iii) The heat rejected to sink,
$Q_L = Q_H - W = 400 - 183.6 = \mathbf{216.4\ kJ}$

Example 6.2 *A cyclic heat engine operates between a source temperature of 800°C and a sink temperature of 30°C. What is the least rate of heat rejection per kW net output of the engine?*

Solution

Given A cyclic heat engine with

$T_H = 800°C + 273 = 1073$ K
$T_L = 30°C + 273 = 303$ K,
$\dot{W}_{net} = 1$ kW

To find The least rate of heat rejected from the engine.

Hint The reversible engine rejects minimum heat.

Analysis The efficiency of a reversible heat engine,

$$\eta_{rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{303}{1073} = 0.7176$$

Further, the efficiency of a reversible heat engine can be expressed as

$$\eta_{rev} = \frac{\text{Rate of work done}}{\text{Rate of heat supply}} = \frac{\dot{W}_{net}}{\dot{Q}_H}$$

Heat-supplyra te,

$$\dot{Q}_H = \frac{\dot{W}_{net}}{\eta_{rev}} = \frac{1\text{ kW}}{0.7176} = 1.393\text{ kW}$$

The heat-rejection rate from the engine

$$\dot{Q}_L = \dot{Q}_H - \dot{W} = 1.393 - 1 = \mathbf{0.393\ kW}$$

Example 6.3 *An engine manufacturer claims that he has developed an engine which will produce 210 kW of power, while taking in 0.5 kg/min of fuel of calorific value of 42000 kJ/kg. Further, he states that the engine receives heat at 527°C and rejects heat at a temperature of 77°C. Find if the claim of manufacturer is true or false.*

Solution

Given Claim of a manufacturer to develop an engine with

$T_H = 527°C + 273 = 800$ K
$T_L = 77°C + 273 = 350$ K
$CV = 42000$ kJ/kg
$\dot{W}_{net} = 210$ kW
$\dot{m} = 0.5$ kg/min

To find To justify the claim of the engine manufacturer

Hint According to the Carnot principle, the reversible engine has maximum efficiency.

Analysis The efficiency of a reversible heat engine,

$$\eta_{rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{350}{800} = 0.5625$$

Further, the efficiency of an actual heat engine can be expressed as

$$\eta_{Act} = \frac{\text{Rate of work done}}{\text{Rate of heat supply}} = \frac{\dot{W}_{net}}{\dot{Q}_H}$$

The heat-supply rate,

$\dot{Q}_H = \dot{m} \times CV = (0.5\text{ kg/min}) \times (42000\text{ kJ/kg})$
$= 21000\text{ kJ/min} = 350\text{ kW}$

$$\eta_{Act} = \frac{210\text{ kW}}{350\text{ kW}} = \mathbf{0.6}$$

The effciency η_{Act} of the actual engine is greater than that of reversible heat engine. The claimed heat engine is impossible to operate under given conditions. *Hence the claim is baseless.*

Example 6.4 *An inventor claims to have invented a refrigeration machine operating between –23°C and 27°C. It consumes 1 kW electrical power and gives 21600 kJ of refrigeration effect in one hour. Comment on his claim.*

Solution

Given Claim of a manufacturer to develop a refrigerating machine with

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$$T_H = 27°C + 273 = 300 \text{ K}$$
$$T_L = -23°C + 273 = 250 \text{ K}$$
$$\dot{Q}_L = 21600 \text{ kJ/h} = 6 \text{ kW}$$
$$\dot{W}_{in} = 1 \text{ kW}$$

To find To compare the *COP* of the claimed refrigerator with the reversible one.

Hint According to the Carnot principle, the reversible refrigerator has maximum *COP*.

Analysis The *COP* of a reversible refrigerator,

$$(COP)_{R,\,rev} = \frac{T_L}{T_H - T_L} = \frac{250}{300 - 250} = 5.0$$

Further, the *COP* of an actual refrigerator can also be expressed as

$$(COP)_{R,\,Act} = \frac{\text{Heat removal rate}}{\text{Rate of work input}} = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{6 \text{ kW}}{1 \text{ kW}} = 6.0$$

The $(COP)_{R,\,Act}$ of the actual refrigerator is greater than that of reversible one. The claimed refrigerator is *impossible to operate.*

Example 6.5 *A domestic food freezer maintains a temperature of –15°C. The ambient temperature is 30°C. The heat leaks into the freezer at 1.75 kJ/s. What is the minimum power necessary to pump this heat out?*

Solution

Given A domestic food freezer with

$$T_H = 30°C + 273 = 303 \text{ K}$$
$$T_L = -15°C + 273 = 258 \text{ K}$$
$$\dot{Q}_L = 1.75 \text{ kJ/s or kW.}$$

To find The minimum power required by food freezer.

Hint Reversible refrigerator requires minimum power.

Analysis The *COP* of reversible refrigerator,

$$(COP)_{R,\,rev} = \frac{T_L}{T_H - T_L}$$
$$= \frac{258}{303 - 258} = 5.73$$

Further, the *COP* of the refrigerator can also be expressed as

$$(COP)_R = \frac{\text{Heat removal rate}}{\text{Rate of work input}} = \frac{\dot{Q}_L}{\dot{W}_{in}}$$

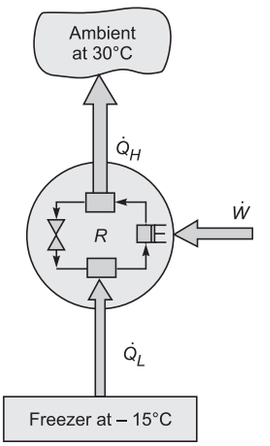

Fig. 6.16 *Schematic diagram of a food freezer*

or
$$\dot{W}_{in} = \frac{\dot{Q}_L}{(COP)_R}$$
$$= \frac{1.75 \text{ kW}}{5.73} = \mathbf{0.305 \text{ kW}}$$

Example 6.6 *A Carnot cycle receives heat at 527°C, causing an increase in entropy equal to 5 kJ/kg·K. The engine delivers 2000 kJ/kg of work. Determine cycle efficiency and lowest temperature in the cycle.*

Solution

Given A Carnot cycle with

$$T_H = 527°C + 273 = 800 \text{ K,}$$
$$\Delta s = 5 \text{ kJ/kg·K}$$
$$w = 2000 \text{ kJ/kg}$$

To find

(i) Carnot cycle efficiency, and
(ii) Lowest temperature in the cycle.

Analysis The heat supply q_H to heat engine can be obtained

$$q_H = T_H(\Delta s) = 800 \times 5$$
$$= 4000 \text{ kJ/kg}$$

Thermal efficiency of the Carnot cycle

$$\eta_{th} = \frac{w}{q_H} = \frac{2000}{4000} = 0.5$$

Further, $$\eta_{rev} = 1 - \frac{T_L}{T_H}$$

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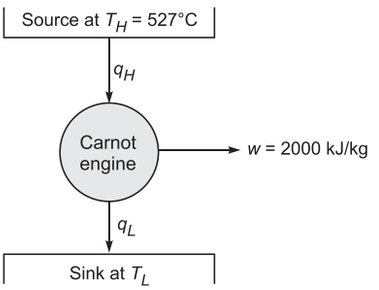

Fig. 6.17

or $$0.5 = 1 - \frac{T_L}{800}$$

or $$T_L = 800 \times 0.5 = \mathbf{400\ K = 123°C}$$

Example 6.7 *A heat pump is used to maintain an auditorium hall at 25°C, when the atmospheric temperature is 10°C.The heat load of the hall is 1500 kJ/min. Calculate the power required to run the heat pump, if its COP is 30% of COP of the Carnot heat pump, working between the same temperatures.*

Solution

Given A heat pump with

$$T_H = 25°C + 273 = 298\ K$$
$$T_L = 10°C + 273 = 283\ K$$
$$\dot{Q}_H = 1500\ kJ/min = 25\ kW$$
$$(COP)_{HP} = 0.3 \times (COP)_{HP,\,rev}$$

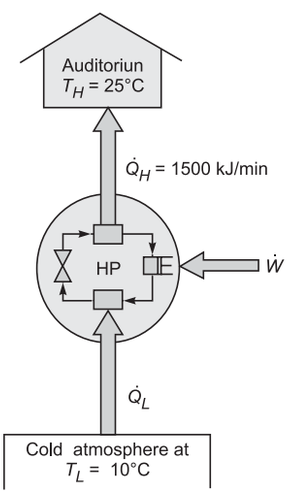

Fig. 6.18 *Schematic of heat pump*

To find The power required by the heat pump.

Analysis The *COP* of the Carnot heat pump,

$$(COP)_{HP,\,rev} = \frac{T_H}{T_H - T_L} = \frac{298}{298 - 283} = 19.86$$

The *COP* of the actual heat pump is

$$(COP)_{HP,\,act} = 0.3 \times 19.86 = 5.96$$

The *COP* of an actual heat pump can also be expressed as

$$(COP)_{HP,\,act} = \frac{\text{Heat supply rate}}{\text{Work input rate}} = \frac{\dot{Q}_H}{\dot{W}_{in}}$$

or $$\dot{W}_{in} = \frac{\dot{Q}_H}{(COP)_{HP,\,act}} = \frac{25\ kW}{5.96} = \mathbf{4.19\ kW}$$

Example 6.8 *A heat pump maintains a space at 22°C on a day, when the outdoor air temperature is 0°C. The heating requirement of the space is 100,000 kJ/h and power consumed by the pump is 5 kW. Calculate the rate at which heat is extracted from the outside air and COP of the heat pump. Also, calculate the maximum COP. Does the heat pump violate the second law of thermodynamics?*

Solution

Given For a heat pump;

$$\dot{Q}_H = 100{,}000\ kJ/h = 27.78\ kW,$$
$$T_H = 22°C + 273 = 295\ K$$
$$T_L = 0°C + 273 = 273\ K,$$
$$\dot{W}_{in} = 5\ kW$$

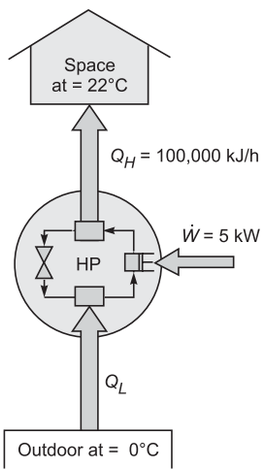

Fig. 6.19 *Schematic of a heat pump*

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To find

(i) Heat extraction rate $\dot{Q}_L$,

(ii) Actual *COP* of heat pump, and

(iii) Maximum *COP* of heat pump.

Analysis

(i) Heat extraction rate from outdoor air;

$$\dot{Q}_L = \dot{Q}_H - \dot{W}_{in}$$
$$= 27.78 - 5 = \mathbf{22.78\ kW}$$

(ii) The actual *COP* of heat pump,

$$(COP)_{HP} = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{27.78\ \text{kW}}{5\ \text{kW}} = \mathbf{5.56}$$

(iii) Maximum *COP* of heat pump,

$$(COP)_{HP,\,rev} = \frac{T_H}{T_H - T_L} = \frac{295\ \text{K}}{295\ \text{K} - 273\ \text{K}}$$
$$= \mathbf{13.4}$$

The $(COP)_{HP,\,rev}$ of the reversible heat pump is greater than the $(COP)_{HP}$ of the actual heat pump. Thus *it does not violate the second law of thermodynamics*.

Example 6.9 *A heat pump delivers 2 kW of heat to a room maintained at 25°C and receives heat from a reservoir at –10°C. If the actual coefficient of performance is 50% of that of an ideal heat pump operating between the same temperature limits, what is the actual power required in kW to run the heat pump?*

Solution

Given A heat pump:

$$Q_H = 2\ \text{kW}$$
$$T_H = 25°\text{C} = 298\ \text{K}$$
$$(COP)_{HP,\,act} = 0.5(COP)_{HP,\,rev}$$
$$T_L = -10°\text{C} = 263\ \text{K}$$

To find Power input to heat pump.

Analysis The *COP* of the Carnot heat pump

$$(COP)_{HP,\,rev} = \frac{T_H}{T_H - T_L} = \frac{298}{298 - 263} = 8.514$$

Actual *COP* of heat pump

$$(COP)_{HP,\,act} = 0.5 \times 8.514 = 4.257$$

Further, the *COP* of heat pump is given as

$$(COP)_{HP,\,act} = \frac{\text{Heat supply rate}}{\text{Work input rate}} = \frac{\dot{Q}_H}{\dot{W}_{in}}$$

$$\text{Power input, } \dot{W}_{in} = \frac{2\ \text{kW}}{4.257} = \mathbf{0.47\ kW}$$

Example 6.10 *A reversed Carnot cycle operating as a refrigerator has a refrigerating capacity of 100 kJ/s while operating between temperature limits of –20°C and 35°C. Determine*

(a) Power input, and (b) COP.

If the system is used for heating purpose only, find its COP.

What would be its efficiency if it runs as an engine?

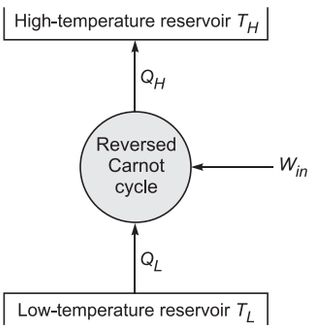

Fig. 6.20

Solution

Given A reversed Carnot cycle

$$\dot{Q}_L = 100\ \text{kJ/s}$$
$$T_H = 35°\text{C} + 273 = 308\ \text{K}$$
$$T_L = -20°\text{C} + 273 = 253\ \text{K}$$

To find For refrigerator

(i) Power input W_{in},

(ii) $(COP)_R$,

(iii) For heat pump: $(COP)_{HP}$, and

(iv) For reversible engine: thermal efficiency.

Analysis

(i) When the reversed Carnot cycle operates as a reversible refrigerator

$$(COP)_{R,\,rev} = \frac{T_L}{T_H - T_L}$$
$$= \frac{253}{308 - 253} = \mathbf{4.6}$$

$$\text{Further, } (COP)_R = \frac{\text{Refrigerating effect}}{\text{Work input rate}} = \frac{\dot{Q}_L}{\dot{W}_{in}}$$

$$\text{or} \quad \dot{W}_{in} = \frac{\dot{Q}_L}{(COP)_R} = \frac{100\ \text{kJ/s}}{4.6} = 21.74\ \text{kJ/s}$$
$$= \mathbf{21.74\ kW}$$

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(ii) When the reversed Carnot cycle operates as a reversible heat pump

$$(COP)_{HP,\,rev} = \frac{T_H}{T_H - T_L} = \frac{308}{308 - 253} = \mathbf{5.6}$$

(iii) When the cycle operates as a reversible heat engine

$$\eta_{rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{253}{308} = 0.178 \text{ or } \mathbf{17.8\%}$$

Example 6.11 *A substance executes a reversed Carnot cycle during which it receives 105.5 kJ/min of heat. Determine the work required in kW, if the adiabatic compression process triples the initial absolute temperature.*

Solution

Given A reversed Carnot cycle

$\dot{Q}_L = 105.5$ kJ/min,

$T_H = 3T_L$

To find Work input in kW.

Assumption The reversed Carnot cycle as a reversible refrigerator.

Analysis The *COP* of a reversible refrigerator can be obtained as

$$(COP)_{R,\,rev} = \frac{T_L}{T_H - T_L} = \frac{T_L}{3T_L - T_L} = \frac{1}{2} = 0.5$$

Further $(COP)_R = \dfrac{\text{Refrigerating effect}}{\text{Power input}} = \dfrac{\dot{Q}_L}{\dot{W}_{in}}$

or $$\dot{W}_{in} = \frac{\dot{Q}_L}{(COP)_R} = \frac{105.5}{0.5} = 211 \text{ kJ/min}$$

$$= \mathbf{3.51\ kW}$$

(It can also be solved by assuming reversed Carnot cycle as a reversible heat engine)

Example 6.12 *300 kg of fish at 5°C is to be frozen at –2°C. The specific heat of fish above freezing point is 4.182 kJ/kg·K and the latent heat of fusion is 234.5 kJ/ kg. Freezing point is –2°C. A refrigerator is used for this purpose which rejects heat in the ambient at 40°C. The COP of the refrigerator is 60% of the COP of a Carnot refrigerator operating between the same temperatures limits. How much power must be required to remove the heat in 10 hours?*

Solution

Given Preservation of fish in a freezer

$m = 300$ kg,

$T_i = 5$°C

$T_H = 40$°C + 273 = 313 K,

$T_L = -2$°C + 273 = 271 K,

$C_p = 4.182$ kJ/kg·K,

$h_{fg} = 234.5$ kJ/kg,

$(COP)_R = 0.6 \times (COP)_{R,\,rev}$,

$\Delta t = 10$ h = 36000 s

To find The power required to remove heat from the fish freezer in 10 hours.

Analysis The $(COP)_{R,\,rev}$ of a reversible refrigerator,

$$(COP)_{R,\,rev} = \frac{T_L}{T_H - T_L} = \frac{271}{313 - 271} = 6.452$$

The $(COP)_R$ of an actual refrigerator;

$$(COP)_R = 0.6 \times (COP)_{R,\,rev}$$
$$= 0.6 \times 6.452 = 3.87$$

The $(COP)_R$ of an actual refrigerator is expressed as

$$(COP)_R = \frac{\text{Refrigerating effect}}{\text{Work input}} = \frac{Q_L}{W_{in}}$$

The refrigeration effect from fish

$$Q_L = m \times [\text{sensible heat} + \text{latent heat of fusion}]$$
$$= m \times [C_p(T_i - T_L) + h_{fg}]$$
$$= 300 \times [4.182 \times \{5 - (-2)\} + 234.5]$$
$$= 300 \times 263.774 = 79132.2 \text{ kJ}$$

Thus work required,

$$W_{in} = \frac{Q_L}{(COP)_R} = \frac{79132.2 \text{ kJ}}{3.87}$$
$$= 20447.6 \text{ kJ}$$

This heat is to be removed in a period of 10 hours, thus rate of work (power input);

$$\dot{W}_{in} = \frac{W_{in}}{\Delta t} = \frac{20447.6 \text{ kJ}}{3600\ s} = \mathbf{0.567\ kW}$$

Example 6.13 *A one-tonne (3.51 kW) reversible refrigerator maintains a cold space at –13°C, while the surroundings are at 40°C. Determine the power consumed by the refrigerator.*

If the same refrigerator is used as a freezer maintaining a space at a temperature of –23°C, while its surroundings remain at 40°C, how much refrigeration will be produced? Assume same power consumption in both cases.

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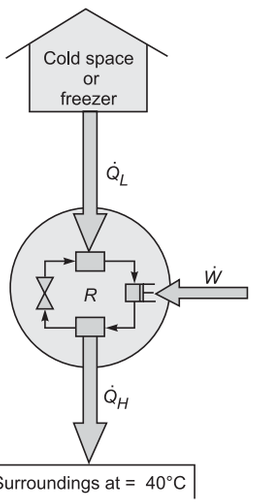

Fig. 6.21 *Schematic of refrigerator or freezer*

Solution

Given A 1-tonne refrigerator

$RE = \dot{Q}_{L_1} = 1$ TR $= 3.51$ kW,
$T_{L_1} = -13°C = 260$ K,
$T_H = 40°C = 313$ K,
$T_{L_2} = -23°C = 250$ K,
$W_1 = W_2 = W$ (say).

To find

(i) The power consuption by a reversible refrigerator when it operates between 260 K and 313 K, and

(ii) The refrigerartion effect when the refrigerator operates between 250 K and 313 K.

Analysis

(i) The $(COP)_{R, rev}$ of a reversible refrigerator,

$$(COP)_{R, rev} = \frac{T_{L_1}}{T_H - T_{L_1}} = \frac{260}{313 - 260} = 4.905$$

The $(COP)_R$ of a refrigerator can also be expressed as

$$(COP)_R = \frac{\text{Refrigerating effect}}{\text{Power input}} = \frac{\dot{Q}_L}{\dot{W}_{in}}$$

The power consumption,

$$\dot{W}_{in} = \frac{3.51 \text{ kW}}{4.905} = \mathbf{0.7155\ kW}$$

(ii) For a freezer operating between 250 K and 313 K,

$$(COP)_{R, rev} = \frac{T_{L_2}}{T_H - T_{L_2}} = \frac{250}{313 - 250} = 3.968$$

Refrigeration effect,

$$\dot{Q}_{L_2} = (COP)_{R, rev} \times \dot{W}_{in} = 3.968 \times 0.7155 = 2.84 \text{ kW}$$

Rating of refrigerator

$$= \frac{2.84 \text{ kW}}{3.51 \text{ kW/TR}} = \mathbf{0.808\ TR}$$

Example 6.14 *A reversible heat engine operates between two reservoirs at 600°C and 40°C. The engine drives a reversible refrigerator which operates between the same 40°C reservoir and a reservoir at –18°C. The heat transfer to the heat engine is 2100 kJ and there is a net work output of 370 kJ from the combined plant. Evaluate the heat transfer to the refrigerator and the net heat transfer to the 40°C reservoir.*

Solution

Given A heat engine drives a refrigerator:

$T_1 = 600 + 273 = 873$ K
$T_2 = 40 + 273 = 313$ K
$T_3 = -18 + 273 = 255$ K
$Q_1 = 2100$ kJ
$W_{net} = 370$ kJ

To find

(i) Refrigeration effect, and

(ii) Net heat transfer to reservoir at 40°C.

Analysis The efficiency of a reversible heat engine

$$\eta_{rev} = 1 - \frac{T_2}{T_1} = 1 - \frac{313}{873} = 0.642 = 64.2\%$$

Further, thermal efficiency for any engine can also be expressed as

$$\eta_{th} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{W_1}{Q_1}$$

Work done by engine,

$$W_1 = \eta_{th} Q_1 = 0.642 \times 2100 = 1348.0 \text{ kJ}$$

But net work,

$$W_{net} = W_1 - W_2 = 370$$

or $$W_2 = W_1 - 370 = 1348 - 370 = 978 \text{ kJ}$$

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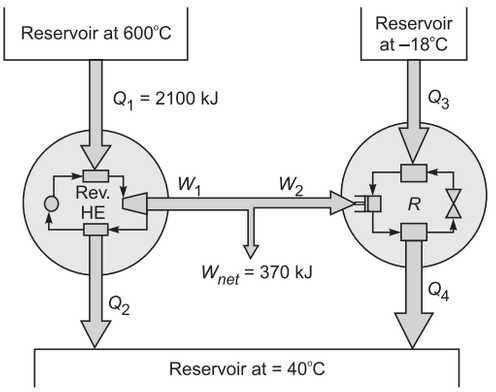

Fig. 6.22 *Heat engine and refrigerator together*

Heat rejected by engine, $Q_2 = 2100 - 1348 = 754$ kJ

COP of a reversible refrigerator,

$$(COP)_{R,\,rev} = \frac{T_3}{T_2 - T_3} = \frac{255}{313 - 255} = 4.4$$

Further, $$(COP)_R = \frac{\text{Referigeration effect}}{\text{Work input}} = \frac{Q_3}{W_2}$$

Refrigeration effect,

$$Q_3 = (COP)_R \times W_2 = 4.4 \times 978$$
$$= \mathbf{4300\ kJ}$$

Heat rejected by the refrigerator to the 40°C reservoir,

$$Q_4 = Q_3 + W_2 = 4300 + 978 = 5278 \text{ kJ}$$

Hence total heat supplied to the reservoir at 40°C

$$Q_{total} = Q_2 + Q_4 = 754 + 5278 = \mathbf{6032\ kJ}$$

Example 6.15 *A heat engine is used to drive a heat pump. The heat transfers from heat pump and heat engine is used to heat the water circulating through a building. The effciency of the heat engine is 27% and the COP of the heat pump is 4. Evaluate the ratio of heat transfer to the circulating water to heat transfer to the engine.*

Solution

Given A heat engine drives a heat pump:

$\eta_{th} = 0.27$, $(COP)_{HP} = 4$.

To find The ratio of heat supplied to circulating water to heat supplied to engine.

Analysis The efficiency of a reversible heat engine

$$\eta_{th} = \frac{W}{Q_1} = 0.27$$

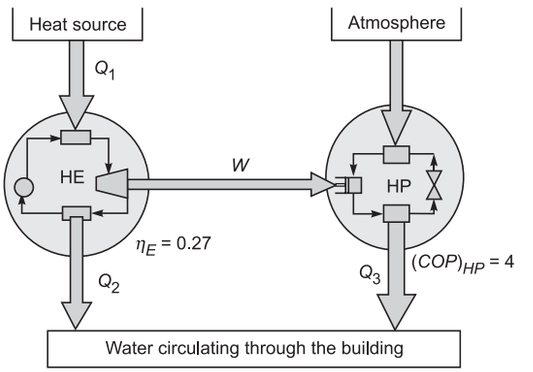

Fig. 6.23 *Heat engine and heat pump together*

$$W = 0.27Q_1$$

and $$Q_2 = Q_1 - W = (1 - 0.27)Q_1 = 0.73\,Q_1$$

Further, the *COP* of the heat pump is expressed as

$$(COP)_{HP} = \frac{\text{Heat supplied}}{\text{Work input}} = \frac{Q_3}{W} = 4$$

Heat supplied,

$$Q_3 = (COP)_{HP} \times W = 4 \times 0.27Q_1$$
$$= 1.08Q_1$$

Since the circulating water receives heat from the engine as rejected heat Q_2 and heat pump as Q_3, thus the total heat supplied to the circulating water

$$Q_{Total} = Q_2 + Q_3 = 0.73Q_1 + 1.08Q_1 = 1.81Q_1$$

Hence, the ratio of heat supplied to the circulating water and heat supplied to engine

$$\frac{Q_{Total}}{Q_1} = \frac{1.81Q_1}{Q_1} = \mathbf{1.81}$$

Example 6.16 *A reversible refrigerator is used in an ice-making plant. It absorbs heat energy from water at 0°C and rejects heat energy to the ambient at 27°C. If a reversible heat engine operates between a source at 500°C and the same ambient at 27°C is used to drive the heat pump, calculate*

(a) Energy removed as a heat from the water by the refrigerator for each kilojoule of energy taken in by the engine

(b) Energy rejected to the ambient at 27°C for each kilojoule of energy absorbed by the engine from the source.

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Solution

Given A reversible refrigerator driven by a reversible heat engine;

For a reversible heat engine:

$$T_H = 500°C + 273 = 773 \text{ K}$$
$$T_L = 27°C + 273 = 300 \text{ K}$$
$$Q_H = 1 \text{ kJ}$$

For a reversible refrigerator:

$$T_R = 0°C + 273 = 273 \text{ K},$$
$$T_L = 27°C + 273 = 300 \text{ K}$$
$$W_{in} = W_{Engine}$$

To find

(i) Refrigerating effect in a reversible refrigerator for each kJ of heat input in reversible heat engine.

(ii) Heat energy rejected to the ambient by a reversible refrigerator and a reversible heat engine for each kJ of heat input to engine.

Analysis The efficiency of a reversible heat engine

$$\eta_{rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{773} = 0.612 = 61.2\%$$

For each kJ of heat input to the reversible heat engine

$$\eta_{rev} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{W_{Engine}}{Q_H}$$

Work done by engine,

$$W_{Engine} = \eta_{rev} \times Q_H = 0.612 \times 1 = 0.612 \text{ kJ}$$

Heat rejected by the engine

$$Q_L = Q_H - W_{Engine} = 1 - 0.612 = 0.388 \text{ kJ}$$

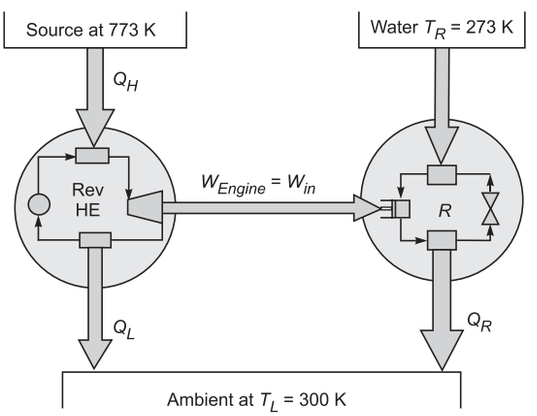

Fig. 6.24 *Schematic of reversible heat engine and reversible refrigerator together*

COP of a reversible refrigerator,

$$(COP)_{R,rev} = \frac{T_R}{T_L - T_R} = \frac{273}{300 - 273} = 10.11$$

Since the engine directly supplies work to the reversible refrigerator, thus

$$W_{in} = W_{Engine} = 0.612 \text{ kJ}$$

The cooling effect of the reversible refrigerator

$$RE = (COP)_{R,rev} \times W_{in} = 10.11 \times 0.612$$
$$= \mathbf{6.188 \text{ kJ}}$$

Heat rejected by the refrigerator to the 27°C reservoir,

$$Q_R = RE + W_{in} = 6.188 + 0.612 = 6.8 \text{ kJ}$$

The total heat rejected to the ambient by the reversible refrigerator and the reversible heat engine for each kJ input to engine;

$$Q_{Total} = Q_R + Q_L = 6.8 + 0.388 = \mathbf{7.188 \text{ kJ}}$$

Example 6.17 *A reversible refrigerator is used to maintain a space at the temperature of 0°C, when it rejects heat to the surroundings at 27°C. If the heat removal rate from the refrigerator is 90 MJ/h, determine the COP of the system.*

If the required input to run the refrigerator is supplied by a reversible engine which receives heat at 400°C and rejects heat to the surroundings, determine the overall COP of the system.

Solution

Given For a reversible refrigerator,

$$T_L = 0°C = 273 \text{ K},$$
$$T_H = 27°C = 300 \text{ K}$$

and $$\dot{Q}_L = 90 \text{ MJ/h} = 25 \text{ kW}$$

For a reversible engine,

$$T_1 = 400°C = 673 \text{ K}$$

To find

(i) $(COP)_R$, and

(ii) Overall *COP* of combined system.

Analysis For a reversible refrigerator,

$$(COP)_{R,rev} = \frac{T_L}{T_H - T_L} = \frac{273}{300 - 273} = \mathbf{10.11}$$

Further; $$(COP)_R = \frac{\text{Heat removal rate}}{\text{Power input}} = \frac{\dot{Q}_L}{\dot{W}_{in}}$$

or $$\dot{W}_{in} = \frac{25}{10.11} = 2.473 \text{ kW}$$

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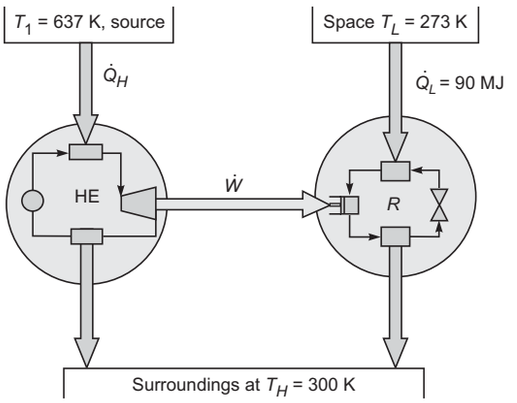


Fig. 6.25

The power input $\dot{W}_{in}$ of 2.473 kW is produced by a reversible heat engine operating between 673 K and 300 K. The efficiency of a reversible engine

$$\eta_{rev} = 1 - \frac{T_H}{T_1} = 1 - \frac{300}{673} = 0.5542$$

Further, efficiency, $\eta_{rev} = \dfrac{\text{Power produced}}{\text{Heat supply rate}} = \dfrac{\dot{W}}{\dot{Q}_H}$

Therefore, $\dot{Q}_H = \dfrac{2.473}{0.5542} = 4.462$ kW

The overall *COP* of the system

$$(COP)_{Overall} = \frac{\text{Sole effect from the system}}{\text{Energy input to the system}}$$

$$= \frac{\text{Refrigeration effect}}{\text{Heat input rate}} = \frac{\dot{Q}_L}{\dot{Q}_H} = \frac{25}{4.462}$$

$$= \mathbf{5.60}$$

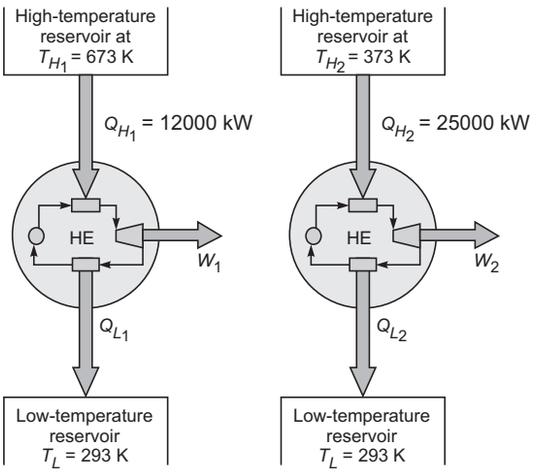
Example 6.18 *A reversible heat engine operates in two environments. In the first operation, it draws 12000 kW from a source at 400 °C and in the second operation, it draws 25000 kW from a source at 100 °C. In both operations, the engine rejects heat to a thermal sink at 20 °C. Detemine the operation in which the engine delivers more power.*

Solution

Given A reversible heat engine operates with two sources separately:

Operation 1

T_{H_1} = 400°C + 273 = 673 K
T_L = 20°C + 273 = 293 K
$\dot{Q}_{H_1}$ = 12000 kW



(a) Operation 1 *(b) Operation 2*

Fig. 6.26

Operation 2

T_{H_2} = 100°C + 273 = 373 K
T_L = 20°C + 273 = 293 K
Q_{H_2} = 25000 kW

To find Operation in which the engine will develop more power.

Analysis

Operation 1

The efficiency of a reversible heat engine

$$\eta_1 = 1 - \frac{T_L}{T_{H_1}} = 1 - \frac{293}{673} = 0.5646$$

Power developed by the engine in the operation 1,

$$P_1 = \eta_1 \times \dot{Q}_H$$
$$= 0.5646 \times 12000 = 6775.63 \text{ kW}$$

Operation 2

The efficiency of a reversible heat engine

$$\eta_2 = 1 - \frac{T_L}{T_{H_2}} = 1 - \frac{293}{373} = 0.2144$$

Power developed by the engine in the operation 2,

$$P_1 = \eta_2 \times Q_H$$
$$= 0.2144 \times 25000 = 5361.93 \text{ kW}$$

The engine will develop more power in the operation All engines develop more power when heat is supplied at higher temperature. Heat energy at higher temperature has more work capability.

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Example 6.19 *A household refrigerator maintains a space at a temperature of 0°C. Every time the door is opened, warm material is placed inside, introducing an average 400 kJ of heat, but making only a small change in temperature of the refrigerator. The door is opened 25 times a day and the refrigerator operates at 25% of ideal COP. The cost of work is Rs.3.50 per kWh. What is the monthly bill of this refrigerator? The atmospheric temperature is at 30°C.*

Solution

Given Working of a household refrigerator

$T_H = 30°C + 273 = 303$ K

$T_L = 0°C + 273 = 273$ K

$(COP)_{Act} = 0.25(COP)_{R,rev}$

Unit cost = Rs 3.50/kWh

Refrigeration effect,

$Q_2 = 400$ (kJ/time) $\times$ 25 (times/day) $\times$ 30 (days)

$= 300\,000$ kJ

To find The monthly bill of the refrigerator

Analysis The *COP* of the reversible refrigerator

$$(COP)_{R,rev} = \frac{T_L}{T_H - T_L} = \frac{273}{303 - 273} = 9.1$$

Actual *COP* of the refrigerator,

$$(COP)_{Act} = 0.25 \times 9.1 = 2.275$$

Further, $(COP)_{Act} = \dfrac{\text{Heat removed}}{\text{Work input}} = \dfrac{Q_L}{W_{in}}$

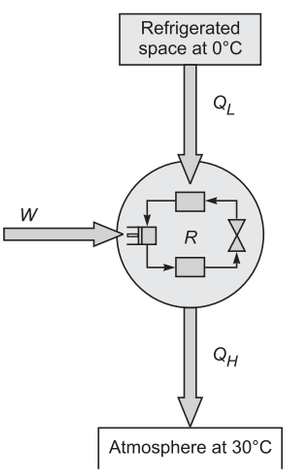

Fig. 6.27 *Schematic of household refrigerator*

or $$W_{in} = \frac{Q_L}{(COP)_{Act}} = \frac{300\,000}{2.275}$$

$= 131868.13$ kJ

1 electric unit = 1 kWh = 3600 kJ

Thus, the power consumed by the refrigerator

$$= \frac{131868.13}{3600} = 36.63 \text{ kWh}$$

The monthly bill of the refrigerator

= Power consumption $\times$ Unit cost

= 36.63 $\times$ Rs. 3.50 = **Rs. 128.20**

Example 6.20 *Two Carnot engines are working in series between a source and a sink. The first engine receives heat from a reservoir at a temperature of 1000 K and rejects the waste heat to another reservoir at the temperature T_2. The second heat engine receives the heat energy rejected by the first engine. It converts some of energy into useful work and rejects the rest to a reservoir at temperature of 300 K.*

(a) If both engines deliver equal power, determine the efficiency of each engine.

(b) If thermal efficiency of both engines are same, determine the intermediate temperature.

Solution

Given Two Carnot heat engines in series

Source temperature, $T_1 = 1000$ K

Sink temperature, $T_3 = 300$K

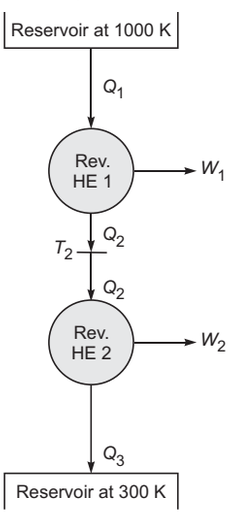

Fig. 6.28 *Schematic of two Carnot engine in series*

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To find

(i) If $W_1 = W_2$ then efficiency is η_1 and η_2.

(ii) If $\eta_1 = \eta_2$ then intermediate temperature is T_2.

Analysis The efficiencies of the reversible engines HE_1 and HE_2 are given by

$$\eta_1 = \frac{T_1 - T_2}{T_1} = \frac{W_1}{Q_1} = \frac{W_1}{Q_2 + W_1} \quad ...(i)$$

and $$\eta_2 = \frac{T_2 - T_3}{T_2} = \frac{W_2}{Q_2} \quad ...(ii)$$

(i) *If both engines deliver equal work*

From Eq. (i), we get

$$W_1 = (Q_2 + W_1) \times \frac{T_1 - T_2}{T_1}$$

$$W_1\left[1 - \frac{T_1 - T_2}{T_1}\right] = Q_2\left[\frac{T_1 - T_2}{T_1}\right]$$

or $$W_1 \times \frac{T_2}{T_1} = Q_2\left[\frac{T_1 - T_2}{T_1}\right]$$

or $$W_1 = Q_2\left[\frac{T_1 - T_2}{T_2}\right] \quad ...(iii)$$

and from Eq. (ii), we get

$$W_2 = Q_2\left[\frac{T_2 - T_3}{T_2}\right] \quad ...(iv)$$

Equating Eq. (iii) and Eq. (iv), we get

$$T_1 - T_2 = T_2 - T_3$$

or $$2T_2 = T_1 + T_3$$

or $$T_2 = \frac{T_1 + T_2}{2}$$

or $$T_2 = \frac{1000 + 300}{2} = \mathbf{650\ K}$$

Hence when both reversible engines are in series and have equal work output, the intermediate temperature is the arithmetic mean of source and sink temperature.

Efficiency of the first engine

$$\eta_1 = \frac{T_1 - T_2}{T_1} = \frac{1000 - 650}{1000} = \mathbf{0.35}$$

Efficiency of the second engine

$$\eta_2 = \frac{T_2 - T_3}{T_2} = \frac{650 - 300}{650} = \mathbf{0.538}$$

(ii) *If both reversible engines have equal thermal efficiency*

Equating (i) and (ii), we get

$$\frac{T_1 - T_2}{T_1} = \frac{T_2 - T_3}{T_2}$$

After cross multiplication, we get

$$T_1T_2 - T_2^2 = T_1T_2 - T_1T_3$$

or $$T_2 = \sqrt{T_1\,T_3} = \sqrt{1000 \times 300}$$

$$= \mathbf{547.72\ K}$$

If both reversible engines are in series and have equal efficiency then the intermediate temperature is the geometric mean of source and sink temperature.

Example 6.21

(a) Two reversible heat engines operate in series between a high-temperature and low-temperatutre reservoirs, T_1 and T_2. Engine A receives 500 kJ of heat and it rejects heat to the engine B, which in turn rejects heat to a low-temperature reservoir. The high-temperature reservoir supplies heat to the engine A at 1000 K, while the low-temperature reservoir receives heat at 400 K and both engines are equally efficient. Determine (i) temperature of heat rejection by the engine A, (ii) the work done by engines A and B, and (iii) the heat rejected by the engine B.

(b) Use the above arrangements of equal efficient engine and consider the work done by the engine B is 527.7 kJ, the low-temperature reservoir is at 305 K and heat received by the engine A is 2110 kJ. Determine (i) the thermal efficiency of each engine, (ii) the temperature of heat supply to the engine A, (iii) the work done by the engine A, (iv) the heat rejected by the engine B, and (v) the temperature of heat addition to the engine B.

Solution

Given Two equal efficient engines are in series arrangement as shown in Fig. 6.29.

Source temperature $T_1 = 1000$ K,

Sink temperature $T_2 = 400$ K,

Heat supply to the engine A $Q_1 = 500$ kJ,

$\eta_A = \eta_B$

To find

(i) When both engines are equally efficient then

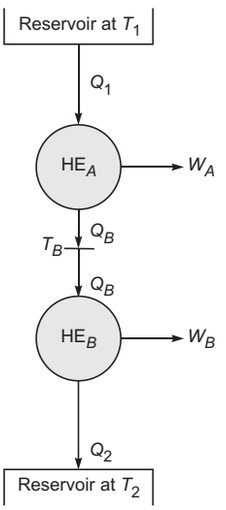

Fig. 6.29 *Schematic of reversible heat engines in series*

(ii) Temperature T_B of heat rejection by engine A,
(iii) The work done by engine A and engine B,
(iv) The heat rejected by engine B.

Assumptions

(i) Intermediate temperature between two reversible engines is T_B.
(ii) The heat rejected by the engine A and received by the engine B is Q_B.

Analysis

(i) The efficiency of the reversible engines HE_A and HE_B are given by:

$$\eta_A = \frac{T_1 - T_B}{T_1} = \frac{W_A}{Q_1} = \frac{W_A}{Q_B + W_A} \quad ...(i)$$

and $$\eta_B = \frac{T_B - T_2}{T_B} = \frac{W_B}{Q_B} \quad ...(ii)$$

If both reversible engines have equal thermal efficiencies:

Equating (i) and (ii) in terms of temperature only,

$$\frac{T_1 - T_B}{T_1} = \frac{T_B - T_2}{T_B}$$

After cross multiplication, we get

$$T_1 T_B - T_B^2 = T_1 T_B - T_1 T_2$$

or $$T_B = \sqrt{T_1 T_2} = \sqrt{1000 \times 400}$$

$$= \mathbf{632.45\ K}$$

(ii) The work done by the reversible engine A can be calculated with the help of effciency as

$$\eta_A = \frac{T_1 - T_B}{T_1} = \frac{1000 - 632.45}{1000} = 0.3675$$

Further, the effciency of the engine A is also expressed as

$$\eta_A = \frac{W_A}{Q_1}$$

or $$W_A = \eta_A \times Q_1$$
$$= 0.3675 \times 500\ \text{kJ} = \mathbf{183.775\ kJ}$$

The heat rejected by the engine A:

$$Q_B = Q_1 - W_A = 500 - 183.775$$
$$= 316.225\ \text{kJ}$$

Since both engines have equal effiencies, thus

$$\eta_B = \eta_A = 0.3675$$

Further, the effciency of the engine B can also be expressed as

$$\eta_B = \frac{W_B}{Q_B}$$

Thus, the work delivered by the engine B:

$$W_B = \eta_B \times Q_B = 0.3675 \times 316.225\ \text{kJ}$$
$$= \mathbf{116.2\ kJ}$$

(iii) The heat rejected by the engine B

$$Q_2 = Q_B - W_B = 316.225 - 116.2$$
$$= \mathbf{200\ kJ}$$

(b) With arrangement of engines as in part (a),

Work done by the engine B

$$W_B = 527.5\ \text{kJ}$$

Sink temperature, $T_2 = 305$ K

Heat supplied to the engine A

$$Q_1 = 2110\ \text{kJ}$$
$$\eta_A = \eta_B$$

To find

(i) thermal efficiency of each engine,
(ii) temperature of heat supply to the engine A,
(iii) work done by the engine A,
(iv) heat rejected by the engine B, and
(v) temperature of heat addition to the engine B.

Analysis

(i) Since both engines have equal efficiencies, thus

$$\eta_A = \frac{Q_1 - Q_B}{Q_1} = \frac{W_B}{Q_B}$$

or $$\frac{2110 - Q_B}{2110} = \frac{527.5}{Q_B}$$

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or $2110Q_B - Q_B^2 = 1113025$

$Q_B^2 - 2110Q_B + 1113025 = 0$

It is a quadratic equation and its solution is

$$Q_B = 1055 \text{ kJ}$$

The thermal efficiency of each engine is

$$\eta_A = \frac{2110 - 1055}{2110}$$

$$= 0.5 \text{ or } \mathbf{50\%}$$

(ii) The temperature of heat supply to the engine *A*

The effciency of either reversible engine is expressed as

$$\eta_B = \frac{T_B - T_2}{T_B}$$

or $$0.5 = \frac{T_B - 305}{T_B}$$

The intermediate temperature, $T_B = 710$ K

Further, the efficiency for the engine *A* is

$$\eta_A = \frac{T_1 - T_B}{T_1}$$

or $$0.5 = \frac{T_1 - 710}{T_1}$$

The temperature of heat supply,

$$T_1 = \mathbf{1420\ K}$$

(iii) Now the work done by the engine *A*;

$$W_A = Q_1 - Q_B = 2110 - 1055$$

$$= \mathbf{1055\ kJ}$$

(iv) The heat rejected by the engine *B*:

$$Q_2 = Q_B - W_B = 1055 - 527.5$$

$$= \mathbf{527.5\ kJ}$$

(v) Intermediate temperature

$$T_B = \mathbf{710\ K}$$

Example 6.22 *Two Carnot refrigerators A and B are arranged in series. Prove that the overall COP of the combined system is given by*

$$(COP)_{R,\ Overall} = \frac{(COP)_A \times (COP)_B}{1 + (COP)_A + (COP)_B}$$

where $(COP)_A$ *and* $(COP)_B$ *are the coefficient of performance of Carnot refrigerator A and B, respectively.*

Solution Two Carnot refrigerators *A* and *B* are in series as shown in Fig. 6.30.

$$(COP)_A = \frac{Q_3}{W_A} = \frac{Q_3}{Q_2 - Q_3}$$

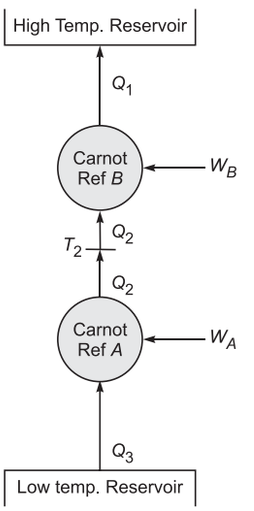

Fig. 6.30 *Two Carnot refrigerators are in series*

or $$Q_3 = (COP)_A\, Q_2 - (COP)_A Q_3$$

It gives $$Q_2 = \frac{1 + (COP)_A}{(COP)_A} \times Q_3 \qquad ...(i)$$

Similarily, $$Q_1 = \frac{1 + (COP)_B}{(COP)_B} \times Q_2 \qquad ...(ii)$$

Using Q_2 from Eq. (i) into Eq. (ii), we get

$$Q_1 = \frac{1 + (COP)_B}{(COP)_B} \times \frac{1 + (COP)_A}{(COP)_A} \times Q_3$$

$$\therefore \quad \frac{Q_1}{Q_3} = \frac{[1 + (COP)_B][1 + (COP)_A]}{(COP)_A \times (COP)_B} \qquad ...(iii)$$

For a Carnot refrgerator working between low-and high-temperature reservoirs

$$(COP)_{R,\ overall} = \frac{Q_3}{W_A + W_B} = \frac{Q_3}{(Q_2 - Q_3) + (Q_1 - Q_2)}$$

$$= \frac{Q_3}{(Q_1 - Q_3)} = \frac{1}{\dfrac{Q_1}{Q_3} - 1}$$

Substituting the value of $\dfrac{Q_1}{Q_3}$ from Eq. (iii), we get

$$(COP)_{R,\ overall} = \frac{1}{\dfrac{[1 + (COP)_B][1 + (COP)_A]}{(COP)_A \times (COP)_B} - 1}$$

$$= \frac{(COP)_A \times (COP)_B}{[1 + (COP)_B][1 + (COP)_A] - (COP)_A \times (COP)_B}$$

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or $$(COP)_{R,\ overall} = \frac{(COP)_A \times (COP)_B}{1 + (COP)_A + (COP)_B}$$

which is the required expression.

Example 6.23 *Three Carnot engines C_1, C_2 and C_3 operate in series between two heat reservoirs, which are at temperatures of 1000 K and 400 K. Calculate the temperature of the intermediate resrvoirs if the amount of work produced by these engines in the proportion of 5:4:3.*

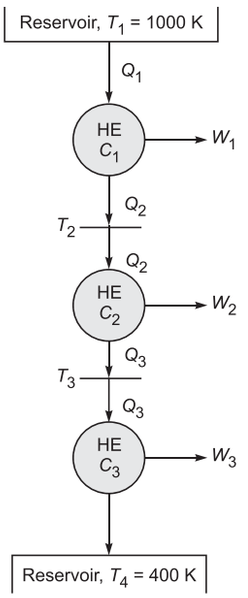

Fig. 6.31 *Schematic of three Carnot engines in series*

Solution

Given Three Carnot engines C_1, C_2 and C_3;
Source temperature $T_1 = 1000$ K
Sink temperature $T_4 = 400$ K
$W_1 : W_2 : W_3 = 5 : 4 : 3$

To find Intermediate temperatures T_2, T_3.

Analysis The efficiency of the Carnot engine C_1:

$$\eta_1 = \frac{T_1 - T_2}{T_1} = \frac{W_1}{Q_1}$$

But $$Q_1 = Q_2 + W_1$$

Using the above $$\eta_1 = \frac{1000 - T_2}{1000} = \frac{W_1}{Q_2 + W_1} \quad ...(i)$$

$$(1000 - T_2) \times (Q_2 + W_1) = 1000\ W_1$$

or $$(1000 - T_2)\ Q_2 = W_1 T_2$$

or $$Q_2 = \frac{W_1 T_2}{1000 - T_2} \quad ...(ii)$$

For the Carnot engine C_2;

$$\eta_2 = \frac{T_2 - T_3}{T_2} = \frac{W_2}{Q_2} \quad ...(iii)$$

or $$W_2 = \frac{Q_2 (T_2 - T_3)}{T_2}$$

Using the value of Q_2 from Eqn. (*ii*), we get

$$W_2 = \frac{W_1 T_2 (T_2 - T_3)}{(1000 - T_2) T_2}$$

or $$\frac{W_2}{W_1} = \frac{T_2 - T_3}{1000 - T_2} \quad ...(iv)$$

Given that

$$\frac{W_1}{W_2} = \frac{5}{4} \quad \text{or} \quad \frac{W_2}{W_1} = \frac{4}{5}$$

Using in Eq. (iv), we get

$$4 \times (1000 - T_2) = 5 \times (T_2 - T_3)$$

$$4000 - 4T_2 = 5T_2 - 5T_3$$

or $$T_2 = \frac{4000 + 5T_3}{9} \quad ...(v)$$

Again $$Q_2 = Q_3 + W_2$$

Substituting in Eq. (iii), we get

$$\frac{W_2}{W_2 + Q_3} = \frac{T_2 - T_3}{T_2}$$

or $$W_2 T_2 = W_2 (T_2 - T_3) + Q_3 (T_2 - T_3)$$

or $$W_2 = \frac{Q_3 (T_2 - T_3)}{T_3} \quad ...(vi)$$

Further, the efficiency of the third engine C_3;

$$\eta_3 = \frac{T_3 - T_4}{T_3} = \frac{W_3}{Q_3}$$

or $$W_3 = \frac{Q_3 (T_3 - T_4)}{T_3}$$

Given that $\frac{W_2}{W_3} = \frac{4}{3}$ or $3\ W_2 = 4\ W_3$

Using the values of W_2 and W_3 in the above equation;

$$3 \times \frac{Q_3 (T_2 - T_3)}{T_3} = 4 \times \frac{Q_3 (T_3 - T_4)}{T_3}$$

or $$3 \times (T_2 - T_3) = 4 \times (T_3 - T_4)$$

Using the value of T_2 from Eq. (v) and $T_4 = 400$ K;

$$3 \times \frac{4000 + 5T_3}{9} - 3T_3 = 4 \times (T_3 - 400)$$

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or $4000 + 5T_3 - 9T_3 = 12T_3 - 4800$

or $16T_3 = 8800$

or $T_3 = \mathbf{550\ K}$

The intermediate temperature T_2 between engine C_1 and C_2 from Eq. (v);

$$T_2 = \frac{4000 + 5 \times 550}{9} = \mathbf{750\ K}$$

Example 6.24 *A house is maintained at a temperature of 20°C by means of a heat pump in the winter by pumping heat from atmosphere. Heat losses through the walls of the house are estimated at 0.65 kJ/K temperature difference between inside of the house and outside atmosphere.*

(a) If the atmospheric temperature is –10°C, what is the minimum power required to drive the heat pump?

(b) It is proposed to use the same heat pump to cool the house in summer. If the same power is supplied to heat pump, what is the maximum permissible atmospheric temperature?

Solution

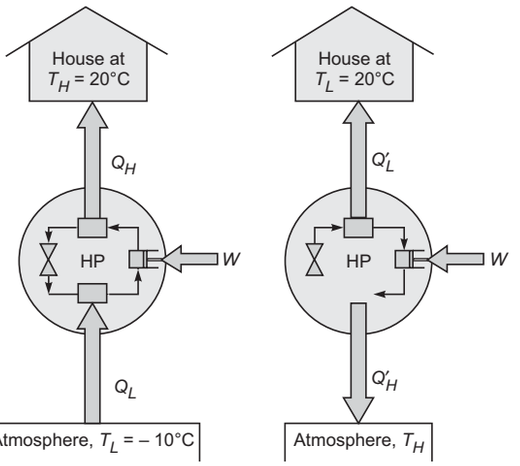
Given

(i) For winter

House temperature $T_H = 20°C = 293\ K$,

Atmospheric temperature $T_L = -10°C = 263\ K$,

Heat load of the house $Q_H = 0.65\ kJ/K$



(a) Schematic of heat pump supplying heat to a house

(b) Schematic of heat pump extracting heat from a house

Fig. 6.32

To find Minimum power input to heat pump.

Analysis The *COP* of the reversible heat pump

$$(COP)_{HP,\,rev} = \frac{T_H}{T_H - T_L} = \frac{293}{293 - 263} = 9.766$$

The *COP* of a heat pump is also expressed as

$$(COP)_{HP} = \frac{Q_H}{W_{in}}$$

or $$W_{in} = \frac{Q_H}{(COP)_{HP}}$$

where $Q_H = 0.65\ kJ/K \times \Delta T$

$= 0.65 \times (293 - 263)$

$= 19.5\ kJ$

Using the values of *COP* and Q_H;

$$W_{in} = \frac{19.5\ kJ}{9.766} = \mathbf{1.996\ kW}$$

(ii) For summer:

Given House temperature, $T'_L = 293\ K$

Power input to heat pump, $W_{in} = 1.996\ kW$

To find Surrounding temperature T_H.

Analysis The *COP* of a reversible refrigerator:

$$(COP)_{R,\,rev} = \frac{T'_L}{T'_H - T'_L} = \frac{293}{T'_H - 293}$$

The refrigeration effect, $Q'_L = 0.65 \times (T'_H - 293)$

$$(COP)_R = \frac{Q'_L}{W_{in}} = \frac{0.65 \times (T'_H - 293)}{1.996}$$

Equating the two equations for *COP* of a refrigerator

$$\frac{293}{(T'_H - 293)} = \frac{0.65 \times (T'_H - 293)}{1.996}$$

or $(T'_H - 293)^2 = 900$

or $T'_H = 293 + 30 = 323\ K = \mathbf{50°C}$

Example 6.25 *A working fluid undergoes a Carnot cycle of operation. The upper absolute temperature of the fluid is θ_1 and the lower absolute temperature is θ_2. The amount of heat taken in and rejected by the working fluids are H_1 and H_2, respectively. On account of losses of heat due to conduction etc. the heat source temperature T_1 is higher than θ_1 and the heat sink temperature T_2 is lower than θ_2.*

If $T_1 = \theta_1 + KH_1$, and $T_2 = \theta_2 - KH_2$

where K is some constant for both the equations, show that the efficiency of the plant is given by

$$\eta = 1 - \frac{T_2}{T_1 - 2KH_1}$$

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Solution The schematic for the given arrangement is shown in Fig. 6.33.

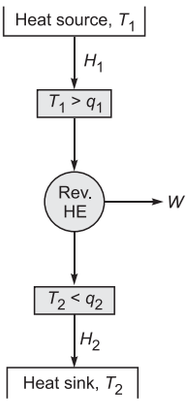

Fig. 6.33 *Schematic of a reversible heat engine*

The efficiency of a reversible engine is given by

$$\eta_{rev} = 1 - \frac{H_2}{H_1} = 1 - \frac{\theta_2}{\theta_1} \qquad ...(i)$$

By absolute temperature scale

$$\frac{\theta_1}{\theta_2} = \frac{H_1}{H_2} \qquad ...(ii)$$

From the given relation,

$$H_1 = \frac{T_1 - \theta_1}{K}$$

and $$H_2 = \frac{\theta_2 - T_2}{K}$$

Using values of H_1 and H_2 in Eq. (ii), we get

$$\frac{\theta_1}{\theta_2} = \frac{T_1 - \theta_1}{\theta_2 - T_2}$$

or $$\theta_1\theta_2 - \theta_1 T_2 = \theta_2 T_1 - \theta_1\theta_2$$

or $$2\theta_1\theta_2 - \theta_2 T_1 = \theta_1 T_2$$

or $$\theta_2(2\theta_1 - T_1) = \theta_1 T_2$$

or $$\theta_2 = \frac{\theta_1 T_2}{2\theta_1 - T_1}$$

Using $\theta_1 = T_1 - KH_1$ in the denominator above, we get

$$\theta_2 = \frac{\theta_1 T_2}{2T_1 - 2KH_1 - T_1} = \frac{\theta_1 T_2}{T_1 - 2KH_1}$$

Substituting in Eq. (i) for efficiency

$$\eta_{rev} = 1 - \frac{\theta_1 T_2}{\theta_1 (T_1 - 2KH_1)}$$

or $$\eta_{rev} = \mathbf{1 - \frac{T_2}{(T_1 - 2KH_1)}}$$

Example 6.26 *A reversed Carnot cycle operates as either a refrigerator or a heat pump. In either case, the power input is 20.8 kW. Calculate the quantity of heat extracted from the cold body for either type of machine. In both cases, 3500 kJ/min heat is delivered by the machine. In case of the refrigerator, the heat is transferred to the surroundings while in case of the heat pump, the space is to be heated. What are their respective coefficient of performances?*

If the temperature of the cold body is 0°C for the refrigerator and 5°C for the heat pump, what are the respective temperatures of the surrounding for refrigerator and heated space for heat pump? What reduction in heat-rejection temperatures would be achieved by doubling the COP for the same cold body temperature?

Solution

Given A reversed Carnot cycle with

$\dot{W}_{in}$ = 20.8 kW
$\dot{Q}_1$ = 3500 kJ/min = 58.33 kW
T_2 = 0°C + 273 = 273 K
T_3 = 5°C + 273 = 278 K

To find

(i) $(COP)_{R, rev}$,
(ii) $(COP)_{HP, rev}$,
(iii) Temperature at which reversible refrigerator rejects heat,
(iv) Temperature at which reversible heat pump supply heat,
(v) Temperature at which refrigerator rejects heat, if its $(COP)_{R, rev}$ is doubled.

Analysis For a reversed Carnot cycle, the heat extracted from low temperature is

$$\dot{Q}_2 = \dot{Q}_1 - \dot{W}_{in} = 58.33 - 20.8$$
$$= 37.53 \text{ kW}$$

(i) $$(COP)_{R, rev} = \frac{\dot{Q}_2}{\dot{W}_{in}} = \frac{37.53 \text{ kW}}{20.8 \text{ k}} = \mathbf{1.804}$$

(ii) $$(COP)_{HP, rev} = \frac{\dot{Q}_1}{\dot{W}_{in}} = \frac{58.53 \text{ kW}}{20.8 \text{ kW}} = \mathbf{2.804}$$

(iii) The $(COP)_{R, rev}$ of a reversible refrigerator can also be expressed in terms of the surrounding temperature T_1 and the cold space temperature T_2(= 273 K) as

$$(COP)_{R, rev} = \frac{T_2}{T_1 - T_2}$$

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or $1.804 \times (T_1 - 273) = 273$

or $1.804 T_1 = 273(1 + 1.804)$

$= 765.92$ K

or $T_1 = 424.33$ K = **151.3°C**

(iv) The temperature T_1 at which heat is supplied to heated space by a reversible heat pump, when it absorbs heat from surroundings at 5°C (= 278 K):

$$(COP)_{HP,\,rev} = \frac{T_1}{T_1 - T_2}$$

or $2.804 \times (T_1 - 278) = T_1$

or $(2.804 - 1) T_1 = 278 \times 2.804 = 779.51$ K

or $T_1 = 432.13$ K = **159.1°C**

(v) When the $(COP)_{R,\,rev}$ of the reversible refrigerator is doubled then

$$(COP)_{R_1,\,rev} = 2(COP)_{R,\,rev}$$

$= 2 \times 1.804 = 3.608$

Further, $(COP)_{R_1,\,rev} = \dfrac{T_2}{T_1' - T_2}$

or $3.608 \times (T_1' - 273) = 273$

or $3.608 T_1' = 273(1 + 3.608)$

$= 1258$ K

or $T_1' = 348.66$ K = **75.66°C**

Example 6.27 *A reversible heat engine works between three reservoirs A, B and C. The engine absorbs an equal amount of heat from the thermal reservoir A and B kept at temperatures T_A and T_B, respectively , and rejects heat to the thermal reservoir C at T_C. The efficiency of the engine is α times the efficiency of the reversible engine operating between the thermal reservoir A and C. Prove that*

$$\frac{T_A}{T_B} = (2\alpha - 1) + 2(1 - \alpha)\frac{T_A}{T_C}$$

Solution Consider the reversible heat engine operating between reservoirs *A*, *B* and *C*. Let the heat supplied to the engine be Q_H and heat rejection from the engine be Q_L as shown in Fig. 6.34.

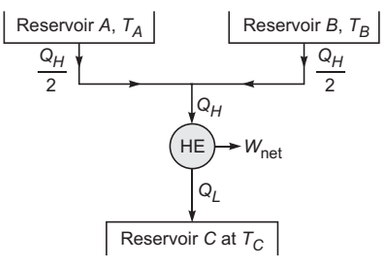

Fig. 6.34 *Schematic*

The thermal efficiency of the engine,

$$\eta_A = 1 - \frac{Q_L}{Q_H} \qquad ...(i)$$

Since the reversible heat engine receives equal amount of heat from reservoirs *A* and *B*, therefore, it will absorb $Q_H/2$ from each reservoir and rejects heat Q_L to the thermal reservoir *C*. According to absolute temperature scale;

$$\frac{Q_H}{2T_A} + \frac{Q_H}{2T_B} = \frac{Q_L}{T_C}$$

or $$Q_H \left[\frac{T_A + T_B}{2.T_A\,T_B}\right] = \frac{Q_L}{T_C}$$

or $$Q_L = \frac{Q_H\,(T_A T_C + T_B\,T_C)}{2\,T_A\,T_B} \qquad ...(ii)$$

Substituting the value of Q_L in Eq. (i), then

$$\eta_A = 1 - \frac{Q_H\,(T_A T_C + T_B\,T_C)}{Q_H \times 2\,T_A\,T_B}$$

$$\eta_A = 1 - \frac{T_A T_C + T_B T_C}{2T_A T_B} \qquad ...(iii)$$

Now considering the engine operating between the thermal reservoirs *A* and *C*, the thermal efficiency of such engine is expressed as

$$\eta_C = \frac{T_A - T_C}{T_A} \qquad ...(iv)$$

But the efficiencies in two arrangements are related as

$$\eta_A = \alpha\,\eta_C$$

Therefore,

$$1 - \frac{T_A T_C + T_B T_C}{2T_A T_B} = \alpha\left[\frac{T_A - T_C}{T_A}\right]$$

or $2T_A T_B - T_A T_C - T_B T_C = 2\alpha T_A T_B - 2\alpha T_B T_C$

Dividing both sides by $T_B T_C$, we get

$$\frac{2T_A}{T_C} - \frac{T_A}{T_B} - 1 = 2\alpha\frac{T_A}{T_C} - 2\alpha$$

Rearranging we get,

$$\frac{T_A}{T_B} = \mathbf{(2\alpha - 1) + 2(1 - \alpha)}\,\frac{T_A}{T_C}$$

Example 6.28 *A reversible engine supplied heat from two constant temperature sources at 800 K and 400 K and rejects heat to a constant temperature sink at 200 K. If the engine executes a number of cycles, while developing 100 kW and rejecting 3500 kJ of heat per*

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mintue. Determine the heat supplied by each source per minute, and efficiency of the engine.

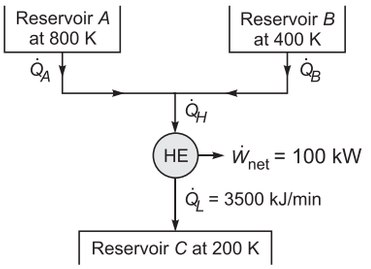

Fig. 6.35

Solution

Given A reversible engine between three reservoirs

$T_A = 800$ K

$T_B = 400$ K

$T_C = 200$ K

$\dot{W}_{net} = 100$ kW $= 6000$ kJ/min

$\dot{Q}_L = 3500$ kJ/min

To find

(i) Heat supplied by each source, and

(ii) Efficiency of the engine.

Analysis

(i) The heat supply rate to engine by two sources

$$\dot{Q}_H = \dot{W}_{net} + \dot{Q}_L$$
$$= 6000 + 3500 = 9500 \text{ kJ/min}$$

The quantity $\dot{Q}_H$ is supplied by two resevoir A and B, thus

$$\dot{Q}_A + \dot{Q}_B = \dot{Q}_H = 9500 \text{ kJ/min} \quad ...(i)$$

and according to the absolute temperature scale

$$\frac{\dot{Q}_A}{T_A} + \frac{\dot{Q}_B}{T_B} = \frac{\dot{Q}_L}{T_C}$$

or $$\frac{\dot{Q}_A}{800} + \frac{\dot{Q}_B}{400} = \frac{3500}{200}$$

or $$\dot{Q}_A + 2\dot{Q}_B = 14000 \quad ...(ii)$$

Solving Eq. (i) and (ii), we get

$\dot{Q}_A =$ **5000 kJ/min** and $\dot{Q}_B =$ **4500 kJ/min**

(ii) Thermal efficiency of the reversible heat engine,

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{6000 \text{ kJ/min}}{9500 \text{ kJ/min}}$$
$$= 0.631 \text{ or } \mathbf{63.1\%}$$

Example 6.29 *A reversible engine is used to operate a satellite between a hot reservoir at T_H and a radiating pannel at T_L. The radiation from the pannel is proportional to its area and the fourth power to temperature T_L. For a given work output and fixed temperature T_H, show that the area will be minimum when $T_L/T_H = 0.75$.*

Solution Let the reversible engine receives heat Q_H from reservoir at T_H and rejects heat Q_L to radiating pannel at T_L.

The radiation heat Q_L from pannel

$$Q_L \propto A T_L^4$$

or $$Q_L = \sigma A T_L^4 \quad ...(i)$$

where σ is constant of proportionality.

The efficiency of a reversible heat engine

$$\eta_{rev} = \frac{T_H - T_L}{T_H} = \frac{W}{Q_H}$$

Equating 2nd and 3rd terms in the above equation, we get

$$\frac{Q_H}{T_H} = \frac{W}{T_H - T_L} \quad ...(ii)$$

According to the absolute temperature scale;

$$\frac{Q_H}{T_H} = \frac{Q_L}{T_L} \quad ...(iii)$$

Thus, Eq. (ii) can be written as

$$\frac{Q_L}{T_L} = \frac{W}{T_H - T_L}$$

Using Q_L from Eq. (i)

$$\frac{\sigma A T_L^4}{T_L} = \frac{W}{T_H - T_L}$$

or $$A = \frac{W}{\sigma (T_H T_L^3 - T_L^4)}$$

For minimum area, differentiating the above equation with respect to T_L, by treating W, T_H constant.

$$\frac{dA}{dT_L} = \frac{W(3T_H T_L^2 - 4T_L^3)}{[\sigma (T_H T_L^3 - T_L^4)]^2} = 0$$

or $$3T_H T_L^2 = 4T_L^3$$

or $$\frac{T_L}{T_H} = \frac{3}{4} = \mathbf{0.75}$$

Example 6.30 *0.5 kg of air (ideal gas) executes a Carnot cycle with thermal efficiency of 50%. The heat transfer to air during isothermal expansion is 40 kJ. At the beginning of isothermal expansion, the pressure and*

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volume of air are 7 bar and 0.12 m^3, respectively. Find the maximum and minimum temperature of the cycle.

Take C_p = 1.008 kJ/kg·K, C_v = 0.721 kJ/kg·K.

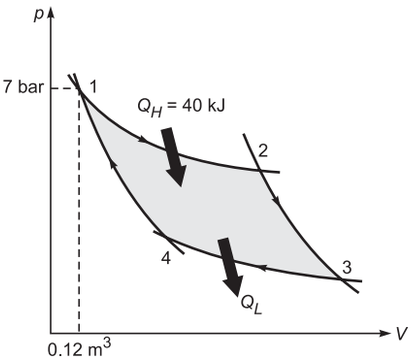

Fig. 6.36 *Carnot cycle*

Solution

Given A Carnot engine with its efficiency

$m = 0.5$ kg $\quad \eta_{rev} = 0.5$

$Q_H = 40$ kJ $\quad p_1 = 7$ bar = 700 kPa

$V_1 = 0.12$ m^3 $\quad C_p = 1.008$ kJ/kg·K

$C_v = 0.721$ kJ/kg·K

To find Source and sink temperatures.

Analysis The specific gas constant for air

$$R = C_p - C_v$$
$$= 1.008 - 0.721 = 0.287 \text{ kJ/kg·K}$$

Using characteristic gas equation at state 1;

$$p_1 V_1 = mR T_H$$

Maximum temperature

$$T_H = \frac{p_1 V_1}{mR} = \frac{700 \times 0.12}{0.5 \times 0.287}$$
$$= \mathbf{585.36\ K} \quad \text{or} \quad \mathbf{312.36°C}$$

For Carnot engine, the efficiency is given by

$$\eta_{rev} = 1 - \frac{T_L}{T_H}$$

or $$0.5 = 1 - \frac{T_L}{585.36}$$

Minimum Temperature

$$T_L = 292.68 \text{ K or } \mathbf{19.68°C}$$

Example 6.31 *The efficiency of Carnot engine is 20%. The efficiency gets doubled, when the sink temperature is reduced by 60°C. Estimate the source and sink temperatures.*

Solution

Given A Carnot engine with its efficiency

$\eta_{rev_1} = 0.2 \quad \eta_{rev_2} = 0.4$

$T_{L_2} = (T_{L_1} - 60)$ K

To find Source and sink temperatures.

Analysis Carnot engine efficiency is given by

$$\eta_{rev_1} = 1 - \frac{T_{L_1}}{T_H} \qquad \text{...(i)}$$

or $$0.2 = 1 - \frac{T_{L_1}}{T_H}$$

or $$T_{L_1} = 0.8 T_H$$

When the sink temperature is lowered by 60°C, then

$$\eta_{rev_2} = 0.4,$$

or $$0.4 = 1 - \frac{T_{L_1} - 60}{T_H}$$

or $$T_{L_1} - 60 = 0.6 T_H \qquad \text{...(ii)}$$

Sustituting T_L from Eq. (i) into Eq. (ii), we get

$$0.2 T_H = 60$$

Source temperature T_H = **300 K**

and sink temperature T_{L_1} = **240 K**

Summary

- The second law of thermodynamics states that the processes occur in certain direction only. A process is not possible, if it does not satisfy both statements of the second law and first law of thermodynamics. The two statements of the second law are the following:
  1. According to Clausius's statement, *the heat energy cannot be transferred from a low-temperature body to a high-temperature body without addition of external work.*

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2. According to the Kelvin–Planck statement, *a cyclic heat engine cannot convert all heat into work, while exchanging heat from a single-temperature reservoir.*

These two statements are equivalent in their consequences. If any of these is taken as a starting point, the second law can be deduced. Any machine that violates the first or second law of thermodynamics is called the *perpetual motion machine*.

- The work is the *high-grade energy* and can be converted directly in any form of energy like heat, etc. But the heat is low-grade energy and cannot be converted into work without a heat engine. The thermal efficiency of heat engines is defined as

$$\eta_{th} = \frac{W_{net}}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

For a reversible heat engine, the efficiency can also be expressed as

$$\eta_{rev} = 1 - \frac{T_L}{T_H}$$

- The refrigerators and heat pumps are the devices that absorb heat from low-temperature reservoir at T_L and rejects heat to high temperature reservoir at T_H. The performance of a refrigerator and a heat pump is expressed in term of *coefficient of performance*, which is defined as

$$COP = \frac{\text{Desired effect}}{\text{Energy input as work}}$$

The *COP* of a refrigerator is given as

$$(COP)_R = \frac{\text{Refrigeration effect}}{\text{Work input}} = \frac{Q_L}{W_{in}} = \frac{Q_L}{Q_H - Q_L}$$

For a reversible refrigerator, *COP* can also be expressed as

$$(COP)_{R,\,rev} = \frac{T_L}{T_H - T_L}$$

Similarly, the *COP* of a heat pump is defined as

$$(COP)_{HP} = \frac{\text{Heat supplied}}{\text{Work input}} = \frac{Q_H}{W_{in}} = \frac{Q_H}{Q_H - Q_L}$$

For a reversible heat pump, *COP* can also be expressed as

$$(COP)_{HP,\,rev} = \frac{T_H}{T_H - T_L}$$

- A reversible process is one, after it has occurred, both system and surroundings can be restored to their original states. A process may be reversible in absence of friction, non quasi-equilibrium expansion or compression and heat transfer through a finite temperature difference. All actual processes are irreversible.
- The Carnot cycle is a reversible cycle, consisting of two reversible isothermal and two isentropic processes.

According to Carnot principle, *all reversible heat engines operating between same temperature limits are equally efficient* and no heat engine can be more efficient than a reversible heat engine operating between same two temperature limits.

- A temperature scale that is independent of physical properties of any substance is called absolute temperature scale.According to this scale, the heat transfers by a reversible device between high and low-temperature reservoirs can be related as

$$\frac{Q_H}{Q_L} = \frac{T_H}{T_L}$$

Glossary

Thermal reservoir A hypothetical body of infinite heat capacity

Heat engine A cyclic work-producing device, which converts the heat energy into useful work

Thermal efficiency Ratio of net work done and energy input as heat

Refrigerator A machine which maintains a body at a lower temperature than the atmospheric temperature

Heat pump A machine which maintains a body at a higher temperature than the atmospheric temperature

COP Ratio of desired output to energy input for refrigerator or heat pump

Reversibility Ability of a process to operate in reverse direction without leaving any change on atmosphere

Irreversibility Characteristics of actual process, which make it irreversible

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Carnot engine A hypothetical engine which gives highest efficiency among all engines operating between same two temperature limits

Absolute temp. scale A temperature scale which is independent of properties of working medium

Review Questions

1. What are the limitations of the first law of thermodynamics?
2. State the importance of the second law of thermodynamics.
3. Define thermal reservoir, heat engine, refrigerator and heat pump.
4. What is a thermal energy reservoir? Define in terms of heat source and heat sink.
5. What are the characteristic features of a heat engine?
6. What is a heat pump. How does it differ from a refrigerator?
7. What do you mean by coefficient of performance? Show that $(COP)_{HP} = (COP)_R + 1$.
8. What do mean by thermal efficiency? Derive an expression for thermal efficiency of a heat engine.
9. State the Kelvin–Planck and Clausius statements of the second law of thermodynamics.
10. Prove that the Kelvin Plank and Clausius's statements of the second law of thermodynamics are equivalent to each other.
11. Is it possible for a heat engine to operate without rejecting any amount of heat. Explain with proper sketch.
12. Prove that violation of Kelvin–Planck statement leads to violation of Clausius statement.
13. Prove that violation of the Clausius statement leads to violation of the Kelvin–Planck statement.
14. Explain reversibility and irreversibility. State the factors responsible for irreversibility of a process.
15. List suitable examples of reversible and irreversible processes.
16. What are the conditions to be a reversible process?
17. What is a thermodynamic temperature scale?
18. Explain the establishment of a thermodynamic temperature scale. Why is the thermodynamic temperature scale called absolute temperature scale?
19. Sketch the Carnot cycle on a p–v diagram.
20. Explain Carnot cycle and prove that
$$\eta_{carnot} = 1 - \frac{T_L}{T_H} \ .$$
21. Is it possible to operate a heat engine on a Carnot cycle? What purpose does a Carnot cycle serve?
22. State and prove the Carnot theorem.
23. Prove that a reversible engine is more efficient than an irreversible engine operating between the same temperature limits.
24. Prove that all reversible engines operating between same temperature limits are equally efficient.
25. A reversible heat engine absorbs an equal amount of heat from two thermal reservoirs kept at temperature T_1 and T_2, respectively, and rejects heat to the thermal reservoir at T_2. The efficiency of this engine is b times the efficiency of the reversible engine operating between thermal reservoir at T_1 and T_3. Prove that
$$\beta = \frac{1}{2}\left[\frac{T_2 - T_3}{T_1 - T_3} + \frac{T_2}{T_1}\right] \times \frac{T_1}{T_2}$$

Problems

1. A steam power plant with a power output of 100 MW, consumes coal at a rate of 40 tonnes/h. If the calorific value of coal 30 MJ/kg, determine the thermal efficiency of this plant.

[30.0 percent]

2. An engine manufacturer claims to have developed an engine that takes in 500 kJ of heat at a temperature of 720 K and rejects 200 kJ of heat at a temperature of 360 K, while producing 260 kJ of work. Is his claim true? Explain. *[Claim is false]*

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3. Determine the heat input to a Carnot engine that operates between 400°C and 47°C and produces 110 kJ of work. *[209.7 kJ]*
4. An experimentalist claims to have developed a heat engine that takes in 300 kJ of heat from a source at 500 K, produces 160 kJ of work and rejects the rest of the heat as waste to a sink at 300 K. Are these measurements reasonable? Why? *[False claim]*
5. A Carnot engine receives 220 MJ/h of heat from a source at 350°C and rejects 130 MJ/h to another cold reservoir. What is the temperature of the cold reservoir? *[95.1°C]*
6. Determine the minimum heat input to a heat engine that operates between 350°C and 25°C and produces 100 kJ of work. *[191.7 kJ]*
7. A Carnot heat engine receives 500 kJ of heat from a source of unknown temperature and rejects 200 kJ of it to a sink at 17°C. Determine (a) the temperature of source, and (b) the thermal efficiency of the heat engine. *[725 K, 60%]*
8. In a Carnot engine using air as working medium, the air enters the engine at 500 kPa, 95°C and occupies volume of a 0.2 m^3 at the beginning of the isothermal expansion. The volume doubles during isothermal expansion. The temperature at the end of adiabatic expansion is 0°C. Determine (a) the heat added, and (b) the heat rejected. *[(a) 69.13 kJ (b) 51.2 kJ]*
9. A Carnot heat engine receives heat from a reservoir at 900°C at a rate of 700 kJ/min and rejects waste heat to the ambient at 27°C. The entire work output of the heat engine is used to drive a refrigerator that removes heat from a space at a temperature of –5°C and transfers it to the same ambient at 27°C. Determine (a) the maximum rate of heat removal from the refrigerated space, and (b) the total heat rejection to ambient air. *[(a) 4338.25 kJ/min; (b) 5038.25 kJ/min]*
10. An inventor claims to have developed a refrigeration system that removes heat from a cooled region at –5°C and transfers it to the surrounding air at 22°C, while maintaining its *COP* of 7.5. Is this claim reasonable? Why? *[Reasonable]*
11. A Carnot refrigerator is working between reservoirs of –13°C and 34°C. What is the coefficient of performance? If the actual refrigerator performs 75% of the Carnot refrigerator, estimate the refrigerating effect per kW of power consumption. *[5.53, 4.149 kW]*
12. The cooling load on an air-conditioner is 25 kW, when the outdoor temperature is 42°C and the indoor air is maintained at 22°C. Determine the minimum power requirement to operate the air-conditioner. *[1.695 kW]*
13. A Carnot refrigerator removes heat at a rate of 40.0 kW from a cold storage space at –20°C. The refrigerator is driven by a Carnot engine that takes heat from a reservoir at 400°C and discharges, heat to the atmosphere at 25°C. Determine (a) the power required to drive the refrigerator, and (b) the total rate of heat rejected to atmosphere by the combined devices. *[(a) 11 kW (b) 52.76 kW]*
14. An air-conditioning system is used to maintain a house at a constant temperature of 20°C. The house is gaining heat from outdoors at a rate of 20,000 kJ/h and the heat generated in the house from the people, lights and appliances amounts to 8000 kJ/h. For a *COP* of 2.5, determine the required power input for this air-conditioning system. *[3.11 kW]*
15. A household refrigerator, having a power input of 450 W and a *COP* of 2.5, is used to cool five watermelons of 10 kg each to 8°C. If the watermelons are initially at 20°C, determine how long it will take for the refrigerator to cool them. The properties of watermelon can be taken as that of water, i.e., specific heat as 4.2 kJ/kg·K and latent heat of freezing as 335 kJ/kg. *[2240 s]*
16. A 2-kg ice-cream mix at 27°C is placed in a refrigerator for freezing. The specific heat of the ice-cream mix above its freezing temperature is 4.2 kJ/kg·K and its freezing temperature is 0°C. The latent heat of freezing is 330 kJ/kg. The refrigerator operates at 60% of *COP* of the Carnot refrigerator when placed in an ambient of 37°C. How much work in kWh is required for removing this heat? *[0.056 kWh]*
17. A heat pump used to heat a house runs about one-third of the time. The house is losing heat at an average of 15,000 kJ/h. If the *COP* of the heat pump is 4.5, determine the power input to the heat pump. *[13.33 kW]*

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18. A heat pump is used to maintain a house at a constant temperature of 22°C. The house is losing heat to the outside air through the walls and windows at a rate of 40 MJ/h, while energy generated within the house from the people, lights and appliances amount to 6000 kJ/h. For a *COP* of 2.4, determine the required power input to the heat pump. *[3.93 kW]*

19. A heat pump with a *COP* of 3.2 is used to heat a house. When it is operated, it consumes a power at a rate of 5 kW. If the temperature in the house is maintained at 7°C, when heat pump is turned on, how long will it take to raise the temperature of the house to 22°C? Is this answer realistic or optimistic? Explain. Assume the entire mass within the house (air, furniture, etc.) is equivalent to 1500 kg of air for which $C_v = 0.72$ kJ/kg · K and $C_p = 1.0$ kJ/kg · K. *[23.44 min.]*

20. A Carnot heat pump is used to heat and maintain a residential building at 22°C. An energy analysis of the house reveals that it loses heat at a rate of 2500 kJ/h°C temperature difference between indoors and outdoors. For an outdoor temperature of 4°C, determine (a) the *COP*, and (b) the power required to run the heat pump.

[(a) 16.4, (b) 0.762 kW]

21. A Carnot refrigerator removes heat at a rate of 40.0 kJ/s from a cold storage room at –2°C. The refrigerator is driven by a Carnot engine that takes heat from a reservoir at 400°C and discharges heat to the atmosphere at 25°C. Determine (a) power required to drive the refrigerator, and (b) the rate at which heat is supplied to the Carnot engine from the high-temperature reservoir.

[(a) 3.985 kW, (b) 7.152 kW]

22. An ice plant is working on a reversible heat pump and produces 15 tonnes of ice per day. The ice is formed from water at 0°C. The heat is rejected to the atmosphere at 25°C. The heat pump used to run the ice plant, is coupled to a Carnot heat engine, which absorbs heat from a reservoir at 220°C, burning liquid fuel of 44500 kJ/kg heating value and the engine rejects heat to atmosphere. Determine (a) power developed by heat engine, and (b) fuel consumed per hour. Take the enthalpy of fusion of ice as 334.5 kJ/kg.

[(a) 5.3 kW, (b) 1.082 kg/h]

23. A reversible heat engine receives heat from two thermal reservoirs maintained at constant temperatures of 750 K and 500 K. The engine develops 100 kW of power and rejects 3600 kJ/min of heat to a heat sink at 250 K. Determine the thermal efficiency of the engine and heat supplied by each thermal reservoir.

[62.5%, 7200 kJ/min and 2400 kJ/min]

24. A Carnot heat engine absorbs heat from three thermal reservoirs at 1000 K, 800 K and 600 K. The engine develops 10 kW of power and rejects 400 kJ/min of heat to a sink at 300 K. If the heat supplied by reservoir at 1000 K is 60% of the heat supplied by a reservoir at 600 K, calculate the quantity of heat absorbed by each reservoir.

[$Q_1 = 312.5$ kJ/min, $Q_2 = 500$ kJ/min, and $Q_3 = 187.5$ kJ/min]

25. A reversed Carnot cycle operating as a refrigerator has a refrigerating capacity of 100 kW, while operating between temperature limits of –20°C and 35°C. Determine,

(a) power nput, and (b) *COP*.

If the system is used for heating purposes only, find its *COP*. What would be its efficiency if it runs as an engine?

26. A heat pump working on a reversed Carnot cycle takes in heat energy from a reservoir maintained at 5°C and delivers it to another reservoir at 77°C. The heat pump derives the power from a reversible heat engine operating between temperatures of 1077°C and 77°C. For 100 kJ/kg of energy supplied to the reservoir at 77°C, calculate the heat energy supplied to reversible heat engine.

[25.9 kJ/kg]

27. A heat engine receives half of its heat at a temperature of 1000 K and the rest at 500 K, while rejecting heat to a sink at 300 K. What is the maximum possible thermal eficiency of this heat engine?

$$\left[\text{Hint : Refer example 6.27} \quad \eta_{rev} = \frac{T_A T_C + T_B T_C}{2 T_A T_B} = 55\%\right]$$

28. A refrigerator transfers 120 kJ of heat from a cold space, and needs 40 kJ of work input. Calculate its coefficient of performance. *[3]*

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Objective Questions

1. The second law of thermodynamics deals with
   (a) direction of process and quality of energy
   (b) energy balance
   (c) balance of internal energy
   (d) system efficiency
2. It is impossible to construct an engine which while operating in a cycle, produces no other effect except to extract the heat from a single-temperature reservoir and do equivalent amount of work.
   (a) It refers to Clasius statement.
   (b) It refers to Kelvine–Planck's statement.
   (c) It refers to Carnot's theorem.
   (d) It refers to Clasius's theorem.
3. A Carnot cycle comprises of
   (a) two isothermal and two isentropic processes
   (b) two constant volume and two isentropic processes
   (c) two constant pressure and two isentropic processes
   (d) one constant volume, one constant pressure and two isentropic processes
4. In a thermal power plant, turbine does 10,000 kJ of work, pump consumes 10 kJ of work. The boiler receives 30,000 kJ of heat. Thermal efficiency of the plant is
   (a) 27% (b) 33.3%
   (c) 35% (d) 40%
5. A refrigerator and heat pump operates between same temperature limits. If the *COP* of the refrigerator is 4, what is the *COP* of heat pump?
   (a) 3 (b) 5
   (c) 4 (d) 3.4
6. A Carnot cycle operates between temperatures of 727°C and 227°C, the efficiency of the engine is
   (a) 40% (b) 50%
   (c) 60% (d) 45%
7. A heat engine receives heat from a source at 1000°C and rejects the waste heat to a sink at 50°C. If the heat is supplied to the engine at the rate of 100 kW. The maximum power output of this engine is
   (a) 25.48 (b) 55.44
   (c) 74.62 (d) 79.85
8. A heat pump is absorbing heat from a cold outdoor at 5°C and supplying heat to a house at 22°C at a rate of 18000 kJ/h. The power consumed by the heat pump is 2.5 kW, the coefficient of performance of heat pump is
   (a) 0.5 (b) 1.0
   (c) 2.0 (d) 5.0
9. The efficiency of the Carnot engine using an ideal gas as working substance is
   (a) $\frac{T_H - T_L}{T_H}$ (b) $\frac{T_H - T_L}{T_H T_L}$
   (c) $\frac{T_H}{T_H - T_L}$ (d) $\frac{T_L}{T_H - T_L}$
10. An air-conditioning system operating on the reversed Carnot cycle is required to remove heat from the house at a rate of 32 kW to maintain the temperature constant at 20°C. If the outdoor temperature is 35°C, the power required to operate the air-conditioning system is
    (a) 1.638 kW (b) 3.2 kW
    (c) 1.56 kW (d) 2.26 kW

Answers

1. (a) 2. (b) 3. (a) 4. (b) 5. (b) 6. (b) 7. (c) 8. (c)
9. (a) 10. (a)

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7

Entropy

Introduction

The second law of thermodynamics gives a precise definition of a property called *entropy*. Entropy can be thought of as a measure of the disorder in the system. The second law states that the entropy—that is, the disorder—of an isolated system can never decrease.

This chapter starts with the concept of entropy, Clausius' theorem, Clausius inequality, increase of entropy principle, entropy transfer and entropy generation. The temperature–entropy diagram and the third law of thermodynamics are also discussed in this chapter.

7.1 DEFINITION

The second law of thermodynamics deals with reversible and irreversible processes. *Entropy* is a property of reversible processes and is an *abstract property*, which cannot be defined precisely. It may be understood by studying its uses in engineering processes.

The amount of heat supplied to an engine is not important but the temperature at which heat supplied is important, because the heat supplied at high temperatures has a greater possibility of conversion into work than the heat supplied at lower temperatures.

Entropy is a function of quality of heat (i.e., temperature), which leads the conversion of heat into work. Thus, the increase in entropy is small when heat is transferred at higher temperature and change in entropy is large when heat is transferred at lower temperatures. Therefore, for maximum entropy there is minimum possibility of conversion of heat into work and for minimum entropy, there is maximum possibility of conversion.

7.2 TWO ISENTROPIC LINES CANNOT INTERSECT EACH OTHER

Let us assume two reversible adiabatic (isentropic) lines *AC* and *BC* intersecting each other at the point *C*. Let a reversible isothermal line *AB* join the two isentropic lines at *A* and *B* as shown in Fig. 7.1.

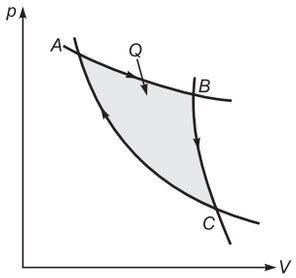

Fig. 7.1 *Two isentropic lines and one isothermal line constitute an impossible cycle*

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The reversible processes *AB*, *BC* and *CA* with suitable direction, together constitute a reversible cycle. The area enclosed by the three processes on a *p–V* plot is work output.

But the operation of any device in such a cycle is impossible because it exchanges heat during a single isothermal process *AB* and produces the net work. Thus this cycle violates the Kelvin–Planck statement of the second law. Thus we can conclude that the two reversible adiabatic lines cannot intersect each other.

Two constant-volume lines, two constant-pressure lines or two constant-temperature lines cannot intersect each other, because each line contains a property whose magnitude differs from the other line. Similarly, we may imagine that each reversible adiabatic line must have some property, whose magnitude differs in two lines, thus they are not intersecting. This property of reversible adiabatic lines is referred as *entropy*. Thus, a reversible adiabatic process is called an isentropic process.

7.3 CLAUSIUS' THEOREM

It states "a reversible line can be replaced by two reversible adiabatic lines and a reversible isothermal line."

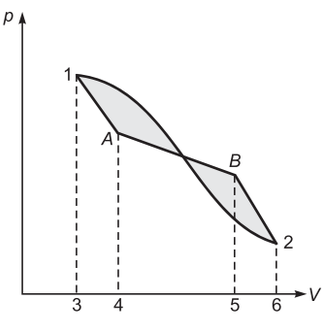

Fig. 7.2 *A reversible line replaced by two isentropic and one isothermal line*

Consider a reversible process 1–2. Let two reversible adiabatic lines 1–*A* and *B*–2 and a reversible isothermal line *A–B* replace the reversible line 1–2 in such a way that the area under 1–*A*–*B*–2 is equal to the area under the curve 1–2 as shown in Fig. 7.2.

Applying the first law of thermodynamics to the reversible process 1–2;

$$Q_{1-2} = W_{1-2} + U_2 - U_1 \quad \text{... (i)}$$

For processes 1–*A*, *A*–*B* and *B*–2

$$Q_{1-A-B-2} = W_{1-A-B-2} + U_2 - U_1 \quad \text{...(ii)}$$

Since $W_{1-2} = W_{1-A-B-2}$

Therefore, $Q_{1-2} = Q_{1-A-B-2}$

$$= Q_{1-A} + Q_{A-B} + Q_{B-2}$$

But isentropic (reversible adiabatic) lines have no heat transfer, i.e.,

$$Q_{1-A} = 0 \quad \text{and} \quad Q_{B-2} = 0$$

Thus $Q_{1-2} = Q_{A-B}$

The heat transfer during the reversible process 1–2 is equal to the heat transfer during the isothermal process *A–B*.

Consider a smooth closed curve representing a reversible cycle. Let the large number of reversible adiabatic and reversible isothermal lines be drawn in such a way that the original reversible cycle is approximated by a number of small Carnot cycles as shown in Fig. 7.3. The sum of the heat transferred during all small Carnot cycles should be equal to the heat transferred during the reversible closed cycles.

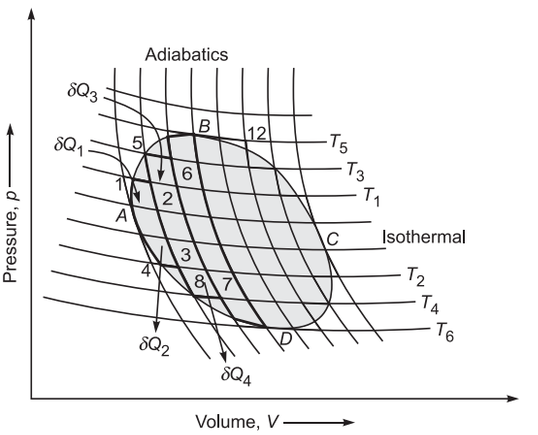

Fig. 7.3 *A reversible cycle approximated by a number of small Carnot cycles*

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For a small Carnot cycle 1–2–3–4, δQ_1 is the heat supplied at the constant temperature T_1 and δQ_2 is the heat rejection at constant temperature T_2. Thus according to absolute temperature scale for a reversible cycle.

$$\frac{\delta Q_1}{T_1} = \frac{\delta Q_2}{T_2}$$

Considering sign convention for heat transfer, i.e., –ve sign for heat rejection.

Then $$\frac{\delta Q_1}{T_1} + \frac{\delta Q_2}{T_2} = 0 \qquad ...(i)$$

Similarly, for the small Carnot cycle 5–6–7–8, transfser of heat δQ_3 at T_3 and δQ_4 at T_4,

$$\frac{\delta Q_3}{T_3} + \frac{\delta Q_4}{T_4} = 0 \qquad ...(ii)$$

and so on.

For all small Carnot cycles approximating a reversible cycle.

$$\left(\frac{\delta Q_1}{T_1} + \frac{\delta Q_2}{T_2}\right) + \left(\frac{\delta Q_3}{T_3} + \frac{\delta Q_4}{T_4}\right) + \ldots = 0$$

or $$\oint \frac{\delta Q}{T} = 0 \text{ for a reversible cycle} \qquad ...(7.1)$$

This is *Clausius' theorem* and it states that the algebraic summation of all $\frac{\delta Q}{T}$ is always zero for a reversible cycle.

7.4 CLAUSIUS, INEQUALITY

The second law of thermodynamics often leads to inequalities. According to Carnot's theorem, an actual (irreversible) heat engine is always less efficient than a reversible one, operating between the same two temperature limits. Similarly, the *COP* of an actual heat pump or refrigerator is always less than a reversible one operating between the same two thermal reservoirs. Another inequality of thermodynamics is the Clausius inequality, that is expressed as

$$\oint \frac{\delta Q}{T} \leq 0 \qquad ...(7.2)$$

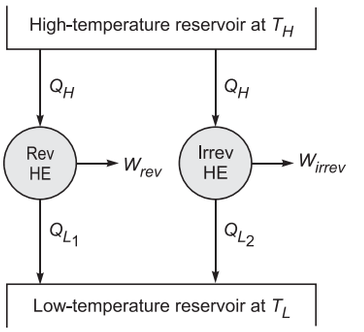

Fig. 7.4 *A reversible and an irreversible heat engines operating between the same two-temperature resevoir*

The cyclic integral or summation of $\frac{\delta Q}{T}$ for a cycle is always less than or equal to zero. This inequality is true for all types of cycles, reversible or irreversible.

Consider the reversible and irreversible heat engines, both operating between a high temperature reservoir at T_H and a low-temperature reservoir at T_L as shown in Fig. 7.4. The amount of heat supplied, Q_H, to both heat engines is same. The reversible heat engine rejects heat Q_{L_1} and does work W_{rev}.

First, considering the reversible heat engine

$$\oint \left(\frac{\delta Q}{T}\right)_{rev} = \int \frac{\delta Q_H}{T_H} - \int \frac{\delta Q_{L_1}}{T_L}$$

$$= \frac{1}{T_H}\int \delta Q_H - \frac{1}{T_L}\int \delta Q_{L_1}$$

$$= \frac{Q_H}{T_H} - \frac{Q_{L_1}}{T_L}$$

According to the Kelvin temperature (absolute) scale,

$$\frac{Q_H}{T_H} = \frac{Q_{L_1}}{T_L}$$

Thus $$\oint \left(\frac{\delta Q}{T}\right)_{rev} = \frac{Q_H}{T_H} - \frac{Q_{L_1}}{T_L} = 0 \qquad ...(7.3)$$

Equation (7.3) is valid for a totally reversible heat engine as well as an internally reversible heat

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engine. For an internally reversible heat engine, T_H and T_L are temperatures of the working fluid at locations where heat transfer take place, not the temperature of thermal reservoir.

Now consider an irreversible heat engine delivering less work W_{irrev} than the reversible engine. Thus it rejects more heat Q_{L_2},

Therefore, $Q_{L_2} > Q_{L_1}$

Expressing as $Q_{L_2} = Q_{L_1} + Q_{diff}$

Then cyclic integral for irreversible heat engine

$$\oint \frac{\delta Q}{T} = \int \frac{\delta Q_H}{T_H} - \int \frac{\delta Q_{L_2}}{T_L}$$

or $$\oint \frac{\delta Q}{T} = \frac{Q_H}{T_H} - \frac{Q_{L_1}}{T_L} - \frac{Q_{diff}}{T_L}$$

But $$\frac{Q_H}{T_H} - \frac{Q_{L_1}}{T_L} = 0 \text{ (refer Eq. 7.3)}$$

Thus $$\oint \frac{\delta Q}{T} = -\frac{Q_{diff}}{T_L} \neq 0$$

The quantity of heat Q_{diff} is positive, therefore,

$$\oint \frac{\delta Q}{T} < 0 \qquad ...(7.4)$$

The Clausius inequality is obtained by combining Eqs. (7.3) and (7.4),

$$\oint \frac{\delta Q}{T} \leq 0$$

which is valid for all cycles. Similarly, the Clausius inequality can also be obtained for refrigeators and heat pumps. But for any machine

$$\oint \frac{\delta Q}{T} > 0 \qquad ...(7.5)$$

is impossible.

Example 7.1 *A heat engine receives 600 kJ of heat from a high-temperature reservoir at 1000 K during a cycle. It converts 150 kJ of this heat to net work and rejects the remaining 450 kJ to a low-temperature sink at 300 K. Determine if this heat engine violates the second law of thermodynamics on the basis of (a) the Clausius inequality, and (b) the Carnot principle.*

Solution

Given A heat engine with different operating conditions:

Heat supplied,	$Q_H = 600$ kJ
Source temperature	$T_H = 1000$ K
Work done	$W = 150$ kJ
Heat rejected	$Q_L = 450$ kJ
Sink Temperature	$T_L = 300$ K

To find

(i) Cyclic $\int \left(\frac{\delta Q}{T}\right)$.

(ii) To compare the reversible and actual efficiency of the engine.

Analysis

(i) $$\oint \left(\frac{\delta Q}{T}\right) = \frac{Q_H}{T_H} - \frac{Q_L}{T_L}$$

$$= \left(\frac{600}{1000}\right) - \left(\frac{450}{350}\right) = -0.9$$

Since the cyclic $\int \left(\frac{\delta Q}{T}\right)$ is less than zero, therefore, the *cyclic heat engine satisfies* the *Clausius inequality* of the second law of thermodynamics.

(ii) The reversible efficiency of the heat engine

$$\eta_{rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{1000} = 0.7 \text{ or } 70\%$$

Actual efficiency of the heat engine

$$\eta_{act} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{450}{600} = 0.25 \text{ or } 25\%$$

The actual efficiency of the heat engine is less than the Carnot efficiency, therefore, the *Carnot principle is satisfied*.

Example 7.2 *A system undergoes a reversible cycle by exchanging heat Q_1, Q_2, and Q_3 with three thermal reservoirs at T_1 T_2, and T_3, respectively. Show that*

$$\frac{Q_1}{\frac{1}{T_2} - \frac{1}{T_3}} = \frac{Q_2}{\frac{1}{T_3} - \frac{1}{T_1}} = \frac{Q_3}{\frac{1}{T_1} - \frac{1}{T_2}}$$

Solution According to the first law of thermodynamics

$$Q_1 + Q_2 + Q_3 = 0 \qquad ...(i)$$

According to Clausius' theorem for a reversible cycle

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \frac{Q_3}{T_3} = 0 \qquad ...(ii)$$

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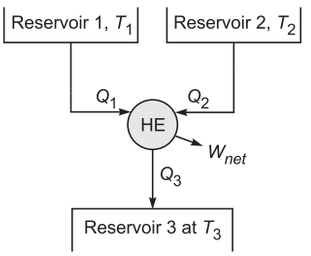

Fig. 7.5 *Schematic*

From Eq. (i), we get $Q_3 = -(Q_1 + Q_2)$

Substituting the value of Q_3 in Eq. (ii), we get

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} - \left(\frac{Q_1 + Q_2}{T_3}\right) = 0$$

or $$Q_1\left(\frac{1}{T_1} - \frac{1}{T_3}\right) = Q_2\left(\frac{1}{T_3} - \frac{1}{T_2}\right)$$

or $$\frac{Q_1}{\left(\frac{1}{T_2} - \frac{1}{T_3}\right)} = \frac{Q_2}{\left(\frac{1}{T_3} - \frac{1}{T_1}\right)} \quad \text{...(iii)}$$

Again from Eq. (i), we get $Q_2 = -(Q_1 + Q_3)$

Substituting the value of Q_2 in Eq. (ii), we get

$$\frac{Q_1}{T_1} - \left(\frac{Q_1 + Q_3}{T_2}\right) + \frac{Q_3}{T_3} = 0$$

or $$Q_1\left(\frac{1}{T_1} - \frac{1}{T_2}\right) = Q_3\left(\frac{1}{T_2} - \frac{1}{T_3}\right)$$

or $$\frac{Q_1}{\left(\frac{1}{T_2} - \frac{1}{T_3}\right)} = \frac{Q_3}{\left(\frac{1}{T_1} - \frac{1}{T_2}\right)} \quad \text{...(iv)}$$

From the identity of Eqs. (iii) and (iv), we get

$$\frac{\boldsymbol{Q_1}}{\left(\frac{\mathbf{1}}{\boldsymbol{T_2}} - \frac{\mathbf{1}}{\boldsymbol{T_3}}\right)} = \frac{\boldsymbol{Q_2}}{\left(\frac{\mathbf{1}}{\boldsymbol{T_3}} - \frac{\mathbf{1}}{\boldsymbol{T_1}}\right)} = \frac{\boldsymbol{Q_3}}{\left(\frac{\mathbf{1}}{\boldsymbol{T_1}} - \frac{\mathbf{1}}{\boldsymbol{T_2}}\right)}$$

which is the required expression.

Example 7.3 *A heat engine is supplied with 1130 kW of heat at a constant temperature of 292°C and it rejects heat at 5°C. The following results were recorded:*

(a) 834 kW heat is rejected, (b) 556 kW heat is rejected, and (c) 278 kW heat is rejected.

Determine whether results report a reversible cycle, irreversible or impossible cycle.

Solution

Given A heat engine operates with a different quantity of heat rejection

$Q_1 = 1130$ kW

$T_1 = 292°C + 273 = 565$ K

$T_2 = 5°C + 273 = 278$ K

To find Whether the result indicates a reversible, irreversible or an impossible cycle.

Analysis According to Clausius inequality to the cyclic process.

$$\oint \frac{\delta Q}{T} \leq 0$$

(i) Heat rejection is 834 kW.

$$\oint \frac{\delta Q}{T} = \left(\frac{1130}{565}\right) - \left(\frac{834}{278}\right) = 2 - 3 = -1$$

Therefore, the cycle is an *irreversible cycle*.

(ii) Heat rejection is 556 kW.

$$\oint \frac{\delta Q}{T} = \left(\frac{1130}{565}\right) - \left(\frac{556}{278}\right) = 2 - 2 = 0$$

Therefore, the cycle is a *reversible cycle*.

(iii) The engine rejects 278 kW of heat. Then

$$\oint \frac{\delta Q}{T} = \left(\frac{1130}{565}\right) - \left(\frac{278}{278}\right) = 2 - 1 = 1$$

Therefore, the cycle is an *impossible cycle*.

7.5 ENTROPY: A PROPERTY OF THE SYSTEM

The mathematical quantity $\int \frac{\delta Q}{T}$ discussed above is the property of a reversible line, and is thus a point function. The cyclic integral of any property is always zero. To demonstrate entropy as a property, we consider a cycle of two reversible processes 1–*A*–2 and 2–*B*–1 as shown in Fig. 7.6.

The cyclic integral for the reversible process 1–*A*–2 and 2–*B*–1,

$$\oint \frac{\delta Q}{T} = \int_1^2 \left(\frac{\delta Q}{T}\right)_A + \int_2^1 \left(\frac{\delta Q}{T}\right)_B$$

For a reversible cycle,

$$\oint \frac{\delta Q}{T} = 0$$

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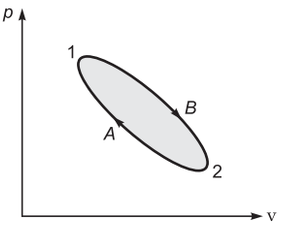

Fig. 7.6 *Entropy change between two states is same, when the process is reversible*

Thus

$$\int_1^2 \left(\frac{\delta Q}{T}\right)_A + \int_2^1 \left(\frac{\delta Q}{T}\right)_B = 0$$

Changing the limit of the second integral and rearranging, we get

$$\int_1^2 \left(\frac{\delta Q}{T}\right)_A = \int_1^2 \left(\frac{\delta Q}{T}\right)_B \qquad ...(7.6)$$

Since processes 1–*A*–2 and 2–*B*–1 are internally reversible processes between states 1 and 2 and this integral depends on end states, between which the process takes place not on the path followed by the process. Thus, the quantity $\left(\frac{\delta Q}{T}\right)_{int,\,rev}$ is a property. Clausius stated that he had discovered a new thermodynamic property and he named this property as *entropy*. It is designated as *s* and is defined as

$$dS = \left(\frac{\delta Q}{T}\right)_{int,\,rev} \text{ (kJ/K)} \qquad ...(7.7)$$

The entropy is an extensive property of the system, and therefore, sometimes, it is also referred as *total entropy*. The entropy per unit mass, called *specific entropy* and designated as *s*, is an intensive property, measured in kJ/kg · K.

7.6 CHANGE OF ENTROPY IN A REVERSIBLE PROCESS

The entropy change of a system during a process can be obtained by integrating Eq. (7.7) between initial and final states.

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{int,\,rev} \text{ (kJ/K)} \qquad ...(7.8)$$

Engineers are usually concerned with the change in entropy, not in the absolute value of entropy. The absolute value of entropy can be determined on the basis of the third law of thermodynamics.

The integration $\frac{\delta Q}{T}$ gives us the value of entropy change for an internally reversible process only. The integral $\frac{\delta Q}{T}$ along an irreversible process is not a property and different values are obtained when the integration is carried out along different irreversible paths. Therefore, for an irreversible process, the entropy change should be calculated by carrying out the integration along some convenient imaginary internally reversible path between the two states.

7.7 TEMPERATURE–ENTROPY DIAGRAM

If entropy is taken along the *x*-axis and absolute temperature along the *y*-axis, the diagram so obtained is called a temperature–entropy (*T*–*S*) diagram. Such a diagram for a process 1–2 is shown in Fig. 7.7. Consider the equation of entropy

$$dS = \left(\frac{\delta Q}{T}\right)_{Int,\,rev}$$

or $$(dQ)_{int,\,rev} = TdS \qquad ...(7.9)$$

The differential area *dA* under the process on a *T*–*S* diagram represents the differential amount

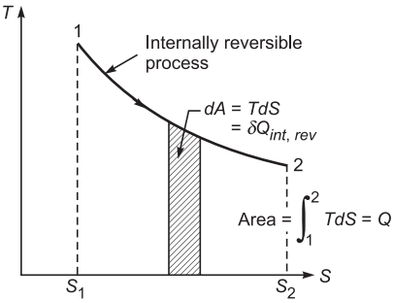

Fig. 7.7 *Area under a T–S diagram for a process represents the heat transfer for an internally reversible process*

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of heat transfer (δQ) for an internally reversible process.

The total heat transferred during an internally reversible process can be obtained on integration of Eq. (7.9).

$$Q_{int,\,rev} = \int_1^2 T\,dS\ (\text{kJ}) \qquad ...(7.10)$$

Therefore, we can conclude that, the area under the process curve on a *T–S* diagram represents the internally reversible heat transfer. It is analogous to a reversible boundary work being represented by the area under the process curve on a *p–V* diagram.

On a unit-mass basis

$$\delta q_{int,\,rev} = T\,ds\ (\text{kJ/kg})$$

and

$$q_{int,\,rev} = \int_1^2 T\,ds\ (\text{kJ/kg}) \qquad ...(7.11)$$

7.8 THE INCREASE OF ENTROPY PRINCIPLE

Consider an irreversible cycle, consisting of two process, one internally reversible and the other irreversible as shown in Fig. 7.8.

From the Clausius inequality,

$$\oint \frac{\delta Q}{T} \le 0$$

For an irreversible cycle 1–*A*–2–*B*–1,

$$\int_1^2 \left(\frac{\delta Q}{T}\right)_A + \int_2^1 \left(\frac{\delta Q}{T}\right)_B < 0$$

The first integral in the above relation is for an internally reversible process and can be written as the entropy change $S_2 - S_1$. Therefore,

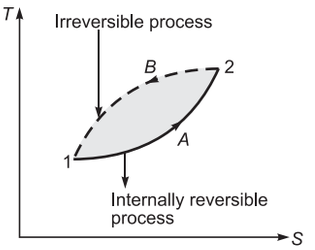

Fig. 7.8 *A cycle composed of a reversible and an irreversible process*

$$S_2 - S_1 + \int_2^1 \left(\frac{\delta Q}{T}\right)_{irrev} < 0$$

It can be rearranged as

$$S_2 - S_1 > \int_1^2 \left(\frac{\delta Q}{T}\right)_{irrev} \qquad ...(7.12)$$

In the differential from

$$dS > \left(\frac{\delta Q}{T}\right)_{irrev} \qquad ...(7.13)$$

We can conclude that the entropy change of a closed system during an irreversible process is greater than the integral of $\frac{\delta Q}{T}$, evaluated for that process.

If the process 1–*A*–2 is considered reversible then

$$dS = \frac{\delta Q}{T}$$

and

$$S_2 - S_1 = \int_1^2 \frac{\delta Q}{T}$$

Thus, the general form of Eq. (7.14), becomes

$$S_2 - S_1 \ge \int_1^2 \frac{\delta Q}{T} \qquad ...(7.14)$$

The quantity $S_2 - S_1$ represents the entropy change of the system during a reversible process and is equal to $\int_1^2 \frac{\delta Q}{T}$, which represents entropy transfer with heat. That is, for a reversible process,

$$\Delta S = S_2 - S_1 \quad \text{or} \quad S_1 = S_2$$

or

$$S = \text{Constant} \qquad ...(7.15)$$

The inequality sign in Eq. (7.13) represents that the entropy change of a closed system during an irreversible process is always greater than the entropy transfer for a reversible process. That is, some entropy is generated or created during an irreversible process due to presence of irreversibilies. The entropy generated during a process is called *entropy generation*, and is designated as S_{gen}. Thus for an irreversible process;

$$S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{gen} \qquad ...(7.16)$$

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The entropy generation S_{gen} is always a positive quantity for an irreversible process and zero for a reversible process, or

$$S_{gen} \begin{cases} > \\ = \\ < \end{cases} \begin{cases} 0 \text{ irreversible process} \\ 0 \text{ reversible process} \\ 0 \text{ impossible process} \end{cases} \quad ...(7.17)$$

Further, the increase of entropy principle states that the entropy of an isolated system, (the heat transfer is zero) either increases or remains constant.

$$\Delta S_{isolated} \geq 0 \quad ...(7.18)$$

Thus, the entropy change of an isolated system can never decrease, if a system and surroundings together (universe) constitute an isolated system. Thus, for all possible processes in the universe,

$$\Delta S_{universe} \geq 0 \quad ...(7.19)$$

or $$\Delta S_{system} + \Delta S_{surroundings} \geq 0 \quad ...(7.20)$$

Since no actual process in the universe is truly reversible, some entropy is generated during each process and therefore, the entropy of the universe (an isolated system) is continuously increasing. The more irreversibilities involved in a process, the larger the entropy generation during that process.

7.9 ENTROPY TRANSFER

The entropy can be transferred to or from a system by heat transfer and mass transfer. The entropy transfer is recognised at the boundary of the system and entropy is either gained or lost during a process.

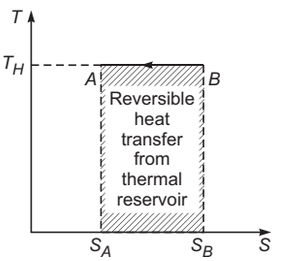
Heat is a disorganised form of energy and some disorganisation (entropy) always flows with heat transfer. The heat transfer to a system increases the entropy of that system and thus the level of molecular disorder and randomness also increase. The heat rejection from a system decreases the entropy of the system. Hence, the entropy transfer takes place in the direction of heat flow.

The entropy does not transfer with work transfer, because work is an organised form of energy, and thus can perform tasks such as raising a weight or generating electricity. The first law of thermodynamics makes no difference between heat and work transfer, two forms of energy. But the second law explains that *an energy transfer with entropy transfer is heat transfer and an energy transfer without entropy transfer is work transfer.*

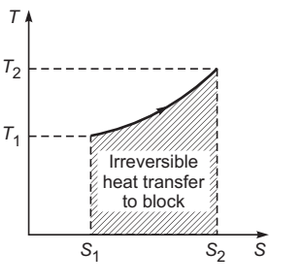
The total entropy is a product of mass and specific entropy. When the mass transfers within an open system, entropy transfer also takes place. Both energy and entropy transport takes place into or out of the system with a stream of mass flow. The closed system does not involve any mass transfer, and thus no entropy transport.

7.10 ENTROPY GENERATION

All actual processes involve irreversibilities like friction, mixing of fluid streams, chemical reactions, heat transfer through finite temperature difference, unrestrained expansion, non quasi-static compression or expansion. These irreversibilities always cause entropy of a system to increase. This



(a) The entropy decrease of the reservoir is $S_B - S_A$



(b) The entropy increase of the block $S_2 - S_1$ is greater than $S_B - S_A$

Fig. 7.9

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increase in entropy of a system is called *entropy generation* and is designated S_{gen}.

For a reversible process, the entropy generation is always zero, and thus entropy change of a system is always equal to entropy transfer into the system.

Consider the heating of a steel block from temperature T_1 to T_2. The heat is supplied from a constant-temperature reservoir at the temperature $T_H (T_H > T_2)$. The block and reservoir together constitute an isolated system. The transfer of heat from the reservoir to the block through the finite temperature difference is irreversible, and thus entropy of an isolated system increases. The entropy increase of the block is greater than the entropy decrease of the reservoir. The difference of two quantities of entropy is entropy generation. It is illustrated in Fig. 7.9.

7.11 ENTROPY BALANCE

7.11.1 Entropy Balance for Closed Systems

A closed system does not involve mass transfer across its boundaries thus its entropy change during a process is the sum of entropy transfer due to heat transfer and entropy generation within a system. Thus,

$$S_2 - S_1 = \int \frac{\delta Q}{T} + S_{gen}$$

or
$$S_2 - S_1 = \sum \frac{Q}{T} + S_{gen} \qquad ...(7.21)$$

For an adiabatic process ($Q = 0$), the above expression reduces to

$$S_2 - S_1 = S_{gen}$$
$$(\Delta S)_{adiabatic} = S_{gen} \qquad ...(7.22)$$

7.11.2 Entropy Change for Open Systems

An open system involves mass transfer and heat transfer across its boundaries. Therefore, the rate of entropy change of an open system is the sum of net rate of entropy transport due to mass flow, entropy-transfer rate due to heat transfer at its boundaries and the net rate of entropy generation within the system due to irreversibilities involved in the process. Thus, for one-dimensional flow.

$$\frac{dS}{dt} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}}{T} + \dot{S}_{gen} \qquad ...(7.23)$$

For a steady flow device, $\frac{dS}{dt} = 0$

$$\dot{S}_{gen} = \dot{m}(s_e - s_i) - \sum \frac{\dot{Q}}{T} \qquad ...(7.24)$$

7.12 PHYSICAL CONCEPT OF ENTROPY

Entropy is considered as a measure of *molecular disorder* or *molecular randomness* in the matter. When the entropy of a system increases, the positions of molecules become less predictable, or the molecules become highly disordered.

All the solids have fixed positions for their molecules. Thus molecules can only oscillate about their mean positions and the entropy change for solids is lowest. It is highest in the gaseous phase. The molecules in the gaseous phase possess a considerable amount of kinetic energy. Thus they randomly collide with each other, vibrate and can move in any direction. When heat is added to a gaseous substance, the uncertainty of molecules further increase with increase in their energy level. But these disorganised molecules as shown in Fig. 7.10 are unable to do any work. They cannot rotate a paddle wheel. Probably the different directions of molecules nullify their net effect causing the wheel to remain motionless.

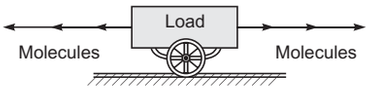

Fig. 7.10 *Unorganised energy of molecules cannot create useful effect*

Now consider a paddle wheel working on an amount of gas in a closed container as shown in Fig. 7.11. The work done by the stirrer on the gas is converted into internal energy of the gas causing rise in temperature and entropy, creating higher level of molecular disorder. The organised paddle-wheel energy is converted to a highly disorganized

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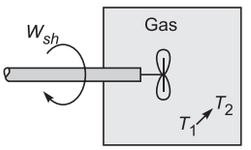

Fig. 7.11 *The energy is degraded during the process, and entropy increases*

form of energy, which cannot be converted back to the paddle wheel by lowering the temperature of the gas, although a part of this energy can be converted into useful work by using a heat-engine cycle. Therefore, the energy is degraded, entropy increases during this process and the ability to do work is reduced.

The quantity of energy is always preserved in either form during an actual process (the first law), but the quality of energy decreases (the second law). This degradation of energy always causes an increase in entropy.

The concept of entropy as a measure of disorganisation can be applied to all areas. Efficient people have low-entropy (highly organised) lives. Therefore, they perform their jobs with minimum delay. On the other hand, inefficient people are disorganised and lead high entropy lives. They perform their jobs in disorganised manner, and hardly meet their goals in the stipulated time.

We can conclude that all natural disorganised processes are in the direction of increase in molecular disorder. Further, all processes in the universe are irreversible, molecular disorder increases day by day, and thus increases of the entropy of the universe.

7.13 *Tds* RELATIONS

The differential form of the conservation-of-energy equation is

$$\delta Q = \delta W + dU$$

For an internally reversible process, Eq. (7.9)

$$\delta Q = T\,dS$$

and $$\delta W = p\,dV$$

Using, we get

$$T\,dS = p\,dV + dU \qquad ...(7.25)$$

For a unit-mass system

$$T\,ds = p\,dv + du \qquad ...(7.26)$$

Equation (7.26) is called the first *Tds relation* or *Gibb's relation*.

Further, the specific enthalpy is expressed as

$$h = u + pv$$

In differential form,

$$dh = du + p\,dv + v\,dp$$

or $$du + p\,dv = dh - v\,dp$$

Using in Eq. (7.26), we get

$$T\,ds = dh - v\,dp \qquad ...(7.27)$$

This equation is the *second Tds relation*.

Thus, the entropy change during a quasi-equilibrium process can be obtained by integrating either relation given below:

$$ds = \frac{du}{T} + \frac{p\,dv}{T} \qquad ...(7.28)$$

and $$ds = \frac{dh}{T} - \frac{v\,dp}{T} \qquad ...(7.29)$$

7.13.1 Entropy Change for an Ideal Gas

$$pv = RT \quad \text{and} \quad du = C_v\,dT$$

or $$\frac{p}{T} = \frac{R}{v}$$

Using in Eq. (7.28) and integrating, we get

$$\Delta s = C_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right) \qquad ...(7.30)$$

Similarly, for an ideal gas

$$\frac{v}{T} = \frac{R}{p}, \quad \text{and} \quad dh = C_p\,dT$$

Using in Eq. (7.29) and integrating, we get

$$\Delta s = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right) \qquad ...(7.31)$$

7.13.2 The Entropy Change of Solids and Liquids

Solids and most of the liquids are considered as incompressible since their volume remains constant. Thus, for solids and liquids, $dv = 0$ and Eq. (7.26) reduces to

$$T\,ds = du = C_v\,dT \qquad ...(7.32)$$

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Since $C_v = C_p = C$ for solids and incompressible fluids, therefore $du = C\,dT$. The entropy for a process whose specific heat depends on temperature is

$$s_2 - s_1 = \int_1^2 C(T)\,\frac{dT}{T}\ (\text{kJ/kg}\cdot\text{K}) \quad ...(7.33)$$

For constant specific heat,

$$s_2 - s_1 = C_{av} \ln\left(\frac{T_2}{T_1}\right) \quad ...(7.34)$$

7.14 THIRD LAW OF THERMODYNAMICS

As discussed above, entropy is a measure of the uncertainty of molecular position in matter in either phase. This uncertainty of molecular position in matter is a function of temperature and it decreases as temperature decreases. It is due to different energy levels of molecules in the matter. The molecules of a substance become motionless at absolute zero temperature, thus having zero entropy. Therefore, *the entropy of a pure substance at absolute zero temperature is zero.* This statement is known as the *third law of thermodynamics*.

The third law of thermodynamics is an independent principle and it cannot be deduced from the first and second laws of thermodynamics or any other principle of nature. But it is important that the third law of thermodynamics provides an absolute reference point ($S = 0$ at $T(\text{K}) = 0$) for the determination of entropy.

Example 7.4 *Air expands in a non-flow system reversibily from 200 kPa, 167°C to 100 kPa, 112°C. Calculate the change in entropy for the following processes:*

(a) A reversible constant-volume process followed by a reversible constant-pressure process

(b) A reversible adiabatic process followed by a reversible constant-pressure process

(c) A reversible isothermal process followed by a reversible constant-pressure process

(d) If air expands reversibily between two states assuming C_p and C_v are constants and are 1.015 and 0.728 kJ/kg·K respectively.

Solution

(i) 1–*a*–2; Reversible constant volume followed by constant-pressure process.

(ii) 1–*b*–2; Reversible adiabatic process followed by constant-pressure process.

(iii) 1–*c*–2; Isothermal process followed by isobaric process.

(iv) 1–2; Reversible process.

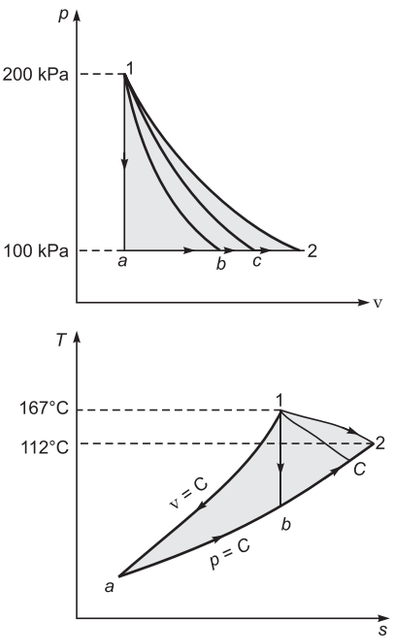

Fig. 7.12 *p–v and T–s diagrams*

Given Reversible expansion of air

State 2: $p_1 = 200$ kPa

$T_1 = 167°\text{C} + 273 = 440$ K

State 2: $p_2 = 100$ kPa

$T_2 = 112°\text{C} + 273 = 385$ K

$C_p = 1.015$ kJ/kg·K

$C_v = 0.728$ kJ/kg·K

To find Entropy change in each combination of processes.

Analysis

(i) *The reversible constant-volume process followed by reversible constant-pressure process,* i.e., *path 1–a–2*

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$$\Delta s = \int_1^2 \frac{\delta q}{T} = \int_1^a \frac{\delta q}{T} + \int_a^2 \frac{\delta q}{T}$$

$$= C_v \int_1^a \frac{dT}{T} + C_p \int_a^2 \frac{dT}{T}$$

$$= C_v \ln\left(\frac{T_a}{T_1}\right) + C_p \ln\left(\frac{T_2}{T_a}\right) \quad ...(A)$$

During constant-volume process, $V_1 = V_2$ and

$$\frac{p_1}{T_1} = \frac{p_a}{T_a} = \frac{p_2}{T_a}$$

or $$T_a = T_1\left(\frac{p_2}{p_1}\right) = 440 \times \left(\frac{100}{200}\right)$$

$$= 220 \text{ K}$$

Using numerical values,

Using $$\Delta s = 0.728 \times \ln\left(\frac{220}{440}\right) + 1.015 \times \ln\left(\frac{385}{220}\right)$$

$$= -0.5046 + 0.568 = \mathbf{0.0634 \text{ kJ/kg} \cdot \text{K}}$$

(ii) *Isentropic process followed by isobaric process during path 1–b–2*

Ratio of specific heats,

$$\gamma = \frac{C_p}{C_v} = \frac{1.015}{0.728} = 1.394$$

Temperature after isentropic expansion

$$\frac{T_b}{T_1} = \left(\frac{p_b}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_b = 440 \times \left(\frac{100}{200}\right)^{\frac{1.394-1}{1.394}} = 361.7 \text{ K}$$

The entropy change during path 1–*b*–2,

$$\Delta s = \int_1^2 \frac{\delta Q}{T} = \int_1^b \frac{\delta Q}{T} + \int_b^2 \frac{\delta Q}{T}$$

$$= 0 + C_p \ln\left(\frac{T_2}{T_b}\right) \quad ...(B)$$

$$= 0 + 1.015 \times \ln\left(\frac{385}{361.7}\right)$$

$$= \mathbf{0.0634 \text{ kJ/kg} \cdot \text{K}}$$

(iii) *When isothermal process is followed by isobaric process,* i.e., *path 1–c–2*

Specific gas constant,

$$R = C_p - C_v = 1.015 - 0.728$$

$$= 0.287 \text{ kJ/kg} \cdot \text{K}$$

During isothermal process ($T_1 = T_c$) and

$$\frac{V_c}{V_1} = \frac{p_1}{p_c} = \frac{p_1}{p_2} = \frac{200}{100} = 2$$

and $$T_c = T_1 = 440 \text{ K}$$

The entropy change during the path 1–*c*–2,

$$\Delta s = R \ln\left(\frac{V_c}{V_1}\right) + C_p \ln\left(\frac{T_2}{T_c}\right) \quad ...(C)$$

$$= 0.287 \times \ln(2) + 1.015 \times \ln\left(\frac{385}{440}\right)$$

$$= \mathbf{0.0634 \text{ kJ/kg} \cdot \text{K}}$$

(iv) *When air expands reversibly between two states 1 and 2*

For an ideal gas, the change in specific entropy is expressed as

$$\Delta s = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right)$$

$$= 1.015 \times \ln\left(\frac{385}{440}\right) - 0.287 \times \ln\left(\frac{100}{200}\right)$$

$$= \mathbf{0.0634 \text{ kJ/kg} \cdot \text{K}}$$

It proves that *entropy is a property*, and therefore, change in its value depends on end states not on path followed during the process.

Example 7.5 *A thermal energy source at 800 K loses 2000 kJ of heat to a sink at (a) 500 K, and (b) 750 K. Determine which heat transfer process is more irreversible.*

Solution

Given $T_{source} = 800$ K, $Q_{source} = 2000$ kJ

To find More irreversible process.

(i) $T_{sink} = 500$ K,

(ii) $T_{sink} = 750$ K.

Analysis The entropy change for each reservoir can be determined as $\Delta S = \frac{Q}{T}$

(i) *When heat is transferred to a sink at 500 K*

$$\Delta S_{source} = \frac{Q_{source}}{T_{source}} = \frac{-2000 \text{ kJ}}{800}$$

$$= -2.5 \text{ kJ/kg}$$

$$\Delta S_{sink} = \frac{Q_{sink}}{T_{sink}} = \frac{+2000 \text{ kJ}}{500} = +4 \text{ kJ/K}$$

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and $\Delta S_{Total} = \Delta S_{sourse} + \Delta S_{sink}$
$= -2.5 + 4.0 = \mathbf{+1.5\ kJ/K}$

(ii) *When heat is transferred to a sink at 750 K*

$\Delta S_{source} = -2.5$ kJ/K

$\Delta S_{sink} = \dfrac{2000\,\text{kJ}}{750\,\text{K}} = 2.7$ kJ/K

and $\Delta S_{Total} = -2.5 + 2.7 = \mathbf{+0.2\ kJ/K}$

The total entropy change for the *process (a)* is large and, therefore, *more irreversible*.

Example 7.6 *1.5 kg of air at 1 bar, 300 K is contained in a rigid insulated tank. During the process, 18 kJ of work is done on the gas through a paddle-wheel mechanism. Determine the final temperature, final pressure of air in the tank and change in entropy. Assume specific heats of air to be constant.*

Solution

Given $m = 1.5$ kg
$p_1 = 1$ bar $= 100$ kPa
$T_1 = 300$ K
$W = -18$ kJ (Work done on the system)
$V =$ constant (rigid tank)
$Q = 0$ (insulated tank)

To find

(i) Final temperature of air in the tank,
(ii) Final pressure of air in the tank, and
(iii) Change in entropy of air.

Assumptions

(i) Air is an ideal gas.
(ii) Specific heats of air as
$C_p = 1.005$ kJ /kg·K, $C_v = 0.716$ kJ/kg·K.

Analysis For an insulated and rigid tank, heat transfer and boundary work are zero. However, the paddle-wheel work is added to a system of air. Hence, according to the first law of thermodynamics, for a closed system,

(i) $Q = W_{paddle} + \Delta U$
or $0 = -18\text{ kJ} + \Delta U$
or $\Delta U = 18$ kJ
But $\Delta U = m C_v (T_2 - T_1)$
$\therefore$ $18 = 1.5 \times 0.716 \times (T_2 - 300)$
or $T_2 = 316.76$ K $= \mathbf{43.76°C}$

(ii) Further, for constant volume process

$$\frac{p_2}{p_1} = \frac{T_2}{T_1}$$

or $p_2 = \dfrac{316.76}{300} \times (1\text{ bar}) = \mathbf{1.0588\ bar}$

(iii) **Change in entropy:** It can be calculated as

$$\Delta S = m C_v \ln\left(\frac{T_2}{T_1}\right) + m R \ln\left(\frac{V_2}{V_1}\right)$$

Since volume remains constant, thus

$$\Delta S = m C_v \ln\left(\frac{T_2}{T_1}\right)$$
$$= 1.5 \times 0.716 \times \ln\left(\frac{316.76}{300}\right)$$
$$= \mathbf{0.0584\ kJ/K}$$

Example 7.7 *Two kg of a certain gas with $C_p = 0.85$ kJ/kg·K and $C_v = 0.70$ kJ/kg·K is allowed to expand adiabatically through a partly open valve, whereby its volume changes from 0.0113 m^3 to some higher volume and during the process the entropy increases by 0.8 kJ/K. Find the volume after expansion of gas.*

Solution

Given $m = 2$ kg,
$C_p = 0.85$ kJ/kg·K
$C_v = 0.70$ kJ/kg·K
$V_1 = 0.0113\ \text{m}^3$
$Q = 0$ (Since adiabatic)
$W = 0$ (Since throttling process)
$\Delta S = 0.8$ kJ/K

To find Final volume of gas.

Analysis The entropy change can be calculated by

$$\Delta S = m C_v \ln\left(\frac{T_2}{T_1}\right) + m R \ln\left(\frac{V_2}{V_1}\right)$$

According to the first law of themodynamics,

$Q = W + \Delta U$

For free expansion $Q = 0$, $W = 0$

Thus, $\Delta U = 0$

Therefore, $T_1 = T_2$

The gas constant $R = C_p - C_v = 0.85 - 0.7$
$= 0.15$ kJ/kg·K

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Then, $\Delta S = mR \ln\left(\frac{V_2}{V_1}\right)$

or $0.8 = 2 \times 0.15 \times \ln\left(\frac{V_2}{0.0113}\right)$

or $\ln\left(\frac{V_2}{0.0113}\right) = 2.667$

or $V_2 = 0.0113 \times \exp(2.667)$
$= \mathbf{0.1627\ m^3}$

Example 7.8 *A 50-kg block of iron casting at 500 K is thrown into a large lake which is at a temperature of 285 K. After the iron block reaches thermal equilibrium with the lake water, determine*

(a) Entropy change of the iron block,
(b) Entropy change of the lake water,
(c) The total entropy change during this process.

Assume average specific heat for the iron block as 0.45 kJ/kg·K.

Solution

Given $m = 50$ kg $\quad T_s = 500$ K
$T_{lake} = 285$ K

To find

(i) Entropy change of the iron block,
(ii) Entropy change of the lake, and
(iii) Total entropy change during the process.

Analysis

(i) *The entropy change of the iron block*

$$\Delta S_{iron} = \int \frac{\delta Q}{T} = \int m C_{av} \frac{dT}{T}$$
$$= m C_{av} \ln\left(\frac{T_2}{T_1}\right)$$
$$= 50 \times 0.45 \times \ln\left(\frac{285}{500}\right)$$
$$= \mathbf{-12.65\ kJ/K}$$

(ii) *The entropy change of the lake water*

The heat receive a by the lake water at constant temperature
= The heat rejected by the iron block
$Q_{lake} = m C_{av}(T_s - T_{lake})$
$= 50 \times 0.45 \times (500 - 285)$
$= 4837.5$ kJ

and $\Delta S_{lake} = \frac{Q_{lake}}{T_{lake}} = \frac{4837.5}{285} = \mathbf{16.97\ kJ/K}$

The total entropy change for this process is the sum of these two processes. Since the block and lake together form an isolated system,

$\Delta S_{Total} = \Delta S_{iron} + \Delta S_{lake} = -12.65 + 16.97$
$= \mathbf{4.32\ kJ/K}$

Example 7.9 *One kg of ice at –20°C is exposed to the atmosphere which is at 20°C. The ice melts and comes into thermal equilibrium with the atmosphere. Calculate the entropy increase of the universe. Take C_p of ice as 2.093 kJ/kg·K and latent heat of the fusion of ice as 334.5 kJ/kg.*

Solution

Given $m = 1$ kg of ice
$T_1 = -20°C + 273 = 253$ K
$T_A = 20 + 273 = 293$ K
Latent heat of fusion $= 334.5$ kJ/kg
Sp. heat of ice, $C_{p,\,ice} = 2.093$ kJ/kg·K

To find Entropy change of the universe.

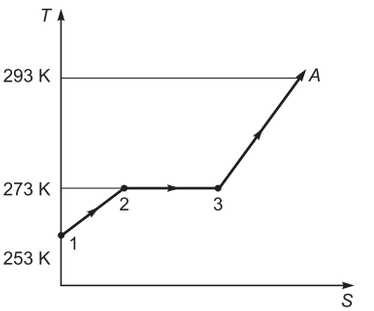

Fig. 7.13

Assumption Sp. heat of water, $C_{pw} = 4.187$ kJ/kg·K

Analysis Entropy change during heating of ice from –20°C to 0°C

Process 1-2;

$$\Delta S_{1-2} = \int_1^2 \left(\frac{\delta Q}{T}\right) = m \int_1^2 C_{p,\,ice} \frac{dT}{T}$$
$$= m C_{p,\,ice} \ln\left(\frac{T_2}{T_1}\right) = 1 \times 2.093 \times \ln\left(\frac{273}{253}\right)$$
$$= \mathbf{0.1592\ kJ/K}$$

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Entropy change during fusion of ice (Process 2-3) ($Q = m h_{fg}$)

$$\Delta S_{2-3} = \int_2^3 \left(\frac{\delta Q}{T}\right) = \frac{Q}{T_2} = \frac{1 \times 334.5}{273} = \mathbf{1.2253\ kJ/K}$$

Entropy change during sensible heating of water from 0°C to 20°C,

$$\Delta S_{3-4} = \int_3^4 \left(\frac{\delta Q}{T}\right)$$

$$= m \int_3^4 C_{pw} \left(\frac{dT}{T}\right) = m C_{pw} \ln\left(\frac{T_4}{T_3}\right)$$

$$= 1 \times 4.187 \times \ln\left(\frac{293}{273}\right) = \mathbf{0.296\ kJ/K}$$

Total entropy change of the ice-and-water system

$$\Delta S_{1-4} = \Delta S_{1-2} + \Delta S_{2-3} + \Delta S_{3-4}$$
$$= 0.1592 + 1.2253 + 0.296 = \mathbf{1.680\ kJ/K}$$

Heat absorbed by the ice–water system from the atmosphere to reach 20°C

= Heat absorbed in warming of ice
+ Latent heat of fusion
+ Heat absorbed during sensible heating of water to raise its temperature from 0°C to 20°C

$$Q = m[C_{p,\,ice}\,\Delta T_{warming} + h_{fg} + C_{pw}\,\Delta T_{heating}]$$
$$= (1\text{ kg}) \times [(2.093\text{ kJ/kg·K}) \times (0 - (-20))\,(°C) + 334.5\text{ kJ/kg}] + (4.187\text{ kJ/kg·K}) \times (20 - 0)\,(°C)$$
$$= 460\text{ kJ}$$

Entropy decrease of the atmosphere

$$\Delta S_{atm} = \frac{Q}{T} = \frac{460}{293} = 1.570\text{ kJ/K}$$

Entropy change of the universe

= Entropy increase of the ice–water system
– Entropy decrease of atmosphere
= 1.680 – 1.570 = **0.109 kJ/K**

Example 7.10 *4 kg of water at 27°C is mixed with 1 kg of ice at 0°C. Assuming adiabatic mixing, determine the final temperature of the mixture of water and ice. Calculate the net change of entropy. Assume enthalpy of fusion of ice as 335 kJ/kg.*

Solution

Given Water mixing with ice

m_1 = 4 kg of water
T_1 = 27°C or 300 K
m_2 = 1 kg of ice
T_2 = 0°C
Latent heat of fusion = 335 kJ/kg

To find The net change of entropy.

Assumption Specific heat of water, C_{pw} = 4.187 kJ/kg·K

Analysis On mixing of ice and water, say final temperature is T_f.

Heat loss by water
= Heat gain by the ice–water system
= Heat gain in fusion of ice + heating of water

$m_1 C_{pw}(T_1 - T_f)$
$= m_2$ [latent heat of fusion + $C_{pw}(T_f - 0)$]

Since temperature difference is involved in the relation, therefore, either temperature scale can be used. Using Celsius scale for calculation of T_f of mixture,

$$4 \times 4.187 \times (27 - T_f) = 1 \times (335 + 4.187 T_f)$$
$$452.196 - 16.742\,T_f = 335 + 4.187\,T_f$$
$$20.929\,T_f = 117.196$$
$$T_f = 5.5999°C \approx 5.6°C = 278.6\text{ K}$$

Entropy change using temperature in K

(i) Entropy decrease of 4 kg of water from 27°C to 5.6°C

$$\Delta S_{water\,1} = m_1 C_{pw} \ln\left(\frac{T_f}{T_1}\right)$$
$$= 4 \times 4.187 \times \ln(278.6/300)$$
$$= -1.239\text{ kJ/K}$$

(ii) Entropy change of 1 kg of ice = Entropy increase during fusion of ice

$$\Delta S_{ice} = m_2 \frac{\text{Latent heat}}{T_0} = 1 \times \left[\frac{335}{273}\right]$$
$$= 1.227\text{ kJ/K}$$

(iii) Entropy increase of 1 kg of water from 0°C to 5.6°C

$$\Delta S_{water\,2} = C_{pw} \ln\left(\frac{T_f}{T_0}\right)$$
$$= 1 \times \left[4.187 \times \ln\left(\frac{278.6}{273}\right)\right]$$
$$= 0.0850\text{ kJ/K}$$

Total entropy increase of 1 kg of ice–water system

$$= \Delta S_{ice} + \Delta S_{water\,2}$$
$$\Delta S_{total} = 1.227 + 0.0850 = 1.312\text{ kJ/K}$$

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Net entropy change due to mixing of 1 kg of ice and 4 kg of water

$$\Delta S_{net} = \Delta S_{total} + \Delta S_{water} = 1.312 - 1.239$$
$$= \mathbf{0.730\ kJ/K}$$

Example 7.11 *4 kg of air is compressed from 40°C and 125 kPa to 250°C and 875 kPa. It is then throttled to 257 kPa. Finally, it is cooled to a pressure of 125 kPa and 180°C. Calculate the overall change in entropy and also for each process.*

Take $C_p = 1.005\ kJ/kg \cdot K$ *and* $C_v = 0.717\ kJ/kg \cdot K$

Solution

Given Compression, throttling and cooling of air.

$m = 4$ kg
$C_p = 1.005$ kJ/kg K
$C_v = 0.717$ kJ/kg K

State 1: $p_1 = 125$ kPa
$T_1 = 40°C + 273 = 313$ K

State 2: $p_2 = 875$ kPa
$T_2 = 250°C + 273 = 523$ K

State 3: $p_3 = 257$ kPa
$h_2 = h_3$

State 4: $p_4 = 125$ kPa
$T_4 = 180°C + 273 = 453$ K

To find Change in entropy for each process and net change.

Analysis The gas constant for air

$$R = C_p - C_v$$
$$= 1.005 - 0.717 = 0.288\ \text{kJ/kg K}$$

(i) *The process 1–2, a polytropic process*

$$\Delta S_{1-2} = m\left\{C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)\right\}$$
$$= 4 \times \left\{1.005 \times \ln\left(\frac{523}{313}\right) - 0.288 \times \ln\left(\frac{875}{125}\right)\right\}$$
$$= 4 \times [0.516 - 0.5604] = \mathbf{-0.178\ kJ/K}$$

(ii) *The process 2–3, throttling process*

$$h_2 = h_3$$

or $$T_2 = T_3$$

$$\therefore \quad \Delta S_{2-3} = m\left\{-R \ln\left(\frac{p_3}{p_2}\right)\right\}$$
$$= 4 \times \left\{-0.288 \times \ln\left(\frac{257}{875}\right)\right\}$$
$$= \mathbf{1.411\ kJ/K}$$

(iii) *The process 3–4, a polytropic cooling process*

$$\Delta S_{3-4} = m\left\{C_p \ln\left(\frac{T_4}{T_3}\right) - R \ln\left(\frac{p_4}{p_3}\right)\right\}$$
$$= 4 \times \left\{1.005 \times \ln\left(\frac{453}{523}\right) - 0.288 \times \ln\left(\frac{125}{257}\right)\right\}$$
$$= 4 \times [-0.144 + 0.2075] = \mathbf{0.2527\ kJ/K}$$

Overall change in entropy,

$$\Delta S_{Total} = \Delta S_{1-2} + \Delta S_{2-3} + \Delta S_{3-4}$$
$$= -0.178 + 1.411 + 0.2527$$
$$= \mathbf{1.485\ kJ/K}$$

Example 7.12 *Show that for an ideal gas, the slope of the constant-pressure line is less than that of the constant-volume line.*

Solution For 1 kg of ideal gas, first *Tds* relation.

$$Tds = du + pdv = C_v dT + pdv$$

For a constant-volume process, $dv = 0$

$$Tds = C_v dT$$

or $$\frac{T}{C_v} = \left(\frac{dT}{ds}\right)_{v=c} \quad \text{... (i)}$$

Again from the second *Tds* relation

$$Tds = dh - vdp$$

For a constant-pressure process, $dp = 0$

$$Tds = dh = C_p dT$$

or $$\frac{T}{C_p} = \left(\frac{dT}{ds}\right)_{p=c} \quad \text{...(ii)}$$

The quantity $\left(\frac{dT}{ds}\right)$ represents the slope of the line on *T–s* diagram and $C_p > C_v$ for ideal gases, therefore

$$\left(\frac{dT}{ds}\right)_{p=c} < \left(\frac{dT}{ds}\right)_{v=c}$$

Therefore, as shown in Fig. 7.14, the slope of constant pressure line is less than the constant volume line.

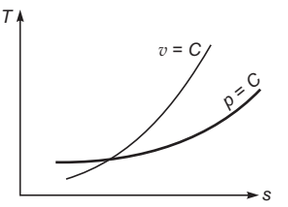

Fig. 7.14

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Example 7.13 *Two identical bodies of same heat capacity are at the same initial temperature T_1 . A refrigerator operates between these two bodies, until one body is cooled to the temperature T_2. If the bodies are at constant temperatures and do not undergo any change of phase, prove that the minimum amount of work needed by the refrigerator is*

$$C_p\left[\frac{T_1^2}{T_2}+T_2-2T_1\right]$$

Solution Consider two bodies A and B. Heat is removed from the body A to bring it to T_2.

Heat removed per kg from the body A

$$q_L = C_p(T_1 - T_2)$$

Heat discharged per kg by refrigerator to the body B

$$q_H = q + w = C_p(T - T_1)$$

where T becomes the temperature of the body B after heat addition.

Work input to the refrigerator,

$$w = C_p(T - T_1) - C_p(T_1 - T_2)$$
$$= C_p(T - 2T_1 + T_2) \quad \text{...(i)}$$

The decrease of entropy of the body A

$$\Delta s_A = \int_1^2 C_p\left(\frac{dT}{T}\right) = C_p \ln\left(\frac{T_2}{T_1}\right)$$

The entropy increase of the body B

$$\Delta s_B = \int_1^2 C_p\left(\frac{dT}{T}\right) = C_p \ln\left(\frac{T}{T_1}\right)$$

Net entropy change of the reversible refrigeration cycle = 0

Thus, cyclic $\int \frac{\delta q}{T} = \Delta s_A + \Delta s_B = 0$

Therefore, $C_p \ln\left(\frac{T_2}{T_1}\right) + C_p \ln\left(\frac{T}{T_1}\right) = 0$

or $\ln\left(\frac{T_2 T}{T_1^2}\right) = 0$

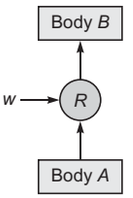

Fig. 7.15

or $\left(\frac{T_2 T}{T_1^2}\right) = 1$

or $T = \frac{T_1^2}{T_2}$

Substituting in Eq. (i)

$$w = C_p\left[\left(\frac{T_1^2}{T_2}\right) + T_2 - 2T_1\right] \textbf{ (Proved)}$$

Example 7.14 *A mass m of fluid at a temperature T_1 is mixed with an equal mass of the same fluid at a temperature T_2. Prove that the resultant change in entropy of the universe is*

$$\frac{2mC_p \ln(T_1 + T_2)}{2\sqrt{T_1 T_2}}$$

Solution If equal masses of two fluids are mixed then the equilibrium temperature T (say) can be obtained as

$$m_1 C_p T_1 + m_2 C_p T_2 = mC_p T$$

or $T = \frac{(m_1 T_1 + m_2 T_2)}{m} = \frac{(T_1 + T_2)}{2}$

$\because$ $m_1 = m_2$ and $m = 2m_1$

The entropy change can be expressed as

$$\Delta S = mC_p \int_{T_1}^{T} \frac{dT}{T} - mC_p \int_{T}^{T_2} \frac{dT}{T}$$
$$= mC_p \ln\left(\frac{T}{T_1}\right) - mC_p \ln\left(\frac{T_2}{T}\right)$$
$$= mC_p \ln\left(\frac{T^2}{T_1 T_2}\right)$$

Substituting the value of T in the above relation, we get

$$\Delta S = mC_p \ln[(T_1 + T_2)^2/(4T_1 T_2)]$$
$$= mC_p \ln\left\{\left(\frac{T_1 + T_2}{2\sqrt{T_1 T_2}}\right)^2\right\}$$
$$= 2mC_p \ln\left(\frac{T_1 + T_2}{2\sqrt{T_1 T_2}}\right) \textbf{ Proved}$$

The arithmetic mean $\frac{(T_1 + T_2)}{2}$ is always greater than geometric mean $\sqrt{(T_1 T_2)}$. Therefore, $\ln\left\{\left(\frac{T_1 + T_2}{2\sqrt{T_1 T_2}}\right)^2\right\}$ is always positive.

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Example 7.15 *A heat engine having a working substance of mass m and specific heat C_p works between a source at a temperature T_1 and a sink at a temperature T_2. Prove that the maximum obtainable work from such an engine is given by*

$$W_{max} = mC_p \left\{\sqrt{T_1} - \sqrt{T_2}\right\}^2$$

Solution The reversible engine gives maximum work. Since it works between the source temperature T_1 and the sink temperature T_2. Let the engine produces work at the temperature T.

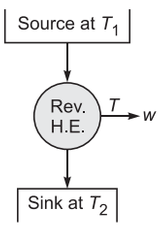

Fig. 7.15

Net entropy change at the engine = Entropy decrease of source – Entropy increase of sink

$$\Delta S = \int_{T_1}^{T} \frac{\delta Q}{T} - \int_{T}^{T_2} \frac{\delta Q}{T}$$

$$= mC_p \ln\left(\frac{T}{T_1}\right) - mC_p \ln\left(\frac{T_2}{T}\right)$$

For a reversible engine operating in a cycle

$$\Delta S = 0$$

Therefore,

$$mC_p \ln\left(\frac{T}{T_1}\right) - mC_p \ln\left(\frac{T_2}{T}\right) = 0$$

or $$\ln\left(\frac{T}{T_1}\right) = \ln\left(\frac{T_2}{T}\right)$$

or $$\frac{T}{T_1} = \frac{T_2}{T} \quad \text{or} \quad T^2 = T_1 T_2$$

or $$T = \sqrt{T_1 T_2}$$

Now the work done by the engine

= Heat supplied – heat rejected

$$W_{min} = mC_p(T_1 - T) - mC_p(T - T_2)$$

$$= mC_p(T_1 - 2T + T_2)$$

$$= mC_p\left(T_1 - 2\sqrt{T_1 T_2} + T_2\right)$$

or $$W_{min} = mC_p\left(\sqrt{T_1} - \sqrt{T_2}\right)^2 \quad \textbf{(Proved)}$$

Example 7.16 *Show that the entropy change between states1 and 2 in a polytropic process pv^n = constant is given by the following relation*

(a) $$s_2 - s_1 = \frac{n-\gamma}{(\gamma-1)(n-1)} R \ln\left(\frac{T_2}{T_1}\right), \text{ and}$$

(b) $$s_2 - s_1 = \frac{n-\gamma}{n(\gamma-1)} R \ln\left(\frac{p_2}{p_1}\right)$$

Solution For a polytropic process,

$$p_1 v_1^n = p_2 v_2^n$$

and $$\frac{T_2}{T_1} = \left(\frac{v_2}{v_1}\right)^{1-n} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

Taking natural log on both sides

$$\ln\left(\frac{T_2}{T_1}\right) = (1-n)\ln\left(\frac{v_2}{v_1}\right) = \frac{n-1}{n}\ln\left(\frac{p_2}{p_1}\right) \quad ...(i)$$

The general relations for change in entropy of a system

$$s_2 - s_1 = C_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right) \quad ...(ii)$$

and $$s_2 - s_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right) \quad ...(iii)$$

Using $\ln\left(\frac{v_2}{v_1}\right) = -\left(\frac{1}{n-1}\right)\ln\left(\frac{T_2}{T_1}\right)$ from Eq. (i) in Eq. (ii), we get

$$s_2 - s_1 = C_v \ln\left(\frac{T_2}{T_1}\right) - \frac{R}{(n-1)}\ln\left(\frac{T_2}{T_1}\right)$$

we have $C_v = \frac{R}{\gamma - 1}$, then

$$s_2 - s_1 = \frac{R}{\gamma-1}\ln\left(\frac{T_2}{T_1}\right) - \frac{R}{(n-1)}\ln\left(\frac{T_2}{T_1}\right)$$

or $$= \left(\frac{1}{\gamma-1} - \frac{1}{n-1}\right) R \ln\left(\frac{T_2}{T_1}\right)$$

or $$s_2 - s_1 = \frac{n-\gamma}{(\gamma-1)(n-1)} R \ln\left(\frac{T_2}{T_1}\right) \quad \textbf{Proved (a)}$$

Using $\ln\left(\frac{T_2}{T_1}\right) = \left(\frac{n-1}{n}\right)\ln\left(\frac{p_2}{p_1}\right)$ from Eq. (i) in Eq. (iii), we get

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$$s_2 - s_1 = \frac{n-1}{n} C_p \ln\left(\frac{p_2}{p_1}\right) - R \ln\left(\frac{p_2}{p_1}\right)$$

$$= \left[\left(\frac{n-1}{n}\right)\left(\frac{\gamma R}{\gamma - 1}\right) - R\right] \ln\left(\frac{p_2}{p_1}\right) \quad \left[\because \; C_p = \frac{\gamma R}{\gamma - 1}\right]$$

$$= \left[\frac{n\gamma - \gamma - n\gamma + n}{n(\gamma - 1)}\right] R \ln\left(\frac{p_2}{p_1}\right)$$

$$= \frac{n - \gamma}{n(\gamma - 1)} R \ln\left(\frac{p_2}{p_1}\right) \qquad \textbf{Proved (b)}$$

Example 7.17 *Calculate the minimum work required to chill 2 kg of drinking water from a temperature of 25°C to 2°C. The C_p of water as 4.184 kJ/kg·K.*

Solution

Given $m = 2$ kg of water
$T_{atm} = T_H = 25°C = 298$ K
$T_L = 2°C = 275$ K,
$C_p = 4.184$ kJ/kg·K

To find The minimum (or reversible) work required.

Assumption Assuming constant pressure during cooling process.

Analysis *During cooling process of drinking water*

Entropy decrease of water,

$$\Delta S_1 = m C_p \ln\left(\frac{T_H}{T_L}\right)$$

And heat removed from the water,

$$Q_L = m C_p (T_H - T_L)$$

Heat supplied to the atmosphere,

$$Q_H = W_{in} + Q_L$$

or $$Q_H = W_{in} + m C_p (T_H - T_L)$$

The entropy increase of atmosphere

$$= \frac{W_{in} + m C_p (T_H - T_L)}{T_H}$$

For a refrigerator operating on the reversible cycle the net entropy change is zero. Therefore,

$$\frac{W_{in} + m C_p (T_H - T_L)}{T_H} - m C_p \ln\left(\frac{T_H}{T_L}\right) = 0$$

or $$W_{in} = m C_p \ln\left(\frac{T_H}{T_L}\right) T_H - m C_p (T_H - T_L)$$

$$W_{in} = m C_p \left[T_H \ln\left(\frac{T_H}{T_L}\right) - (T_L - T_L)\right]$$

Using numerical values

$$W_{in} = 2 \times 4.184 \left[298 \times \ln\left(\frac{295}{275}\right) - (298 - 275)\right]$$

$$= \mathbf{7.833\ kJ}$$

Example 7.18 *A novel reversible heat engine plot on a T–s diagram is as circle. The maximum and minimum temperatures are 1100 K and 200 K, respectively and the maximum entropy change in the cycle is 2 kJ/K. Calculate the heat added to the cycle, heat rejected, net work output and the thermal efficiency of the cycle.*

Solution

Given *T–s* diagram is a circle as shown in Fig. 7.16.

$T_1 = 1100$ K
$T_2 = 200$ K
$\Delta S = 2$ kJ/K

To find

(i) Heat addition to engine,
(ii) Heat rejection by engine,
(iii) Efficiency of the engine.

Analysis Mean temperature

$$T_3 = \frac{(T_1 + T_2)}{2}$$

$$T_3 = \frac{(1100 + 200)}{2} = 650 \text{ K}$$

From Fig. 7.16,

Area 1231 = area 1341
= area of semicircle

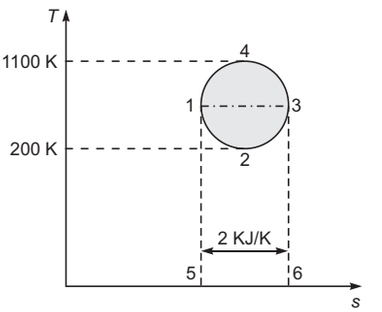

Fig. 7.16

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$$= 0.5 \times \left(\frac{\pi}{4}\right) \times D^2$$

$$= 0.5 \times \left(\frac{\pi}{4}\right) \times (\Delta s \times \Delta T)$$

where $\Delta s = 2$ kJ/K

and $\Delta T = 1100 - 200 = 900$ K

or Area 1231 $= 0.5 \times \left(\frac{\pi}{4}\right) \times 2 \times 900 = 706.85$ kJ

Heat added = Upper semicircle area + lower rectangle area

Thus, heat added = area (1341) + area (1365)

$= 706.85 + 2 \times 650 =$ **2006.85 kJ**

Heat rejection = Area of rectangle 1365 – area of semicircle 1321

$= 2 \times 650 - 706.85 =$ **593.15 kJ**

The work done in the cycle

= Head added – heat rejection

$= 2006.85 - 593.15 = 1413.7$ kJ

Thermal efficiency of cycle

$$= \frac{\text{Work done}}{\text{Heat supplied}}$$

$$\eta_{th} = \frac{1413.7}{2006.85} = \textbf{0.7044 or 70.44\%}$$

Example 7.19 *(a) 1 kg water at 0°C is brought into contact with a heat reservoir at 90°C. When water has reached 90°C, find*

(i) Entropy change of water,

(ii) Entropy change of the reservoir,

(iii) Entropy change of the universe.

(b) If water is heated from 0°C to 90°C by first bringing it in contact with the reservoir at 40°C and then with a reservoir at 90°C, what will be the entropy change of the universe?

(c) Explain how water might be heated from 0°C to 90°C with almost no change in entropy of the universe?

Solution

Given $m = 1$ kg

$T_1 = 0°C = 273$ K $\qquad T_2 = 90°C = 363$ K

To find

(i) Entropy change of water,

(ii) Entropy change of reservoir,

(iii) Entropy change of universe.

Assumption The specific heat of water as 4.187 kJ/kg·K.

Analysis

(i) Heat transferred to water from reservoir,

$$Q = mC_p(\Delta T)$$

or $Q = 1 \times 4.187 \times (90 - 0) = 376.83$ kJ

(a) Entropy change of water,

$$\Delta S_w = mC_p \ln\left(\frac{T_2}{T_1}\right)$$

$$= 1 \times 4.187 \times \ln\left(\frac{363}{273}\right) = \textbf{1.193 kJ/K}$$

(b) Entropy change of reservoir,

$$(\Delta S)_{reservoir} = \frac{Q}{T_2} = \frac{376.83}{363} = \textbf{1.038 kJ/K}$$

(c) Entropy change of universe,

$\Delta S_{universe} = \Delta S_2 - \Delta S_{reservoir}$

$= 1.193 - 1.038 =$ **0.155 kJ/K**

(ii) **Given**

$m = 1$ kg

$T_1 = 273$,

$T_2 = 40 + 273 = 313$ K,

$T_3 = 90 + 273 = 363$ K

Heat transferred to water from reservoir at 40°C

$$Q_1 = mC_p(T_2 - T_1)$$

$= 1 \times 4.187 \times (40 - 0) = 167.48$ kJ

Heat transferred to water from reservoir at 90°C

$$Q_2 = mC_p(T_3 - T_1)$$

$= 1 \times 4.187 \times (90 - 40) = 209.35$ kJ

Now entropy change of the two reservoirs

$$(\Delta S)_{res} = \frac{Q_1}{T_2} + \frac{Q_2}{T_1}$$

$$(\Delta S)_{res} = \frac{167.48}{313} + \frac{209.35}{363}$$

$= 0.535 + 0.576$

$= 1.111$ kJ/kg

The entropy of water will remain same for the same temperature difference. Thus entropy change of the universe,

$\Delta S_{universe} = (\Delta S)_w - (\Delta S)_{res}$

$= 1.193 - 1.111$

$=$ **0.0842 kJ/K**

(iii) If water is heated reversibly, then net entropy change of universe will be *zero*.

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Summary

- Entropy is an abstract property. It is the quantitative measure of molecular disorder within a system. The entropy change dS is defined as

$$dS = \left(\frac{\delta Q}{T}\right)_{int,\, rev} \quad \text{(kJ/K)}$$

- The total entropy (extensive property) change during a process is obtained by integrating the above relation.

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_{int,\, rev} \quad \text{(kJ/K)}$$

- The entropy per unit mass s is called specific entropy and it is an intensive property. The change in its value during a process is

$$\Delta s = \int_1^2 \left(\frac{\delta q}{T}\right)_{int,\, rev} \quad \text{(kJ/Kg} \cdot \text{K)}$$

- The entropy change for a cyclic process is explained by Clausius inequality as

$$\oint \frac{\delta Q}{T} \leq 0$$

where equality sign holds for an internally or totally reversible process and the inequality for irreversible process. The entropy for all actual processes always increases and its increase of entropy principle

$$dS \geq \frac{\delta Q}{T}$$

or $$\Delta S_{isolated} \geq 0$$

- The entropy generated during an irreversible process is called *entropy generation*, S_{gen}.

$$S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{gen}$$

- The entropy change of a system or its surroundings can be negative during a process, but entropy generation cannot. The increase of entropy principle can be summarised as

$$S_{gen} \begin{cases} > 0 & \text{irreversible process} \\ = 0 & \text{reversible process} \\ < 0 & \text{impossible process} \end{cases}$$

- The two Tds relations are

$$Tds = du + pdv$$
$$Tds = dh - vdp$$

- For ideal gas, the entropy change during a process can be obtained by integrating these relations, and we get

$$\Delta s = C_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)$$

and $$\Delta s = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right)$$

- For solids and incompressible liquids, $C_p = C_v = C_{av}$ and $v_1 = v_2$, thus

$$\Delta S = C_{av} \ln\left(\frac{T_2}{T_1}\right)$$

- The process during which entropy remains constant is called an *isentropic process*.

Glossary

Entropy A measure of *molecular disorder* or *molecular randomness* in matter

Specific entropy Entropy per unit mass

Entropy generation Entropy generated during a process

Third law of thermodynamics Entropy of a pure substance at absolute zero temperature is zero

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Review Questions

1. Define entropy and prove that entropy is a property.
2. Prove that two isentropic lines cannot intersect each other.
3. State and prove Clausius' theorem.
4. What is the Clausius inquality? Discuss.
5. What is entropy generation? Explain.
6. State and prove Clausius inequality.
7. What is the concept of entropy? Explain.
8. Prove that $dS = \frac{\delta Q}{T}$ for a reversible process and show that the entropy is a property of a system.
9. Derive a general expression for the change in entropy of a system during a process.
10. Derive an expression for change in the entropy of a system during a constant-volume process.
11. Derive an expression for change in the entropy of a system during a constant-pressure process.
12. Derive an expression for change in the entropy of a system during an isothermal process.
13. Derive an expression for change in the entropy of a system during a polytropic process.

Problems

1. 300 kW of heat is supplied at a constant temperature of 290°C to a heat engine. The engine rejects heat at 8.5°C. The following results were recorded:

(a) 215 kW is rejected.
(b) 150 kW is rejected.
(c) 75 kW is rejected.

Classify which of the results reports a reversible, irreversible or impossible cycle.

[(a) irreversible cycle, (b) reversible cycle, (c) impossible cycle]

2. Air expands steadily through a turbine from 200 kPa, 60°C to 90 kPa, 15°C. The entropy of the surroundings decreases by 0.04 kJ/kg K. Would such a process be irreversible, reversible or impossible? Why? *[impossible process]*

3. A refrigerator removes heat from a cold space at 2°C at a rate of 300 kJ/min and rejects heat to the kitchen air at 26°C at a rate of 345 kJ/min. Determine whether the refrigerator violates the second law on the basis of (a) Clausius inequality, and (b) Carnot principle.

[Voilates (a), Satisfy (b)]

4. Heat is transferred from a very large mass of water at 90°C to 0.1 kg of air that expands irreversibly from 400 kPa, 60°C to 150 kPa, 35°C, doing 4.00 kJ of work. Calculate the heat transferred to air and the entropy change of the universe.

[2.2 kJ, 0.016 kJ/kg]

5. 1 kg of air expands from 400 kPa, 550°C to 100 kPa. During the process, the entropy change of the surroundings is 0.135 kJ/K. Determine the minimum possible temperature of air in the system.

6. A 10-kg metal piece with constant specific heat of 0.9 kJ/kg K at 200°C is dropped into an insulated tank which contains 100 kg of water at 20°C. Determine the final equilibrium temperature and total change in entropy for this process.

[23.8°C, 1.183 kJ/K]

7. 1 MJ heat is supplied from a thermal reservoir at 800 K to a thermal reservoir at 400 K. Calculate the entropy change of the universe resulting from this heat-exchange process. *[1.25 kJ/K]*

8. During isothermal heat addition process of a Carnot cycle, 800 kJ heat is added to the working fluid from a source of 527°C. Determine

(a) change in entropy of the working fluid,
(b) change in entropy of the source,
(c) total entropy change during the process.

[(a) 1 kJ/K, (b) –1 kJ/K, (c) 0]

9. A rigid tank contains air at 35°C, which is being stirred by a paddle wheel. The paddle wheel does

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500 kJ of work on the air. During the stirring process, the air temperature remains constant, owing to transfer of heat between the system and surroundings, which is at 15°C. Calculate

(a) change in entropy of air in the tank, and

(b) change in entropy of the surroundings.

Does this process satisfy the increase-of-entropy principle?

10. Air expands irreversibly from 3 bar, 200°C to 1.5 bar, 105°C. Calculate the specific entropy change of air. *[– 4.242 kJ/kg K]*

11. Air flows through an insulated horizontal duct. The pressure and temperatures are measured at two different stations *A* and *B*. They are p_A = 0.2 MPa; T_A = 150°C; p_B = 0.18 MPa, and T_B = 130°C. Determine the specific entropy change of air. *[– 0.533 kJ/kg·K]*

12. A rigid cylinder containing 0.006 m^3 of nitrogen (R = 0.296 kJ/kg·K) at 1.04 bar, 150°C is heated reversibly until the temperature reaches 90°C. Calculate the change of entropy and heat supplied. Take γ= 1.4. *[0.00125 kJ/K, 0.406 kJ]*

13. Heat is transferred from a very large mass of water at 90°C to 0.1 kg of oxygen that expands irreversibly in a cylinder fitted with a piston from 400 kPa, 60°C to 150 kPa, 350°C. The oxygen does 4.0 kJ of work. Calculate (a) the heat transfer to the oxygen, and (b) entropy change of the universe. *[(a) 2.34 kJ (b) 0.0118 kJ/K]*

14. Air expands irreversibly in a cylinder from 280 kPa, 60°C to 140 kPa. The air does 30 kJ/kg of work and 14 kJ/kg of heat is removed from the air during the expansion. The initial volume of air is 0.00878 m^3. Calculate the specific entropy change of air. *[0.0095 kJ/k]*

15. 3 kg of an ideal gas is expanded from a pressure of 7 bar and a volume of 1.5 m^3 to a pressure of 1.4 bar and a volume of 4.5 m^3. Determine: (a) Specific gas constant, (b) initial and final temperature, and (c) change in entropy during heat exchange.

Take C_p = 1.05 kJ/kg·K

[(a) 0.3 kJ/kg·K, (b) 893.67°C, 427°C, (c) –0.1606 kJ/K]

16. Two boilers discharge an equal amount of steam into a steam main. The steam from one boiler is at 18 bar and 380°C and from the other boiler, it is 0.95 dry at 18 bar. Determine

(a) equilibrium condition after mixing,

(b) the loss of entropy by high temperature steam,

(c) gain of entropy by low temperature steam,

(d) net increase or decrease of entropy.

[(a) 63.7°C (b) 0.383 kJ/kg·K (c) 0.470 kJ/kg·K (d) 0.854 kJ/kg·K]

Objective Questions

1. Entropy is
   (a) an extensive property
   (b) an abstract property
   (c) a function of quality of heat
   (d) all of the above
2. Entropy is a function of
   (a) work transfer (b) volume
   (c) temperature (d) pressure
3. For any thermodynamic process, the area under the *T*–*s* curve represents
   (a) work transfer
   (b) heat transfer
   (c) pressure volume product
   (d) none of the above
4. When heat is absorbed by a gas, the change in entropy of gas is considered to be
   (a) zero (b) positive
   (c) negative (d) none of the above
5. The condition of reversibility of a cycle is
   (a) $\oint \frac{\delta Q}{T} < 0$ (b) $\oint \frac{\delta Q}{T} > 0$
   (c) $\oint \frac{\delta Q}{T} = 0$ (d) none of the above

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6. The condition of irreversibility of a cycle is
   (a) $\oint \frac{\delta Q}{T} < 0$ (b) $\oint \frac{\delta Q}{T} > 0$
   (c) $\oint \frac{\delta Q}{T} = 0$ (d) none of the above
7. If $\oint \frac{\delta Q}{T} > 0$ then the cycle is
   (a) reversible (b) irreversible
   (c) impossible (d) none of the above
8. During reversible adiabatic process, the entropy
   (a) increases (b) decreases
   (c) remains constant (d) none of the above
9. During a throttling process, the entropy
   (a) increases (b) decreases
   (c) remains constant (d) none of the above
10. On a *T*–*s* diagram, the slope of constant-pressure lines in comparison with constant-volume lines is
   (a) more (b) less
   (c) equal (d) none of the above
11. For an irreversible process, the entropy of the system is
   (a) zero (b) increased
   (c) decreased (d) none of the above
12. Entropy of the universe always tends to
   (a) zero (b) increase
   (c) decrease (d) none of the above
13. Entropy of an isolated system increases when a process is
   (a) reversible (b) irreversible
   (c) ideal (d) none of the above
14. Entropy of water at 0°C is assumed to be
   (a) 0 (b) 1
   (c) –1 (d) none of the above

Answers

1. (d) 2. (c) 3. (b) 4. (b) 5. (c) 6. (a) 7. (c) 8. (c)
9. (a) 10. (b) 11. (b) 12. (b) 13. (b) 14. (a)

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8

Availability and Irreversibility

Introduction

The second law of thermodynamics asserts that energy is degraded during a process; entropy is generated which reduces capacity to do work. The *availability* (exergy) concept is also derived from the second law of thermodynamics and it is closely related with reversibility and entropy. The availability is the maximum useful work which can be obtained from the system when it operates reversibly at a given state in a given environment. The irreversibility (also called exergy destruction) is the waste work potential during a process as a result of irreversibilities. Here, we have developed the availability function and availability balance relation for closed and open systems. The second law efficiency for various system is also taken up in this chapter.

8.1 SOURCES OF ENERGY

It becomes clear from the first and second laws of thermodynamics that energy has quantity as well as quality. Therefore, the sources of energy can be divided into two categories as high-grade energy and low-grade energy.

High-Grade Energy

(a) Mechanical work
(b) Electrical energy
(c) Water power or hydraulic energy
(d) Wind energy
(e) Tidal energy
(f) Kinetic energy of a jet

Low Grade Energy

(a) Thermal energy
(b) Heat obtained from nuclear fission or fusion
(c) Heat obtained from combustion of fossil fuels
(d) Geothermal energy

High-grade energy can be converted directly in any from of energy with its equivalent quantity, but *low-grade energy* such as heat cannot be converted directly into high-grade energy. The conversion of low-grade energy into high-grade energy requires a cyclic process and its complete conversion is impossible.

According to the second law of thermodynamics, the part of low-grade energy that can be converted into work is known as *available energy* and the part which is rejected as waste is known as *unavailable energy*.

Further, the quality of energy is always degraded during a process, hence entropy is generated, and the opportunities to produce work are reduced.

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8.2 AVAILABLE AND UNAVAILABLE ENERGY

As discussed above, the quality of energy is a measure of the work potential of a given quantity of energy. A reversible heat engine operating between a source temperature T_H and a sink temperature T_L has a work potential as

$$W_{rev,\,net} = \eta_{rev} \times Q_H = \left[1 - \frac{T_L}{T_H}\right] Q_H \qquad ...(8.1)$$

where Q_H is the amount of heat supplied to the reversible heat engine by the source at T_H.

We can increase the work potential either by increasing the source temperature T_H or by lowering the sink temperature T_L. However, the low temperature T_L has a large impact on the work potential, but the lowest temperature cannot be below the atmospheric temperature T_0. Then Eq. (8.1) becomes

$$W_{rev,\,net} = Q_H \left(1 - \frac{T_0}{T_H}\right) \qquad ...(8.2)$$

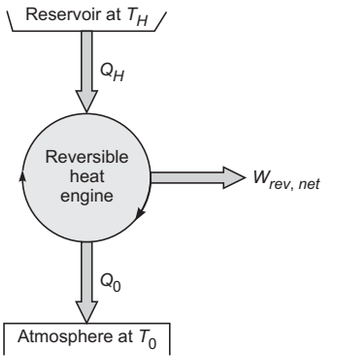
The quantity given at the right side of Eq. (8.2) is the available portion of the total energy quantity Q_H. Therefore, the available and unavailable energy can be defined as follows:

The available energy (A) is that portion of the amount of heat energy supplied to a reversible engine, which could be converted into useful work.

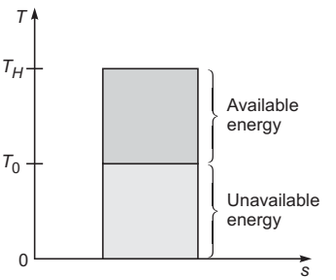
The maximum work can only be obtained from a reversible heat engine, because no engine is more efficient than a reversible engine, operating between the same temperature limits.

The unavailable energy is that portion of energy which cannot be converted into useful work by any means.

The unavailable energy is always present with the total energy; it is low-quality energy and its upgradation is not possible. For example, the energy in the atmosphere is huge in quantity but no part of it can be converted into work. Further, all the man-made and natural process are irreversible, and thus some portion of the available energy is converted to unavailable energy to overcome the irrerversibilities associated with processes, thus increasing the unavailable energy of universe.



(a) A reversible heat engine converts a constant energy source Q_H into network $W_{rev,net}$.



(b) T–s diagram for constant energy source as available and unavailable energy portions

Fig. 8.1

8.2.1 Dead State

If the state of a system is brought close to the atmosphere, the opportunity for developing work diminishes and ceases completely, when the system reaches thermodynamic equilibrium with the atmosphere. This state of the system is called the *dead state*. At the dead state, both the system and atmosphere possess energy, but the capacity to do work is zero, because there is no possibility of any interaction between the system and the atmosphere. The properties at the dead state are designated with subscript 0, such as pressure p_0, temperature T_0, volume V_0, etc.

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8.2.2 Useful Work

The useful work is defined as the network done by the system. It does not include work done on the atmosphere. When a gas expands in a piston cylinder device from an initial volume V_i to final volume V_f, the gas does work W. But all the work W would not be available for delivery, a part of this work is used to push the atmospheric air at pressure p_0 through a volume change of $V_f - V_i$ as shown in Fig. 8.2 and the work done on the atmosphere equals to

$$W_{surr} = p_0(V_f - V_i)$$

and the difference between gas work W and surroundings work W_{surr} is called the *useful work* given as

$$W_{useful} = W - W_{surr} = W - p_0(V_f - V_i) \quad ...(8.3)$$

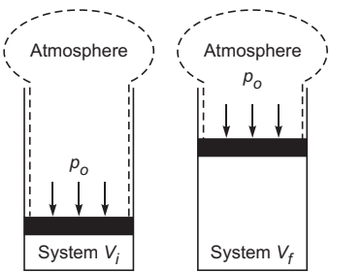

Fig. 8.2 *As a closed system expands some work is required to push the atmospheric air out.*

8.2.3 Maximum Work–Availablility

The work is only produced by a system; if it is initially not in dead state. Further, the maximum useful work can be produced, if a system approaches the dead state through a reversible process. Therefore, we conclude that a system can produce *maximum possible work*, when it undergoes a change of state in a reversible manner from an initial state to the state of its surroundings, i.e., the dead state. This useful maximum work is referred as availability of the system. In simple words, the availability is the useful work potential of the energy contained in a system of a specified state.

Thus, the *availability* of a system is defined as the *maximum useful work* that can be obtained from a system, when it undergoes a reversible process from an initial state to dead state. The availability of a system is sometimes referred as *exergy*.

8.2.4 Irreversibility

When an actual process occurs, it produces certain effects, therefore, the process cannot be reversed and the system and its surroundings cannot be restored to their initial states. During an irreversible process, the total energy remains constant but capacity to do work is lost due to degradation of some portion of available energy.

This degradation of energy is responsible for *entropy generation* within the system during a process. The entropy generation is always equal to irreversibilities involved in the process.

Simply the *irreversibility* is defined as the *difference between* the reversible work W_{max} and actual work W_{useful} during a process. It is designated as I and expressed as

$$I = W_{max} - W_{useful} \text{ (kJ)} \quad ...(8.4)$$

or

$$i = w_{max} - w_{useful} \text{ (kJ/kg)} \quad ...(8.5)$$

The irreversibility generated during a process per unit time is called the *irreversibility rate* $\dot{I}$ and is given by

$$\dot{I} = \dot{W}_{max} - \dot{W}_{useful} \text{ (kW)} \quad ...(8.6)$$

For a reversible process W_{max} is always equal to the useful work W_{useful}, thus the irreversibility is zero.

Example 8.1 *A heat engine receives heat from a source at 1200 K at a rate of 500 kW and rejects the waste heat to a medium at 300 K. The power output of the heat engine is 180 kW. Determine the reversible power and the irreversibility rate from this process.*

Solution

Given A heat engine operates between

$T_H = 1200$ K

$T_L = 300$ K

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$\dot{Q}_H$ = 500 kW
$\dot{W}_{useful}$ = 180 kW

To find

(i) Reversible work, and
(ii) Irreversibility rate in the process.

Analysis

(i) Reversible work: Carnot engine work.

$$\eta_{Carnot} = 1 - \frac{T_H}{T_H} = \frac{\dot{W}_{rev}}{\dot{Q}_H}$$

or $$\dot{W}_{rev} = \left(1 - \frac{300}{1200}\right) \times 500 = \mathbf{375\ kW}$$

(ii) Irreversibility rate

$\dot{I}$ = Reversible power – Useful (actual) power
$\dot{I}$ = 375 kW – 180 kW = **195 kW**

Here, $\dot{Q}_H - \dot{W}_{rev}$ = 500 – 375
= 125 kW is unavailable energy.

8.3 AVAILABILITY OF ENERGY ENTERING A SYSTEM

Energy may enter a system either as heat or work across its boundary. In actual practice, as heat is added to a system, its temperature rises from T_1 to T_2 as shown in Fig. 8.3. If this amount of heat is directly given to a reversible heat engine instead to a system, that would produce maximum amount of work. This work denotes the amount of heat as available energy.

The available portion of the heat energy that crosses the boundary of a system may be determined by assuming that the reversible engine receives the heat in the same manner as a system receives.

Consider a differential amount of heat δQ_H is added at a temperature T to a reversible heat engine. Thus, the reversible engine operates between a temperature level T and the temperature of the ambient T_0. The maximum work produced by the engine becomes the available portion of δQ_H and is given by

$$\delta W_{max} = \delta Q_H \left(\frac{T - T_0}{T}\right) \qquad ...(8.7)$$

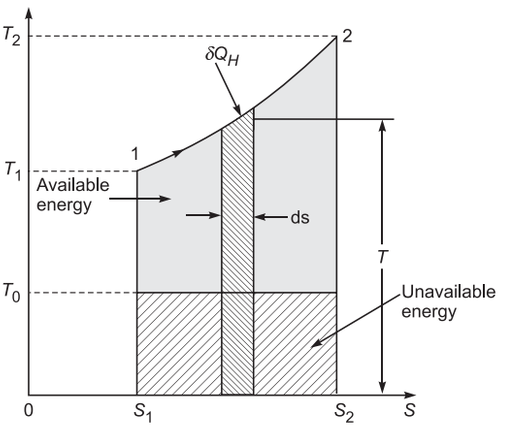

Fig. 8.3 *Addition of heat to a system*

$$\text{Unavailable portion} = \delta Q_H - \delta Q_H \left(\frac{T - T_0}{T}\right)$$

$$= \delta \dot{Q}_H \frac{T_0}{T} \qquad ...(8.8)$$

The hatched protion in Fig. 8.3 represents the unavailable portion of energy added.

$$\text{Unavailable energy added} = T_0 \int_1^2 \frac{\delta Q}{T}$$

We have $\frac{\delta Q}{T} = dS$

Thus

$$\text{unavailable energy} = T_0(S_2 - S_1)$$

An available portion of energy added

$$A = Q_H - T_0(S_2 - S_1) \qquad ...(8.9)$$

Since a finite temperature difference must be needed to cause a heat flow from a body to another, but as heat flows from a high-temperature source to a relatively lower temperature body, the temperature of heat energy decreases from source to destination, thus the capacity to producing work is lost. Figure 8.4 shows the loss of available energy during heat transfer.

The source transfers heat energy irreversibly, its temperature decreases from T_1 to T_2. As a result of heat addition to a system, its temperature increases from T_3 to T_4. The average temperature of heat transfer to a system is less than the average temperature of heat addition from the source,

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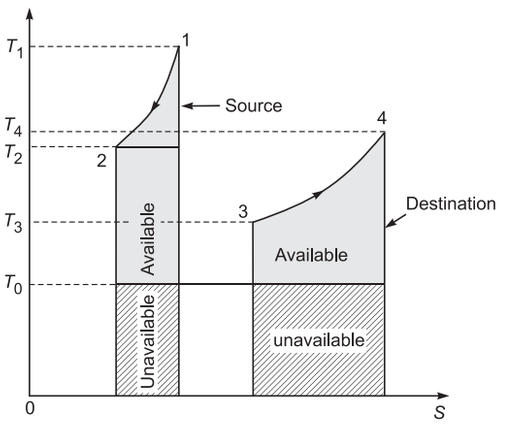

Fig. 8.4 *Loss of availability due to irreversible heat transfer*

thus available energy is reduced during such heat transfer process.

8.3.1 Availability Aspect

1. Availability is a measure of work potential of a system that a system can deliver when it is brought to atmospheric conditions.
2. The value of availability cannot be negative. If a system is at any state other than the dead state, the system would be able to change its condition spontaneously towards the dead state and this tendency is completely ceased when the system reaches the dead state. Accordingly, at least zero work is being developed, and thus the availability (maximum work) cannot be negative.
3. Availability is not conserved, but is destroyed by irreversibilities. As a limiting case, the availability of a system may be destroyed completely if there is no provision to obtain the work.
4. The availability is thus viewed as the maximum theoretical work obtainable, when a system passes a given state to dead state, while interacting with the atmosphere only.
5. The availability is an extensive property.

Example 8.2 *Air at 1 bar and 27°C is heated in a non-flow system at constant pressure to 177°C. Heat is supplied from a constant temperature reservoir at 577°C. The atmospheric temperature is 20°C. What percentage of heat added per kg of air is the available energy?*

Solution

Given $p = 1$ bar
$T_1 = 27°C$,
$T_2 = 177°C$
$T_H = 577°C = 850$ K
$T_0 = 20° = 293$ K

To find Percentage available energy of heat supplied.

Assumption The specific heat of air at constant pressure is 1.005 kJ/kg·K.

Analysis The heat transfer to air at constant pressure

$$Q_H = C_p(T_2 - T_1)$$
$$= 1.005 \times (177 - 27) = 150.75 \text{ kJ/kg}$$

Unavailable energy $Q_{unav} = Q_H \times \dfrac{T_0}{T_H}$

$$= 150.75 \times \frac{293}{850} = 51.96 \text{ kJ/kg}$$

Therefore, the available energy

$$A = Q_H - Q_{unav} = 150.75 - 51.96$$
$$= 98.8 \text{ kJ/kg}$$

Percentage of heat added as available energy

$$= \frac{A}{Q_H} \times 100 = \frac{98.8 \times 100}{150.75}$$
$$= \mathbf{65.53\%}$$

Example 8.3 *The exhaust gas leaves an internal combustion engine at 800°C and 1 atm, after having done 1050 kJ of work per kg of gas in the engine. Take C_p of gas as 1.1 kJ/kg·K and temperature of the surroundings as 30°C.*

(a) How much available energy per kg of gas is lost by throwing away exhaust gases?

(b) What is the ratio of lost available energy to the engine work?

Solution

Given Exhaust coming out of an IC engine
$p = 1$ atm

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$T_H = 800°C = 1073$ K
$T_0 = 30° = 303$ K
$w_{net} = 1050$ kJ/kg
$C_{pg} = 1.1$ kJ/kg·K

To find

(i) Loss of available energy in exhaust, and
(ii) Ratio of lost available energy to work of engine.

Assumption Exhaust process at constant pressure.

Analysis

(i) *Loss of available energy*

Total available energy in exhaust gases above the surrounding temperature

$$A = C_{pg}(T_H - T_0)$$
$$= 1.1 \times (1073 - 303) = 847 \text{ kJ/kg}$$

Change of entropy of exhaust gases at constant pressure

$$\Delta s = C_{pg} \ln\left(\frac{T_H}{T_0}\right) = 1.1 \times \ln\left(\frac{1073}{303}\right)$$
$$= 1.39 \text{ kJ/kg·K}$$

Unavailable energy due to change in entropy

$$q_{unav} = T_0 \Delta s = 303 \times 1.39$$
$$= 421.45 \text{ kJ/kg}$$

Therefore, the loss in available energy

$$q_2 = A - q_{unav} = 847 - 421.45$$
$$= \mathbf{425.55 \text{ kJ/kg}}$$

(ii) *Ratio of lost energy to engine work*

$$= \frac{q_2}{w_{net}} = \frac{425.55}{1050} = \mathbf{0.4014}$$

Example 8.4 *The exhaust gas at 650°C from a boiler is used to heat water. The rate of gas flow is 1510 kg/min and the rate of water flow is 1890 kg/min. The water enters the heat exchanger at 35°C, and the gases leave the exchanger at 145°C. Assume that the mean specific heat of gases and water are 1.088 kJ/kg·K and 4.27 kJ/kg·K respectively. The atmospheric temperature is 27°C. Determine the loss of available energy resulting from heat transfer.*

Given The flow across a heat exchanger

$T_1 = 650°C = 923$ K
$T_2 = 145°C = 418$ K
$C_{pg} = 1.088$ kJ/kg·K
$\dot{m}_g = 1510$ kg/min

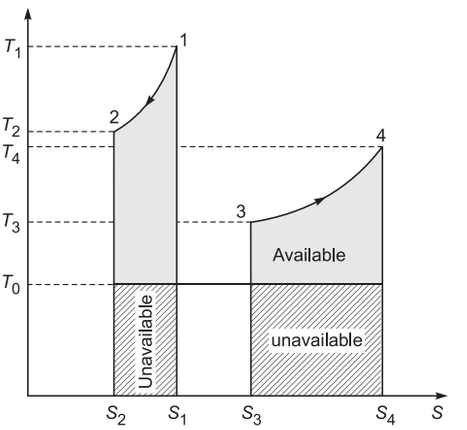

Fig. 8.5 *Irreversible heat transfer*

$\dot{m}_w = 1890$ kg/min
$C_{pw} = 4.27$ kJ/kg·K
$T_3 = 35°C = 308$ K
$T_0 = 27°C = 300$ K
$T_4 = ?$

To find Loss of available energy due to irreversible heat transfer.

Assumption The heat exchanger at constant pressure in heat exchanger.

Analysis For flow of fluids across a heat exchanger, applying energy balance on heat exchanger

$$\dot{m}_g C_{pg}(T_1 - T_2) = \dot{m}_w C_{pw}(T_4 - T_3)$$

using numerical values

$$1510 \times 1.088 \times (650 - 145) = 1890 \times 4.27 \times (T_4 - 35)$$

or $T_4 = 137.8°C = 410.8$ K

The unavailable evergy removed from the gases

$$\dot{Q}_{unav_1} = T_0(S_1 - S_2) = T_0 \dot{m}_g C_{pg} \ln\left(\frac{T_1}{T_2}\right)$$
$$= 300 \times 1510 \times 1.088 \times \ln\left(\frac{923}{418}\right)$$
$$= 390, 421 \text{ kJ/min}$$

The unavailable energy added to water

$$\dot{Q}_{unav_2} = T_0(S_4 - S_3) = T_0 \dot{m}_w C_{pw} \ln\left(\frac{T_4}{T_3}\right)$$
$$= 300 \times 1890 \times 4.27 \times \ln\left(\frac{410.8}{308}\right)$$
$$= 697, 290 \text{ kJ/min.}$$

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Loss in available energy = $\dot{Q}_{unav_2} - \dot{Q}_{unav_1}$
= 697, 290 – 390, 421
= **306869 kJ/min**

Example 8.5 *1000 kJ of heat is supplied by hot gases at 1400°C from a fire box. This heat is used to generate the steam at 250°C. The atmospheric temperature is 20°C. Calculate the energy as available and unavailable portion*

(a) as it leaves the hot gases,
(b) as it enters the system.

Prove that $\Delta I = T_0(\Delta S)_{universe}$ *holds good.*

Solution

Given $Q_H = 1000$ kJ
$T_H = 1400°C = 1673$ K
$T_L = 250°C = 523$ K
$T_0 = 20°C = 293$ K

To find Available and unavailable portion of energy
(i) as it leaves the hot gases,
(ii) as it enters the system,
To prove $\Delta I = T_0(\Delta S)_{universe}$.

Analysis The decrease of entropy of source during heat transfer

$$\Delta S_1 = \frac{Q_H}{T_H} = -\frac{1000 \text{ kJ}}{1673 \text{ K}} = -0.598 \text{ kJ/K}$$

Change of entropy of steam during heat absorption;

$$\Delta S_2 = \frac{Q_H}{T_L} = \frac{1000 \text{ kJ}}{523 \text{ K}} = 1.912 \text{ kJ/K}$$

Net change in entropy
$(\Delta S)_{universe} = \Delta S_1 + \Delta S_2$
$= -0.598 + 1.912 = 1.314$ kJ/K

(i) *As energy leaves the hot gases*
Available energy of hot gases
$A = Q_H - T_0 \Delta S_1$
$= 1000 - 293 \times (0.598)$
= **824.786 kJ**
Unavailable portion of energy in hot gases
$= T_0(\Delta S_1)$
$= 293 \times (0.598)$
= **175.214 kJ**

(ii) *As energy enters the system*
Available energy $A = Q_H - T_0(\Delta S_2)$
$= 1000 - 293 \times (1.912)$
= **439.78 kJ**
Unavailable portion of energy
$= T_0(\Delta S_2)$
$= 293 \times 1.912$
= **560.2 kJ**
To prove $\Delta I = T_0(\Delta S)_{universe}$
The change in irreversibility
$\Delta I = -175.214 + 560.21$
= **385 kJ**
and $T_0(\Delta S)_{universe} = 293 \times 1.314$
= **385 kJ**
The two quantities are equal, thus
$\Delta I = T_0(\Delta S)_{universe}$ **Proved**

8.3.2 Availability of a System

Consider a thermodynamic system which undergoes a process from a state 1 to a state of ambient p_0, T_0. The system is exchanging heat with its ambient only. The first law and second law of thermodynamics for such a process can be expressed as the first law

$$Q - W = \Delta E = E_2 - E_1$$

Second law $$S_{gen} = S_2 - S_1 + \frac{Q_{surr}}{T_0}$$

where
$Q_{surr} = -Q$ (heat transferred to surroundings)
S_{gen}= entropy generations during the process.
Rearranging the above equation as

$$W = E_1 - E_2 + Q$$

and $$T_0 S_{gen} = T_0(S_2 - S_1) - Q$$

or $$Q = T_0(S_2 - S_1) - T_0 S_{gen}$$
$$= -T_0 S_{gen} - T_0(S_1 - S_2)$$

Combining these equations, we get

$$W = E_1 - E_2 - T_0 S_{gen} - T_0(S_1 - S_2) \quad ...(8.10)$$

It is the actual work done during a process undergoing a change of state from states 1 to 2. If the volume of the system is also changing during the process, then useful work

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$$W_{useful} = W - W_{surr}$$
$$= W - p_0(V_2 - V_1)$$

or $$W_{useful} = E_1 - E_2 - T_0 S_{gen} - T_0(S_1 - S_2) + p_0(V_1 - V_2)$$

For a reversible process $S_{gen} = 0$, then

$$W_{rev} = W_{max} = E_1 - E_2 - T_0(S_1 - S_2) + p_0(V_1 - V_2) \quad ...(8.11)$$

It is the maximum useful work that can be produced during a reversible process, thus becomes the definition availability.

8.4 AVAILABILITY OF A CLOSED SYSTEM

The maximum useful work potential of a closed system is designated as Φ and is obtained from Eq. (8.11) by replacing the state 1 by parameters without a subscript and the state 2 by the dead state with the subscript '0'. Further for a closed system, the changes in kinetic and potential energies are negligible, and ($E = U$) thus the availability of a closed system becomes

$$\Phi = (U - U_0) - T_0(S - S_0) + p_0(V - V_0) \text{ kJ} \quad ...(8.12)$$

For unit mass

$$\phi = (u - u_0) - T_0(s - s_0) + p_0(v - v_0) \text{ kJ/kg} \quad ...(8.13)$$

Since the availability of a closed system at the dead state is zero then Eq. (8.13) yields to

$$a = u - T_0 s + p_0 v \quad ...(8.14)$$

This expression represents the *availability function* '*a*'of the closed system. The availability function of the closed system is the composite properties of the system and surroundings.

If the system undergoes a change of state from 1 to 2, then change in availability is referred as work transfer of the system.

$$W_{rev} = (\Phi_1 - \Phi_0) - (\Phi_2 - \Phi_0)$$
$$= \Phi_1 - \Phi_2 \text{ (kJ)} \quad ...(8.15)$$

or $$w_{rev} = \phi_1 - \phi_2 = a_1 - a_2 \quad ...(8.16)$$

If a closed system exchanges heat with a constant-temperature reservoir during a process, then maximum work done during a process from the state 1 to the state 2 can be expressed as

$$W_{rev} = \Phi_1 - \Phi_2 + \text{Heat transfer to surroundings}$$
$$= \Phi_1 - \Phi_2 + Q_R\left(1 - \frac{T_0}{T_R}\right) \quad ...(8.17)$$

where

T_0 = Absolute temperature of surroundings,

T_R = Absolute temperature of reservoir,

Q_R = Heat transfer between system and reservoir: – ve to reservoir and + ve from reservoir

The net entropy generation during a process

$$S_{gen} = (S_2 - S_1)_{sys} + \frac{Q_{surr}}{T_0} + \frac{Q_R}{T_R} \quad ...(8.18)$$

Example 8.6 *Prove that the availability of a closed system always decreases in an irrevesible process.*

Analysis The availability of a closed system is expressed as

$$\phi = (u - u_0) - T_0(s - s_0) + p_0(v - v_0) \quad ...(i)$$

For any isolated system, due to internal irreversibilities

$$\Delta s > 0$$

The first law of thermodynamics applied to an isolated system

$$q = w + \Delta u$$

But $q = 0$ for isolated system

and $$w = -\Delta u \quad ...(ii)$$

where w is the total work transfer and it is expressed as

$$w = w_{useful} + p_0(v - v_0)$$
$$w_{useful} = -\Delta u - p_0(v - v_0)$$
$$= -(u - u_0) - p_0(v - v_0) \quad ...(iii)$$

It is the gain in availability from the surroundings.

Therefore, the change in availability of a closed system

$(\Delta A)_{closed}$ = Availability of the system + gain in availability

$$= (u - u_0) - T_0(s - s_0) + p_0(v - v_0) + [-(u - u_0) - p_0(v - v_0)]$$
$$= -T_0(s - s_0)$$

But $(s - s_0)$ is a positive quantity, therefore for an irreversible process

$$(\Delta A)_{closed} = -\text{ve} \quad \text{or} \quad < 0$$

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Example 8.7 *Air in a piston–cylinder arrangement is heated at constant pressure by addition of 100 kJ/kg of air. The air is initially at 28°C while the surroundings is at 21°C. Calculate the change in availability per kg of air.*
Take $C_p = 1.005$ *kJ/kg·K*
and atmospheric air pressure is 1 bar.

Solution

Given A piston–cylinder arrangement

$q = 100$ kJ/kg
$T_1 = 28°C = 301$ K
$T_0 = 21°C = 294$ K
$C_p = 1.005$ kJ/kg·K
$p_0 = 1$ bar $= 100$ kPa

To find The change in availability per kg of air.

Assumptions

(i) The initial pressure of air in the cylinder at the atmospheric pressure of 100 kPa,
(ii) Air as an ideal gas,
(iii) The specific gas constant for air,
$R = 0.287$ kJ/kg·K.

Analysis Heat addition at constant pressure is given by

$$q = C_p(T_2 - T_1)$$
or $$100 = 1.005 \times (T_2 - 301)$$
or $$T_2 = 400.5 \text{ K}.$$

The initial specific volume

$$v_1 = \frac{RT_1}{p_0} = \frac{0.287 \times 301}{100} = 0.864 \text{ m}^3/\text{kg}$$

The final specific volume,

$$v_2 = \frac{RT_2}{p_0} = \frac{0.287 \times 400.5}{100} = 1.15 \text{ m}^3/\text{kg}$$

The change in specific entropy

$$\Delta s = C_p \ln\left(\frac{T_2}{T_1}\right)$$
$$= 1.005 \times \ln\left(\frac{400.5}{301}\right) = 0.287 \text{ kJ/kg·K}.$$

The specific heat at constant volume

$$C_v = C_p - R$$
$$= 1.005 - 0.287 = 0.718 \text{ kJ/kg·K}$$

The change in availability can be calculated as

$$\phi_2 - \phi_1 = (u_2 - u_1) - T_0(s_2 - s_1) + p_0(v_2 - v_1)$$
$$= C_v(T_2 - T_1) - T_0(\Delta s) + p_0(v_2 - v_1)$$
$$= 0.718 \times (400.5 - 301) - 294 \times 0.287 + 100 \times (1.15 - 0.864)$$
$$= \mathbf{15.66 \text{ kJ/kg}}$$

Example 8.8 *A rigid cylinder with a volume of 2.5 m^3 contains air at 150 kPa and 27°C. The heat is transferred to air from a constant-temperature heat source at 1500 K and air in the cylinder is heated to 700 K. The atmosphere is at 1 bar and 17°C. Calculate the initial and final availability of air, maximum useful work and irreversibility.*

Solution

Given A rigid cylinder with

$V = 2.5 \text{ m}^3$
$p_1 = 150$ kPa
$T_1 = 27°C = 300$ K
$T_H = 1500$ K
$T_2 = 700$ K
$T_0 = 17°C = 290$ K
$p_0 = 1$ bar $= 100$ kPa

To find

(i) Initial availability of air,
(ii) Final availability of air,
(iii) Maximum useful work, and
(iv) Irreversibility.

Assumptions

(i) A non-flow process,
(ii) Specific heat at constant volume is 0.717 kJ/kg·K,
(iii) The gas constant of air as 0.287 kJ/kg·K,
(iv) $\Delta ke = 0$ and $\Delta pe = 0$.

Analysis

The initial availability of air in a closed system.

$$\Phi_1 = m[(u_1 - u_0) + p_0(v_1 - v_0) - T_0(s_1 - s_0)]$$

Here $$m = \frac{p_1 V}{RT_1} = \frac{150 \times 2.5}{0.287 \times 300} = 4.35 \text{ kg}$$

$$v_0 = \frac{RT_0}{p_0} = \frac{0.287 \times 290}{100}$$
$$= 0.832 \text{ m}^3/\text{kg}$$

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$$v_1 = \frac{V}{m} = \frac{2.5}{4.35} = 0.574 \text{ m}^3/\text{kg}$$

$$u_1 - u_0 = C_v(T_1 - T_0) = 0.717 \times (300 - 290) = 7.17 \text{ kJ/kg}$$

Calculating each item separately

$$p_0(v_1 - v_0) = 100 \times (0.574 - 0.832) = -25.7 \text{ kJ/kg}$$

$$s_1 - s_0 = C_v \ln\left(\frac{T_1}{T_0}\right) + R\ln\left(\frac{v_1}{v_0}\right)$$

$$= 0.717 \times \ln\left(\frac{300}{290}\right) + 0.287 \times \ln\left(\frac{0.574}{0.832}\right)$$

$$= -0.082 \text{ kJ/kg·K}$$

Therefore, the initial availability

$$\Phi_1 = 4.35 \times [(7.17) + (-25.7) - 290 \times (-0.082)]$$

$$= \mathbf{22.83 \text{ kJ}}$$

(ii) The final availability of air

$$\Phi_2 = m[(u_2 - u_0) + p_0(v_2 - v_0) - T_0(s_2 - s_0)]$$

For a constant-volume cylinder

$$v_1 = v_2 = 0.574 \text{ m}^3/\text{kg}$$

$$p_2 = p_1 \frac{T_2}{T_1} = 150 \times \frac{700}{300} = 350 \text{ kPa}$$

$$u_2 - u_0 = C_v(T_2 - T_0) = 0.717 \times (700 - 290) = 293.97 \text{ kJ/kg}$$

$$s_2 - s_0 = C_v \ln\left(\frac{T_2}{T_0}\right) + R\ln\left(\frac{v_2}{v_0}\right)$$

$$= 0.717 \times \ln\left(\frac{700}{290}\right) + 0.287 \times \ln\left(\frac{0.574}{0.832}\right)$$

$$= 0.525 \text{ kJ/kg·K}$$

$$p_0(v_2 - v_0) = -25.7 \text{ kJ/kg}$$

$$\therefore \quad \Phi_2 = 4.35 \times [293.97 - 25.7 - 290 \times 0.525]$$

$$= \mathbf{504.68 \text{ kJ}}$$

(iii) The air receives heat from a constant-temperature reservoir at 1500 K, thus the maximum useful work

$$W_{max} = \Phi_1 - \Phi_2 + Q_R\left(1 - \frac{T_0}{T_R}\right)$$

Here, $Q_R = m(u_2 - u_1) = mC_v(T_2 - T_1)$

$$= 4.35 \times 0.717 \times (700 - 300)$$

$$= 1247.58 \text{ kJ}$$

Therefore,

$$W_{max} = (22.83 - 504.68) + 1247.58 \times \left(1 - \frac{290}{1500}\right)$$

$$= -481.85 + 1006.38 = \mathbf{524.53 \text{ kJ}}$$

The irreversibility of the closed system

$$I = W_{max} - W_{act}$$

For a constant-volume process $W_{act} = 0$

Thus $I = W_{max} = \mathbf{524.53 \text{ kJ}}$

8.5 AVAILABILITY IN A STEADY FLOW PROCESS

The availability in a steady flow process can be obtained from Eq. (8.10) by using

$$E = \dot{m}\left(u + \frac{\mathcal{V}^2}{2} + zg\right)$$

and writing the equation on a time-rate bases.

$$\dot{W}_{rev} = \dot{m}_1\left[u_1 + \frac{\mathcal{V}_1^2}{2} + z_1 g\right] - \dot{m}_2\left[u_2 + \frac{\mathcal{V}_2^2}{2} + z_2 g\right] - T_0(S_1 - S_2) - p_0(V_2 - V_1)$$

For steady flow $\dot{m}_1 = \dot{m}_2$, thus

$$\dot{W}_{rev} = \dot{m}\left[u_1 + p_0 v_1 + \frac{\mathcal{V}_1^2}{2} + z_1 g - T_0 s_1\right] - \dot{m}\left[u_2 + p_0 v_2 + \frac{\mathcal{V}_2^2}{2} + z_2 g - T_0 s_2\right]$$

$$= \dot{m}\left[h_1 - h_2 + \frac{\mathcal{V}_1^2 - \mathcal{V}_2^2}{2} + (z_1 - z_2)g - T_0(s_1 - s_2)\right] \text{(kJ)} \quad ...(8.19)$$

For a unit-mass flow rate

$$w_{rev} = (h_1 - h_2) + \frac{\mathcal{V}_1^2 - \mathcal{V}_2^2}{2} + (z_1 - z_2)g - T_0(s_1 - s_2) \text{ (kJ/kg)} \quad ...(8.20)$$

It can be interpreted as

$$w_{rev} = T_0 \Delta s - \Delta h - \Delta ke - \Delta pe \text{ (kJ/kg)} \quad ...(8.21)$$

The availability of the fluid stream in an open system is called the *stream availability* and is

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denoted by ψ. It is obtained from Eq. (8.20) by replacing inlet parameters without subscript and the exit parameter by dead state with subscript '0' with $\mathcal{V}_0 = 0$, $z_0 = 0$:

$$\psi = (h - h_0) - T_0(s - s_0) + \frac{\mathcal{V}^2}{2} + zg \text{ (kJ/kg)} \quad ...(8.22)$$

The reversible work for a process between states 1 and 2 can be expressed as

$$\dot{W}_{rev} = \dot{m}(\psi_1 - \psi_2) \text{ kW} \quad ...(8.23)$$

or $$w_{rev} = \psi_1 - \psi_2 \text{ (kJ/kg)} \quad ...(8.24)$$

The rate of irreversibility $\dot{I}$ or irreversibility per unit mass flow rate i associated with a steady flow process is

$$\dot{I} = \dot{W}_{rev} - \dot{W}_{useful} = T_0 \dot{S}_{gen} \quad ...(8.25)$$

or $$i = w_{rev} - w_{useful} = T_0 s_{gen} \quad ...(8.26)$$

When a steady flow device exchanges heat with a thermal reservoir at T_R at a rate of Q_R, then reversible work.

$$\dot{W}_{rev} = \dot{m}(\psi_1 - \psi_2) + \dot{Q}_R \left(1 - \frac{T_0}{T_R}\right) \text{ (kW)} \quad ...(8.27)$$

where the sign of Q_R is taken with respect to reservoir.

Example 8.9 *Determine the maximum work that can be produced by a steam turbine which has an inlet state of 3 MPa and 450°C and an outlet state as a dry saturated steam at 0.1 MPa. The heat transfer with surroundings is at 25°C.*

Given A steam turbine with

Inlet state: $p_1 = 3$ MPa $= 3000$ kPa
$T_1 = 450°C = 723$ K

Exit state: $p_2 = 0.1$ MPa $= 100$ kPa
$T_0 = 25°C = 298$ K

To find Maximum work.

Assumptions

(i) Negligible change in kinetic energy.
(ii) Negligible change in potential energy.
(iii) Steadily expansion of steam in the turbine.

Analysis Maximum reversible work of turbine, Eq. (8.21)

$$w_{max} = T_0 \Delta s - \Delta h$$

Properties of steam At 3 MPa and 450°C

$h_1 = 3344$ kJ/kg
$s_1 = 7.0833$ kJ/kg·K

At 0.1 MPa, dry steam

$h_2 = 2675.52$ kJ/kg
$s_2 = 7.3593$ kJ/kg·K
$\Delta s = s_2 - s_1 = 7.3593 - 7.0833 = 0.276$ kJ/kg·K
$\Delta h = h_2 - h_1 = 2675.52 - 3344 = -668.48$ kJ/kg.

Then $w_{max} = 298 \times 0.276 - (-668.48) =$ **750.72 kJ/kg**

Example 8.10 *The steam at 1000 kPa, 275°C enters a steady flow system with negligible velocity and leaves at 100 kPa, 120°C with a velocity of 160 m/s. The steam flow rate is 9500 kg/h. Heat is exchanged with only surroundings at 15°C. Determine the maximum possible power output.*

Solution

Given Steam in a steady flow system.

$p_1 = 1000$ kPa
$p_2 = 100$ kPa
$T_1 = 275°C$
$T_2 = 120°C$
$\mathcal{V}_1 = 0$
$\mathcal{V}_2 = 160$ m/s
$\dot{m}_s = 9500$ kg/h
$T_0 = 15°C$

To find Maximum power output from the system.

Schematic

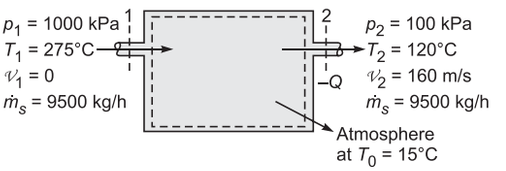

Fig. 8.6

Assumptions

(i) No information about inlet and exit elevations is provided, thus we neglect the potential energy change.
(ii) Constant properties.

Analysis The availability of a steady flow system for unit mass flow rate, Eq (8.21) is

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$$W_{rev} = W_{max} = T_0 \Delta s - \Delta h - \Delta ke \text{ (kJ/kg)}$$

Properties of steam

At 1000 kPa, 275°C;

h_1 = 2996.6 kJ/kg,

s_1 = 7.0257 kJ/kg·K

At 100 kPa, 120°C;

h_2 = 2716.3 kJ/kg,

s_2 = 7.4656 kJ/kg·K

Then, $w_{rev} = w_{max}$

$$= 288 \times (7.4656 - 7.0257) - (2716.3 - 2996.6) - \frac{160^2}{2000}$$

= 394 kJ/kg

For $\dot{m}_s$ = 9500 kg/h, the maximum possible power output

$$w_{max} = \dot{m}_s w_{max} = \frac{9500}{3600} \times 394 = \mathbf{1040\ kW}$$

Example 8.11 *1.5 kg of gas flows through a gas turbine unit from its initial pressure and temperature of 600 kPa and 1300 K, respectively and exhausts at a pressure of 102 kPa and a temperature of 600 K to the atmosphere. The atmospheric pressure and temperature are 100 kPa and 298 K. Calculate availability at the entrance to the gas turbine and exhaust of the gas turbine. Take necessary assumptions.*

Solution

Given An air compression process with

p_1 = 600 kPa

T_1 = 1300 K

p_2 = 102 kPa

T_2 = 600 K

T_0 = 298 K

p_0 = 100 kPa

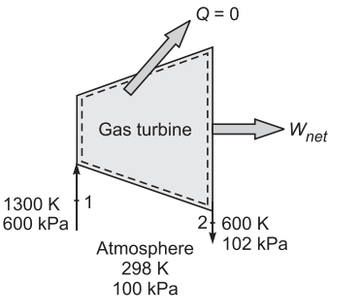

Fig. 8.7 *Schematic*

To find

(i) Availability at the entrance to the gas turbine.

(ii) Availability at the exit from the gas turbine.

Assumptions

(i) No heat transfer from the turbine during expansion of gas.

(ii) No change in potential and kinetic energies.

(ii) Air as ideal gas with C_p = 1.005 kJ/kg·K and R = 0.287 kJ/kg·K.

Analysis

(i) *Availability at the turbine inlet*

Enthalpy of gas above atmospheric temperature at the turbine inlet

$$h_1 - h_0 = C_p(T_1 - T_0)$$
$$= 1.005 \times (1300 - 298)$$
$$= 1007 \text{ kJ/kg}$$

Change in entropy;

$$s_1 - s_0 = C_p \ln\left(\frac{T_1}{T_0}\right) - R \ln\left(\frac{p_1}{p_0}\right)$$
$$= 1.005 \times \ln\left(\frac{1300}{298}\right) - 0.287 \times \ln\left(\frac{600}{100}\right)$$
$$= 0.966 \text{ kJ/kg}$$

The availability of gas at the turbine inlet

$$\psi_{inlet} = (h_1 - h_0) - T_0(s_1 - s_0)$$
$$= 1007 - 298 \times 0.966$$
$$= \mathbf{719.08\ kJ/kg\ of\ gas}$$

(ii) *Availability at the turbine exit*

Enthalpy of gas above atmospheric temperature at turbine exit

$$h_2 - h_0 = C_p(T_2 - T_0)$$
$$= 1.005 \times (600 - 298)$$
$$= 303.51 \text{ kJ/kg}$$

Change in entropy;

$$s_2 - s_0 = C_p \ln\left(\frac{T_2}{T_0}\right) - R \ln\left(\frac{p_2}{p_0}\right)$$
$$= 1.005 \times \ln\left(\frac{600}{298}\right) - 0.287 \times \ln\left(\frac{102}{100}\right)$$
$$= 0.6976 \text{ kJ/kg}$$

The availability of gas at the turbine inlet

$$\psi_{exit} = (h_2 - h_0) - T_0(s_2 - s_0)$$
$$= 303.51 - 298 \times 0.6976$$
$$= \mathbf{95.61\ kJ/kg\ of\ gas}$$

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Example 8.12 *Air enters a compressor in steady flow manner at 140 kPa, 17°C and 70 m/s and leaves it at 350 kPa, 127°C and 110 m/s. The environment is at 100 kPa and 7°C. Calculate per kg of air:*

(a) Actual amount of work required,
(b) The minimum work required, and
(c) The irreversibility of the process.

Solution

Given An air-compression process with

$p_1 = 140$ kPa
$T_1 = 17°C = 290$ K
$\mathcal{V}_1 = 70$ m/s
$p_2 = 350$ kPa
$T_2 = 127°C = 400$ K
$\mathcal{V}_2 = 110$ m/s
$p_0 = 100$ kPa
$T_0 = 7°C = 280$ K

To find

(i) Actual work input to compressor,
(ii) Minimum work input, and
(iii) Irreversibility in the compression process.

Assumptions

(i) Adiabatic compression process,
(ii) No change in potential energies,
(iii) Air as ideal gas with $C_p = 1.005$ kJ/kg·K and $R = 0.287$ kJ/kg·K.

Analysis Change in enthalpy

$$h_1 - h_2 = C_p(T_1 - T_2) = 1005 \times (17 - 127)$$
$$= -110.55 \text{ kJ/kg}$$

Change in kinetic energy

$$\frac{\mathcal{V}_1^2 - \mathcal{V}_2^2}{2} = \frac{70^2 - 110^2}{2} = -3600 \text{ J/kg}$$
$$= -3.6 \text{ kJ/kg}$$

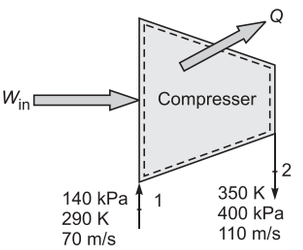

Fig. 8.8 *Schematic*

Change in entropy;

$$s_1 - s_2 = C_p \ln\left(\frac{T_1}{T_2}\right) - R \ln\left(\frac{p_1}{p_2}\right)$$
$$= 1.005 \times \ln\left(\frac{290}{400}\right) - 0.287 \times \ln\left(\frac{140}{350}\right)$$
$$= -0.0602 \text{ kJ/kg·K}$$

(i) *Actual work input*

For a unit mass flow rate through the compressor in absence of change in potential energy, the steady flow energy equation flow yields to

$$w_{act} = (h_1 - h_2) + \frac{\mathcal{V}_1^2 - \mathcal{V}_2^2}{2}$$
$$= -110.55 - 3.6 = \mathbf{-114.15 \text{ kJ/kg}}$$

(ii) Minimum (reversible) work of compression

$$w_{rev} = (h_1 - h_2) + \frac{\mathcal{V}_1^2 - \mathcal{V}_2^2}{2} - T_0(s_1 - s_2)$$
$$= -110.55 - 3.6 - 280 \times (-0.0602)$$
$$= \mathbf{-97.29 \text{ (kJ/kg)}}$$

(iii) *Irreversibility of the process*

$$I = w_{rev} - w_{rev}$$
$$= -97.29 - (-114.15)$$
$$= \mathbf{16.86 \text{ kJ/kg}}$$

Example 8.13 *An air compressor receives air at 1 bar and 27°C. It compresses the air to a pressure of 10 bar, while its temperature reaches 267°C. The compressor rejects 50 kJ/kg of heat from its body during compression process. Find the actual and reversible work transfer, and irreversibility in the process.*

Solution

Given An air compression process with

$p_1 = 1$ bar $= 100$ kPa
$T_1 = 27°C = 300$ K
$p_2 = 10$ bar $= 1000$ kPa
$T_2 = 267°C = 540$ K
$q = -50$ kJ/kg

To find

(i) Actual work input to compressor,
(ii) Reversible work input, and
(iii) Irreversibility in the compression process.

Assumptions

(i) Non-adiabatic compression process.

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(ii) No change in kinetic and potential energies.
(iii) Air as ideal gas with $C_p = 1.005$ kJ/kg·K and $R = 0.287$ kJ/kg·K.

Analysis The schematic of air compressor is shown in Fig. 8.9.

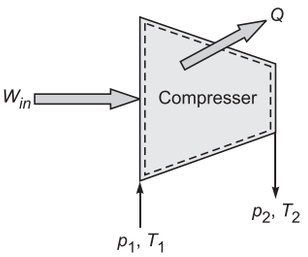

Fig. 8.9 *Schematic*

(i) For a unit-mass flow rate through the compressor in the absence of change in kinetic and potential energies, steady flow energy equation flow

$$w_{act} = (h_1 - h_2) + q$$
$$= C_p(T_1 - T_2) + q$$
$$= 1.005 \times (27 - 267) - 50$$
$$= \mathbf{-291.2\ kJ/kg}$$

(ii) The reversible work transfer during compression

$$w_{rev} = (h_1 - h_2) - T_0(s_1 - s_2)$$

For a non-adiabatic process.

$$s_2 - s_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)$$
$$= 1.005 \times \ln\left(\frac{540}{300}\right) - 0.287 \times \ln\left(\frac{10}{1}\right)$$
$$= 0.5907 - 0.6608 = -0.070\ \text{kJ/kg·K}.$$

and $h_1 - h_2 = 1.005 \times (300 - 540) = -241.2$ kJ

Then $w_{rev} = -241.2 - 300 \times (-0.070)$
$= \mathbf{-220.2\ kJ/kg}$

(iii) Rate of irreversibility

$$i = w_{rev} - w_{act} = -220.2 - (-291.2)$$
$$= \mathbf{71\ kJ/kg}$$

Example 8.14 *A feedwater heater has 5 kg/s water at 5 MPa, 40°C flowing through it. The water is heated from two constant temperature sources. One source adds 900 kW from a 100°C reservoir and the other source adds heat from a 200°C reservoir in such a way that water is heated to 180°C. Calculate the reversible work and rate of irreversibility. Take atmospheric temperature as 27°C.*

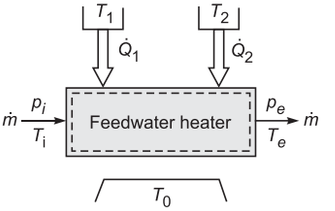

Fig. 8.10 *Schematic*

Solution

Given A feed water heated with two heat sources.

$\dot{m} = 5$ kg/s, $p_i = 5$ MPa
$T_i = 40°C$, $p_e = 5$ MPa
$T_e = 180°C$, $\dot{Q}_1 = +900$ kw
$T_1 = 100°C$, $T_2 = 200°C$
$T_0 = 27°C = 300$ K

To find

(i) Reversible work, and
(ii) Irreversibility rate.

Assumptions

(i) Constant-pressure heat addition,
(ii) No change in kinetic and potential energies,
(iii) Mean specific heat of water as 4.18 kJ/kg·K.

Analysis The energy equation for feedwater heater

$$h_1 + q_1 + q_2 = h_e$$

or
$$\dot{Q}_2 = \dot{m}(h_e - h_i) - \dot{Q}_1$$
$$= \dot{m}C_{pw}(T_e - T_i) - \dot{Q}_1$$
$$= 5 \times 4.18 \times (180 - 40) - 900$$
$$= 2026\ \text{kW}$$

The reversible work with heat transfer from two constant temperature reservoirs.

$$\dot{W}_{rev} = \dot{m}(\psi_1 - \psi_2) + \dot{Q}_1\left(1 - \frac{T_0}{T_1}\right) + \dot{Q}_2\left(1 - \frac{T_0}{T_2}\right)$$

where,

$$\psi_1 - \psi_2 = (h_i - h_e) - T_0(s_i - s_e)$$
$$= C_{pw}(T_i - T_e) - T_0 C_{pw} \ln\left(\frac{T_i}{T_e}\right)$$
$$= 4.18 \times \left[(40 - 180) - 300 \times \ln\left(\frac{40 + 273}{180 + 273}\right)\right]$$
$$= -121.6\ \text{kJ/kg}$$

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Therefore,

$$\dot{W}_{rev} = (5\text{ kg/s}) \times (-121.6\text{ kJ/kg}) + (900\text{ kW})\left(1 - \frac{300}{100+273}\right) + 2026\left(1 - \frac{300}{200+273}\right)$$

$$= -608.05\text{ kW} + 176.14 + 741.0$$

$$= \mathbf{309.1\text{ kW}}$$

The rate of irreversibility

$$\dot{I} = \dot{W}_{rev} = \mathbf{309.1\text{ kW}}$$

Example 8.15 *25 kg of water at 90°C is mixed with 40 kg of water at 40°C at constant pressure. The atmospheric pressure is 1 bar and 20°C. Calculate the decrease in available anergy.*

Solution

Given The mixing of water,

$m_1 = 25$ kg $\quad m_2 = 40$ kg

$T_1 = 90°C \quad T_2 = 40°C$

$T_0 = 20°C \quad p = 1$ bar

To find Loss in availability.

Assumptions

(i) No heat loss from mixture to surroundings,

(ii) The specific heat of water as 4.187 kJ/kg·K,

(iii) The change in potential and kinetic energies is negligible.

Analysis The availability of energy can be calculated as

$$\psi = (h_i - h_0) - T_0(s_i - s_0) = m C_{pw}\left[(T_i - T_0) - T_0 \ln\left(\frac{T_i}{T_0}\right)\right]$$

The availability of 25 kg water at 90°C

$$\psi_1 = 25 \times 4.187 \times \left[(90 - 20) - 293 \times \ln\left(\frac{363}{293}\right)\right]$$

$$= 104.67 \times (70 - 62.77) = 756.8\text{ kJ}$$

The availability of 40 kg water at 40°C

$$\psi_2 = 40 \times 4.187 \times \left[(40 - 20) - 293 \times \ln\left(\frac{313}{293}\right)\right]$$

$$= 109.37\text{ kJ}$$

Total available energy

$\psi_m = \psi_1 + \psi_2 = 756.8 + 109.37 = 866.17$ kJ

Total mass of water after mixing

$m_3 = m_1 + m_2 = 25 + 40 = 65$ kg

The steady flow mixing of two streams of water leads to

$$m_3 h_3 = m_1 h_1 + m_2 h_2$$

or $m_3 C_{pw} T_m = m_1 C_{pw} T_1 + m_2 C_{pw} T_2$

$65 \times 4.18\, T_m = 25 \times 4.18 \times 90 + 40 \times 4.18 \times 40$

Temperature T_m of water after mixing

$$T_m = \frac{25 \times 90 + 40 \times 40}{65} = 59.2°C$$

The availability of 65 kg water at 59.2°C

$$\psi_3 = 65 \times 4.187 \times \left[(59.2 - 20) - 293 \ln\left(\frac{59.2 + 273}{293}\right)\right]$$

$= 655.77$ kJ

The loss in availability done to mixing.

$= \psi_m - \psi_3$

$= 866.17 - 655.77 = \mathbf{210.39\text{ kJ}}$

Example 8.16 *2 kg of water at 50°C is mixed with 3 kg of water at 100°C in a steady flow process. Calculate the temperature of resulting mixture, state whether the mixing is isentropic?*

If not, what is the entropy change and unavailable energy with respect to surroundings at 50°C?

Solution

Given The mixing of two streams of water,

$m_1 = 2$ kg $\quad m_2 = 3$ kg

$T_1 = 50°C \quad T_2 = 100°C$

$T_0 = 50°C$

To find

(i) State of water after mixing,

(ii) Entropy change of mixture,

(iii) Unavailble energy.

Assumptions

(i) No heat loss to surroundings during mixing of two streams of water.

(ii) The specific heat of water as 4.187 kJ/kg·K.

(iii) The change in potential and kinetic energies is negligible.

Analysis

(i) *Mixing of two streams of water*

Total mass of water after mixing

$m_3 = m_1 + m_2 = 2 + 3 = 5$ kg

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The steady flow mixing of two streams of water leads to

$$m_3 h_3 = m_1 h_1 + m_2 h_2$$

or $m_3 C_{pw} T_m = m_1 C_{pw} T_1 + m_2 C_{pw} T_2$

$5 \times 4.18\, T_m = 2 \times 4.18 \times 50 + 3 \times 4.18 \times 100$

Temperature T_m of water after mixing

$$T_m = \frac{2 \times 50 + 3 \times 100}{5}$$

$$= 80°C \text{ or } 353 \text{ K}$$

Thus the mixing is not isentropic.

(ii) *Entropy change*

Entropy change during mixing process

$$\Delta s = m_1 C_{pw} \ln\left(\frac{T_m}{T_1}\right) + m_2 C_{pw} \ln\left(\frac{T_m}{T_2}\right)$$

$$= 2 \times 4.187 \times \ln\left(\frac{353}{323}\right) + 3 \times 4.187 \times \ln\left(\frac{353}{373}\right)$$

$$= 0.7437 - 0.692 = \mathbf{0.0517\ kJ/kg \cdot K}$$

(iii) *Unavailable energy*

$q_{unavail} = T_0 \Delta s = 323 \times (0.517) = \mathbf{16.71\ kJ/kg}$

Example 8.17 *The moment of inertia of a flywheel is 0.54 kg·m² and it rotates at a speed of 3000 rpm in a large insulated system at 15°C. The kinetic energy of the flywheel is distributed as frictional heat at the shaft bearings. The water equivalent of the shaft bearings is 2 kg. Find the rise in temperature of the shaft bearing when flywheel has come to rest. Determine the maximum possible heat, which may return to the flywheel as high-grade energy. Calculate how much amount of kinetic energy becomes unavailable. What would be the final rpm of the flywheel if it is set in motion with this available energy?*

Solution

Given A flywheel with

$I = 0.54 \text{ kg} \cdot m^2$

$N_1 = 3000$ rpm

$m = 2$ kg

$T_0 = 15°C = 288$ K

To find

(i) Temperature rise of shaft bearings,

(ii) Maximum possible heat returned to flywheel as available energy,

(iii) Unavailable kinetic energy

(iv) Final rpm of flywheel

Assumption The specific heat of water as 4.187 kJ/kg·K.

Analysis

(i) The initial angular velocity of the flywheel

$$\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi \times 3000}{60} = 314.2 \text{ rad/s}$$

Initial available energy of the flywheel = initial kinetic energy

$$A_1 = \frac{1}{2} I \omega_1^2$$

$$= \frac{1}{2} \times (0.54 \text{ kg m}^2) \times (314.2 \text{ rad/s})^2$$

$$= 2.66 \times 10^4 \text{ Nm} = 26.6 \text{ kJ}$$

When this kinetic energy is dissipated as frictional heat then rise in temperature of shaft and bearings

$$\text{K.E.} = m C_{pw} (\Delta T)$$

$$26.6 = (2 \text{ kg}) \times (4.187 \text{ kJ/kg} \cdot \text{K.})\ (\Delta T\ °C)$$

or $$\Delta T = \frac{26.2}{2 \times 4.187} = 3.12°C$$

Thus the final temperature of bearings

$$T_2 = T_0 + \Delta T = 15°C + 3.12°C = 18.12°C$$

(ii) *The maximum energy returned to flywheel as available energy*

$$a_2 = \Delta h - T_0 \Delta s$$

and $$A_2 = m C_{pw} \left\{ (T_2 - T_0) - T_0 \ln\left(\frac{T_2}{T_0}\right) \right\}$$

$$= 2 \times 4.187 \times \left[(3.12) - 288 \ln\left(\frac{18.12 + 273}{288}\right) \right]$$

$$= \mathbf{1.80\ kJ}$$

(iii) *The amount of kinetic energy becoming unavailable*

$$UE = A_1 - A_2 = 26.6 - 1.80 = \mathbf{24.8\ kJ}$$

(iv) *The final rpm of fly wheel*

Available energy retained by flywheel $= \frac{1}{2} I \omega_2^2$

$$= 1.80 \times 10^3 \text{ Nm} = \frac{1}{2} \times 0.54 \times \omega_2^2$$

or $$\omega_2 = 81.6 \text{ rad/s}$$

Further $$\omega_2 = \frac{2\pi N_2}{60}$$

or $$N_2 = \frac{\omega_2 \times 60}{2\pi} = \frac{81.6 \times 60}{2\pi} = \mathbf{779.2\ rpm}$$

Example 8.18 *Refrigerant 134 a, initially a saturated vapour at –30°C, is contained in a rigid insulated vessel. The vessel is fitted with a paddle wheel connected to a pulley and suspended mass. As the mass descends a*

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certain distance, the refrigerant is stirred until it attains a state where the pressure is 1.4 bar. The only significant changes of state are experienced by the falling mass and the refrigerant. If the mass of the refrigerant is 1.11 kg, determine

(a) initial availability, final availability and change in availability of the refrigerant in kJ,

(b) change in availability of suspended mass in kJ,

(c) change in availability in isolated system of the vessel and pulley–mass assembly, in kJ.

Take atmospheric temperature and pressure as 293 K and 1 bar, respectively.

Solution

Given Refrigerant 134*a* in a rigid, insulated container is stirred by a paddle wheel connected by a pulley–mass assembly.

$T_0 = 293\ \text{K} = 20°\text{C}$

$p_0 = 1\ \text{bar}, \quad p_1 = 1.4\ \text{bar}$

$T_1 = -30°\text{C} = 247\ \text{K}$

$m = 1.11\ \text{kg}$

To find

(i) Initial, final and change in availability of refrigerant.

(ii) Change in availability of suspended mass.

(iii) Change in availability of an isolated system of the vessel and pulley–mass assembly.

Assumptions

(i) The refrigerant in rigid insulated tank, suspended mass and isolated system are under consideration. For an isolated system, $Q = W = 0$.

(ii) Due to paddle work addition, only significant change experienced by refrigerant and suspended mass.

Analysis

(i) *The initial and final availability of the refrigerant*

From Table B-6

Properties of Refrigerant 134*a*, saturated vapour at initial state of –30°C

$u_1 = u_g = 360.63\ \text{kJ/kg},$

$v_1 = v_g = 0.22402\ \text{m}^3/\text{kg}$

$s_1 = s_g = 1.7493\ \text{kJ/kg·K}$

From Table B-8 at 1 bar and 20°C,

$v_0 = 0.23392\ \text{m}^3/\text{kg}, \ h_0 = 420.05,$

$s_0 = 1.8869\ \text{kJ/kg·K}$

$u_0 = h_0 - p_0 = 420.05 - 100\ \text{kPa} \times 0.23392$

$= 396.658\ \text{kJ/kg}$

Initial availability of refrigerant

$\Phi_1 = m[(u_1 - u_0) - T_0(s_1 - s_0) + p(v_1 - v_0)]\ \text{kJ}$

$= (1.11\ \text{kg}) \times [(360.63 - 396.658)$

$- (293\ \text{K}) \times (1.7493 - 1.8869) + (100\ \text{kPa})$

$\times (0.22402 - 0.23392)]$

$= 1.11 \times [-36.028 + 40.3168 - 0.99]$

$= \mathbf{3.66\ kJ/kg}$

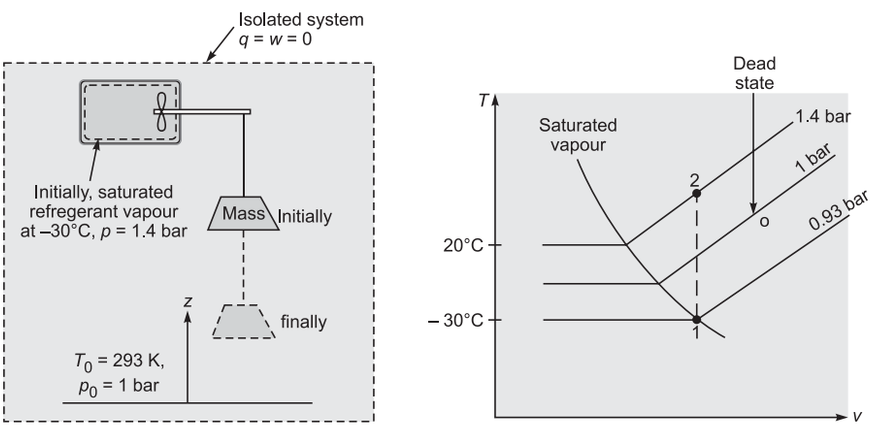

Fig. 8.11 *Schematic and T-v diagram*

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Final state of refrigerant, at $p_2 = 1.4$ bar and $v_2 = v_1$ by interpolation.

$u_2 = 450.148$ kJ/kg,

$s_2 = 2.041$ kJ/kg·K

Final availability of refrigerant,

$$\Phi_2 = 1.11 \times [(450.148 - 396.658) - 293 \times (2.0410 - 1.8669) + 100 \times (0.22402 - 0.23392)]$$
$$= 1.11 \times [53.49 - 45.15 - 0.99]$$
$$= \mathbf{8.1585\ kJ}$$

Change in availability of refrigerant 134*a*

$$\Phi_2 - \Phi_1 = 8.1585 - 3.66 = \mathbf{4.4985\ kJ}$$

(ii) *Change in availability of suspended mass*

Initial availability of mass

$$\dot{m}\,\psi_1 = \dot{W}_{rev}$$
$$= m[\Delta h + \Delta ke + \Delta pe - T_0 \Delta s]\ \text{kJ}$$

For a suspended mass system

$\Delta h = 0$,

$\Delta ke = 0$,

$\Delta s = 0$, thus

$\therefore \quad \Delta\Phi = m(\Delta pe) = \Delta PE$

The change in potential energy of the suspended mass can be obtained from an energy balance for the isolated system which is the sum of the energy changes of the refrigerant and suspended mass. Thus,

$$\cancel{(\Delta KE}^{0} + \cancel{\Delta PE}^{0} + \Delta U)_{refrigerant} + \cancel{(\Delta KE}^{0} + \Delta PE + \cancel{\Delta U)}^{0}_{mass} = \cancel{Q}^{0} - \cancel{W}^{0}$$

$$\Delta PE_{mass} = -\Delta U_{refrigerant}$$
$$= -1.11 \times [450.148 - 360.63]$$
$$= \mathbf{-99.365\ kJ}$$

The availability of mass decreases, because mass descends.

(iii) *Change in availability of an isolated system of vessel and pulley mass assembly*

$$\Phi = (\Delta\Phi)_{refrigerant} + (\Delta\Phi)_{mass}$$
$$= 3.66\ \text{kJ} + -99.365\ \text{kJ} = \mathbf{-95.7\ kJ}$$

The availability of the isolated system decreases.

Example 8.19 *Compressed air enters a counter-flow heat exchanger operating at steady state at 340°C, 10 bar and exits at 590°C, 9.7 bar. The hot combustion gases enters as a separate stream at 750°C, 1.1 bar and exit at 1 bar. Each stream has a mass-flow rate of 90 kg/s. Heat transfer between outer surface of heat exchanges and surroundings can be ignored. Kinetic and potential energy effects are negligible. Assuming that the combustion gas stream has the properties of air, and using ideal gas model for both streams, determine for the heat exchanger:*

(a) The exit temperature of combustion gas in °C,

(b) Net change in flow availability rate from inlet to exit of each stream is MW,

(c) Rate of availability loss in MW.

Take atmospheric pressure and temperature as 1 bar and 27°C respectively.

Solution

Given The steady flow operation in a heat exchanger

Cold fluid: air Hot fluid: combustion gases

$T_1 = 340°C = 613$ K

$T_3 = 750°C = 1023$ K

$T_2 = 590°C = 863$ K

$T_4 = ?$ $\quad p_1 = 10$ bar, $p_2 = 9.7$ bar

$\dot{m}_c = 90$ kg/s $\quad p_3 = 1.1$ bar, $p_4 = 1$ bar

$\dot{m}_h = 90$ kg/s

$p_0 = 1$ bar, $\quad T_0 = 27°C = 300$ K

To find

(i) Exit temperature of combustion gases,

(ii) Net change in flow availability rate from inlet to exit each stream,

(iii) Rate of availability loss in MW.

Schematic

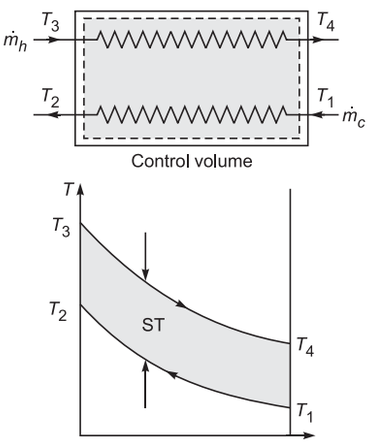

Fig. 8.12

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Assumptions

(i) The specific heat of air as $C_p = 1.005$ kJ/kg·K

(ii) For control volume $\dot{Q} = 0$, $\dot{W} = 0$

Analysis

(i) The exit temperature of hot combustion gases can be evaluated by making energy balancy on the heat exchanger:

Heat given by hot fluid = Heat gain by cold fluid

$$\dot{m}_h C_{p,h}(T_3 - T_4) = \dot{m}_c C_{p,c}(T_2 - T_1)$$

Here $\dot{m}_h = \dot{m}_c$

and $C_{p,h} = C_{p,c} = C_p$ of air

Therefore, $T_4 = T_3 + T_1 - T_2$

$= 750 + 340 - 590$

$=$ **500°C (773 K)**

(ii) The net change in flow availability rate from inlet to exit. The change in availability kg of a steady flow system is given by

$$\psi_1 - \psi_2 = T_0 \Delta s - \Delta h - \Delta ke - \Delta pe \text{ (kJ/kg)}$$

Here $\Delta pe = \Delta ke = 0$,

Thus $\psi_1 - \psi_2 = T_0 \Delta s - \Delta h$

For compressed air stream

$$\Delta h = h_2 - h_1 = C_p(T_2 - T_1)$$

$$= 1.005(863 - 613) = 251.25 \text{ kJ/kg}$$

$$\Delta s = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right)$$

$$= 1.005 \ln\left(\frac{863}{613}\right) - 0.287 \times \ln\left(\frac{9.7}{10}\right)$$

$= 0.3525$ kJ/kg·K

Change in availability per kg

$$\psi_1 - \psi_2 = 300 \times 0.3525 - 251.25$$

$= -145.5$ kJ/kg

The total change in availability of compressed air

$$= \dot{m}(\psi_1 - \psi_2) = 90 \times (-145.5)$$

$=$ **–13.095 MW**

–ve sign indicates increase in availability rate.

Change in availability rate of hot combustion gases

$$\Delta h = h_4 - h_3 = C_p(T_4 - T_3)$$

$$= 1.005 \times (773 - 1023)$$

$= -251.25$ kJ/kg

$$\Delta s = C_p \ln\left(\frac{T_4}{T_3}\right) - R \ln\left(\frac{p_4}{p_3}\right)$$

$$= 1.005 \times \ln\left(\frac{773}{1023}\right) - 0.287 \times \ln\left(\frac{1}{1.1}\right)$$

$= -0.2542$ kJ/kg·K

Change in availability

$$\psi_3 - \psi_4 = 300 \times (-0.2542) + 251.25$$

$= 174.99$ kJ/kg

Total change in availability rate

$$= \dot{m}(\psi_3 - \psi_4) = 90 \times 173.99$$

$= 15749.1$ kW

$=$ **15.75 MW**

It is decrease in availability rate of combustion gases.

(iii) Rate of availability loss within control volume

$= 15.75$ MW – 13.095 MW

$=$ **2.655 MW**

Example 8.20 *Steam enters a turbine with a pressure of 30 bar, a temperature of 400°C, a velocity of 160 m/s. The steam exists as saturated vapour at 100°C with a velocity of 100 m/s. At steady state, the turbine develops work at the rate of 540 kJ/kg of steam flowing through the turbine. The heat transfer between turbine and surroundings occurs at an average outer surface temperature of 350 K. Develop an accounting of net availability carried in by steam, per unit mass of steam. Neglect the change in potential energy between inlet and exit. Let $T_0 = 25°C$, $p_0 = 1$ atm.*

Solution

Given The steam expands through a turbine in steady state manner.

Inlet: $p_1 = 30$ bar

$T_1 = 400°C = 673$ K

$\mathcal{V}_1 = 160$ m/s

Exit: $T_2 = 100°C = 373$ K

$\mathcal{V}_2 = 100$ m/s

$w_{1-2} = 540$ kJ/kg

$T_s = 350$ K

$T_0 = 25°C = 298$, $p_0 = 1$ atm

$\Delta pe = 0$

To find Net change of availability enters, flow and exit the system.

Analysis From steam tables, A-14,

At 30 bar, 400°C,

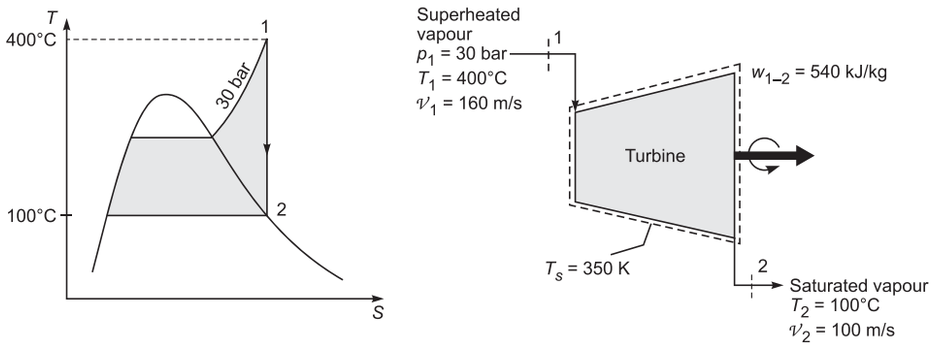

Fig. 8.13 *Schematic and T-s diagram*

$h_1 = 3230.9$ kJ/kg,

$s_1 = 6.9212$ kJ/kg·K

At 100°C

$h_2 = 2676.1$ kJ/kg,

$s_2 = 7.3549$ kJ/kg·K

The net available heat carried per unit mass of steam flowing through the turbine.

$$\psi_1 - \psi_2 = T_0 \Delta s - \Delta h - \Delta ke - \Delta pe$$

where

$$\Delta h = h_2 - h_1 = 2676.1 - 3230.9 = -554.8 \text{ kJ/kg}$$

$$T_0 \Delta s = 298 \times (7.3549 - 6.9212) = 129.034 \text{ kJ/kg·K}$$

$$\Delta ke = \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} = \frac{100^2 - 160^2}{2} = -7800 \text{ J/kg}$$

$$= -7.8 \text{ kJ/kg}$$

$$\Delta pe = 0 \text{ (given)}$$

Then $\psi_1 - \psi_2 = 129.034 - (-554.8) - (-7.8) - 0$

$$= \mathbf{691.634 \text{ kJ/kg}}$$

During a steady-state process, the heat transfer rate to surroundings

$$q - w = \Delta h + \Delta ke + \Delta pe$$

or $q = 540 + (-554.8) + (-7.8) + 0$

$$= -22.6 \text{ kJ/kg}$$

The loss of availability with this heat rejection is the maximum work capacity of this heat quantity

$$= \left(1 - \frac{T_0}{T_s}\right) q = \left(1 - \frac{298}{350}\right)(-22.6)$$

$$= \mathbf{-3.36 \text{ kJ/kg}}$$

It is loss of availability with heat transfer to surroundings.

8.6 THE SECOND-LAW EFFICIENCY

The reversible work or maximum work done by an engine is exactly equal to decrease in availability during a process. But the irreversibility is always involved in actual process. Therefore, the actual work done by an engine is always less than the reversible work. The *second-law efficiency* is related with actual and reversible work transfer by the devices such as engines, turbines, compressors, etc. The second-law efficiency is also called *effectiveness* and it is denoted as η_{II}.

8.6.1 Second-Law Efficiency of a Turbine or an Engine

It is defined as the ratio of actual thermal efficiency to reversible thermal efficiency under the same working conditions.

$$\eta_{II} = \frac{\eta_{th,act}}{\eta_{th,rev}} \tag{8.28}$$

It is sometimes also referred as *turbine effectiveness*.

It can also be expressed as the ratio of the actual work output and maximum possible work output

$$\eta_{II} = \frac{W_{act}}{W_{rev}} = \frac{W_{useful}}{W_{rev}} \tag{8.29}$$

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8.6.2 Second-Law Efficiency of Compressors and Pumps

The second-law efficiency of a power-consuming device such as a compressor, pump or a blower can be expressed as the ratio of minimum reversible work input to actual work input

$$\eta_{II} = \frac{W_{rev}}{W_{act}} \qquad (8.30)$$

8.6.3 Second-Law Efficiency of Refrigerators and Heat Pumps

The performance of a refrigerator or a heat pump is measured in terms of coefficient of performance. Therefore, the second-law efficiency for such devices is expressed as

$$\eta_{II} = \frac{(COP)_{act}}{(COP)_{rev}} \qquad ...(8.31)$$

8.6.4 Second-Law Efficiency or Effectiveness of Heat Exchanger

The heat exchanger shown in Fig. 8.14 operates at steady state with no heat transfer with its surroundings and both streams are separately flowing and at a temperature above T_0. For such a heat exchanger, the efficiency is expressed as

$$\eta_{II} = \varepsilon = \frac{\text{Gain in availability of system}}{\text{Loss in availability of surroundings}}$$

$$= \frac{\text{Actual heat transferred for use}}{\text{Total heat rejected by hot fluid}}$$

$$= \frac{\dot{m}_c(\psi_4 - \psi_3)}{\dot{m}_h(\psi_1 - \psi_2)}$$

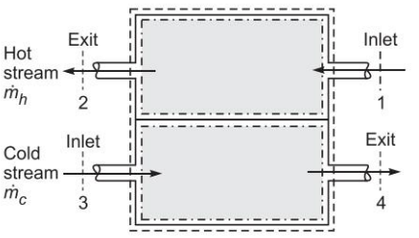

Fig. 8.14 *Counter flow heat exchanger*

where $\dot{m}_c$ and $\dot{m}_h$ are the mass flow rates for cold and hot fluids, respectively.

For direct contact, a heat exchanger as shown in Fig. 8.15 operates at steady state with no heat transfer with its surroundings. The energy balance for such a heat exchanger is expressed as

$$\dot{m}_1 h_2 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

where $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$ for a mass rate balance. The second-law efficiency or effectiveness is given by

$$\varepsilon = \frac{\text{Heat transferred by hot fluid}}{\text{Heat absorbed by cold fluid}}$$

$$= \frac{\dot{m}_3 \psi_3}{\dot{m}_1 \psi_1 + \dot{m}_2 \psi_2} \qquad ...(8.32)$$

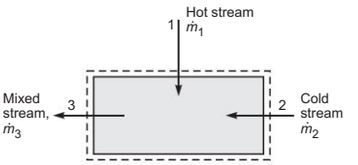

Fig. 8.15 *Direct contact heat exchanger*

Example 8.21 *1 kg water initially at 25°C is heated to 90°C using an electric heating coil. Assuming that the heat losses to the surroundings at 300 K are negligible. Calculate the first law and second law efficiencies of the process.*

Solution

Given Heating of water

$m = 1$ kg

$T_1 = 25°C$, $T_2 = 90°C$

$T_0 = 300$ K $Q = 0$

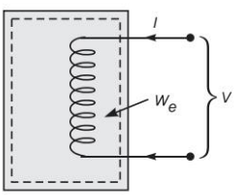

Fig. 8 *Schematic*

To find

(i) First law efficiency,

(ii) Second law efficiency.

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Assumption Specific heat of water, $C_{pw} = 4.187$ kJ/kg · K.

Analysis The system is illustrated in Fig. 8.16.

(i) First law efficiency

Applying first law of thermodynamics

$$-W_e = U_2 - U_1 = m(u_2 - u_1)$$
$$= m\,C_{pw}\,(T_2 - T_1)$$
$$= 1 \times 4.187 \times (90 - 25) = 272.15 \text{ kJ}$$

First law efficiency can be expressed as

$$\eta_I = \frac{\text{Desired energy output}}{\text{Required energy input}}$$
$$= \frac{U_2 - U_1}{-W_e} = \frac{272.15}{272.15} \times 100 = \mathbf{100\%}$$

(ii) Second law efficiency

Increase in availability of water

$$\phi_2 - \phi_1 = (u_2 - u_1) - T_0(s_2 - s_1) + p_0(v_2 - v_1)$$
$$= (h_2 - h_1) - T_0(s_2 - s_1)$$

From steam table A-12

$h_1 = 105$ kJ/kg, $s_1 = 0.367$ kJ/kg · K

$h_2 = 377$ kJ/kg $s_2 = 1.193$ kJ/kg · K

Then $\phi_2 - \phi_1$

$$= (377 - 105) - 300 \times (1.193 - 0.367)$$
$$= 24.2 \text{ kJ/kg}$$

Total availability increase of water

$$\phi_2 - \phi_1 = m(\phi_2 - \phi_1) = 24.2 \text{ kJ}$$

Second law efficiency of the process

$$\eta_{II} = \frac{\text{Increase of availability of water}}{\text{Energy input}}$$
$$= \frac{24.2}{272.15} = 0.089 \text{ or } \mathbf{8.9\%}$$

Example 8.22 *The steam enters a steam turbine steadily at 3 MPa and 450°C at a rate of 8 kg/s and exits at 0.2 MPa and 150°C. The steam is losing heat to the surroundings at 100 kPa and 25°C at a rate or 300 kW. The changes in kinetic and potential energies are negligible.Calculate*

(a) actual power output,
(b) the reversible work,
(c) second law efficiency,
(d) the irreversibility, and
(e) the availability of steam at inlet conditions.

Solution

Given A steam turbine with

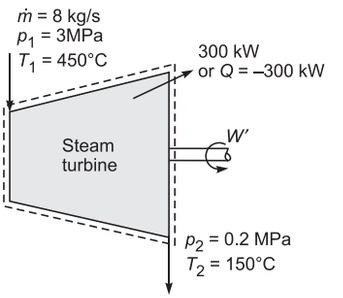

Fig. 8.16

$T_0 = 25°C$

$p_0 = 100$ kPa

To find

(i) Actual power output,
(ii) Reversible work,
(iii) Second-law efficiency,
(iv) Irreversibility, and
(v) Availability of steam at inlet conditions.

Analysis *The properties of steam*

Inlet condition $p_1 = 3$ MPa,
$T_1 = 450°C$
$s_1 = 7.0834$ kJ/kg · K
$h_1 = 3344.0$ kJ/kg.

Exit condition $p_2 = 0.2$ MPa
$T_2 = 150°C$
$h_2 = 2768.8$ kJ/kg,
$s_2 = 7.2795$ kJ/kg · K

Dead state $p_0 = 100$ kPa
$T_0 = 25°C$,
$h_0 = h_f = 104.89$ kJ/kg
$s_0 = s_f = 0.3674$ kJ/kg · K

(i) The actual power output from steam turbine

$$\dot{Q} - \dot{W} = \dot{m}(\Delta h + \Delta ke + \Delta pe) = \dot{m}(\Delta h)$$

Using numerical value

$$-300 \text{ kW} - \dot{W}_{act}$$
$$= (8 \text{ kg/s}) \times (2768.8 - 3344.0) \text{ (kJ/kg)}$$

or $\dot{W}_{act} = \mathbf{4301.6 \text{ kW}}$

(ii) Reversible work done by the turbine

$$\dot{W}_{rev} = \dot{m}\,[(h_1 - h_2) - T_0(s_1 - s_2)]$$
$$= 8 \times [(3344.0 - 2768.8)$$
$$- 298 \times (7.0834 - 7.2795)]$$
$$= \mathbf{5069 \text{ kW}}$$

(iii) The second-law efficiency

$$\eta_{II} = \frac{\dot{W}_{act}}{\dot{W}_{rev}} = \frac{4301.6}{5067} = \mathbf{84.86\%}$$

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(iv) Irreversibility

$$\dot{I} = \dot{W}_{rev} - \dot{W}_{act}$$
$$= 5069 - 4301.6 = \mathbf{767.4\ kW}$$

(v) Availability of steam at the inlet condition

$$\psi_1 = (h_1 - h_0) - T_0(s_1 - s_0)$$
$$= (3344.0 - 104.89) - 298 \times (7.0834 - 0.3674)$$
$$= \mathbf{1238\ kJ/kg}$$

Example 8.23 *In a mine, the used air motor requires minimum 7 bar air pressure for its operation and delivering 105 kJ/kg of work from air supplied. A compressed air bottle of 1.66 m^3 volume containing air at 20 bar and 300 K is available for operation of air motor.*

If the ambient conditions are 1 atm and 300 K, Calculate:

(a) Maximum work output of air motor, operated from one bottle of compressed air, and

(b) Second law efficiency of air motor.

Solution

Given Operation of air motor.

$V = 1.66\ m^3, p_1 = 20$ bar $= 2000$ kPa, $T = 300$ K

Air motor: $p_2 = 700$ kPa, $w_{out} = 105$ kJ/kg,

Dead state $p_0 = 1$ atm $= 101.3$ kPa, $T_0 = 300$ K

To find

(i) Maximum work delivered by air motor using one bottle of compressed air,

(ii) Second law efficiency of the system.

Assumption The specific gas constant of air, $R = 0.287\ kJ/kg \cdot K$.

Analysis The initial mass of compressed air in the bottle

$$m_1 = \frac{p_1 V}{R T_1}$$
$$= \frac{2000 \times 1.66}{0.287 \times 300} = 38.5\ kg$$

The mass of air remaining in bottle, when its pressure reaches to 7 bar

$$m_2 = \frac{p_2 V}{R T_2} = \frac{700 \times 1.66}{0.287 \times 300} = 13.5\ kg$$

The mass of air supplied to air motor

$$m = m_1 - m_2 = 38.5 - 13.5 = 25\ kg$$

Total work delivered by air motor

$$W = mw = 25 \times 105 = 2625\ kJ$$

(i) Maximum work possible in air motor

= Change in availability of compressed air

$$\phi_1 - \phi_0 = u_1 - u_0 + p_0(v_1 - v_0) - T_0(s_1 - s_0)$$

where $u_1 - u_0 = 0$

$$p_0(v_1 - v_0) = p_0\left[\frac{RT}{p_1} - \frac{RT}{p_0}\right]$$
$$= 101.3 \times 0.287 \times 300 \times \left[\frac{1}{2000} - \frac{1}{101.3}\right]$$
$$= -81.749\ kJ/kg$$

$$T_0(s_1 - s_0) = T_0\left[C_v \ln\frac{T_1}{T_0} - R\ln\left(\frac{p_1}{p_0}\right)\right]$$
$$= 300 \times \left[0 - 0.287 \times \ln\left(\frac{2000}{101.3}\right)\right]$$
$$= -256.82\ kJ/kg$$

$$\therefore \quad \phi_1 - \phi_0 = -81.749 - (-256.82)$$
$$= 175\ kJ/kg$$

Total availability of compressed air

$$W_{max} = m(\phi_1 - \phi_0)$$
$$= 38.5 \times 175 = 6740.6\ kJ$$

(ii) Second law efficiency

$$\eta_{II} = \frac{\text{Actual work delivered}}{\text{Maximum possible work}}$$
$$= \frac{2625\ kJ}{6740.6\ kJ} = 0.389 \text{ or } \mathbf{38.9\%}$$

Example 8.24 *A steam power plant is designed to operate on an ideal Rankine cycle. The boiler pressure is 40 bar and condenser pressure is 7.5 kPa. The mass flow rate of water is 38 kg/s and enthalpy rise of generated steam is 2630 kJ/kg. While change in entropy is 5.5 kJ/kg·K.*

The heating for steam generation in the boiler unit is provided by a steady stream of hot gases initially at 2000 K and 1 atm pressure. The hot gas exhausts at 450 K to the ambient at 300 K and 1 atm. The heating rate in the boiler is 100 MW. Calculate

(a) the mass flow rate of hot flue gases,

(b) Availability of hot flue gases at inlet conditions,

(c) Availability of hot flue gases at boiler exit,

(d) Change in availability of hot flue gases in steam generation process, and

(e) second law efficiency in the entire process.

Take $C_{pg} = 1.1$ kJ/kg·K for hot flue gases.

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Solution

Given Generation of steam with the help of hot flue gases.

Boiler pressure $p_1 = 40$ bar $\dot{m}_s = 38$ kg/s
Condenser pressure, $p_2 = 7.5$ kPa $\Delta h_w = 2630$ kJ/kg
Heating rate, $\dot{Q} = 100$ MW = 1,00,000 kW
Hot flue gases, $T_i = 2000$ K, $T_e = 450$ K,
$C_{pg} = 1.1$ kPa
$p_i = 1$ atm, $p_e = 1$ atm.
Dead state: $p_0 = 1$ atm = 101.3 KPa
$T_0 = 300$ K.
$\Delta S_w = 5.5$ kJ/kg·K

To find

(i) Mass flow rate of flue gases,
(ii) Availability of hot flue gases at boiler inlet condition,
(iii) Availability of hot flue gases at boiler at boiler exit conditions,
(iv) Decrease of availability of hot flue gases in the boiler, and
(v) Second law efficiency in entire process.

Analysis

(i) The mass flow rate of hot flue gases for 100 MW heat supply to boiler

$$\dot{Q} = \dot{m}_g C_{pg} (T_i - T_e)$$
$$1,00,000 = \dot{m}_g \times 1.1 \times (2000 - 450)$$
or $\dot{m}_g = 58.65$ kg/s or $\mathbf{2.11 \times 10^5}$ **kg/h**

(ii) Availability of hot flue gases at boiler inlet

$$\psi_1 = (h_i - h_0) - T_0 (s_1 - s_0)$$

where $h_i - h_0 = C_{pg} (T_i - T_0)$
$= 1.1 \times (2000 - 300) = 1870$ kJ/kg

$$s_i - s_0 = C_{pg} \ln\left(\frac{T_i}{T_0}\right) = 1.1 \times \ln\left(\frac{2000}{300}\right)$$
$= 2.087$ kJ/kg·K

$\therefore \quad \psi_1 = 1870 - 300 \times 2.087$
$= 1243.95$ kJ/kg

Total availability at inlet conditions
$= \dot{m}_g \psi_1 = 58.65 \times 1243.95$
$= 72,957.67$ or **72.95 MW**

(iii) Availability of flue gases at boiler exit

$$\dot{m}_g \psi_2 = \dot{m}_g [(h_e - h_0) - T_0 (s_e - s_0)]$$
$= 58.65$
$$\times \left[1.1 \times (450 - 300) - 300 \times 1.1 \times \ln\left(\frac{450}{300}\right)\right]$$
$= 1829.6$ kW or **1.83 MW**

(iv) Decrease in availability of hot flue gases
$= \dot{m}_g (\psi_1 - \psi_2)$
$= 72.95 - 1.83 =$ **71.12 MW**

(v) Second law efficiency
Availability increase of water during steam generation

$$\dot{m}_s \psi_s = \dot{m}_s [\Delta h_w - T_0 \Delta s_w]$$
$= 38 \times [2630 - 300 \times 5.5]$
$= 37,240$ or 37.24 MW

$$\therefore \quad \eta_{II} = \frac{\text{Availability rise of water}}{\text{Availability of flue gases}} \times 100$$
$$= \frac{37.24}{72.95} \times 100 = \mathbf{51.05\%}$$

Summary

- The energy contains quantity as well as quality. The quality of energy is always degraded during a process, hence the entropy is generated, and the opportunities to produce work are lost.
- The available energy is that portion of heat energy applied to a reversible engine, which could be converted into useful work.
- The maximum useful work potential of a system at the specified state is called *availability*. Availability is a composite property of system and surroundings.
- At dead state, i.e., state of surroundings, where the availability becomes zero.
- The useful work available as output is always less than the work produced by the system because a portion of work produced by the system is lost to displace the atmosphere through a change of volume. Thus

$$W_{useful} = W_{sys} - W_{surr} = W - p_0 (v_2 - v_1)$$

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- The surrounding work W_{surr} is zero for cyclic devices, steady flow devices and systems with fixed boundaries.
- The irreversibility is a difference between reversible work W_{rev} and actual useful work W_{useful} for a process. For any system,

$$I = W_{rev} - W_{useful}\ (\text{kJ}) = T_0 S_{gen}\ (\text{kJ})$$

or $$i = w_{rev} - w_{useful}\ (\text{kJ/kg})$$
$$= T_0 s_{gen}\ (\text{kJ/kg})$$

or $$\dot{I} = \dot{W}_{rev} - \dot{W}_{useful}\ (\text{kW})$$
$$= T_0\ \dot{S}_{gen}\ (\text{kW})$$

- Due to irreversibilities in the process, the entropy S_{gen} is generated. For a totally reversible process, entropy generation and irreversibilities are zero.
- The availability of a closed system (ϕ) and a steady flow system ψ are given as

$$\phi = (u - u_0) - T_0(s - s_0) + p_0(v - v_0)\ (\text{kJ/kg})$$

$$\psi = (h - h_0) - T_0(s - s_0) + \frac{v_2^2 - v_1^2}{2} + (z_2 - z_1)\, g\ (\text{kJ/kg})$$

- For a system undergoing a change of state from 1 to 2, the change in availability is referred as work transfer during the process and it is expressed as

$$W_{rev} = m(\phi_1 - \phi_2)\ (\text{kJ})$$

or $$\dot{W}_{rev} = \dot{m}\,(\psi_1 - \psi_2)\ \text{kW}$$

- If a system exchanges heat Q_R with a constant temperatrue reservoir at T_R during a process then the maximum work done from states 1 to 2 is

$$W_{rev} = \Phi_1 - \Phi_2 + Q_R\left(1 - \frac{T_0}{T_R}\right)\ (\text{kJ})$$

or $$\dot{W}_{rev} = \dot{m}(\psi_1 - \psi_2) + \dot{Q}_R\left(1 - \frac{T_0}{T_R}\right)\ \text{kW}$$

- For work-producing devices like heat engines and turbines, the second-law efficiency is defined as

$$\eta_{II} = \frac{\eta_{th}}{\eta_{rev}} = \frac{W_{useful}}{W_{rev}}$$

- For work absorbing devices, like refrigerators, pump, compressors etc., the second law efficiency is

$$\eta_{II} = \frac{COP}{(COP)_{rev}} = \frac{W_{rev}}{W_{act}}$$

Glossary

High-grade energy Energy that can be converted directly into useful work and other forms of energy with its equivalent quantity

Low-grade energy Energy that cannot be converted into useful work and other forms of energy with its equivalent quantity

Available energy The part of energy that can be converted into useful work

Unavailable energy The part of energy which cannot be converted into useful work and is rejected as waste

Exergy Work potential of energy, also called availabilty

Dead state State of system when it is in thermodynamic equilibrium with the atmosphere

Useful work The net work done by the system; System work–work done against atmosphere

Irreversibility The difference between the reversible work and actual work during a process

Availability function Availability of a closed system: composite properties of the system and surroundings

Stream availability Availability of the fluid stream in an open system

Second law efficiency Ratio of reversible output to actual output for engines, turbines, etc., and ratio of reversible input to actual input for compressors, pumps, etc.

Review Questions

1. Define availabilty and unavailability.
2. What is the available energy?
3. Define dead state and useful work, maximum work.

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4. What is irreversibility? What are the effects of irreversibility on work output of a system?
5. Discuss the availability of energy entering a system.
6. Discuss the availability of a closed system.
7. Discuss the availability of a steady flow system.
8. Discuss the steady flow availability function.
9. Explain the loss in availabilty.
10. Derive an expression for available energy from finite energy source at temperature T, when the environment temperature is T_0.
11. What is meant by quality of energy?
12. What do you understand by energy and exergy?
13. Define second-law efficiency of turbine, pump and heat exchanger.
14. How are the concept of entropy and unavailable energy related to each other?
15. An ideal gas is stored in a closed vessel at pressure p and temperature T.
    (a) If $T = T_0$, derive an expression for the specific availability in terms of p, p_0, T_0 and the gas constant R.
    (b) If $p = p_0$, derive an expression for the specific availability in terms of T, T_0, and the specific heat C_p, which can be taken as constant. Neglect the effect of motion and gravity.

    (b) $C_p T_0 \left[\frac{T}{T_0} - 1 - \ln\left(\frac{T}{T_0} \right) \right]$

Problems

1. How much is the available energy of 1 kg of air increased by heating it reversibily at constant pressure of 1.5 bar from 40°C to 250°C with a lowest available temeprature of 20°C?

[60 kJ/kg]

2. A constant temperature source is maintained at 727°C, while the surrounding temperature is 17°C. If the heat transferred from the source is 4000 kJ in a reversible manner; calculate the availability of heat energy and unavailable heat.

[2840 kJ, 1160 kJ]

3. A heat engine receives heat from a source at 1200 K at a rate of 500 kW and rejects the waste heat to its surroundings at 300 K. The power output of the heat engine is 180 kW. Determine maximum power output and irreversibility rate for the process. *[375 kW, 195 kW]*

4. The heat is supplied to a reversible engine from a finite heat reservoir in a reversible manner. During the heat supply process, the temperature of the working fluid increases from 523 K to 873 K. The water equivalent is 100 kJ/K. The heat rejection during the cycle takes place at a surrounding temperature of 288 K. All processes are reversible. Determine the total heat abstracted, availability and the loss of availability.

[35000 kJ, 20244.3 kH, 14755.7 kJ]

5. In a boiler, the saturated water at 250°C is evaporated into dry saturated steam at the rate of 5 kg/s by heat transfer from hot flue gases. The gases are cooled during the process from 1127°C to 527°C. The surrounding temperature is at 27°C. Find the following:
   (a) Mass flow rate of hot gases,
   (b) Entropy change of hot gases, water and net change in entropy,
   (c) Availability of hot gases and steam,
   (d) Unavailable energy of hot gases and steam,
   (e) Increase in unavailable energy due to irreversible heat transfer.

   Assume latent heat of steam at 250°C, h_{fg} = 1716.2 kJ/kg, specific heat of hot gases C_{pg} = 1.00 kJ/kg.

   [(a) 14.3 kg/s, (b) –8.002 kW/K, 16.4 kW/K, 8.40 kW/K (c) 6180.25 kW, 3658.81 kW, (d) 2400.75 kW, 4922.2 kW, (e) 2521.44 kW]

6. A crater lake has a base area of 10,000 m^2 and is 12 m deep. The ground of the crater is nearly flat and is 140 m below the base of the lake. Determine the maximum amount of electric work that can be generated by feeding this water to a hydroelectric power plant. *[45,780 kWh]*

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7. A steam turbine receives steam at 6 MPa, 800°C. It has a heat loss of 49.7 kJ/kg. For an exit pressure of 15 kPa and surroundings at 20°C, find the actual work and reversible work, if the turbine has an isentropic efficiency of 90%.

[1483.9 kJ/kg, 1636.8 kJ/kg]

8. 2 kg of air at 500 kPa, 80°C expands isentropically in a non-flow system until its volume doubles, the temperature and pressure approach to ambient condition at 5°C and 100 kPa. Determine (a) maximum work, (b) the change in availability, and (c) irreversibility.

For air, take

$C_v = 0.718$ kJ/kg·K, $R = 0.287$ kJ/kg·K.

[(a) 122.72 kJ, (b) 82.2 kJ, (c) 15.2 kJ]

9. A 500-kg iron block initially at 200°C is allowed to cool to 27°C by transferring heat to its surrounding air at 27° C. Determine the reversible work and irreversibility for the process.

[8191 kJ, 8191 kJ]

10. A 5-kg iron block initially at 350°C is quenched in an insulated tank which contains 100 kg of water at 30°C. Assume the water vapourised during the process condenses back in the tank. The ambient conditions are 20°C and 100 kPa. Determine (a) final equilibrium temperature, (b) the availability of the combined system at initial and final states, and (c) the wasted work potential during this process.

[(a) 31.7°C (b) 95.7 kJ (c) 219.3 kJ]

11. An insulated steam turbine receives 30 kg of steam per second at 3 MPa and 350°C. The steam expands in the turbine to 0.5 MPa, where steam at the rate of 5 kg/s is bled off. The temperature of this steam is 200°C. The remaining steam is further expanded to 15 kPa and 90% quality, and then it is exhausted. Calculate the availability per kg of steam entering and at both points at which steam leaves the turbine and the second-law efficiency.

[1109.6 kJ/kg, 755.3 kJ/kg, 195 kJ/kg, 0.817%]

12. A refrigerator has a second-law efficiency of 45% and it removes heat at the rate of 3 kW. If the refrigerated space is maintained at 3°C, while the surrounding air temperature is at 27°C, determine the power input to the refrigerator. *[0.58 kW]*

13. A 10-kg iron disk brake on a car is initially at 10°C. Suddenly the brake is applied and the brake temperature rises by friction to 110°C. Determine the availability of the disk and energy depletion of the car gas, if the car engine has a thermal efficiency of 35%. *[64.6 kJ, 1285 kJ]*

14. Air flows at 1500 K, 100 kPa through a constant-pressure heat exchanger, giving energy to a heat engine and comes out at 500 K. At what constant temperature should the same heat be delivered to provide same availability? *[924 K]*

15. The exhaust gases from a gas turbine are used to heat water. The gases leave the turbine at 650°C and may be cooled to 145°C. The rate of the gas flow is 1510 kg/min. and the rate of water flow is 1890 kJ/min. The water enters at 35°C. Assume the mean specific heat of the gases and water as 1.088 kJ/kg·K and 4.187 kJ/kg·K, respectively. The atmospheric temperature is 32°C. Determine the loss of available energy resulting from the heat transfer. *[310164.5 kJ/min]*

16. In a steady flow system, air enters at 10 bar, 200°C and a velocity of 200 m/s and the exit is at 1.5 bar, 30°C and a velocity of 100 m/s. The ambient conditions are 1 bar, 30°C. For unit mass flow rate, determine the irreversibility and the effectiveness. Assume $C_p = 1$ kJ/kg·K and $R = 0.287$ kJ/kg·K. *[30 kJ/kg, 0.86]*

17. Air enters in an air turbine at a pressure of 6 bar, 327°C with a velocity of 100 m/s and leaves at 1 bar, 177°C and at 60 m/s. The flow is adiabatic and surrounding air temperature is 300 K. Calculate

(a) Actual work done per kg of air,
(b) Reversible work done per kg of air,
(c) Availability of air entering and leaving the turbine.

Assume $C_p = 1.0$ kJ/kg·K, $C_v = 0.71$ kJ/kg·K and steady flow conditions.

[(a) 154.2 kJ/kg, (b) 222.78 kJ/kg, (c) 252.94 kJ/kg, 30.16 kJ/kg]

18. Calculate the availability of following closed systems.

(a) 5 kg water at 1 bar and 90°C,
(b) 2 kg ice at 1 bar and –10°C,
(c) 0.1 kg of steam at 40 bar and 500°C, and
(d) 0.5 kg of wet steam at 0.1 bar, 0.85 dry

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Consider atmospheric pressure is 1 bar and temperature is 27°C.

[(a) 122 kJ, (b) 81.2 kJ, (c) 98.7 kJ (d) 623 kJ]

19. A flywheel with a moment of intertia of 6.74 kg·m^2, rotates at 3000 rpm. As the flywheel is braked to rest, its kinetic energy is converted entirely to internal energy of the brake lining. The brake lining has a mass of 2.27 kg and has a specific heat of 4.19 kJ/kg·K. Heat transfer to surroundings is negligible.

(a) Determine final temperature of brake lining in °C, if its initial temperature is 16°C.

(b) Maximum possible rotational speed in rpm, that could be attained by the flywheel using energy stored in the brake lining after flywheel has been braked to rest. Assume T_0 = 16°C. *[51°C, 711 rpm]*

20. Water at 25°C, 1 bar is drawn from a mountain lake 1 km above a valley and allowed to flow through a hydraulic turbine generator to a pond on the valley floor for steady-state operation. Calculate minimum mass-flow rate required to generate electricity at a rate of 1 MW. Let T_0 = 25°C and p_0 = 1 bar. *[101.9 kJ/s]*

21. Saturated steam at 0.008 MPa and a mass flow rate of 2.6 × 10^5 kg/h enters the condenser of a 100 MW power plant and exits a saturated liquid at 0.008 MPa. The cooling water enters at 15°C and leaves at 35°C with negligible pressure drop. For a steady state operation, determine

(a) the rate of energy leaving the plant with the cooling water stream in MW,

(b) the rate of availability leaving the plant with the cooling water stream in MW.

[173.3 MW, 2.8 MW]

22. Air at 7 bar, 1000°C enters a turbine and expands to 1.5 bar, 665°C with a mass flow rate of 5 kg/s. The turbine operates at steady state with negligible heat transfer with its surroundings. Assume the ideal gas model with 1.35 and neglecting kinetic and potential energy changes, determine

(a) isentropic turbine efficiency, and

(b) second-law efficiency.

Take T_0 = 25°C, p_0 = 1 atm. *[80%, 92.3%]*

23. A thermal energy storage system (A) composed of water receives 1000 kJ of heat and heats up water from 27°C to 100°C. The system remains in liquid state throughout the heating process.

An alternative storage system (B) uses water at 27°C that becomes dry saturated steam at 100°C and 1 atm, while receiving 1000 kJ of heat. The atmospheric conditions are 1 atm and 300 K. Calculate the availability change in two processes. *[105 kJ, 183.5 kJ]*

24. A counterflow heat exchanger operating in steady state has oil and liquid water flowing in separate steams. The oil is cooled from 440 K to 320 K, while the water temperature increases from 290 K to 305 K. No stream experiences pressure drop. The mass flow of oil is 500 kg/h. The oil and water can be treated as incompressible fluids with constant specific heats of 2 and 4 kJ/kg·K, respectively. Assume no heat is transferred to the surroundings. Determine

(a) mass-flow rate of water in kg/h

(b) second-law efficiency of heat exchanges

(c) hourly cost of loss of available energy if available energy is valued at Rs 10 per kWh

Take T_0 = 17°C, p_0 = 1 atm.

[2000 kg/h, 89%, Rs. 69.6]

25. A geothermal energy source is the underground high temperature water or steam which is heated by the earth's interior. Calculate the steady flow availability per unit mass of water at 150°C and 0.6 MPa. *[83.7 kJ/kg]*

26. Calculate irreversibilty per unit mass for a steady flow steam turbine operating between 3 MPa, 450°C and 0.1 MPa. The work produced by the turbine is 650 kJ/kg. *[100.6 kJ/kg]*

27. Air enters a compressor of a gas turbine at 0.9 bar, 15°C with a velocity of 120 m/s. The air leaves the compressor at a pressure of 4 bar, 200°C with a velocity of 60 m/s. The process is adiabatic. Calculate the reversible work and irreversibility per kg of gas during the process. Take atmospheric temperature as 15°C, C_p = 1.004 kJ/kg·K and R = 0.287 kJ/kg·K. *[–159.48 kJ/kg, 20.86 kJ/k]*

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Objective Questions

1. The available energy is
   (a) high-grade energy
   (b) portion of energy as useful work
   (c) theoretical maximum amount of work
   (d) none of the above
2. Availability function of a closed system is
   (a) $\phi = u + p_0 v - T_0 s$
   (b) $\phi = u + p_0 dv - T_0 ds$,
   (c) $\phi = du + p_0 dv - T_0 ds$,
   (d) $\phi = u + p_0 v + T_0 s$.
3. Availability function of a steady flow system
   (a) $\psi = h - T_0 s$ (b) $\psi = h - T_0 ds$
   (c) $\psi = h + p_0 dv - T_0 ds$ (d) $\psi = h + T_0 s$
4. Which one of the follwing represents availability?
   (a) $\Delta Q - T_0(\Delta S_0)$ (b) $\Delta Q - T(\Delta S)$
   (c) $\Delta Q - T_0(\Delta S)$ (d) $\Delta Q - T(\Delta S_0)$
5. Which one of the follwing represents unavailability?
   (a) $T_0(\Delta S_0)$ (b) $T(\Delta S)$
   (c) $T_0(\Delta S)$ (d) $T(\Delta S_0)$
6. Which one of the follwing represents irreversibility of a closed system?
   (a) $TdS - \delta Q$ (b) $TdS_0 - \delta Q$
   (c) $T_0 dS_0 - \delta Q$ (d) $T_0 dS - \delta Q$
7. To improve the work capacity of energy transfer as heat from high temperature to low temperature,
   (a) lower temperature should be lowered, keeping the temperature difference same
   (b) higher temperature should be increased keeping the temperature difference same
   (c) temperature difference should be increased
   (d) temperature difference should be decreased
8. If heat is transferred to a system, which of the following statement is not correct?
   (a) Δs_{system} decreases
   (b) Δs_{system} increases
   (c) $(\Delta s_{system} + \Delta s_{system})$ decreases
   (d) $(\Delta s_{system} + \Delta s_{system})$ increases
9. The degradation of energy is responsible for
   (a) *entropy generation* within the system
   (b) decrease of entropy within the system
   (c) maximum work done by the system
   (d) none of the above

Answers

1. (b) 2. (a) 3. (a) 4. (c) 5. (c) 6. (d) 7. (c) 8. (b)
9. (a)

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9

Thermodynamic Relations

Introduction

In the previous chapters, we have studied a number of properties like pressure, specific volume, temperature, mass, internal energy, enthalpy, entropy and specific heats. Some of these are measured experimentally and the remaining are calculated from experimental data. In order to calculate the properties that cannot be measured directly, certain thermodynamic relations are required. These relations are developed and discussed in this chapter.

9.1 HELMHOLTZ AND GIBBS FUNCTION: GIBBSIAN EQUATIONS

The Helmholtz and Gibbs functions are new functions used in problems related to chemical equilibrium and combustion. The Helmholtz and Gibbs functions are extensive properties. These are expressed as

Helmholtz Function $F = U - TS$ ...(9.1)

Gibbs Function $G = H - TS$ ...(9.2)

The corresponding specific values

$$f = u - Ts \quad ...(9.3)$$

$$g = h - Ts \quad ...(9.4)$$

In order to derive Gibbsian equation using first and second law of thermodynamics,

$$\delta Q - \delta W = \delta U$$

where $\delta Q = TdS$

and $\delta W = pdV$

Thus $dU = TdS - pdV$ ...(9.5)

Enthalpy $H = U + pV$

$$dH = dU + pdV + Vdp$$

Using Eq. (9.5)

$$dH = TdS - pdV + pdV + Vdp$$

$$= TdS + Vdp \quad ...(9.6)$$

Further differentiating Eq. (9.1) and Eq. (9.2)

$$dF = dU - TdS - SdT$$

$$= -pdV - SdT \quad ...(9.7)$$

and $dG = dH - TdS - SdT = Vdp - SdT$ ...(9.8)

Rewritting the above derived equations, we get the following set of *Gibbsian relations*:

$$dU = TdS - pdV \quad ...(9.5)$$

$$dH = TdS + Vdp \quad ...(9.6)$$

$$dF = -pdV - SdT \quad ...(9.7)$$

$$dG = Vdp - SdT \quad ...(9.8)$$

9.2 PRINCIPAL EXACT DIFFERENTIALS

If z is the property expressed in the form

$$z = f(x, y)$$

Its partial derivative is

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy \quad ...(9.9)$$

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It can be expressed as

$$dz = Mdx + Ndy \qquad ...(9.10)$$

where $M = \left(\frac{\partial z}{\partial x}\right)_y$ and $N = \left(\frac{\partial z}{\partial y}\right)_x$

The coefficient M is the partial derivative of z with respect to x, when the variable y is kept constant. Similarly, N is the partial derivative of z with respect to y, when the variable x is being constant.

Taking the partial derivative of M with respect to y and N with respect to x

$$\left(\frac{\partial M}{\partial y}\right)_x = \left[\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)_y\right]_x = \frac{\partial^2 z}{\partial x\,\partial y} \qquad ...(9.11)$$

and $$\left(\frac{\partial N}{\partial x}\right)_y = \left[\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)_x\right]_y = \frac{\partial^2 z}{\partial x\,\partial y} \qquad ...(9.12)$$

Since z is a property, thus it is an exact differential. Therefore, the two relations above are identical.

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y \qquad ...(9.13)$$

It is an important relation for partial derivatives and is used to test a function whether it is an *exact* or *inexact* differentiation. An *exact differentiable function is always a property or point function.*

9.3 PARTIAL DERIVATIVE RELATIONS

The function $z = f(x, y)$ can also be expressed as $x = \phi(z, y)$, then

$$dx = \left(\frac{\partial x}{\partial z}\right)_y dz + \left(\frac{\partial x}{\partial y}\right)_z dy \qquad ...(9.14)$$

Using Eq. (9.14) in Eq. (9.9), we get

$$dz = \left(\frac{\partial z}{\partial x}\right)_y \left\{\left(\frac{\partial x}{\partial z}\right)_y dz + \left(\frac{\partial x}{\partial y}\right)_z dy\right\} + \left(\frac{\partial z}{\partial y}\right)_x dy$$

$$dz = \left[\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z + \left(\frac{\partial z}{\partial y}\right)_x\right] dy + \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial z}\right)_y dz$$

Rearranging,

$$\left[\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z + \left(\frac{\partial z}{\partial y}\right)_x\right] dy = \left[1 - \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial z}\right)_y\right] dz$$

But y and z are independent of each other. If z is *constant,* $dz = 0$. Then

$$\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z + \left(\frac{\partial z}{\partial y}\right)_x = 0$$

or $$\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z = -\left(\frac{\partial z}{\partial y}\right)_x$$

or $$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1 \qquad ...(9.15)$$

It is called the *cyclic relation.*

9.4 MAXWELL RELATIONS

The equations that relate the partial derivative of properties of p, v, T and s of a compressible fluid are called *Maxwell relations.*

The four Gibbsian relations for a unit mass are

$$du = Tds - pdv \qquad ...(i)$$

$$dh = Tds + vdp \qquad ...(ii)$$

$$df = -pdv - sdT \qquad ...(iii)$$

$$dg = -sdT + vdp$$

Since u, h, f and g are the properties thus point functions and the above relations can be expressed in the form

$$dz = Mdx + Ndy$$

with $$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

Applying the cyclic relation as

$$Mdx + Ndy \rightarrow \left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

Thus replacing M, N, y and x by T, p, v, s of each of Gibbsian equations in cyclic order, we get the following:

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The first relation leads to

$$\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_v \quad ...(9.16)$$

The second relation leads to

$$\left(\frac{\partial T}{\partial p}\right)_s = \left(\frac{\partial v}{\partial s}\right)_p \quad ...(9.17)$$

The third relation leads to

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T \quad ...(9.18)$$

And the fourth relation leads to

$$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p \quad ...(9.19)$$

These derived relations are called the first, second, third and fourth *Maxwell relations*, respectively. These are extremely important in thermodynamics and provide means of determining change in entropy.

9.5 THERMODYNAMIC SQUARE

Figure 9.1 shows a thermodynamic square, a mnemonic diagram which is very useful to obtain the Maxwell and Gibbsian relations. The corners are levelled with properties T, p, s and v, respectively. The sequence of properties T, p, s, v can be kept in mind by remembering a single word तपस्वी (*Tpsv*), which starts from left top corner of the square and runs clockwise.

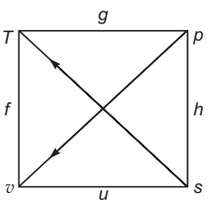

Fig. 9.1 *Thermodynamic square*

The sides of the square are levelled by potential u, f, g and h, starting from the bottom side and run clockwise. The square is also stamped with two diagonal arrows heading towards the left corners.

9.5.1 Gibbsian Equations

The Gibbsian relations can be obtained by using the following rule:

(i) Each potential is flanked with its natural variable as $u = f(v, s)$, $h = \phi(p, s)$, etc. The differential of these variables are multiplied by its properties lying on their diagonally opposite corners. For example,
$du = Tds - p\,dv$

(ii) If the arrow is pointing away from the natural variable then a positive sign is used before the quantity and if the arrow is pointing towards the quantity then a negative sign should be used before the quantity. For example, if the arrow is pointing away from s, a positive sign is considered for Tds, and the arrow is pointing towards v, a negative sign for $p\,dv$.

9.5.2 Maxwell Relations

The Maxwell relations can also be written by using thermodynamic square or mnemonic diagram as follows:

(i) The partial derivative of the properties lying on a corner of a side with respect to the other corner property is equated with partial derivative of the property on a corner opposite with respect to the other corner property of the opposite side. For example,

$$\left(\frac{\partial T}{\partial p}\right)_s = \left(\frac{\partial v}{\partial s}\right)_p$$

(ii) If the partial derivative of properties lying on the horizontal line are equated then a positive sign is used in the relation. If the partial derivatives of properties lying on vertical lines are equated then a negative sign appears on either side of equations. For example, partial derivative lying on vertical lines

$$\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_v$$

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Table 9.1 *Gibbsian and Maxwell relations derived by using a thermondynamic square*

Sr. No	*Gibbsian Relations*	*Maxwell Relations*
1.	$du = Tds - pdv$	$\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_v$
2.	$dh = Tds + vdp$	$\left(\frac{\partial T}{\partial p}\right)_s = \left(\frac{\partial v}{\partial s}\right)_p$
3.	$df = -sdT - pdv$	$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v$
4.	$dg = -sdT + vdp$	$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$

9.6 VOLUMETRIC EXPANSIVITY, ISOTHERMAL AND ISENTROPIC COMPRESSIBILITY

The equation of state for any thermodynamic system is expressed as

$$f(p, v, T) = 0$$

Any two properties of the system are independent and the value of the third property is fixed by the values of the other two properties. The cyclic relation for their partial derivative can be written as

$$\left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_v = -1 \qquad ...(9.20)$$

9.6.1 Coefficient of Volumetric Expanssion

The *coefficient of volumetric expansion* β is defined as the ratio of fractional change in volume to change in temperature when pressure is kept constant. It is expressed as

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p \qquad ...(9.21)$$

In other words the coefficient of volumetric expansion is an indication of change in volume with respect to change in temperature, while pressure remains constant. It is also called *coefficient of thermal expansion.*

The quantity $\left(\frac{\partial v}{\partial T}\right)_v$ is the slope of the curve on v-T diagram.

9.6.2 Coefficient of Isothermal Compressibility

The coefficient of isothermal compressibility, κ is defined as the ratio of fractional change in volume to change in pressure when temperature remains constant or in other words, the isothermal compressibility is an indication of the change in volume with change in pressure, when the temperature is kept constant. It is expressed as

$$\kappa = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T \qquad ...(9.22)$$

where the quantity $\left(\frac{\partial v}{\partial p}\right)_T$ is the slope of the curve on a v–p diagram. Since volume decreases with an increase in pressure, therefore, negative sign is used in the above relation, which makes κ positive for all substances in all phases.

The coefficient of volumetric expansion and isothermal compressibility are thermodynamic properties like specific volume. the value of β and κ for liquid water at 1 atm for different temperatures are given in Table 9.2.

Table 9.2 *Coefficient of volumetric expansion* β *and isothermal compressibility* κ *water at 1 atm*

T(°C)	*Density* ρ(kg/m^3)	$\beta \times 10^6$ K^{-1}	$\kappa \times 10^6$ (bar)$^{-1}$
0	999.84	–68.14	50.89
4	1000	0	50.01
10	999.70	87.90	47.81
20	998.21	206.6	45.90
30	995.65	303.1	44.77
40	992.22	385.4	44.24
50	988.04	457.8	44.18

9.6.3 Isentropic Compressibility

The isentropic compressibility α is the ratio of fractional change in volume to change in pressure, when entropy remains constant. It is an indication of change in volume with change in pressure for an isentropic process. It is given as

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$$\alpha = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_s \qquad ...(9.23)$$

The isentropic compressibility is measured in kPa^{-1}, a reciprocal of pressure unit.

9.6.4 β and κ for an Ideal Gas

For a perfect gas

$$pv = RT$$

Its differential form

$$p\,dv + v\,dp = R\,dT \qquad ...(i)$$

When pressure p is kept constant, $dp = 0$

$$p\,dv = R\,dT$$

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} \qquad ...(ii)$$

and from ideal gas relation

$$\frac{1}{v} = \frac{p}{RT} \qquad ...(iii)$$

The product of Eq. (ii) and (iii) yields coefficient of volumetric expansion β

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p = \frac{p}{RT}\left(\frac{R}{p}\right) = \frac{1}{T} \qquad ...(9.24)$$

Thus, for an ideal gas, the coefficient of volumetric expansion β is the inverse of absolute temperature of gas. Similarly, when temperature T is kept constant, $dT = 0$, then Eq. (i) gives

$$\left(\frac{\partial v}{\partial p}\right)_T = -\frac{v}{p}$$

The coefficient of isothermal expansion becomes

$$\kappa = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$$

$$= -\frac{1}{v}\left(-\frac{v}{p}\right) = \frac{1}{p} \qquad ...(9.25)$$

Thus for an ideal gas, the isothermal compressibility κ is inverse of absolute pressure of gas.

9.6.5 Relation between β and κ

The cyclic relation for properties p, v, T

$$\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial T}{\partial v}\right)_p \left(\frac{\partial v}{\partial p}\right)_T = -1$$

or

$$\left(\frac{\partial p}{\partial T}\right)_v = -\left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p$$

$$= -\frac{(\partial v/\partial T)_p}{(\partial v/\partial p)_T} = \frac{\beta}{\kappa} \qquad ...(9.26)$$

9.7 INTERNAL ENERGY, ENTHALPY, AND ENTROPY

The internal energy, enthalpy and entropy are properties of the substance and they cannot be measured directly.

But these properties are calculated from measured properties considering a simple compressible system.

9.7.1 Change in Internal Energy

We assume that the internal energy is function of temperature and volume, i.e., $u = u(T, v)$ and

$$du = \left(\frac{\partial u}{\partial T}\right)_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv \qquad ...(9.27)$$

using the definition of $C_v = \left(\frac{\partial u}{\partial T}\right)_v$

Thus

$$du = C_v\,dT + \left(\frac{\partial u}{\partial v}\right)_T dv \qquad ...(9.28)$$

We also have $du = Tds - p\,dv$

Now we assume entropy is function of temperature and volume

$$s = f(T, v) \text{ and}$$

$$ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv$$

Substituting ds above, we get

$$du = T\left(\frac{\partial s}{\partial T}\right)_v dT + \left\{\left(\frac{\partial s}{\partial v}\right)_T T - p\right\} dv \qquad ...(9.29)$$

Equating the coefficient of dT and dv in Eq. (9.28) and (9.29) we get

$$C_v = T\left(\frac{\partial s}{\partial T}\right)_v \text{ and}$$

$$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial s}{\partial v}\right)_T - p \qquad ...(9.30)$$

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Using the third Maxwell relation

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v$$

We get $$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - p \qquad ...(9.31)$$

Then Eq. (9.28) becomes

$$du = C_v dT + \left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right] dv \qquad ...(9.32)$$

The change in internal energy of a simple compressible fluid can be calculated as

$$u_2 - u_1 = \int_{T_1}^{T_2} C_v\, dT + \int_{v_1}^{v_2} \left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right] dv \qquad ...(9.33)$$

9.7.2 Change in Enthalpy

Assuming the enthalpy as a function of temperature and pressure, i.e., $h = h(T, p)$ and

$$dh = \left(\frac{\partial h}{\partial T}\right)_p dT + \left(\frac{\partial h}{\partial p}\right)_T dp$$

Using the definition of C_p

$$C_p = \left(\frac{\partial h}{\partial T}\right)_p$$

Thus $$dh = C_p dT + \left(\frac{\partial h}{\partial p}\right)_T dp \qquad ...(9.34)$$

We have $dh = Tds + vdp$

Further, choosing the entropy as function of temperature and pressure,

$$s = f(T, p)$$

and $$ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp$$

Using above, we get

$$dh = T\left(\frac{\partial s}{\partial T}\right)_p dT + \left[T\left(\frac{\partial s}{\partial p}\right)_T + v\right] dp \qquad ...(9.35)$$

Comparing Eq. (9.34) with Eq. (9.35), we have

$$C_p = T\left(\frac{\partial s}{\partial T}\right)_p \quad \text{and}$$

$$\left(\frac{\partial h}{\partial p}\right)_T = T\left(\frac{\partial s}{\partial p}\right)_T + v \qquad ...(9.36)$$

Using the fourth Maxwell relation

$$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$$

Using in Eq. (9.34), we get

$$dh = C_p dT + \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] dp \qquad ...(9.37)$$

The change in enthalpy of a simple compressible fluid can be calculated as

$$h_2 - h_1 = \int_{T_1}^{T_2} C_p\, dT + \int_{p_1}^{p_2} \left[v - T\left(\frac{\partial p}{\partial T}\right)_p\right] dp \qquad ...(9.38)$$

9.7.3 Change in Entropy

The entropy is a property of the system, it cannot be measured directly. but it is calculated in terms of other easily measurable properties, the pressure, temperature and specific volume. The entropy can be expressed in the functional form as

$$s = f(T, v) \qquad ...(i)$$

$$s = \phi(p, T) \qquad ...(ii)$$

$$s = \varphi(p, v) \qquad ...(iii)$$

Using Eq. (i) and taking its partial derivative

$$ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv$$

and $$Tds = T\left(\frac{\partial s}{\partial T}\right)_v dT + T\left(\frac{\partial s}{\partial v}\right)_T dv$$

For a constant volume process.

$$ds = C_v \frac{dT}{T}$$

or $$C_v = T\left(\frac{\partial s}{\partial T}\right)_v \qquad ...(9.39)$$

From Maxwell's third relation

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v$$

Therefore, $$Tds = C_v dT + T\left(\frac{\partial p}{\partial T}\right)_v dv \qquad ...(9.40)$$

Equation (9.40) is known as the *first Tds relation*, Re-writing (Eq. (9.40)) in the form

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$$ds = C_v \frac{\partial T}{T} + \left(\frac{\partial p}{\partial T}\right)_v dv \quad ...(9.41)$$

For an ideal gas $pv = RT$

or $$\frac{R}{v} = \frac{p}{T}$$

For a constant-volume process,

$$\frac{R}{v} = \left(\frac{\partial p}{\partial T}\right)_v$$

or $$ds = C_v \frac{\partial T}{T} + \frac{R}{v} dv \quad ...(9.42)$$

Further, the term $\left(\frac{\partial p}{\partial T}\right)_v$ of Eq. (9.41) can be expressed as

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial v}{\partial T}\right)_p \Big/ \left(\frac{\partial v}{\partial p}\right)_T = \frac{\beta}{\kappa}$$

Therefore, Eq. (9.41) can be written as

$$ds = C_v \frac{dT}{T} + \frac{\beta}{\kappa} dv \quad ...(9.43)$$

Using functional relation for entropy as in Eq. (ii) and taking its partial derivative

$$ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp$$

and $$Tds = T\left(\frac{\partial s}{\partial T}\right)_p dT + T\left(\frac{\partial s}{\partial p}\right)_T dp \quad ...(9.44)$$

For a constant-pressure process

$$ds = C_p \frac{dT}{T}$$

or $$C_p = T\left(\frac{\partial s}{\partial T}\right)_p \quad ...(9.45)$$

Further, from Maxwell's fourth relation

$$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$$

Using in Eq. (9.44), we get

$$Tds = C_p dT - T\left(\frac{\partial v}{\partial T}\right)_p dp \quad ...(9.46)$$

The Eq. (9.46) is known as the *second Tds relation*.

The change in entropy

$$ds = C_p \frac{dT}{T} - \left(\frac{\partial v}{\partial T}\right)_p dp \quad ...(9.47)$$

For an ideal gas

$$\frac{R}{p} = \frac{v}{T}$$

For constant-pressure process

$$\frac{R}{p} = \left(\frac{\partial v}{\partial T}\right)_p$$

Thus $$ds = C_p \frac{dT}{T} - \frac{R}{p} dp \quad ...(9.48)$$

Further, $$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p$$

or $$\beta v = \left(\frac{\partial v}{\partial T}\right)_p$$

Therefore, Eq. (9.47) can also be expressed in the form

$$ds = C_p \frac{dT}{T} - \beta v dp \quad ...(9.49)$$

Now using $s = \varphi(p, v)$

Its partial derivative

$$ds = \left(\frac{ds}{dp}\right)_v dp + \left(\frac{\partial s}{\partial v}\right)_v dv$$

and $$Tds = T\left(\frac{\partial s}{\partial p}\right)_v dp + \left(\frac{\partial s}{\partial v}\right)_p Tdv \quad ...(9.50)$$

Using chain relation of derivatives of properties

$$\left(\frac{\partial s}{\partial p}\right)_v \left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial T}{\partial s}\right)_v = 1$$

or $$\left(\frac{\partial s}{\partial p}\right)_v = \left(\frac{\partial s}{\partial T}\right)_v \left(\frac{\partial T}{\partial p}\right)_v = \frac{C_v}{T}\left(\frac{\partial T}{\partial p}\right)_v$$

We have

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p$$

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and $$\kappa = \frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$$

or $$\frac{\kappa}{\beta} = \left(\frac{\partial T}{\partial p}\right)_v$$

Therefore,

$$\left(\frac{\partial s}{\partial p}\right)_v = \frac{C_v}{T}\,\frac{\kappa}{\beta} \qquad ...(9.51)$$

Similarly, from chain relation at constant pressure

$$\left(\frac{\partial v}{\partial s}\right)_p \left(\frac{\partial s}{\partial T}\right)_p \left(\frac{\partial T}{\partial v}\right)_p = 1$$

and $$\left(\frac{\partial s}{\partial v}\right)_p = \left(\frac{\partial s}{\partial T}\right)_p \left(\frac{\partial T}{\partial v}\right)_p = \frac{C_p}{T}\left(\frac{\partial T}{\partial v}\right)_p$$

$$= \frac{C_p}{T}\,\frac{1}{\beta v} \qquad ...(9.52)$$

Therefore, we get *third Tds relation* in the form

$$Tds = C_v\frac{\kappa}{\beta}dp + C_p\frac{1}{\beta v}dv \qquad ...(9.53)$$

9.8 SPECIFIC HEATS

Recalling the definitions of C_p and C_v,

$$C_p = \left(\frac{\partial h}{\partial T}\right)_p$$

and $$C_v = \left(\frac{\partial u}{\partial T}\right)_v$$

We have also obtained for a simple compressible fluid, Eq. (9.36)

$$C_p = \left(\frac{\partial h}{\partial T}\right)_p = T\left(\frac{\partial s}{\partial T}\right)_p \qquad ...(9.54)$$

Similarly, from Eq. (9.30)

$$C_v = \left(\frac{\partial u}{\partial T}\right)_v = T\left(\frac{\partial s}{\partial T}\right)_v \qquad ...(9.55)$$

9.8.1 Evaluating $C_p - C_v$

An expression for the difference between $C_p - C_v$ can be obtained by equating the two differential equations for entropy given by Eqs. (9.40) and (9.46).

$$C_p dT - T\left(\frac{\partial v}{\partial T}\right)_p dp = C_v dT + T\left(\frac{\partial p}{\partial T}\right)_v dv$$

or $$(C_p - C_v)\,dT = T\left(\frac{\partial p}{\partial T}\right)_v dv + T\left(\frac{\partial v}{\partial T}\right)_p dp$$

or $$dT = \frac{T\left(\frac{\partial p}{\partial T}\right)_v dv}{C_p - C_v} + \frac{T\left(\frac{\partial v}{\partial T}\right)_p dp}{C_p - C_v} \qquad ...(9.56)$$

For an equation of state $T = f(p, v)$ and

$$dT = \left(\frac{\partial T}{\partial v}\right)_p dv + \left(\frac{\partial T}{\partial p}\right)_v dp \qquad ...(9.57)$$

Comparing Eq. (9.56) and Eq. (9.57), we get

$$\left(\frac{\partial T}{\partial v}\right)_p = \frac{T\left(\frac{\partial p}{\partial T}\right)_v}{C_p - C_v} \text{ and } \left(\frac{\partial T}{\partial p}\right)_v = \frac{T\left(\frac{\partial v}{\partial T}\right)_p}{C_p - C_v}$$

Both these equations yield to

$$C_p - C_v = T\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial v}{\partial T}\right)_p \qquad ...(9.58)$$

From cyclic relation for these properties

$$\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial T}{\partial v}\right)_p \left(\frac{\partial v}{\partial p}\right)_T = -1$$

$$\left(\frac{\partial p}{\partial T}\right)_v = -\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial p}{\partial v}\right)_T$$

Thus, $$C_p - C_v = -T\left(\frac{\partial v}{\partial T}\right)_p^2 \left(\frac{\partial p}{\partial v}\right)_T \qquad ...(9.59)$$

It is very important equation in thermodynamics. The following facts can be concluded:

(i) The slope $\left(\frac{\partial v}{\partial T}\right)$ may be positive or negative, but its square is always positive. But the slope $\left(\frac{\partial p}{\partial v}\right)_T$ for any substance is always negative, therefore, $C_p - C_v$ is always positive. The temperature T is the absolute temperature, which is always positive.

(ii) As the absolute temperature approaches zero, then

$$C_p \approx C_v$$

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(iii) When $\left(\frac{\partial v}{\partial T}\right)_p = 0$ for an incompressible fluids, then the two specific heats are identical

i.e. $C_p = C_v = C$

(iv) For an ideal gas

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} = \frac{v}{T}$$

and $$\left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT}{v^2}$$

Using in Eq. (9.59), we get

$$C_p - C_v = -T\left(\frac{v}{T}\right)^2 \times \left(-R\frac{T}{v^2}\right) = R \quad ...(9.60)$$

Equation (9.59) can also be expressed in terms of coefficient of volumetric expansion and isothermal compressibility

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p$$

and $$\kappa = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$$

Equation (9.59) takes the form

$$C_p - C_v = \frac{vT\beta^2}{\kappa} \quad ...(9.61)$$

9.8.2 Evaluating C_p/C_v

An expression for ratio of two specific heats of an ideal gas can be obtained from Eqs. (9.30) and (9.36) and rearranging to obtain

$$\frac{C_v}{T} = \left(\frac{\partial s}{\partial T}\right)_v \quad ...(i)$$

$$\frac{C_p}{T} = \left(\frac{\partial s}{\partial T}\right)_p \quad ...(ii)$$

Using cyclic relation for $s = f(v, T)$

$$\left(\frac{\partial s}{\partial T}\right)_v \left(\frac{\partial T}{\partial v}\right)_s \left(\frac{\partial v}{\partial s}\right)_T = -1$$

Thus $$\left(\frac{\partial s}{\partial T}\right)_v = -\frac{1}{\left(\frac{\partial T}{\partial v}\right)_s \left(\frac{\partial v}{\partial s}\right)_T} \quad ...(iii)$$

Similarly, for $s = \phi(p, T)$

$$\left(\frac{\partial s}{\partial T}\right)_v = -\frac{1}{\left(\frac{\partial T}{\partial p}\right)_s \left(\frac{\partial p}{\partial s}\right)_T} \quad ...(iv)$$

Using Eq. (iii) in Eq. (i) and Eq. (iv) in Eq. (ii); we get

$$\frac{C_v}{T} = -\frac{1}{\left(\frac{\partial T}{\partial v}\right)_s \left(\frac{\partial v}{\partial s}\right)_T} \quad ...(v)$$

and $$\frac{C_p}{T} = -\frac{1}{\left(\frac{\partial T}{\partial p}\right)_s \left(\frac{\partial p}{\partial s}\right)_T} \quad ...(vi)$$

Dividing (vi) by (v),

$$\frac{C_p}{C_v} = \frac{\left(\frac{\partial T}{\partial v}\right)_s \left(\frac{\partial v}{\partial s}\right)_T}{\left(\frac{\partial T}{\partial p}\right)_s \left(\frac{\partial p}{\partial s}\right)_T} \quad ...(9.62)$$

Rearranging Eq. (9.62), we get

$$\frac{C_p}{C_v} = \left[\left(\frac{\partial p}{\partial T}\right)_s \left(\frac{\partial T}{\partial v}\right)_s\right]\left[\left(\frac{\partial v}{\partial s}\right)_T \left(\frac{\partial s}{\partial p}\right)_T\right] \quad ...(9.63)$$

The chain rule allows us to write

$$\left(\frac{\partial v}{\partial p}\right)_T = \left(\frac{\partial v}{\partial s}\right)_T \left(\frac{\partial s}{\partial p}\right)_T$$

and $$\left(\frac{\partial p}{\partial v}\right)_s = \left(\frac{\partial p}{\partial T}\right)_s \left(\frac{\partial T}{\partial v}\right)_s$$

Then Eq. (9.63) takes the form

$$\frac{C_p}{C_v} = \left(\frac{\partial v}{\partial p}\right)_T \left(\frac{\partial p}{\partial v}\right)_s \quad ...(9.64)$$

Where ratio $C_p/C_v = \gamma$ (gamma) for ideal gases.

From Eqs. (9.22) and (9.23), the above expression can be written in terms of the isothermal and isentropic compressibilities as

$$\gamma = \frac{C_p}{C_v} = \frac{\kappa}{\alpha} \quad ...(9.65)$$

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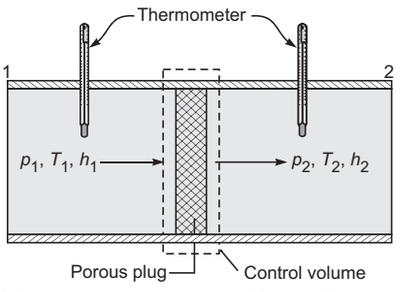
9.9 JOULE–THOMPSON COEFFICIENT—THE POROUS PLUG EXPERIMENT

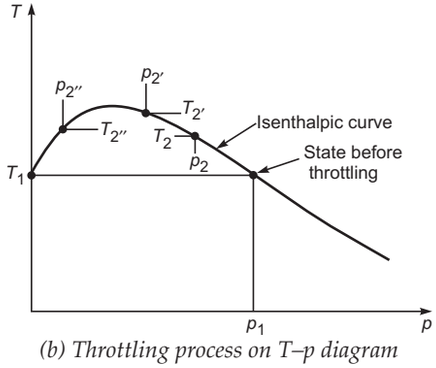
The porous plug experiment was designed to measure the resulting temperature change, when a fluid flows steadily through a porous plug, which is inserted in a thermally insulated horizontal pipe as shown in Fig. 9.2(a). The steady flow through such a restricted passage is called *throttling process* or *isenthalpic process.* It is an irreversible process.

A gas at initial pressure p_1, temperature T_1 and enthalpy h_1 flows steadily through a porous plug located in an insulated, horizontal tube and emerges into a space, which is maintained at pressure p_2. Consider the region enclosed by dotted lines as control volume. Since the pipe is insulated and it does not experience any work transfer; $\dot{Q} = 0$, and $\dot{W} = 0$. If the kinetic and potential energy changes are negligible, then the steady-flow energy equation reduces to

$$h_1 = h_2$$



(a) Porus plug experiment of Joule–Thompson



(b) Throttling process on T–p diagram

Fig. 9.2

Joule–Thompson conducted experiments on porous plug for the same gas with given initial pressure p_1 and temperature T_1 but with different downstream pressures $p_2' p_2'$, p_2'', etc. The different exit temperatures were obtained after expansion as T_2, T_2', T_2'', respectively. The downstream pressure was altered by controlling the opening of the throttle valve. The plot of exit pressures and temperatures for a gas yields an isenthalpic curve as shown in Fig. 9.2(b). During the throttling process, the temperature may decrease, increase or remains constant. The *Joule–Thompson coefficient* is the slope of isenthalpic curve. It is designated as μ and defined as

$$\mu = \left(\frac{\partial T}{\partial p}\right)_h \qquad ...(9.66)$$

Joule and Thomson conducted a series of experiments for real gases on a porous plug with constant inlet pressure and temperature. The gas is forced to flow through different sizes of porous plugs, each giving a different set of T_2 and p_2. When these experimental data are plotted on a T–p diagram, it gives a family of curves as shown in Fig. 9.3. It is observed that each curve passes through a maximum point referred as *inversion point*, A curve passing through the inversion point is known as an *inversion curve*.

The temperature on the point is known as *inversion temperature*. The slope of isenthalpic curve $\mu = \left(\frac{\partial T}{\partial p}\right)_h$ is positive. If the state after throttling falls within the inversion curve then the throttling produces a drop in temperature with decrease in pressure. Therefore, the region within the inversion curve is known as the *cooling zone.* At room temperature and at low moderate pressures, the state of most gases after throttling falls inversion curve.

Similarly, if the state after throttling falls out of the inversion curve where the slope isenthalpic curve $\mu = \left(\frac{\partial T}{\partial p}\right)_h$ is negative, the *temperature increases* with decrease in pressure and there is a heating effect after throttling. Therefore, the region

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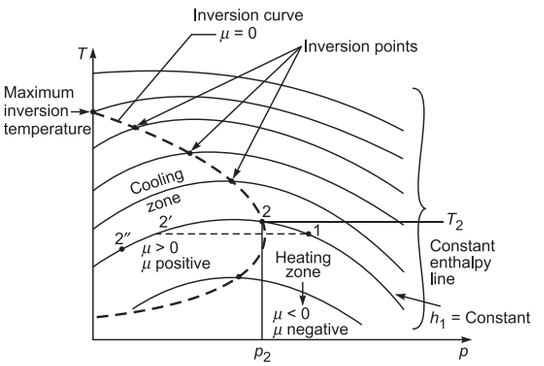

Fig. 9.3 *Family of isenthalpic lines and inversion curve*

outside the inversion curve is known as *heating zone.* The Joule–Thomson coefficient μ decides the heating or cooling effect with decrease in pressure.

$$\mu \begin{cases} < 0 & T \text{ increases} \\ = 0 & T \text{ remains constant} \\ > 0 & T \text{ decreases} \end{cases} \qquad ...(9.67)$$

The state of any ideal gas after throttling always falls at the point on the curve, where slope $\mu = \left(\frac{\partial T}{\partial p}\right)_h = 0$. Therefore, the temperature remains constant after throttling for ideal gases. ($\because h = const.$, $\therefore T = const.$)

9.9.1 Joule–Thomson Coefficient in Terms of p, v, T and C_p data

We have Eq. (9.37)

$$dh = C_p dT + \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] dp$$

For constant enthalpy process. $dh = 0$, then

$$-C_p\left(\frac{dT}{dp}\right)_h = v - T\left(\frac{\partial v}{\partial T}\right)_p$$

or
$$\mu = \left(\frac{\partial T}{\partial p}\right)_h = -\frac{1}{C_p}\left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] \qquad ...(9.68)$$

For an ideal gas

$$\frac{v}{T} = \frac{R}{p}$$

$$\therefore \quad \left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} = \frac{v}{T} \qquad ... (9.69)$$

Using in Eq. (9.68), for an ideal gas, we get

$$\mu = -\frac{1}{C_p}\left\{v - T\frac{v}{T}\right\} = 0$$

[Thomson later became Lord Kelvin. Therefore, the coefficient is also referred as *Joule–Kelvin coefficient*].

9.9.2 The Coefficient of Constant Temperature

When the data obtained from porous plug experiments are plotted on a an $h - p$ diagram. The slope of curve gives the constant temperature coefficient, it is denoted by C_T and is expresed as

$$C_T = \left(\frac{\partial h}{\partial p}\right)_T \qquad ...(9.70)$$

Using the cyclic relationship to properties h, p and T

$$\left(\frac{\partial h}{\partial p}\right)_T \left(\frac{\partial p}{\partial T}\right)_h \left(\frac{\partial T}{\partial h}\right)_p = -1$$

$$C_T \frac{1}{\mu} \frac{1}{C_p} = -1$$

or
$$\mu = -\frac{C_T}{C_p} \qquad ...(9.71)$$

For a perfect gas, $\mu = 0$, and at constant temperature $dh = 0$, thus $C_T = 0$.

9.10 CLAUSIUS–CLAPEYRON EQUATION

The *Clausius–Clapeyron equation* relates saturation pressure, saturation temperature, enthalpy of vaporisation and specific volume of two phases of saturated fluid during phase change.

Considering entropy as a function of temperature and specific volume $s = f(T, v)$ and

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$$ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv \qquad ...(9.72)$$

During the phase change of a pure substance, the saturated temperature remains constant, thus $dT = 0$ and the above equation reduces to

$$ds = \left(\frac{\partial s}{\partial v}\right)_T dv$$

or $$\frac{ds}{dv} = \left(\frac{\partial s}{\partial v}\right)_T \qquad ...(9.73)$$

Using the third Maxwell relation, we get

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v = \frac{ds}{dv} \qquad ...(9.74)$$

For two phases of a pure substance in equilibrium, the pressure and temperature are independent of specific volume. Thus the derivative in Eq, (9.74) may be written as

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{dp}{dT}\right)_{sat}$$

and $$\left(\frac{dp}{dT}\right)_{sat} = \frac{ds}{dv} = \frac{s_g - s_f}{v_g - v_f} = \frac{s_{fg}}{v_{fg}} \qquad ...(9.75)$$

the pressure and temperature during phase change, remain constant, thus

$$q = h_2 - h_1 = h_g - h_f = h_{fg}$$

and $q = Ts_{fg}$ for a constant temperature process

$$\therefore \quad s_{fg} = \frac{h_{fg}}{T} \qquad ...(9.76)$$

Using Eq. (9.75), we get

$$\left(\frac{dp}{dT}\right)_{sat} = \frac{h_{fg}}{Tv_{fg}} \qquad ...(9.77)$$

This equation is known as *Clapeyron equation.* The derivative dp/dT represents the slope on the pressure–temperature curve. Equation (9.76) can also be applied to solid–gas and solid–liquid phase changes.

At temperatures relatively high but below critical point, $v_g >> v_f$ and pure substance follows the ideal gas behaviour

$$v_{fg} = v_g = \frac{RT}{p}$$

The above equation becomes

$$\left(\frac{dp}{dT}\right)_{sat} = \frac{ph_{fg}}{RT^2} \qquad ...(9.78)$$

This equation is referred as *Clausius Clapeyron equation.* Rearranging the above equation,

$$\frac{dp}{p} = \frac{h_{fg}}{R}\frac{dT}{T^2}$$

On integration,

$$\text{In}\left(\frac{p_2}{p_1}\right) = \frac{h_{fg}}{R}\left[\frac{1}{T_1} - \frac{1}{T^2}\right] \qquad ...(9.79)$$

This equation can be used more frequently.

The change in specific entropy during a phase change from saturated liquid to saturated vapour can be obtained from Eq. (9.66).

The change in specific internal energy can be determined by using

$$dh = du + pdv$$

Integration over a phase change leads to

$$h_g - h_f = u_g - u_f + p(v_g - v_f)$$
$$h_{fg} = u_{fg} + pv_{fg}$$

or $$u_{fg} = h_{fg} - pv_{fg} \qquad ... (9.80)$$

Thus change in specific internal energy during a phase change at constant temperature and pressure can be determined from Eq. (9.80).

Example 9.1 *The students of a thermodynamics class went to a hill station on a trip. When they reached the top of a hill they interested to know the approximate height of the hill from sea level. Since they had not carried any instrument with them, they decided to use their thermodynamics knowledge to calculate the height of the hill. In this connection, they boiled the water on the hill and found it boiled at 90°C. Assume the atmospheric temperature is 27°C and the pressure variation is governed by $pv = p_0v_0$ where p_0 is sea level pressure, 101kPa. Estimate the height of the hill.*

Solution

Given $T_b = 90°C = 363$ K, $p_0 = 101$ kPa, $T_0 = 27°C = 300$K

To find The height of hill.

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Assumptions

(i) The normal boiling temperature of water is as 100°C = 373 K.

(ii) The latent of vapourisation is 2257 kJ/kg.

(iii) Acceleration due to gravity $g = 9.81$ m/s^2.

Analysis The Clausius–Clapeyron equation

$$\ln\left(\frac{p_2}{p_1}\right) = \frac{h_{fg}}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$R \text{ for water} = \frac{R_u}{\mathcal{M}} = \frac{8.314}{18} = 0.461 \text{ (kJ/kg·K)}$$

Using the values, we get

$$\ln\left(\frac{p_2}{p_1}\right) = \frac{(2257 \text{ kJ/kg})}{(0.461 \text{ kJ/kg.K})} \times \left[\frac{1}{373} - \frac{1}{363}\right]$$

$$= -0.361 \qquad ...(i)$$

Further the pressure difference across a vertical fluid column, measured from top is given as

or $$dp = -\rho g dh = -g\frac{dh}{v} \qquad ...(ii)$$

Using relation $pv = p_0 v_0$

or $$v = \frac{p_0 v_0}{p}$$

Using in Eq. (ii), we get

or $$dp = -p\frac{g\,dh}{p_0 v_0}$$

or $$\frac{dp}{p} = -\frac{g\,dh}{p_0 v_0}$$

Integrating on both sides

$$\ln\left(\frac{p_2}{p_1}\right) = -\frac{gh}{p_0 v_0}$$

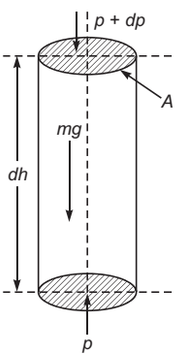

Fig. 9.4

For air $p_0 v_0 = RT_0$

Thus $$\ln\left(\frac{p_2}{p_1}\right) = -\frac{gh}{RT_0} \qquad ...(ii)$$

Using the value

$$-0.361 = \frac{-(9.81 \text{ m/s}^2)h}{(287 \text{ J/kg K}) \times (300 \text{ K})}$$

or $$h = \mathbf{3168.4 \text{ m}}$$

Example 9.2 *Calculate the enthalpy of vaporisation of water at 300°C. Use the steam table for required data. Compare the value obtained with the Clapeyron equation.*

Solution

Given $T = T_s = 300°C = 573$ K

To find The enthalpy of vaporisation.

Analysis From steam table at 300°C,

$p = 85.81$ bar,
$h_{fg} = 1404.9$ kJ/kg
$v_g = 0.02167$ m^3/kg,
$v_f = 0.001404$ m^3/kg

For calculating the slope $\left(\frac{dp}{dT}\right)_{300°C}$, we take

$T_{s_1} = 295°C$ $\quad p_1 = 79.93$ bar
$T_{s_2} = 305°C$ $\quad p_2 = 92.02$ bar
$v_{fg} = v_g - v_f = 0.02167 - 0.001404 = 0.02027$ m^3/kg

Then slope $$\left(\frac{\partial p}{\partial T}\right)_{300°C} = \frac{p_2 - p_1}{T_{s_2} - T_{s_1}}$$

$$= \frac{92.02 - 79.93}{305 - 295}$$

$= 1.209$ bar/°C or 120.9 kPa/K

From Clapeyron equation

$$\left(\frac{dp}{dT}\right)_{sat} = \frac{h_{fg}}{Tv_{fg}}$$

Therefore,

$h_{fg} = (120.9 \text{ kPa/K}) \times (573\text{K}) \times (0.02027 \text{ m}^3\text{/Kg})$
$= 1404.6$ kJ/kg

The approximated value is slightly different from tabulated value, this difference may be due to approximation used for calculation of slope $\frac{dp}{dT}$.

Example 9.3 *Using p-v-T data for saturated water, calculate at 100°C (a) h_{fg}, (b) u_{fg}, and (c) s_{fg}. Compare the results with respective steam table values.*

Solution

Given Saturated water at 100°C.

To find Using saturation data at 100°C,

(i) Specific enthalpy of evaporation,

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(ii) Specific internal energy, and

(iii) Specific entropy.

Assumption

(i) Unit mass of water.

(ii) Saturation data at 100°C (= 373 K)

$p_s = 1.014$ bar

$v_f = 1.0435 \times 10^{-3}$ m^3/kg,

$v_g = 1.673$ m^3/kg.

Analysis The Clapeyron equation can be expressed as

(i) $h_{fg} = T(v_g - v_f)\left(\frac{dp}{dt}\right)_{sat}$

This equation requires slope $\left(\frac{dp}{dt}\right)_{sat}$, we take from steam tables

$T_1 = 95°C \qquad p_1 = 0.8455$ bar

$T_2 = 110°C \qquad p_2 = 1.433$ bar

$$\left(\frac{dp}{dt}\right)_{sat} = \frac{p_2 - p_1}{T_2 - T_1} = \frac{1.433 - 0.8455}{110 - 95}$$

$= 0.03916$ bar/°C $= 3.9167$ kPa/°C

Using numerical values

$$h_{fg} = 373 \times (1.673 - 1.0435 \times 10^{-3}) \times 3.9167$$

$$= \mathbf{2442.6\ kJ/kg}$$

which is 8% higher, due to poor estimation of $\left(\frac{dp}{dt}\right)$.

(ii) The change in specific internal energy u_{fg}

$$u_{fg} = h_{fg} - p(v_g - v_f)$$

$$= 2442.6 - 1.014 \times 100 \times (1.673 - 1.0435 \times 10^{-3})$$

$$= \mathbf{2273.06\ kJ/kg}$$

which is also higher than the tabulated value.

(iii) The change in specific entropy during evaporation.

$$s_{fg} = \frac{h_{fg}}{T_s} = \frac{2442.6}{373}$$

$$= \mathbf{6.5485\ kJ/kg \cdot K}$$

which is again higher than the tabulated value.

Example 9.4 *Calculate the per-cent error in C_v that would result if $C_p = C_v$ is assumed for liquid water at 1 atm and 27°C.*

Solution

Given Liquid water at

$p = 1$ atm $\quad T = 27°C = 300$ K

To find Per-cent error in C_v if $C_p = C_v$ is assumed.

Analysis Using Eq. (9.61) for obtaining difference between C_p and C_v

$$C_p - C_v = \frac{vT\beta^2}{\kappa} = \frac{\beta^2 T}{\rho\kappa}$$

From Table 9.2, by interpolation at 27°C (300 K)

$\rho = 996.42$, $\beta = 274.15 \times 10^{-6}$ K^{-1},

$\kappa = 45.11 \times 10^{-6}$ (bar)$^{-1}$

Using numerical values.

$$C_p - C_v = \frac{274.15 \times 10^{-6} \times 300 \times 100}{996.42 \times 45.11 \times 10^{-6}}$$

$= 0.0501$ kJ/kg·K

The specific heat of water at 27°C, (from standard tables)

$C_p = 4.179$ kJ/kg·K)

Then $C_v = 4.179 - 0.0501 = 4.129$ kJ/kg·K

Per-cent error in approximation

$$= \left(\frac{C_p - C_v}{C_v}\right) \times 100 = \frac{0.0501}{4.129} \times 100 = \mathbf{1.21\%}$$

Example 9.5 *Prove that*

$$\left(\frac{\partial u}{\partial T}\right)_p = C_p - pv\beta$$

Solution Using equation

$$du = Tds - pdv$$

At constant pressure, it can be written as

$$\left(\frac{\partial u}{\partial T}\right)_p = T\left(\frac{\partial s}{\partial T}\right)_p - p\left(\frac{\partial v}{\partial T}\right)_p$$

From Eq. (9.45)

$$T\left(\frac{\partial s}{\partial T}\right)_p = C_p$$

and Eq. (9.21) gives

$$\left(\frac{\partial v}{\partial T}\right)_p = v\beta$$

Thus

$$\left(\frac{\partial u}{\partial T}\right)_p = C_p - pv\beta$$ **Proved**

Example 9.6 *Prove that*

$$\left(\frac{\partial u}{\partial v}\right)_p = \frac{C_p}{v\beta} - p.$$

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Solution Using equation for internal energy change

$$du = Tds - pdv$$

Partial differentiation with respect to specific volume at constant pressure

$$\left(\frac{\partial u}{\partial v}\right)_p = T\left(\frac{\partial s}{\partial v}\right)_p - p\left(\frac{\partial v}{\partial v}\right)_p \qquad \text{(i)}$$

The quantity $\left(\frac{\partial s}{\partial v}\right)_p$ can be written as

$$\left(\frac{\partial s}{\partial v}\right)_p = \left(\frac{\partial s}{\partial T}\right)_p \left(\frac{\partial T}{\partial v}\right)_p = \frac{C_p}{\beta v} \qquad \text{(ii)}$$

Using Eq. (ii) in Eq. (i), we get

$$\left(\frac{\partial u}{\partial v}\right)_p = \frac{Cp}{v\beta} - p \qquad \textbf{Proved}$$

Example 9.7 *If a gas obeys the following equation of state*

$$p = \frac{RT}{v} - \frac{a}{v^2}$$

calculate the change in enthalpy at constant temperature.

Solution The change in enthalpy is expressed by Eq. (9.37) as

$$dh = C_p dT + \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] dp$$

At constant temperature, $dT = 0$, then

$$dh = \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] dp \qquad \text{...(i)}$$

Let $p = f(v, T)$, then

$$dp = \left(\frac{\partial p}{\partial v}\right)_T dv + \left(\frac{\partial p}{\partial T}\right)_v dT$$

$\therefore \quad dT = 0$

$\therefore \quad dp = \left(\frac{\partial p}{\partial v}\right)_T dv \qquad \text{...(ii)}$

Substituting Eq. (ii), in Eq. (i). we get

$$dh = \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right]\left(\frac{\partial p}{\partial v}\right)_T dv$$

$$= \left[v \times \left(\frac{\partial p}{\partial v}\right)_T - T\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial p}{\partial v}\right)_T\right] dv \qquad \text{...(iii)}$$

The cyclic relation for properties p, v and T

$$\left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_v = -1$$

or

$$\left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial p}{\partial T}\right)_v$$

Using the above in Eq. (iii), we get

$$dh = \left[v\left(\frac{\partial p}{\partial v}\right)_T + T\left(\frac{\partial p}{\partial T}\right)_v\right] dv$$

The equation of state of the gas is

$$p = \frac{RT}{v} - \frac{a}{v^2}$$

and

$$\left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT}{v^2} + \frac{a}{v^3}$$

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v} - 0$$

Using in Eq. (iv)

$$dh = \left[-\frac{vRT}{v^2} + \frac{av}{v^3} + \frac{RT}{v}\right] dv = \frac{a}{v^2} dv$$

Integrating both sides,

$$\int_1^2 dh = h_2 - h_1 = \int_{v_1}^{v_2} \frac{a}{v^2} dv = a\left[\frac{v^{-2+1}}{-2+1}\right]_{v_1}^{v_2}$$

$$= a\left[\frac{1}{v_1} - \frac{1}{v_2}\right] = a\left[\frac{v_2 - v_1}{v_1 v_2}\right]$$

Example 9.8 *Prove that for a real gas, whose equation of state is*

$$p(v-b)\, e^{\frac{a}{R_u T v}} = R_u T$$

the critical pressure is given by

$$p_c = \frac{a}{4e^2 b^2}, \; v_c = 2b, \; T_c = \frac{a}{4R_u b}$$

Take $\left(\frac{\partial p}{\partial v}\right)_T = 0$ and $\left(\frac{\partial^2 p}{\partial v^2}\right)_T = 0$

Solution The equation of state

$$p(v-b)\, e^{\frac{a}{R_u T v}} = R_u T$$

or

$$p = \frac{R_u T}{v-b}\, e^{-\frac{a}{R_u T v}} \qquad \text{...(i)}$$

Its partial derivative at constant temperature

$$\left(\frac{\partial p}{\partial v}\right)_T = \frac{R_u T}{v-b}\left\{e^{-\frac{a}{R_u T v}}\left(-\frac{a}{R_u T}\right)\left(-\frac{1}{v^2}\right)\right\}$$

$$- e^{-\frac{a}{R_u T v}} \frac{R_u T}{(v-b)^2}$$

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$$= \frac{R_u T}{(v-b)} e^{-a/R_u T v} \frac{a}{R_u T v^2} - \frac{R_u T}{(v-b)^2} e^{-a/R_u T v}$$

$$= \frac{a}{v^2(v-b)} e^{-a/R_u T v} - \frac{R_u T}{(v-b^2)} e^{-a/R_u T v}$$

For maximum pressure

$$\left(\frac{\partial p}{\partial v}\right)_T = 0$$

then $$\frac{a}{v^2} = \frac{R_u T}{(v-b)}$$

or $$R_u T = \frac{a(v-b)}{v^2} \quad ...(ii)$$

Taking second-order partial derivative and equating it to zero

$$\left(\frac{\partial^2 p}{\partial v^2}\right)_T = 0$$

$$= \frac{av^2\ (v-b)\left\{e^{-a/R_u T v}\left(\frac{a}{R_u T v^2}\right)\right\} - a(3v^2 - 2vb)e^{-a/R_u T v}}{v^4 (v-b)^2}$$

$$- \frac{R_u T}{(v-b^2)} \cdot e^{-a/R_u T v} \cdot \left(-\frac{a}{R_u T}\right)\left(-\frac{1}{v^2}\right) + \frac{2R_u T e^{-a/R_u T v}}{(v-b)^3}$$

Dividing both sides by $e^{(-a/R_u T v)}$, we get

$$0 = \frac{a[(v-b)\cdot(a/R_u T) - 3v^2 + 2vb]}{v^4 (v-b)^2} - \frac{a}{v^2(v-b)^2} + \frac{2R_u T}{(v-b)^3}$$

or $$0 = \frac{a^2}{R_u T v^4 (v-b)} - \frac{a(3v-2b)}{v^3(v-b)^2} - \frac{a}{v^2(v-b)^2} + \frac{2R_u T}{(v-b)^3}$$

Dividing both sides by $(v-b)$, we get

$$0 = \frac{a^2}{R_u T v^4} - \frac{a(3v-2b)}{v^3(v-b)} - \frac{a}{v^2(v-b)} + \frac{2R_u T}{(v-b)^2}$$

Using the value of $R_u T$ from Eq. (ii)

$$0 = \frac{a^2 v^2}{a(v-b)v^4} - \frac{3av - 2ab}{v^3(v-b)} - \frac{a}{v^2(v-b)} + \frac{2a(v-b)}{(v-b)^2 v^2}$$

$$0 = \frac{a}{v^2(v-b)} - \frac{3a}{v^2(v-b)} + \frac{2ab}{v^3(v-b)} - \frac{a}{v^2(v-b)} + \frac{2a}{v^2(v-b)}$$

or $$\frac{2ab}{v^3(v-b)} = \frac{a}{v^2(v-b)}$$

or $$b = v/2 \quad \text{or} \quad v_c = 2b \quad ...(iii)$$

Using the value of v_c in Eq. (ii)

$$R_u T_c = \frac{a(2b-b)}{4b^2} = \frac{ab}{4b^2} = \frac{a}{4b}$$

$\therefore$ $$T_c = \frac{a}{4R_u b}$$

Using the value of T_c and v_c in Eq. (i)

$$p_c = \frac{R_u a}{4R_u b(2b-b)} e^{-\frac{a4R_u b}{R_u a 2b}}$$

$$= \frac{\boldsymbol{a}}{\boldsymbol{4b^2 e^2}}$$ **Proved**

Example 9.9 *A gas obeys the equation of state as $p(v-b) = RT$*

(a) Does $(C_p - C_v)$ change or remain constant?

(b) Will the temperature of this gas change during the throttling process? If it does, does it increase or decrease?

(c) Does the internal energy change depend upon the change in pressure?

Prove all the conclusions.

Solution

An equation of state $p(v-b) = RT$ or $p = \frac{RT}{v-b}$ ...(i)

and $$v = \frac{RT}{p} + b \quad ...(ii)$$

(i) The change in specific heats is given by Eq. (9.59)

$$C_p - C_v = -T\left(\frac{\partial v}{dT}\right)_p^2 \left(\frac{\partial p}{\partial v}\right)_T \quad ...(iii)$$

Now from Eq. (i)

$$\left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT}{(v-b)^2} \quad ...(iv)$$

and from Eq. (ii)

$$\left(\frac{\partial v}{dT}\right)_p = \frac{R}{p} \quad ...(v)$$

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Using the values in relation Eq. (iii), the

$$C_p - C_v = -T\left(\frac{R}{p}\right)^2 \left\{-\frac{RT}{(v-b)^2}\right\}$$

$$= \frac{T^2R^3}{p^2(v-b)^2}$$

$$= \frac{R^2T^2R}{p^2(v-b)^2}$$

Using the value of RT from equation of state as $p(v-b)$, then

$$\boldsymbol{C_p - C_v = \frac{p^2(v-b)^2}{p^2(v-b)^2} R = R}$$

Thus the gas constant R does not change

(ii) During throttling process, the change in enthalpy $dh = 0$

Using in Eq. (9.37);

$$dh = 0 = C_p dT + \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] dp$$

Using the value of $\left(\frac{\partial v}{dT}\right)_p$ from Eq. (v)

$$0 = C_p dT + \left[v - T\frac{R}{p}\right] dp$$

Using the value of v from Eq. (ii)

$$0 = C_p dT + \left[\frac{RT}{p} + b - \frac{RT}{p}\right] dp$$

$$= C_p dT + b dp$$

or $C_p dT = -b dp$

Rearranging, we get

$$\left(\frac{\partial p}{\partial T}\right)_h = -\frac{C_p}{b}$$

or $$\mu = -\frac{\boldsymbol{C_p}}{\boldsymbol{b}}$$

The Joule–Thomson coefficient is negative, thus temperature will increase with decrease in pressure.

(iii) The change in internal energy of a real gas is given by Eq. (9.32)

$$du = C_v dT + \left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right] dv$$

Partial derivative of Eq. (i) with respect to T

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v-b}$$

Then $$du = C_v dT + \left[\frac{RT}{v-b} - p\right] dv$$

Using the value of RT from equation of state as $p(v-b)$

We get $du = C_v dT + [p - p]dv = C_v dT$

Thus, change in internal energy is function of temperature only, it does not depend on pressure variation.

Example 9.10 Van der waals equation is given as

$$\left(p + \frac{a}{v^2}\right)(v-b) = RT$$

Prove that $C_p - C_v = \dfrac{R}{1 - \dfrac{2a(v-b)^2}{RTv^3}}$

Solution The change in specific heats is given by

$$C_p - C_v = -T\left(\frac{\partial v}{dT}\right)_p^2 \left(\frac{\partial p}{\partial v}\right)_T = \frac{T\beta^2 v}{\kappa} \quad ...(i)$$

The van der waals equation

$$\left(p + \frac{a}{v^2}\right)(v-b) = RT$$

or $$p = \frac{RT}{v-b} - \frac{a}{v^2} \quad ...(ii)$$

and $$\left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT}{(v-b)^2} + \frac{2a}{v^3} \quad ...(iii)$$

From cyclic relationship for properties p, v, T

$$\left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial v}{dT}\right)_p \left(\frac{\partial T}{\partial p}\right)_v = -1$$

and $$\left(\frac{\partial v}{dT}\right)_p = -\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial v}{dp}\right)_T$$

For given equation of state

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v-b}$$

Using the value in Eq. (i)

$$C_p - C_v = -T\left(\frac{\partial p}{\partial T}\right)_v^2 \left(\frac{\partial v}{dp}\right)_T^2 \left(\frac{\partial p}{\partial v}\right)_T$$

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$$= -T\left(\frac{\partial p}{\partial T}\right)_v^2 \left(\frac{\partial v}{dp}\right)_T$$

$$= -T\frac{R^2}{(v-b)^2} \times \frac{1}{-\frac{RT}{(v-b)^2}+\frac{2a}{v^3}}$$

$$= -\frac{R^2T}{(v-b)^2} \times \frac{1}{-\frac{RTv^3+2a(v-b)^2}{(v^3)(v-b)^2}}$$

$$= -\frac{R^2T\,v^3}{-RTv^3+2a(v-b)^2}$$

$$= \frac{\boldsymbol{R}}{\boldsymbol{1-\frac{2a(v-b)^2}{RTv^3}}}$$

Example 9.11 *Derive an expression for change in internal energy and change in entropy for van der waal's equation.*

Solution The van der waals Equation is

$$\left(p+\frac{a}{v^2}\right)(v-b) = RT$$

The change in internal energy, Eq. (9.32)

$$du = C_v dT + \left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right] dv$$

The van der waals equation can be written as

$$p = \frac{RT}{v-b} - \frac{a}{v^2}$$

and $$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v-b}$$

Thus $$du = C_v dT + \left[\frac{RT}{v-b} - p\right] dv$$

Using the value of $\frac{RT}{v-b}$ from equation of state as $p + a/v^2$, we get

$$du = C_v dT + \left[\frac{a}{v^2}\right] dv$$

Integrating both sides

$$\int_1^2 du = u_2 - u_1 = \int_{T_1}^{T_2} C_v dT + \int_{v_1}^{v_2} \frac{a}{v^2} dv$$

$$= C_v(T_2 - T_1) + a\left[\frac{1}{v_1} - \frac{1}{v_2}\right] + u_0$$

where u_0 is constant of integration and can be evaluated from boundary conditions.

The change in entropy is given by Eq. (9.40)

$$ds = C_v \frac{dT}{T} + \left(\frac{\partial p}{\partial T}\right)_v dv$$

$$= C_v \frac{dT}{T} + \left(\frac{R}{v-b}\right) dv$$

On integration

$$\Delta s = s_2 - s_1 = C_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2-b}{v_1-b}\right) + s_0$$

where s_0 is constant of integration.

Example 9.12 Prove that

$$\left(\frac{\partial u}{\partial p}\right)_T = pv\kappa - Tv\beta$$

Using the method of partial differential.

Solution We have

$$du = Tds - pdv \qquad ...(i)$$

Differentiating it partially w.r.t. p, treating T as constant:

$$\left(\frac{\partial u}{\partial p}\right)_T = T\left(\frac{\partial s}{\partial p}\right)_T - p\left(\frac{\partial v}{\partial p}\right)_T$$

Introducing the fourth Maxwell's relation

$$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$$

$$\left(\frac{\partial u}{\partial p}\right)_T = -T\left(\frac{\partial v}{\partial T}\right)_p - p\left(\frac{\partial v}{\partial p}\right)_T \qquad ...(ii)$$

Using the definition of volumetric expansion coefficient

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p$$

and coefficient of isothermal compressibility

$$\kappa = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$$

Equation (ii) leads to

$$\left(\frac{\partial u}{\partial p}\right)_T = -Tv\beta + pv\kappa$$ **Proved**

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Summary

- The thermodynamic relations are used for those properties which cannot be measured directly.
- The Gibbsian relations are

$$du = Tds - pdv$$
$$dh = Tds + vdp$$
$$dh = -pdv - sdT$$
$$dg = vdp - sdT$$

- The Maxwell relations are equations that relate the partial derivative of properties p, v, T and s of a simple compressible fluid to each other. These Maxwell relations derived from Gibbsian relations are listed below:

$$\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_v$$
$$\left(\frac{\partial T}{\partial p}\right)_s = \left(\frac{\partial v}{\partial s}\right)_p$$
$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T$$
$$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$$

- The coefficient of volumetric expansion β is defined as

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p$$

- The coefficient of isothermal compressibility is defined as

$$\kappa = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$$

- The coefficient of isentropic compressibility is defined as

$$\alpha = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_s$$

and $\left(\frac{\partial p}{\partial T}\right)_v = \frac{\beta}{\kappa}$

- The general expressions for internal energy, enthalpy and entropy in terms of p, v, T and specific heat data

$$du = C_v dT + \left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right] dv$$

$$dh = C_p dT + \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] dp$$

$$ds = C_v \frac{dT}{T} + \left(\frac{\partial p}{\partial T}\right)_v dv$$

$$= C_p \frac{dT}{T} - \left(\frac{\partial v}{\partial T}\right)_p dp$$

- The general relation for C_p and C_v

$$C_p = T\left(\frac{\partial s}{\partial T}\right)_p$$

$$C_v = T\left(\frac{\partial s}{\partial T}\right)_v$$

$$C_p - C_v = T\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial v}{\partial T}\right)_p$$

$$C_p - C_v = -T\left(\frac{\partial v}{\partial T}\right)_p^2 \left(\frac{\partial p}{\partial v}\right)_T$$

$$= \frac{vT\beta^2}{\kappa}$$

$$\frac{C_p}{C_v} = \left(\frac{\partial v}{\partial p}\right)_T \left(\frac{\partial p}{\partial v}\right)_s$$

- The Joule–Thomson coefficient

$$\mu = \left(\frac{\partial T}{\partial p}\right)_h$$

The Joule–Thomson coefficient μ decides the heating or cooling effect after throttling

$$\mu = \begin{cases} < 0 & T \text{ increases} \\ = 0 & T \text{ remains constat} \\ > 0 & T \text{ decreases} \end{cases}$$

- For an ideal gas, during throttling

$h = C$, thus T = Constant

- The coefficient of constant temperature is defined as

$$C_T = \left(\frac{\partial h}{\partial p}\right)_T$$

- The Clausius–Clapeyron equation yields

$$\frac{dp}{dT} = \frac{h_{fg}}{T v_{fg}}$$

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Review Questions

1. What are Maxwell relations and why they are important in thermodynamics?
2. Sketch the thermodynamic mnemonic diagram and explain its use to obtain Gibbsian equations and Maxwell relations.
3. Define Helmholtz and Gibbs functions.
4. Show that the work done by the system during an adiabatic process is equal to decrease in internal energy of the system.
5. What is Clapeyron equation?
6. What are assumptions made in obtaining Clausius equation from Clapeyron equation?
7. State Helmholtz and Gibbs function and then derive Gibbsian relations.
8. State Gibbsian relations and then derive Maxwell relations.
9. Define
   (a) Coefficient of volumetric expansion,
   (b) Coefficient of isothermal compressibility.
   (c) Isentropic compressibility.
10. Establish the relationship between β and κ for an ideal gas.
11. Considering specific entropy as function of pressure, sp. volume and temperature, derive three Tds relations.
    Using Tds relations, prove that
$$C_p - C_v = T\left(\frac{\partial v}{\partial T}\right)_p^2 \left(\frac{\partial p}{\partial v}\right)_T$$
12. What is Joule Thompson coefficient. Discuss the zone of heating and cooling with help of inversion curve.
13. Considering $u = f(T, v)$, prove that
$$du = CvdT + \left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right] dv$$

Problems

1. Develop an expression for the change in entropy of a gas for which the equation of state is
$$p = \frac{RT}{v-b} - \frac{a}{v^2}$$

2. Show that the latent heat of vaporisation may be expressed as
$$h_{fg} = T\int_{v_f}^{v_g}\left(\frac{\partial p}{\partial T}\right)_v dv$$

3. A certain gas follows the equation of state $p(v - b) = RT$. Show that for a reversible adiabatic process of this gas
$$T(v-b)^{R/C_v} = \text{constant}$$

4. Determine the Joule–Thomson coefficient of water at
(a) 70 kPa, 150°C and (b) 20.5 MPa. 400°C.

5. Prove that for a van der Waals gas the inversion temperature is given by
$$T = \frac{2a}{bR}\left(1 - \frac{b}{v}\right)^2$$

6. A rigid vessel of 0.3 m^3 volume contains 10 kg of air at 300 K. Using (a) the perfect gas equation, (b) the van der Waals equation of state, and (c) the generalised compressibility chart, determine pressure extracted by air on the vessel.
[Ans. (a) 28.67 bar (b) 28.13 bar (c) 28.32 bar]

7. At pressures above 22 MPa, an increase in pressure at constant temperature of 10°C produces a decrease in entropy of water. What must happen to the specific volume under these conditions?

8. For a pure substance $s = f(p, v)$. Prove that
$$Tds = \left(\frac{\kappa C_v}{\beta}\right)dp + \left(\frac{C_p}{v\beta}\right)dv$$

9. Prove that
$$\left(\frac{\partial f}{\partial p}\right)_s = -\frac{p\kappa}{\beta} \quad \text{and} \quad \left(\frac{\partial g}{\partial v}\right)_T = -\frac{1}{\kappa}$$

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10. Prove that the slope of an isentropic process on a *T–p* diagram is given by

$$\left(\frac{\partial T}{\partial p}\right)_s = \frac{\beta v T}{C_p}$$

11. Calculate the rise in pressure of a liquid, when it is heated at constant volume from 25°C to 75°C. Take $\beta = 2.07 \times 10^{-4}\ K^{-1}$, and $\kappa = 4.85 \times 10^{-4}\ (MPa)^{-1}$ for this liquid.

12. For the van der Waals equation, prove that

$$\mu = \frac{v}{C_p}\left[\frac{2a(v-b)^2 - TR_u b v^2}{R_u T v^2 - 2a(v-b)^2}\right]$$

Objective Questions

1. Maxwell's Thermodynamic relations are valid for
   (a) all processes
   (b) a closed system
   (c) a thermodynamic system in equilibrium.
   (d) an open system.
2. The coefficient of volumetric expansion β is given by
   (a) $\frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p$ (b) $\frac{1}{p}\left(\frac{\partial v}{\partial T}\right)_p$
   (c) $\frac{1}{T}\left(\frac{\partial p}{\partial T}\right)_v$ (d) $\frac{1}{p}\left(\frac{\partial v}{\partial T}\right)_T$
3. Isothermal compressibility κ of a substance is
   (a) $-\frac{1}{p}\left(\frac{\partial p}{\partial v}\right)_T$ (b) $-\frac{1}{p}\left(\frac{\partial v}{\partial p}\right)_T$
   (c) $-\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$ (d) none of the above
4. The specific heat relation is
   (a) $C_p - C_v = \frac{vT\beta^2}{\kappa}$ (b) $C_p - C_v = \frac{vT\kappa}{\beta}$
   (c) $C_p - C_v = \frac{vT\kappa}{\beta^2}$ (d) $C_p - C_v = \frac{v^2 T\beta}{\kappa}$
5. When z is the function of two independent variables x and y, then
   (a) $dz = \frac{\partial y}{\partial x}dz + \frac{\partial x}{\partial y}dz$
   (b) $dz = \frac{\partial y}{\partial x}dz - \frac{\partial x}{\partial y}dz$
   (c) $dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dz$
   (d) $dz = \frac{\partial z}{\partial x}dx - \frac{\partial z}{\partial y}dy$
6. The cyclic relation of three variable is given by
   (a) $\left(\frac{\partial x}{\partial y}\right)_z\left(\frac{\partial y}{\partial z}\right)_x\left(\frac{\partial z}{\partial x}\right)_y = 1$
   (b) $\left(\frac{\partial x}{\partial y}\right)_z\left(\frac{\partial y}{\partial z}\right)_x\left(\frac{\partial z}{\partial x}\right)_y = -1$
   (c) $\left(\frac{\partial z}{\partial x}\right)_y\left(\frac{\partial z}{\partial y}\right)_x = 1$
   (d) $\left(\frac{\partial z}{\partial x}\right)_y\left(\frac{\partial z}{\partial y}\right)_x = -1$
7. The *Tds* equation is
   (a) $Tds = C_v dT + \frac{T\beta}{\kappa}dv$
   (b) $Tds = C_v dT - \frac{T\beta}{\kappa}dv$
   (c) $Tds = C_v dT + \frac{T\kappa}{\beta}dv$
   (d) $Tds = C_v dT + \frac{T\beta}{\kappa}dp$
8. The *Tds* relation is also given by
   (a) $Tds = \frac{\kappa C_v}{\beta}dv + \frac{C_p}{v\beta}dv$
   (b) $Tds = \frac{\kappa C_v}{\beta}dp + \frac{C_p}{v\beta}dv$
   (c) $Tds = \frac{\kappa C_p}{\beta}dv + \frac{C_p}{v\beta}dv$
   (d) $Tds = \frac{\kappa C_v}{\beta}dv + \frac{C_v}{v\beta}dv$
9. The specific heat at constant pressure is given by
   (a) $T\left(\frac{\partial s}{\partial T}\right)_p$ (b) $T\left(\frac{\partial T}{\partial s}\right)_p$

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(c) $T\left(\frac{\partial v}{\partial T}\right)_p$ (d) $T\left(\frac{\partial p}{\partial T}\right)_v$

10. A relation of vapour pressure to enthalpy of vaporisation is expressed in
    (a) van der waals equation
    (b) Maxwell's relations
    (c) Carrier's equation
    (d) Clausius–Claypeyron equation
11. Joule–Thompson coefficient μ is given as
    (a) $\frac{k}{\alpha}$ (b) $\left(\frac{\partial T}{\partial p}\right)_h$
    (c) $\left(\frac{\partial v}{\partial p}\right)_s$ (d) $\left(\frac{\partial s}{\partial p}\right)_T$
12. When $\mu > 0$, the temperature of gas with decrease in pressure
    (a) decreases (b) increases
    (c) remains constant (d) none of the above
13. The coefficient of constant temperature C_T is given by
    (a) $\left(\frac{\partial p}{\partial T}\right)_h$ (b) $\left(\frac{\partial h}{\partial p}\right)_T$
    (c) $\left(\frac{\partial T}{\partial h}\right)_p$ (d) $\left(\frac{\partial v}{\partial T}\right)_p$
14. μ, C_p and C_T are related as
    (a) $\mu = C_T C_p$ (b) $\mu = \frac{C_T}{C_p}$
    (c) $\mu = -\frac{C_T}{C_p}$ (d) $C_T = -\frac{\mu}{C_p}$

Answers

1. (c) 2. (a) 3. (c) 4. (a) 5. (c) 6. (b) 7. (a) 8. (b)
9. (a) 10. (d) 11. (b) 12 (a) 13. (b) 14. (c)

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10

Compressible Fluid Flow

Introduction

When the density of fluid is the function of pressure, then the fluid is called *compressible fluid*. The compressible fluid flow involves motion of fluid with Mach number greater than 0.3. It also includes dynamic, thermal and viscous effects. The fluid dynamics of compressible flows is generally referred as *gas dynamics.* The knowledge of gas dynamics is essential in the design of turbo-machines.

10.1 STATIC PROPERTIES

Whenever the kinetic and potential energies of a fluid are negligible, the properties of the fluid are referred as *static properties.* For example, the energy of a fluid system can be expressed by *enthalpy* as combination of internal energy and flow energy of the fluid as

$$h = u + pv \qquad ...(10.1)$$

If the kinetic energy of the fluid system becomes negligible then this enthalpy is referred as *static enthalpy.*

10.2 STAGNATION PROPERTIES

For high speed fluid flow, the potential energy of the fluid is negligible but kinetic energy is significant. In such cases, each property of the fluid is influenced by kinetic energy, thus it is convenient to choose a property with kinetic energy in a single term to form a stagnation property. Hence, a *stagnation state* is defined as a state in a fluid flow field, when the fluid is brought to rest isentropically.

10.2.1 Stagnation Enthalpy

The stagnation enthalpy is the total energy of a steadily flowing fluid. It is designated as h_0 and expressed as

$$h_{stagnation} = h_0 = h + \frac{\mathcal{V}^2}{2} \text{ (J/kg)} \qquad ...(10.2)$$

The stagnation enthalpy h_0 is made of a *static* part h and a *dynamic* part $\frac{\mathcal{V}^2}{2}$. In the absence of

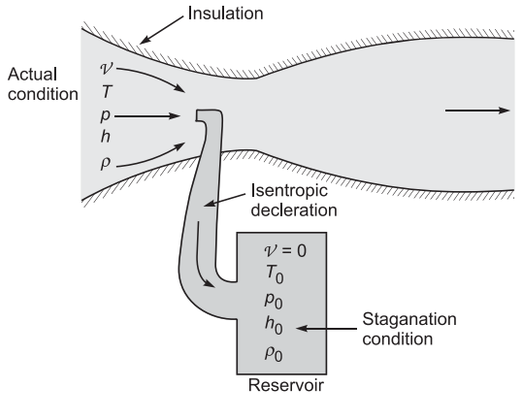

Fig. 10.1 *Concept of stagnation state*

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potential energy change, the steady-flow energy equation reduces to

$$q - w = h_2 + \frac{\mathcal{V}_2^2}{2} - \left(h_1 + \frac{\mathcal{V}_1^2}{2}\right)$$

$$= h_{02} - h_{01} \qquad ...(10.3)$$

It represents the total energy of the flowing fluid system and if the fluid velocity is brought to zero isentropically; ($q = 0$, $w = 0$), then

$$h_{02} = h_{01} \qquad ...(10.4)$$

or $$h_1 + \frac{\mathcal{V}_1^2}{2} = h_2 + 0 = h_{02} \qquad ...(10.5)$$

Thus, the stagnation enthalpy or *total enthalpy* represents the enthalpy of the fluid when it is brought to rest isentropically.

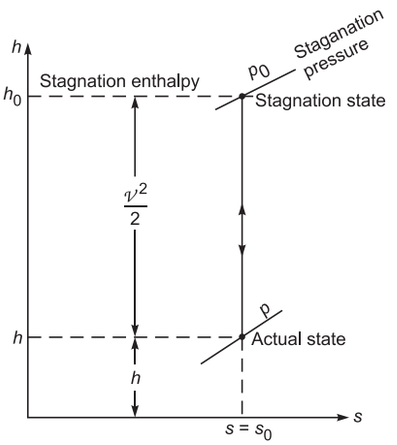

Fig. 10.2 *Defintion of staganation enthalpy*

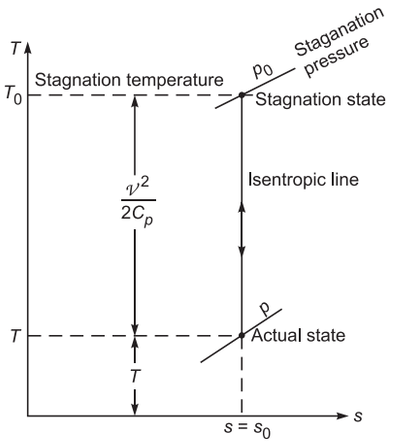

Fig. 10.3 *Definition of staganation temperature*

10.2.2 Stagnation Temperature

For an ideal gas, the enthalpy is a function of temperature only. The stagnation enthalpy can be expressed as

$$C_p T_0 = C_p T + \frac{\mathcal{V}^2}{2}$$

or $$T_0 = T + \frac{\mathcal{V}^2}{2C_p} (°C) \qquad ...(10.6)$$

where T_0 is the *stagnation* or *total temperature* of the flowing fluid and it represents the temperature that a fluid attains when it is brought to rest isentropically. It should be expressed in °C. The term $\frac{\mathcal{V}_1^2}{2C_p}$ represents temperature rise during such isentropic process and thus it is called *dynamic temperature* or *velocity temperature.*

10.2.3 Stagnation Pressure

Total pressure or *stagnation pressure* of a flowing fluid is the pressure that it attains when the fluid is brought to rest isentropically. The stagnation pressure exceeds the static pressure by the pressure equivalent of the velocity. For an ideal gas with constant specific heat, the stagnation pressure p_0 is related to static pressure p as

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma-1}{\gamma}} \qquad ...(10.7)$$

10.2.4 Stagnation Density

The *stagnation density* or *total desnity* of a flowing fluid at any location is the density corresponding to stagnation pressure at that location. From the equation of state for an ideal gas

$$\rho_0 = \frac{p_0}{RT_0}$$

For isentropic deceleration process, the stagnation density ρ_0 is related to the actual density ρ of the flowing fluid as

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{\gamma-1}} \qquad ...(10.8)$$

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Example 10.1 *Air is flowing isentropically through a nozzle at 27°C and 0.8 bar with a velocity of 120 m/s. Calculate the stagnation enthalpy, stagnation temperature, stagnation pressure and stagnation density of air.*

Solution

Given The isentropic flow of air through a nozzle with

$T = 27°C + 273 = 300$ K

$p = 0.8$ bar $= 80$ kPa

$\mathcal{V} = 120$ m/s

To find

(i) Stagnation enthalpy,
(ii) Stagnation temperature,
(iii) Stagnation pressure, and
(iv) Stagnation density.

Assumptions For air:

(i) Specific gas constant $R = 0.287$ kJ/kg·K.
(ii) Specific heat at constant pressure, $C_p = 1005$ J/kg·K.
(iii) Ratio of specific heat $\gamma = 1.4$.

Analysis

(i) *Stagnation Enthalpy*

$$h_0 = h + \frac{\mathcal{V}^2}{2} = C_p T + \frac{\mathcal{V}^2}{2}$$

$$= 1005 \times 300 + \frac{(120)^2}{2}$$

$$= \mathbf{308.87 \times 10^3\ J/kg}$$

(ii) *Stagnation temperature*

$$T_0 = T + \frac{\mathcal{V}_1^2}{2C_p}$$

$$= 300 + \frac{(120)^2}{2 \times 1005} = 300 + 7.16$$

$$= \mathbf{307.16\ K}$$

(iii) *Stagnation pressure*

$$p_0 = p\left(\frac{T_0}{T}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 0.8 \times \left(\frac{307.16}{300}\right)^{\frac{1.4-1}{1.4}} = \mathbf{0.868\ bar}$$

(iv) *Stagnation density*

$$\rho_0 = \rho\left(\frac{T_0}{T}\right)^{\frac{1}{\gamma-1}} = \frac{p}{RT}\left(\frac{T_0}{T}\right)^{\frac{1}{\gamma-1}}$$

$$= \frac{80}{0.287 \times 300} \times \left(\frac{307.16}{300}\right)^{\frac{1}{1.4-1}}$$

$$= \mathbf{0.985\ kg/m^3}$$

Example 10.2 *Atmospheric air at 101 kPa and 27°C is accelerated isentropically in a nozzle from 13.5 m/s to 223.8 m/s. Find the changes in (a) temperature, (b) pressure, (c) density, (d) stagnation temperature and stagnation pressure.*

Solution

Given Isentopic expansion of air through a nozzle

$p_1 = 101$ kPa $\quad T_1 = 27°C = 300$ K

$\mathcal{V}_1 = 13.5$ m/s $\quad \mathcal{V}_2 = 223.8$ m/s

To find Change in

(i) Temperature,
(ii) Pressure,
(iii) Density, and
(iv) Stagnation temperature and stagnation pressure.

Assumptions

(i) Air is an ideal gas with constant specific heats, thus

$R = 0.287$ kJ/kg·K

$C_p = 1.005$ kJ/kg·K,

$\gamma = 1.4$

(ii) Changes in potential energy is negligible.

Analysis

(i) *Change in static temperature*

Applying steady-flow energy equation between two states

$$h_1 + \frac{\mathcal{V}_1^2}{2} = h_2 + \frac{\mathcal{V}_2^2}{2}$$

or $$C_p T_1 + \frac{\mathcal{V}_1^2}{2} = C_p T_2 + \frac{\mathcal{V}_2^2}{2}$$

or $$T_2 = T_1 + \frac{\mathcal{V}_1^2 - \mathcal{V}_2^2}{2C_p}$$

$$= 300 + \frac{(13.5)^2 - (223.8)^2}{2 \times 1005}$$

$$= 275.18\ \text{K}$$

Change in temperature,

$$\Delta T = T_2 - T_1 = 275.18 - 300$$

$$= \mathbf{-24.82\ K\ (or\ °C)}$$

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(ii) *Change in static pressure*

For isentropic expansion

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= \left(\frac{275.18}{300}\right)^{\frac{1.4}{1.4-1}} = 0.739$$

or $p_2 = 0.739 \times (101 \text{ kPa})$

$= 74.65 \text{ kPa}$

Change in pressure

$$\Delta p = p_2 - p_1 = 74.65 - 101$$

$$= \mathbf{-26.5 \text{ kPa}}$$

(iii) *Change in density of air*

$$\text{Initial density, } \rho_1 = \frac{p_1}{RT_1} = \frac{101}{0.287 \times 300}$$

$= 1.173 \text{ kg/m}^3$

Final density of air,

$$\rho_2 = \frac{p_2}{RT_2} = \frac{74.65}{0.287 \times 275.18}$$

$= 0.945 \text{ kg/m}^3$

Change in density,

$$\Delta\rho = \rho_2 - \rho_1 = 0.945 - 1.173$$

$$= \mathbf{0.228 \text{ kg/m}^3}$$

(iv) *Change in stagnation temperature of air*

Staganation temperature at the state 1

$$T_{01} = T_1 + \frac{\mathcal{V}_1^2}{2C_p}$$

$$= 300 + \frac{(13.5)^2}{2 \times 1005} = 300.09 \text{ K}$$

Stagnation temperature after expansion

$$T_{02} = T_2 + \frac{\mathcal{V}_2^2}{2}$$

$$= (275.18 \text{ K}) + \frac{(223.8)^2}{2 \times 1005}$$

$$= \mathbf{300.09 \text{ K}}$$

No change in stagnation temperature. The stagnation temperature remains constant throghout the nozzle for isentropic flow.

(v) *Change in stagnation pressure*

Initial *stagnation pressure* of air,

$$p_{01} = p_1\left(\frac{T_{02}}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= 101 \times \left(\frac{300.09}{300}\right)^{\frac{1.4}{0.4}}$$

$$= \mathbf{101.10 \text{ kPa}}$$

Stagnation pressure after expansion

$$p_{02} = p_2\left(\frac{T_{02}}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= 74.65 \times \left(\frac{300.09}{275.18}\right)^{\frac{1.4}{0.4}}$$

$$= \mathbf{101.1 \text{ kPa}}$$

There is no change in stagnation pressure. The stagnation pressure remains constant throghout the nozzle for isentropic flow.

10.3 VELOCITY OF SOUND: SONIC VELOCITY

It is the velocity with which the sound travels through a medium. The velocity at which a small pressure wave propagates in a fluid is called *velocity of sound* or *sonic velocity*. It is designated as *a*.

Consider a gas at the rest within a constant area duct as shown in Fig. 10.4. The piston moves to the right with a small and constant incremental velocity

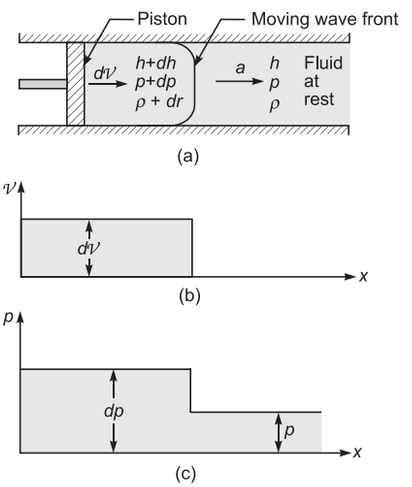

Fig. 10.4 *Propagation of small pressure wave along a duct*

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$d\mathcal{V}$, creating a sonic wave. The wave front travels to the right through the gas at the sonic velocity a and it separates the moving gas adjacent to the piston from the gas which is still at rest.

Consider a control volume that encloses the wave front and moves with it as shown in Fig. 10.5. The flow pattern can be better studied, when an observer moves with the wave front. The stagnation gas to the right, will appear to be moving towards the wave front with a velocity a and the gas to the left will appear to be moving away from the wave front with a velocity $a - d\mathcal{V}$. Thus, the wave can be considered stationary within the control volume and its flow from right to left.

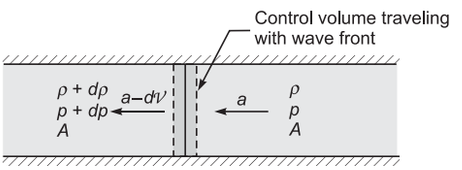

Fig. 10.5 *Observer moving with a small pressure wave*

The continuity equation

$$\dot{m} = \rho A a \qquad ...(10.9)$$

For a steady flow system, it can be expressed as

$$\dot{m}_{left} = \dot{m}_{right}$$

$$(\rho + d\rho)(a - d\mathcal{V})A = \rho A a$$

At the left of the wave front, the gas experiences an incremental change in its properties, while on the right side of the wave front, the gas maintains its original properties as shown in Fig. 10.5.

$$(\rho + d\rho)(a - d\mathcal{V}) = \rho a$$

or $$\rho a - \rho d\mathcal{V} + a d\rho - d\mathcal{V} d\rho = \rho a$$

Neglecting the higher order term $d\mathcal{V} d\rho$, we get

$$\rho d\mathcal{V} = a d\rho \qquad ...(10.10)$$

Further, the rate of change of momentum of gas will be equal to applied forces. Hence

$$A[p - (p + dp)] = \dot{m}[(a - d\mathcal{V}) - a]$$

or $$A\,dp = \dot{m}\,d\mathcal{V}$$

Using the continuity equation (10.9), we get

$$dp = \rho a d\mathcal{V}$$

or $$d\mathcal{V} = \frac{dp}{\rho a} \qquad ...(10.11)$$

Substituting $d\mathcal{V}$ in Eq. (10.10),

$$\rho \frac{dp}{\rho a} = a d\rho$$

or $$a^2 = \frac{dp}{d\rho} \qquad ...(10.12)$$

The intensity of the sonic wave is very small and does not cause any significant change in the pressure and temperature of the gas. Hence, the propagation of the wave can be considered as isentropic. Then

$$a = \left(\sqrt{\frac{\partial p}{\partial \rho}}\right)_{s=C} \qquad ...(10.13)$$

For an isentropic process,

$$\frac{p}{\rho^{\gamma}} = C \text{ (a constant)}$$

Differentiating with respect to ρ, we get

$$\frac{dp}{d\rho} = \left(\frac{\partial p}{\partial \rho}\right)_{s=C}$$

$$= \gamma C \rho^{\gamma - 1} = \frac{\gamma p}{\rho} \qquad ...(10.14)$$

Using $$\frac{p}{\rho} = R T$$

where R is the gas constant. Combining Eqs (10.13) and (10.14)), we get

$$a = \sqrt{\gamma R T} \qquad ...(10.15)$$

In Eq. (10.15), the quantities γ and R are constants. Thus the sonic velocity is the function of temperature only. For a given gas, the speed of the sound depends only on the square root of the absolute temperature. Equation (10.15) can be written as

$$a = \sqrt{\frac{\gamma R_u T}{\mathcal{M}}} \qquad ...(10.16)$$

where $\mathcal{M}$ is the molecular weight of the gas and R_u is the universal gas constant (= 83144 J/kmol·K). The γ is almost constant for all gases, and hence, speed of sound at a given temperature is inversely proportional to its molecular weight.

Some typical values for the speed of sound at 0°C are given in Table 10.1.

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Table 10.1

Gas	*M Molecular, weight (kg/kmol)*	*γ*	*a, Speed of sound at 0°C. (m/s)*
Air	28.96	1.404	331
Argon	39.94	1.667	308
Carbon dioxide (CO_2)	44.01	1.300	258
Freon 12 (CCI_2F_2)	120.90	1.139	146
Helium (He)	4.00	1.667	970
Hydrogen (H_2)	2.01	1.407	1270

10.4 MACH NUMBER

The Mach number at a point in a flow field is defined as the *ratio of local fluid velocity to the sonic or acoustic velocity* in the medium. It is a non-dimensional number and it is designated as M and expressed as

$$M = \frac{\text{Fluid velocity}}{\text{Speed of sound}} = \frac{\mathcal{V}}{a} \quad ...(10.17)$$

The definition of Mach number can also be interpreted as the square root of of the ratio of the *inertia force* due to flow to the *elastic force* of fluid.

$$M = \sqrt{\frac{\text{Inertia force of flow}}{\text{Elastic force of fluid}}}$$

The Mach number may be regarded as a measure of the ratio of kinetic energy of the flowing fluid to the kinetic energy of random molecular motion in the fluid.

10.4.1 Different Regimes of Compressible Fluid Flow

The Mach number is a very important parameter in the analysis of compressible fluid flow. The compressible regions have been classified according to Mach number of flow as

Mach number	*Classification*
$M < 1$	Subsonic flow
$M = 1$	Sonic flow
$M > 1$	Supersonic flow
$M > 5$	Hypersonic flow

Subsonic Flow If the fluid flows with a Mach number less than unity (that is, $\mathcal{V} < a$) then the flow is called a *subsonic flow*. Such a flow is characterised by smooth streamlines and continuously varying properties. Mach number ranges from 0.3 to 1.

Sonic Flow If the Mach number is equal to 1 (that is, $\mathcal{V} = a$) then the flow is regarded as a *sonic flow*. If fhe flow is with a Mach number around unity ($0.8 < M < 1.2$), it is said to be a *transonic flow*.

Supersonic Flow If the fluid flows with Mach number greater than unity (that is, $\mathcal{V} > a$) then the flow is called a *supersonic flow*. Such a flow is observed with an oblique shock in the flow field and a significant change in the properties of the fluid during flow. Mach number ranges from 1 to 5.

Hypersonic Flow If the fluid flows with a very high Mach number (that is, $M > 5$) then the flow is called a *hypersonic flow*. Such a flow is observed with a severe shock in the flow field and abrupt change in properties of fluid during flow.

10.4.2 Physical Difference Between Flow Regimes

When the pressure field is created by a point disturbance in a stagnant gas, the pressure pulses generated from the point of disturbance move in all directions as spherical sound waves with a sonic velocity in the gas medium.

In order to illustrate the effect of a velocity of body relative to the speed of sound in the flow field, consider a small object moving at a uniform linear velocity $\mathcal{V}$ in a stationary compressible gas. This small object, as it moves, tends to divert the stationary gas. The diverted fluid pushes the surrounding gas, and this local disturbance creates pressure pulses which propagate further into the surrounding gas.

Figures 10.6, 10.7 and 10.8 show the pressure-pulse pattern for different speeds of the moving object in a gas. In each pattern, the point A (at $t = 0$) represents the present position of the object. The points B, C, D are respective positions of the object before time intervals t, $2t$, $3t$, respectively.

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The spherical sound wave originates from the point source and grows in all directions. The distance travelled by the object is represented by $\mathcal{V}t$ in time t. While the distance travelled by the wave front generated at that time is marked with respective radii (sonic velocity × time) at, $2at$, $3at$, respectively, where, at represents the distance travelled by the wave front in time t, $2at$ in time $2t$ and $3at$ in time $3t$ drawn from locations B, C and D respectively.

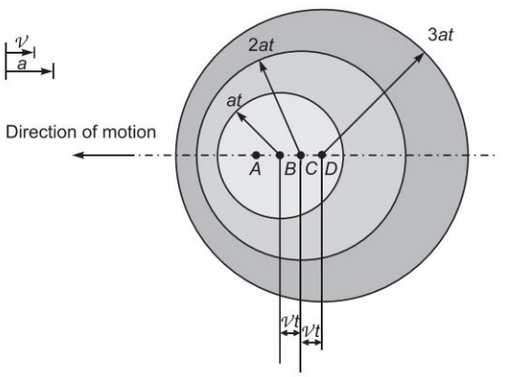

Fig. 10.6 *Spread of waves from source of disturbance at a subsonic speed*

When the point source velocity $\mathcal{V}$ is less than the sonic velocity a, (subsonic $M < 1$), the sound waves are generated and travel in all directions around the originating point. Since the sonic velocity a is always greater than the velocity of the object $\mathcal{V}$, the wavefront generated by the moving object is always ahead of the object as shown in Fig. 10.6. Consequently, if the body moves in a compressible fluid with a subsonic speed, the fluid ahead of the body becomes aware of the presence of the body. For example, all the automobiles travel with subsonic velocities, thus the horn is heard before the vehicle reaches a person standing on the road.

When the point source travels with sonic velocity ($M = 1$), the front of the pressure disturbance moves the same distance as a small object as shown in Fig. 10.7. Consequently, all the disturbances are created by the object at the present position A of the object, and all wave fronts are contained in the half plane to the right of the object.

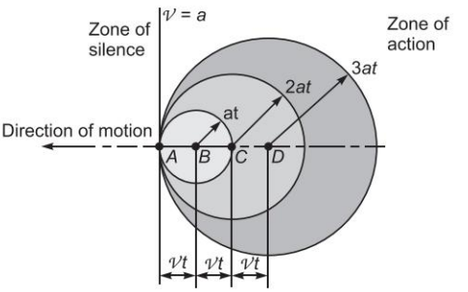

Fig. 10.7 *Spread of waves from source of disturbance at a sonic speed*

Since the wave front cannot overtake the body, the fluid ahead of the body does not receive any information about the body. The region ahead of the the point source is called a *zone of silence* and the sound cannot be heard in this zone. The zone behind the point source, where sound waves are traversed, is called the *zone of action*. The two zones are separated by a line perpendicular to the direction of the motion of the body.

When the point source travels with supersonic velocity ($M > 1$), the wave front circles will not exist at the point source as shown in Fig. 10.8. The wave front originating before time interval t, $2t$ and $3t$ are drawn with their originating locations. Since the object is moving with a speed greater than the sonic velocity, the wave front of disturbance created by the body lags behind the body and the wavefront cannot overtake the moving body. The zone of action takes the form of a cone.

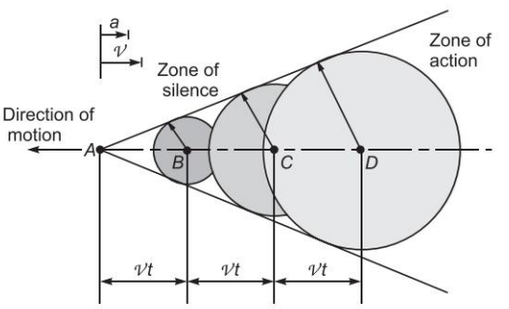

Fig. 10.8 *Spread of waves from the source of disturbance at a supersonic speed*

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10.4.3 Mach Cone and Mach Angle

When the speed of an object is greater than the speed of sound, the object is always ahead of the spherical wavefronts generated by the object. The circles of wavefronts are formed along a straight line, known as *Mach line*. The angle between the Mach line and the direction of flow is known as the *Mach angle*. All the wavefronts are confined to the region within a cone, called the *Mach cone*. It is the zone of action as shown in Fig. 10.8. The atmosphere outside the Mach cone is called the *zone of silence.* Hence, the sound of a jet plane is heard after it passes forward.

It is observed that for supersonic flow, all the waves lie within the Mach cone, which has its vertex at the point source, A. The semivertex angle of the Mach cone is the *Mach angle*, denoted by α and given as

$$\sin \alpha = \frac{a}{\mathcal{V}} = \frac{1}{M}$$

or
$$\alpha = \sin^{-1}\left(\frac{1}{M}\right) \qquad ...(10.18)$$

Example 10.3 *A gas has a molecular weight of 44 and a specific heat ratio of 1.3. Calculate the speed of the sound in this gas if the temperature is –23°C. If this gas is flowing at a velocity of 450 m/s, calculate the Mach number and Mach angle.*

Solution

Given

$\mathcal{M} = 44$ $\qquad$ $\gamma = 1.3$

$T = -23°C = 250$ K $\qquad$ $\mathcal{V} = 450$ m/s

To find

(i) Speed of sound,
(ii) Mach number, and
(iii) Mach angle.

Analysis The specific gas constant for the given gas;

$$R = \frac{R_u}{\mathcal{M}} = \frac{8314}{44} = 188.95 \text{ J/kg·K}$$

(i) The speed of sound is given by Eq. (10.15);

$$a = \sqrt{\gamma RT} = \sqrt{1.3 \times 188.95 \times 250} = \mathbf{247.8 \text{ m/s}}$$

(ii) Mach number is given by Eq. (10.17);

$$M = \frac{\mathcal{V}}{a} = \frac{450}{247.8} = \mathbf{1.816}$$

(iii) Mach angle is given by Eq. (10.18);

$$\alpha = \sin^{-1}\left(\frac{1}{M}\right) = \sin^{-1}\left(\frac{1}{1.816}\right) = \mathbf{33.5°}$$

Example 10.4 *An air plane travels at a Mach number of 1.5 at an elevation where the temperature is –37°C. Determine the velocity of the plane in kmph. Assume $\gamma = 1.4$.*

Solution

Given Flight of an air plane

$M = 1.5$

$T = -37°C = 236$ K

$\gamma = 1.4$

To find Velocity of the air plane in kmph

Assumption

(i) Air as a perfect gas,
(ii) The specific gas constant $R = 287$ J/kg·K,
(iii) Steady state operation.

Analysis The sonic velocity,

$$a = \sqrt{\gamma RT}$$
$$= \sqrt{1.4 \times 287 \times 236} = 307.94 \text{ m/s}$$

Actual velocity of the plane, using Eq. (10.17)

$$\mathcal{V} = Ma$$
$$= 1.5 \times 307.94 = 461.9 \text{ m/s}$$
$$= 461.9 \times 10^{-3} \times 3600$$
$$= \mathbf{1662.8 \text{ kmph}}$$

Example 10.5 *A supersonic aircraft is flying at an altitude of 3 km with a constant flight speed of 2000 km/h. The aircraft passes over a ground observation post. Find the time taken to hear the sound waves from the aircraft at the observation post after it has past directly over it. Assume the average temperature of atmospheric air at the obervation post to be 27°C.*

Solution

Given Supersonic aircraft

$H = 3$ km $= 3000$ m

$\mathcal{V} = 2000$ km/h

$T_1 = 27°C = 300$ K

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To find Time taken by sound waves to reach the ground.

Assumptions

(i) The gas constant $R = 287$ J/kg·K.
(ii) The ratio of specific heats, $\gamma = 1.4$.

Analysis The flight velocity;

$$\mathcal{V} = \frac{2000 \times 1000}{3600} = 555.56 \text{ m/s}$$

Velocity of sound in the atmospheric air;

$$a = \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 300}$$
$$= 347.2 \text{ m/s}$$

Flight mach number; $M = \frac{\mathcal{V}}{a} = \frac{555.56}{347.2} = 1.6$

Mach angle is given by Eq. (10.18);

$$\alpha = \sin^{-1}\left(\frac{1}{M}\right) = \sin^{-1}\left(\frac{1}{1.6}\right)$$
$$= 38.68°$$

From Fig. 10.9, distance

$$AB = \mathcal{V}t = \frac{\text{Height } BC}{\tan \alpha} = \frac{3000 \text{ m}}{\tan(38.68°)}$$
$$= 3747.3 \text{ m}$$

Time $$t = \frac{\text{Distance } AB}{\mathcal{V}} = \frac{3747.3 \text{ m}}{555.56 \text{ m/s}}$$
$$= 6.75 \text{ s}$$

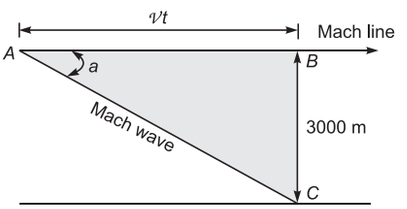

Fig. 10.9 *Mach wave with supersonic aircraft*

Example 10.6 *The air at 100 m/s initially is accelerated isentropically in a nozzle. If the temperature at the initial state is 400 K and the Mach number at the final state is 2.0, determine (a) initial Mach number, and (b) final temperature.*

Solution

Given Acceleration of air in a nozzle;

$\mathcal{V}_1 = 100$ m/s, $T_1 = 400$ K
$M_2 = 2.0$, $s_1 = s_2$

To find

(i) The initial Mach number.
(ii) Final temperature of air.

Assumptions

(i) The gas constant $R = 287$ J/kg·K.
(ii) The specific heat $C_p = 1005$ J/kg·K.
(iii) The ratio of specific heats, $\gamma = 1.4$.

Analysis The velocity of sound at the initial state

$$a = \sqrt{\gamma RT_1} = \sqrt{1.4 \times 287 \times 400}$$
$$= 400.9 \text{ m/s}$$

(i) *Initial mach number*

$$M_1 = \frac{\mathcal{V}_1}{a_1} = \frac{100}{400.9} = \mathbf{0.249}$$

(ii) *The final temperature of air*

The initial stagnation temperature

$$T_{01} = T_1 + \frac{\mathcal{V}_1^2}{2C_p}$$
$$= 400 + \frac{(100)^2}{2 \times 1005} = 404.97 \text{ K}$$

For isentropic flow; $T_{01} = T_{02}$

Sonic velocity of air at exit

$$a_2 = \sqrt{\gamma RT_2}$$
$$= \sqrt{1.4 \times 287 \times T_2} = 20.04\sqrt{T_2}$$

Exit velocity of air;

$$\mathcal{V}_2 = M_2 a_2 = 2.0 \times 20.04 \sqrt{T_2}$$
$$= 40.08 \sqrt{T_2}$$

Finalt emperature;

$$T_2 = T_{02} - \frac{\mathcal{V}_2^2}{2C_p}$$
$$= 404.97 - \frac{(40.08\sqrt{T_2})^2}{2 \times 1005}$$

or $$T_2 = 404.97 - 0.8\,T_2$$

or $$T_2 = \frac{404.97}{1.8} = \mathbf{224.98 \text{ K}}$$

Example 10.7 *Air at 110 kPa, 90°C, with a velocity of 180 m/s is to be expanded isentropically through a convergent–divergent nozzle, until its Mach number becomes 1.5. The mass flow rate of air is 0.15 kg/s. Determine the final pressure and cross-sectional area at the nozzle exit.*

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Solution

Given Expansion of air in a convergent–divergent nozzle

$p_1 = 110$ kPa $\quad T_1 = 90°C = 363$ K

$\mathcal{V}_1 = 180$ m/s $\quad s_1 = s_2$

$M_2 = 1.5 \quad \dot{m} = 0.15$ kg/s

To find

(i) Final pressure, and

(ii) Cross-sectional area at the exit.

Assumption

(i) The gas constant $R = 0.287$ kJ/kg·K.

(ii) The specific heat of air, $C_p = 1.005$ kJ/kg·K.

(iii) $\gamma = 1.4$.

Analysis Using steady-flow energy equation in the absence of potential energy change.

$$h_1 - h_2 = \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2}$$

or $$C_p(T_1 - T_2) = \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2}$$

$$1005 \times (363 - T_2) = \frac{\mathcal{V}_2^2 - 180^2}{2}$$

or $$\mathcal{V}_2 = \sqrt{32400 + 2010 \times (363 - T_2)} \quad ...(i)$$

Now using $$M_2 = \frac{\mathcal{V}_2}{a_2} = \frac{\mathcal{V}_2}{\sqrt{\gamma R T_2}}$$

or $$\mathcal{V}_2 = 1.5 \times \sqrt{1.4 \times 287\, T_2} = 30.06\sqrt{T_2} \quad ...(ii)$$

Equating the two equations, we get

$$30.06\sqrt{T_2} = \sqrt{32400 + 2010 \times (363 - T_2)}$$

$$903.6\, T_2 = 32400 + 2010 \times (363 - T_2)$$

or $903.6\, T_2 + 2010\, T_2 = 32400 + 729630$

or $2913.6\, T_2 = 762030$

or $T_2 = 261.5$ K

and velocity; $\mathcal{V}_2 = 30.06 \times \sqrt{261.5} = 486.1$ m/s

Using $$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

or $$p_2 = 110 \times \left(\frac{261.5}{363}\right)^{\frac{1.4}{1.4-1}} = \mathbf{34.9\ kPa}$$

Further, $$\rho_2 = \frac{p_2}{RT_2} = \frac{34.9}{0.287 \times 261.5}$$

$$= 0.465 \text{ kg/m}^3$$

Using continuity equation,

$$\dot{m} = \rho_2 A_2 \mathcal{V}_2$$

$$0.15 = 0.465 \times A_2 \times 486.1$$

or $$A_2 = 6.636 \times 10^{-4} \text{ m}^2 = \mathbf{6.636\ cm^2}$$

Example 10.8 *Static air at 10 bar and 800°C flows through a duct. Calculate the temperature, velocity and Mach number at points where pressures are 5 bar and 1 bar, respectively. Assume flow to be isentropic and C_p = 1.005 kJ/kg·K and γ = 1.4 for air.*

Solution

Given Isentropic flow through a duct

$p_1 = 10$ bar $\quad T_1 = 800°C = 1073$ K

$p_2 = 5$ bar $\quad C_p = 1.005$ kJ/kg·K

$p_3 = 1$ bar $\quad \gamma = 1.4$

$\mathcal{V}_1 = 0$

To find

(i) Temperature at two stations,

(ii) Velocity at two stations, and

(iii) Mach number at two stations, where pressures are 5 bar and 1 bar, respectively.

Analysis Considering the station where pressure is 5 bar

(i) Static temperature after isentropic flow,

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 1073 \times \left(\frac{5}{10}\right)^{\frac{1.4-1}{1.4}} = 880.22 \text{ K}$$

$$= \mathbf{607.22°C}$$

(ii) For static air, $\mathcal{V}_1 = 0$

Using steady-flow energy equation in the absence of potential energy change ($q = w = 0$)

$$0 = \Delta h + \Delta ke = h_2 - h_1 + \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2}$$

or $$2C_p(T_1 - T_2) = \mathcal{V}_2^2$$

$$\text{Velocity } \mathcal{V}_2 = \sqrt{2 \times 1005 \times (1073 - 880.22)}$$

$$= \mathbf{622.5\ m/s}$$

(iii) The speed of sound at the station, where

$p_2 = 5$ bar

$$a_2 = \sqrt{\gamma R T_2} = \sqrt{1.4 \times 287 \times 880.22}$$

$$= 594.7 \text{ m/s}$$

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Mach number, $M_2 = \dfrac{\mathcal{V}_2}{a_2} = \dfrac{622.5}{594.7} = \mathbf{1.046}$

Similarly at the station where the pressure is 1 bar,

$T_3 = 55.75\ \text{K} = \mathbf{282.75°C}$,
$\mathcal{V}_3 = \mathbf{1019.63\ m/s}$,
$M_3 = \mathbf{2.157}$

10.5 PROPERTY RELATIONS FOR ISENTROPIC FLOW THROUGH A DUCT

Consider isentropic flow through a duct as shown in Fig. 10.10. For isentropic flow between states 1 and 2, the properties are related as

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma-1}$$

The steady-flow energy equation applied to isentropic flow through duct ($q = 0$, $w = 0$, $\Delta pe = 0$), gives

$$h_1 + \frac{\mathcal{V}_1^2}{2} = h_2 + \frac{\mathcal{V}_2^2}{2}$$

or $$C_p T_1 + \frac{\mathcal{V}_1^2}{2} = C_p T_2 + \frac{\mathcal{V}_2^2}{2}$$

or $$T_1\left(1 + \frac{\mathcal{V}_1^2}{2C_p T_1}\right) = T_2\left(1 + \frac{\mathcal{V}_2^2}{2C_p T_2}\right)$$

or $$\frac{T_2}{T_1} = \frac{1 + \left(\dfrac{\mathcal{V}_1^2}{2C_p T_1}\right)}{1 + \left(\dfrac{\mathcal{V}_2^2}{2C_p T_2}\right)}$$

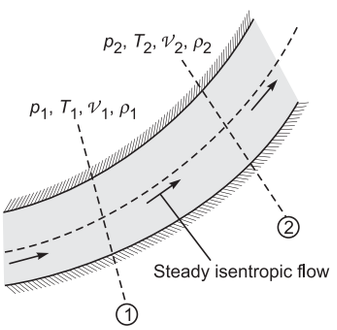

Fig. 10.10 *Steady isentropic flow through a duct*

But $$C_p = \frac{\gamma R}{\gamma - 1} \quad \text{and} \quad a^2 = \gamma RT$$

$$\therefore \quad \frac{\mathcal{V}^2}{2C_p T} = \frac{\mathcal{V}^2(\gamma - 1)}{2\gamma RT}$$

$$= \frac{\mathcal{V}^2}{a^2} \times \frac{\gamma - 1}{2} = \frac{\gamma - 1}{2} M^2$$

Therefore $$\frac{T_2}{T_1} = \frac{1 + \dfrac{\gamma - 1}{2} M_1^2}{1 + \dfrac{\gamma - 1}{2} M_2^2} \quad ...(10.19)$$

The relation between pressures and densities are

$$\frac{p_2}{p_1} = \left[\frac{1 + \dfrac{\gamma - 1}{2} M_1^2}{1 + \dfrac{\gamma - 1}{2} M_2^2}\right]^{\frac{\gamma}{\gamma-1}} \quad ...(10.20)$$

and $$\frac{\rho_2}{\rho_1} = \left[\frac{1 + \dfrac{\gamma - 1}{2} M_1^2}{1 + \dfrac{\gamma - 1}{2} M_2^2}\right]^{\frac{1}{\gamma-1}} \quad ...(10.21)$$

Example 10.9 *Air at 28°C and 700 kPa enters a nozzle with a velocity of 80 m/s. The nozzle has inlet area of 10 cm^2. The air leaves the nozzle of a pressure of 250 kPa. Determine*

(a) mass flow rate of air through the nozzle, and
(b) velocity at the exit of nozzle, assuming one-dimensional isentropic flow.

Solution

Given Air flow through nozzle as shown in Fig. 10.11.

To Find

(i) Mass flow rate of air,

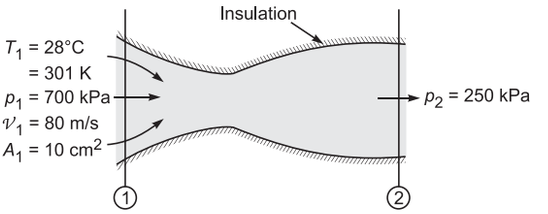

Fig. 10.11 *Air flow through nozzle*

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(ii) Velocity of air at nozzle exit.

Assumptions

(i) Air as an ideal gas.

(ii) For air, $R = 287$ J/kg·K, and $\gamma = 1.4$.

Analysis

(i) *Mass flow rate of air through nozzle*

At the section 1, the density of air

$$\rho_1 = \frac{p_1}{RT_1} = \frac{700 \times 10^3}{287 \times 301} = 8.1 \text{ kg/m}^3$$

The continuity equation gives

$$\dot{m} = \rho_1 A_1 \mathcal{V}_1 = 8.1 \times (10 \times 10^{-4}) \times 80$$
$$= \mathbf{0.648 \text{ kg/s}}$$

(ii) *Exit velocity of air*

Sonic velocity at the section 1

$$a_1 = \sqrt{\gamma RT}$$
$$= \sqrt{1.4 \times 287 \times 301} = 347.77 \text{ m/s}$$

Mach number $M_1 = \frac{\mathcal{V}_1}{a_1} = \frac{80 \text{ m/s}}{347.77 \text{ m/s}} = 0.23$

For isentropic flow through a duct using Eq. (10.21)

$$\frac{p_2}{p_1} = \left[\frac{1 + \frac{\gamma - 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_2^2}\right]^{\frac{\gamma}{\gamma - 1}}$$

or $$\left[\frac{1 + \frac{1.4 - 1}{2} \times (0.23)^2}{1 + \frac{1.4 - 1}{2} M_2^2}\right]$$

$$= \left(\frac{250}{700}\right)^{\frac{1.4-1}{1.4}} = 0.74.5$$

or $1 + 0.2 M_2^2 = 1.356$

or $M_2 = 1.335$

Further, for isentropic flow

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} = 301 \times \left(\frac{250}{700}\right)^{\frac{1.4-1}{1.4}}$$
$$= 224.24 \text{ K}$$

Sonic velocity at nozzle exit

$$a_2 = \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 224.24}$$
$$= 300.17 \text{ m/s}$$

Actual exit velocity of air from the nozzle

$$\mathcal{V}_2 = M_2 a_2 = 1.335 \times 300.17$$
$$= \mathbf{400.72 \text{ m/s}}$$

10.6 PROPERTY RELATIONS AT STAGNATION CONDITIONS

The equations derived above for isentropic flow through a duct are applicable at stagnation conditions. At stagnation state, the Mach number is zero and properties are denoted by the subscript 0, and the static properties are represented without subscript. Then the stagnation temperature of an ideal gas is related to static temperature

$$\frac{T_0}{T} = 1 + \left(\frac{\gamma - 1}{2}\right) M^2 \qquad ...(10.22)$$

The relation between stagnation pressure p_0 and the static pressure p is

$$\frac{p_0}{p} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]^{\frac{\gamma}{\gamma - 1}} \qquad ...(10.23)$$

and

$$\frac{\rho_0}{\rho} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]^{\frac{1}{\gamma - 1}} \qquad ...(10.24)$$

The properties of fluid at a location, where Mach number becomes unity (throat), are called *critical properties* and the relations are called critical ratios. Assuming the superscript (*) represents the critical values and setting $M = 1$, the critical equations are

$$\frac{T_0}{T^*} = 1 + \left(\frac{\gamma - 1}{2}\right) = \frac{\gamma + 1}{2} \qquad ...(10.25)$$

$$\frac{p_0}{p^*} = \left[1 + \frac{(\gamma - 1)}{2}\right]^{\frac{\gamma}{\gamma - 1}} = \left(\frac{\gamma + 1}{2}\right)^{\frac{\gamma}{\gamma - 1}} \qquad ...(10.26)$$

$$\frac{\rho_0}{\rho^*} = \left[1 + \frac{(\gamma - 1)}{2}\right]^{\frac{1}{\gamma - 1}} = \left(\frac{\gamma + 1}{2}\right)^{\frac{1}{\gamma - 1}} \qquad ...(10.27)$$

Example 10.10 *Air at 500 kPa and 330 K, enters a convergent nozzle with negligible velocity. The nozzle discharges into a receiver, where a pressure of 120 kPa is maintained. Assuming isentropic flow, calculate the velocity at the nozzle exit.*

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Solution

Given Flow of air in a convergent nozzle

$p_1 = 500$ kPa $\quad\quad$ $\mathcal{V}_1 = 0$ m/s

$\therefore$ $\quad T_0 = 330$ K $\quad\quad$ $p_b = 120$ MPa

To find The velocity at the exit of the nozzle.

Assumptions

(i) Isentropic flow,

(ii) Air as ideal gas with specific heat $C_p = 1005$ J/kg·K, $\gamma = 1.4$ and $R = 0.287$ kJ/kg·K.

Analysis Since the inlet velocity of the fluid is negligible, thus

$$T_1 = T_{01} \quad \text{and} \quad p_1 = p_{01}$$

For isentropic flow through a nozzle, the velocity reaches sonic velocity at the nozzle exit and temperature and pressure become critical.

The critical temperature,

$$T^* = T_{01}\left(\frac{2}{\gamma+1}\right)$$

$$= 330 \times \frac{2}{(1.4+1)} = 275 \text{ K}$$

The critical pressure at the throat

$$p^* = p_{01}\left(\frac{T^*}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= (500 \text{ kPa}) \times \left(\frac{275 \text{ K}}{330 \text{ K}}\right)^{\frac{1.4}{1.4-1}}$$

$$= 264 \text{ kPa}$$

Since the receiver pressure is lower than the critical pressure p^*, the pressure at the throat of the convergent nozzle will be p^*. The velocity of the flow at the throat will be sonic velocity;

$$\mathcal{V}_2 = a = \sqrt{\gamma RT^*} = \sqrt{1.4 \times (287 \text{ J/kg·K}) \times (275 \text{ K})}$$

$$= \mathbf{332 \text{ m/s}}$$

Example 10.11 *Air at 1 MPa and 500°C enters a convergent section with a velocity of 150 m/s. Determine the mass flow rate through the nozzle, for its throat area of 50 cm^2 and back pressure of 0.7 MPa.*

Solution

Given Flow of air in a convergent nozzle

$p_1 = 1$ MPa $\quad\quad$ $T_1 = 500°\text{C} = 773$ K

$\mathcal{V}_1 = 150$ m/s $\quad\quad$ $A = 50$ cm^2

$p_b = 0.7$ MPa

To find The mass flow rate through the nozzle.

Assumptions

(i) Isentropic flow,

(ii) Air as ideal gas with specific heat $C_p = 1005$ J/kg·K and $\gamma = 1.4$.

Analysis The stagnation pressure and temperature can be calculated as

$$T_{01} = T_1 + \frac{\mathcal{V}_1^2}{2C_p} = 773 \text{ K} + \frac{(150 \text{ m/s})^2}{2 \times (1005 \text{ J/kg·K})}$$

$$= 784.2 \text{ K}$$

$$p_{01} = p_1\left(\frac{T_{01}}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = (1 \text{ MPa}) \times \left(\frac{784.2}{773}\right)^{\frac{1.4}{1.4-1}}$$

$$= 1.05 \text{ MPa}$$

and the critical temperature,

$$T^* = T_{01}\left(\frac{2}{\gamma+1}\right) = 784.2 \times \frac{2}{(1.4+1)} = 653.5 \text{ K}$$

The critical pressure ratio at the throat

$$\frac{p^*}{p_{01}} = \left(\frac{T^*}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{653.5 \text{ K}}{784.2 \text{ K}}\right)^{\frac{1.4}{1.4-1}} = 0.528$$

Since flow through the nozzle is assumed isentropic, the stagnation pressure and temperature remain constant.

The pressure at the throat

$p^* = 0.528 \times 1$ MPa $= 0.528$ MPa $= 528$ bar

Back-pressure ratio $\frac{p_b}{p_{01}} = \frac{0.7 \text{ MPa}}{1.05 \text{ MPa}} = 0.667$

Since the back pressure ratio is greater than the critical pressure ratio, the exit pressure will be equal to the back pressure.

From Table 10.2, at a back pressure ratio of 0.667,

$M^* = 0.778$, and temperature ratio at the throat

$T^*/T_0 = 0.892$

Thus, temperature at the throat,

$$T^* = 0.892 \times 784.2 = 699.5 \text{ K}$$

Density of air at the throat,

$$\rho^* = \frac{p^*}{RT^*} = \frac{700 \text{ kPa}}{(0.287 \text{ kJ/kg·K}) \times (699.5 \text{ K})}$$

$$= 3.484 \text{ kg/m}^3$$

The velocity of air at the throat,

$$\mathcal{V}^* = M^* a = M^* \sqrt{\gamma RT^*}$$

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$$= (0.778) \times \sqrt{1.4 \times (287 \text{ J/kg} \cdot \text{K}) \times (699.5 \text{ K})}$$
$$= 412.5 \text{ m/s}$$

The mass flow rate at the throat,

$$\dot{m}^* = \rho^* A^* \mathcal{V}^*$$
$$= (3.484 \text{ kg/m}^3) \times (50 \times 10^{-4} \text{ m}^2) \times (412.5 \text{ m/s})$$
$$= \mathbf{7.185 \text{ kg/s}}$$

Example 10.12 *Air at 8.6 bar, and 190°C expands at a rate of 4.5 kg/s through a convergent–divergent nozzle to an atmospheric pressure of 1.013 bar. Assuming that the inlet velocity is negligble, calculate the throat and exit cross-section areas of the nozzle.*

Solution

Given Isentropic flow of air through a nozzle with

$\dot{m} = 4.5$ kg/s $\quad \mathcal{V}_1 = 0$

$p_0 = 8.6$ bar $\quad T_0 = 190°\text{C} = 463$ K

$p_2 = 1.013$ bar

To find

(i) The throat area of the nozzle, and

(ii) The exit area of the nozzle.

Schematic

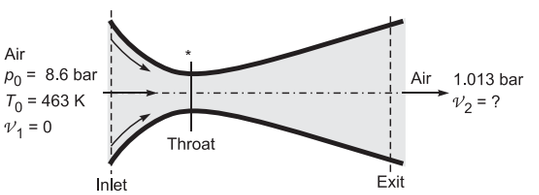

Fig. 10.12

Assumptions

(i) Air as a perfect gas,

(ii) The specific gas constant of air as 0.287 kJ/kg·K,

(iii) Change in potential energy is negligible.

Analysis

(i) The critical pressure ratio for flow through a nozzle is given by

$$\frac{p^*}{p_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{2}{1.4+1}\right)^{\frac{1.4}{1.4-1}}$$
$$= 0.528$$

or $\quad p^* = 0.528 \times 8.6 = 4.543$ bar

The critical temperature

$$T^* = T_0\left(\frac{2}{\gamma+1}\right) = 463 \times \frac{2}{1.4+1}$$
$$= 385.8 \text{ K}$$

The specific volume of air at the throat

$$v^* = \frac{RT^*}{p^*} = \frac{287 \times 385.8}{(4.543 \times 10^5)}$$
$$= 0.244 \text{ m}^3/\text{kg}$$

The velocity of air at the throat

$$\mathcal{V}^* = \sqrt{\gamma RT^*} = \sqrt{1.4 \times 287 \times 385.8}$$
$$= 393.7 \text{ m/s}$$

Using continuity equation to find the throat area

$$A^* = \frac{\dot{m} v^*}{\mathcal{V}^*} = \frac{4.5 \times 0.244}{393.7}$$
$$= 0.002788 \text{ m}^2 = \mathbf{2788.8 \text{ mm}^2}$$

(ii) For isentropic expansion process, the temperature at the nozzle exit

$$\frac{T_2}{T_0} = \left(\frac{p_2}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1.013}{8.6}\right)^{\frac{1.4-1}{1.4}}$$
$$= 0.5424$$

or $\quad T_2 = 0.5424 \times 463 = 252.14$ K

The specific volume at the exit

$$v_2 = \frac{287 \times 251.14}{1.013 \times 10^5} = 0.711 \text{ m}^3/\text{kg}$$

Using steady-flow energy equation

$$h_0 = h_2 + \frac{\mathcal{V}_2^2}{2}$$

or $\quad \mathcal{V}_2 = \sqrt{2(h_0 - h_2)} = \sqrt{2C_p(T_0 - T_2)}$

$$= \sqrt{2 \times 1005 \times (463 - 251.14)}$$
$$= 652.56 \text{ m/s}$$

The exit area

$$A_2 = \frac{\dot{m} v_2}{\mathcal{V}_2} = \frac{4.5 \times 0.711}{652.56}$$
$$= 0.00490 \text{ m}^2 \quad \text{or} \quad \mathbf{4903 \text{ mm}^2}$$

Example 10.13 *A gas which has a molar mass of 35.9 and the specific heat ratio of 1.60 is discharged from a large chamber in which the pressure is 470 kPa, the temperature is 27°C and the velocity is effectively zero through a nozzle. Assuming one-dimensional isentropic flow, find*

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(a) if the pressure at a certain section in the nozzle is 75 kPa, and calculate the Mach number, temperature and velocity at this section

(b) if the nozzle has circular cross-section and its diameter is 12 mm at the section discussed in (a) above, and calculate the mass flow rate through the nozzle.

Solution

Given The steady one-dimensional isentropic flow through a nozzle with

$\mathcal{M} = 35.9$ kg/kmol $\quad \gamma = 1.60$

$p_0 = 470$ kPa $\quad T_0 = 27°C = 300$ K

$\mathcal{V}_0 = 0$

$\therefore \quad M = 0$

$p = 75$ kPa

$d = 12$ mm $= 12 \times 10^{-3}$ m

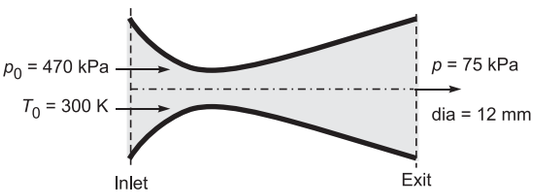

Fig. 10.13

To find

(i) Mach number, temperature and velocity, when pressure is 75 kPa at some point in the nozzle.

(ii) When the nozzle has diameter of 12 mm, the mass flow rate through the nozzle.

Analysis

(i) For isentropic flow, we have the relation

$$\frac{p_0}{p} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{\gamma - 1}}$$

It follows the Mach number

$$M = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{p_0}{p}\right)^{\frac{\gamma - 1}{\gamma}} - 1\right]}$$

Hence, $M = \sqrt{\frac{2}{0.6} \times \left[\left(\frac{470}{75}\right)^{\frac{1.6 - 1}{1.6}} - 1\right]} = \mathbf{1.816}$

Temperature at exit

$$T = T_0\left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1}$$

$$= (300 \text{ K}) \times \left(1 + \frac{0.6}{2} \times (1.816)^2\right)^{-1}$$

$$= 150.8 \text{ K} = \mathbf{-122.2°C}$$

The specific gas constant for air

$$R = \frac{R_u}{M} = \frac{8314}{\mathfrak{Z}\ .9} = 231.58 \text{ J/kg} \cdot \text{K}$$

The sonic velocity corresponds to this temperature

$$a = \sqrt{\gamma RT} = \sqrt{1.6 \times 231.58 \times 150.8}$$

$$= 236.38 \text{ m/s}$$

Hence fluid velocity,

$$\mathcal{V} = a M = 236.38 \times 1.816$$

$$= 429.28 \text{ m/s}$$

(ii) Mass flow rate;

The density $\rho = \frac{p}{RT} = \frac{75 \times 10^3}{231.58 \times 150.8}$

$= 2.15$ kg/m^3

Hence

$$\dot{m} = \rho A \mathcal{V} = 2.15 \times \frac{\pi}{4} \times (12 \times 10^{-3})^2 \times 429.28$$

$$= \mathbf{0.104 \text{ kg/s}}$$

10.7 ONE -DIMENSIONAL ISENTROPIC FLOW

The flow through the ducts, where the flow cross-sectional area varies gradually is of practical interest. The nozzles and diffusers are most common examples of flow devices in which passage of cross-sectional area changes in the flow direction. In such devices, the heat transfer may be considered negligible. If the frictional effects in such devices are negligibly small then the flow through such devices can be treated as one-dimensional isentropic flow with good accuracy. In addition, when a fluid flows through a varying cross-section area duct, the pressure, temperature, and velocity of fluid vary contiuously. These variations are analyzed in this section.

10.7.1 Effects of Area Variation

Consider a control volume of a varying cross-sectional area duct in which the fluid flows steadily

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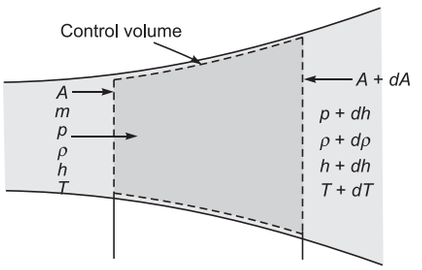

Fig. 10.14 *Control volume to analyse the effect of area variation*

and isentropically.

The continuity equation for a steady one-dimensional flow process:

$$\dot{m} = \rho A \mathcal{V} = \text{constant}$$

Differentiating and dividing the resultant equation by continuity equation, we get,

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{d\mathcal{V}}{\mathcal{V}} = 0 \qquad ...(10.28)$$

The steady flow energy equation with negligible potential energy change ($w = 0$, $q = 0$, $\Delta pe = 0$ can be expressed as

$$h_1 + \frac{\mathcal{V}_1^2}{2} = h_2 + \frac{\mathcal{V}_2^2}{2}$$

In differential form,

$$dh + \mathcal{V} d\mathcal{V} = 0 \qquad ...(10.29)$$

Using *Tds* relation,

$$Tds = dh - v\,dp$$

For isentropic flow ($Tds = 0$),

$$dh = v\,dp = \frac{dp}{\rho} \qquad ...(10.30)$$

Substituting dh in Eq. (10.29), we get

$$\mathcal{V} d\mathcal{V} + \frac{dp}{\rho} = 0 \qquad ...(10.31)$$

This relation is the differential form of *Bernoulli's equation*.

Substituting $d\mathcal{V}$ from Eq. (10.31) into Eq. (10.28), we get

$$\frac{d\rho}{\rho} + \frac{dA}{A} - \frac{dp}{\mathcal{V}^2 \rho} = 0$$

or

$$\frac{dA}{A} = \frac{dp}{\rho}\left(\frac{1}{\mathcal{V}^2} - \frac{d\rho}{dp}\right) \qquad ...(10.32)$$

Substituting the definition of velocity of sound in an isentropic flow, Eq. (10.12), we get

$$\frac{dA}{A} = \frac{dp}{\rho}\left(\frac{1}{\mathcal{V}^2} - \frac{1}{a^2}\right)$$

$$= \frac{dp}{\rho \mathcal{V}^2}\left(1 - \frac{\mathcal{V}^2}{a^2}\right)$$

Using the definition of Mach number, Eq. (10.17),

$$\frac{dA}{A} = \frac{dp}{\mathcal{V}^2 \rho}(1 - M^2) \qquad ...(10.33)$$

Further, using Eq. (10.31) for the value of dp/ρ in Eq. (10.33), we get

or

$$\frac{dA}{A} = -\frac{d\mathcal{V}}{\mathcal{V}}(1 - M^2) \qquad ...(10.34)$$

Further, using $dp = a^2 d\rho$ in Eq. (10.33), to get area variation in terms of density change

$$\frac{dA}{A} = \frac{a^2 d\rho}{\rho \mathcal{V}^2}(1 - M^2)$$

$$= \frac{d\rho}{\rho}\left(\frac{1 - M^2}{M^2}\right) \qquad ...(10.35)$$

Equations (10.33), (10.34) and (10.35) are important relations for isentropic flow through a duct, since these equations describe the variation of pressure, velocity and density with change of duct area.

10.7.2 Types of Nozzles

A *nozzle* is a device, used to increase the velocity of fluid passing through it at the expense of pressure drop. Hence for a nozzle, the quantity $\frac{d\mathcal{V}}{\mathcal{V}}$ must be positive and the quantity $\frac{dp}{p}$ must be negative in the direction of flow.

Subsonic Nozzle For subsonic flow, $M < 1$, $M^2 < 1$; $d\mathcal{V} > 0 \Rightarrow dA < 0$, the term $(1 - M^2)$ is always positive. It indicates that when a duct converges in the direction of flow, i.e., flow area decreases ($dA < 0$), the fluid pressure decreases, Eq. (10.33), while fluid velocity increases, Eq. (10.34) as shown in Fig. 10.15(a).

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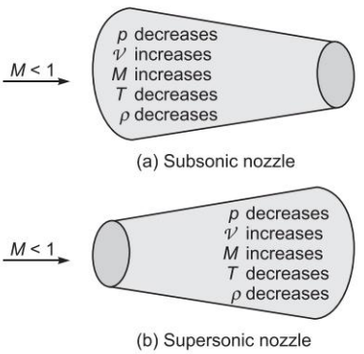

Fig. 10.15 *Subsonic flow*

Supersonic Nozzle For supersonic flow, $M > 1$, $M^2 > 1$, $d\mathcal{V} > 0 \Rightarrow dA > 0$. The term $(1 - M^2)$ is always negative. It indicates that when the duct diverges in the direction of flow, i.e., flow area increases ($dA > 0$), the fluid pressure decreases, Eq. (10.33), while fluid velocity increases Eq. (10.34) as shown in Fig. 10.15(b). Hence to obtain nozzle action in supersonic stream, the passage must be diverging.

Whenever highest velocity is desired, then a converging nozzle is used at subsonic velocities and a diverging nozzle is used at supersonic velocities. Hence, the nozzles are constructed with three parts, *converging duct*, *throat* and *diverging duct* as shown in Fig. 10.16. In the converging duct, the velocity of fluid increases up to sonic velocity (highest attainable in convergent portion) and then fluid flows in the diverging section via the throat, to further accelerate the velocity.

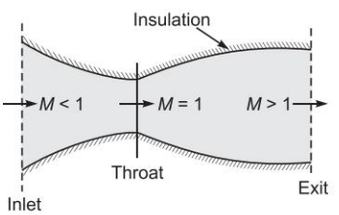

Fig. 10.16 *Converging–diverging nozzle*

10.7.3 Types of Diffusers

A *diffuser* is a device, used to increase the pressure of the fluid at the expense of velocity drop. For a diffuser, $d\mathcal{V}$ must be negative and dp must be positive.

Subsonic Diffuser For subsonic flow, $M < 1$, $M^2 < 1$; $d\mathcal{V} < 0 \Rightarrow dA > 0$: the term $(1 - M^2)$ is positive. It indicates that when duct diverges in the direction of flow, i.e., flow area increases ($dA > 0$), the fluid pressure increases, Eq. (10.33), and fluid velocity decreases, Eq. (10.34), as shown in Fig. 10.17(b).

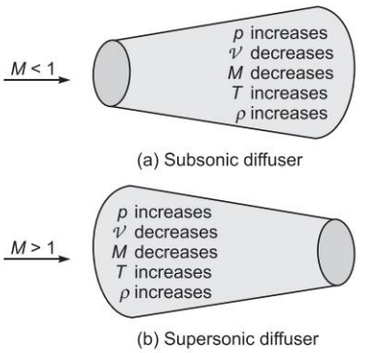

Fig. 10.17 *Supersonic flow*

Supersonic Diffuser For supersonic flow, $M > 1$, $M^2 > 1$; $d\mathcal{V} < 0 \Rightarrow dA < 0$: the term $(1 - M^2)$ is negative. It indicates that when duct converges in the direction of flow, i.e., flow area decreases ($dA < 0$), the fluid pressure increases, Eq. (10.33), while fluid velocity decreases, Eq. (10.34) as shown in Fig. 10.17(b).

Whenever a compressible fluid is desired to decelerate from supersonic speed to subsonic speed, the fluid must pass a converging–diverging duct of shape as shown in Fig. 10.18. To get the diffuser action in the duct, the fluid flow at the throat must be sonic.

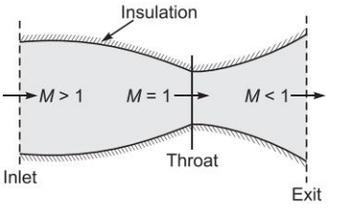

Fig. 10.18 *Converging–diverging diffuser*

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10.8 EFFECT OF BACK PRESSURE ON MASS FLOW RATE

The *back pressure* is the pressure in the exhaust region outside the nozzle. The case of a converging nozzles is taken up first and then converging–diverging nozzle is considered.

10.8.1 Flow Through a Convegent Nozzle

Consider a convergent nozzle is attached to a large reservoir as shown in Fig. 10.19. Since the gas velocity at the inlet to nozzle will be relatively small and flow through the nozzle is isentropic, hence the stagnation pressure and the stagnation temperature at any section of the nozzle will be equal to gas pressure and temperature in the reservoir, and thus, may be designated as p_0 and T_0, respectively. The pressure at the exit plane of the nozzle is p_e and the back pressure is p_b.

1. When the back pressure $p_b = p_0$, there is no mass flow $\dot{m} = 0$ through the nozzle. This corresponds to the case 1 of Fig. 10.19.
2. If the back pressure p_b is reduced to p_2 as in the case 2, it causes the pressure to decrease in flow direction which results in a certain mass flow rate of gases through the nozzle.
3. Further, decrease in back pressure results in greater mass flow rate. In this case, the velocity is subsonic through the nozzle and the exit pressure p_e equals the back pressure p_b. The Mach number increases as p_b decreases.
4. If the back pressure p_b is further reduced to $p_3 = p^*$, the mass flow rate reaches its maximum possible value. This case is represented by 3 on Fig. 10.19. The Mach number reaches unity and it cannot be increased further in conversing section. p^* is the pressure corresponding to sonic velocity, thus called *critical pressure.*
5. If the back pressure is further reduced to a value p_4 less than p^* as in the case 4, the velocity at the nozzle exit remains sonic velocity. Neither the mass flow rate nor pressure variation in the nozzle is affected and the nozzle is said to be *chocked*. For a *chocked nozzle*, the mass-flow rate reaches the maximum possible value for given stagnation conditions.
6. For p_b less than p^* the flow expands outside the nozzle to match the lower back pressure.

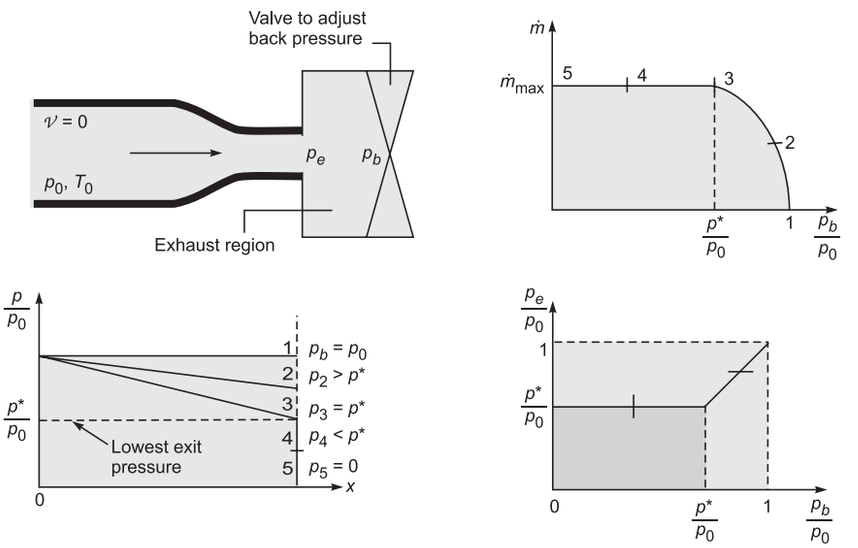

Fig. 10.19 *Effect of back pressure on pressure distribution mass flow rate and exit pressure along a nozzle*

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10.8.2 Flow Through a Convergent–Divergent Nozzle

The highest velocity to which a fluid can be subjected in a convergent nozzle is limited to sonic velocity ($M = 1$), which occurs at the exit plane of the nozzle. However, the fluid flow can be accelerated from subsonic to supersonic by attaching a diverging duct to the subsonic nozzle at the throat. For given inlet conditions, the flow through a convergent–divergent nozzle is governed by the back pressure as explained below:

Consider an isentropic fluid flow from an infinite reservoir, through a convergent–divergent nozzle to an exhaust chamber as shown in Fig. 10.20. Since the inlet velocity $\mathcal{V}$ to the nozzle is very small, hence, the conditions at the inlet to the nozzle are at the stagnation state. Let the inlet conditions be p_0, T_0, and ρ_0. The nozzle exit pressure be p_e, while p_b is the back pressure (pressure in the chamber).

1. When the back pressure p_b is equal to p_0 (p_A in the case A) then the pressure in the nozzle will be uniform throughout and there will no fluid flow through the nozzle.
2. When the back pressure p_b is just slightly below inlet pressure p_0 (p_B in the case B), the fluid velocity increases in the converging section and reaches maximum at the throat ($M < 1$), but decreases in the diverging section. There is a decrease in the pressure in converging section, which becomes minimum at the throat and increases in the diverging section at the expense of the velocity. The flow becomes subsonic throughout the nozzle.
3. As the back pressure p_b is further reduced (p_C in the case C), the pressure at the throat becomes the critical pressure p^*, the fluid velocity becomes sonic at the throat. The fluid mass flow reaches maximum and the flow is said to be *choked*. But the diverging section acts as diffuser and hence, the fluid velocity decreases and pressure increases in this section as shown by the curve C.
4. When the back pressure p_b is further reduced (p_D in the case D), the fluid flow rate at the throat remains constant. The fluid velocity continues to increase and accelerate in the

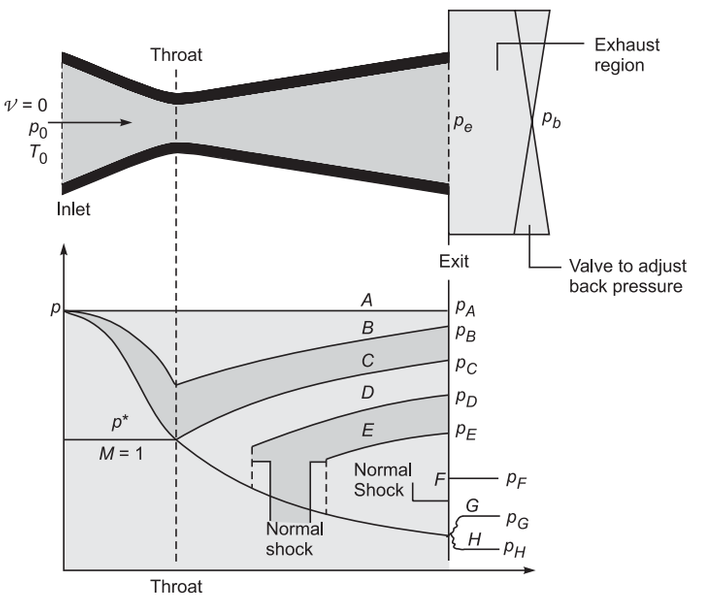

Fig. 10.20 *The effects of back pressure on the flow through a converging–diverging nozzle*

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diverging section of the nozzle and its velocity becomes supersonic as shown by the curve *D*.

However, at a section in downstream, a discontinuity occurs in the flow and abrupt increase in pressure is noticed and fluid flow decelerates from supersonic to subsonic. This discontinuity in the flow is called *normal shock*. The flow through the shock is steady, irreversible and adiabatic.

5. If the back pressure p_b is further reduced to p_E, the shock shifts towards downstream, approaching the nozzle exit as shown by the curve *E*.
6. When the back pressure p_b is at the value represented by the point *F*, the normal shock stands in the exit plane of the nozzle. The flow through the duct is isentropic; subsonic in convergent nozzle, sonic at the throat and supersonic in the diverging portion. However, the fluid passes through the shock as it leaves the nozzle and hence, jet leaving the nozzle becomes subsonic.
7. When the back pressure p_b is slightly below p_F, the fluid expands isentropically up to the nozzle exit plane and no shock forms within or outside the nozzle and one-dimensional supersonic jet leaves the nozzle.

For any further reduction in back pressure p_b, the flow within and outside, the nozzle remains the same.

10.9 MASS FLOW RATE THROUGH AN ISENTROPIC NOZZLE

Since the fluid flow through the nozzle is isentropic, therefore, the stagnation enthalpy remains constant. The velocity at any cross section in the nozzle;

$$\mathcal{V} = \sqrt{2(h_0 - h)}$$

Using $h = C_p T$; $\mathcal{V} = \sqrt{2C_p(T_0 - T)}$

Using $C_p = \dfrac{\gamma R}{\gamma - 1}$

$$\mathcal{V} = \sqrt{\frac{2\gamma R}{\gamma - 1}(T_0 - T)}$$

or $$\mathcal{V} = \sqrt{\frac{2\gamma RT_0}{\gamma - 1}\left(1 - \frac{T}{T_0}\right)} \quad ...(10.36)$$

For isentropic process, using

$$\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}$$

then $$\mathcal{V} = \sqrt{\frac{2\gamma RT_0}{\gamma - 1}\left\{1 - \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right\}} \quad ...(10.37)$$

The mass flow rate

$$\dot{m} = \rho A \mathcal{V}$$

or $$\dot{m} = \rho A \sqrt{\frac{2\gamma RT_0}{\gamma - 1}\left\{1 - \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right\}} \quad ...(10.38)$$

Using $\rho = \dfrac{p}{RT}$

and $T = T_0\left(\dfrac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}$

Thus $\rho = \dfrac{p}{RT_0}\left(\dfrac{p_0}{p}\right)^{\frac{\gamma-1}{\gamma}} = \dfrac{p_0}{RT_0}\left(\dfrac{p}{p_0}\right)^{\frac{1}{\gamma}}$

Substituting in to Eq. (10.38)

$$\dot{m} = \frac{Ap_0}{RT_0}\sqrt{\frac{2\gamma RT_0}{\gamma - 1}\left\{\left(\frac{p}{p_0}\right)^{\frac{2}{\gamma}} - \left(\frac{p}{p_0}\right)^{\frac{\gamma+1}{\gamma}}\right\}}$$

or $$\dot{m} = \frac{Ap_0}{\sqrt{RT_0}}\sqrt{\frac{2\gamma}{\gamma - 1}\left\{\left(\frac{p}{p_0}\right)^{\frac{2}{\gamma}} - \left(\frac{p}{p_0}\right)^{\frac{\gamma+1}{\gamma}}\right\}} \quad ...(10.39)$$

or $$\dot{m} = C\left\{\left(\frac{p}{p_0}\right)^{\frac{2}{\gamma}} - \left(\frac{p}{p_0}\right)^{\frac{\gamma+1}{\gamma}}\right\}^{1/2} \quad ...(10.40)$$

where C is constant and an equivalent of constant quantities γ, R, T_0, p_0 and A in the above equation.

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For maximum discharge rate, differentiating the above equation with respect to throat pressure p and equating it to zero;

$$\frac{d\dot{m}}{dp} = 0$$

we get, $$\frac{p^*}{p_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \quad ...(10.41)$$

The properties of a fluid at throat ($M = 1$) are called the *critical properties* and denoted by the subscript asterisk (*). The pressure p^* corresponds to maximum discharge, thus it is called *critical pressure*. For all back pressures lower than p^*, the pressure of the exit plane of the convergent nozzle is equal to p^*.

The velocity at the throat can be obtained by substituting the above Eq. (10.41) in Eq. (10.37) as

$$\mathcal{V}^* = \sqrt{\frac{2\gamma RT_0}{\gamma-1}\left\{1-\left(\frac{2}{\gamma+1}\right)\right\}} \quad ...(10.42)$$

or $$\mathcal{V}^* = \sqrt{\frac{2\gamma RT_0}{\gamma+1}} \quad ...(10.43)$$

or $$\mathcal{V}^* = \sqrt{\gamma RT^*} = a^* \quad ...(10.44)$$

Where $$T^* = \frac{2}{\gamma+1}T_0 \quad ...(10.45)$$

The temperature T^* is the *critical temperature* at the throat. The fluid has sonic velocity at the throat when the mass flow rate is maximum.

The convergent–divergent nozzle has minimum area at the throat, thus the fluid mass flow rate is maximum at the throat. Under steady flow condition, the mass flow rate through the nozzle is constant and can be expressed as

$$\dot{m} = \rho A\mathcal{V} = A\frac{p}{RT}(M\sqrt{\gamma RT})$$
$$= AMp\sqrt{\frac{\gamma}{RT}}$$

Using terms of T_0 and p_0 from Eqs. (10.22) and (10.23), the above equation can be expressed as

$$\dot{m} = \frac{AMp_0}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}} \times \sqrt{\frac{\gamma}{RT_0}} \times \left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{1}{2}}$$

or $$\dot{m} = \frac{AMp_0}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}\sqrt{\frac{\gamma}{RT_0}} \quad ...(10.46)$$

For choked flow, $M = 1$ and $A = A^*$, and the flowing fluid is air ($\gamma = 1.4$), the maximum flow rate through the nozzle is obtained as

$$\dot{m}_{max} = \frac{0.6847\,p_0A^*}{\sqrt{RT_0}} \quad ...(10.47)$$

Thus, for any ideal gas, $\dot{m}_{max}$ through the nozzle is controlled by p_0 and T_0, hence, a convergent nozzle can be used as a flowmeter.

Example 10.14 *Helium gas at 6.9 bar and 93°C enters a convergent nozzle with negligible velocity and expands isentropically to a pressure of 3.6 bar. Calculate the mass flow rate per square metre of the exit area.*

Take C_p for helium gas as 5.19 kJ/kg·K and molar mass as 4 kg/kmol.

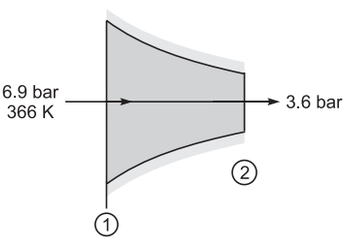

Fig. 10.21

Solution

Given Isentropic flow of helium gas through a convergent nozzle

$p_1 = 6.9$ bar $\quad T_1 = 93°C = 366$ K

$p_2 = 3.6$ bar $\quad \mathcal{V}_1 = 0$

$C_p = 5.19$ kJ/kg·K $\quad \mathcal{M} = 4$ kg/kmol

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To find The mass flow rate of helium per square metre of the nozzle exit area.

Assumptions

(i) Helium as a perfect gas.
(ii) Change in potential energy is zero.

Analysis The specific gas constant for helium

$$R = \frac{R_u}{\mathcal{M}} = \frac{8314}{4} = 2078.5 \text{ J/kg·K.}$$

and $$C_p = \frac{\gamma R}{\gamma - 1}$$

or $$\frac{\gamma - 1}{\gamma} = \frac{R}{C_p} = \frac{2078.5}{5.19 \times 10^3} = 0.4$$

Thus $$\gamma = \frac{1}{1 - 0.4} = 1.667$$

The critical pressure ratio for helium gas flow through the nozzle

$$\frac{p^*}{p_1} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}} = \left(\frac{2}{1.667 + 1}\right)^{\frac{1.667}{1.667 - 1}} = 0.487$$

or $p^* = 0.487 \times 6.9 = 3.360$ bar

which is less than the exit pressure and hence the fluid does not reach the critical conditions and the nozzle is not chocked.

Temperature T_2 at the nozzle exit

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} = 366 \times \left(\frac{3.6}{6.9}\right)^{\frac{1.667 - 1}{1.667}}$$
$$= 282.14 \text{ K}$$

The velocity at the exit of the nozzle

$$\mathcal{V}_2 = \sqrt{2C_p(T_1 - T_2)}$$
$$= \sqrt{2 \times 5.19 \times 10^3 \times (366 - 282.14)}$$
$$= 933 \text{ m/s}$$

The specific volume at the nozzle exit

$$v_2 = \frac{RT_2}{p_2} = \frac{2.0785 \times 282.14}{3.6 \times 10^2} = 1.63 \text{ m}^3\text{/kg}$$

The mass flow rate of helium for 1 m^2 exit area

$$\dot{m} = \frac{A\mathcal{V}_2}{v_2} = \frac{1 \times 933}{1.63} = \mathbf{572.75 \text{ kg/s}}$$

Example 10.15 *A convergent–divergent nozzle has an exit area to throat area ratio of 2. Air enters this nozzle with a stagnation pressure of 1 MPa and a stagnation temperature of 360 K. The throat area is 500 mm². Determine the mass flow rate, exit pressure, exit temperature, exit Mach number and exit velocity for the following conditions:*

(a) Sonic velocity at the throat, diverging section acting as a nozzle,
(b) Sonic velocity at the throat, diverging section acting as a diffuser.

Solution

Given A convergent–divergent nozzle

$p_0 = 1$ MPa $\qquad T_0 = 360$ K
$A/A^* = 2 \qquad A^* = 500 \text{ mm}^2$

To find

(i) Mass flow rate,
(ii) Exit temperature,
(iii) Exit pressure,
(iv) Exit Mach number, and
(v) Exit velocity.

Assumptions

(i) Isentropic flow, and
(ii) Air as ideal gas with $C_p = 1005$ J/kg·K, $\gamma = 1.4$ and $R = 0.287$ kJ/kg·K.

Analysis

(i) For $A/A^* = 2$, we get two Mach numbers, greater than unity and less than unity. When diverging section acts as supersonic nozzle, then $M > 1$. From Table A-10;

$$\frac{A}{A^*} = 2 \quad M_e = \mathbf{2.197} \quad \frac{p_e}{p_0} = 0.0938$$
$$\frac{T_e}{T_0} = 0.5089$$

Therefore, $p_e = 0.0938 \times (1 \text{ MPa}) = \mathbf{0.0938 \text{ MPa}}$,
$T_e = 0.5089 \times (360 \text{ K}) = \mathbf{183.2 \text{ K}}$

$$a_e = \sqrt{\gamma RT_e}$$
$$= \sqrt{1.4 \times (287 \text{ J/kg·K}) \times (183.2 \text{ K})}$$
$$= 271.3 \text{ m/s}$$
$$\mathcal{V}_e = M_e a_e = 2.197 \times (271.3 \text{ m/s})$$
$$= \mathbf{596.1 \text{ m/s}}$$

The mass flow rate is determined at throat condition. Here, the velocity at the throat is sonic velocity ($M = 1$). Thus, the density of air at the throat,

$$\rho_e = \frac{p_e}{RT_e} = \frac{(0.0938 \times 1000 \text{ kPa})}{(0.287 \text{ kJ/kg·K}) \times (183.2 \text{ K})}$$
$$= 1.784 \text{ kg/m}^3$$

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The mass flow rate at the throat,

$$\dot{m} = \rho_e A_e \mathcal{V}_e$$
$$= (1.784 \text{ kg/m}^3) \times (500 \times 10^{-6} \text{ m}^2) \times (271.3 \text{ m/s})$$
$$= \mathbf{0.242 \text{ kg/s}}$$

(ii) When the diverging section acts as a diffuser nozzle then $M < 1$. From Table A-10;

$$\frac{A}{A^*} = 2, \quad M_e = \mathbf{0.308},$$
$$\frac{p_e}{p_0} = 0.936, \quad \frac{T_e}{T_0} = 0.9812$$

Therefore, $p_e = 0.308 \times (1 \text{ MPa}) = \mathbf{0.308 \text{ MPa}}$,

$$T_e = 0.9812 \times (360 \text{ K}) = \mathbf{353.3 \text{ K}}$$
$$a_e = \sqrt{\gamma R T_e} = \sqrt{1.4 \times (287 \text{ J/kg} \cdot \text{K}) \times (353.3 \text{ K})}$$
$$= 376.8 \text{ m/s}$$
$$\mathcal{V}_e = M_e a_e = 0.308 \times (376.8 \text{ m/s}) = \mathbf{116 \text{ m/s}}$$

The mass flow rate remains the same, since $M = 1$.

Example 10.16 *Air flows through a nozzle which has an inlet area of 10 cm^2. If air has a velocity of 80 m/s, a temperature of 28°C, and a pressure of 700 kPa at the inlet section and a pressure of 250 kPa at the exit, find the mass flow rate through the nozzle and the velocity at the exit, when flow conditions are isentropic.*

Solution

Given

$p_i = 700$ kPa, $T_i = 28°\text{C} = 301$ K
$\mathcal{V}_i = 80$ m/s, $A_i = 10$ cm^2
$p_e = 250$ kPa, $s_i = s_e$

To find

(i) The mass flow rate, and
(ii) The exit velocity.

Assumptions

(i) Isentropic flow
(ii) Air as an ideal gas with $C_p = 1005$ J/kg·K, $\gamma = 1.4$ and $R = 0.287$ kJ/kg·K.

Analysis

(i) The mass flow rate is given by

$$\dot{m} = \rho_i A_i \mathcal{V}_i = \frac{p_i}{RT_i} \times A_i \times \mathcal{V}_i$$
$$\dot{m} = \frac{(700 \text{ kPa})}{(0.287 \text{ kJ/kg} \cdot \text{K}) \times (301 \text{ K})} \times (10 \times 10^{-4} \text{ m}^2) \times (80 \text{ m/s})$$
$$= \mathbf{0.648 \text{ kg/s}}$$

(ii) The sonic velocity at the inlet can be calculated as

$$a_i = \sqrt{\gamma R T_i} = \sqrt{1.4 \times (287 \text{ J/kg} \cdot \text{K}) \times (301 \text{ K})}$$
$$= 347.7 \text{ m/s}$$

The inlet Mach number is

$$M_i = \frac{\mathcal{V}_i}{a_i} = \frac{80 \text{ m/s}}{347.7 \text{ m/s}} = 0.23$$

For isentropic expansion through the nozzle

$$\frac{p_e}{p_i} = \left[\frac{1 + \frac{\gamma - 1}{2} M_i^2}{1 + \frac{\gamma - 1}{2} M_e^2} \right]^{\frac{\gamma}{\gamma - 1}}$$

or
$$\frac{250}{700} = \left[\frac{1 + \frac{1.4 - 1}{2} (0.23)^2}{1 + \frac{1.4 - 1}{2} M_e^2} \right]^{\frac{1.4}{1.4 - 1}}$$

or $M_e = 1.335$

The exit temperature

$$T_e = T_i \left(\frac{p_e}{p_i} \right)^{\frac{\gamma - 1}{\gamma}}$$
$$T_e = 301 \times \left(\frac{250}{700} \right)^{\frac{1.4 - 1}{1.4}} = 224.3 \text{ K}$$

The exit velocity,

$$\mathcal{V}_e = M_e \times a_e = M_e \sqrt{\gamma R T_e}$$
$$= 1.335 \times \sqrt{1.4 \times 287 \times 224.3}$$
$$= \mathbf{400.8 \text{ m/s}}$$

10.10 SHOCK WAVE

The shock wave is an extremely thin region in the flow field, in which transition from supersonic velocity to relatively low velocity occurs. The normal waves exist due to abrupt changes in fluid properties. A shock wave may be viewed similar to a hydraulic jump which occurs in free surface liq-

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uid flows. The shock waves are generally curved. However, many shock waves that occur in practical are straight, being at right angles to flow direction (*normal shock wave*), at an angle to upstream flow (*oblique shock wave*).

Figure 10.22 shows a control surface that includes normal shock. The changes in fluid properties across a normal shock are illustrated in Fig. 10.22. The fluid velocity decreases, while the pressure, temperature, density, etc., increase after normal shock occurs.

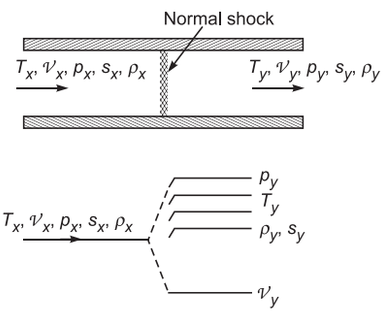

Fig. 10.22 *Flow across a normal shock wave*

The fluid is assumed in thermal equilibrium before and after the shock wave. The property changes across a normal shock can be obtained from continuity equation $\rho\mathcal{V} = G$ (a constant) and the momentum equation $p + \rho\mathcal{V}^2 = F$ (a constant).

10.10.1 Normal Shocks in Nozzle Flow

Assuming steady flow with no work and heat interactions and no potential energy changes, let the subscripts x and y denote the conditions upstream (inlet) and downstream (exit) of shock, respectively as shown in Fig. 10.22.

The steady-state energy equation;

$$h_x + \frac{\mathcal{V}_x^2}{2} = h_y + \frac{\mathcal{V}_y^2}{2}$$

$$h_{0x} = h_{0y} \qquad ...(10.48)$$

The continuity equation for constant area duct;

$$\frac{\dot{m}}{A} = \rho_x \mathcal{V}_x = \rho_y \mathcal{V}_y \qquad ...(10.49)$$

The momentum equation;

$$A(p_x - p_y) = \dot{m}(\mathcal{V}_x - \mathcal{V}_y) \qquad ...(10.50)$$

The second law of thermodynamics;

$$s_y - s_x \geq 0 \qquad ...(10.51)$$

If we combine energy and continuity equation into a single equation and plot it on an h–s diagram using property relations, the resultant curve is called *Fanno line*. The momentum and continuity equations in combination give *Rayleigh line*. Both these lines are shown in Fig. 10.23 on an h–s diagram, simultaneously. The points a and b correspond to maximum entropy for $M = 1$. The lower part of each curve corresponds to supersonic velocities, and the upper part corresponds to subsonic velocities.

The two points, where all three equations are satisfied are the points x and y, the point x being in a supersonic region and the *point y* in subsonic region. For any adiabatic process, $s_y - s_x \geq 0$ according to the second law of thermodynamics. Hence, the normal shock can proceed from x to y. The supersonic velocity of flow becomes subsonic after normal shock.

Since the stagnation enthalpy remains constant across the shock, thus for an ideal gas

$$T_{0x} = T_{0y} \qquad ...(10.52)$$

That is, the stagnation temperature of an ideal gas also remains constant across a normal shock.

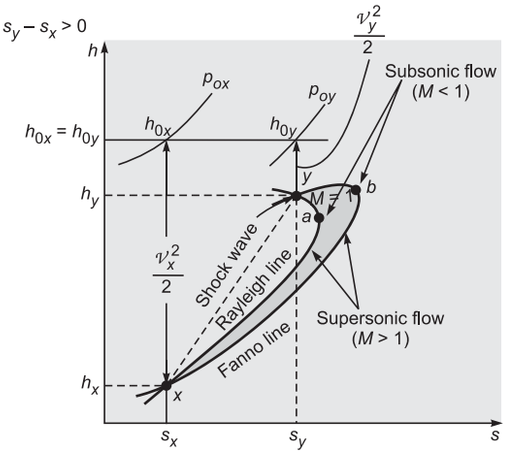

Fig. 10.23 *Fanno and Rayleigh lines across a shock*

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10.10.2 Properties Relation Across a Shock

The actual flow process inside the shock wave is non-isentropic, but the flow process ahead of and after the shock can be considered isentropic. Using Eq. (10.22) across the shock

$$\frac{T_{0x}}{T_x} = 1 + \frac{(\gamma - 1)}{2} M_x^2$$

and $$\frac{T_{0y}}{T_y} = 1 + \frac{(\gamma - 1)}{2} M_y^2$$

Dividing the first equation by the second equation;

$$\frac{T_y}{T_x} = \frac{1 + \frac{(\gamma - 1)}{2} M_x^2}{1 + \frac{(\gamma - 1)}{2} M_y^2} \qquad ...(10.53)$$

For an ideal gas,

$$\rho_x = \frac{p_x}{RT_x} \quad \text{and} \quad \rho_y = \frac{p_y}{RT_y}$$

Substituting it into the continuity equation,

$$\rho_x \mathcal{V}_x = \rho_y \mathcal{V}_y$$

$$\frac{p_x \mathcal{V}_x}{RT_x} = \frac{p_y \mathcal{V}_y}{RT_y}$$

Rearranging the above equation and using

$$M = \mathcal{V}/a \text{ and } a = \sqrt{\gamma RT}$$

$$\frac{T_y}{T_x} = \frac{p_y \mathcal{V}_y}{p_x \mathcal{V}_x} = \frac{p_y M_y a_y}{p_x M_x a_x}$$

$$= \frac{p_y M_y \sqrt{\gamma RT_y}}{p_x M_x \sqrt{\gamma RT_x}} = \frac{p_y M_y \sqrt{T_y}}{p_x M_x \sqrt{T_x}}$$

Therefore, $$\left(\frac{p_y}{p_x}\right)^2 = \left(\frac{M_x}{M_y}\right)^2 \times \frac{T_y}{T_x} \qquad ...(10.54)$$

Combining Eqs. (10.53) and (10.54), i.e., combining energy and continuity equation, gives pressure ratio across the shock;

$$\frac{p_y}{p_x} = \frac{M_x \sqrt{1 + \frac{\gamma - 1}{2} M_x^2}}{M_y \sqrt{1 + \frac{\gamma - 1}{2} M_y^2}} \qquad ...(10.55)$$

It is also called the *equation of Fanno line*.

The momentum and continuity equations can be combined to give an equation of Rayleigh line.

$$p_x - p_y = \frac{\dot{m}}{A}(\mathcal{V}_x - \mathcal{V}_y)$$

$$= \rho_y \mathcal{V}_y^2 - \rho_x \mathcal{V}_x^2$$

or $$p_x + \rho_x \mathcal{V}_x^2 = p_y + \rho_y \mathcal{V}_y^2$$

or $$p_x + \frac{p_x}{RT_x} \mathcal{V}_x^2 = p_y + \frac{p_y}{RT_y} \mathcal{V}_y^2$$

Using $$a^2 = \gamma RT$$

We get $$p_x(1 + \gamma M_x^2) = p_y(1 + \gamma M_y^2)$$

or $$\frac{p_y}{p_x} = \frac{1 + \gamma M_x^2}{1 + \gamma M_y^2} \qquad ...(10.56)$$

It is the *equation of Rayleigh line*.

The Eqs. (10.55) and (10.56) in combination give

$$M_y^2 = \frac{M_x^2 + \frac{2}{\gamma - 1}}{\frac{2\gamma}{\gamma - 1} M_x^2 - 1} \qquad ....(10.57)$$

It represents the intersection of Fanno and Rayleigh lines and relates the Mach number upstream of the shock to downstream of the shock.

These shocks occur in hypersonic nozzles as well as ahead of the supersonic air craft or bullets as shown in Fig. 10.24. If we consider a stationary aircraft and movement of air with supersonic velocity towards the aircraft, the air follows the stagnation process and it stops when it reaches the nose of the aircraft. The normal shock occurs where velocity of air suddenly decelerates to subsonic. It creates a sonic boom.

The entropy change across the shock can be calculated as

$$s_y - s_x = C_p \ln \frac{T_y}{T_x} - R \ln \frac{p_y}{p_x} \qquad ...(10.58)$$

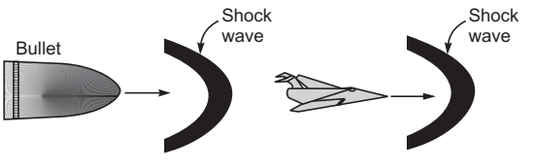

Fig. 10.24 *Shock waves across a supersonic aircraft and bullets*

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10.11 FLOW THROUGH ACTUAL NOZZLES AND DIFFUSERS

The flow through the nozzles and diffusers are not reversible adiabatic (isentropic) but irreversible adiabatic. Thus, three important parameters are used by which the actual flow can be compared with isentropic flow. These are *nozzle efficiency*, *velocity coefficient* and *discharge coefficient.*

The *nozzle efficiency* (η_N) can be expressed as

$$\eta_N = \frac{\text{Actual kinetic energy at nozzle exit}}{\text{Kinetic energy at nozzle exit for isentropic flow to the same exist pressure}}$$

$$= \frac{\mathcal{V}_2^2}{\mathcal{V}_{2s}^2} \qquad ...(10.59)$$

The nozzle efficiency can also be defined in terms of actual enthalpy drop and isentropic enthalpy drop. It can be expressed as

$$\eta_N = \frac{h_{01} - h_2}{h_{01} - h_{2s}} \qquad ...(10.60)$$

where h_{01} is stagnation enthalpy of the fluid at the nozzle inlet and h_2 is the enthalpy of fluid at the exit for actual nozzle, while h_{2s} is the exit enthalpy for a nozzle under isentropic conditions. The nozzle efficiency varies from 90% to 99%. The larger nozzle has more efficiency than smaller nozzle. Figure 10.25 shows the effect of irreversibility.

The *nozzle velocity coefficient* (C_V) is also an important parameter and can be expressed as

$$C_V = \frac{\text{Actual velocity at the nozzle exit}}{\text{Velocity at the nozzle exit with isentropic flow and same exit pressure}}$$

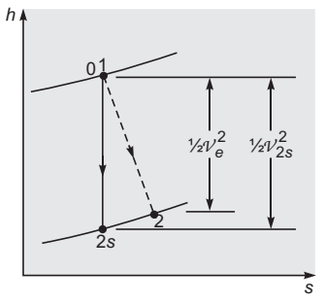

Fig. 10.25 *Isentropic and actual flow through a nozzle*

$$= \frac{\mathcal{V}_2}{\mathcal{V}_{2s}} \qquad ...(10.61)$$

It follows that the velocity coefficient is equal to the square root of the nozzle efficiency;

$$C_V = \sqrt{\eta_N} \qquad ...(10.62)$$

The mass flow rate through the nozzle is the design consideration, because it is affected by irreversibilities. An important parameter, *coefficient of discharge* (C_D) relates the actual mass flow with mass flow under the isentropic conditions for the same nozzle.

$$C_D = \frac{\text{Actual mass flow rate}}{\text{Mass flow rate with isentropic flow}}$$

$$= \frac{\dot{m}}{\dot{m}_s} \qquad ...(10.63)$$

Diffusers are designed to increase the pressure of a fluid by decelerating it. The *diffuser efficiency* η_D is based on its ability to convert the kinetic energy of the fluid into a useful pressure rise.

$$\eta_D = \frac{\text{Actual kinetic energy available for pressure rise}}{\text{Maximum kinetic energy available}}$$

Refer Fig. 10.26, the *h–s* diagram states 1 and 01 are actual and stagnation states at the diffuser inlet, states 2 and 02 are actual and stagnation states at the diffuser exit. The state 02s is a fictitious diffuser exit state, which would be attained for isentropic process in diffuser. The states 2 and 02 become identical for zero exit velocity of diffuser.

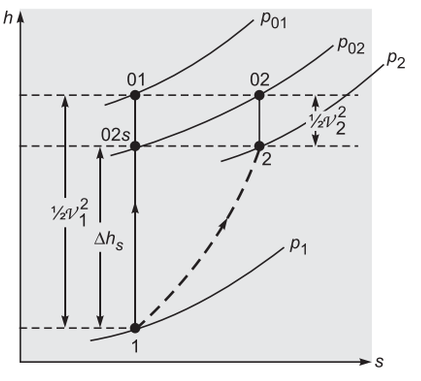

Fig. 10.26 *Parameters for evaluating diffuser efficiency*

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The maximum kinetic energy at the diffuser inlet

$$\frac{\mathcal{V}_1^2}{2} = h_{01} - h_1$$

Actual enthalpy drop, which can be converted to pressure rise

$$\Delta h_s = h_{02s} - h_1$$

Then the diffuser efficiency can be expressed as

$$\eta_D = \frac{h_{02s} - h_1}{h_{01} - h_1} \qquad ...(10.63)$$

The diffuser efficiency varies from 90 to 100 per cent for subsonic diffusers and it deceases with increasing Mach numbers.

Another parameter to measure the diffuser ability to increase the pressure of the fluid is the *pressure recovery factor*, F_P, which is defined as the ratio of actual stagnation pressure at the diffuser exit to isentropic stagnation pressure.

$$F_P = \frac{p_{02}}{p_{01}} \qquad ...(10.64)$$

A third parameter to evaluate diffuser's performance is the *pressure-rise coefficient* C_{PR}. It is defined as the ratio of actual pressure rise in the diffuser to that corresponding to isentropic process. Thus,

$$C_{PR} = \frac{\text{Actual pressure rise}}{\text{Isentropic pressure rise}} = \frac{p_2 - p_1}{p_{01} - p_1} \qquad ...(10.65)$$

The pressure rise coefficient depends on flow characteristics and shape of the diffuser and its value is usually less than 0.8.

Example 10.17 *A certain diffuser having an efficiency of 90% is used to reduce the velocity of an air stream initially at 150 m/s, 300 K, and 100 kPa down to 60 m/s. Calculate the ratio of exit area to inlet area to accomplish this reduction. What is the value of pressure rise coefficient for this diffuser?*

Solution

Given A diffuser with

$\eta_D = 0.9$ $\quad \mathcal{V}_1 = 150$ m/s

$T_1 = 300$ K, $\quad p_1 = 100$ kPa

$\mathcal{V}_2 = 60$ m/s

To Find

(i) Ratio of exit area to inlet area.

(ii) Pressure rise coefficient.

Assumptions

(i) The fluid is an ideal gas.

(ii) The specific heat $C_p = 1005$ J/kg·K.

(iii) Adiabatic diffuser.

Analysis For adiabatic diffuser

$$T_{01} = T_{02}$$

The stagnation temperature at the diffuser inlet

$$T_{01} = T_1 + \frac{\mathcal{V}_1^2}{2C_p} = 300 \text{ K} + \frac{(150)^2}{2 \times 1005} = 311.2 \text{ K}$$

and

$$T_2 = T_{02} - \frac{\mathcal{V}_2^2}{2C_p} = T_{01} - \frac{\mathcal{V}_2^2}{2C_p} = 311.2 - \frac{60^2}{2 \times 1005} = 309.4 \text{ K}$$

Then the stagnation pressure at the diffuser inlet

$$p_{01} = p_1 \left(\frac{T_{01}}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = (100 \text{ kPa}) \times \left(\frac{311.2}{300}\right)^{\frac{1.4}{0.4}}$$

$$= 113.68 \text{ kPa}$$

For air as an ideal gas, the diffuser efficiency can be expressed as

$$\eta_D = \frac{h_{02s} - h_1}{h_{01} - h_1} = \frac{T_{02s} - T_1}{T_{01} - T_1}$$

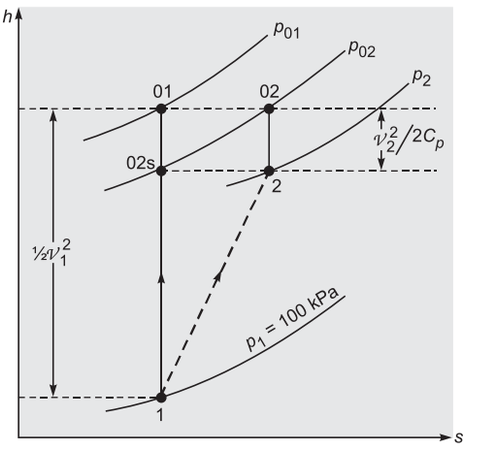

Fig. 10.27

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or $$0.9 = \frac{T_{02s} - 300}{311.2 - 300} \Rightarrow T_{02s} = 310.08 \text{ K}$$

and $$p_{02s} = p_1\left(\frac{T_{02s}}{T_1}\right) = (100 \text{ kPa}) \times \left(\frac{310.08}{300}\right)^{\frac{1.4}{0.4}}$$
$$= 112.26 \text{ kPa}$$

Refering Fig. 10.27, $p_{02s} = p_{02}$, then

$$p_2 = p_{02}\left(\frac{T_2}{T_{02}}\right)^{\frac{\gamma}{\gamma-1}} = 112.26 \times \left(\frac{309.4}{311.2}\right)^{\frac{1.4}{0.4}}$$
$$= 110.01 \text{ kPa}$$

The pressure rise coefficient

$$C_{PR} = \frac{p_2 - p_1}{p_{01} - p_1} = \frac{110.01 - 100}{113.68 - 100} = \mathbf{0.732}$$

Exit-area-to-inlet-area ratio is determined by use of continuity equation

$$\dot{m} = \rho_1 \mathcal{V}_1 A_1 = \rho_2 \mathcal{V}_2 A_2$$

Using $$\rho = \frac{p}{RT}$$

we get $$\frac{A_2}{A_1} = \frac{p_1 T_2 \mathcal{V}_1}{T_1 p_2 \mathcal{V}_2} = \frac{100 \times 309.4 \times 150}{300 \times 110.01 \times 60} = \mathbf{2.343}$$

10.12 EFFECT OF IRREVERSIBILITIES ON NOZZLE EFFICIENCY

In the preceeding section of this chapter, the flow through the nozzles and diffusers is approximated isentropic. Actual expansion through nozzles is non-isentropic flow. It is due to presence of irreversibilies at the surface of flow and within the fluid itself. The primary cause of irreversibly in nozzles (and diffusers) is the presense of frictional effects, which are due to

(i) friction between fluid and wall surface of nozzle, and
(ii) friction within the fluid itself.

The friction between the wall surface and fluid molecules makes the expansion adiabatic but not isentropic. The energy lost in overcoming the friction is used to reheat the fluid and the enthalpy and entropy of fluid increase during the process.

The convergent portion of the nozzle is smaller than the divergent portion. Thus the wall friction is small in the convergent portion as compared to divergent portion. The fluid friction is also small in the convergent portion than in the divergent portion, since the fluid velocity in the convergent portion is also small. Thus, *most of the friction occurs in the divergent portion of the nozzle* and the *h–s* diagram takes the shape as shown in Fig. 10.28. The nozzle efficiency for such a nozzle is given as

$$\eta_{N,\,divergent} = \frac{h^* - h_e}{h^* - h_{es}} \qquad ...(10.66)$$

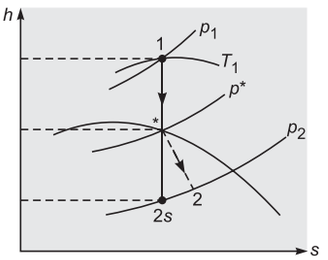

Fig. 10.28 *Effect of friction in divergent portion of nozzle*

Another cause of irreversibility in the nozzles and diffusers is flow separation, which induces strong turbulence near the nozzle wall. Flow separation occurs when the angle of divergence in a convergent–divergent nozzle is made too large. Consequently, the flow area increases faster than the fluid expands. Thus the included (cone) angle of the divergent duct is usually kept less than 20°C.

The frictional losses in the nozzle depends upon the material of construction, size, shape and surface because the wall surface in the large nozzles occupies a smaller potion of total flow volume.

The *effect of friction in a nozzle* can be summarized as

(i) Reduction in enthalpy drop,
(ii) Reheating of fluid,
(iii) Reduction in exit velocity,
(iv) Increase in specific volume, and
(v) Decrease in mass flow rate.

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Summary

- This chapter deals with high-speed fluid flow. The *stagnation enthalpy* is the combination of enthalpy and kinetic energy of fluid

$$h_0 = h + \frac{\mathcal{V}^2}{2} \text{(kJ/kg)}$$

- The properties at a stagnation state are called the stagnation properties and are designated by the subscript 0. The stagnation temperature of an ideal gas with constant specific heat is

$$T_0 = T + \frac{\mathcal{V}^2}{2C_p}$$

which is the temperature when an ideal gas is brought to rest isentropically.

- The stagnation properties of an ideal gas are related as

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{(\gamma-1)}}$$

- The velocity of sound in a gas medium is given by

$$a = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_{s=c}} = \sqrt{\gamma RT}$$

- The Mach number is defined as

$$M = \frac{\mathcal{V}}{a}$$

- The flow is called sonic, when $M = 1$; subsonic when $M < 1$; supersonic when $M > 1$; and hypersonic when $M > 5$.
- The nozzles whose flow area decreases in flow direction are called *convergent nozzles.* The nozzles whose flow area first decreases and then increases in flow direction are called *convergent–divergent nozzles.* The location of the smallest flow area of a nozzle is called *throat.*
- In a convergent nozzle, the fluid can only be accelerated to sonic velocity. Acceleration of fluid to supersonic velocity is only possible in a convergent–divergent nozzle.
- The ratio of stagnation properties to static properties for ideal gases with constant specific heats are

$$\frac{T_0}{T} = 1 + \left(\frac{\gamma-1}{2}\right)M^2$$

$$\frac{p_0}{p} = \left[1 + \left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho_0}{\rho} = \left[1 + \left(\frac{\gamma-1}{2}\right)M^2\right]^{1/(\gamma-1)}$$

- When $M = 1$, the above relations are termed as critical ratio, and critical properties are denoted by a superscript * (asterisk):

$$\frac{T^*}{T_0} = \frac{2}{\gamma+1}$$

$$\frac{p^*}{p_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho^*}{\rho_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{(\gamma-1)}}$$

- The pressure outside the exit plane of a nozzle is called the back pressure. For all back pressures lower than the critical pressure p^*, the pressure at the exit plane of the convergent nozzle is equal to p^* and Mach number at the exit plane is unity and mass flow rate is maximum (or choked)
- Under steady flow conditions, the mass flow rate through a nozzle is given by

$$\dot{m} = \frac{AMp_0\sqrt{\frac{\gamma}{RT_0}}}{\left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

- For some range of back pressure, the gas attains the sonic velocity at the throat of a convergent–divergent nozzle and is accelerated to supersonic velocity. In the divergent section-experiences a normal shock, which causes a sudden rise in pressure and temperature and a sudden drop in velocity. The flow through shock is irreversible. The properties of an ideal gas with constant specific heats before (subscript x) and after (subseript y) a shock are related as

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$$T_{0x} = T_{0y}$$

$$\frac{p_y}{p_x} = \frac{M_x\sqrt{1+\frac{\gamma-1}{2}M_x^2}}{M_y\sqrt{1+\frac{\gamma-1}{2}M_y^2}}$$

$$M_y^2 = \frac{M_x^2 + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_x^2 - 1}$$

- The entropy change across a shock is given by

$$s_y - s_x = C_p \ln\left(\frac{T_y}{T_x}\right) - R\ln\left(\frac{p_y}{p_x}\right)$$

- The presence of friction on nozzle wall surface and within the fluid makes the fluid flow non-isentropic but adiabatic. The nozzle efficiency is defined as

$$\eta_N = \frac{h_{01}-h_2}{h_{01}-h_{2s}} = \frac{\mathcal{V}_2^2}{\mathcal{V}_{2s}^2}$$
$$= \frac{\text{Actual enthalpy drop}}{\text{Isentropic enthalpy drop}}$$

- The nozzle velocity coefficient;

$$C_V = \frac{\mathcal{V}_2}{\mathcal{V}_{2s}}$$
$$= \frac{\text{Actual velocity at nozzle exit}}{\text{Velocity at nozzle exit for isentropic flow}}$$

- Coefficient of discharge

$$C_D = \frac{\dot{m}}{\dot{m}_s}$$
$$= \frac{\text{Actual mass flow rate}}{\text{mass flow rate for isentropic expansion}}$$

Glossary

Incompressible Fluid A fluid whose density does not change during a flow process

Compressible Fluid A fluid whose density changes during a flow process

Static properties Properties of fluid with negligible kinetic energy

Stagnation Properties Properties of fluid with significant kinetic energy

Sonic velocity The velocity at which a small pressure wave propagates in a fluid

Mach number Ratio of local fluid velocity to the sonic velocity in the medium

Subsonic flow The fluid flows with Mach number less than unity (that is, $\mathcal{V} < a$)

Sonic flow The fluid flow with Mach number is equal to 1 (that is, $\mathcal{V} = a$)

Transonic flow The fluid flow with Mach number around unity ($0.8 < M < 1.2$)

Supersonic flow The fluid flows with Mach number greater than unity (that is, $\mathcal{V} > a$).

Hypersonic flow The fluid flows with a very high Mach number (that is, $M > 5$)

Mach line A straight line along which circles of wave front are formed

Mach angle The angle between the Mach line and the direction of fluid flow

Mach cone A cone, which confines all the wave front

Zone of silence Region outside the mach cone

Critical properties The properties of fluid at a location where the Mach number becomes unity (throat)

Chocked Flow Flow of fluid condition when the mass flow rate reaches the maximum possible value

Fanno line A curve on h–s plot which represents energy and continuity equations both

Rayleigh line A curve which represents momentum and continuity equations in combination

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Review Questions

1. Discuss the shapes of nozzles required for the subsonic and supersonic flow.
2. Derive the required condition for maximum mass flow rate in case of nozzles.
3. Define sonic velocity and prove that
$$a = \sqrt{\gamma RT}$$
where a = sonic velocity
γ = ratio of specific heats
R = Specific gas constant
T = absolute temperature of gas.
4. Define Mach number and classify various types of flow on the basis of Mach number.
5. Explain why a horn is heard before an automabile reaches a person?
6. Why is the sound of a jet plane heard after it has passed forward?
7. Define (a) stagnation enthalpy, (b) stagnation temperature, and (c) stagnation pressure.
8. Prove that the stagnation temperature and stagnation pressure remain constant for an isentropic flow through the nozzle.
9. Prove that for an isentropic flow through a duct,
$$\frac{T_2}{T_1} = \frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}$$
10. Prove that for sonic flow at throat
$$\frac{T^*}{T_0} = \frac{2}{\gamma+1}$$
and $$\frac{p^*}{p_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}$$
11. Discuss the effects of area changes on fluid velocity, pressure and Mach number for
    (a) subsonic nozzle,
    (b) subsonic diffuser,
    (c) supersonic nozzle, and
    (d) supersonic diffuser,
12. Discuss the effect of back pressure on convergent nozzle.
13. Prove that the velocity at the throat is sonic under the conditions of maximum mass flow rate in case of nozzles.
14. What do you understand by the term 'critical pressure' as applied to steam nozzles.
15. In case of steam nozzles, show that the pressure ratio of steam at throat to inlet is given by the equation
$$\frac{p^*}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{k}{k-1}}$$
under maximum mass flow rate conditions
Hence, derive the equations for critical velocity and maximum mass flow rate.
16. Prove that for an isentropic flow of an ideal gas with constant specific heat ratio γ, the ratio of temperature T^* to the stagnation temperature T_0 is
$$\frac{T^*}{T_0} = \frac{2}{\gamma+1}$$
17. Discuss the effect of friction in case of nozzles with the help of Mollier's diagram.
18. Discuss the effect of variation of back pressure in case of convergent–divergent nozzles.
19. What is shock wave? Define normal shock and oblique shock.
20. Define
    (a) Nozzle efficiency,
    (b) Nozzle velocity coefficient,
    (c) Coefficient of discharge, and pressure recovery factor.

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Problems

1. Air leaves a compressor in a pipe with a stagnation temperature and pressure of 150°C, 300 K, and a velocity of 125 m/s. The cross-sectional area is 0.02 m^2. Determine the static pressure, temperature and mass flow rate.

[281 kPa, 425.4 K, 5.9 kg/s]

2. A convergent nozzle has an exit area of 500 mm^2. Air enters the nozzle with stagnation pressure of 10 bar and a stagnation temperature of 360 K. Determine the mass-flow rate for the back pressure of 800 kPa, 528 kPa and 300 kPa, assuming isentropic flow.

[0.8712 kg/s, 1.0646 kg/s, 1.0646 kg/s]

3. A convergent–divergent nozzle has a throat diameter of 0.05 m and an exit diameter of 0.1 m. The inlet stagnation state is 500 kPa, 500 K. Find the back pressure that will lead to maximum possible flow rate and the mass flow rate for air.

4. A gas with a molar mass of 4 and a specific heat ratio of 1.3 flows through a variable area duct. At some location in the flow, the velocity is 150 m/s, the pressure is 100 kPa and the temperature is 15°C. Find the Mach number at this location of flow. At some other location of flow, the temperature is found to be –10°C. Find the Mach number, pressure and velocity at this location in the flow, assuming the flow to be isentropic and one-dimensional.

[$M_1 = 0.17$, $M_2 = 0.8157$, $p_2 = 67.5$ kPa, $\mathcal{V}_2 = 687.6$ m/s]

5. A nozzle is designed for an isentropic flow with a Mach number of 2.6. The air flow is with stagnation pressure and temperature of 2 MPa and 150°C. The mass-flow rate is 5 kg/s. Take the ratio of specific heats as 1.4. Determine the exit pressure, temperature, area and the throat area. Suppose that the back pressure at the nozzle exit is raised to 1.4 MPa and that the flow remains isentropic except for a normal shock wave. Determine the exit Mach number and the temperature, and the mass-flow rate through the nozzle.

6. Air expanded in a nozzle from 700 kPa, 200°C, to 150 kPa in a nozzle having an efficiency of 90%. The mass flow rate is 4 kg/s. Determine the exit area of the nozzle, the exit velocity, and increase of entropy per kg of air.

7. Calculate the throat and exit area of the nozzle to expand air at the rate of 4.5 kg/s from 8.3 bar, 327°C into a space at 1.38 bar. Neglect the inlet velocity and assume isentropic flow.

[3290 mm^2, 4840 mm^2]

8. It is required to produce a stream of helium at a rate of 0.1 kg/s, travelling at sonic velocity at a temperature of 15°C. Assume negligible inlet velocity, isentropic flow, and back pressure of 1.013 bar. Determine:

(a) required inlet pressure and temperature,

(b) exit area of the nozzle.

For helium, take molar mass as 4 kg/k mol and $\gamma = 1.66$. *[(a) 2.077 bar, 110°C, (b) 593 mm^2]*

9. A gas expands isentropically through a converging nozzle from a large tank at 10 bar, 600 K. Assuming ideal gas behaviour, calculate the critical pressure p^* in bar and the corresponding temperature in kelvins, if the gas is

(a) air,

(b) oxygen (O_2),

(c) water vapour.

10. Air enters a nozzle operating at steady state at 3 bar, 400 K, with a velocity of 145 m/s, and expands isentropically to an exit velocity of 460 m/s. Determine

(a) exit pressure in a bar,

(b) ratio of exit area to inlet area,

(c) whether the nozzle is diverging only, converging only, or converging and diverging in cross-section.

11. Air as an ideal gas with $\gamma = 1.4$ undergoes a normal shock. The upstream conditions are $p_x = 0.5$ bar, $T_x = 280$ K, and $M_x = 1.8$. Determine

(a) the pressure p_y in bar,

(b) the stagnation pressure p_{0x} in bar,

(c) the stagnation temperature T_{0x} in K,

(d) the change in specific entropy across the shock, in kJ/kg·K.

[(a) 1.81 bar, (b) 2.87 bar, (c) 461 K, (d) 5.94×10^{-3} kJ/kg·K]

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12. Air expands in a convergent divergent nozzle from 6.89 bar and 427°C into a space at 1 bar. The throat area of the nozzle is 650 mm^2 and the exit area is 975 mm^2. The exit velocity is found to be 680 m/s, when the inlet velocity is negligible. Assuming that friction in the convergent portion is negligible, determine

(a) Mass flow rate through the nozzle stating whether nozzle is expanding or over-expanding,

(b) Nozzle efficiency and coefficient of velocity.

[(a) 0.684 kg/s, under expanding,
(b) p_2 = 1.39 bar; 89.5%, 0.946]

Objective Questions

1. A fluid is a compressible fluid when its density
   (a) decreases with pressure
   (b) increases with pressure
   (c) increases with temperature
   (d) both (a) and (b)
2. The Mach number is defined as
   (a) $\frac{\text{Distance}}{\text{Flight velocity}}$ (b) $\frac{\text{Sonic velicty}}{\text{Flight distance}}$
   (c) $\frac{\text{Local velocity}}{\text{Sonic velocity}}$ (d) None of the above
3. Flow of fluid is called transonic when
   (a) $M > 1$ (b) $M = 1$
   (c) $M < 1$ (d) $0.8 > M > 1.2$
4. The sound of an automobile is heard
   (a) before it passes (b) after it passes
   (c) when it reaches (d) none of the above
5. A nozzle is designed for
   (a) maximum pressure at outlet
   (b) minimum pressure at outlet,
   (c) maximum discharge at outlet
   (d) maximum discharge and maximum pressure at outlet
6. When a fluid is coming out of a duct at a higher pressure than it enters, the duct is called a/an
   (a) orifice (b) nozzle
   (c) diffuser (d) venturi
7. Friction loss in a nozzle
   (a) increases the exit velocity
   (b) decrease the exit velocity
   (c) has no effect on exit velocity
   (d) none of above
8. Whole friction loss in a convergent-divergent nozzle occurs in
   (a) divergent portion
   (b) convergent portion
   (c) at throat
   (d) none of the above
9. The exit pressure from a nozzle is equal to or more than the critical pressure when it is
   (a) in a divergent portion,
   (b) convergent
   (c) convergent–divergent
   (d) none of the above
10. If the exit pressure from a nozzle is less than critical pressure, it is
    (a) divergent
    (b) convergent
    (c) convergent–divergent
    (d) none of the above
11. Critical pressure in a convergent–divergent nozzle is defined as the ratio of
    (a) exit pressure and inlet pressure
    (b) inlet pressure and outlet pressure
    (c) throat pressure and inlet pressure
    (d) none of the above
12. If the exit pressure from a nozzle is less than the critical pressure, the mass flow rate will be
    (a) decreasing (b) increasing
    (c) constant (d) none of the above
13. At critical pressure ratio for a nozzle, the velocity at the outlet will be
    (a) more than the sonic velocity
    (b) less than the sonic velocity

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(c) equal to the sonic velocity
(d) none of the above

14. Nozzle efficiency is defined as the ratio of
    (a) actual enthalpy drop to isentropic enthalpy drop
    (b) isentroic enthalpy drop to actual enthalpy drop
    (c) product of isentropic enthalpy drop and actual enthalpy drop
    (d) square root of isentropic enthalpy drop to actual enthalpy drop
15. Frictional loss in a nozzle
    (a) increases the enthalpy drop
    (b) decreases the enthapy drop
    (c) has no effect on the enthalpy drop
    (d) none of the above
16. By increasing the length of the nozzle, the nozzle efficiency
    (a) decreases
    (b) remains constant
    (c) increases
    (d) is affected by frictional resistance
17. Coefficient of discharge is designed as
    (a) $\dfrac{\text{Actual kinetic energy change}}{\text{Isentropic kinetic energy change}}$
    (b) $\dfrac{\text{Actual enthalpy drop}}{\text{Isentropic enthalpy drop}}$
    (c) $\dfrac{\text{Actual nozzle exit velocity}}{\text{Nozzle exit velocity in isentropic flow}}$
    (d) $\dfrac{\text{Actual mass flow rate}}{\text{Mass flow rate with isentropic flow}}$

Answers

1. (d)	2. (c)	3. (d)	4. (a)	5. (c)	6. (c)	7. (b)	8. (a)
9. (b)	10. (c)	11. (c)	12. (c)	13. (c)	14. (a)	15. (b)	16. (c)
17. (d)							

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11

Gas Power Cycles

Introduction

The devices producing net power output are called *engines*, and the thermodynamic cycles on which they operate are called *power cycles*. The thermodynamic power cycles can be categorized as *gas power* cycles and *vapour power* cycles.

Two principal types of engines are used: rotary and reciprocating engines. The reciprocating internal-combustion engines operate on the Otto-cycle, Diesel cycle and dual cycle. The Otto-cycle engine, named after its inventor, the German technician *Nikolaus August Otto*, is the familiar gasoline engine used in automobiles and airplanes. The Diesel engine, named after the French-born German engineer *Rudolf Christian Karl Diesel*, operates on a different principle and usually uses oil as a fuel. It is employed in electric-generating and marine-power plants, in trucks and buses, and in some automobiles. Both Otto-cycle and Diesel-cycle engines are manufactured in two-stroke and four-stroke cycle models.

11.1 DEFINITIONS

Figures 11.1 and 11.2 show sketches of an internal combustion engine, a piston that reciprocates within a cylinder fitted with two valves and a spark plug or fuel injector. The sketches are lebelled with some special terms, defined below:

1. Bore It is the internal diameter of the cylinder of the reciprocating engine. It is denoted by d and expressed in mm.

2. Stroke It is the linear distance through which the piston travels between the top dead centre (*TDC*) and the bottom dead centre (*BDC*) in the cylinder. It is also called *stroke length*. It is denoted by L and expressed in mm.

3. Swept Volume It is also called *piston displacement volume* or *stroke volume.* It is the volume created or displaced by the piston during its one stroke travel, i.e., travel of the piston from one dead centre to other dead centre. It is denoted by V_s and calculated for one cycle as

$$V_s = (\pi/4)\, d^2 L \qquad ...(11.1)$$

4. Cubic Capacity or Engine Capacity It is the product of swept volume of a cylinder and the number of cylinders in an engine, i.e., total volume displaced in each cycle of an engine. For k number of cylinders,

$$Cubic\ Capacity = k V_s$$

5. Clearance Volume It is the volume left in the cylinder when the piston reaches the top dead centre. It is denoted by V_c.

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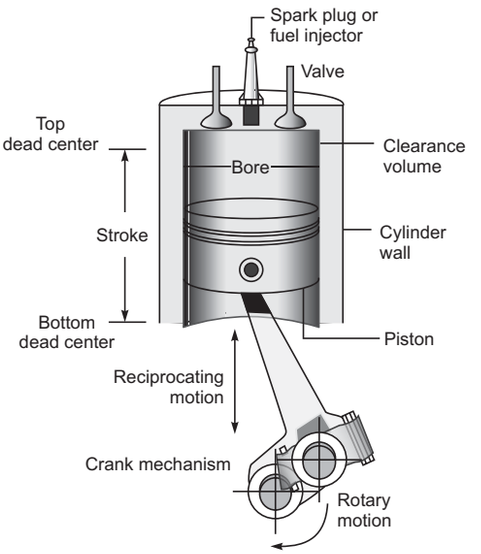
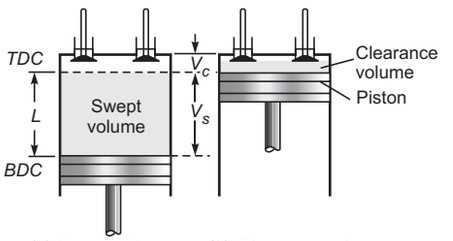


Fig. 11.1 *Nomenclature for reciprocating volume engine*



(a) Swept volume *(b) Clearance volume*

Fig. 11.2 *Swept volume and clearance of a reciprocating engine*

6. Total Volume It is the maximum volume in the cylinder and is the sum of swept volume and clearance volume. It is denoted by V_1.

$$V_1 = V_s + V_c \qquad ...(11.2)$$

7. Compression Ratio It is the ratio of maximum possible volume to clearance (minimum) volume in the cylinder. It is denoted by r and expressed as

$$r = \frac{V_{max}}{V_{max}} = \frac{V_1}{V_2} = \frac{V_s + V_c}{V_c} \qquad ...(11.3)$$

8. Clearance Ratio It is the ratio of clearance volume to swept volume. It is usually kept 3–5% of the swept volume. It is denoted by c and expressed as

$$c = \frac{V_c}{V_s} \qquad ...(11.4)$$

9. Cut-off Ratio It is the ratio of cut-off volume to swept volume. It is denoted by ρ. This term is used in Diesel-cycle and dual-cycle-engines.

10. Expansion Ratio It is the ratio of maximum volume to minimum volume during expansion in the cylinder. It is denoted by r_e.

11. Dead Centres There are two fixed positions in the cylinder, between which the piston reciprocates. The uppermost position is called *top dead centre* (*TDC*), while the bottom-most position is called the *bottom dead centre* (*BDC*).

12. Charge It is air-petrol mixture for petrol engines and only air for Diesel engines, which is introduced during suction stroke of the engine.

13. Mean Effective Pressure (p_m) It is a *hypothetical average pressure*, which if acted on the piston during the entire power stroke, will produce the same power output as produced during the actual cycle.

Net work done in a cycle,

$$W_{net} = p_m \times (\text{piston area}) \times (\text{stroke})$$
$$= p_m \times (\text{displacement volume})$$
$$= p_m \times V_s$$

Thus
$$p_m = \frac{W_{net}}{V_s} \qquad ...(11.5)$$

The actual indicator diagram of an engine and its corresponding mean effective pressure for the same power output is shown in Fig. 11.3. For any particular engine, under specific operating conditions, there will be an indicated mean effective pressure (*imep* or p_{mi}) and a corresponding brake mean effective pressure (*bmep* or p_{mb}). From a given indicator diagram, the indicated mean effective pressure can be obtained as

$$p_{mi} = \frac{\text{Area of indicator diagram (mm}^2\text{)}}{\text{Length of the indicator diagram (mm)}} \times \text{Spring constant (kPa/mm)} \qquad ...(11.6)$$

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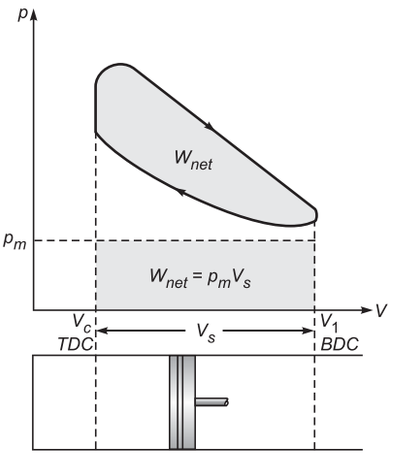

Fig. 11.3 *The network of a cycle is a product of the mean effective pressure and swept volume*

The mean effective pressure is used to compare the output of similar engines of different sizes.

11.2 AIR STANDARD ANALYSIS

The simplest models for both spark ignition and compression ignition engines are air standard cycles shown in Fig. 11.4. The processes of these ideal cycles are selected such that they should be similar to actual cycles, as much as possible, because the analysis of actual cycles is very much complex. The assumptions made in the analysis of air standard cycles are the following:

1. The system is closed, thus the cycle is a complete thermodynamic cycle and the same fluid is used repeatedly.
2. Air as an ideal gas is used as working fluid in the cycle.
3. The compression and expansion processes are reversible, adiabatic.
4. The combustion process is replaced by a reversible heat-addition process from an external source.
5. The exhaust process is replaced by a reversible constant-volume, heat-rejection process to the surroundings to restore the system back to its initial state.

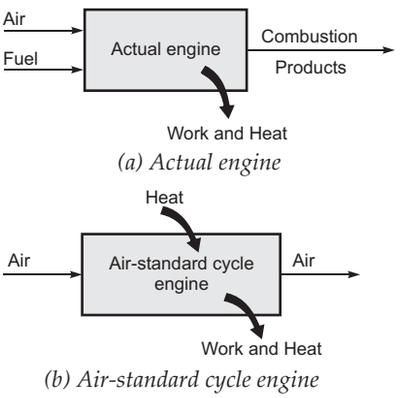

Fig. 11.4

6. In addition, in cold air standard analysis, the specific heats of air are assumed to be constant at their ambient temperature value.

$$C_p = 1.005 \text{ kJ/kg} \cdot \text{K}$$
$$C_v = 0.717 \text{ kJ/kg} \cdot \text{K}$$
$$R = 0.287 \text{ kJ/kg} \cdot \text{K}$$
$$\gamma = 1.4$$

11.3 CARNOT CYCLE

Basic operational principle of Carnot cycle is discussed in Chapter 6. It consists of four reversible processes: isothermal heat addition, isentropic expansion, isothermal heat rejection and isentropic compression. The p–v and T–s diagrams of Carnot cycle are replotted in Fig. 11.5.

From Fig. 11(b),

Heat supplied per kg of air during isothermal process 3–4

$$q_{in} = T_H(s_4 - s_3)$$

Heat rejected per kg of air during isothermal process 1–2;

$$q_{out} = T_L(s_1 - s_2)$$

Process 2–3 and 4–1 are isentropic processes

$$\therefore \quad q_{2-3} = 0 \text{ and } q_{4-1} = 0$$

and $s_1 = s_4$ and $s_2 = s_3$

The net work done per kg of air in the cycle

$$w_{net} = \Sigma q = q_{in} - q_{out}$$
$$= T_H(s_1 - s_2) - T_L(s_1 - s_2)$$
$$= (T_H - T_L)(s_1 - s_2)$$

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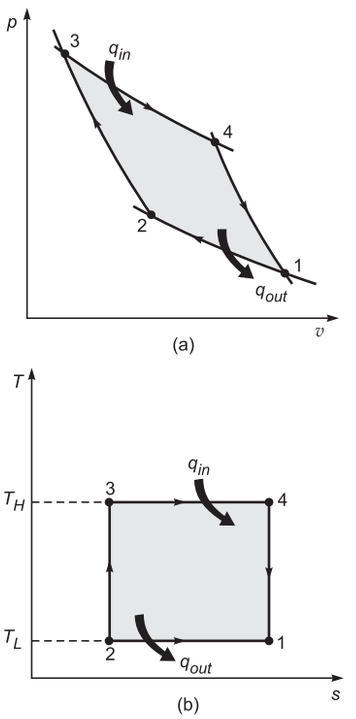

Fig. 11.5 *p–v and T–s diagrams of Carnot cycle*

Thermal efficiency of Carnot cycle

$$\eta_{Carnot} = \frac{w_{net}}{q_{in}} = \frac{(T_H - T_L)(s_1 - s_2)}{T_H(s_1 - s_2)}$$

$$= 1 - \frac{T_L}{T_H} \qquad ...(11.7)$$

In actual practice, the transfer of heat energy at constant temperature is very difficult to achieve, because it would require very large heat exchangers and the heat would be transfer with extremely slow rate to be isothermal.

Further, the isentropic processes are very fast processes, therefore, it is impractical to design an engine which would operate with very fast and extremely slow processes in succession thus no thermodynamic cycle closely approximate Carnot cycle.

Thermal efficiency of Carnot engine is the function of source and sink temperatures only and it is more than that of all actual and ideal cycles operating between same two temperature limits.

Moreover, the thermal efficiency of Carnot cycle conveys an important message that is applicable to all types of engines. Thermal efficiency increases with an increase in the average temperature at which heat is added to the system or with a decrease in average temperature at which heat is rejected from the system.

11.4 STIRLING CYCLE

This is an air standard cycle developed by Er. James Stirling and Dr. Robert Stirling. It consists of two isothermal processes and two reversible constant volume processes.

Similar to Carnot cycle, the heat is added at constant temperature T_H and heat is rejected at constant temperature T_L. The *p–v* and *T–s* diagrams of an ideal Stirling cycle are shown in Fig. 11.6

If two constant volume processes of Stirling cycle are made regenerative i.e. heat rejected by

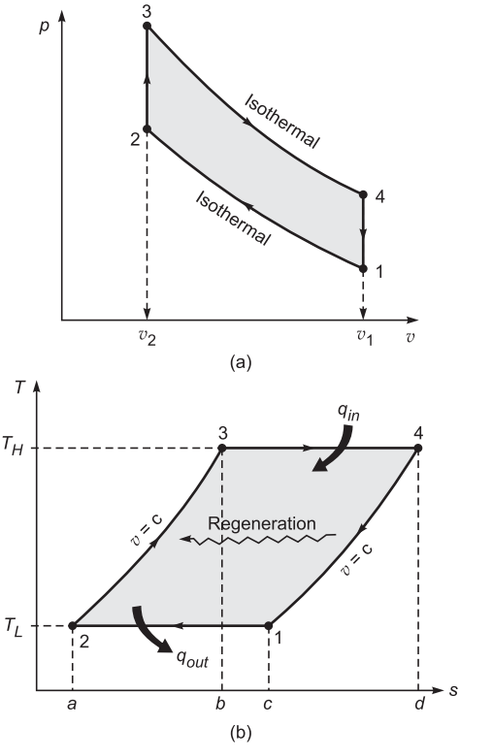

Fig. 11.6 *Stirling cycle on p–v and T–s diagrams*

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process 2–3 is given to process 4–1. Then net heat transfer with these two constant volume processes becomes zero,

Heat supplied per kg of air in the cycle

$$q_{in} = q_{3-4} = p_3 v_3 \ln\left(\frac{v_4}{v_3}\right) = RT_H \ln\left(\frac{v_4}{v_3}\right) \quad ...(i)$$

Heat rejected per kg of air in the cycle

$$q_{out} = q_{1-2} = p_1 v_1 \ln\left(\frac{v_1}{v_2}\right) = RT_L \ln\left(\frac{v_1}{v_2}\right)$$

Net work done in the cycle with ideal regeneration

$$w_{net} = \Sigma q = q_{in} - q_{out}$$
$$= RT_H \ln\left(\frac{v_4}{v_3}\right) - RT_L \ln\left(\frac{v_1}{v_2}\right)$$

Since processes 2–3 and 4–1 are constant volume processes

$$v_2 = v_3 \quad \text{and} \quad v_1 = v_4$$

$$\therefore \quad w_{net} = R(T_H - T_L) \ln\left(\frac{v_1}{v_2}\right)$$

Thermal efficiency of Stirling cycle

$$\eta_{Stirling} = \frac{w_{net}}{q_{in}} = \frac{R(T_H - T_L) \ln\left(\frac{v_1}{v_2}\right)}{RT_H \ln\left(\frac{v_1}{v_2}\right)}$$

$$= \frac{T_H - T_L}{T_H} = 1 - \frac{T_L}{T_H} \quad ...(11.8)$$

same as Carnot engine efficiency.

The Stirling engine was used earlier for hot air engines. But as Otto and Diesel cycles are developed and came into use, Stirling engine became obsolete. The design of Stirling engine involves practical difficulties of isothermal heat transfer. Still there is an interest in this cycle because it has potential of high thermal efficiency.

11.5 ERICSSON CYCLE

This cycle was also designed for hot air engine by John Ericsson by replacing two constant volume lines of Stirling cycle by two reversible constant pressure lines. Thus the Ericsson cycle consists of two isothermal and two reversible constant pressure processes. Heat addition and heat rejection take place at constant pressure as well at constant temperature processes. Fig 11.7 shows *p–v* and *T–s* diagrams of an Ericsson cycle.

It is evident from Fig. 11.7(b), that process 2–3 and 4–1 are parallel to each other on *T–s* diagram, therefore, the heat rejected during process 4–1 can be supplied regenerative to process 2–3. External heat is supplied during process 3–4 and heat is rejected to ambient during process 1–2. Then the efficiency of an Ericsson cycle is equal to Stirling or Carnot cycle engine i.e.

$$\eta_{Ericsson} = \eta_{Stirling} = \eta_{Carnot} = 1 - \frac{T_L}{T_H} \quad ...(11.9)$$

The Ericsson cycle uses smaller pressure ratio than Stirling and Carnot cycle. Ericsson cycle does not find any practical application in piston cylinder engines but it is approached by a gas turbine using a large number of stages with heat exchangers, and reheaters.

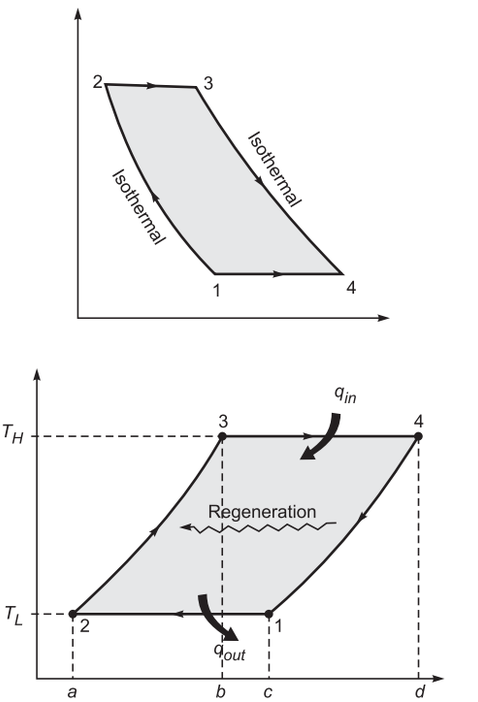

Fig. 11.7 *p–v and T–s diagrams of Ericsson cycle*

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11.6 OTTO CYCLE

The Otto cycle is an ideal air standard cycle for the gasolene (petrol) engine, gas engines and high-speed engines. The operations of the Otto cyle is shown in Fig. 11.8 and the *p–v* and *T–s* diagrams are shown in Fig. 11.9. The Otto cycle consists of four processes:

Process 1–2 Isentropic compression,

Process 2–3 Reversible constant volume heat addition,

Process 3–4 Isentropic expansion, and

Process 4–1 Reversible constant-volume heat rejection.

In an actual engine, the working substance becomes the air–fuel mixture and heat transfer processes are replaced by combustion process and exhaust process.

Analysis

For Otto cycle, per unit mass of air

Heat supplied, $q_{in} = q_{2-3} = C_v(T_3 - T_2)$
(a positive quantity)

Heat rejected $q_{out} = q_{4-1} = C_v(T_4 - T_1)$
(a negative quantity)

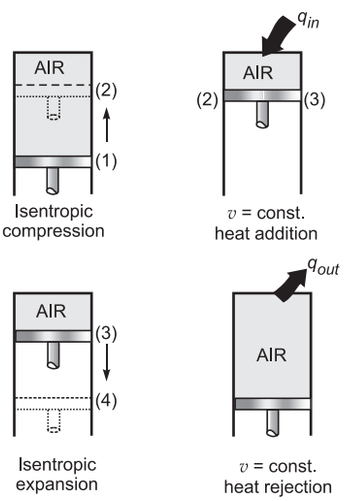
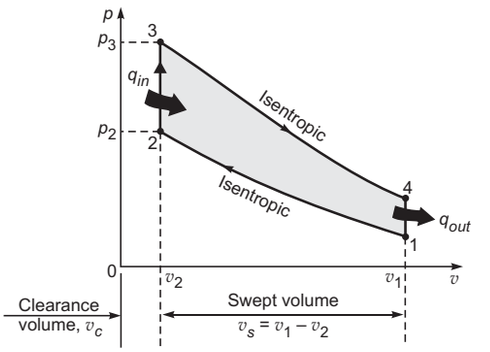
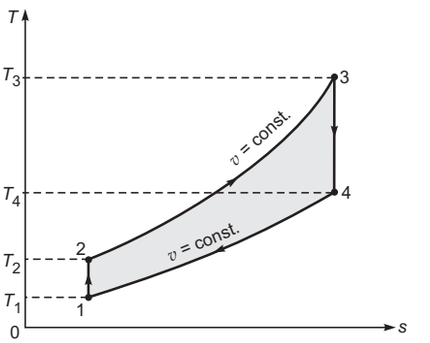


Fig. 11.8 *Operation*



(a) p–v diagram



(b) T–s diagram for air standard Otto Cycle

Fig. 11.9 *Otto cycle*

Net work done $= \Sigma q$ for a cycle

$$w_{net} = q_{in} - q_{out}$$
$$= C_v(T_3 - T_2) - C_v(T_4 - T_1)$$

The thermal efficiency of the cycle

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{q_{in} - q_{out}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

$$= 1 - \frac{C_v(T_4 - T_1)}{C_v(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

$$= 1 - \frac{T_1\left(\frac{T_4}{T_1} - 1\right)}{T_2\left(\frac{T_3}{T_2} - 1\right)} \quad \text{...(i)}$$

Considering the isentropic process 1–2,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = r^{\gamma - 1} \quad \text{...(ii)}$$

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where $r = \frac{v_1}{v_2}$, is called compression ratio.

For the isentropic process 3–4,

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma-1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = r^{\gamma-1} \quad ...(iii)$$

Equating Eqs. (ii) and (iii), we get

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{or} \quad \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

Using in Eq. (i), we get

$$\eta_{Otto} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{\frac{T_2}{T_1}} = 1 - \frac{1}{(r)^{\gamma-1}} \quad ...(11.10)$$

For air, the ratio of specific heats γ (gamma) is assumed to be constant, and thus the efficiency of the Otto cycle given by Eq. (11.10) is the function of compression ratio only. The efficiency of the Otto cycle increases with increase in compression ratio.

11.7 DIESEL CYCLE

The Diesel cycle is an ideal cycle for reciprocating *compression ignition* (*CI*) *engines*. It was proposed by Rudolph Diesel in 1892.

In gasolene engines, a mixture of air and fuel is compressed during the compression stroke, and thus the compression ratio is limited by engine knock (auto-ignition of charge). In a Diesel engine, only air is compressed during the compression stroke, thus eliminating the possibility of auto-ignition. Therefore, Diesel engines are designed to operate at a very high compression ratio, typically between 12 and 24.

In the air-standard Diesel cycle (shown in Fig. 11.10) the combustion process is approximated as a constant-pressure, heat-addition process. The remaining three processes are same for both Otto and Diesel cycles.

Process 1–2 Isentropic compression,

Process 2–3 Reversible constant pressure heat addition,

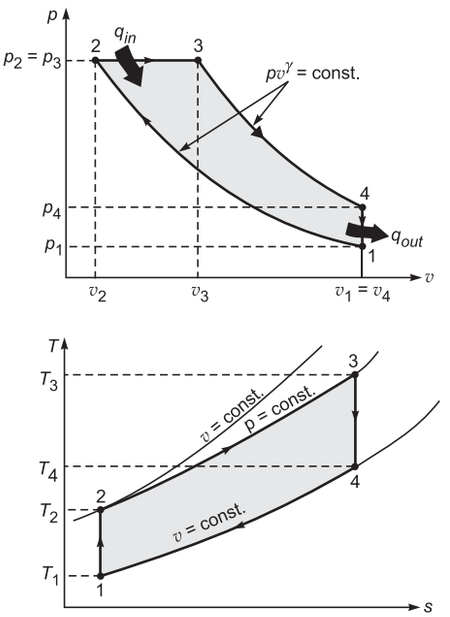

Fig. 11.10 *Air-Standard Diesel cycle*

Process 3–4 Isentropic expansion, and

Process 4–1 Reversible constant volume heat rejection.

Analysis

For 1 kg of air undergoing a Diesel cycle,

Heat supplied $q_{in} = q_{2-3} = C_p(T_3 - T_2)$

Heat rejected $q_{out} = q_{4-1} = C_v(T_4 - T_1)$

The efficiency of the cycle

$$\eta = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{C_v(T_4 - T_1)}{C_p(T_3 - T_2)}$$

$$\eta_{Diesel} = 1 - \left(\frac{T_4 - T_1}{\gamma(T_3 - T_2)}\right) \quad ...(11.11)$$

The efficiency of the Diesel cycle may be expressed in the following terms:

Compression ratio

$$r = \frac{V_1}{V_2} = \frac{v_1}{v_2}$$

$$= \frac{\text{Volume before compression}}{\text{Volume after compression}}$$

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Cut-off ratio

$$\rho = \frac{V_3}{V_2} = \frac{v_3}{v_2}$$

$$= \frac{\text{Volume after heat supply}}{\text{Volume after compression}}$$

Expansion ratio

$$r_e = \frac{V_4}{V_3} = \frac{v_4}{v_3}$$

$$= \frac{\text{Volume after expansion}}{\text{Volume before expansion}}$$

It can be proved that

$$r = r_e\, \rho \text{ as } \left(\frac{v_4}{v_3} \times \frac{v_3}{v_2} = \frac{v_1}{v_2} = r \right) \quad ...(11.12)$$

Now for the isentropic process 1–2,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2} \right)^{\gamma - 1} = r^{\gamma - 1}$$

or $$T_2 = T_1 r^{\gamma - 1} \quad ...(i)$$

For the constant-pressure heat-addition process 2–3,

$$\frac{T_3}{T_2} = \frac{v_3}{v_2} = \rho$$

or $$T_3 = \rho T_2 = \rho r^{\gamma - 1} T_1 \quad ...(ii)$$

For the isentropic process 3–4

$$\frac{T_4}{T_3} = \left(\frac{v_3}{v_4} \right)^{\gamma - 1}$$

$$= \left(\frac{v_3}{v_1} \right)^{\gamma - 1} = \left(\frac{v_3}{v_2} \times \frac{v_2}{v_1} \right)^{\gamma - 1}$$

or $$T_4 = \left(\frac{\rho}{r} \right)^{\gamma - 1} T_3 = \left(\frac{\rho}{r} \right)^{\gamma - 1} \rho r^{\gamma - 1}\, T_1$$

$$= \rho^{\gamma} T_1 \quad ...(iii)$$

Using the values of T_2, T_3 and T_4 from Eqs. (i), (ii) and (iii), respectively in Eq. (11.11), we get

$$\eta_{Diesel} = 1 - \frac{(\rho^{\gamma} - 1) T_1}{\gamma (\rho r^{\gamma - 1} - r^{\gamma - 1}) T_1}$$

$$= 1 - \frac{1}{r^{\gamma - 1}} \left[\frac{\rho^{\gamma} - 1}{\gamma (\rho - 1)} \right] \quad ...(11.13)$$

11.7.1 Effect of Cut-off Ratio on Diesel Cycle Efficiency

Since the cut-off ratio ρ is always greater than unity then the quantity

$\frac{1}{\gamma} \left(\frac{\rho^{\gamma} - 1}{\rho - 1} \right)$ is also greater than unity. Therefore, the efficiency of the Diesel cycle is less than that of the Otto cycle for the same compression ratio. However, since Diesel engines employ much higher compression ratio, thus their thermal efficiency is higher.

As cut-off ratio decreases, (Fig. 11.13) the efficiency of the Diesel cycle increases. For a cut-off ratio $\rho \approx 1$, the quantity in the bracket of Eq. (11.13) approaches unity and the efficiency of Otto and Diesel cycles becomes identical.

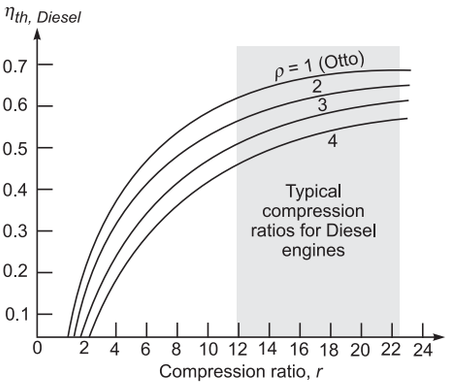

Fig. 11.11 *Effect of cut-off ratio and compression ratio on thermal efficiency of Diesel cycle*

11.8 DUAL CYCLE

In the Otto cycle, the combustion is assumed at constant volume, while in the Diesel cycle, the combustion is at constant pressure. In actual Diesel engine, the fuel injection starts before the end of compression stroke and thus a part of heat is added at constant volume and rest at constant pressure as shown in Fig. 11.12. Such cycle is referred as air standard *dual cycle* or *mixed cycle* or *limited pressure cycle*.

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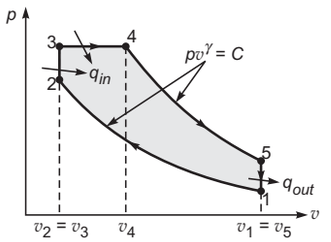
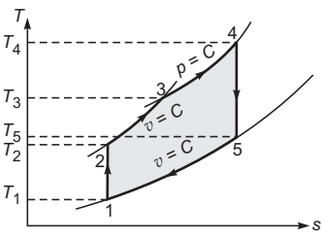


(a) p–v diagram



(b) T–s diagram

Fig. 11.12 *Air Standard Dual cycle*

Heat supplied, $q_{in} = C_v(T_3 - T_2) + C_p(T_4 - T_3)$

Heat rejected, $q_{out} = C_v(T_5 - T_1)$

Thermal efficiency $\eta = 1 - \dfrac{q_{out}}{q_{in}}$

$$= 1 - \frac{C_v(T_5 - T_1)}{C_v(T_3 - T_2) + C_p(T_4 - T_3)}$$

$$= 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)} \quad \ldots(11.14)$$

Considering the following ratio to express the efficiency of Dual cycle

Compression ratio $r = \dfrac{v_1}{v_2}$ …(i)

Cut-off ratio, $\rho = \dfrac{v_4}{v_3}$ …(ii)

Pressure ratio, $r_p = \dfrac{p_3}{p_2}$ …(iii)

Expansion ratio, $r_e = \dfrac{v_5}{v_4}$ …(iv)

For isentropic compression process 1–2;

$$T_2 = T_1 r^{\gamma-1} \quad \ldots(v)$$

For constant volume process 2–3,

$$\frac{T_3}{T_2} = \frac{p_3}{p_2} = r_p$$

or $$T_3 = r_p r^{\gamma-1} T_1 \quad \ldots(vi)$$

For constant pressure process 3–4,

$$\frac{v_4}{T_4} = \frac{v_3}{T_3}$$

or $$T_4 = \frac{v_4}{v_3} T_3 = \rho T_3$$

$$= \rho r_p r^{\gamma-1} T_1 \quad \ldots(vii)$$

For isentropic expansion process 4–5

$$\frac{T_5}{T_4} = \left(\frac{v_4}{v_5}\right)^{\gamma-1} = \left(\frac{v_4}{v_1}\right)^{\gamma-1} = \left(\frac{v_4}{v_3} \times \frac{v_3}{v_2} \times \frac{v_2}{v_1}\right)^{\gamma-1}$$

$$= \left(\frac{\rho}{r}\right)^{\gamma-1} \quad (\because v_3 = v_2)$$

or $$T_5 = \left(\frac{\rho}{r}\right)^{\gamma-1} \rho r_p r^{\gamma-1} T_1 = \rho^\gamma r_p T_1 \quad \ldots(viii)$$

Using the values for T_2, T_3, T_4 and T_5 in Eq. (11.14)

$$\eta_{Dual} = 1 - \frac{T_1(\rho^\gamma r_p - 1)}{T_1(r_p r^{\gamma-1} - r^{\gamma-1}) + \gamma T_1(\rho r_p r^{\gamma-1} - r_p r^{\gamma-1})}$$

$$= 1 - \frac{1}{r^{\gamma-1}}\left[\frac{r_p \rho^\gamma - 1}{(r_p - 1) + \gamma r_p(\rho - 1)}\right] \quad \ldots(11.15)$$

Therefore, the efficiency of Dual cycle given by Eq. (11.15) is function of r, ρ, r_p and γ. When

(i) $r_p = 1$, then $\eta_{Dual} = \rho_{Diesel}$

(ii) $\rho = 1$, $r_p = 1$, $\eta_{Dual} = \eta_{Otto}$

(iii) For same compression ratio and cut-off ratio as pressure r_p increases, thermal efficiency of Dual cycle increases.

Hence, modern Diesel engines are designed to operate more closely to Dual cycle.

11.9 COMPARISON OF OTTO AND DIESEL CYCLES

The three cycles can be compared on the basis of either the same compression ratio or the same maximum pressure and temperature.

11.9.1 For Same Compression Ratio

Figure 11.13 shows the comparison of three cycles on *p–v* and *T–s* diagrams.

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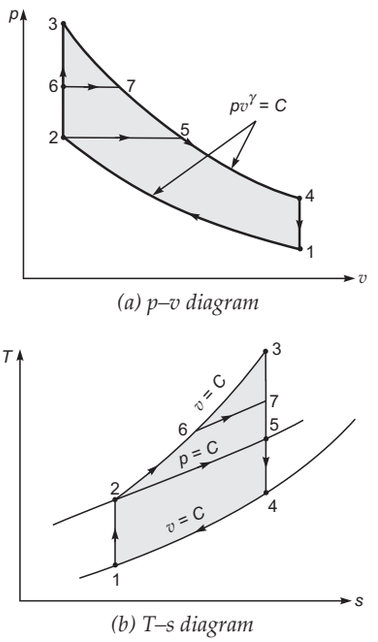

Fig. 11.13 *Comparison of Otto and Diesel cycles for same compression ratio*

Here,

Cycle 1–2–3–4–1 is Otto cycle
Cycle 1–2–5–4–1 is Diesel cycle
Cycle 1–2–6–7–5–4–1 is Dual cycle

It is evident from *p–v* and *T–s* diagrams, that the heat-rejection process 4–1 for Otto, Diesel and Dual cycles is same, but heat supply (area under 2–3 on the *T–s* diagram) and work done (area under 3–4 on the *p–v* diagram) are highest for Otto cycle. Thus, it is the most efficient cycle. Heat supply (process 2–5) and work done (process 2–5–4) for Diesel cycle are least, and thus it is the least efficient among the three cycles. Therefore,

$$\eta_{Otto} > \eta_{Dual} > \eta_{Diesel}$$

11.9.2 For Same Maximum Pressure and Temperature

Figure 11.14 shows the Otto cycle and Diesel cycle on *p–v* and *T–s* diagrams.

Cycle 1–2–5–4–1 is Otto cycle
Cycle 1–2–3–5–4–1 is Diesel cycle
Cycle 1–2–6–7–5–4–1 is Dual cycle

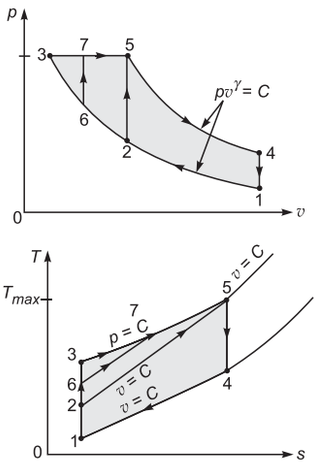

Fig. 11.14 *Comparison of Otto and Diesel cycles for same maximum pressure and temperature*

The peak pressure ($p_3 = p_5$), peak temperature T_5 and heat rejection (area under the curve 4–1 on the *T–s* diagram) are same for the three cycles.

The heat input for the Diesel cycle is the area under the curve 3–5, for Dual cycle, area under 6–7–5 and for the Otto cycle, it is the area under the curve 2–5 on the *T–s* diagram. It is evident that the heat supply in a Diesel cycle is more, and hence, it is the most efficient. The Otto cycle receives the least heat supply, and thus is the least efficient for same maximum pressure and temperature. Therefore,

$$\eta_{Diesel} > \eta_{Dual} > \eta_{Otto}$$

Otto Cycle

Example 11.1 *Calculate the ideal air-standard cycle efficiency of a petrol engine operating on Otto cycle. The cylinder bore is 50 mm, a stroke is of 75 mm and the clearance volume is of 21.3 cm^3.*

Solution

Given An air-standard Otto cycle engine

d = 50 mm = 5 cm
L = 75 mm 7.5 cm
V_c = 21.3 cm^3

To find The thermal efficiency of the cycle.

Assumptions

(i) The working substance is air, an ideal gas.
(ii) Cold air standard assumptions with $\gamma = 1.4$.

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Analysis The swept volume of the cylinder

$$V_s = \frac{\pi}{4} d^2 L$$

$$= \frac{\pi}{4} \times (5 \text{ cm})^2 \times (7.5 \text{ cm})$$

$$= 147.26 \text{ cm}^3$$

Total volume of the cylinder

$$V_1 = V_s + V_c$$

$$= 147.26 + 21.3 = 168.56 \text{ cm}^3$$

The compression ratio of the engine

$$r = \frac{V_1}{V_c} = \frac{168.56 \text{ cm}^3}{21.3 \text{ cm}^3} = 7.913$$

The thermal efficiency of the air-standard Otto cycle

$$\eta_{Otto} = 1 - \frac{1}{r^{\gamma - 1}}$$

$$= 1 - \frac{1}{(7.913)^{1.4-1}} = 1 - \frac{1}{2.287}$$

$$= \mathbf{0.56.28 \text{ or } 56.28\%}$$

Example 11.2 *In an engine working on an ideal Otto cycle, the temperatures at the beginning and at the end of compression are 27°C and 327°C. Find the compression ratio and air-standard efficiency of the engine.*

Solution

Given An engine working on ideal cycle with

$T_1 = 27°\text{C} = 300 \text{ K}$

$T_2 = 327°\text{C} = 600 \text{ K}$

To find

(i) Compression ratio of engine.

(ii) Thermal efficiency of the engine.

Assumptions

(i) Working substance is air.

(ii) Constant specific heats and its ratio as $\gamma = 1.4$.

Analysis

(i) The compression ratio is expressed as

$$r = \frac{V_1}{V_2} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma - 1}} = \left(\frac{600}{300}\right)^{\frac{1}{1.4-1}} = (2)^{2.5}$$

$$= \mathbf{5.65}$$

(ii) Thermal efficiency of Otto cycle

$$\eta_{Otto} = 1 - \frac{1}{r^{\gamma - 1}} = 1 - \frac{1}{(5.65)^{1.4-1}} = 0.5$$

$$= \mathbf{50\%}$$

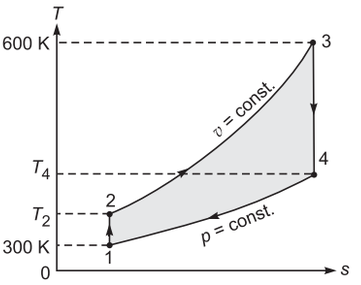

Fig. 11.15

Example 11.3 *The pressure and temperature of air at the beginning of compression in an Otto cycle is 103 kPa and 27°C, respectively. The heat added per kg of air is 1850 kJ. The compression ratio is 8. Determine maximum temperature, maximum pressure, thermal efficiency.*

Solution

Given An engine operates on Otto cycle with

$p_1 = 103 \text{ kPa}$,

$T_1 = 27°\text{C} = 300 \text{ K}$

Heat supplied, $q_{in} = 1850 \text{ kJ/kg}$

Compression ratio $r = 8$

To find

(i) Maximum temperature in the cycle.

(ii) Maximum pressure in the cycle.

(iii) Thermal efficiency of the cycle.

Assumptions

(i) Constant specific heat of air, $C_v = 0.718 \text{ kJ/kg K}$.

(ii) Ratio of two specific heats, $\gamma = 1.4$ and $R = 0.287 \text{ kJ/kg} \cdot \text{K}$.

(iii) 1 kg air in the cycle.

Analysis

(i) The temperature after isentropic compression

$$T_2 = T_1 r^{\gamma - 1} = 300 \times 8^{1.4-1} = 689.2 \text{ K}$$

Temperature of air after heat addition at constant volume

$$q_{in} = C_v (T_3 - T_2)$$

$$1850 = 0.718 \times (T_3 - 689.2)$$

or $T_3 = 3265.8 \text{ K} = 2992.8°\text{C}$

The maximum temperature in the cycle is **2992.8°C**

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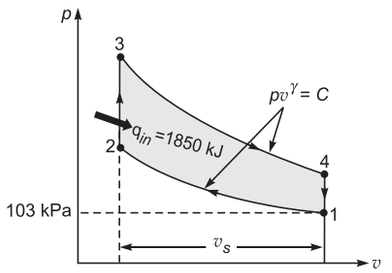

Fig. 11.16

(ii) The pressure after isentropic compression

$$p_2 = p_1\left(\frac{v_1}{v_2}\right)^{\gamma} = 103 \times (8)^{1.4} = 1893 \text{ kPa}$$

The pressure after constant-volume heat addition

$$p_3 = \frac{p_2}{T_2} \times T_3 = \frac{1893}{689.2} \times 3265.8$$

$$= 8970.3 \text{ kPa} \gg 89.7 \text{ bar}$$

The maximum pressure in the cycle is **89.7 bar**.

(iii) Thermal efficiency of the Otto cycle

$$\eta_{Otto} = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{8^{1.4-1}} = 0.564$$

$$= \mathbf{56.4\%}$$

Example 11.4 *A four-stroke, four-cylinder petrol engine of 250-mm bore and 375-mm stroke works on Otto cycle. The clearance volume is 0.01052 m^3. The initial pressure and temperature are 1 bar and 47°C. If the maximum pressure is limited to 25 bar, find the following:*

(a) The air standard efficiency of the cycle,

(b) The mean effective pressure.

Solution

Given Four-stroke petrol engine on Otto cycle

$d = 250 \text{ mm} = 0.25 \text{ m}$

$L = 375 \text{ mm} = 0.375 \text{ m}$

$V_c = 0.01052 \text{ m}^3$

$p_1 = 1 \text{ bar}$ $\quad T_1 = 47°\text{C} = 320 \text{ K}$

$p_3 = 25 \text{ bar}$ $\quad k = 4$

To find

(i) Air standard efficiency, and

(ii) The mean effective pressure.

Assumptions

(i) Air as working fluid in the engine.

(ii) Cold air assumptions, i.e.,

$C_p = 1.005 \text{ kJ/kg} \cdot \text{K}$

$C_v = 0.718 \text{ kJ/kg} \cdot \text{K}$

$\gamma = 1.4$

$R = 0.287 \text{ kJ/kg} \cdot \text{K}.$

Analysis The swept volume of all cylinders

$$V_s = (\pi/4)d^2Lk$$

$$= (\pi/4) \times (0.25 \text{ m})^2 \times (0.375 \text{ m}) \times 4$$

$$= 0.0736 \text{ m}^3$$

The total volume of the cylinder

$$V_1 = V_s + V_c = 0.0736 + 0.01052$$

$$= 0.08412 \text{ m}^3$$

The compression ratio

$$r = \frac{V_1}{V_c} = \frac{0.08412}{0.01052} = 8$$

(i) Air standard efficiency

$$\eta_{Otto} = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{(8)^{1.4-1}}$$

$$= 0.565 \quad \text{or} \quad \mathbf{56.5\%}$$

(ii) Temperature after isentropic compression

$$T_2 = T_1 r^{\gamma-1} = 320 \times (8)^{1.4-1}$$

$$= 735.17 \text{ K}.$$

Pressure after isentropic compression

$$p_2 = p_1 r^{\gamma} = (1 \text{ bar}) \times (8)^{1.4}$$

$$= 18.34 \text{ bar}$$

Temperature after heat addition at constant volume

$$T_3 = \frac{p_3}{p_2} \times T_2 = \left(\frac{25 \text{ bar}}{18.34 \text{ bar}}\right) \times (735.17 \text{ K})$$

$$= 1000 \text{ K}$$

The mass of air in the cylinders

$$m = \frac{p_1 V_1}{RT_1} = \frac{(100 \text{ kPa}) \times (0.08412 \text{ m}^3)}{(0.287 \text{ kJ/kg} \cdot \text{K}) \times (320 \text{ K})}$$

$$= 0.0922 \text{ kg}$$

The amount of heat added at constant volume

$$Q_{in} = m C_v (T_3 - T_2)$$

$$= 0.0922 \times 0.718 \times (1000 - 735.17)$$

$$= 17.53 \text{ kJ}$$

The work done per cycle

$$W_{net} = \eta_{Otto} Q_{in} = 0.565 \times 17.53$$

$$= 9.9 \text{ kJ}.$$

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The mean effective pressure of the cycle

$$p_m = \frac{W_{net}}{V_s} = \frac{9.9 \text{ kJ}}{0.0736 \text{ m}^2}$$

$$= 134.6 \text{ kN/m}^2 = \mathbf{1.346 \text{ bar}}$$

Example 11.5 *In an SI engine working on the ideal Otto cycle, the compression ratio is 5.5. The pressure and temperature at the beginning of compression are 1 bar and 27°C , respectively. The peak pressure is 30 bar. Determine the pressure, temperature at the salient points, the air standard efficiency, and mean effective pressure.*

Assume ratio of specific heat to be 1.4 for air.

Solution

Given An air-standard Otto cycle

$r = 5.5$ $\quad p_1 = 1 \text{ bar} = 100 \text{ kPa}$

$T_1 = 27°\text{C} = 300 \text{ K}$ $\quad p_2 = 30 \text{ bar}$

$\gamma = 1.4$

To find

(i) The pressure, temperature and volume at the salient points in the cycle.

(ii) Air standard efficiency.

(iii) Mean effective pressure.

Assumptions $C_p = 1.005 \text{ kJ/kg} \cdot \text{K}$, $C_v = 0.717 \text{ kJ/kg} \cdot \text{K}$ and $R = 0.287 \text{ kJ/kg} \cdot \text{K}$

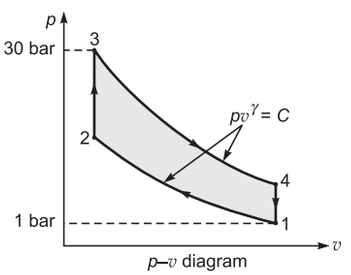

Fig. 11.17

Analysis

(i) *Pressure and temperature at salient points*

Isentropic compression process 1–2

Temperature $T_2 = T_1(r)^{\gamma-1} = 300 \times (5.5)^{1.4-1}$

$= 593.28 \text{ K}$ or $\mathbf{320.28°C}$

Pressure $p_2 = p_1 r^{\gamma} = 1 \times (5.5)^{1.4}$

$= \mathbf{10.877 \text{ bar}}$

Process 2–3: Constant-volume heat addition

$$T_3 = \frac{p_3}{p_2} T_2 = \frac{30}{10.877} \times 593.28$$

$= 1636.33 \text{ K}$ or $\mathbf{1363.33°C}$

$p_3 = p_{max} = 30 \text{ bar}$

Process 3–4: Isentropic expansion

$$T_4 = \frac{T_3}{(r)^{\gamma-1}} = \frac{1636.33}{(5.5)^{1.4-1}}$$

$= 827.42 \text{ K}$ or $\mathbf{554.42°C}$

$$p_4 = p_3\left(\frac{1}{r}\right)^{\gamma} = 30 \times \left(\frac{1}{5.5}\right)^{1.4} = \mathbf{2.758 \text{ bar}}$$

(ii) *The air-standard efficiency of an Otto Cycle*

$$\eta_{Otto} = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{(5.5)^{1.4-1}}$$

$= 0.494$ or $\mathbf{49.4\%}$

The heat supplied per kg of air

$q_{in} = C_v(T_3 - T_2)$

$= 0.717(1636.33 - 593.28)$

$= 747.86 \text{ kJ/kg}$

The net work done per kg of air

$w_{net} = \eta_{Otto}\, q_{in} = 0.494 \times 747.86$

$= 369.44 \text{ kJ/kg}$

The total volume of the cylinder at beginning of compression

$$v_1 = \frac{RT_1}{p_1} = \frac{0.287 \times 300}{100} = 0.861 \text{ m}^3/\text{kg}$$

$$p_m = \frac{w_{net}}{v_1 - v_2} = \frac{w_{net}}{v_1\left(1 - \frac{1}{r}\right)}$$

$$= \frac{369.44}{0.861 \times \left(1 - \frac{1}{5.5}\right)}$$

$= 524.44 \text{ kPa}$ or $\mathbf{5.24 \text{ bar}}$

Example 11.6 *An engine working on Otto cycle has a total volume of 0.45 m³, pressure 1 bar and temperature 27°C at the beginning of the compression stroke. At the end of the compression stroke, the pressure is 11 bar, and 210 kJ of heat is added at constant volume. Calculate*

(a) the pressure, temperature and volume at the salient points in the cycle,

(b) percentage clearance volume,

(c) net work done per cycle,

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(d) the ideal power developed by the engine if the number of working cycles per minute is 210

Assume $C_p = 1.005\ kJ/kg \cdot K$

and $C_v = 0.717\ kJ/kg \cdot K.$

Solution

Given An air-standard Otto cycle

$V_1 = 0.45\ m^3$ $\quad p_1 = 1\ bar = 100\ kPa$

$T_1 = 27°C = 300\ K,$ $\quad Q_{in} = 210\ kJ$

$p_2 = 11\ bar$

No. of working cycles = 210 per min.

$C_p = 1.005\ kJ/kg \cdot K$

$C_v = 0.717\ kJ/kg \cdot K$

To find

(i) The pressure, temperature and volume at the salient points in the cycle,

(ii) Percentage clearance volume,

(iii) Net work done per cycle,

(iv) The ideal power developed by the engine if the number of working cycles per minute is 210.

Analysis The ratio of specific heats

$$\gamma = \frac{C_p}{C_v} = \frac{1.005}{0.717} = 1.4$$

Specific gas constant

$$R = C_p - C_v = 1.005 - 0.717$$
$$= 0.288\ kJ/kg \cdot K$$

The mass of the air present in the cycle

$$m = \frac{p_1 V_1}{RT_1} = \frac{(100\ kPa) \times (0.45\ m^3)}{(0.288\ kJ/kg \cdot K) \times (300\ K)}$$
$$= 0.52\ kg/cycle.$$

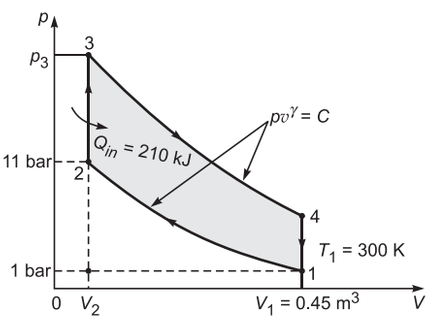

Fig. 11.18

(i) *Pressure, temperature and volume at salient points*

Isentropic compression process 1–2

$$V_2 = V_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} = 0.45 \times \left(\frac{1}{11}\right)^{\frac{1}{1.4}} = \mathbf{0.081\ m^3}$$

The compression ratio

$$r = \frac{V_1}{V_2} = \frac{0.45}{0.081} = 5.55$$

$$T_2 = T_1(r)^{\gamma - 1} = 300 \times (5.55)^{1.4-1}$$
$$= \mathbf{595.68\ K}\ \text{or}\ \mathbf{322.68°C}$$

Process 2–3: Constant volume heat addition

$$Q_{in} = m C_v (T_3 - T_2)$$

Rearranging and using numerical value

$$T_3 = 595.68 + \frac{210}{0.52 \times 0.717}$$
$$= \mathbf{1159\ K}\ \text{or}\ \mathbf{886°C}$$

For constant-volume process

$$V_3 = V_2 = \mathbf{0.081\ m^3}$$

and $\quad \frac{p_3}{T_3} = \frac{p_2}{T_2}$

or $\quad p_3 = \frac{11}{595.68} \times 1159 = \mathbf{21.4\ bar}$

Process 3–4: Isentropic expansion

$$T_4 = \frac{T_3}{(r)^{\gamma-1}} = \frac{1159}{(5.55)^{1.4-1}} = \mathbf{584\ K}$$

$$p_4 = p_3\left(\frac{V_3}{V_4}\right)^{\gamma} = p_3\left(\frac{1}{r}\right)^{\gamma} = 21.4 \times \left(\frac{1}{5.55}\right)^{1.4}$$
$$= \mathbf{1.94\ bar}$$

$$V_4 = V_1 = \mathbf{0.45\ m^3}$$

(ii) *Percentage clearance volume*

Swept volume, $\quad V_s = V_1 - V_2 = 0.45 - 0.081$
$$= \mathbf{0.369\ m^3}$$

Clearance volume, $\quad V_c = V_2 = 0.081\ m^3$

Percentage clearance volume,

$$c = \frac{V_c}{V_s} = \frac{0.081}{0.369} = \mathbf{0.219\ or\ 2.19\%}$$

(iii) *Net work done per cycle*

The thermal efficiency of an Otto Cycle

$$\eta_{Otto} = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{(5.55)^{1.4-1}} = 0.496$$

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The network done per cycle

$$W_{net} = \eta_{Otto}\, Q_{in} = 0.496 \times 210 = \mathbf{104.2\ kJ}$$

(iv) *Ideal power developed*

Power = Workdone per cycle × No. of working cycles per second.

$$= 104.2 \times \left(\frac{210}{60}\right) = \mathbf{364.68\ kW}$$

Example 11.7 *(a) An engine working on an air standard Otto cycle operates between upper and lower temperature limits T_3 and T_1. Prove that for maximum output from the cycle, the intermediate temperature is given by*

$$T_2 = T_4 = \sqrt{T_1 T_3}$$

(b) If the above engine has an upper temperature of 1450 K and a lower temperature of 310 K, calculate the maximum power developed by the engine, if air circulation per minute is 0.38 kg.

Solution

(a) Given An ideal Otto cycle with

Lower temperature = T_1

Upper temperature = T_3

To prove For maximum work output from the cycle

$$T_2 = T_4 = \sqrt{T_1 T_3}$$

Analysis The heat supplied per kg of air during the process 2–3 is

$$q_{in} = C_v(T_3 - T_2)$$

Heat rejected per kg of air during the process 4–1 is

$$q_{out} = C_v(T_4 - T_1)$$

Net work done in the cycle

$$w_{net} = \Sigma q = q_{in} - q_{out}$$
$$= C_v(T_3 - T_2) - C_v(T_4 - T_1)$$

where $T_2 = T_1 r^{\gamma-1}$ and $T_4 = \dfrac{T_3}{(r)^{\gamma-1}}$ ...(i)

Using, we get

$$w_{net} = C_v\left[T_3 - T_1 r^{\gamma-1} - \frac{T_3}{r^{\gamma-1}} + T_1\right]$$

Differentiating with respect to r and equating to zero

$$\frac{dw_{net}}{dr} = C_v[0 - (\gamma - 1)T_1 r^{\gamma-2} - T_3(1-\gamma)r^{1-\gamma-1} + 0]$$
$$= 0$$

$$(\gamma - 1)\, T_1 r^{\gamma-2} = (\gamma - 1)\, T_3 r^{-\gamma}$$

or $$r^{2\gamma-2} = \frac{T_3}{T_1}$$

or $$r = \left(\frac{T_3}{T_1}\right)^{\frac{1}{2(\gamma-1)}} \quad \text{...(ii)}$$

Substituting the value of r in Eq. (i)

$$T_2 = T_1\left(\frac{T_3}{T_1}\right)^{\frac{\gamma-1}{2(\gamma-1)}}$$
$$= T_1^{\frac{1}{2}}\, T_3^{\frac{1}{2}} = \sqrt{\mathbf{T_1 T_3}}$$

Similarly,

$$T_4 = \frac{T_3}{\left(\frac{T_3}{T_1}\right)^{\frac{\gamma-1}{2(\gamma-1)}}} = \sqrt{\mathbf{T_1 T_3}} \quad \textbf{Proved.}$$

(b) Given

$T_3 = 1450$ K,

$T_1 = 310$ K, $\dot{m} = 0.38$ kg/min

To find Maximum power of the cycle

Analysis The net work output per kg of the cycle

$$w_{net} = C_v(T_3 - T_2) - C_v(T_4 - T_1)$$

using $T_2 = T_4 = \sqrt{T_1 T_3} = \sqrt{310 \times 1450} = 670.44$ K

$C_v = 0.717$ kJ/kg·K

$$w_{net} = 0.717 \times (1450 - 670.44) - 0.717 \times (670.44 - 310)$$
$$= 300.5 \text{ kJ/kg}$$

The power developed by the cycle

$$P = \dot{m} w_{net}$$
$$= \left(\frac{0.38}{60}\text{kg/s}\right) \times (300.5\ \text{kJ/kg}) = \mathbf{1.90\ kW}$$

Example 11.8 *In an air-standard Otto cycle the pressure ratio during the compression is 15. The temperature of air at the beginning of compression is 37°C and maximum temperature attained in the cycle is 1950°C. Determine*

(a) Compression ratio,

(b) Thermal efficiency of the cycle.

(c) Work done.

Take $\gamma = 1.4$, $C_v = 0.717$ kJ/kg·K.

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Solution

Given An air-standard Otto cycle with

$p_2 = 15p_1$ $T_1 = 37°C = 310$ K

$T_3 = 1950°C = 2223$ K $\gamma = 1.4$

To find

(i) Compression ratio,

(ii) Thermal efficiency of the cycle

(iii) Work done.

Analysis

(i) Compression ratio

$$r = \frac{V_1}{V_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}}$$

$$= (15)^{\frac{1}{1.4}} = 6.919$$

(ii) Thermal efficiency of the cycle

$$\eta_{Otto} = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{(6.919)^{1.4-1}} = 0.5387$$

$$= \mathbf{53.8\%}$$

(iii) Work done

$$T_2 = T_1 r^{\gamma-1} = 310 \times (6.919)^{0.4} = 672 \text{ K}$$

Heat supplied per kg of air

$$q_s = C_v(T_3 - T_2)$$

$$= 0.717 \times (2223 - 672) = 1112 \text{ kJ/kg}$$

Work done = $\eta_{Otto} \times q_s$

$$= 0.538 \times 1112 = \mathbf{598.3\ kJ/kg}$$

Example 11.9 *(a) Prove that the compression ratio corresponding to the maximum work in the Otto cycle between upper and lower limits of absolute temperature T_3 and T_1, respectively is given by*

$$r = \left(\frac{T_3}{T_1}\right)^{\frac{1}{2(\gamma-1)}}$$

(b) Calculate the air standard efficiency of cycle, when it develops maximum work and operates between 300 K and 1200 K as minimum and maximum temperatures, respectively with working fluid as air.

(c) Calculate the change in efficiency, if helium is used in Otto cycle? The cycle operates between same temperature limits for maximum work output.

For air $\gamma = 1.4$

For helium take $C_p = 5.22$ kJ/kg·K, $C_v = 3.13$ kJ/kg·K

Solution

(a) Given An air standard Otto cycle operating for maximum work output between minimum temperature T_1 and maximum temperature T_3.

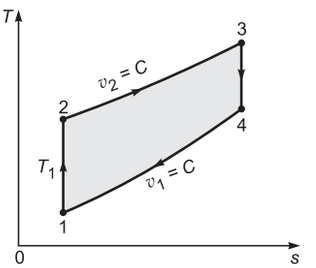

Fig. 11.19

To find To prove compression ratio as

$$r = \left(\frac{T_3}{T_1}\right)^{\frac{1}{2(\gamma-1)}}$$

Analysis The work done per kg of air in an Otto cycle

$$W_{net} = q_{in} - q_{out}$$

$$= C_v(T_3 - T_2) - C_v(T_4 - T_1)$$

$$= C_v(T_3 - T_2 - T_4 + T_1)$$

$$= C_v T_1\left(\frac{T_3}{T_1} - \frac{T_2}{T_1} - \frac{T_4}{T_1} + 1\right)$$

Here, $\frac{T_2}{T_1} = r^{\gamma-1}$

$$T_4 = \frac{T_3}{r^{\gamma-1}}$$

For given T_1 and T_3, the work done will be maximum, when

$$\frac{dw_{net}}{dr} = 0$$

$$0 = \frac{d}{dr}\left[C_v T_1\left\{\frac{T_3}{T_1} - r^{\gamma-1} - \frac{T_3}{T_1}\cdot\frac{1}{r^{\gamma-1}} + 1\right\}\right]$$

$$= C_v T_1\left\{0 - (\gamma-1)r^{\gamma-2} - \frac{T_3}{T_1}(1-\gamma)r^{-\gamma} + 0\right\}$$

or $r^{\gamma-2} = \frac{T_3}{T_1} r^{-\gamma}$

or $r^{2(\gamma-1)} = \frac{T_3}{T_1}$

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or $$r = \left(\frac{T_3}{T_1}\right)^{\frac{1}{2(\gamma-1)}}$$ **Proved**

(b) Given For air $T_1 = 300$ K,
$T_3 = 1200$ K,
$\gamma = 1.4$

To find Air standard efficiency of Otto cycle.

Analysis The compression ratio for maximum work

$$r = \left(\frac{T_3}{T_1}\right)^{\frac{1}{2(\gamma-1)}}$$

$$= \left(\frac{1200}{300}\right)^{\frac{1}{2(1.4-1)}} = 5.656$$

The air standard efficiency of Otto cycle

$$\eta_{Otto} = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{(5.656)^{0.4}}$$

$$= 0.5 = \mathbf{50\%}$$

(c) Given For helium

$T_1 = 300$ K, $T_3 = 1200$ K,
$C_p = 5.22$ kJ/kg·K $C_v = 3.13$ kJ/kg·K

To find Air standard efficiency of Otto cycle.

Analysis The ratio of specific heats

$$\gamma = \frac{C_p}{C_v} = \frac{5.22}{3.13} = 1.667$$

The compression ratio for maximum work

$$r = \left(\frac{T_3}{T_1}\right)^{\frac{1}{2(\gamma-1)}}$$

$$= \left(\frac{1200}{300}\right)^{\frac{1}{2(1.667-1)}} = 2.823$$

The air standard efficiency of Otto cycle

$$\eta_{Otto} = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{(2.823)^{0.667}}$$

$$= 0.5 = \mathbf{50\%}$$

There is no change in air standard efficiency of Otto cycle by changing the fluid.

Diesel Cycle

Example 11.10 *Determine the air-standard efficiency of the Diesel engine having a cylinder with a bore of 250 mm, a stroke of 375 mm and a clearance volume of 1500 cc, with fuel cut off occurring at 5% of the stroke. Assume $\gamma = 1.4$.*

Solution

Given Diesel engine with
Bore $d = 250$ mm $= 25$ cm
Stroke $L = 375$ mm $= 37.5$ cm
Clearance volume $V_c = V_2 = 1500$ cc
$\gamma = 1.4$
Fuel cut–off = 5% of stroke

To find Air-standard efficiency of Diesel cycle.

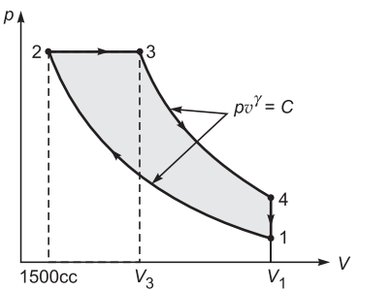

Fig. 11.20

Analysis The piston displacement (swept volume)

$$V_s = (\pi/4)\, d^2 L$$

$$= (\pi/4) \times (25)^2 \times (37.5) = 18408 \text{ cc}$$

Total volume $V_1 = V_s + V_c = 18408 + 1500$

$$= 19908 \text{ cc}$$

Compression ratio, $r = \frac{V_1}{V_2} = \frac{19908}{1500} = 13.27$

Since $V_3 = V_2 + 0.05\, V_s$

$$= 1500 + 0.05 \times 18408$$

Cut-off volume $V_3 = 2420.4$

Cut-off ratio $\rho = \frac{V_3}{V_2} = \frac{2420.4}{1500} = 1.613$

The air standard efficiency of the cycle

$$\eta_{Diesel} = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{\rho^\gamma - 1}{\gamma(\rho-1)}\right]$$

$$= 1 - \frac{1}{(13.27)^{1.4-1}} \times \left[\frac{(1.613)^{1.4} - 1}{1.4 \times (1.613 - 1)}\right]$$

$$= 0.6052 = \mathbf{60.52\%}$$

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Example 11.11 *A Diesel engine has a compression ratio of 18 and cut off takes place at 5% of the stroke. Calculate the air-standard efficiency. Take $\gamma = 1.4$.*

Solution

Given An air-standard Diesel cycle

$r = 18$

Cut off $= 0.05$ of stroke (V_s)

$\gamma = 1.4$

To find Air-standard efficiency of the Diesel cycle.

Analysis The compression ratio is given as

$$r = \frac{V_1}{V_2} \quad \text{or} \quad V_1 = rV_2$$

$$\therefore \quad V_1 = 18V_2$$

The swept volume

$$V_s = V_1 - V_2 = 18V_2 - V_2 = 17V_2$$

The cut-off volume

$$V_3 = V_2 + 0.05V_s$$
$$= V_2 + 0.05 \times 17V_2 = 1.85V_2$$

The cut-off ratio, $r = \dfrac{V_1}{V_2} = 1.85$

The air standard Diesel cycle efficiency, Eq. (11.13)

$$\eta_{Diesel} = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{\rho^{\gamma}-1}{\gamma(\rho-1)}\right]$$

$$= 1 - \frac{1}{(18)^{1.4-1}} \times \left[\frac{(1.85)^{1.4}-1}{1.4 \times (1.8\ 5-1)}\right]$$

$$= \mathbf{0.6387} \text{ or } \mathbf{63.877\%}$$

Example 11.12 *An air-standard Diesel cycle has a compression ratio of 14. The pressure at the beginning of the compression stroke is 1 bar and the temperature is 300 K. The maximum cycle temperature is 2500 K. Determine the cut-off ratio and thermal efficiency.*

Solution

Given An air-standard Diesel Cycle

$T_1 = 300$ K, $p_1 = 1$ bar

$r = 14$ $T_3 = 2500$ K

To find

(i) Cut-off ratio

(ii) Air-standard efficiency of the Diesel cycle

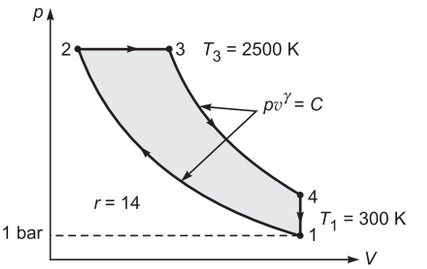

Fig. 11.21

Analysis The temperature after isentropic compression

$$T_2 = T_1(r)^{\gamma-1}$$
$$= 300 \times (14)^{1.4-1} = 862.13 \text{ K}.$$

(i) The cut-off ratio after constant-pressure heat addition

$$\rho = \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{2500}{862.13}$$
$$= \mathbf{2.9}$$

(ii) The thermal efficiency of an air-standard Diesel Cycle.

$$\eta_{Diesel} = 1 - \frac{1}{(r)^{\gamma-1}}\left[\frac{\rho^{\gamma}-1}{\gamma(\rho-1)}\right]$$

$$= 1 - \frac{1}{(14)^{1.4-1}} \times \left[\frac{2.9^{1.4}-1}{1.4 \times (2.9-1)}\right]$$

$$= 0.55 \text{ or } \mathbf{55\%}$$

Example 11.13 *An engine works on a Diesel cycle with an inlet pressure and temperature of 1 bar and 17°C. The pressure at the end of the adiabatic compression is 35 bar. The ratio of expansion, i.e., after constant-pressure heat addition is 5. Calculate the heat addition, heat rejection and efficiency of the cycle.*

Assume $\gamma = 1.4$

$C_p = 1.004$ kJ/kg·K

$C_v = 0.717$ kJ/kg·K

Solution

Given An engine works on the Diesel cycle

$p_1 = 1$ bar $T_1 = 17°\text{C} = 290$ K

$p_3 = p_2 = 35$ bar $r_e = 5$

$\gamma = 1.4$ $C_p = 1.004$ kJ/kg·K.

$C_v = 0.717$ kJ/kg·K

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To find

(i) Heat addition in the cycle,
(ii) Heat rejection in the cycle, and
(iii) Efficiency of the cycle.

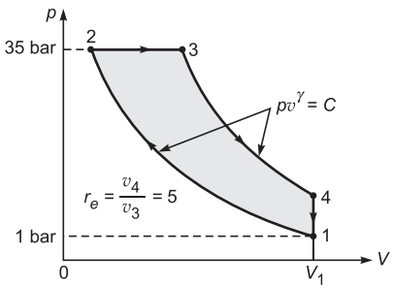

Fig. 11.22 *p–V diagram*

Analysis The temperature after adiabatic compression

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 290 \times \left(\frac{35}{1}\right)^{\frac{1.4-1}{1.4}} = 801 \text{ K}$$

The pressure after heat addition

$$p_3 = p_2 = 35 \text{ bar}$$

The pressure after adiabatic expansion

$$p_4 = \frac{p_3}{r_e^{\gamma}} = \frac{35 \text{ bar}}{(5)^{1.4}} = 3.677 \text{ bar}$$

For the constant-volume process 4 – 1

$$T_4 = \frac{p_4}{p_1}T_1 = \frac{3.677}{1} \times 290 = 1066.37 \text{ K}$$

For the isentropic expansion process 3–4

$$T_3 = T_4(r_e)^{\gamma-1} = 1066.37 \times (5)^{1.4-1} = 2030 \text{ K}$$

(i) The heat addition per kg of air in the cycle

$$q_{in} = C_p(T_3 - T_2)$$
$$= 1.004 \times (2030 - 801)$$
$$= \mathbf{1233.9 \text{ kJ/kg}}$$

(ii) Heat rejection per kg of air in the cycle

$$q_{out} = C_v(T_4 - T_1)$$
$$= 0.717 \times (1066.37 - 290)$$
$$= \mathbf{556.67 \text{ kJ/kg}}$$

(iii) Efficiency of the cycle

$$\eta_{Cycle} = \frac{\text{Net work done}}{\text{Heat sypplied}} = \frac{q_{\text{in}} - q_{\text{out}}}{q_{\text{in}}}$$
$$= 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{556.67}{1233.9}$$
$$= 0.588 \text{ or } \mathbf{54.88\%}$$

Example 11.14 *Calculate the percentage loss in air standard efficiency of a Diesel engine with compression ratio 14 and if fuel cut-off is delayed from 5% to 8%.*

Solution

Given A Diesel engine working on an air standard Diesel cycle.

$$r = 14 \quad \text{and} \quad \frac{v_3 - v_2}{v_s} = 0.05 \text{ and } 0.08$$

To find Percentage loss in air standard Diesel efficiency.

Assumption

(i) The working fluid in piston cylinder assembly as a closed system.
(ii) Compression and expansion are isentropic.
(iii) Working substance is air as an ideal gas with $\gamma = 1.4$.
(iv) Heat addition at constant pressure.

Analysis When fuel cut-off takes place at 5% of the swept volume

$$\frac{v_3 - v_2}{v_s} = \frac{v_3 - v_2}{v_1 - v_2} = 0.05$$

or

$$\frac{\frac{v_3}{v_2} - 1}{\frac{v_1}{v_2} - 1} = \frac{\rho - 1}{r - 1} = 0.05$$

or

$$\rho = 1 + 0.05 \times (14 - 1) = 1.65$$

The air standard Diesel cycle efficiency

$$\eta_{Diesel} = 1 - \frac{1}{r^{\gamma-1}} \times \left(\frac{\rho^{\gamma} - 1}{\gamma(\rho - 1)}\right)$$
$$= 1 - \frac{1}{14^{0.4}} \times \left(\frac{1.65^{1.4} - 1}{1.4 \times (1.6\ 5 - 1)}\right)$$
$$= 0.611 \text{ or } 61.1\%$$

When fuel cut-off takes place at 8% of the swept volume

$$\rho = 1 + 0.08(14 - 1) = 2.04$$

Cycle and air-standard Diesel cycle efficiency is

$$\eta_{Diesel} = 1 - \frac{1}{14^{0.4}} \times \left(\frac{2.04^{1.4} - 1}{1.4 \times (2.04 - 1)}\right)$$
$$= 0.590 = 59\%$$

Percentage change = 61.1% – 59% = **2.1%**

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Example 11.15 *An engine operates on air standard Diesel cycle. The pressure and temperatures at the beginning of compression are 100 kPa and 27°C. The compression ratio is 18. The heat added per kg of air is 1850 kJ. Determine maximum pressure, maximum temperature, thermal efficiency, network done and mean effective pressure of the cycle. Assume $\gamma = 1.4$ and $C_p = 1.005$ kJ/kg·K.*

Solution

Given An engine operates on air standard Diesel cycle with

$p_1 = 100$ kPa, $T_1 = 27°C = 300$ K,
$r = 18$, $q_{in} = 1850$ kJ/kg,
$\gamma = 1.4$ $C_p = 1.005$ kJ/kg·K

To find

(i) Maximum pressure in the cycle,
(ii) Maximum temperature in the cycle,
(iii) Thermal efficiency,
(iv) The net work done of the cycle, and
(v) The mean effective pressure.

Schematic *p–v* and *T–s* diagrams are shown in Fig. 11.23.

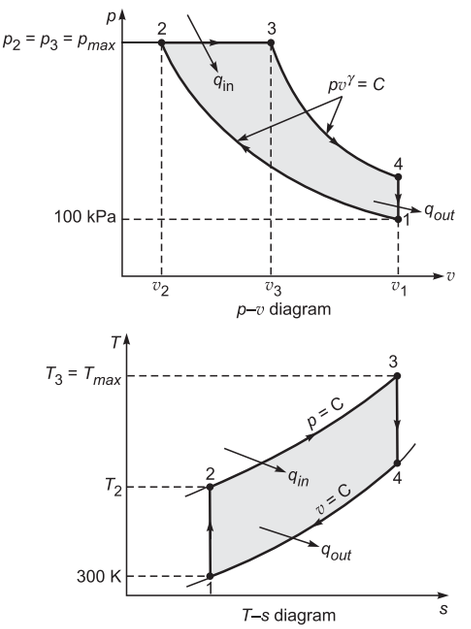

Fig. 11.23

Analysis

(i) Maximum pressure in the cycle as $p_2 = p_3 = p_{max}$

$$p_2 = p_1\left(\frac{v_1}{v_2}\right)^\gamma = p_1 r^\gamma$$
$$= (100 \text{ kPa}) \times (18)^{1.4} = 5720 \text{ kPa.}$$
$$= \mathbf{57.2 \text{ bar}}$$

(ii) Maximum temperature in the cycle, T_3

Temperature after compression

$$T_2 = T_1 r^{\gamma-1} = 300 \times (18)^{1.4-1}$$
$$= 953.3 \text{ K.}$$

Since 1850 kJ heat is added per kg of air at constant pressure, thus,

$$q_{in} = C_p(T_3 - T_2)$$

or $$1850 = 1.005 \times (T_3 - 953.3)$$

or $$T_3 = \frac{1850}{1.005} + 953.3$$
$$= 2794 \text{ K} = 2511°C$$

Thus, the maximum temperature is **2521°C**

(iii) Thermal efficiency of Diesel cycle

For constant pressure process.

$$\frac{v_2}{T_2} = \frac{v_3}{T_3}$$

or $$\rho = \frac{v_3}{v_2} = \frac{T_3}{T_2} = \frac{2794}{953.5} = 2.93$$

Using Eq. (11.13)

$$\eta_{Diesel} = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{\rho^\gamma - 1}{\gamma(\rho - 1)}\right]$$
$$= 1 - \frac{1}{(18)^{1.4-1}} \times \left[\frac{(2.93)^{1.4} - 1}{1.4 \times (2.9\,3 - 1)}\right]$$
$$= \mathbf{0.592 \text{ or } 58.2\%}$$

(iv) The net work done of the Diesel cycle

$$w_{net} = \eta_{Diesel} \times q_{in} = 0.592 \times 1850$$
$$= \mathbf{1095 \text{ kJ/kg}}$$

(v) The mean effective pressure

The specific heat at constant volume

$$C_v = \frac{C_p}{\gamma} = \frac{1.005}{1.4} = 0.718 \text{ kJ/kg·K}$$

The gas constant for air,

$$R = C_p - C_v = 1.005 - 0.718$$
$$= 0.287 \text{ kJ/kg·K}$$

Initial specific volume of air,

$$v_1 = \frac{RT_1}{p_1} = \frac{0.287 \times 300}{100}$$
$$= 0.861 \text{ m}^3\text{/kg}$$

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The volume after compression,

$$v_2 = \frac{v_1}{18} = \frac{0.861}{18} = 0.478 \text{ m}^3/\text{kg}$$

Swept volume,

$$v_s = v_1 - v_2 = 0.861 - 0.0478 = 0.813 \text{ m}^3/\text{kg}$$

Using Eq. (11.5);

$$p_m = \frac{w_{net}}{v_s} = \frac{1095}{0.813}$$

$$= 1346.5 \text{ kPa} \approx \mathbf{13.46 \text{ bar}}$$

Example 11.16 *In an air-standard Diesel engine cycle with a compression ratio of 14, the condition of air at the start of the compression stroke are 1 bar and 300 K. After addition of heat at constant pressure, the temperature rises to 2775 K. Determine the thermal efficiency of the cycle, net work done per kg of air and the mean effective pressure.*

Given An air-standard Diesel cycle

$T_1 = 300$ K, $p_1 = 1$ bar

$r = 14$ $T_3 = 2775$ K

To find

(i) Air-standard efficiency of the Diesel cycle,

(ii) Work done per kg of air, and

(iii) Mean effective pressure.

Assumptions

(i) Air as working fluid in the engine.

(ii) Cold air assumptions, i.e.,

$C_p = 1.005$ kJ/kg·K

$C_v = 0.718$ kJ/kg·K

$\gamma = 1.4$

$R = 0.287$ kJ/kg·K

Analysis The temperature after isentropic compression

$$T_2 = T_1(r)^{\gamma-1}$$

$$= 300 \times (14)^{1.4-1} = 862.13 \text{ K}.$$

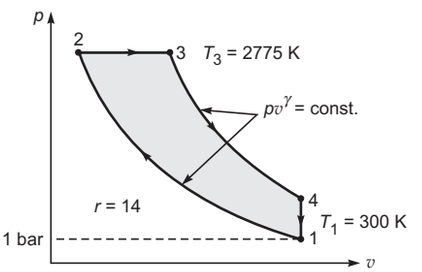

Fig. 11.24

The cut-off ratio after constant-pressure heat addition

$$\rho = \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{2775}{862.13} = 2.638$$

(i) *The thermal efficiency of an air-standard Diesel cycle*

$$\eta_{Diesel} = 1 - \frac{1}{(r)^{\gamma-1}}\left[\frac{\rho^\gamma - 1}{\gamma(\rho - 1)}\right]$$

$$= 1 - \frac{1}{(14)^{1.4-1}} \times \left[\frac{(2.638)^{1.4} - 1}{1.4 \times (2.638 - 1)}\right]$$

$$= 0.561 \text{ or } \mathbf{56.1\%}$$

The heat addition per kg of air in the cycle

$$q_{in} = C_p(T_3 - T_2)$$

$$= 1.005 \times (2775 - 862.13)$$

$$= 1922.43 \text{ kJ/kg}$$

(ii) *Work done per kg of air*

$$w_{net} = \eta_{Diesel}\, q_{in} = 0.561 \times 1922.43$$

$$= \mathbf{1078.48 \text{ kJ/kg}}$$

(iii) *Mean effective pressure*

The total volume of the cylinder at the beginning of compression

$$v_1 = \frac{RT_1}{p_1} = \frac{0.287 \times 300}{100}$$

$$= 0.861 \text{ m}^3/\text{kg}$$

$$p_m = \frac{w_{net}}{v_1 - v_2} = \frac{w_{net}}{v_1\left(1 - \frac{1}{r}\right)}$$

$$= \frac{1078.48}{0.861\left(1 - \frac{1}{14}\right)}$$

$$= 1348.94 \text{ kPa or } \mathbf{13.49 \text{ bar}}$$

Example 11.17 *Consider an ideal Diesel cycle. At the beginning of the compression process, the cylinder volume is 1500 cm^3 and at the end of the heat addition process, it is 150 cm^3. The compression ratio is 15. Air is at 101 kPa and 20°C at the beginning of the compression process. Calculate*

(a) Pressure at the beginning of heat rejection process,

(b) Net work per cycle in kJ,

(c) The mean effective pressure of the cycle.

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Solution

Given An ideal Diesel cycle with

$p_1 = 101$ kPa $\quad T_1 = 20°C = 293$ K

$r = 15 \quad V_1 = 1500$ cm^3

$V_3 = 150$ cm^3

To find

(i) The pressure at end of heat rejection,

(ii) The net work done in kJ, and

(iii) Mean effective pressure.

Assumptions

(i) Air as working fluid in the engine.

(ii) Cold air assumptions, i.e.,

$C_p = 1.005$ kJ/kg·K

$C_v = 0.718$ kJ/kg·K

$\gamma = 1.4$

$R = 0.287$ kJ/kg·K

Schematic *p–V* diagram is shown in Fig. 11.25.

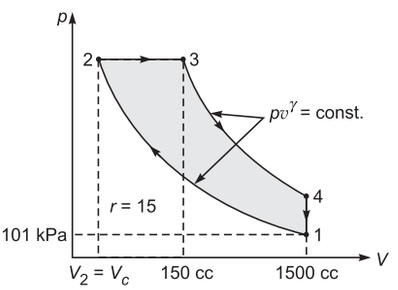

Fig. 11.25

Analysis The mass of the air inducted per cycle

$$m = \frac{p_1 V_1}{R T_1} = \frac{(101\text{ kPa}) \times (1500 \times 10^{-6}\text{ m}^3)}{(0.287\text{ kJ/kg}\cdot\text{K}) \times (293\text{ K})}$$

$$= 1.80 \times 10^{-3}\text{ kg/cycle.}$$

The volume at the end of compression

$$V_2 = \frac{V_1}{r} = \frac{1500\text{ cc}}{15} = 100\text{ cc}$$

The cut-off ratio $\rho = \dfrac{V_3}{V_2} = \dfrac{150\text{ cc}}{100\text{ cc}} = 1.5$

Temperature after isentropic compression 1–2;

$$T_2 = T_1 r^{\gamma - 1} = 293 \times (15)^{1.4-1} = 865.57\text{ K.}$$

The pressure at the end of compression;

$$p_2 = p_1 \left(\frac{v_1}{v_2}\right)^\gamma = p_1 r^\gamma$$

$$= (101\text{ kPa}) \times (15)^{1.4}$$

$$= 4475.58\text{ kPa or } 44.75\text{ bar}$$

Pressure at the end of heat addition;

$p_3 = p_2 = p_{max} = 44.75$ bar

Temperature T_3 at the end of constant-volume heat addition 2–3;

$$T_3 = \rho T_2 = 1.5 \times 865.57 = 1298.37\text{ K.}$$

Temperature T_4 at the end of isentropic expansion;

$$T_4 = \rho^\gamma T_1 = (1.5)^{1.4} \times 293 = 516.88\text{ K.}$$

(i) *The pressure at the end of isentropic expansion 3–4*

$$p_4 = p_3 \left(\frac{v_3}{v_4}\right)^\gamma = p_2 \left(\frac{v_3}{v_2} \times \frac{v_2}{v_1}\right)^\gamma = p_2 \left(\frac{\rho}{r}\right)^\gamma$$

$$= 44.75 \times \left(\frac{1.5}{15}\right)^{1.4} = \mathbf{1.78\text{ bar}}$$

(ii) *The net work done of the cycle*

Heat supplied per kg of air

$$q_{in} = C_p(T_3 - T_2)$$

$$= 1.005 \times (1298.37 - 865.57)$$

$$= 434.96\text{ kJ/kg}$$

Heat rejected per kg of air

$$q_{out} = C_p(T_4 - T_1)$$

$$= 1.005 \times (516.88 - 293)$$

$$= 225\text{ kJ/kg}$$

Net work done per cycle

$$W_{net} = m(q_{in} - q_{out})$$

$$= 1.8 \times 10^{-3} \times (434.95 - 225)$$

$$= \mathbf{0.38\text{ kJ}}$$

(iii) *The mean effective pressure*

$$p_m = \frac{W_{net}}{V_s} = \frac{W_{net}}{V_1 - V_2}$$

$$= \frac{0.38\text{ kJ}}{(1500 - 100) \times 10^{-6}}$$

$$= 269.95 \approx \mathbf{2.7\text{ bar}}$$

Example 11.18 *An engine working on an air standard Diesel cycle operates between minimum and maximum temperatures of the cycle as T_1 and T_3, respectively. If this engine operate for its maximum power, find the expression for its compression ratio in terms of the given parameters.*

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Solution The net work done in the Diesel cycle is given as

$$w_{net} = C_p(T_3 - T_2) - C_v(T_4 - T_1) \quad ...(i)$$

where $T_2 = T_1 r^{\gamma-1}$

and $$T_4 = \frac{T_3}{(r_e)^{\gamma-1}} = T_3\left(\frac{\rho}{r}\right)^{\gamma-1}$$

Substituting in Eq. (i)

$$w_{net} = C_p[T_3 - T_1 r^{\gamma-1}] - C_v\left[T_3\left(\frac{\rho}{r}\right)^{\gamma-1} - T_1\right] \quad ...(ii)$$

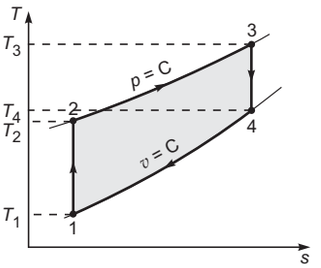


Fig. 11.26

For maximum work output of the engine, differentiating Eq. (ii) with respect to r, and equating it to zero,

$$\frac{dw_{net}}{dr} = -C_p T_1(\gamma - 1)r^{\gamma-2} - C_v T_3(\rho)^{\gamma-1}(1-\gamma)r^{1-\gamma-1} = 0$$

or $$\frac{C_p}{C_v}T_1(\gamma-1)r^{\gamma-2} = T_3(\rho)^{\gamma-1}(\gamma-1)r^{-\gamma}$$

or $$r^{\gamma-2+\gamma} = \frac{T_3}{T_1}\frac{\rho^{\gamma-1}}{\gamma} \qquad \left[\because \frac{C_p}{C_v} = \gamma\right]$$

or $$r = \left[\frac{T_3}{T_1}\frac{\rho^{\gamma-1}}{\gamma}\right]^{\frac{1}{2(\gamma-1)}}$$

It is the required relation for compression ratio, when cycle produces maximum power.

Dual Cycle

Example 11.19 *In an air standard dual cycle, the pressure and temperature are 0.1 MPa and 27°C. The compression ratio is 18. The pressure ratio for the constant volume part of heating process is 1.5 and the volume ratio for the constant pressure part of heating is 1.2, determine, (a) thermal efficiency, (b) mean effective pressure in MPa.*

Solution

Given An air standard dual cycle with

$p_1 = 0.1$ MPa = 100 kPa $\quad T_1 = 27°C = 300$ K

$r = \frac{v_1}{v_2} = 18 \quad r_p = \frac{p_3}{p_2} = 1.5$

$\rho = \frac{v_4}{v_3} = 1.2$

To find

(i) Thermal efficiency, and

(ii) Mean effective pressure.

Assumptions

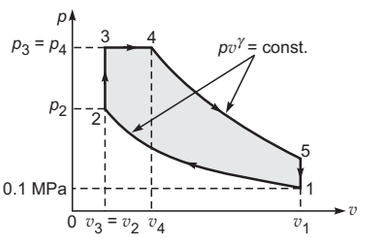
(i) Working, fluid air is modeled as an ideal gas.

(ii) Air in piston cylinder arrangement is the closed system.

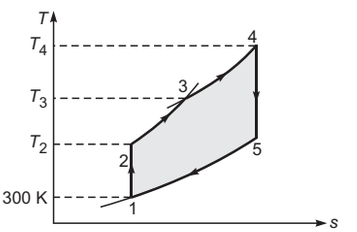
(iii) All processes are internally reversible.

(iv) The specific heat ratio for air, $\gamma = 1.4$, $R = 0.287$ kJ/kg·K, $C_p = 1.005$ kJ/kg·K, $C_v = 0.718$ kJ/kg·K

Schematic



(a) p–v diagram for Dual cycle



(b) T–s diagram for Dual cycle

Fig. 11.27

Analysis

(i) The air-standard efficiency of a dual cycle is given by

$$\eta_{dual} = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{r_p\rho^{\gamma} - 1}{(r_p - 1) + \gamma r_p(\rho - 1)}\right]$$

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Using numerical values,

$$\eta_{dual} = 1 - \frac{1}{(18)^{1.4-1}}\left[\frac{1.5\times(1.2)^{1.4}-1}{(1.5-1)+1.4\times1.5\times(1.2-1)}\right]$$

$$= 1 - \frac{1}{3.178}\times\left[\frac{0.936}{0.5+0.4\,2}\right]$$

$$= 0.68 \approx \mathbf{68\%}$$

(ii) Mean effective pressure

$$p_m = \frac{\text{Net work done}}{\text{Swept volume}} = \frac{w_{net}}{v_s} = \frac{\eta_{dual}\times q_{in}}{v_1 - v_2}$$

Here, $v_1 = \dfrac{RT_1}{p_1} = \dfrac{0.287\times300}{100} = 0.861$ m^3/kg

$$v_2 = \frac{v_1}{18} = 0.0478 \text{ m}^3/\text{kg}$$

$$T_2 = T_1 r^{\gamma-1} = 300\times(18)^{1.4-1}$$

$$= 953.3 \text{ K}$$

$$T_3 = T_2 r_p = 953.3\times1.5 = 1430 \text{ K}$$

$$T_4 = T_3\rho = 1430\times1.2 = 1716 \text{ K}$$

The heat supplied per kg of air:

$$q_{in} = C_v(T_3 - T_2) + C_p(T_4 - T_3)$$

$$= 0.718\times(1430 - 953.3) + 1.005\times(1716 - 1430)$$

$$= 630.6 \text{ kJ/kg}$$

Then $$p_m = \frac{0.68\times630.6}{0.861-0.0478}$$

$$= 5.22 \text{ kPa} \approx \mathbf{0.527\ MPa}$$

Example 11.20 *In an air standard dual cycle, the pressure and temperature at the beginning of compression are 1 bar and 57°C, respectively. The heat supplied in the cycle is 1250 kJ/kg, two third of this being added at constant volume and rest at constant pressure.*

If the compression ratio is 16. Determine the maximum pressure, temperature in the cycle, thermal efficiency and mean effective pressure.

Solution

Given An air standard dual cycle with

$p_1 = 1$ bar $= 100$ kPa, $T_1 = 57°C = 330$ K

$q_{in} = 1250$ kJ/kg, $q_{2-3} = \frac{2}{3}\,q_{in}$, $q_{3-4} = \frac{1}{3}\,q_{in}$

To find

(i) Maximum pressure in the cycle,

Schematic

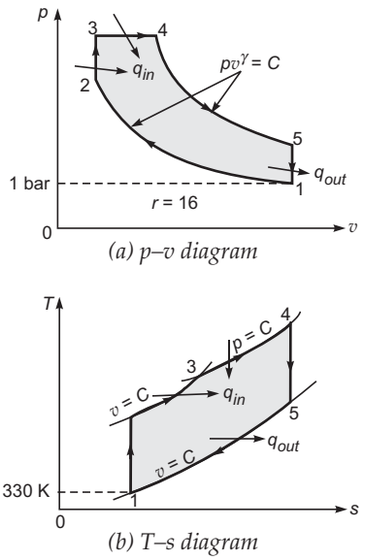

Fig. 11.28

(ii) Maximum temperature in the cycle,

(iii) Thermal efficiency, and

(iv) Mean effective pressure.

Assumption

(i) Working substance is air.

(ii) Compression and expansion are isentropic.

(iii) Heat addition and rejection are reversible.

(iv) Constant specific heats, $C_p = 1.005$ kJ/kg·K, $C_v = 0.718$ kJ/kg·K, $R = 0.287$ kJ/kg·K, and $\gamma = 1.4$.

Analysis

(i) The pressure and temperature after isentropic compression

$$p_2 = p_1 r^{\gamma} = (1 \text{ bar})\times(16)^{1.4} = 48.5 \text{ bar}$$

$$T_2 = T_1 r^{\gamma-1} = (330 \text{ K})\times(16)^{1.4-1}$$

$$= 1000.3 \text{ K}$$

Heat added at constant volume,

$$q_{2-3} = \frac{2}{3}\,q_{in} = \frac{2}{3}\times1250$$

$$= 833.33 \text{ kJ/kg}.$$

Heat added at constant pressure,

$$q_{3-4} = \frac{1}{3}\,q_{in} = \frac{1250}{3} = 416.67 \text{ kJ/kg}$$

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During process 2–3

$$q_{2-3} = C_v(T_3 - T_2)$$

or $833.33 = 0.718 \times (T_3 - 1000.3)$

or $T_3 = 2161$ K

and $\dfrac{p_3}{T_3} = \dfrac{p_2}{T_2}$

or pressure ratio

$$r_p = \frac{p_3}{p_2} = \frac{T_3}{T_2} = \frac{2161}{1000.3} = 2.16$$

and $p_3 = 2.16 p_2 = 2.16 \times 48.5$

$= \mathbf{104.77\ bar}$

It is the maximum pressure attained in the cycle.

(ii) During process 3 – 4,

Heata ddition $q_{3-4} = C_p(T_4 - T_3)$

or $416.67 = 1.005 \times (T_4 - 2161)$

or $T_4 = 2575.6$ K = **2302.6°C**

It is the *maximum temperature* reached in the cycle.

$$\text{cut-off ratio } \rho = \frac{v_4}{v_3} = \frac{T_4}{T_3} = \frac{2575.6}{2161} = 1.191$$

$$\text{Therefore, } \frac{T_5}{T_4} = \left(\frac{v_4}{v_5}\right)^{\gamma-1} = \left(\frac{v_4}{v_3} \times \frac{v_2}{v_1}\right)^{\gamma-1}$$

$$= \left(\frac{\rho}{r}\right)^{\gamma-1} = \left(\frac{1.191}{16}\right)^{0.4} = 0.353$$

$$T_5 = 2575.6 \times 0.353 = 911.16 \text{ K}$$

Heat rejected

$$q_{5-1} = C_v(T_5 - T_1)$$
$$= 0.718 \times (911.16 - 330)$$
$$= 417.2 \text{ kJ/kg}$$

(iii) Thermal efficiency

$$\eta_{dual} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{417.2}{1250}$$
$$= 0.6667 = \mathbf{66.67\%}$$

(iv) Net work done in the cycle,

$$w_{net} = q_{in} - q_{out} = 1250 - 417.2$$
$$= 832.8 \text{ kJ/kg}$$

The initial specific volume of air,

$$v_1 = \frac{RT_1}{p_1} = \frac{0.287 \times 330}{100}$$
$$= 0.9471 \text{ m}^3/\text{kg}$$

The clearance volume,

$$v_2 = \frac{v_1}{r} = \frac{0.9471}{16} = 0.0592 \text{ m}^3/\text{kg}$$

The piston displacement volume, per kg of air

$$v_s = v_1 - v_2 = 0.9471 - 0.0592$$
$$= 0.888 \text{ m}^3/\text{kg}$$

The mean effective pressure,

$$p_m = \frac{w_{net}}{v_s} = \frac{832.8}{0.888} = 938 \text{ kPa}$$
$$= \mathbf{9.38\ bar}$$

Example 11.21 *The pressure and temperature at the beginning of compression in an air-standard dual cycle are 1 bar and 30°C, respectively. The compression ratio is 9. The maximum pressure in the cylinder is limited to 60 bar. The heat is added during constant pressure process upto to 4% of the stroke. Assuming cylinder bore and stroke as 250 mm and 300 mm, respectively, determine:*

(a) Air standard efficiency of the dual cycle,

(b) Power developed, if the number of working cycles are 3 per second.

For air take, $C_v = 0.71$ kJ/kg·K and $C_p = 1.0$ kJ/kg·K

Solution

Given An air standard dual cycle with

$p_1 = 1$ bar, $p_{max} = 60$ bar

$T_1 = 30°C = 303$ K, $r = 9$

$d = 250$ mm $L = 300$ mm

$C_v = 0.71$ kJ/kg·K $C_p = 1.0$ kJ/kg·K

$V_4 - V_3 = 0.04\, v_s$

$rps = 3$(No. of cycle per second)

Schematic With given data

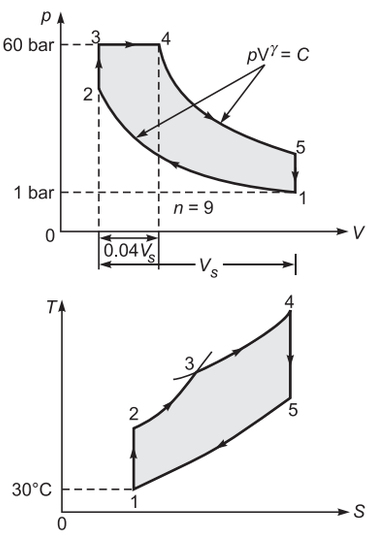

Fig. 11.29

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To find

(i) Air standard efficiency of dual cycle.

(ii) Power developed.

Assumptions

(i) The air in piston-cylinder assembly as a closed system.

(ii) Air as an ideal gas.

Analysis

(i) Air standard efficiency:

Sweptv olume $V_s = \frac{\pi}{4} d^2 L$

$$= \frac{\pi}{4} \times (0.25)^2 \times 0.3 = 0.0147 \text{ m}^3$$

The compression ratio

$$r = \frac{V_1}{V_2} = \frac{V_s + V_c}{V_c} = 1 + \frac{V_s}{V_c}$$

or $$V_c = V_2 = V_3 = \frac{V_s}{r-1} = \frac{0.0147}{8}$$

$$= 0.0184 \text{ m}^3$$

and $$V_1 = V_s + V_c = 0.0147 + 0.00184$$

$$= 0.0165 \text{ m}^3$$

The ratio of specific heats

$$\gamma = \frac{C_p}{C_v} = \frac{1}{0.71} = 1.408$$

Pressure after isentropic compression 1–2:

$$p_2 = p_1, r^{\gamma} = (1 \text{ bar}) \times (9)^{1.408}$$

$$= 22.0 \text{ bar}$$

The temperature after isentropic compression

$$T_2 = T_1 r^{\gamma - 1} = 303 \times (9)^{0.408} = 735.2 \text{ K}$$

After constant volume heat addition 2–3:

$$r_p = \frac{p_3}{p_2} = \frac{60}{22} = 2.727$$

$$T_3 = T_2 r_p = 735.2 \times 2.727 = 2005.1 \text{ K}$$

Constant pressure heat addition upto 4% of stroke volume i.e.,

$$V_4 - V_3 = 0.04 (V_1 - V_2)$$

or $$\frac{V_4}{V_3} = 1 + 0.04 \left(\frac{V_1}{V_2} - 1 \right) \quad (\because V_2 = V_3)$$

or $$\rho = 1 + 0.04 (9 - 1) = 1.32$$

For constant pressure process

$$T_4 = \rho T_3 = 1.32 \times 2005.1$$

$$= 2646.73 \text{ K}$$

Temperature after isentropic expansion 4–5

$$\frac{T_4}{T_5} = \left(\frac{V_5}{V_4} \right)^{\gamma - 1} = \left(\frac{V_1}{V_4} \right)^{\gamma - 1} = \left(\frac{V_1}{V_2} \times \frac{V_3}{V_4} \right)^{\gamma - 1}$$

$$= \left(\frac{r}{\rho} \right)^{\gamma - 1}$$

or $$T_5 = \frac{2646.3}{\left(\frac{9}{1.32} \right)^{0.4}} = 1228.1 \text{ K}$$

Heat supplied per kg of air

$$q_{in} = q_{2-3} + q_{3-4} = C_v (T_3 - T_2) + C_p (T_4 - T_3)$$

$$= 0.71 (2005.1 - 735.2) +$$

$$1.0 \times (2646.73 - 2005.1)$$

$$= 901.63 + 641.63 = 1543.2 \text{ kJ/kg}$$

Heat rejected per kg of air during process 4–5

$$q_{4-5} = q_{out} = C_v (T_5 - T_1) = 0.71 \times (1228.1 - 303)$$

$$= 656.82 \text{ kJ/kg}$$

$$\eta_{air\text{-}standard} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{656.82}{1543.2}$$

$$= 0.574 = \mathbf{57.4\%}$$

(ii) Power developed by the engine:

Specific gas constant for air,

$$R = C_p - C_v = 1 - 0.71 = 0.29 \text{ kJ/kg} \cdot \text{K}$$

Mass of air

$$m = \frac{p_1 V_1}{R T_1} = \frac{100 \times 0.0165}{0.29 \times 303}$$

$$= 0.0187 \text{ kg}$$

Work done per cycle,

$$W = m (q_{in} - q_{out})$$

$$= 0.0187 (1543.2 - 656.82)$$

$$= 16.64 \text{ kJ}$$

Power developed

= Work done per cycle × No. of cycles/ second

$$= 16.64 \text{ cycle/s} = \mathbf{49.93 \text{ kW}}$$

Example 11.22 *The compression and expansion ratios of an oil engine working on air standard dual cycle are 9 and 5, respectively. The initial pressure and temperature are 1 bar and 30°C, respectively. The heat liberated at constant pressure is twice the heat liberated at constant volume. The expansion and compression follow the law $pV^{1.25}$ = const. Determine:*

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(a) Pressure and temperature at all salient points.
(b) The mean effective pressure of the cycle.
(c) Thermal efficiency of the cycle.
(d) Power developed of the engine, if eight cycles complete in a second.

Take cylinder bore = 250 mm and stroke = 400 mm

Solution

Given An oil engine works on air standard dual cycle

$r = 9$, $r_e = 5$
$p_1 = 1$ bar $T_1 = 30°C = 303$ K
$q_{3-4} = 2q_{2-3}$
$pV^{1.25} = \text{const.}$ $n = 1.25$ $N = 8$ cycles/s
$d = 250$ mm $L = 400$ mm

To find

(i) Pressure and temperature at all salient points,
(ii) The mean effective pressure of the cycle,
(iii) Thermal efficiency of the cycle, and
(iv) Power developed of the engine, if eight cycles complete in a second.

Schematic

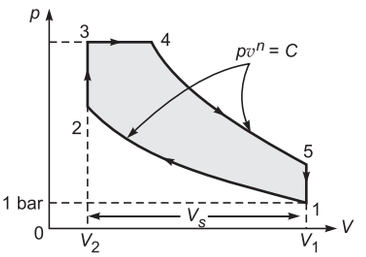

Fig. 11.30

Assumptions

(i) The working substance in the piston cylinder assembly as a closed system.
(ii) The working substance is air, modeled as an ideal gas, with $\gamma = 1.4$, $C_p = 1.005$ kJ/kg·K, $C_v = 0.717$ kJ/kg·K

Analysis

(i) Pressure and temperature at all salient points:

After polytropic compression 1–2;

$$T_2 = T_1(r)^{n-1} = (303\text{ K}) \times (9)^{1.25-1}$$
$$= \mathbf{524.81\ K}$$

$$p_2 = p_1(r)^n = (1\text{ bar}) \times (9)^{1.25}$$
$$= \mathbf{15.58\ bar}$$

Givent hat $q_{3-4} = 2q_{2-3}$

$$C_p(T_4 - T_3) = 2C_v(T_3 - T_2) \quad \ldots(i)$$

We have $r_e = \dfrac{r}{\rho}$

Thus, $\rho = \dfrac{r}{r_e} = \dfrac{9}{5} = 1.8$

and $\rho = \dfrac{V_4}{V_3} = \dfrac{T_4}{T_3}$

or $T_4 = \rho T_3 = 1.8\ T_3 \quad \ldots(ii)$

Using in Eq. (i), we get

$1.005 \times (1.8\,T_3 - T_3)$
$= 2 \times 0.717 \times (T_3 - 524.81)$

or $0.804\,T_3 = 1.434\,T_3 - 752.57$

Temperature at state 3:

$$T_3 = \mathbf{1194.56\ K}$$

Pressure at state 3:

$$p_3 = p_2 \times \frac{T_3}{T_2}$$
$$= 15.58 \times \frac{1194.56}{524.81} = 35.46\text{ bar}$$

At state 4: $p_4 = p_3 = \mathbf{35.46\ bar}$

$$T_4 = 1.8\,T_3 = \mathbf{2150.22\ K}$$

After polytropic expansion 4–5:

$$p_5 = p_4\left(\frac{V_4}{V_5}\right)^n = p_4\left(\frac{1}{r_e}\right)^n$$
$$= 35.46 \times \left(\frac{1}{5}\right)^{1.25} = \mathbf{4.74\ bar}$$

$$T_5 = \frac{T_4}{(r_e)^{n-1}} = \frac{2150.22}{(5)^{0.25}} = \mathbf{1438\ K}$$

(ii) Thermal efficiency of the cycle

Heat supplied per kg of air

$$q_{in} = C_v(T_3 - T_2) + C_p(T_4 - T_3)$$
$$= 0.717 \times (1194.56 - 524.81)$$
$$+ 1.005 \times (2150.22 - 1194.56)$$
$$= 480.21 + 960.44 = 1440.65\text{ kJ/kg}$$

Heat rejected per kg of air

$$q_{out} = C_v(T_5 - T_1)$$
$$= 0.717 \times (1438 - 303)$$
$$= 813.75\text{ kJ/kg}$$

Net work done per kg of air

$$w_{net} = q_{in} - q_{out} = 1440.65 - 813.75$$
$$= 626.9\text{ kJ/kg}$$

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Thermal efficiency

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{626.9}{1440.65}$$
$$= 0.435 \text{ or } \mathbf{43.5\%}$$

The specific volume of air inducted

$$v_1 = \frac{p_1}{RT_1} = \frac{100}{0.287 \times 303}$$
$$= 1.15 \text{ m}^3/\text{kg}$$
$$v_2 = \frac{v_1}{r} = \frac{1.15}{9} = 0.1277 \text{ m}^3/\text{kg}$$
$$v_3 = v_1 - v_2 = 1.15 - 0.1277$$
$$= 1.0222 \text{ m}^3/\text{kg}$$

(iii) The mean effective pressure

$$p_m = \frac{w_{net}}{v_s} = \frac{626.9}{1.0222}$$
$$= 613.27 \text{ kPa or } \mathbf{6.13 \text{ bar}}$$

Swept volume

$$V_s = \frac{\pi}{4} d^2 L = \frac{\pi}{4} \times (0.25)^2 \times 0.4$$
$$= 0.0196 \text{ m}^3$$

Mass of air

$$m = \frac{V_s}{v_s} = \frac{0.0196}{1.0222} = 0.0192 \text{ kg}$$

(iv) Power developed

$$P = m\, w_{net} \times \text{No. of cycle/s}$$
$$= 0.0192 \times 626.9 \times 8 = \mathbf{96.33 \text{ kW}}$$

Example 11.23 *A high-speed oil engine operating on a dual combustion cycle has a pressure of 1 bar and a temperature of 50°C before compression. Air is then compressed isentropically to 1/15th of its original volume. The maximum pressure is twice the pressure at the end of isentropic compression. If the cut-off ratio is 2, determine the temperature at the end of each process and ideal efficiency of the cycle. Take $\gamma = 1.4$.*

Solution

Given A high-speed engine operates on air-standard dual cycle with

$p_1 = 1$ bar, $T_1 = 50°\text{C} = 323$ K,
$r = 15$, $\rho = 2$,
$\gamma = 1.4$ $p_3 = 2p_2$

To find

(i) Temperature at the end of each process, and
(ii) Air standard efficiency of cycle.

Schematic *p–v* and *T–s* diagrams

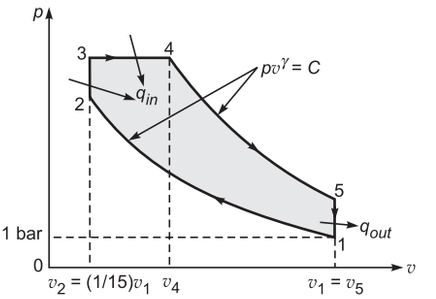

Fig. 11.31

Analysis The specific volume after compression;

$$v_2 = \frac{1}{15^{th}} v_1$$

Temperature after the isentropic compression 1–2;

$$T_2 = T_1 r^{\gamma - 1} = 323 \times (15)^{1.4-1}$$
$$= 954.2 \text{ K or } \mathbf{681.2°C}$$

The pressure at the end of compression;

$$p_2 = p_1 r^{\gamma}$$
$$= (1 \text{ bar}) \times (15)^{1.4} = 44.312 \text{ bar,}$$

Temperature T_3 at the end of constant-volume heat addition 2–3;

$$T_3 = \left(\frac{p_3}{p_2}\right) T_2 = 2 \times 954.2$$
$$= 1908.4 \text{ K or } \mathbf{1635.4°C}$$

Pressure p_3 at the end of constant-volume heat addition;

$$p_3 = p_{max} = 2p_2 = 2 \times 44.312 = 88.625 \text{ bar}$$

Temperature T_4 at end of constant-pressure heat addition 3 –4;

$$T_4 = \rho T_3 = (2) \times 1908.4$$
$$= 3816.8 \text{ K or } \mathbf{3543.8°C}$$

The *temperature* at the end of isentropic expansion 4–5;

$$T_5 = T_4 \left(\frac{\rho}{r}\right)^{\gamma - 1}$$
$$= 3816.8 \times \left(\frac{2}{15}\right)^{1.4-1}$$
$$= 1704.8 \text{ K or } \mathbf{1431.8°C}$$

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(ii) *Air standard efficiency of the cycle*

Heat supplied per kg of air

$$q_{in} = C_v(T_3 - T_2) + C_p(T_4 - T_3)$$

Heat rejected per kg of air

$$q_{out} = C_p(T_5 - T_1)$$

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{C_v(T_5 - T_1)}{C_v(T_3 - T_2) + C_p(T_4 - T_3)}$$

$$= 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)}$$

$$= 1 - \frac{1794.8 - 323}{(1908.4 - 954.2) + 1.4 \times (3816.8 - 1908.4)}$$

$= 0.6189$ or **61.9%**

Example 11.24 *For the same compression ratio, prove that the efficiency of the Otto cycle is greater than that of the Diesel cycle.*

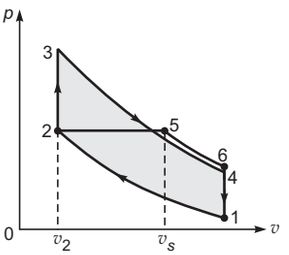
Solution The Fig. 11.32 shows Otto and Diesel cycles on *p–v* and *T–s* diagrams.

Cycle 1–2–3–4–1 Otto cycle

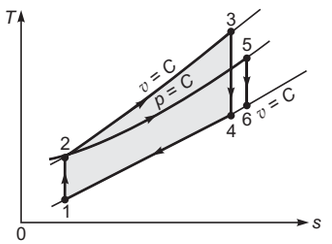
Cycle 1–2–5–6–1 Diesel cycle

The efficiency of the Otto cycle is given as

$$\eta_{Otto} = 1 - \frac{1}{(r)^{\gamma - 1}}$$



(a) Otto and Diesel cycles on p–v diagrams



(b) Otto and Diesel cycles on T–s diagrams

Fig. 11.32

The efficiency of the Diesel cycle is given as

$$\eta_{Diesel} = 1 - \frac{1}{(r)^{\gamma - 1}} \left[\frac{\rho^{\gamma} - 1}{\gamma(\rho - 1)} \right]$$

For both cycles, $r = \frac{V_1}{V_2}$ (same)

For Diesel cycle $\rho = \frac{V_5}{V_2}$

Since V_5, volume after heat addition is always greater than V_2 thus $\rho > 1$

Thus quantity $\frac{\rho^{\gamma} - 1}{\gamma(\rho - 1)}$ will always be greater than 1 and when it is multiplied to $\frac{1}{r^{\gamma - 1}}$, the negative quantity will increase, thus efficiency of Diesel cycle will be less than Otto cycle.

Example 11.25 *A four-cylinder, four-stroke engine has a displacement volume of 300 cc per cylinder. The compression ratio of the engine is 10 and operates at a speed of 3000 rpm. The engine is required to develop an output power of 40 kW at this speed. Calculate thermal efficiency of the cycle, assuming that the engine operates on the Otto cycle and that the pressure and temperature at the inlet condition are 1 bar and 27°C, respectively.*

If the above engine is operating on the Diesel cycle and receiving heat at the same rate, calculate thermal efficiency and maximum temperature of the cycle. Compare the efficiency of Otto and Diesel cycles.

Solution

Given A four-stroke, four-cylinder engine operates on Otto and Diesel cycles.

$n = \frac{N}{2 \times 60}$, Effective suctions $k = 4$

$V_s = 300$ cc $\quad r = 10$

$N = 3000$ rpm $\quad \dot{W}_{net} = 40$ kW

$p_1 = 1$ bar $\quad T_1 = 27°C = 300$ K

To find

(i) The efficiency of the engine when it operates on Otto cycle.

(ii) Thermal efficiency and maximum temperature in cycle, when it operates on Diesel cycle.

Assumptions

(i) Air standard cold air assumptions.

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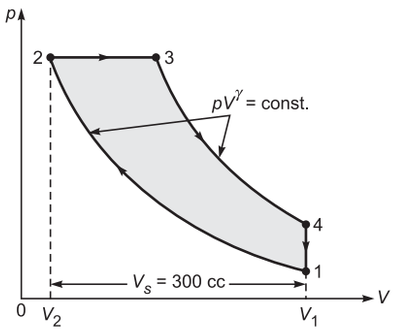

Fig. 11.33

(ii) Ratio of specific heat $\gamma = 1.4$.

(iii) Specific heats $C_p = 1.005$ kJ/kg/K and $C_v = 0.72$ kJ/kg·K.

Analysis

(i) *When the engine operates on the Otto cycle*

Thermal efficiency of the cycle.

$$\eta_{Otto} = 1 - \frac{1}{r^{\gamma-1}}$$

$$= 1 - \frac{1}{(10)^{1.4-1}} = 0.60 \text{ or } \mathbf{60\%}$$

The efficiency is also given as

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}}$$

Heat supply rate,

$$\dot{Q}_{in} = \frac{\dot{W}_{net}}{\eta_{th}} = \frac{40 \text{ kW}}{0.6} = 66.67 \text{ kW}$$

(ii) When the engine operates on the Diesel cycle

$r = 10$

$p_1 = 1$ bar

$T_1 = 300$ K

$N = 3000$ rpm

Temperature after isentropic compression

$$T_2 = T_1 r^{\gamma-1}$$

$$= 300 \times (10)^{1.4-1} = 753.56 \text{ K}$$

The compression ratio is given by

$$r = \frac{V_1}{V_2} = \frac{V_s + V_2}{V_2} = 1 + \frac{V_s}{V_2}$$

or $$\frac{V_s}{V_2} = 10 - 1 = 9$$

or $$V_2 = \frac{V_s}{9} = \frac{300}{9} = 33.33 \text{ cc}$$

Total volume,

$$V_1 = V_s + V_2 = 300 + 33.33$$

$$= 333.33 \text{ cc}$$

$$= 333.33 \times 10^{-6} \text{ m}^3$$

The mass of air in a cylinder/cycle

$$m = \frac{p_1 V_1}{R T_1} = \frac{(100 \text{ kPa}) \times (333.33 \times 10^{-6} \text{ m}^3)}{(0.287 \text{ kJ/kg·K}) \times (300 \text{ K})}$$

$$= 3.87 \times 10^{-4} \text{ kg}$$

The mass flow rate to four-cylinders of the four-stroke engine

$$\dot{m} = m k n = 3.87 \times 10^{-4} \times 4 \times \frac{3000}{2 \times 60}$$

$$= 0.0387 \text{ kg/s}$$

The heat supplied per second to the engine is given by

$$\dot{Q}_{in} = \dot{m} C_p (T_3 - T_2)$$

Using numerical values

$$66.67 = 0.0387 \times 1.005 (T_3 - 753.56)$$

It gives $T_3 = 2468$ K = **2195 °C**

The cut off ratio

$$\frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{2468}{753.56} = 3.27$$

The efficiency of the Diesel cycle

$$\eta_{Diesel} = 1 - \frac{1}{r^{\gamma-1}} \left[\frac{\rho^{\gamma} - 1}{\gamma(\rho - 1)} \right]$$

$$= 1 - \frac{1}{(10)^{1.4-1}} \left[\frac{(3.27)^{1.4} - 1}{1.4 \times (3.27 - 1)} \right]$$

$$= \mathbf{0.467} \text{ or } \mathbf{46.7\%}$$

Comment The efficiency of the Otto cycle is more than that of the Diesel cycle for same compression ratio and same heat input.

11.10 LENOIR CYCLE

The Lenoir cycle is used for pulse jet engines. It consists of three reversible processes:

Process 1–2 Constant-volume heat addition,

Process 2–3 Isentropic expansion, and

Process 3–1 Constant-pressure cooling.

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The p–v and T–s representations of the Lenoir cycle are shown in Fig.11.34.

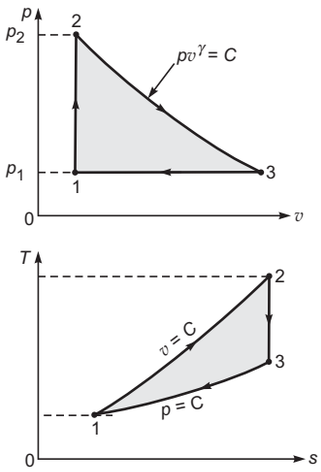

Fig. 11.34 *Lenoir cycle*

For unit mass of air

Heat Supplied,

$$q_{in} = q_{1-2} = C_v(T_2 - T_1)$$

(a positive quantity)

Heat rejected

$$q_{out} = q_{3-1} = C_p(T_3 - T_1)$$

(a negative quantity)

The thermal efficiency of the Lenoir cycle can be expressed as

$$\eta_{Lenoir} = 1 - \frac{q_{out}}{q_{in}}$$

$$1 - \frac{C_p(T_3 - T_1)}{C_v(T_2 - T_1)} = 1 - \gamma\left(\frac{T_3 - T_1}{T_2 - T_1}\right) \quad ...(i)$$

Expressing $r_p = \dfrac{p_2}{p_1}$

For the constant-volume process 1–2;

$$T_2 = T_1 r_p \quad ...(ii)$$

For the isentropic process 2–3;

$$T_3 = T_2\left(\frac{p_3}{p_2}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}$$

$$= T_1 r_p\left(\frac{1}{r_p}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} = T_1 r_p^{\left(\frac{1}{\gamma}\right)} \quad ...(iii)$$

Substituting Eqs. (ii) and (iii) in Eq. (i),

$$\eta_{Lenoir} = 1 - \gamma\left(\frac{T_1 r_p^{\left(\frac{1}{\gamma}\right)} - T_1}{T_1 r_p - T_1}\right)$$

$$= 1 - \gamma\left(\frac{r_p^{\left(\frac{1}{\gamma}\right)} - 1}{r_p - 1}\right) \quad ...(11.16)$$

The net work done per kg of air during Lenoir cycle

Net work done = Σq for a cycle

$$w_{net} = q_{in} - q_{out} = C_v(T_2 - T_1) - C_p(T_3 - T_1)$$

$$= C_v T_1\left[(r_p - 1) - \gamma\left(r_p^{\left(\frac{1}{\gamma}\right)} - 1\right)\right]$$

$$= \frac{R}{\gamma - 1} T_1\left[(r_p - 1) - \gamma\left(r_p^{\left(\frac{1}{\gamma}\right)} - 1\right)\right] \text{ (kJ/kg)}$$

...(11.17)

11.11 ATKINSON CYCLE

The Atkinson cycle is an ideal cycle for Otto engine exhausting to a gas turbine. The constant-volume combustion gas turbine plants operate on Atkinson cycle.

In Atkinson cycle, the heat addition takes place at constant volume, the remaining three processes are same as in air standard Brayton cycle. The p–v and T–s diagrams are shown in Fig. 11.35. The nature of processes are

Process 1–2 Isentropic compression,

Process 2–3 Reversible constant-volume heat addition,

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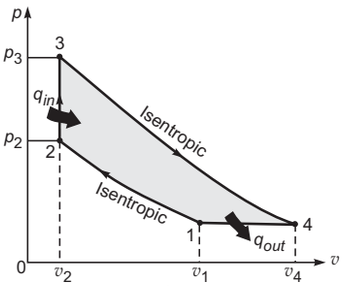
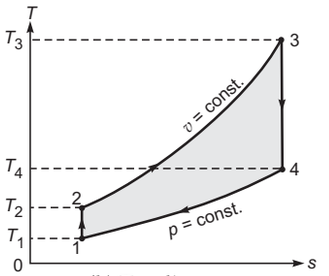


(a) p–v diagram



(b) T–s diagram

Fig. 11.35 *Atkinson cycle*

Process 3–4 Isentropic expansion, and

Process 4–1 Reversible constant-pressure heat rejection.

For unit mass of air

Heat supplied,

$$q_{in} = q_{2-3} = C_v(T_3 - T_2)$$

(a positive quantity)

Heat rejected

$$q_{out} = q_{4-1} = C_p(T_4 - T_1)$$

(a negative quantity)

The thermal efficiency of the cycle

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}}$$

$$= 1 - \frac{C_p(T_4 - T_1)}{C_v(T_3 - T_2)} = 1 - \gamma\left(\frac{T_4 - T_1}{T_3 - T_2}\right) \quad ...(i)$$

The efficiency of the Atkinson cycle may be expressed in the following terms:

Compression ratio

$$r = \frac{v_1}{v_2} = \frac{\text{Volume before compression}}{\text{Volume after compression}}$$

Expansion ratio

$$r_e = \frac{v_4}{v_3} = \frac{\text{Volume after expansion}}{\text{Volume before expansion}}$$

Expressing each temperature in terms of T_1 with the use of above terms,

For the isentropic process 1–2;

$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{\gamma-1} = T_1 r^{\gamma-1} \quad ...(ii)$$

For the constant-pressure process 4–1;

$$T_4 = T_1\left(\frac{v_4}{v_1}\right) = T_1\left(\frac{v_4}{v_3} \times \frac{v_2}{v_1}\right) = T_1\frac{r_e}{r} \quad ...(iii)$$

For the isentropic process 3–4;

$$T_3 = T_4\left(\frac{v_4}{v_3}\right)^{\gamma-1} = T_4 r_e^{\gamma-1}$$

Using T_4 from Eq. (iii), we get

$$T_3 = T_1\frac{r_e}{r}r_e^{\gamma-1} = T_1\frac{r_e^{\gamma}}{r} \quad ...(iv)$$

Using Eq. (ii), (iii) and (iv) in Eq. (i);

$$\eta_{Atkinson} = 1 - \gamma\left(\frac{\frac{r_e}{r} - 1}{\frac{r_e^{\gamma}}{r} - r^{\gamma-1}}\right)$$

$$= 1 - \gamma\left[\frac{r_e - r}{r_e^{\gamma} - r^{\gamma}}\right] \quad ...(11.18)$$

The net work done per kg of air during the Atkinson cycle

Net work done = Σq for a cycle

$$w_{net} = q_{in} - q_{out} = C_v(T_3 - T_2) - C_p(T_4 - T_1)$$

$$= C_v T_1\left[\left(\frac{r_e^{\gamma}}{r} - r^{\gamma-1}\right) - \gamma\left(\frac{r_e}{r} - 1\right)\right]$$

$$= \frac{R}{\gamma - 1}T_1\frac{1}{r}[(r_e^{\gamma} - r^{\gamma}) - \gamma(r_e - r)]$$

...(11.19)

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Example 11.26 *In an ideal Atkinson cycle, the gas is compressed isentropically from 1 bar, 27 °C to 4 bar. The maximum pressure of the cycle is limited to 16 bar. Calculate:*

(a) the work done per kg of cycle,
(b) the thermal efficiency of the cycle,
(c) mean effective pressure of the cycle.

Take $C_p = 0.761$ *kJ/kg·K,*
and $C_v = 0.573$ *kJ/kg·K.*

Solution

Given An ideal Atkinson cycle with

$p_1 = 1$ bar, $p_2 = 4$ bar
$T_1 = T_4 = 27°C = 300$ K
$p_3 = 16$ bar
$C_p = 0.761$ kJ/kg·K
$C_v = 0.573$ kJ/kg·K

To find

(i) The net work done,
(ii) The thermal efficiency of the cycle, and
(iii) Mean effective pressure.

Assumptions

(i) Air standard assumption.
(ii) Constant specific heat of gas.
(iii) Kinetic and potential energy effects are negligible.

Analysis An ideal Atkinson cycle is shown on the *p–v* diagram in Fig. 11.36.

The ratio of two specific heats

$$\gamma = \frac{C_p}{C_v} = \frac{0.761}{0.573} = 1.328$$

The specific gas constant;

$$R = C_p - C_v$$
$$= 0.761 - 0.573 = 0.188 \text{ kJ/kg·K}$$

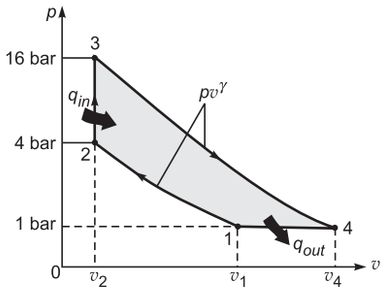

Fig. 11.36

The compression ratio;

$$r = \frac{v_1}{v_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = \left(\frac{4}{1}\right)^{\frac{1}{1.328}} = 2.84$$

Temperature after isentropic compression,

$$T_2 = T_1 r^{\gamma-1} = 300 \times (2.84)^{1.328-1} = 422.5 \text{ K}$$

Temperature after constant-volume heat addition;

$$T_3 = \frac{p_3}{p_2} T_2 = \frac{16}{4} \times 422.5 = 1690 \text{ K}$$

Temperature after isentropic expansion

$$T_4 = T_3 \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = 1690 \times \left(\frac{1}{16}\right)^{\frac{1.328-1}{1.328}} = 852 \text{ K}$$

(i) *Work done per kg of gas*

Heat supplied,

$$q_{in} = q_{2-3} = C_v(T_3 - T_2)$$
$$= 0.573 \times (1690 - 422.5) = 726.3 \text{ kJ/kg}$$

Heatr ejected

$$q_{out} = q_{4-1} = C_p(T_4 - T_1)$$
$$= 0.761 \times (852 - 300) = 420.1 \text{ kJ/kg}$$

$$w_{net} = q_{in} - q_{out}$$
$$= 726.3 - 420.1 = \mathbf{306.2 \text{ kJ/kg}}$$

(ii) *Thermal efficiency of the cycle*

$$\eta_{th} = 1 - \frac{W_{net}}{q_{in}} = \frac{306.2}{726.3}$$
$$= 0.422 \text{ or } \mathbf{42.2\%}$$

(iii) *The mean effective pressure of the cycle*

The swept volume per kg of gas

$$v_s = v_1 - v_2 = v_1\left(1 - \frac{1}{r}\right) = \frac{RT_1}{p_1}\left(1 - \frac{1}{r}\right)$$
$$= \frac{0.188 \times 300}{100} \times \left(1 - \frac{1}{2.84}\right)$$
$$= 0.3654 \text{ m}^3/\text{kg}$$

$$p_m = \frac{w_{net}}{v_s} = \frac{306.2}{0.3654}$$
$$= 838 \text{ kPa or } \mathbf{8.38 \text{ bar}}$$

11.12 BRAYTON CYCLE

The *Brayton Cycle* is also called *Joule Cycle*. It is an ideal cycle for gas turbine plants. The schematic arrangement for a Brayton cycle is shown in Fig. 11.37 and its cooresponding *p–v* and *T–s*

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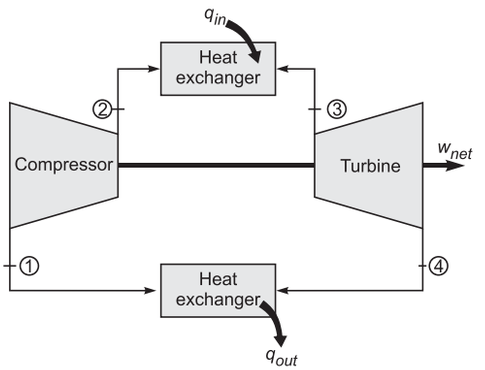

Fig. 11.37 *Schematic arrangement for Brayton cycle*

diagrams are shown in Fig. 11.38. Air as the working fluid is compressed isentropically in the compressor, heated at constant pressure in the heat exchanger, expanded isentropically in the turbine and cooled again to the initial state before re-entering the compressor.

The assumptions made in the Brayton cycle are the:

1. Working fluid is air as an ideal gas.
2. Actual combustion process in the combustion chamber is replaced by constant-pressure heat addition from an external source.
3. Exhaust process is replaced by a constant-pressure heat rejection in a heat exchanger.

The Brayton cycle consists of four internally reversible processes.

Process 1–2 Isentropic compression of air in the compressor,

Process 2–3 Constant-pressure heat addition to air,

Process 3–4 Isentropic expansion of air in the turbine,

Process 4–1 Constant pressure heat rejection from air.

The *p–v* and *T–s* diagrams for an ideal Brayton cycle are shown in Fig. 11.38. All the four processes of the Brayton cycle are executed in a steady flow manner. Thus they are analysed as steady flow

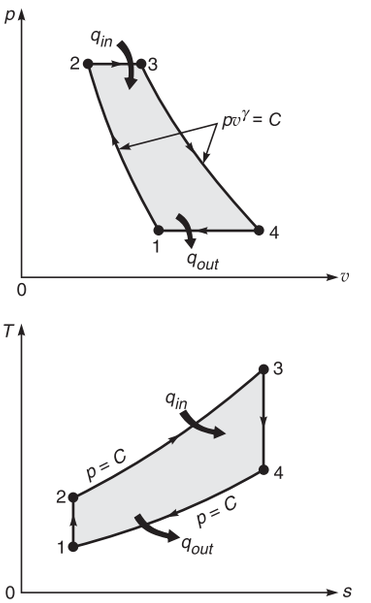

Fig. 11.38 *Air standard Brayton cycle*

processes. In absence of any changes in kinetic and potential energies; the steady-flow energy equation on a unit mass basis, as

$$q - w = \Delta h = h_{exit} - h_{inlet}$$

Assuming constant specific heat of air, the heat supplied to and rejected from air are

$$q_{in} = h_3 - h_2 = C_p(T_3 - T_2) \qquad ...(11.20)$$

$$q_{out} = h_4 - h_1 = C_p(T_4 - T_1)$$

Work developed per kg of air by turbine in isentropic manner

$$w_T = h_3 - h_4 = C_p(T_3 - T_4) \qquad ...(11.21)$$

Work input to compressor

$$w_C = h_2 - h_1 = C_p(T_2 - T_1) \qquad ...(11.22)$$

Net work of the gas turbine plant

$$w_{net} = \Sigma q \quad \text{or} \quad \Sigma w$$

$$= q_{in} - q_{out} \quad \text{or} \quad w_T - w_C$$

Thermal efficiency of the ideal Brayton cycle is

$$\eta_{Brayton} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{C_p(T_4 - T_1)}{C_p(T_3 - T_2)}$$

$$= 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

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For isentropic processes 1–2 and 3–4;

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(r_p\right)^{\frac{\gamma-1}{\gamma}} \qquad (11.23)$$

and $$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(r_p\right)^{\frac{\gamma-1}{\gamma}} \qquad (11.24)$$

Since $p_2 = p_3$ and $p_4 = p_1$, we get

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{or} \quad \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

Substituting in Eq. (i), we get

$$\eta_{Brayton} = 1 - \frac{1}{r_p^{(\gamma-1)/\gamma}} \qquad ...(11.25)$$

where, $r_p = \frac{p_2}{p_1}$, pressure ratio and γ is ratio of two specific heats.

11.12.1 Effect of Pressure Ratio on Efficiency of Brayton Cycle

Equation (11.25) indicates that the thermal efficiency of an ideal Brayton cycle depends on pressure ratio and specific heats ratio.

Figure 11.39(a) illustrates the effect of pressure ratio on thermal efficiency of the Brayton cycle. When pressure, after compression increases from p_2 to p_2', both heat supply q_{in} and net work w_{net} increase by an amount equal to the area 2–2′–3′–3–2, whereas the heat rejection q_{out} from the cycle remains unchanged. Hence, the thermal efficiency of the cycle increases with increase in pressure ratio as shown in Fig. 11.39(b).

11.12.2 Effect of Pressure Ratio for Fixed Temperature Ratio on Thermal Efficiency

From Fig. 11.39, it is evident that as pressure ratio increases from p_2/p_1 to p_2'/p_1, the amount of heat supplied and work done are increased by an area 2–2′–3′–3–2. Hence, the thermal efficiency of the cycle increases.

But as pressure p_2 increases, the maximum temperature T_3 reaching at the end of combustion process, also increases. The turbine blades cannot withstand very high temperature, (not more than 1700 K, limit imposed by metallurgical considerations); thus temperature T_3 is restricted. This also limits the pressure ratio that can be used in the cycle. For fixed T_3, the net work output per cycle increases with pressure ratio, reaches a maximum value and then starts to decrease as shown in Fig. 11.40.

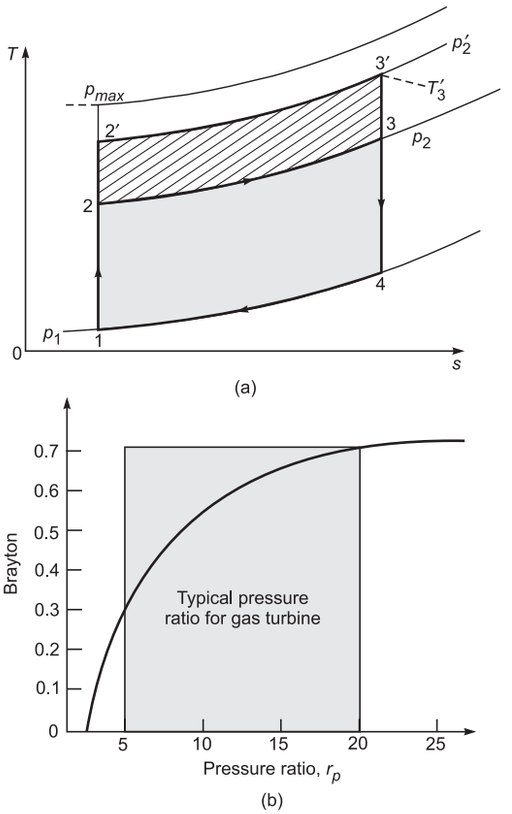

Fig. 11.39 *Thermal efficiency of Brayton cycle as a function of pressure ratio*

Figure 11.40 shows the effect of pressure ratio for fixed maximum temperature on work output of Brayton cycle. The pressure ratio $r_{p_2} > r_{p_1}$ on the effective area of cycle 1–2′–3′– 4′–1 is more than on the area 1–2–3–4, thus more work output is obtained from the cycle. But as pressure ratio increases to r_{p_3}, the area of 1–2″–3″– 4″–1 decreases, hence work output of the cycle also decreases, and thus

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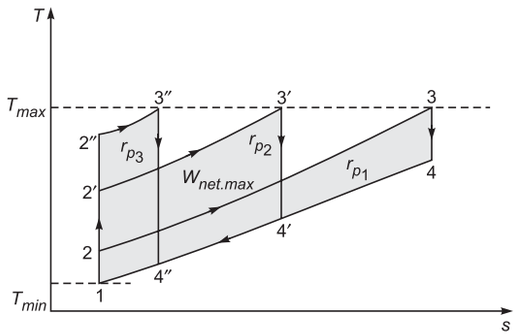

Fig. 11.40 *Effect of pressure ratio for fixed temperature ratio on work output of Brayton cycle*

the cyclic efficiency decreases. Therefore, in a gas turbine power plant, there should be a compromise between the pressure ratio and network output.

11.12.3 Back Work Ratio

The *back work ratio* is an important characteristic of a gas turbine plant. It is defined as the ratio of work input to the compressor to work produced by the turbine. It is denoted as r_{bw} and expressed as

$$r_{bw} = \frac{\text{Compressor work}}{\text{Turbine work}} \qquad ...(11.26)$$

During steady-state operation, the back work ratio is

$$r_{bw} = \frac{h_2 - h_1}{h_3 - h_4} = \frac{C_p(T_2 - T_1)}{C_p(T_3 - T_4)} = \frac{T_2 - T_1}{T_3 - T_4}$$

$$= \frac{T_1\left[T_2/T_1 - 1\right]}{T_3\left[1 - T_4/T_3\right]} \qquad ...(11.27)$$

From Eq. (11.23) and (11.24)

$$\frac{T_2}{T_1} = \left(r_p\right)^{\frac{\gamma-1}{\gamma}}$$

and $\frac{T_3}{T_4} = \left(r_p\right)^{\frac{\gamma-1}{\gamma}}$ or $\frac{T_4}{T_3} = \frac{1}{\left(r_p\right)^{\frac{\gamma-1}{\gamma}}}$

Using we get

$$r_{bw} = \frac{T_1}{T_3}\left[\frac{\left(r_p\right)^{\frac{\gamma-1}{\gamma}} - 1}{1 - \frac{1}{\left(r_p\right)^{\frac{\gamma-1}{\gamma}}}}\right]$$

$$= \frac{T_1}{T_3}\left(r_p\right)^{\frac{\gamma-1}{\gamma}}\left[\frac{\left(r_p\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(r_p\right)^{\frac{\gamma-1}{\gamma}} - 1}\right]$$

$$= \frac{T_1}{T_3}\left(r_p\right)^{\frac{\gamma-1}{\gamma}}$$

Back work ratio $r_{bw} = \frac{T_{min}}{T_{max}}\left(r_p\right)^{\frac{\gamma-1}{\gamma}}$ ...(11.28)

Usually, more than one half of the turbine work output is used to drive the compressor. A gas power plant with a high back work ratio requires a larger turbine to provide additional power requirement of the compressor. Therefore, the turbines used in gas power plants are larger than those used in steam power plants.

11.12.4 Work Ratio

The *work ratio* is another important characteristic of a gas turbine cycle and it is defined as the ratio of the net work of the plant to the turbine work.

$$r_w = \frac{\text{Net work output}}{\text{Turbine work}} \qquad ...(11.29)$$

$$= \frac{\text{Turbine work} - \text{Compressor work}}{\text{Turbine work}}$$

$$= 1 - \frac{\text{Compressor work}}{\text{Turbine work}} = 1 - r_{bw}$$

$$= 1 - \frac{T_{min}}{T_{max}}\left(r_p\right)^{\frac{\gamma-1}{\gamma}} \qquad ...(11.30)$$

In expressions (11.28) and (11.30), the temperature T_3 represents the maximum temperature T_{max} reached in the cycle after heat addition (combustion). The temperature T_1 represents minimum temperature T_{min}, i.e., ambient temperature generally remains constant. Therefore, the work ratio r_w depends on temperature ratio T_{min}/T_{max} and pressure ratio r_p. The size of the gas turbine plant depends on the magnitude of work ratio.

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11.12.5 Air Rate

The air rate is defined as the amount of air required to generate 1 kWh power output of the turbine. It is designated as AR and expressed as

$$AR = \frac{\text{Mass of air required}}{1 \text{ kWh output}} \left(\text{kg/kWh}\right)$$

$$= \frac{3600}{W_{net} \text{ (kW)}} \left(\text{kg/kWh}\right) \qquad ...(11.31)$$

11.12.6 Pressure Ratio for Maximum Work

For an ideal Brayton cycle, the net work output per kg is given by

$$w_{net} = w_T - w_C$$

$$= C_p(T_3 - T_4) - C_p(T_2 - T_1)$$

$$= C_p \left\{ T_3 \left(1 - \frac{T_4}{T_3}\right) - T_1 \left(\frac{T_2}{T_1} - 1\right) \right\} \qquad ...(11.32)$$

For a gas turbine plant, the atmospheric temperature T_1 and turbine inlet temperature T_3 are limiting temperatures. The efficiency and net work output are greatly influenced by these temperatures. Therefore, expressing T_2 and T_4 in terms of T_1 and T_3.

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = \left(r_p\right)^{\frac{\gamma - 1}{\gamma}}$$

Using $$k = \frac{\gamma - 1}{\gamma}$$

then $$w_{net} = C_p \left\{ T_3 \left(1 - \frac{1}{r_p^k}\right) - T_1 (r_p^k - 1) \right\}$$

Differentiating with respect to r_p and equating it to zero for maximization.

$$\frac{dw_{net}}{dr_p} = C_p \left\{ T_3 \frac{k}{r_p^{k+1}} - T_1 k r_p^{k-1} \right\} = 0$$

or $$\frac{T_3}{r_p^{k+1}} = T_1 r_p^{k-1}$$

or $$\frac{T_3}{T_1} = r_p^{2k} = r_p^{2\left(\frac{\gamma - 1}{\gamma}\right)}$$

or $$r_p = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{2(\gamma - 1)}} \qquad ...(11.33)$$

For fixed temperature limits of cycle-T_1 and T_3, the maximum pressure ratio possible for maximum specific work output

$$r_{p,\,max} = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

Inserting in Eq. (11.33), we get

$$r_p = \sqrt{r_{p,\,max}} \qquad ...(11.34)$$

Thus, the pressure ratio for maximum work is the function of limiting temperature ratio.

Example 11.27 *In an ideal Brayton cycle, air is compressed from 1 bar to a pressure ratio of 6. Calculate the cyclic efficiency. If the ratio of lower to upper temperature is 0.3 then calculate the work ratio.*

Solution

Given An ideal Brayton cycle with

$p_1 = 1$ bar

$r_p = 6$

$T_1/T_3 = 0.3$

To find

(i) Brayton cycle efficiency,

(ii) Work ratio.

Assumptions

(i) Cold air standard assumptions,

(ii) Ratio of specific heats $\gamma = 1.4$, $C_p = 1.005$ kJ/kg·K,

(iii) The effect of kinetic and potential energy is negligible.

Analysis

(i) The Brayton cycle efficiency is given by Eq. (11.21)

$$\eta_{Brayton} = 1 - \frac{1}{\left(r_p\right)^{\frac{\gamma - 1}{\gamma}}}$$

$$= 1 - \frac{1}{(6)^{\frac{1.4 - 1}{1.4}}} = 0.40 \text{ or } \mathbf{40\%}$$

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(ii) The work ratio is given by Eq. (11.24)

$$r_w = 1 - \frac{T_1}{T_3}\left(r_p\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 1 - 0.3 \times \left(6\right)^{\frac{1.4-1}{1.4}} = \mathbf{0.5}$$

Example 11.28 *The pressure ratio and maximum temperature of a Brayton cycle are 5:1 and 923 K, respectively. Air enters the compressed at 1 bar and 298 K. Calculate for 1 kg of air flow, the compressor work, turbine work and the efficiency of the cycle.*

Solution

Given An ideal Brayton cycle with

$r_p = 5$, $p_1 = 1$ bar,
$T_1 = 298$ K, $T_3 = 923$ K
$m = 1$ kg

To find

(i) Compressor work,
(ii) Turbine work, and
(iii) The thermal efficiency of the cycle.

Assumptions

(i) Air standard assumption.
(ii) Kinetic and potential energy effects are negligible.
(iii) For air $C_p = 1.005$ kJ/kg·K and $\gamma = 1.4$.

Analysis An ideal Brayton cycle is shown on the *T–s* diagram in Fig. 11.41.

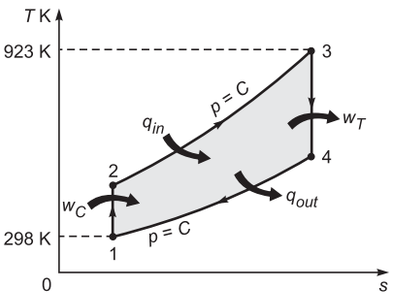

Fig. 11.41

Process 1–2: Isentropic compression of gas in compressor

$$T_2 = \left(r_p\right)^{\frac{\gamma-1}{\gamma}} T_1 = (5)^{\frac{1.4-1}{1.4}} \times 298 = 472 \text{ K}$$

Process 3–4: Isentropic expansion of gas in turbine.

$$T_4 = \frac{T_3}{\left(r_p\right)^{\frac{\gamma-1}{\gamma}}} = \frac{923}{(5)^{\frac{1.4-1}{1.4}}} = 582.76 \text{ K}$$

(i) *Compressor work per kg of air*

$$w_C = h_2 - h_1 = C_p(T_2 - T_1)$$
$$= 1.005 \times (472 - 298)$$
$$= \mathbf{174.87 \text{ kJ/kg}}$$

(ii) *Turbine work per kg of air*

$$w_T = h_3 - h_4 = C_p(T_3 - T_4)$$
$$= 1.005 \times (923 - 582.76)$$
$$= \mathbf{341.94 \text{ kJ/kg}}$$

(iii) *Thermal efficiency of cycle*

The net work output per kg of air

$$w_{net} = w_T - w_C$$
$$= 341.94 - 174.87 = 167.07 \text{ kJ/kg}$$

Heat supplied per kg of air

$$q_{in} = h_3 - h_2 = C_p(T_3 - T_2)$$
$$= 1.005 \times (923 - 472)$$
$$= 453.25 \text{ kJ/kg}$$

Thermal efficiency;

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{167.07}{453.25}$$

$$= 0.3686 \text{ or } \mathbf{36.86\%}$$

Example 11.29 *Air enters the compressor of an air-standard Brayton cycle at 100 kPa, 300 K with a volumetric flow ratio of 5 m^3/s. The compressor pressure ratio is 10. The turbine inlet temperature is 1400 K. Determine (a) thermal efficiency of the cycle, (b) back work ratio. (c) net power developed in kW.*

Solution

Given An air-standard Brayton cycle operates with

$p_1 = 100$ kPa $T_1 = 300$ K
$\dot{V} = 5$ m^3/s $r_p = 10$
$T_3 = 1400$ K

To find

(i) Thermal efficiency,
(ii) Back work ratio, and
(iii) Net power developed.

Assumptions

(i) Each component in the cycle is assumed in steady state.

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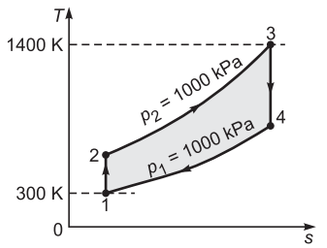

Fig. 11.42 *Schematic with given data*

(ii) Turbine and compressor processes are isentropic.
(iii) No pressure drop in heat exchangers.
(iv) The working fluid is air with $C_p = 1.005$ kJ/kg·K, $\gamma = 1.4$ and $R = 0.287$ kJ/kg·K
(v) Kinetic and potential energy effects are negligible.

Analysis

(i) The thermal efficiency of an air-standard Brayton cycle is given by

$$\eta_{Brayton} = 1 - \frac{1}{r_p^{(\gamma-1)/\gamma}} = 1 - \frac{1}{(10)^{\frac{1.4-1}{1.4}}}$$

$$= 0.482 \quad \text{or} \quad \mathbf{48.2\%}$$

(ii) The back work ratio

$$r_{bw} = \frac{\text{Compressor work}}{\text{Turbine work}} = \frac{T_2 - T_1}{T_3 - T_4}$$

Here $T_2 = T_1 r_p^{\frac{\gamma-1}{\gamma}} = 300 \times (10)^{\frac{1.4-1}{1.4}}$

$= 579.2$ K

$$T_4 = \frac{T_3}{\left(r_p\right)^{\frac{\gamma-1}{\gamma}}} = \frac{1400}{(10)^{\frac{1.4-1}{1.4}}} = 721.1 \text{ K}$$

Then back work ratio

$$r_{bw} = \frac{579.2 - 300}{1400 - 725.1} = 0.417 = \mathbf{41.7\%}$$

(iii) Net power developed

Net work done per kg of air

= Turbine work – Compressor work.

$= C_p(T_3 - T_4) - C_p(T_2 - T_1)$

$= 1.005 \times (1400 - 725.1)$

$\quad - 1.005 \times (579.2 - 300)$

$= 397.67$ kJ/kg

Density of fresh air

$$\rho = \frac{p_1}{RT_1} = \frac{100}{0.287 \times 300}$$

$= 1.161$ kg/m^3

Mass flow rate of air

$= \rho \dot{V} = 1.161 \times 5 = 5.80$ kg/s

Net work developed

= Mass flow rate × Net work done/kg

$= 5.80 \times 397.67 =$ **2309.34 kW**

Example 11.30 *The pressure and temperature at the beginning of compression in an air standard Brayton cycle are 100 kPa and 27°C. The heat added per kg of air is 1850 kJ. The compression ratio is 4. Determine the maximum pressure and temperature, thermal efficiency and mean effective pressure. Assume $\gamma = 1.4$, $C_p = 1.005$ kJ/kg·K*

Solution

Given

$p_1 = 100$ kPa, $T_1 = 27°C = 300$ K
$q_{in} = 1850$ kJ/kg $r = 4$,
$\gamma = 1.4$, $C_p = 1.005$ kJ/kg·K

To find

(i) Maximum pressure in the cycle,
(ii) Maximum temperature in the cycle,
(iii) Thermal efficiency, and
(iv) Mean effective pressure.

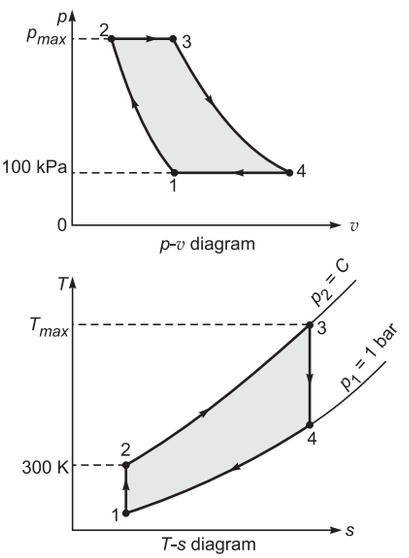

Fig. 11.43

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Assumptions

(i) Working substance is air,
(ii) Compression and expansion are isentropic,
(iii) Heat addition and rejection are reversible.

Analysis

(i) The pressure and temperature after isentropic compression

$$p_{max} = p_2 = p_1 r^{\gamma} = 1 \text{ bar} \times (4)^{1.4}$$
$$= 6.96 \text{ bar}$$

The *maximum pressure* in the cycle is **6.96 bar**

$$T_2 = T_1 r^{\gamma-1} = 300 \times (4)^{1.4-1} = 522.23 \text{ K}$$

(ii) Temperature after heat addition

$$q_{2-3} = C_p(T_3 - T_2)$$
or $$1850 = 1.005 \times (T_3 - 522.23)$$
or $$T_3 = 2363 \text{ K} = 2090°\text{C}$$

The *maximum temperature* in the cycle is **2090°C**

(iii) Thermal Efficiency

Pressure ratio in the cycle

$$r_p = \frac{p_2}{p_1} = \frac{6.96}{1} = 6.96$$

then $$\eta_{Brayton} = 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{(6.96)^{\frac{1.4-1}{1.4}}}$$
$$= 0.4255 \text{ or } \mathbf{42.55\%}$$

(iv) Mean Effective pressure p_m:

$$w_{net} = \eta_{Brayton} \times q_{in} = 0.4255 \times 1850$$
$$= 787.26 \text{ kJ/kg}$$

The specific gas constant for air

$$R = \frac{\gamma - 1}{\gamma} C_p = \frac{1.4 - 1}{1.4} \times 1.005$$
$$= 0.287 \text{ kJ/kg} \cdot \text{K}$$

The initial specific volume of air

$$v_1 = \frac{RT_1}{p_1} = \frac{0.287 \times 300}{100}$$
$$= 0.861 \text{ m}^3/\text{kg}$$

The clearance specific volume

$$v_2 = \frac{v_1}{r} = \frac{0.861}{4} = 4 \text{ m}^3/\text{kg}$$

The swept volume per kg of air

$$v_s = v_1 - v_2 = 0.861 - 4$$
$$= 4 \text{ m}^3/\text{kg}$$

The mean effective pressure

$$p_m = \frac{w_{net}}{v_s} = \frac{787.26}{4} = 4 \text{ kPa}$$
$$= \mathbf{4 \text{ bar}}$$

Example 11.31 *A gas turbine working on an air standard Brayton cycle operates between the temperature limits of 300 K and 1200 K and pressure limits of 101 kPa and 505 kPa. Calculate (a) thermal efficiency of the cycle, (b) compressor work in kJ/kg, (c) turbine work in kJ/kg, and (d) air flow rate for 2.0 kW of net power output.*

Solution

Given A gas turbine operating on air-standard Brayton cycle

$T_1 = 300$ K $T_3 = 1200$ K
$p_1 = 101$ kPa $p_2 = 505$ kPa
$P = 2.0$ kW

To find

(i) Thermal efficiency of the cycle,
(ii) Compressor work in kJ/kg,
(iii) Turbine work in kJ/kg, and
(iv) Mass flow rate of air for power output of 2 kW.

Assumptions

(i) Constant specific heat of air as $C_p = 1.005$ kJ/kg·K.
(ii) Ratio of specific heat as $\gamma = 1.4$.
(iii) Each device in the cycle as steady flow device.

Analysis The air standard Brayton cycle for the given data is shown in Fig. 11.44.

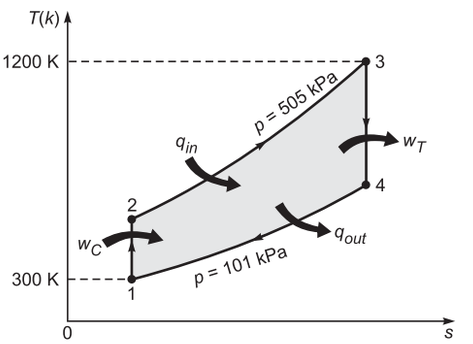

Fig. 11.44

The pressure ratio

$$r_p = \frac{p_2}{p_1} = \frac{505 \text{ kPa}}{101 \text{ kPa}} = 5$$

(i) The efficiency of the Brayton cycle is given as

$$\eta_{Brayton} = 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}$$

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$$= 1 - \frac{1}{(5)^{\frac{1.4-1}{1.4}}} = 0.3686 = \mathbf{36.86\%}$$

Process 1–2: Isentropic compression of air in the compressor;

$$T_2 = T_1\left(r_p\right)^{\frac{\gamma-1}{\gamma}} = 300 \times (5)^{\frac{1.4-1}{1.4}}$$

$$= 475.14 \text{ K}$$

Process 3–4: Isentropic expansion of air in the turbine;

$$T_4 = \frac{T_3}{\left(r_p\right)^{\frac{\gamma-1}{\gamma}}} = \frac{1200}{(5)^{\frac{1.4-1}{1.4}}} = 757.66 \text{ K}$$

(ii) The compressor work per kg of air

$$w_C = h_2 - h_1 = C_p(T_2 - T_1)$$
$$= 1.005 \times (475.14 - 300)$$
$$= \mathbf{176.0 \text{ kJ/kg}}$$

(iii) Turbine work per kg of air

$$w_T = h_3 - h_4 = C_p(T_3 - T_4)$$
$$= 1.005 \times (1200 - 757.66)$$
$$= \mathbf{444.5 \text{ kJ/kg}}$$

The net work output per kg of air

$$w_{net} = w_T - w_C$$
$$= 444.5 - 176 = 268.5 \text{ kJ/kg}$$

(iv) The mass flow rate of air

$$\dot{m}_{air} = \frac{P}{w_{net}} = \frac{2.0 \text{ kW}}{268.5 \text{ kJ/kg}}$$
$$= 7.45 \times 10^{-3} \text{ kg/s} = \mathbf{26.81 \text{ kg/h}}$$

Example 11.32 *A gas turbine plant operates on an air-standard Brayton cycle between the temperature limits of 27°C K and 800°C.*

(a) Find the pressure ratio at which the cycle efficiency approaches the Carnot cycle efficiency.

(b) Find the pressure ratio at which the work done per kg of air would be maximum.

(c) Compare the efficiency at this pressure ratio with Carnot efficiency for the given temperature.

Solution

Given A gas turbine operating on air standard Brayton cycle

$T_{min} = 27°\text{C} = 300 \text{ K}$

$T_{max} = 800°\text{C} = 1073 \text{ K}$

To find

(i) Pressure ratio for $\eta_{Carnot} = \eta_{Brayton}$.

(ii) Pressure ratio for maximum work done.

(iii) Efficiency comparison between η_{Carnot} and $\eta_{Brayton}$.

Analysis

(i) *The pressure ratio at which the cycle efficiency approaches the Carnot cycle efficiency:*

The air-standard Carnot cycle efficiency

$$\eta_{Carnot} = 1 - \frac{T_{min}}{T_{max}} = 1 - \frac{300}{1073} = 0.72$$

The air standard Brayton cycle efficiency is given as

$$\eta_{Brayton} = 1 - \frac{1}{\left(r_p\right)^{\frac{\gamma-1}{\gamma}}}$$

When η_{Carnot} approaches $\eta_{Brayton}$ then

$$0.72 = 1 - \frac{1}{\left(r_p\right)^{\frac{1.4-1}{1.4}}}$$

or $\left(r_p\right)^{\frac{1.4-1}{1.4}} = \frac{1}{0.28}$

Pressure ratio; $r_p = \left(\frac{1}{0.28}\right)^{\frac{1.4}{0.4}} = \mathbf{86.08}$

(ii) *Pressure ratio for maximum work done*

$$(r_p)_{opt} = \left(\frac{T_{max}}{T_{min}}\right)^{\frac{\gamma}{2(\gamma-1)}}$$

$$= \left(\frac{1073}{300}\right)^{\frac{1.4}{2\times(1.4-1)}} = \mathbf{9.3}$$

(iii) *Comparison efficiency for $r_p = 9.3$*

$$\eta_{Brayton} = 1 - \frac{1}{\left(r_p\right)^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{(9.3)^{\frac{1.4-1}{1.4}}} = 0.471$$

Ratio $\frac{\eta_{Brayton}}{\eta_{Carnot}} = \frac{0.471}{0.72} = \mathbf{0.654}$

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Summary

- Internal combustion engine is a heat engine that converts chemical energy of fuel into mechanical energy.
- The compression ratio r of a reciprocating engine is the ratio of maximum possible volume to the clearance volume in the cylinder.

$$r = \frac{V_{max}}{V_{min}} = \frac{V_{BDC}}{V_{TDC}} = \frac{V_1}{V_2}$$

- The mean effective pressure p_m is that pressure, which if it acts on the piston during the entire power stroke, would produce an amount of work equal to that actually produced by the engine.

$$p_m = \frac{W_{net}}{V_{max} - V_c} = \frac{W_{net}}{V_s} \text{ (kPa)}$$

- In an air standard cycle analysis, the working substance is air as an ideal gas. The combustion process of an actual cycle is replaced by heat-addition process and exhaust process by heat-rejection process.
- The Otto cycle is an ideal cycle for spark-ignition reciprocating engines. It consists of four reversible processes—isentropic compression, constant-volume heat adddition, isentropic expansion and constant-volume heat rejection. The thermal efficiency of an Otto cycle is

$$\eta_{Otto} = 1 - \frac{1}{r^{\gamma - 1}}$$

- The Diesel cycle is an ideal cycle for the compression ignition engines. It is very similar to Otto cycle except that the heat is added at constant pressure. Its thermal efficiency is given by

$$\eta_{Diesel} = 1 - \frac{1}{r^{\gamma - 1}} \left[\frac{\rho^{\gamma} - 1}{\gamma(\rho - 1)} \right]$$

- For given compression ratio,

$$\eta_{Otto} > \eta_{Dual} > \eta_{Diesel}$$

- For same maximum pressure and temperature,

$$\eta_{Diesel} > \eta_{Dual} > \eta_{Otto}$$

- The Brayton cycle is an ideal cycle for the gas turbine plants and its thermal efficiency is given as

$$\eta_{Brayton} = 1 - \frac{1}{r_p^{(\gamma - 1)/\gamma}}$$

where, $r_p = \frac{p_2}{p_1}$, pressure ratio and γ is ratio of two specific heats. The thermal efficiency of an ideal Brayton cycle depends on pressure ratio and specific heats ratio.

- The *back work ratio* is an important characteristic of a gas turbine cycle. It is defined as

$$r_{bw} = \frac{\text{Compressor work}}{\text{Turbine work}}$$

- The Atkinson cycle is an ideal cycle for Otto engine exhausting to a gas turbine. Its efficiency is given by

$$\eta_{Atkinson} = 1 - \frac{1}{\gamma} \left[\frac{r_e - r}{r_e^{\gamma} - r^{\gamma}} \right]$$

Glossary

BDC Bottom dead centre

Bore Internal diameter of cylinder

Brake power Power available from the engine for external use

CI engine Compression ignition (Diesel) engine

Clearance volume Volume left in the centre, when piston is at TDC

Compression ratio Ratio of maximum volume to minimum volume in the cylinder

Diesel cycle Theoretical cycle for Diesel and oil engines

Friction power Difference between indicated power and brake power

IC engines Internal combustion reciprocating engines

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Indicated power Power developed on the piston by combustion gases inside the cylinder

Mean effective pressure Ratio of net work done to swept volume in the cycle

Mechanical efficiency Ratio of brake power to the indicated power

Otto cycle Theoretical cycle for petrol engines

SI engine Spark ignition (petrol) engine

Stroke Linear distance between TDC and BDC

Swept volume Piston displacement volume in cylinder

TDC Top dead centre

Thermal efficiency Ratio of brake power to the heat supply rate

Review Questions

1. Define compression ratio.
2. What is mean effective pressure? and what is its significance.
3. Define (a) work ratio (b) swept volume, (c) charge, (d) thermal efficiency.
4. Derive an expression for thermal efficiency of Otto cycle.
5. What are cold air assumptions?
6. Why are engines not operated on Carnot cycle? Explain
7. Write the drawback of Carnot cycle.
8. Discuss the deviation of Stirling and Ericsson cycles from Carnot cycle.
9. Compare Otto, Diesel and Dual cycles for given compression ratio.
10. Prove that the compression ratio corresponds to maximum work in the Otto cycle, is given by

$$r = \left(\frac{T_{max}}{T_{min}}\right)^{\frac{1}{2(\gamma-1)}}$$

11. Discuss the effect of pressure variation on thermal efficiency of Brayton cycle.

Problems

1. The engine of a car has four cylinders of 70-mm bore, and 75-mm stroke. The compression ratio is 8. Determine the cubic capacity of the engine and the clearance volume of each cylinder.

[1154 cc, 41.21 cc]

2. A four-cylinder, four-stroke petrol engine has a bore of 80 mm and a stroke of 80 mm. The compression ratio is 8. Calculate the cubic capacity of the engine and clearance volume of each cylinder. What type of engine is this?

[(a) 1608.4 cc, (b) 57.4 cc, (c) Square engine]

3. The compression ratio in an air standard Otto cycle is 10. At the beginning of compression strokes, the pressure is 100 kPa and temperature is 15°C. The heat transferred to the air per cycle is 1800 kJ/kg of air. Determine:

(a) The pressure and temperatures at the salient points.

(b) The mean effective pressure.

4. A four cylinder spark ignition engine has a compression ratio of 8, each cylinder has a maximum volume of 0.6 litre. At the beginning of the compression process, the air is at 98 kPa and 17°C and the maximum temperature in the cycle is 1800 K. Assume engine operates on ideal Otto cycle, determine: (a) amount of heat supplied per cylinder (b) thermal efficiency.

[2.32 kJ, 56.4%]

5. An ideal Otto cycle has a compression ratio of 9.2 and uses air as a working fluid. At the beginning of the compression process, the air is at 98 kPa and 27°C. The pressure is doubled during heat addition process. Determine: (a) Amount of heat added to air, (b) net work output, (c) thermal efficiency. *[523.3 kJ/kg, 307.9 kJ/kg, 58.8%]*

6. An engine is working on Otto cycle. Air enters the cylinder at 1 bar and 27°C, it is compressed isentropically with a compression ratio of 7 and then heated at constant volume till the temperature

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rises to 2000 K. Find the air standard efficiency, pressure of air at the end of compression and after heat addition process, and the mean effective pressure of the cycle. Assume C_v = 0.718 kJ/kg·K, γ = 1.4, and R = 0.287 kJ/kg·K.

[54.08%, 15.245 bar, 46.66 bar, 708.5 bar]

7. A four stroke engine working on Otto cycle has a swept volume of 0.1 m^3. The compression ratio is 7. The condition at the start of the cycle is at 1 bar, 90°C. The heat addition at the constant volume is 100 kJ/cycle. Find the thermal efficiency, mean effective pressure, the pressure and temperature at salient points in the cycle. Assume air as working substance with C_v = 0.718 kJ/kg·K and γ = 1.4

[46%, 4.592 bar, T_2 = 790.6 K, p_2 = 15.25 bar, p_3 = 39.24 bar, T_3 = 2034.1 K, p_4 = 2.57 bar, T_4 = 934.0 K]

8. An engine of 250 mm bore and 375 mm stroke works on Otto cycle. The clearance volume is 0.00263 m^3. The initial pressure and temperature are 1 bar and 50°C. If the maximum pressure is limited to 25 bar, find the following:

(a) The air standard efficiency of the cycle.

(b) The mean effective pressure for the cycle.

Assume the ideal conditions.

[(a) 56.5%, (b) 1.334 bar]

9. An engine working on Otto cycle has a volume of 0.45 m^3, pressure 1 bar and temperature 30°C at the beginning of compression stroke. At the end of compression stroke, the pressure is 11 bar, 210 kJ of heat is added at constant volume. Determine:

(a) Pressures, temperature and volumes at salient points in the cycle,

(b) Percentage clearance,

(c) Efficiency,

(d) Net work per cycle,

(e) Mean effective pressure,

(f) Ideal power developed by the engine if the number of working cycles per minute is 210.

Assume the cycle is reversible.

[(a) 600 K, 0.081 m^3, 1172 K, 21.48 bar, 0.081 m^3, 1.97 bar, 591.8 K, 0.45 m^3 (b) 21.95%, (c) 0.495, (d) 2.818 bar, (e) 364 kW]

10. The compression ratio in an air-standard Otto cycle is 8. At the beginning of compression process, the pressure is 1 bar and the temperature is 300 K. The heat transfer to the air per cycle is 1900 kJ/kg of air. Calculate:

(a) Thermal efficiency,

(b) The mean effective pressure.

[(a) 56.47%, (b) 14.24 bar]

11. The temperature at the beginning of the compression process of an air standard Otto cycle with a compression ratio of 8 is 300 K, the pressure is 1 atm and the cylinder volume is 560 cc. The maximum temperature in the cycle is 2000 K. Calculate

(a) Temperature and pressure at the end of each process of cycle,

(b) Thermal efficiency of the cycle.

[p_2 = 18.3 atm, p_3 = 53.33 atm, p_4 = 2.9 atm, T_2 = 689.2 K, T_4 = 870.5 K, η_{Otto} = 56.4%]

12. A four-stroke engine having a swept volume of 0.13 m^3 operates on Otto cycle. The compression ratio is 6 and the conditions at the beginning of compression are 1 bar and 60°C. The heat supplied is 150 kJ per cycle. Calculate the work done, efficiency and mean effective pressure.

Take C_p = 1.005 kJ/kg·K.

and C_v = 0.718 kJ/kg·K.

[76.75 kJ/cycle, 51.2%, 5.9 bar]

13. An engine working on a constant volume cycle has a clearance volume of 1 litre and a stroke volume of 6 litres. The suction pressure and temperature are 1 bar and 20°C, respectively. The pressure at the end of heat addition is 25 bar. Determine

(a) Thermal efficiency of cycle,

(b) Work done per cycle.

Take C_v during heat addition = 0.837 kJ/kg·K. C_v during heat rejection = 0.737 kJ/kg·K and γ = 1.4 *[59.5%, 1.693 kJ]*

14. A four-cylinder, four-stroke Diesel engine is to develop 30 kW at 1000 rpm. The stroke is 1.4 times the bore and the indicated mean effective pressure is 6.0 bar. Determine the bore and stroke of the engine. *[176 mm, 246 mm]*

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15. A Diesel engine has an inlet temperature and pressure of 17°C and 1 bar, respectively. The compression ratio is 15 and the maximum cycle temperature is 1400 K. Calculate the air-standard efficiency of the Diesel cycle. Take $\gamma = 1.4$.

[1.634, 62.3%]

16. An engine working on the Otto cycle has a clearance of 17% of stroke volume and the initial pressure of 0.95 bar, and a temperature of 30°C. If the pressure at the end of constant volume heat addition is 28 bar, find

(a) Air standard efficiency
(b) The maximum temperature in the cycle
(c) Ideal mean effective pressure

Assume working fluid to be air.

[(a) 53.7%, (b) 1024.8°C, (c) 3.167 bar]

17. A four-stroke Diesel engine with four cylinders develops an indicated power of 125 kW and delivers brake power of 100 kW. Calculate (a) friction power, and (b) mechanical efficiency of the engine. *[(a) 25 kW, (b) 80%]*

18. An air standard Diesel cycle has a compression ratio of 16 and a cut-of ratio of 2. At the beginning of the compression process the air is at 95 kPa and 27°C. Determine:

(a) temperature after heat addition,
(b) the thermal efficiency, and
(c) mean effective pressure.

19. An ideal Diesel engine has a compression ratio of 20 and uses air as working fluid. At the beginning of compression stroke, the air is at 100 kPa and 20°C. If the maximum temperature in the cycle is not to exceed 2200 K, determine: (a) thermal efficiency, and (b) mean effective pressure.

[(c) 63.5%, (b) 933 kPa]

20. Two engines are operated on ideal Otto and diesel cycles for which the following information is available:

Maximum temperature = 1227°C
Exhaust temperature = 447°C
Ambient condition = 1 bar, 37°C
Air consumption = 2 kg/min

Estimate: (a) Compression ratio, (b) Air standard efficiency, and (c) power output.

Assume $C_p = 1.005\ \text{kJ/kg}\cdot\text{K}$, $C_v = 0.718\ \text{kJ/kg}\cdot\text{K}$.

[Otto cycle (a) 6.8 (b) 53.54%, (c) 11.31 KW Diesel cycle (a) 12.41, (b) 58.21%, (c) 13.67 KW]

21. An engine with 200 mm cylinder diameter and 300 mm stroke works on theoretical Diesel cycle. The initial pressure and temperature of air used are 1 bar and 27°C. The cut off is 8% of the stroke. Determine:

(a) Pressures and temperatures at all salient points.
(b) Theoretical air standard efficiency.
(c) Mean effective pressure.
(d) Power of the engine if the working cycles per minute are 380.

Assume that compression ratio is 15 and working fluid is air. Consider all conditions to be ideal.

[(a) 44.31 bar, 886.2 K, 0.0006728 m^3, 44.31 bar, 0.001426 m^3, 1878.3 K, 2.866 bar, 858.38 K, 0.0101 m^3 (b) 59.8% (c) 7.424 bar, (d) 44.27 kW]

22. An air-standard Diesel cycle has a compression ratio of 18, and the heat transferred to the working fluid per cycle is 1800 kJ/kg. At the beginning of the compression stroke, the pressure is 1 bar and the temperature is 300 K. Calculate: (a) Thermal efficiency, (b) The mean effective pressure.

[(a) 61%, (b) 13.58 bar]

23. A compression ignition engine has a stroke 270 mm, and a cylinder diameter of 165 mm. The clearance volume is 0.000434 m^3 and the fuel ignition takes place at constant pressure for 4.5 per cent of the stroke. Find the efficiency of the engine assuming it works on the Diesel cycle. *[61.7%]*

24. The compression ratio of dual combustion cycle is 14 and the maximum temperature is limited to 1850°C. If the working substance receives equal amount of heat during both constant volume and constant pressure process, determine the air standard efficiency of the cycle. Assume temperature at the start of the cycle is 27°C.

[64.12%]

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Objective Questions

1. Thermal efficiency of a heat engine is defined as
   (a) $\dfrac{\text{Work done}}{\text{Heat supplied}}$ (b) $\dfrac{\text{Indicated Power}}{\text{Brake Power}}$
   (c) $\dfrac{\text{Brake Power}}{\text{Indicated Power}}$ (d) none of the above
2. Air-standard efficiency of Otto cycle is given by
   (a) $1-\left(\dfrac{1}{r}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}$ (b) $1-(r)^{(\gamma-1)}$
   (c) $r^{(\gamma-1)}-1$ (d) $1-\left(\dfrac{1}{r}\right)^{(\gamma-1)}$
3. The mean effective pressure of an engine is defined as
   (a) $\dfrac{\text{Work done}}{\text{Stroke volume}}$
   (b) $\dfrac{\text{Work done/cycle}}{\text{Cylinder volume}}$
   (c) $\dfrac{\text{Work done/cycle}}{\text{Stroke volume}}$
   (d) $\dfrac{\text{Work done/kg}}{\text{Stroke volume}}$
4. Which statement is true for Otto cycle?
   (a) Heat addition at constant volume and heat rejection at constant volume
   (b) Heat addition at constant volume and heat rejection at constant pressure
   (c) Heat addition at constant pressure and heat rejection at constant volume
   (d) Heat addition at constant pressure and heat rejection at constant pressure
5. Air standard Otto cycle consists of
   (a) two isothermal processes and two constant-volume processes
   (b) two isothermal processes and two constant-pressure processes
   (c) two isentropic processes and two constant-volume processes
   (d) two isothermal processes and two isentropic processes
6. Which statement is true for Diesel cycle?
   (a) Heat addition at constant volume and heat rejection at constant volume
   (b) Heat addition at constant volume and heat rejection at constant pressure
   (c) Heat addition at constant pressure and heat rejection at constant volume
   (d) Heat addition at constant pressure and heat rejection at constant pressure
7. Air standard Diesel cycle consists of
   (a) two isothermal and two constant-volume processes
   (b) two isentropic and two constant-pressure processes
   (c) two isentropic and two constant-volume processes
   (d) none of the above
8. The efficiency of air standard Otto cycle depends on
   (a) pressure ratio in the cycle
   (b) temperature ratio in the cycle
   (c) Compression ratio in the cycle
   (d) mean effective pressure
9. The efficiency of air standard Diesel cycle depends on
   (a) cut-off ratio in the cycle
   (b) compression ratio in the cycle
   (c) both (a) and (b)
   (d) none of above
10. Air standard efficiency of Diesel cycle is given by
   (a) $1-\dfrac{1}{(r)^{\gamma-1}}\left[\dfrac{\rho^{\gamma-1}}{\gamma(\rho-1)}\right]$
   (b) $1-\dfrac{1}{(r)^{\gamma-1}}\left[\dfrac{\rho^{\gamma}-1}{\gamma(\rho-1)}\right]$
   (c) $1-\dfrac{1}{(r)^{\gamma-1}}\left[\dfrac{\rho^{\gamma-1}}{(\rho-1)}\right]$
   (d) $1-\dfrac{1}{(r)^{\gamma-1}}\left[\dfrac{\rho^{-1}}{(\rho^{\gamma}-1)}\right]$

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11. Cut-off ratio is defined as
    (a) $\frac{\text{Volume before compression}}{\text{Volume after compression}}$
    (b) $\frac{\text{Volume after heat supply}}{\text{Volume after compression}}$
    (c) $\frac{\text{Volume after expansion}}{\text{Volume after compression}}$
    (d) $\frac{\text{Volume after expansion}}{\text{Volume before expansion}}$
12. Which process is included in air standard Diesel cycle?
    (a) Polytropic compression
    (b) Isochoric heat addition
    (c) Isobaric heat addition
    (d) Isochoric and isobaric heat addition
13. Which one of the following is part of air standard dual cycle?
    (a) Polytropic compression
    (b) Isochoric heat addition
    (c) Isobaric heat addition
    (d) Isochoric and isobaric heat addition
14. Which one of the following is part of air standard Brayton cycle?
    (a) Polytropic compression
    (b) Isochoric heat addition
    (c) Isobaric heat addition
    (d) Isochoric and isobaric heat addition
15. Air standard Stirling cycle consists of
    (a) two isothermal and two constant-volume processes
    (b) two isothermal and two constant-pressure processes
    (c) two isentropic and two constant-volume processes
    (d) two isothermal and two isentropic processes
16. Air standard Ericsson cycle consists of
    (a) two isothermal and two constant-volume processes
    (b) two isothermal and two constant-pressure processes
    (c) two isentropic and two constant-volume processes
    (d) two isothermal and two isentropic processes
17. Which one of the following is part of air standard Stirling cycle?
    (a) Reversible isothermal compression
    (b) Polytropic expansion
    (c) Isobaric heat addition
    (d) Reversible isentropic compression
18. Which one of the following is part of air-standard Ericsson cycle?
    (a) Reversible isothermal compression
    (b) Polytropic expansion
    (c) Isochoric heat rejection
    (d) Reversible isentropic compression
19. Which one of the following is part of air standard Atkinson cycle?
    (a) Isothermal heat addition
    (b) Constant-volume heat rejection
    (c) Constant-volume heat addition
    (d) Constant-pressure heat addition
20. Which one of the following is part of air standard Lenoir cycle?
    (a) Constant volume addition
    (b) Isentropic expansion
    (c) Constant pressure heat rejection
    (d) All of the above
21. Air standard efficiency of Brayton cycle is given by
    (a) $1-\frac{1}{(r)^{\gamma-1}}\left[\frac{\rho^{\gamma-1}}{\gamma(\rho-1)}\right]$
    (b) $1-\frac{1}{(r)^{\gamma-1}}\left[\frac{\rho^{\gamma}-1}{\gamma(\rho-1)}\right]$
    (c) $1-\frac{1}{r_p{}^{(\gamma-1)/\gamma}}$
    (d) $1-\frac{1}{(r)^{\gamma-1}}$
22. Air standard efficiency of Lenoir cycle is given by
    (a) $1-\frac{1}{(r)^{\gamma-1}}\left[\frac{\rho^{\gamma-1}}{\gamma(\rho-1)}\right]$
    (b) $1-\frac{1}{(r)^{\gamma-1}}\left[\frac{\rho^{\gamma-1}-1}{\gamma(\rho-1)}\right]$

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(c) $1-\dfrac{1}{r_p^{(\gamma-1)/\gamma}}$

(d) $1-\gamma\left(\dfrac{r_p^{\left(\frac{1}{\gamma}\right)}-1}{r_p-1}\right)$

23. Air standard efficiency of Atkinson cycle is given by

(a) $1-\dfrac{1}{(r)^{\gamma-1}}\left[\dfrac{\rho^{\gamma-1}}{\gamma(\rho-1)}\right]$

(b) $1-\dfrac{1}{(r)^{\gamma-1}}\left[\dfrac{\rho^{\gamma}-1}{\gamma(\rho-1)}\right]$

(c) $1-\gamma\left[\dfrac{r_e-r}{r_e^{\gamma}-r^{\gamma}}\right]$

(d) $1-\gamma\left(\dfrac{r_p^{\left(\frac{1}{\gamma}\right)}-1}{r_p-1}\right)$

24. For given compression ratio, among Otto, Diesel and dual cycles
   (a) Diesel cycle is most efficient
   (b) Otto cycle is most efficient
   (c) dual cycle is most efficient
   (d) None of the above

25. For same maximum pressure and temperature among Otto, Diesel and dual cycles
   (a) Diesel cycle is most efficient
   (b) Otto cycle is most efficient
   (c) dual cycle is most efficient
   (d) none of the above

26. The efficiency of a cycle is maximum when it is
   (a) Diesel cycle engine
   (b) Otto cycle engine
   (c) dual cycle engine
   (d) reversible engine

27. Gas power cycle is not used for
   (a) Diesel cycle engine
   (b) Otto cycle engine
   (c) Locomotive engine
   (d) Aircraft engine

Answers

1. (a)	2. (d)	3. (c)	4. (a)	5. (c)	6. (c)	7. (d)	8. (c)	9. (c)	10. (b)
11. (b)	12. (c)	13. (d)	14. (c)	15. (a)	16. (b)	17. (a)	18. (a)	19. (c)	20. (d)
21. (c)	22. (d)	23. (c)	24. (b)	25. (a)	26. (d)	27. (c)			

12

Vapour Power Cycles

Introduction

In the vapour power cycle, the phase of working substance changes alternatively. The change of phase allows more energy to be stored in the working substance that can be stored by only sensible heating. The working substance expands as a vapour but it is compressed as a liquid with much smaller specific volume, thus a very little of expansion work is used for compression process. The most common working substance is water and thus the cycle and plant are called *steam power cycle* and *steam power plant*, respectively, even though water is used in the liquid phase during a part of the cycle. Steam power plants generate a major fraction of electric power produced in the world.

12.1 MODELLING A STEAM POWER PLANT

A steam power plant consists of the following main components:

1. Boiler with its mountings and accessories
2. Turbine
3. Condenser
4. Feed-water pump
5. Electric generator
6. Cooling tower
7. Circulating water pump
8. Chimney
9. Draught system
10. Feed-water treatment plant

The basic components of a simplified steam power plant is shown in Fig. 12.1. To facilitate thermodynamic analysis, the whole plant can be divided into four major subsystems identified as is *A, B, C* and *D* on the diagram.

Subsystem (A) The subsystem *A* consists of a furnace and chimney. Its function is to supply heat energy to the boiler for steam generation. The heat energy may be obtained by burning of fossil fuel, nuclear reaction, or by concentrating solar energy.

Subsystem (B) In the subsystem *B*, the working fluid passes through the series of four interconnected components and power is generated in this cycle. This cycle is referred as *steam cycle*.

The subsystem *B* is of prime focus in this chapter, where the heat energy is converted into work. It consists of a boiler (steam generator), a turbine, a condenser, and a feed pump. The steam generated in the boiler is passed through the turbine, where it expands to a lower pressure, thus power is generated. The steam leaving the turbine is passed through the condenser, where it condenses to water

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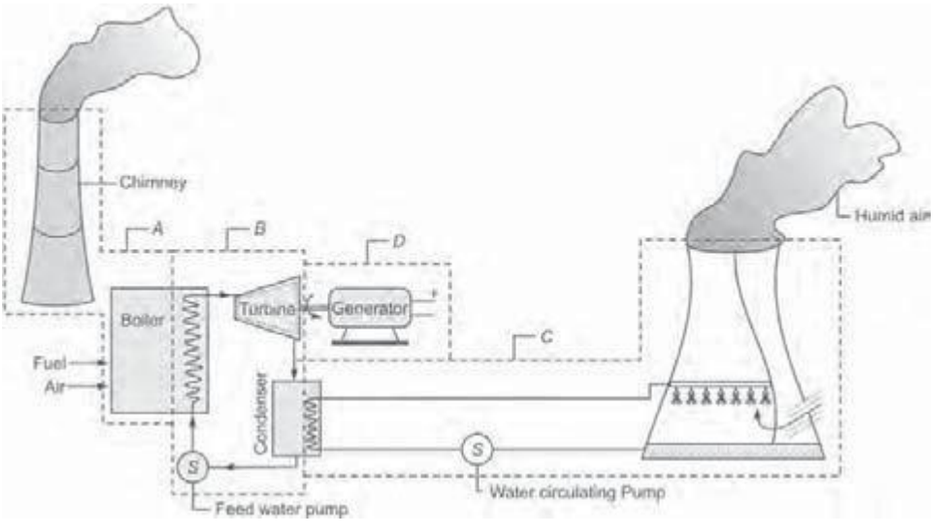

Fig. 12.1 *Components of a simple steam power plant*

and creates a low pressure at the turbine exit. The cooling water is circulated in the condenser with the help of the subsystem *C*. The condensate is then recirculated to the boiler with the help of the feed pump.

Subsystem (C) The subsystem *C* consists of the cooling tower and water-circulation arrangement. The circulated warm water from the condenser is sent to the cooling tower, where its heat energy is rejected to the atmosphere. The cooling water is then recirculated through the condenser.

Subsystem (D) The subsystem *D* pertains to generation of electrical energy, and thus consists of a generator. The generated electricity is supplied to a power grid through the substation.

12.2 PERFORMANCE PARAMETERS OF VAPOUR POWER CYCLE

12.2.1 Thermal Efficiency

The thermal efficiency of any power cycle is expressed as

$$\eta_{th} = \frac{\text{Net work done in the cycle}}{\text{Heat supplied in the cycle}} = \frac{w_{net}}{q_{in}} \qquad ...(12.1)$$

12.2.2 Back Work Ratio

It is another performance parameter for a power plant and is defined as the ratio of pump work input to the work developed by the turbine. It is denoted by r_{bw}. The back work ratio is

$$r_{bw} = \frac{\text{Pump work}}{\text{Turbine work}} = \frac{w_p}{w_T} \qquad ...(12.2)$$

12.2.3 Work Ratio

The *work ratio* for a power plant is defined as ratio of the net work output of the cycle to the work developed by the turbine. It is denoted as r_w; and expressed as

$$r_w = \frac{\text{Net work output of the cycle}}{\text{Turbine work}} = \frac{w_{net}}{w_T} = 1 - bwr \qquad ...(12.3)$$

12.2.4 Steam Rate

It is also called *specific steam consumption*. It relates the power output to amount of steam necessary to produce it. It is the amount of steam required to produce 1 kWh (3600 kJ) of power. It is denoted by *ssc* and is expressed as

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$$ssc = \frac{\text{Mass of steam in kg/h}}{\text{Power output in kW}}$$

$$= \frac{\dot{m}_s\,(\text{kg/h})}{\dot{m}_s\,(\text{kg/s}) \times w_{net}\,(\text{kJ/kg})}$$

$$= \frac{3600\ \text{kJ/kWh}}{w_{net}\,(\text{kJ/kg})} \qquad ...(12.4)$$

12.2.5 Heat Rate

It is the amount of heat required by a power plant to produce 1 kWh of power. It is expressed in kJ/kWh.

$$\text{Heat rate} = \frac{(\text{Heat input in kJ/s}) \times (3600\ \text{s/h})}{\text{Net power output in kW}}$$

$$= \frac{3600}{\eta_{th}}\,(\text{kJ/kWh}) \qquad ...(12.5)$$

12.3 CARNOT VAPOUR POWER CYCLE

When we think of a power cycle of maximum efficiency, the Carnot cycle immediately conjures up in our mind. It is a cycle, which has maximum efficiency, operating between given temperature limits and its efficiency is independent of properties of working fluid.

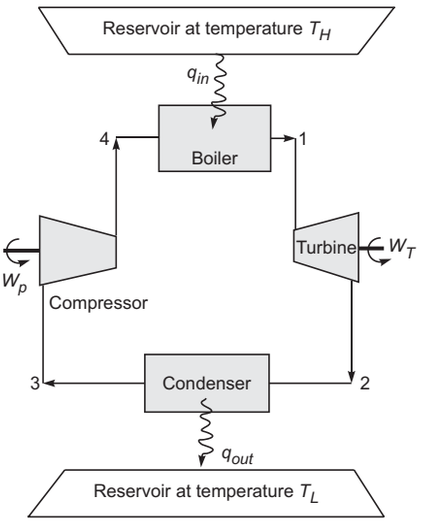
12.3.1 Principal Components and Operation

A Carnot vapour power cycle is executed within saturation dome of a pure substance. It uses a two-phase fluid as the working medium as shown in Fig. 12.2.

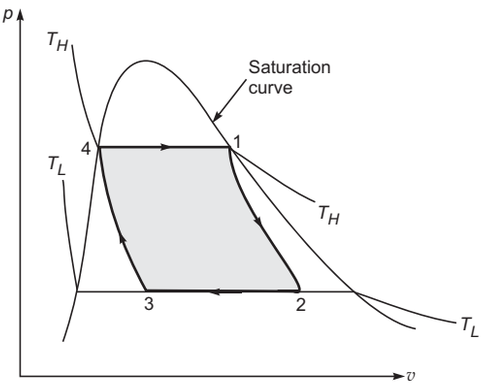
Figure 12.2(a) gives the arrangement of components in the cycle, 12.2(b) shows Carnot vapour power cycle on *p–v* coordinates, 12.2(c) on *T–s* coordinates, and 12.2(d) on *h–s* coordinates. The boundary of the region in which liquid and vapour are both present (the vapour dome) is also indicated.

The four processes in the cycle are as follows:

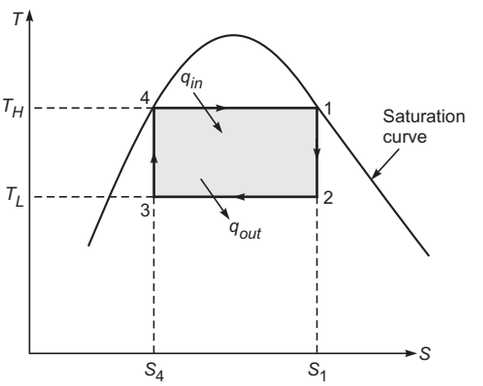
1. Reversible Adiabatic Expansion 1–2 Saturated steam expands in the turbine. The temperature lowers from T_H to T_L. The state 2 is reached in the wet region.



(a) Schematic



(b) p–v as diagram



(c) T–s diagram

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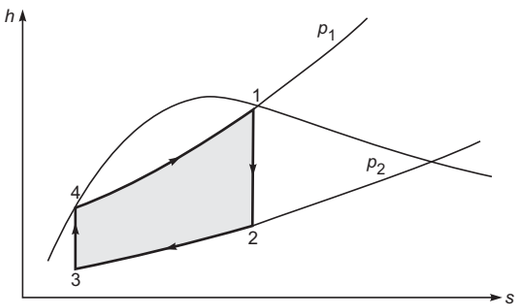

(d) h–s diagram

Fig. 12.2 *Carnot vapour power cycle with two-phase medium*

2. Controlled Condensation 2–3 During this process, the condensation starts from state 2 and stops at state 3 and the heat q_L per unit mass is rejected in the condenser to the sink at T_L.

3. Reversible Adiabatic Compression 3–4 The mixture of liquid and vapour is compressed to the saturation liquid state 4 at boiler pressure.

4. Reversible Isothermal Heat Addition 4–1 During this process, a quantity of heat q_H per unit mass is added in the boiler from the heat source at the temperature T_H.

12.3.2 Analysis of Carnot Vapour Power Cycle

Isothemal heat addition to a vapourising fluid in the boiler;

$$q_{in} = T_H(s_1 - s_4)$$

Isothermal heat rejected by the working substance in the condeser;

$$q_{out} = T_L(s_1 - s_4)$$

The net work done of the cycle;

$$\begin{aligned} w_{net} &= q_{in} - q_{out} \\ &= T_H(s_1 - s_4) - T_L(s_1 - s_4) \\ &= (T_H - T_L)(s_1 - s_4) \end{aligned} \quad ...(12.6)$$

The thermal efficiency of the cycle;

$$\eta_{Carnot} = \frac{w_{net}}{q_{in}} = \frac{(T_H - T_L)(s_1 - s_4)}{T_H(s_1 - s_4)}$$

$$= 1 - \frac{T_H}{T_L} \quad ...(12.7)$$

Equation (12.7) is exactly same as equation derived for an ideal gas. The efficiency of the Carnot vapour power cycle depends on the operating temperatures and is independent of properties of working substance.

12.3.3 Practical Difficulties Associated with Carnot Vapour Power Cycle

However, in spite of the claim of theoretical higher thermal efficiency, the Carnot vapour power cycle has certain practical difficulties as discussed below:

1. Isothermal heat transfer at constant temperature to or from the working substance is not difficult to obtain. It can be achieved by maintaining the working fluid in a two-phase system in the boiler and condenser. But heat-transfer processes in a two-phase system limit the maximum operating temperature in the cycle, thus limiting the thermal efficiency.
2. In the turbine, the dry saturated steam expands isentropically. The quality of steam decreases during expansion. The presence of high moisture content in steam will lead to erosion and wear of the turbine blades.
3. Isentropic compression (process 3–4) involves the compression of liquid–vapour mixture to saturated liquid state. There are three practical difficulties associated with this process:
   (a) Control of condensation at state 3, before reaching to saturated liquid state.
   (b) Compression of two-phase (water + steam) system. Because of large specific volume of vapour than liquid, the compressor size and work input will have to be large.
   (c) Higher compression work will reduce the thermal efficiency of the plant.

These practical difficulties limit the use of Carnot cycle as a suitable model for design of steam power plants.

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Example 12.1 *A steam power plant operates on the Carnot cycle using dry steam at 17.5 bar. The exhaust takes place at 0.075 bar into condenser. The steam consumption is 20 kg/min. Calculate:*

(a) Power developed in the cycle,
(b) The efficiency of the cycle.

Solution

Given A steam power plant operating on Carnot cycle

$p_1 = 17.5$ bar $\qquad p_2 = 0.075$ bar

$\dot{m}_s = 20$ kg/min

To Find

(i) Power developed, and
(ii) Thermal efficiency of the cycle.

Analysis *Properties of steam*

At boiler pressure from Table A-12

$p_1 = 17.5$ bar $= 1750$ kPa

$T_H = 205.76°C = 478.77$ K

$h_4 = h_f = 878.48$ kJ/kg

$h_1 = h_g = 2796.43$ kJ/kg

At condenser pressure from Table A-12,

$p_2 = 0.075$ bar $= 7.5$ kPa

$T_L = 40.29°C = 313.3$ K

The heat supplied in the cycle

$$q_{in} = h_1 - h_4 = 2796.43 - 878.48$$
$$= 1917.95 \text{ kJ/kg}$$

The Carnot efficiency is given as

$$\eta_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{313.3}{478.77}$$
$$= 0.3456 \quad \text{or} \quad \mathbf{34.56\%}$$

The work done per kg in the cycle

$$w_{net} = \eta_{Carnot} \times q_{in} = 0.3456 \times 1917.95$$
$$= 662.84 \text{ kJ/kg}$$

The power developed in the cycle

$$P = \dot{m}_s \, w_{net} = \left(\frac{20}{60} \text{ kg/s}\right) \times (662.84 \text{ kJ/kg})$$
$$= \mathbf{220.95 \text{ kW}}$$

Example 12.2 *A Carnot engine contains 0.1 kg of water. During the heat addition process, saturated liquid is converted into saturated vapour. Heat addition occurs at 12 MPa and heat rejection ocuurs at 30 kPa. Determine:*

(a) Quality of steam at the end of isentropic expansion and at the end of isothermal heat rejection,
(b) Heat added per cycle,
(c) Net work developed in the cycle,
(d) The efficiency of the cycle,
(e) Work ratio.

Solution

Given A Carnot engine operating on vapour power cycle

$p_1 = 12$ MPa $\qquad p_2 = 30$ kPa

$m_s = 0.1$ kg

To Find

(i) Quality of steam at the end of isentropic expansion and at the end of isothermal heat rejection,
(ii) Heat added per cycle,
(iii) Net work developed in the cycle,
(iv) The efficiency of the cycle, and
(v) Work ratio.

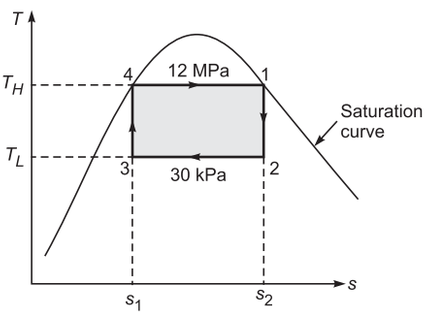

Fig. 12.3 *Schematic*

Analysis *Properties of steam*

At boiler pressure, from Table A-12

$p_1 = 12$ MPa $= 12000$ kPa

$T_H = 324.75°C = 597.75$ K

$h_4 = h_f = 1491.24$ kJ/kg

$h_1 = h_g = 2688.83$ kJ/kg

$s_4 = s_f = 3.4972$ kJ/kg·K

$s_1 = s_g = 5.5$ kJ/kg·K

At condenser pressure, from Table A-12

$p_2 = 30$ kPa

$T_L = 69.13°C = 342.13$ K

$h_f = 289.21$ kJ/kg

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h_{fg} = 2336.07 kJ/kg
s_f = 0.9439 kJ/kg·K
s_{fg} = 6.8247 kJ/kg·K

(i) *Quality of steam at the end of isentropic expansion and at the end of isothermal heat rejection*

The state of steam after isentropic expansion in the turbine;

$$s_1 = s_2 = (s_f + x_2 s_{fg})_{@\ 30\ kPa}$$
$$5.5 = 0.9439 + x_2(6.8247)$$

or $$x_2 = \frac{5.5 - 0.9439}{6.8247} = \mathbf{0.6676}$$

The state of steam before isentropic compression in the compressor;

$$s_4 = s_3 = (s_f + x_3 s_{fg})_{@\ 30\ kPa}$$
$$3.4972 = 0.9439 + x_3(6.8247)$$

or $$x_3 = \frac{3.4972 - 0.9439}{6.8247} = \mathbf{0.374}$$

Specific enthalpy at the state 2

$$h_2 = (h_{f_2} + x_2 h_{fg_2})_{@\ 30\ kPa}$$
$$= 289.21 + 0.6676 \times 2336.07$$
$$= 1848.77\ kJ/kg$$

Specific enthalpy at the state 3

$$h_3 = (h_{f_3} + x_3 h_{fg_3})_{@\ 30\ kPa}$$
$$= 289.21 + 0.374 \times 2336.07$$
$$= 1163.2\ kJ/kg$$

The heat supplied per kg of steam

$$q_{in} = h_1 - h_4 = 2688.83 - 1491.24$$
$$= 1197.59\ kJ/kg$$

(ii) *Heat added per cycle* $Q_{in} = m_s q_{in}$

$$= (0.1\ kg) \times (1197.59\ kJ/kg)$$
$$= \mathbf{119.75\ kJ/cycle}$$

The heat rejected per kg of steam

$$q_{out} = h_2 - h_3 = 1848.77 - 1163.2$$
$$= 685.57\ kJ/kg$$

Net work per kg of steam;

$$w_{net} = q_{in} - q_{out}$$
$$= 1197.59 - 685.57 \approx 512\ kJ/kg$$

(iii) *Net work done* per cycle;

$$W_{net} = m_s w_{net} = (0.1\ kg) \times (512\ kJ/kg)$$
$$= \mathbf{51.2\ kJ/cycle}$$

(iv) *Thermal efficiency of the Carnot cycle*

$$\eta_{Carnot} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{685.57}{1197.59}$$
$$= 0.4275 \quad \text{or} \quad \mathbf{42.75\%}$$

Note: Equation (12.7) will also give same efficiency of Carnot cycle.

(iv) *Work ratio*

Turbine work;

$$w_T = h_1 - h_2 = 2688.83 - 1848.77$$
$$= 840.06\ kJ/kg$$

$$r_w = \frac{w_{net}}{w_T} = \frac{512}{840.06} = \mathbf{0.609}$$

It means 60.9% of turbine workdone will be available as net output of the cycle.

12.4 RANKINE CYCLE

Many of the practical difficulties associated with the Carnot vapour power cycle are eliminated in Rankine cycle. The steam coming out of the boiler is usually in superheated state, and expands in the turbine. After expanding in the turbine, the steam is condensed completely in the condenser. The Rankine Cycle shown in Fig. 12.3 is an ideal vapour power cycle and is used in steam power plants.

12.4.1 Principal Components of Vapour Power Plant

The four basic components of a vapour power plant are shown in Fig. 12.4(a). Each component in the cycle is regarded as a control volume, operating at steady state.

Turbine The vapour leaving the boiler enters the turbine, where it expands isentropically to the condenser pressure at the state 2. The work produced by the turbine is rotary (shaft) work and is used to drive an electric generator or machine.

Condenser The condenser is attached at the exit of the turbine. The vapour leaving the turbine is wet vapour and it is condensed completely in the condenser to the state 3, by giving its latent heat to some other cooling fluid like water.

Pump The liquid condensate leaving the condenser at the state 3 is pumped to the operating pressure of the boiler. The pump operation is considered isentropic.

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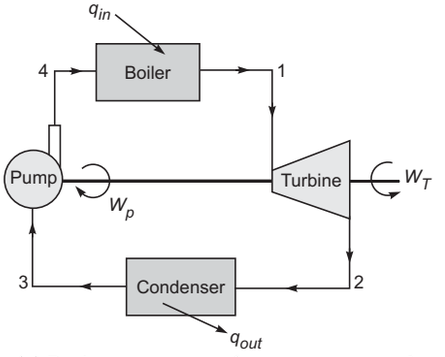
Boiler The heat is supplied to the working fluid (feed water) in the boiler and thus vapour is generated. The vapour leaving the boiler is either saturated at the state 1 or superheated at the state 1′, depending upon the amount of heat supplied in the boiler.

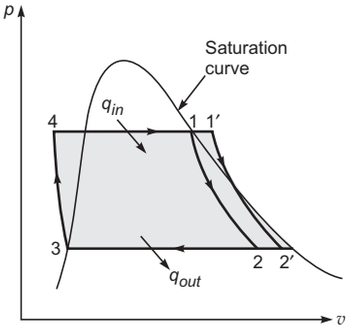
12.4.2 Operation of Rankine cycle

Figure 12.4(b) shows Rankine cycle on *p–v* coordinates, Fig. 12.4(c) on *T–s* coordinates, and Fig. 12.4(d) on *h–s* coordinates. The liquid, vapour, and wet vapour regions are also indicated with the help of saturation curve.

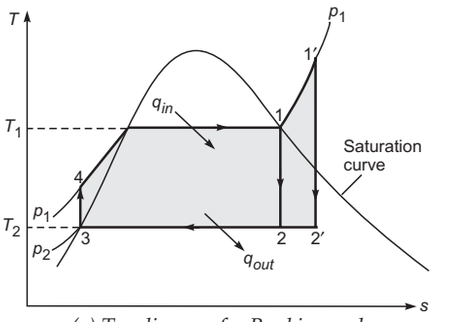
In all three diagrams above, the cycle **1–2–3–4–1** represents an ideal Rankine cycle using *saturated steam* and the cycle **1′–2′–3–4 –1′** represents an ideal Rankine cycle with *superheated steam* at the turbine entry. The Rankine cycle consists of the following four internally reversible processes:



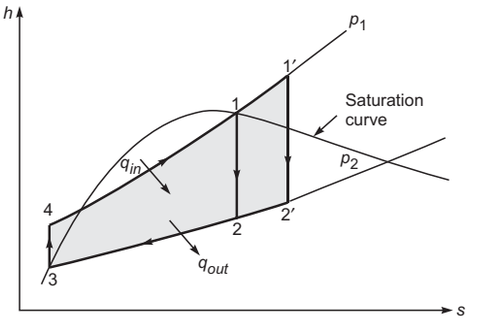
(a) Basic components of vapour power cycle



(b) p–v diagram for Rankine cycle



(c) T–s diagram for Rankine cycle



(d) h–s diagram for Rankine cycle

Fig. 12.4 *Rankine cycle, an ideal vapour power cycle*

Process 1–2 *Isentropic expansion* of the working fluid in the turbine from boiler pressure to condenser pressure.

Process 2–3 *Heat rejection* from the working fluid at *constant pressure* in the condenser till the fluid reaches the saturated liquid state 3.

Process 3–4 *Isentropic compression* of the working fluid in the pump to the boiler pressure at the state 4 in the compressed liquid region.

Process 4–1 *Heat addition* to working fluid at constant pressure in the boiler from state 4 to 1.

12.4.3 Analysis of Rankine Cycle

We assume 1 kg of working substance in the cycle and applying steady flow energy equation to each component in the cycle. If changes of kinetic and potential energy are neglected then the steady-flow energy equation reduces to

$$q - w = \Delta h \qquad ...(12.8)$$

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(i) For isentropic compression ($q = 0$) in the pump (process 3–4);

$$-w_p = h_4 - h_3$$

Taking pump work negative; then

$$w_p = h_4 - h_3 \qquad ...(12.9)$$

where h_3 is h_f enthalpy of liquid at condenser pressure p_2.

h_4 is the enthalpy of water at state 4, calculated as

$$h_4 = h_3 + w_p \qquad ...(12.10)$$

Then the isentropic compression work w_p is obtained as

$$w_p = \int_{p_2}^{p_1} v\,dp = v(p_1 - p_2) \quad ...(12.11)$$

where v_f is the specific volume of liquid at condenser pressure p_2. The negative sign from $-\int_{p_2}^{p_1} v\,dp$ has been dropped to make the pump work positive.

(ii) For constant-pressure heat addition process in the boiler ($w = 0$):

$$q_{2-3} = q_{in} = h_1 - h_4 \qquad ...(12.12)$$

(iii) For isentropic expansion process 1–2 in the turbine ($q = 0$):

$$-w_T = h_2 - h_1$$

or $$w_T = h_1 - h_2 \qquad ...(12.13)$$

(iv) For constant-pressure heat removal process 2–3 in the condenser ($w = 0$):

$$q_{2-3} = q_{out} = h_3 - h_2$$

Taking negative sign for heat rejection, then

$$q_{out} = h_2 - h_3 \qquad ...(12.14)$$

The thermal efficiency of any power cycle is expressed as

$$\eta = \frac{\text{Net work done in the cycle}}{\text{Heat supplied in the cycle}} = \frac{w_{net}}{q_{in}}$$

For Rankine cycle,

$$w_{net} = w_T - w_p$$
$$= (h_1 - h_2) - v_f(p_1 - p_2)$$

For a thermodynamic cycle, the net work is also equal to net heat transfer;

$$w_{net} = q_{in} - q_{out}$$
$$= (h_1 - h_4) - (h_2 - h_3) \quad ...(12.15)$$

Thus the efficiency of Rankine cycle can be expressed as

$$\eta_{Rankine} = \frac{q_{in} - q_{out}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$
$$= 1 - \frac{h_2 - h_3}{h_1 - h_4} \qquad ...(12.16)$$

In order to avoid *errosion* and *corrosion* on turbine blades, the wet steam is never allowed to enter the turbine. Figure 12.5 shows *T–s* and *h–s* (Mollier) diagrams for an ideal Rankine cycle using wet steam at turbine entry.

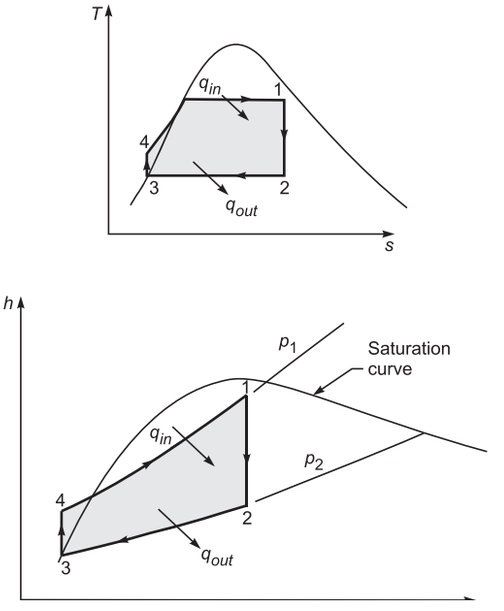

Fig. 12.5 *T–s and h–s diagrams for an ideal Rankine cycle using wet steam*

12.4.4 Relative Efficiency, or Efficiency Ratio

It is defined as the ratio of the actual thermal efficiency of steam power plant to the corresponding Rankine efficiency. It is denoted by '$\eta_{relative}$' and expressed as

$$\eta_{relative} = \frac{\text{Actual thermal efficiency}}{\text{Rankine efficiency}} \qquad ...(12.17)$$

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Example 12.3 *A steam power plant has boiler and condenser pressures of 60 bar and 0.1 bar, respectively. Steam coming out of the boiler is dry and saturated. The plant operates on the Rankine cycle. Calculate thermal efficiency.*

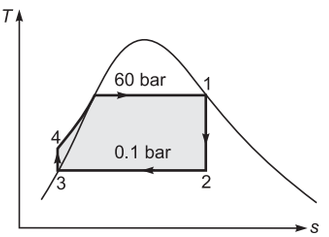

Fig. 12.6 *Rankine cycle using saturated steam*

Solution

Given Rankine cycle with dry saturated steam

$p_1 = 60$ bar $= 6000$ kPa,

$p_2 = 0.1$ bar $= 10$ kPa

To find Thermal efficiency of steam power plant.

Analysis *Properties of steam at principal states*

State 1: Dry saturated steam; from Table A-13

$p_1 = 6000$ kPa,

$h_1 = 2785.10$ kJ/kg

$s_1 = 5.8891$ kJ/kg·K

State 2 : Wet steam;

$p_2 = 10$ kPa

$h_{f_2} = 191.81$ kJ/kg

$h_{fg_2} = 2392.82$ kJ/kg

$s_{f_2} = 0.6492$ kJ/kg·K

$s_{fg_2} = 7.5010$ kJ/kg·K

State 3: Saturated liquid;

$p_3 = 0.1$ bar $= 10$ kPa

$h_3 = h_{f_3} = 191.81$ kJ/kg

$v_{f_3} = 0.001010$ m^3/kg

State 4: Compressed liquid;

$p_4 = 6000$ kPa,

The state 2, after isentropic expansion can be defined by equating entropy at states 1 and 2;

$$s_1 = s_2 = (s_f + x s_{fg})_{@\ 10\ \text{kPa}}$$

$$5.8891 = 0.6492 + x\,(7.5010)$$

or $$x = \frac{5.8891 - 0.6492}{7.5010} = 0.698$$

Specific enthalpy at the state 2

$$h_2 = (h_{f_2} + x h_{fg_2})_{@\ 10\ \text{kPa}}$$
$$= 191.81 + 0.698 \times 2392.82$$
$$= 1863.34 \text{ kJ/kg}$$

The pump work;

$$w_p = v_f (p_1 - p_2) = 0.001010 \times (6000 - 10)$$
$$= 6.05 \text{ kJ/kg}$$

Enthalpy at the state 4;

$$h_4 = h_3 + w_p = 191.81 + 6.05$$
$$= 197.86 \text{ kJ/kg.}$$

Rankine cycle efficiency, Eq. (12.16)

$$\eta_{Rankine} = 1 - \frac{h_2 - h_3}{h_1 - h_4} = 1 - \frac{1863.34 - 191.81}{2785.10 - 197.86}$$
$$= 0.353 \quad \text{or} \quad \mathbf{35.35\%}$$

Example 12.4 *A steam power plant works between pressures of 40 bar and 0.05 bar. If the steam supplied is dry saturated and the cycle of operation is Rankine cycle, find*

(a) Cycle efficiency

(b) Specific steam consumption

Solution

Given Rankine cycle with dry saturated steam

$p_1 = 40$ bar $= 4000$ kPa,

$p_2 = 0.05$ bar $= 5$ kPa

To find

(i) Rankine cycle efficiency, and

(ii) Specific steam consumption.

Analysis *Properties of steam at principal states (From table A-13)*

State 1: Dry saturated steam;

$p_1 = 4000$ kPa

$h_1 = 2800.36$ kJ/kg

$s_1 = 6.0685$ kJ/kg·K

State 2: Wet steam;

$p_2 = 5$ kPa

$h_{f_2} = 137.79$ kJ/kg

$h_{fg_2} = 2423.66$ kJ/kg

$s_{f_2} = 0.4763$ kJ/kg·K

$s_{fg_2} = 7.9187$ kJ/kg·K

State 3: Saturated liquid;

$p_3 = 5$ kPa;

$h_3 = h_{f_3} = 137.79$ kJ/kg

$v_{f_3} = 0.001005$ m^3/kg

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State 4: Compressed liquid;

$$p_4 = 4000 \text{ kPa}$$

$$h_4 = h_3 + w_p$$

The state 2, after isentropic expansion can be defined by equating entropy at states 1 and 2;

$$s_1 = s_2 = (s_f + x s_{fg})_{@\ 5\ \text{kPa}}$$

$$6.0685 = 0.4763 + x\,(7.9187)$$

or $$x = \frac{6.0685 - 0.4763}{7.9187} = 0.7064$$

Specific enthalpy at the state 2

$$h_2 = (h_{f_2} + x h_{fg_2})_{@\ 5\ \text{kPa}}$$

$$= 137.79 + 0.7064 \times 2423.66$$

$$= 1849.84 \text{ kJ/kg}$$

The pump work;

$$w_p = v_f (p_1 - p_2)$$

$$= 0.001005 \times (4000 - 5)$$

$$= 4.015 \text{ kJ/kg}$$

Enthalpy at the state 4;

$$h_4 = h_1 + w_p = 137.79 + 4.015 = 141.8 \text{ kJ/kg}$$

Turbine work;

$$w_T = h_1 - h_2 = 2800.36 - 1849.84$$

$$= 952.52 \text{ kJ/kg}$$

Net work of cycle;

$$w_{net} = w_T - w_p = 952.52 - 4.015$$

$$= 946.5 \text{ kJ/kg}$$

Heat supplied;

$$q_{in} = h_1 - h_4 = 2800.36 - 141.8$$

$$= 2658.56 \text{ kJ/kg}$$

(i) Rankine cycle efficiency,

$$\eta_{Rankine} = \frac{w_{net}}{q_{in}} = \frac{946.50}{2658.56} = 0.3562 \text{ or } \mathbf{35.62\%}$$

(ii) Specific steam consumption by Eq. (12.14);

$$ssc = \frac{3600 \text{ kJ/kWh}}{w_{net} \text{ kJ/kg}} = \frac{3600}{946.50} = \mathbf{3.78 \text{ kg/kWh}}$$

Example 12.5 *In a Rankine cycle, the steam at inlet to a turbine is dry satuarted at a pressure of 35 bar and the exhaust pressure is 0.2 bar. Calculate*

(a) Pump work,
(b) Turbine work,
(c) Rankine efficiency,
(d) Condenser heat flow.
Assume flow rate of 9.5 kg/s.

Solution

Given A Rankine cycle using dry and saturated steam;
Mass flow rate of steam; $\dot{m}_s = 9.5$ kg/s
Turbine inlet pressure; $p_1 = 35$ bar = 3500 kPa,
Condenser pressure, $p_2 = 0.2$ bar

To find

(i) Pump work,
(ii) Turbine work,
(iii) Rankine efficiency,
(iv) Condenser heat flow.

Assumptions

(i) Each component in the cycle is analysed as control volume at steady state.
(ii) Turbine and pump operation are isentropic.
(iii) No pressure drop in boiler and condenser.
(iv) All processes are internally reversible.
(v) Kinetic and potential energy effects in each components are negligible.

Analysis *Properties of steam at principal states*
State 1: Dry saturated steam;

$$p_1 = 3500 \text{ kPa}$$

$$h_1 = 2802.25 \text{ kJ/kg}$$

$$s_1 = 6.122 \text{ kJ/kg·K}$$

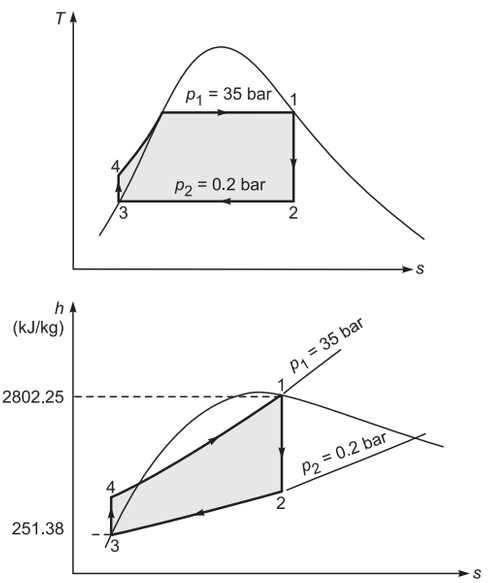

Fig. 12.7 *T–s and h–s diagrams for Ex. 12.5*

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State 2: Wet steam;

$p_2 = 20$ kPa

$h_f = 251.38$ kJ/kg

$h_{fg} = 2358.33$ kJ/kg

$s_f = 0.8319$ kJ/kg·K

$s_{fg} = 7.0766$ kJ/kg·K

State 3: Saturated liquid;

$p_3 = 20$ kPa

$h_3 = h_{f_3} = 251.38$ kJ/kg

$v_{f_3} = 0.001017$ m^3/kg

State 4: Compressed liquid

$p_4 = 3500$ kPa

$h_4 = h_3 + w_p$

The state 2, after isentropic expansion can be defined by equating entropy at states 1 and 2;

$$s_1 = s_2 = (s_f + x s_{fg})_{@\ 3.5\ \text{kPa}}$$

$$6.122 = 0.8319 + x(7.0766)$$

or $$x = \frac{6.122 - 0.8319}{7.0766} = 0.7476$$

Specific enthalpy at the state 2

$$h_2 = (h_{f_2} + x h_{fg_2})_{@20\ \text{kPa}}$$

$$= 251.38 + 0.7476 \times 2358.33$$

$$= 2014.61 \text{ kJ/kg}$$

Pump work per kg of steam;

$$w_p = v_f (p_1 - p_2)$$

$$= 0.001017 \times (3500 - 20) = 3.54 \text{ kJ/kg}$$

State 4; $h_4 = h_3 + w_p = 251.38 + 3.54$

$= 254.92$ kJ/kg

(i) Pump power required;

$$W_p = \dot{m}_s w_p = 9.5 \times 3.54 = \mathbf{33.63\ kW}$$

(ii) Turbine Power output;

$$W_T = \dot{m}_s (h_1 - h_2)$$

$$= 9.5 \times (2802.25 - 2014.61)$$

$$= \mathbf{7482.58\ kW}$$

Net power of cycle;

$$W_{net} = W_T - W_p$$

$$= 7482.58 - 33.63 = 7448.95 \text{ kW}$$

Total heat supplied to steam;

$$Q_{in} = \dot{m}_s (h_1 - h_4)$$

$$= 9.5 \times (2802.25 - 254.92)$$

$$= 24199.63 \text{ kW}$$

(iii) Rankine cycle efficiency,

$$\eta_{Rankine} = \frac{W_{net}}{Q_{in}} = \frac{7482.58}{24199.63}$$

$= 0.3078$ or **30.78%**

(iv) Condenser heat flow;

$$Q_{in} = \dot{m}_s (h_2 - h_3)$$

$$= 9.5 \times (2014.61 - 251.38)$$

$$= \mathbf{16750.6\ kW}$$

Example 12.6 *A steam power plant operates on ideal Rankine cycle. The steam enters the turbine at 3 MPa, 350°C and is condensed in the condenser at a pressure of 75 kPa. Determine thermal efficiency, back work ratio and work ratio of this cycle.*

Solution

Given A steam power plant operating on ideal Rankine cycle.

Turbine inlet condition $p_1 = 3$ MPa $= 3000$ kPa

$T_1 = 350°C$

Condenser pressure, $p_2 = 75$ kPa

To find

(i) Thermal efficiency of the Rankine cycle,

(ii) Back work ratio, and

(iii) Work ratio.

Schematic with given data

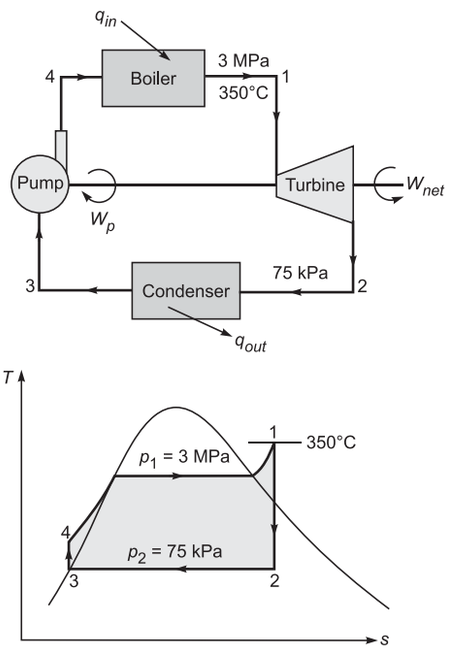

Fig. 12.8

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Analysis *Properties of steam at principal states*

State 1: Superheated steam, Table A-14

$p_1 = 3000$ kPa
$T_1 = 350°C$,
$h_1 = 3115.6$ kJ/kg
$s_1 = 6.7428$ kJ/kg·K

State 2: Wet steam, Table A-13

$p_2 = 75$ kPa
$h_{f_2} = 384.36$ kJ/kg
$h_{fg_2} = 2278.6$ kJ/kg
$s_{f_2} = 1.213$ kJ/kg·K
$s_{fg_2} = 6.2434$ kJ/kg·K

State 3: Saturated liquid

$p_1 = 75$ kPa
$h_3 = h_{f_2} = 384.36$ kJ/kg
$v_{f_3} = 0.001037$ m^3/kg

State 4: Compressed liquid

$p_4 = 3000$ kPa
$h_4 = h_3 + w_p$

The state 2, after isentropic expansion can be defined by equating entropy at states 1 and 2;

$$s_1 = s_2 = (s_f + x s_{fg})_{@\ 75\ \text{kPa}}$$
$$6.7428 = 1.213 + x(6.2434)$$

or
$$x = \frac{6.7428 - 1.213}{6.2434} = 0.885$$

Specific enthalpy at the state 2

$$h_2 = (h_{f_2} + x h_{fg_2})_{@\ 75\ \text{kPa}}$$
$$= 384.36 + 0.885 \times 2278.6$$
$$= 2401.6 \text{ kJ/kg}$$

The pump work;

$$w_p = v_f (p_1 - p_2)$$
$$= 0.001037 \times (3000 - 75) = 3.03 \text{ kJ/kg}$$

Enthalpy at the state 4;

$$h_4 = h_3 + w_p = 384.39 + 3.03$$
$$= 387.40 \text{ kJ/kg}$$

Turbine work;

$$w_T = h_1 - h_2 = 3115.6 - 2401.6$$
$$= 714 \text{ kJ/kg}$$

Net work of cycle;

$$w_{net} = w_T - w_p$$
$$= 714 - 3.03 \approx 711 \text{ kJ/kg}$$

(i) Rankine cycle efficiency, Eq. (12.16)

$$\eta_{Rankine} = 1 - \frac{h_2 - h_3}{h_1 - h_4}$$
$$= 1 - \frac{2401.6 - 383.36}{3115.3 - 387.40} = 0.26 \text{ or } \mathbf{26\%}$$

(ii) Back work ratio;

$$r_{bw} = \frac{\text{Pump work}}{\text{Turbine work}} = \frac{w_p}{w_T}$$
$$= \frac{3.03 \text{ kJ/kg}}{714 \text{ kJ/kg}} = \mathbf{0.0042}$$

(iii) Work ratio; $r_w = \dfrac{w_{net}}{w_T} = \dfrac{711 \text{ kJ/kg}}{714 \text{ kJ/kg}} = \mathbf{0.995}$

Example 12.7 *A steam turbine develops 5 kW, operating on an ideal Rankine cycle. It receives steam at 30 bar, and 300°C and exhausts it to a condenser at a vacuum of 685 mm of Hg. The barometer reads 760 mm of Hg. The condensate is then returned to the boiler by a feed pump. Calculate*

(a) Rankine cycle efficiency,
(b) Dryness fraction of steam entering the condenser,
(c) back work ratio of this cycle,
(d) Mass flow rate of steam.

Solution

Given An ideal Rankine cycle

Turbine inlet condition $p_1 = 30$ bar $= 3000$ kPa,
$T_1 = 300°C$
Condenser vacuum, $h_2 = 685$ mm of Hg
Barometric pressure, $h_{atm} = 760$ mm of Hg

To find

(i) Rankine cycle efficiency,
(ii) Dryness fraction of steam entering the condenser,
(iii) Back work ratio, and
(iv) Mass flow rate of steam.

Analysis The absolute pressure in the condenser in mm of Hg

$$h_{condenser} = h_{atm} - h_2 = 760 - 685 = 75 \text{ mm of Hg}$$

The corresponding pressure in kPa

$$p_2 = (75 \text{ mm of Hg}) \times \frac{101.325 \text{ kPa}}{760 \text{ mm of Hg}} = 10 \text{ kPa}$$

Properties of steam at principal states

State 1: Superheated steam; Table A-14,

$p_1 = 3000$ kPa
$T_1 = 300°C$
$h_1 = 2993.48$ kJ/kg
$s_1 = 6.5389$ kJ/kg·K

State 2: Wet steam; Table A-13,

$p_2 = 10$ kPa

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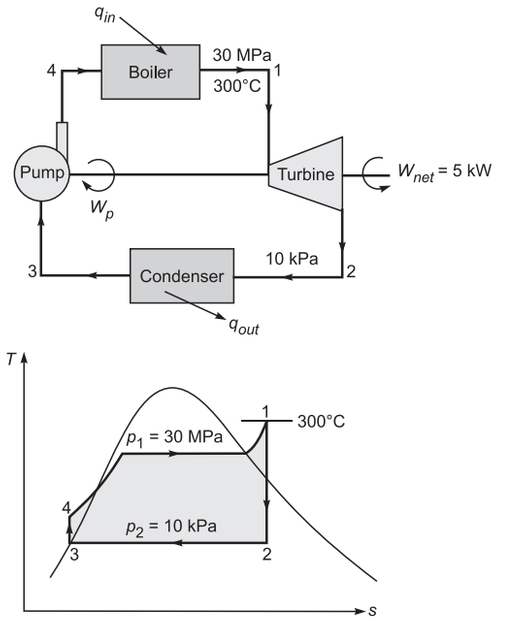

Fig. 12.9 *T–s diagram for Ex. 12.7*

$h_{f_2} = 191.81$ kJ/kg

$h_{fg_2} = 2392.82$ kJ/kg

$s_{f_2} = 0.6492$ kJ/kg·K

$s_{fg_2} = 7.5010$ kJ/kg·K

State 3: Saturated liquid;

$p_3 = 10$ kPa

$h_3 = h_{f_3} = 191.81$ kJ/kg

$v_{f_3} = 0.001001$ m^3/kg

State 4: Compressed liquid;

$p_4 = 3000$ kPa

$h_4 = h_3 + w_p$

The state 2, after isentropic expansion can be defined by equating entropy at states 1 and 2;

$$s_1 = s_2 = (s_f + x s_{fg})_{@\ 10\ \text{kPa}}$$

$$6.5389 = 0.6492 + x(7.5010)$$

or $$x = \frac{6.5389 - 0.6492}{7.5010} = \mathbf{0.785}$$

Specific enthalpy at the state 2

$$h_2 = (h_{f_2} + x h_{fg_2})_{@\ 10\ \text{kPa}}$$
$$= 191.81 + 0.785 \times 2392.82$$
$$= 2070.62 \text{ kJ/kg}$$

The pump work;

$$w_p = v_f (p_1 - p_2)$$
$$= 0.001001 \times (3000 - 10) \approx 3 \text{ kJ/kg}$$

Enthalpy at the state 4;

$$h_4 = h_3 + w_p$$
$$= 191.81 + 3.0 = 194.81 \text{ kJ/kg}$$

Turbine work;

$$w_T = h_1 - h_2$$
$$= 2993.48 - 2070.62 = 922.85 \text{ kJ/kg}$$

Net work of cycle;

$$w_{net} = w_T - w_p$$
$$= 922.85 - 3.0 \approx 919.85 \text{ kJ/kg}$$

Rankine cycle efficiency, Eq. (12.16)

$$\eta_{Rankine} = 1 - \frac{h_2 - h_3}{h_1 - h_4}$$
$$= 1 - \frac{2070.62 - 191.81}{2993.48 - 194.81}$$
$$= 0.3286 \quad \text{or} \quad \mathbf{32.86\%}$$

Back work ratio;

$$r_{bw} = \frac{\text{Pump work}}{\text{Turbine work}}$$
$$= \frac{w_p}{w_T} = \frac{3.0 \text{ kJ/kg}}{922.85 \text{ kJ/kg}} = \mathbf{0.0032}$$

Mass flow rate of steam;

$$\dot{m}_s = \frac{\dot{W}}{w_{net}} = \frac{5 \times 10^3 \text{ kW}}{919.85 \text{ kJ/kg}}$$
$$= \mathbf{5.43 \text{ kg/s} \quad or \quad 19568.4 \text{ kg/h}}$$

Example 12.8 *A Rankine cycle operates with steam as working fluid. The saturated vapour enters the turbine at 8.0 MPa and saturated liquid exists the condenser at a pressure of 0.008 MPa. The net power output of the cycle is 100 MW.*

Determine for the cycle

(a) thermal efficiency,

(b) back work ratio,

(c) mass flow rate of steam in kg/h,

(d) rate of heat transfer to the working fluid in the boiler is MW,

(e) rate of heat rejection in the condenser,

(f) mass flow rate of cooling water in kg/h, if cooling water enters the condenser at 15°C and leaves at 35°C.

Solution

Given A power plant working on Rankine cycle with saturated steam

Turbine entry;

$p_1 = 8.00$ MPa (8000 kPa) as a saturated steam

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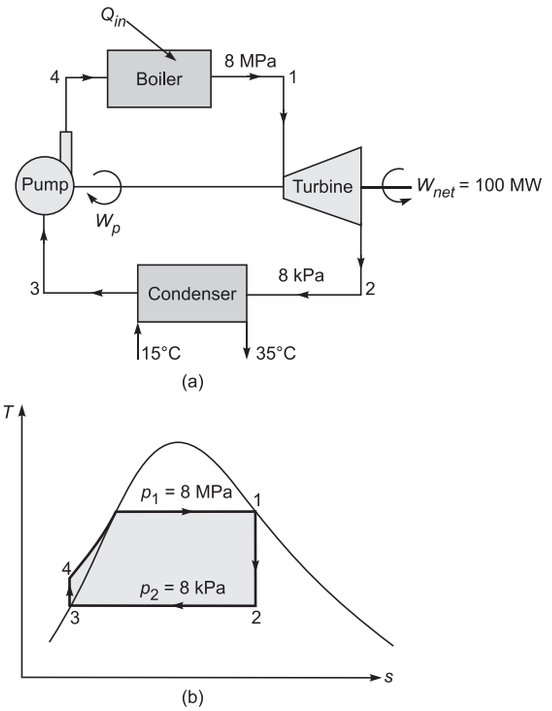

Fig. 12.10 *(a) Schematic. (b) T–s diagram for an ideal Rankine cycle.*

Condenser exit;

$$p_3 = 0.008 \text{ MPa} = 8 \text{ kPa as a saturated liquid}$$

Net power output,

$$P = 100 \text{ MW}$$

To find

(i) Thermal efficiency of Rankine cycle,
(ii) The back work ratio,
(iii) The mass flow rate of steam, kg/h,
(iv) Heat transfer rate to the working fluid in the boiler,
(v) Heat rejection rate in the condenser, and
(vi) The mass flow rate of cooling water for its temperature change from 15°C to 35°C.

Analysis *The properties of steam at principal states (From Table A-13)*

State 1: Saturated steam;

$$p_1 = 8000 \text{ kPa}$$
$$h_1 = 2757.94 \text{ kJ/kg}$$
$$s_1 = 5.7431 \text{ kJ/kg·K}$$

State 2: Wet steam;

$$p_2 = 8 \text{ kPa}$$
$$h_{f_2} = 173.37 \text{ kJ/kg}$$
$$h_{fg_2} = 2403.38 \text{ kJ/kg}$$
$$s_{f_2} = 0.59088 \text{ kJ/kg·K}$$
$$s_{fg_2} = 7.64028 \text{ kJ/kg·K}$$

State 3: Saturated liquid;

$$p_3 = 8 \text{ kPa}$$
$$h_3 = h_{f_3} = 173.94 \text{ kJ/kg}$$
$$v_{f_3} = 0.001008 \text{ m}^3\text{/kg}$$

State 4: Compressed liquid;

$$p_4 = 8000 \text{ kPa},$$
$$h_4 = h_3 + w_p$$

The state of steam after isentropic expansion in the turbine;

$$s_1 = s_2 = (s_f + x s_{fg})_{@\ 8 \text{ kPa}}$$
$$5.7431 = 0.59088 + x(7.64028)$$

or
$$x = \frac{5.7431 - 0.59088}{7.64028} = 0.674$$

Specific enthalpy at the state 2

$$h_2 = (h_{f_2} + x h_{fg_2})_{@\ 8 \text{ kPa}}$$
$$= 173.37 + 0.674 \times 2403.38$$
$$= 1794.09 \text{ kJ/kg}$$

The pump work;

$$w_p = v_f (p_1 - p_2)$$
$$= 0.001008 \times (8000 - 8) = 8.055 \text{ kJ/kg}$$

Enthalpy at the state 4;

$$h_4 = h_3 + w_p$$
$$= 173.37 + 8.055 = 181.43 \text{ kJ/kg}$$

Turbine work;

$$w_T = h_1 - h_2 = 2757.94 - 1794.09$$
$$= 963.85 \text{ kJ/kg}$$

Net work of cycle;

$$w_{net} = w_T - w_p$$
$$= 963.85 - 8.055 \approx 955.79 \text{ kJ/kg}$$

The heat supplied per kg of steam

$$q_{in} = h_1 - h_4 = 2757.94 - 181.43$$
$$= 2576.51 \text{ kJ/kg}$$

(i) Thermal efficiency of Rankine cycle

$$\eta_{Rankine} = \frac{w_{net}}{q_{in}} = \frac{955.79}{2576.51} = \mathbf{0.370 \text{ or } 37\%}$$

(ii) Back work ratio;

$$r_{bw} = \frac{w_p}{w_T} = \frac{8.055 \text{ kJ/kg}}{963.85 \text{ kJ/kg}}$$
$$= \mathbf{0.00835}$$

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(iii) The mass flow rate of steam;

$$\dot{m}_s = \frac{\text{Power output}}{w_{net}}$$

$$= \frac{100 \times 10^3 \text{ kW}}{955.79 \text{ kJ/kg}}$$

$$= 104.625 \text{ kg/s or } \mathbf{376 \times 10^3 \text{ kg/h}}$$

(iv) Heat supply rate to working fluid in the boiler

$$\dot{Q}_{in} = \dot{m}_s q_{in}$$

$$= (104.625 \text{ kg/s}) \times (2576.51 \text{ kJ/kg})$$

$$= \mathbf{269.57 \times 10^3 \text{ kW}}$$

(v) Rate of heat rejection in the condenser;

$$\dot{Q}_{out} = \dot{m}_s (h_2 - h_3)$$

$$= 104.625 \times (1794.09 - 173.378)$$

$$= \mathbf{169.566 \times 10^3 \text{ kW}}$$

(vi) The mass flow rate of cooling water in the condenser

$$\dot{Q}_{out} = \dot{m}_w C_{pw} (T_{out} - T_{in})$$

$$169.566 \times 10^3 = \dot{m}_w \times 4.180 \times (35 - 15)$$

$$\dot{m}_w = \frac{169.556 \times 10^3}{4.180 \times (35 - 15)}$$

$$= 2028.3 \text{ kg/s or } \mathbf{7.30 \times 10^6 \text{ kg/h}}$$

Example 12.9 *Steam enters the turbine as 100% saturated vapour at 6 MPa and saturated liquid enters the pump at a pressure of 0.01 MPa. If the heat rate to boiler is 150 MW, determine*

(a) the thermal efficiency,
(b) the mass flow rate of the steam,
(c) net rate of work transfer from the cycle,
(d) work ratio,
(e) specific steam consumption.

Solution

Given A power plant works on Rankine cycle with saturated steam

Turbine entry:

p_1 = 6.00 MPa (6000 kPa) as a saturated steam

Condenser exit;

p_3 = 0.01MPa (10 kPa) as a saturated liquid

Heat rate to boiler,

$\dot{Q}_{in}$ = 150 MW

To find

(i) Thermal efficiency of Rankine cycle,
(ii) The mass flow rate of steam, kg/h,
(iii) Rate of work transfer from the cycle,
(iv) Work ratio,
(v) Specific steam consumption.

Assumptions

(i) Each components of the cycle is in steady state.
(ii) All processes of the working fluid are internally reversible.
(iii) The turbine and pump operate isentropically ($q = 0$).
(iv) The kinetic and potential energy effects are negligible.

Schematic

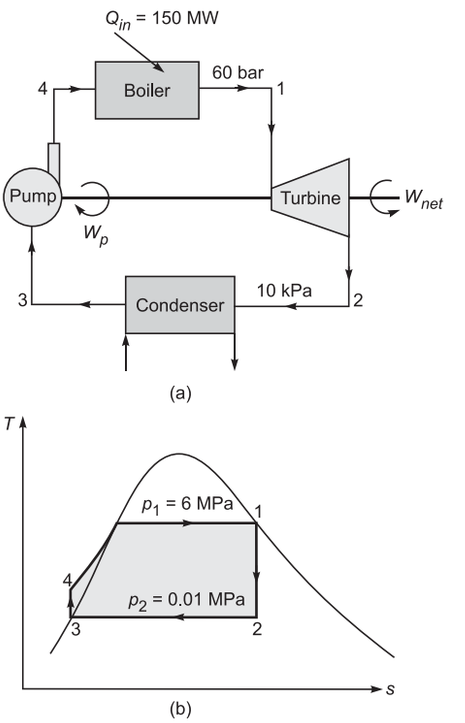

Fig. 12.11 *(a) Schematic. (b) T-s diagram for an ideal Rankine cycle*

Analysis *The properties of steam at principal states*

State 1: Saturated steam;

p_1 = 6000 kPa,
h_1 = 2785.1 kJ/kg
s_1 = 5.8891 kJ/kg·K

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State 2: Wet steam;

$$p_2 = 10 \text{ kPa}$$
$$h_{f_2} = 191.81 \text{ kJ/kg}$$
$$h_{fg_2} = 2392.82 \text{ kJ/kg}$$
$$s_{f_2} = 0.6492 \text{ kJ/kg·K}$$
$$s_{fg_2} = 7.5010 \text{ kJ/kg·K}$$

State 3: Saturated liquid;

$$p_3 = 10 \text{ kPa}$$
$$h_3 = h_{f_3} = 191.81 \text{ kJ/kg}$$
$$v_{f_3} = 0.001001 \text{ m}^3\text{/kg}$$

State 4: Compressed liquid; $p_4 = 6000$ kPa,

$$h_4 = h_3 + w_p$$

The state of steam after isentropic expansion in the turbine;

$$s_1 = s_2 = (s_f + x s_{fg})_{@\ 10 \text{ kPa}}$$
$$5.8891 = 0.6492 + x(7.5010)$$

or
$$x = \frac{5.8891 - 0.6492}{7.5010} = 0.698$$

Specific enthalpy at the state 2

$$h_2 = (h_{f_2} + x\, h_{fg_2})_{@\ 10 \text{ kPa}}$$
$$= 191.81 + 0.698 \times 2392.82$$
$$= 1863.3 \text{ kJ/kg}$$

The pump work;

$$w_p = v_f (p_1 - p_2)$$
$$= 0.001008 \times (6000 - 10) = 6.0 \text{ kJ/kg}$$

Enthalpy at the state 4;

$$h_4 = h_3 + w_p = 191.81 + 6.0 = 197.81 \text{ kJ/kg}$$

Turbine work;

$$w_T = h_1 - h_2 = 2785.1 - 1863.3$$
$$= 921.8 \text{ kJ/kg}$$

Net work of cycle;

$$w_{net} = w_T - w_p = 921.8 - 6.0$$
$$= 915.8 \text{ kJ/kg}$$

The heat supplied per kg of steam

$$q_{in} = h_1 - h_4$$
$$= 2785.1 - 197.81 = 2587.29 \text{ kJ/kg}$$

(a) Thermal efficiency of Rankine cycle;

$$\eta_{Rankine} = \frac{w_{net}}{q_{in}} = \frac{915.8}{2587.29} = \mathbf{0.354 \text{ or } 35.4\%}$$

(b) The mass flow rate of steam;

$$\dot{m}_s = \frac{\text{Heat rate}}{q_{in}}$$
$$= \frac{150 \times 10^3 \text{ kW}}{2587.29 \text{ kJ/kg}} = 57.97 \text{ kg/s}$$
$$= \mathbf{208.71 \times 10^3 \text{ kg/h}}$$

(c) Net rate of work transfer from the cycle

$$p = \dot{m}_s\, w_{net}$$
$$= 57.97 \times 915.8$$
$$= 53094.16 \text{ kW or } \mathbf{53.09 \text{ MW}}$$

(d) Work ratio

$$r_w = \frac{w_{net}}{w_T} = \frac{915.8}{921.8} = \mathbf{0.993}$$

(e) Specific steam consumption

$$ssc = \frac{3600 \text{ kJ/kWh}}{w_{net}}$$
$$= \frac{3600 \text{ kJ/kWh}}{915.8 \text{ kJ/kg}} = \mathbf{3.93 \text{ kg/kWh}}$$

12.5 COMPARISON BETWEEN CARNOT AND RANKINE CYCLES

Figure 12.12 shows the graphical comparison between Rankine and Carnot cycles on *p–v* and *T–s* diagrams.

Cycle 1–2–3′–4′–1 Carnot cycle with saturated steam

Cycle 1″–2′–3′–4′–1 Carnot cycle with superheated steam

Cycle 1–2–3–4–1 Rankine cycle with saturated steam

Cycle 1′–2′–3–4–1 Rankine cycle with superheated steam

The following facts can easily be acknowledged from the above two diagrams:

1. In the Rankine cycle, liquid water is pumped during the process 3–4. Since the specific volume of the working substance after complete condensation at the state 3 becomes very small, therefore, the *back work ratio* in a Rankine cycle is almost negligible. While the specific volume of liquid and vapour mixture at the state 3′ is large, thus a *large compression work* is required in Carnot vapour power cycle.
2. There are higher rates of heat transfer in the boiler and condenser due to long processes. Consequently, the Rankine cycle requires

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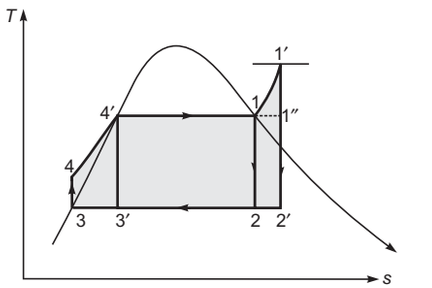
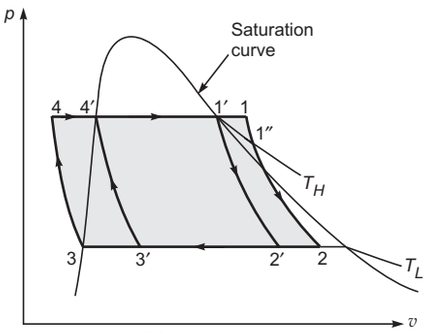


(a) T–s diagram



(b) p–v diagram

Fig. 12.12 *Comparison between Carnot and Rankine cycles*

a *lower steam flow rate* than Carnot cycle. Therefore, the plant size for Rankine cycle is smaller.

3. The Rankine cycle uses a part of heat supplied at constant temperature. Therefore, its efficiency is lower than that of Carnot cycle.
4. In a Rankine cycle, the steam is superheated at constant pressure 1–1′, thus maintaining the pressure ratio in the cycle unaffected. If Carnot vapour power cycle is to operate with superheated steam then the pressure of steam must be reduced to keep the temperature constant. It means the heat is transferred to steam as it proceeds for the expansion during process 1–1″–2′. Such a type heat addition is almost impractical.

12.6 DIFFERENCE BETWEEN CARNOT AND RANKINE CYCLES

Although the Carnot vapour power cycle has maximum efficiency, still the Rankine cycle is commonly used in steam power plants. The differences between the two cycles are tabulated below:

Sr. No	*Aspect*	*Carnot Vapour Power Cycle*	*Rankine Cycle*
1.	Cycle	It is a reversible cycle.	It is an irreversible cycle.
2.	Cyclic efficiency	It has theoretically maximum efficiency.	It has less thermal efficiency than that of the Carnot vapour power cycle.
3.	Heat addition	Heat is added at constant temperature.	Heat is added at constant pressure.
4.	Superheated steam	Use of superheated steam is practically difficult.	It uses superheated steam and performs better.
5.	Condensation	Condensation is terminated before being saturated liquid.	Complete condensation of steam takes place.
6.	State after Condensation	The mixture of water and steam exits at the termination point.	Only liquid water exists after condensation.
7.	Pump work	It requires a large pump work to handle the two phase mixture.	It requires negligible pump work to handle the liquid water only.
8.	Steam quality after expansion	Since it uses saturated steam, the moisture content at the end of expansion is much higher which can lead to blade erosion.	It uses superheated steam in the cycle at the end of expansion, the quality of steam is not objectionable.
9.	Standard of cycle	It is just a theoretical cycle and cannot be used in practice.	Almost all thermal plants operate on Rankine cycle.

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Example 12.10 *The boiler produces dry and saturated steam at 30 bar. The steam expands in the turbine to a condenser pressure of 20 kPa. Compare the cyclic work done and thermal efficiency of Carnot and Rankine cycles for these conditions.*

Solution

Given Dry saturated steam pressure

Boiler pressure $p_1 = 30$ bar

Condenser pressure; $p_2 = 20$ kPa

To find

(i) Work done by Carnot cycle,

(ii) Work done by Rankine cycle,

(iii) Thermal efficiency of Carnot cycle,

(iv) Thermal efficiency of Rankine cycle.

Properties of steam

At $p_1 = 30$ bar;

$h_f = 1008.42$ kJ/kg

$h_g = 2804.2$ kJ/kg

$T_H = 233.9°C$

$s_g = 6.1869$ kJ/kg

At $p_2 = 20$ kPa;

$v_f = 0.001017$ m^3/kg

$T_L = 60.06°C$

$h_f = 251.38$ kJ/kg

$h_{fg} = 2358.33$ kJ/kg

$s_f = 0.8319$ kJ/kg·K

$s_{fg} = 7.0766$ kJ/kg·K

The specific enthalpies

$h_1 = 2804.2$ kJ/kg

$h_4 = 1008.42$ kJ/kg

Carnot Cycle

Analysis

The efficiency of Carnol cycle is given by

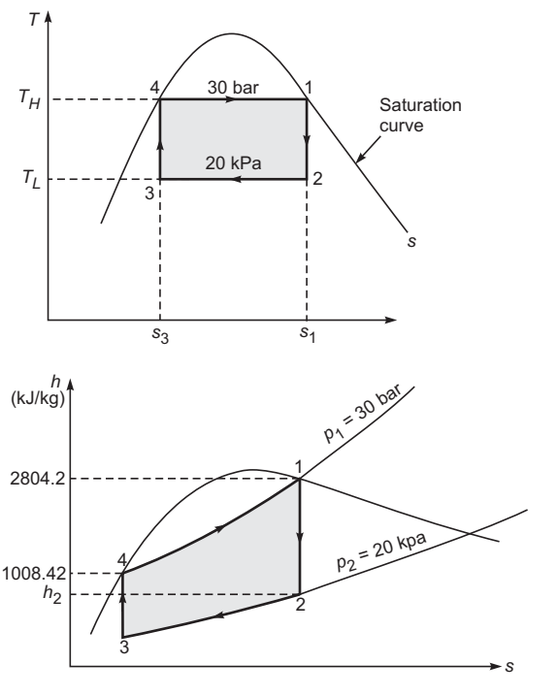
$$\eta_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{(60.06 + 273)}{(233.9 + 273)}$$

$$= 0.343 \text{ or } \mathbf{34.3\%}$$

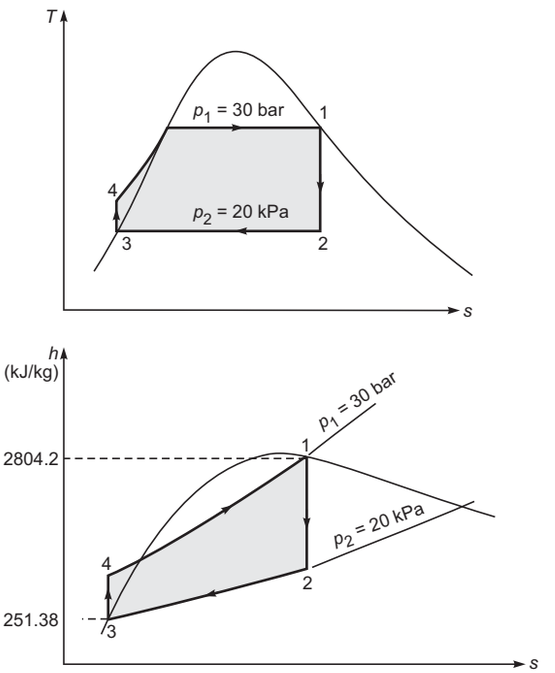
The heat supplied per kg of steam

$$q_{in} = h_1 - h_4 = 2804.2 - 1008.42$$

$$= 1795.78 \text{ kJ/kg}$$



(a) T–s and h–s diagram for Carnot Cycle



(b) T–s and h–s diagram for Rankine Cycle

Fig. 12.13

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The work done per kg of steam

$$w_{net} = \eta_{Carnot}\, q_{in} = 0.343 \times 1795.78$$
$$= \mathbf{615.86\ kJ/kg}$$

Rankine Cycle

Condition of steam at the state 2

$$s_1 = s_2 = (s_f + x s_{fg})_{@20\ kPa}$$
$$6.1869 = 0.8319 + x \times (7.0766)$$

or $x = 0.8319$

Specific enthalpy at principal states;

State 1: $h_1 = 2804.2$ kJ/kg

State 2: $h_2 = (h_f + x\, h_{fg})_{@20\ kPa}$
$= 251.38 + 0.8319 \times 2358.33$
$= 2035$ kJ/kg·K

State 3: $h_3 = h_f = 251.38$ kJ/kg

The pump work;

$$w_p = v_f (p_1 - p_2)$$
$$= 0.001017 \times (3000 - 20) = 3.03\ kJ/kg$$

State 4: $h_4 = h_3 + w_p = 251.38 + 3.03 = 254.41$ kJ/kg

Turbine work per kg of steam;

$$w_T = h_1 - h_2 = 2804.2 - 2035 = 769.2\ kJ/kg$$

Net work done by Rankine cycle

$$w_{net} = w_T - w_p = 769.2 - 3.03 = \mathbf{766.17\ kJ/kg}$$

Heat supplied in Rankine cycle

$$q_{in} = h_1 - h_4 = 2804.2 - 254.41$$
$$= 2549.79\ kJ/kg$$

Thermal efficiency of Rankine cycle

$$\eta_{Rankine} = \frac{w_{net}}{q_{in}} = \frac{766.17}{2549.79} = 0.302 \text{ or } \mathbf{30.2\%}$$

12.7 IRREVERSIBILITIES AND LOSSES IN VAPOUR POWER CYCLE

The Rankine cycle considered in Section 12.3 is an ideal vapour cycle. In actual practice, all four processes of the cycle involve irreversibility and losses and therefore, the efficiency of the actual vapour cycle is less than that of an ideal Rankine cycle. The losses associated with actual processes are shown in Fig. 12.14.

Piping Losses When the working fluid passes through the tubes of the boiler and condenser, the pressure drops due to frictional effects and transfer of heat to surroundings. Both pressure drop and heat

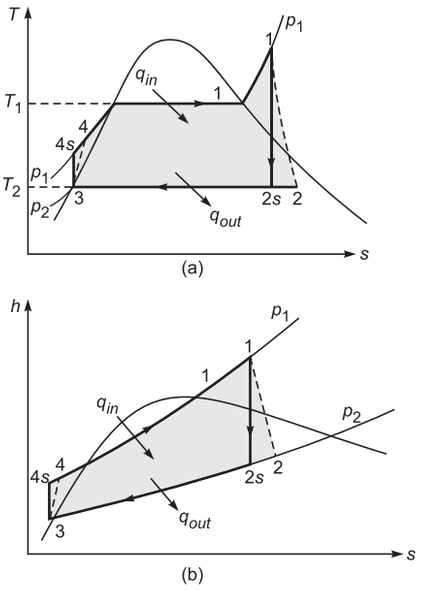

Fig. 12.14 *Actual Rankine cycle on (a) T–s and (b) h–s diagrams showing the effect of irreversibility on pump and turbine operations*

transfer decrease the availability of steam entering the turbine. However, these effects are negligible, and not shown in Fig. 12.14.

Turbine Losses Actual expansion process 1–2 in the turbine is an irreversible process as shown in Fig. 12.14. The entropy inceases during the actual expansion process. Some heat may also transfer to the surroundings. Thus the actual work developed in the turbine is less than the work corresponding to isentropic process 1–2s. The isentropic efficiency of the turbine is given as

$$\eta_T = \frac{\text{Actual work}}{\text{Isentropic work}} = \frac{\text{Actual enthalpy drop}}{\text{Isentropic enthalpy drop}}$$
$$= \frac{h_1 - h_2}{h_1 - h_{2s}} \qquad ...(12.18)$$

Pump Losses The pump losses are very similar to turbine lossses. The actual compression process 3–4 in the pump is an irreversible process. The entropy increases during the actual compression process. Therefore, the actual work input in the pump is more than the work corresponding to

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the isentropic process 3–4*s*. The isentropic efficiency of the pump is given as

$$\eta_p = \frac{\text{Isentropic work input}}{\text{Actual work input}} = \frac{\text{Isentropic enthalpy drop}}{\text{Actual enthalpy drop}} = \frac{h_{4s} - h_3}{h_4 - h_3} \quad ...(12.19)$$

Condenser Losses The condenser losses are relatively small. The cooling of condensate below saturation temperature may be possible in the condenser. Some additional heat is required to bring down the saturation temperature of water.

Further, the air enters into the condenser with exhaust steam and from joints of the condenser. It reduces the back pressure in the condenser. An air pump is required to maintain the correct vacuum in the condenser. Condenser losses are also small, thus not incorporated in *T–s* and *h–s* diagrams of Fig. 12.14.

Example 12.11 *A steam power plant operates on an ideal Rankine cycle between a boiler pressure of 40 bar, 300°C and a condenser pressure of 0.035 bar. Calculate cycle efficiency, work ratio, and specific steam consumption for*

(a) Ideal Rankine cycle

(b) For Rankine cycle, when expansion process has an isentropic efficiency of 80%

Solution

Given A steam power plant operating on Rankine cycle

Boiler conditions; $p_1 = 40$ bar, 300°C

Condenser pressure; $p_2 = 0.035$ bar = 3.5 kPa

To find

(i) Cycle efficiency,

(ii) Work ratio, and

(iii) Specific steam consumption.

Analysis *The properties of steam*

State 1: Superheated steam; Table A-14

$p_1 = 4000$ kPa

$T_1 = 300$°C

$h_1 = 2960.68$ kJ/kg,

$s_1 = 6.3614$ kJ/kg·K

State 2: Wet steam; Table A-13

$p_2 = 3.5$ kPa

$h_f = 111.80$ kJ/kg,

$h_{fg} = 2438.6$ kJ/kg

$s_f = 0.3910$ kJ/kg·K

$s_{fg} = 8.133$ kJ/kg·K

State 3: $p_2 = 3.5$ kPa,

$h_f = 111.80$ kJ/kg,

$v_f = 0.001003$ m^3/kg

State 4: Compressed liquid, $p_1 = 40$ bar

(a) *For an ideal Rankine cycle 1–2s–3–4–1*

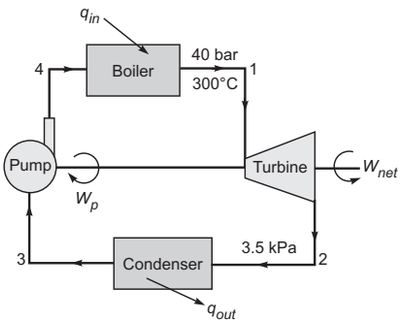
The state 2, after isentropic expansion can be defined by equating entropy at states 1 and 2*s*;

$$s_1 = s_{2s} = (s_f + x s_{fg})_{@\ 3.5\ \text{kPa}}$$

$$6.3614 = 0.3910 + x(8.133)$$

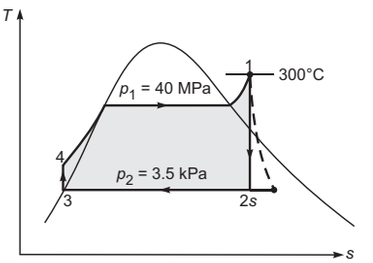
or $$x = \frac{6.3614 - 0.3910}{8.133} = 0.734$$

Schematic



(a) Schematic diagram

(i) Cycle 1–2*s*–3–4–1 is an ideal Rankine cycle



(b) T-s diagram

(ii) Cycle 1–2–3–4–1 is an irreversible Rankine cycle

Fig. 12.15 *Schematic and T–s diagram of Ex. 12.11*

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Specific enthalpy after isentropic expansion at the state 2*s*;

$$h_{2s} = (h_f + x h_{fg})_{@\ 3.5\ kPa}$$
$$= 111.80 + 0.734 \times 2438.6$$
$$= 1901.73\ kJ/kg$$

The pump work;

$$w_p = v_f (p_1 - p_2)$$
$$= 0.001003 \times (4000 - 3.5)$$
$$= 4.0\ kJ/kg$$

Enthalpy at the state 4;

$$h_4 = h_1 + w_p$$
$$= 111.80 + 4.0 = 115.80\ kJ/kg$$

Turbine work,

$$w_T = h_1 - h_{2s}$$
$$= 2960.68 - 1901.73$$
$$= 1058.95\ kJ/kg$$

Heat supplied in the boiler;

$$q_{in} = h_1 - h_4 = 2960.68 - 115.80$$
$$= 2844.88\ kJ/kg$$

Net work done per kg of steam,

$$w_{net} = w_T - w_p$$
$$= 1058.95 - 4.0 = 1054.95\ kJ/kg$$

(i) The Rankine cycle efficiency

$$\eta_{Rankine} = \frac{w_{net}}{q_{in}} = \frac{1054.95}{2844.88}$$
$$= 0.370 \quad \text{or} \quad \mathbf{37\%}$$

(ii) Work ratio;

$$r_w = \frac{w_{net}}{w_T} = \frac{1054.95}{1058.95}$$
$$= \mathbf{0.996 \quad or \quad 99.6\%}$$

(iii) Specific steam consumption rate

$$ssc = \frac{3600\ kJ/kWh}{w_{net}\ kJ/kg} = \frac{3600\ kJ/kWh}{1054.95\ kJ/kg}$$
$$= \mathbf{3.41\ (kg/kWh)}$$

(b) For an irreversible Rankine cycle 1–2–3–4–1, we have

$h_1 = 2960.68\ kJ/kg$
$h_{2s} = 1901.73\ kJ/kg$
$w_p = 4{\cdot}0\ kJ/kg$
$h_4 = 115.80\ kJ/kg$
$q_{in} = 2844.88\ kJ/kg$
$\eta_T = 0.8$

Isentropic efficiency of the turbine is given as

$$\eta_T = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{w_T}{h_1 - h_{2s}}$$

Actual turbine work;

$$w_T = \eta_T (h_1 - h_{2s})$$
$$= 0.8 \times (2960.68 - 1901.73)$$
$$= 847.48\ kJ/kg$$

Net work done per kg of steam;

$$w_{net} = w_T - w_p = 847.48 - 4.0$$
$$= 843.48\ kJ/kg$$

(i) Cycle efficiency

$$\eta_{Cycle} = \frac{w_{net}}{q_{in}} = \frac{843.48}{2844.88}$$
$$= 0.296 \quad \text{or} \quad \mathbf{29.6\%}$$

(ii) Work ratio; $r_w = \dfrac{w_{net}}{w_T} = \dfrac{843.48}{847.48}$

$$= \mathbf{0.9952 \quad or \quad 99.52\%}$$

(iii) Specific steam consumption rate

$$ssc = \frac{3600\ kJ/kWh}{w_{net}\ kJ/kg} = \frac{3600\ kJ/kWh}{843.48\ kJ/kg}$$
$$= \mathbf{4.268\ (kg/kWh)}$$

Example 12.12 *A steam power plant operates on a cycle. The pressure and temperature are designated in Fig.12.16. The turbine efficiency is 90% and pump efficiency is 85%. Calcualte the thermal efficiency of the cycle.*

Solution

Given A steam power plant with schematic as shown in Fig.12.16;

Properties of steam From Tables A-13, A-14

State 1: At 3.8 MPa, 380°C; Superheated steam;

$h_1 = 3169.1\ kJ/kg$
$s_1 = 6.7235\ kJ/kg \cdot K$

State 2: At 10 kPa, wet steam;

$h_f = 191.8\ kJ/kg$
$h_{fg} = 2392.8\ kJ/kg$
$s_f = 0.6493\ kJ/kg \cdot K$
$s_{fg} = 7.5009\ kJ/kg$

State 3: At 10 kPa saturated liquid $v_f = 0.001009\ m^3/kg$

State 4: At 5 MPa; compressed liquid

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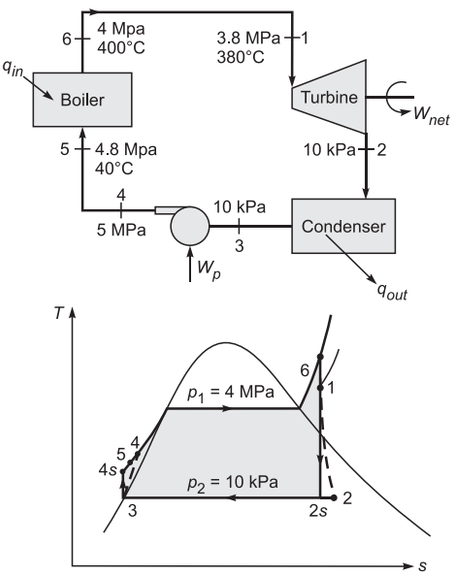

Fig. 12.16 *Schematic and T–s diagram of Ex. 12.12*

State 5: At 4.8 MPa, 40°C Compressed liquid

$$h_5 = h_{f@40°C} = 191.8 \text{ kJ/kg}$$

State 6: At 4 MPa, 400°C Superheated steam

$$h_4 = 3213.6 \text{ kJ/kg}$$

To find Thermal efficiency of the cycle.

Assumptions

(i) Each component in the cycle is analysed as a control volume at steady state.

(ii) Compression and expansion are considered adiabatic.

(iii) Kinetic and potential energy effects are negligible.

Analysis The isentropic enthalpy drop during process 3–4*s*;

$$h_{4s} - h_3 = v_f (p_1 - p_2)$$
$$= 0.001009 \times (5000 - 10) = 5.035 \text{ kJ/kg}$$

The isentropic efficiency of the pump

$$\eta_p = \frac{h_{4s} - h_3}{h_4 - h_3} = \frac{h_{4s} - h_3}{w_p}$$

Actual pump work per kg of steam;

$$w_p = \frac{h_{4s} - h_3}{\eta_p} = \frac{5.035}{0.85} = 5.92 \text{ kJ/kg}$$

The state 2*s*, after isentropic expansion can be defined by equating entropy at states 1 and 2*s*;

$$s_1 = s_{2s} = (s_f + x s_{fg})_{@\ 10 \text{ kPa}}$$

$$6.7235 = 0.6492 + x(7.5010)$$

or
$$x = \frac{6.7235 - 0.6492}{7.5010} = 0.8098$$

Specific enthalpy after isentropic expansion at state 2*s*

$$h_{2s} = (h_f + x h_{fg})_{@\ 10 \text{ kPa}}$$
$$= 191.8 + 0.8098 \times 2392.8$$
$$= 2129.5 \text{ kJ/kg}$$

The isentropic efficiency of the turbine is given as

$$\eta_T = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{w_T}{h_1 - h_{2s}}$$

Actual turbine work per kg of steam;

$$w_T = \eta_T (h_1 - h_{2s}) = 0.9 \times (3169.1 - 2129.5)$$
$$= 935.65 \text{ kJ/kg}$$

Net work done in the cycle per kg of steam

$$w_{net} = w_T - w_p$$
$$= 935.65 - 5.92 = 929.73 \text{ kJ/kg}$$

The heat supplied in the boiler per kg of steam

$$q_{in} = h_6 - h_5$$
$$= 3213.6 - 171.8 = 3041.8 \text{ kJ/kg}$$

The thermal efficiency of the cycle

$$\eta_{cycle} = \frac{w_{net}}{q_{in}} = \frac{929.73}{3041.8} = 0.3056 \text{ or } \mathbf{30.56\%}$$

12.8 EFFECT OF OPERATING VARIABLES ON RANKINE CYCLE

12.8.1 Effect of Exhaust Pressure

The steam enters the condenser as a saturated mixture of vapour and moisture at the turbine back pressure p_2. If this pressure is lowered, the saturation temperature of exhausted steam decreases, and thus, the amount of heat rejection in the condenser also decreases. The efficiency of the Rankine cycle increases by lowering the exhaust pressure. Figure 12.17 illustrates the effect of condenser pressure on the Rankine cycle.

As turbine back pressure p_2 decreases to p_2', the heat rejection decreases by an area 2–3–3′–2′–2. The heat transfer to steam is also increased by an area *a*′–4′–4–*a*–*a*′. Thus, the net work done and efficiency of the cycle increases.

However, there are limitations to this method. These are

1. Lowering the back pressure causes an increase in moisture content of the steam

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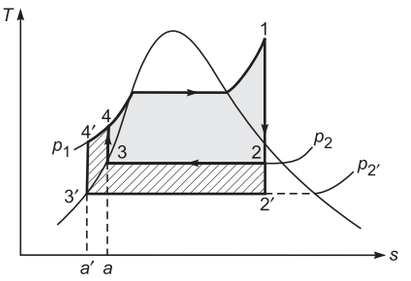

Fig. 12.17 *Effect of lowering the condenser pressure on the Rankine cycle*

leaving the turbine. It is an unfavourable factor, because, if the moisture content of steam in low-pressure stages of the turbine exceeds 10%, there is a decrease in turbine efficiency and erosion of turbine blade may also be a very serious problem.

2. To maintain the high vacuum, the air extraction pump will run continuously and its work input will increase, thus reducing the useful work.

12.8.2 Effect of Superheating

Superheating of steam increases the mean temperature of heat addition. The effect of superheated steam on the performance of the Rankine cycle is shown in Fig. 12.18(a). The increase in superheat is shown by the line 1–1′. The hatched area 1–1′–2′–2–1 represents an increase in net work done during the cycle. The area under the curve 1–1′ represents increase in the heat input. Thus, both the net work done and heat transfer increase as a result of superheating the steam to higher temperature. Therefore, the thermal efficiency of the Rankine cycle increases. It is observed that the specific steam consumption decreases as steam is superheated. The effect of superheating is shown in Fig.12.18(b).

Superheating of steam to higher temperature is desirable, because the moisture content of steam leaving the turbine decreases as indicated by the state 2′ in Fig. 12.18 (a). However, the metallurgical considerations, restrict the superheating of steam to a very high temperature.

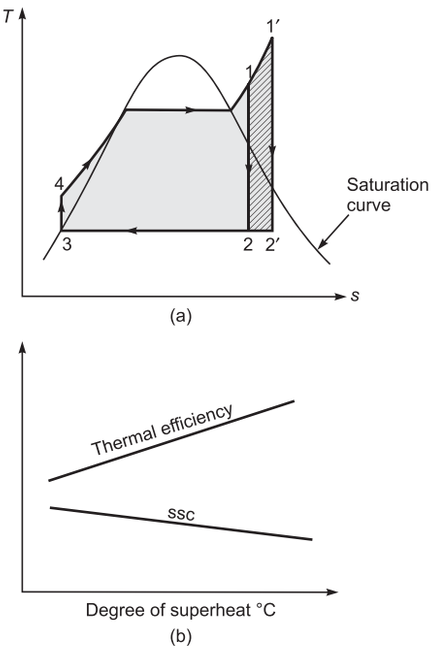

Fig. 12.18 *Effect of superheating on Rankine cycle*

12.8.3 Effect of increase in Boiler Pressure

By increasing the boiler pressure, the mean temperature of heat addition increases, and thus raises the thermal efficiency of the cycle. Figure. 12.19(a) illustrates the effect of boiler pressure on Rankine cycle efficiency.

By keeping the maximum temperature T_{max} and condenser pressure p_2 constant if boiler pressure increases, the heat rejection decreases by an area *b′–2′–2–b–b′*. The net work done by the cycle remains almost same, thus, the Rankine cycle efficiency increases, with an increase in maximum pressure.

Figure 12.19(b) shows the variation of efficiency and specific steam consumption with boiler pressure. The graph shows that the thermal efficiency first increases, reaches to peak value and then decreases.

1. As boiler pressure increases, the saturation temperature of feed water increases and the enthalpy of vapourisation reduces. Thus a

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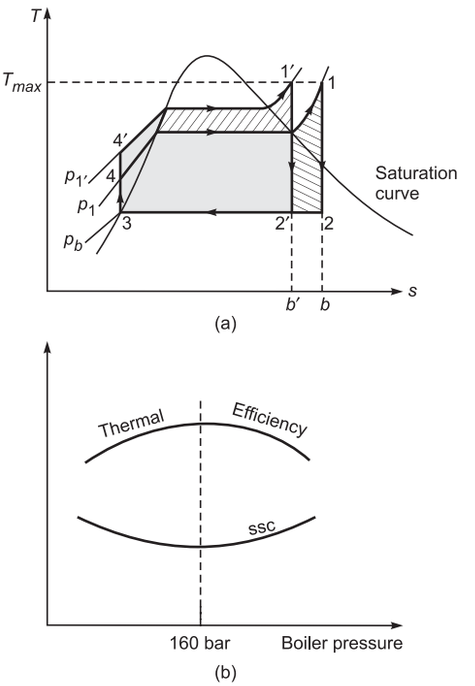

Fig. 12.19 *Effect Boiler pressure on Ranking cycle performance*

larger portion of heat is used to increase the temperature of feed water to its saturation temperature. This non-isothermal process increases irreversibility and thus thermal efficiency decreases.

2. With increase in boiler pressure, the cycle shifts toward left and the moisture contents of exhaust steam increases. It is an undesirable effect.
3. Specific steam consumption also decreases first and then increases after reaching a minimum level at 160 bar.

We can conclude that the efficiency of Rankine cycle can be increased by lowering the condenser pressure, by increasing the boiler pressure and by superheating the steam. The quality of steam leaving the turbine decreases by lowering condenser pressure and by increasing boiler pressure, while it improves by superheating. Table 12.1 presents the summary of the above discussion.

Table 12.1 *Effect of operating variabes on work done and thermal efficiency of Rankine efficiency*

Sr No.	*Operating variable*	*Work done*	*Efficiency*
1.	Decrease in exhaust pressure	Increases	Increases
2.	Increase in boiler pressure	No effect	Increases
3.	Superheating steam	Increases	Increases

Example 12.13 *In a steam power plant operating on an ideal Rankine cycle, the steam enters the turbine at 3 MPa and 400°C and it is exhausted at 10 kPa. Determine*

(a) Thermal efficiency

(b) Thermal effeciency, if the steam is superheated to 500°C at 3 MPa, before it enters the turbine

(c) Thermal efficiency, if steam enters the turbine at 10 MPa and 400°C

Solution

Given An ideal Rankine cycle with different operating variables

To find Thermal efficiency of Rankine cycle in each case

Schematic

Case (a): Turbine inlet: 3 MPa, 400°C: Cycle 1–2–3–4–1

Case (b): Turbine inlet 3 MPa, 500°C: Cycle 1″–2″–3–4–1″

Case (c): Turbine inlet 10 MPa, 400°C: Cycle 1′–2′–3–4–1′

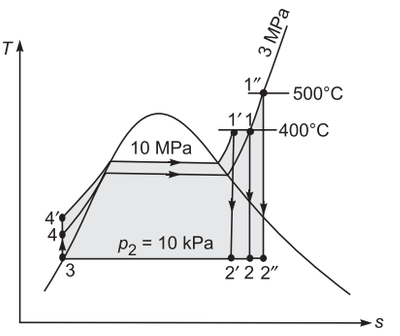

Fig. 12.20

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Analysis

Case (a): Steam enters the turbine at 3 MPa and 400°C.

Ideal Rankine cycle 1-2-3-4-1

From steam tables,

State 1: $p_1 = 3$ MPa,

$T_3 = 400°C$: Superheated steam

$h_1 = 3230.9$ kJ/kg

$s_1 = 6.9212$ kJ/kg·K

State 2: Wet steam

$p_2 = 10$ kPa,

$h_f = 191.83$ kJ/kg

$h_{fg} = 2392.87$ kJ/kg

$s_f = 0.6492$ kJ/kg

$s_{fg} = 7.5010$ kJ/kg·K

State 3: Saturated liquid,

$p_3 = 10$ kPa

$h_3 = h_f = 191.83$ kJ/kg

$v_f = 0.001010$ m^3/kg

State 4: Compressed liquid,

$p_4 = 3$ MPa

The state 2 of steam after isentropic expansion in the turbine;

$$s_1 = s_2 = (s_f + x s_{fg})_{@\ 10\ \text{kPa}}$$

$$6.9212 = 0.6492 + x(7.5010)$$

or $$x = \frac{6.9212 - 0.6492}{7.5010} = 0.836$$

Specific enthalpy at the state 2

$$h_2 = (h_f + x h_{fg})_{@\ 10\ \text{kPa}}$$
$$= 191.81 + 0.836 \times 2392.82$$
$$= 2192.27 \text{ kJ/kg}$$

The pump work;

$$w_p = v_f (p_1 - p_2)$$
$$= 0.001010 \times (3000 - 10) = 3.02 \text{ kJ/kg}$$

Enthalpy at the state 4;

$$h_4 = h_3 + w_p = 191.83 + 3.02 = 194.85 \text{ kJ/kg}$$

Heat supplied in the boiler per kg of steam;

$$q_{in} = h_1 - h_4 = 3230.90 - 194.85$$
$$= 3036.05 \text{ kJ/kg}$$

Heat rejected in the condenser per kg of steam;

$$q_{out} = h_2 - h_3 = 2192.27 - 191.83$$
$$= 2000.44 \text{ kJ/kg}$$

The Rankine or plant efficiency

$$\eta_{plant} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{2000.44}{3036.05}$$
$$= 0.3411 \quad \text{or} \quad \mathbf{34.11\%}$$

Case (b) The steam supplied to the turbine at 3 MPa and 500°C

Rankine cycle 1″–2″–3–4–1″.

The properties of steam at states 3 and 4 will remain same but properties at states 1″ and 2″ will be different.

State 1: Superheated steam at 3 MPa nd 500°C

$h_{1''} = 3456.5$ kJ/kg

$s_{1''} = 7.2338$ kJ/kg·K

For the state 2″

$$s_{1''} = s_{2''} = (s_f + x'' s_{fg})_{@\ 10\ \text{kPa}}$$

$$7.2338 = 0.6492 + x(7.5010)$$

or $$x'' = \frac{7.2338 - 0.6492}{7.5010} = 0.878$$

$$h_{2''} = (h_f + x'' h_{fg})_{@\ 10\ \text{kPa}}$$
$$= 191.81 + 0.878 \times 2392.82$$
$$= 2292.77 \text{ kJ/kg}$$

Heat supplied per kg;

$$q_{in} = h_{1''} - h_4 = 3456.50 - 194.85$$
$$= 3261.65 \text{ kJ/kg}$$

Heat rejected per kg

$$q_{out} = h_{2''} - h_3 = 2292.77 - 191.83 = 2100.94$$

and $$\eta_{plant} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{2100.94}{3261.65}$$
$$= 0.3559 \quad \text{or} \quad \mathbf{35.59\%}$$

Comment: The increase in superheating temperature results into increase in thermal efficiency of the plant as well steam quality after expansion in the turbine.

Case (c) With increase in boiler pressure to 10 MPa and 400°C

Rankine cycle 1′–2′–3–4′–1′

State 1: $h_{1'} = 3096.5$ kJ/kg

$s_{1'} = 6.2120$ kJ/kg·K

State 2: After isentropic expansion;

$$x' = \frac{6.2120 - 0.6492}{7.5010} = 0.742$$

$$h_{2'} = (h_f + x' h_{fg})_{@\ 10\ \text{kPa}}$$
$$= 191.83 + 0.742 \times 2392.87$$
$$= 1967.34 \text{ kJ/kg}$$

and $$h_{4'} = h_3 + v_f (p_1 - p_2)$$
$$= 191.83 + 0.001010 \times (10000 - 10)$$
$$= 201.92 \text{ kJ/kg}$$

$$q_{in} = h_{1'} - h_{4'} = 3096.5 - 201.92$$
$$= 2894.58 \text{ kJ/kg}$$

$$q_{out} = h_{2'} - h_3 = 1967.34 - 191.83$$
$$= 1775.51 \text{ kJ/kg}$$

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$$\eta_{plant} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{1775.51}{2894.58}$$
$$= 0.3866 \quad \text{or} \quad \mathbf{38.66\%}$$

Comment: The increase in boiler pressure results in an increase in plant efficiency from 34.11% to 38.66%

12.9 REHEATING OF STEAM

If the steam expands completely in a single stage then steam coming out the turbine is very wet. The wet steam carries suspended moisture particles, which are heavier than the vapour particles, thus deposited on the blades and causing its erosion. In order to increase the life of the turbine blades, it is necessary to keep the steam dry during its expansion. It is done by allowing the steam to expand to an intermediate pressure in a high-pressure turbine, and then taking it out and sending back to the boiler, where it is reheated at constant pressure, until it reaches the inlet temperature of the first stage as shown in Fig. 12.21. This process is called *reheating* during which heat is added to the steam. The reheated steam then further expands in the next stage of the turbine. Due to reheating, the work output of the turbine increases, thus improving the thermal efficiency.

12.9.1 Reheat Rankine Cycle

The reheat cycle is designed to take advantage of higher boiler pressure by eliminating the problem of excessive moisture content in the exhaust steam.

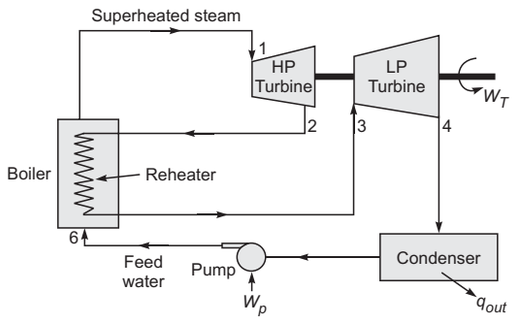
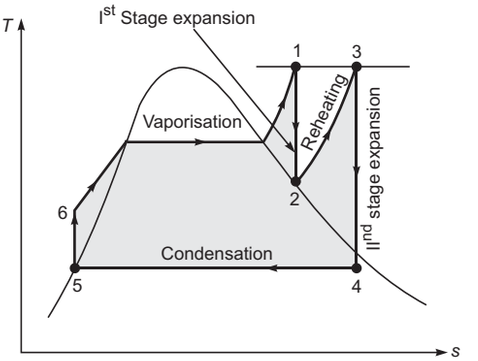


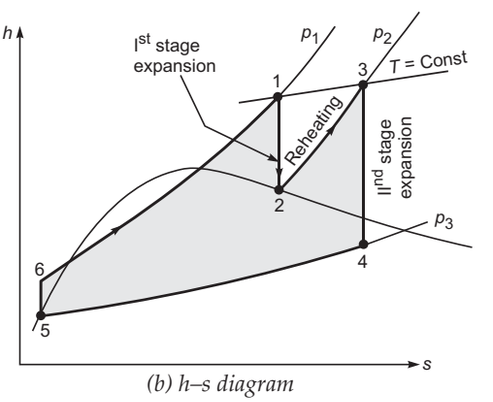
Fig. 12.21 *Schematic of a power plant with reheating and multistage expansion*

In a reheat Rankine cycle, the steam is expanded in a number of stages. After each stage of expansion, the steam is reheated in the boiler. Then, it expands in the next stage of turbine and is finally exhausted to the condenser.

Figure 12.21 shows the reheating process schematically. Figures 12.22(a) and (b) show the reheating process 2–3 on *T–s* and *h–s* (Mollier) diagrams, respectively. The steam at state 1 enters in the first stage of turbine and expands isentropically to the state 2. The quality of steam at the state 2, is either just dry or slightly wet and thus it is taken back in the boiler and is reheated to original superheat temperature T_3 (= T_1) at constant pressure p_2. Then this reheated steam at the state 3 enters the next stage turbine and further expands to back pressure at the state 4.



(a) T–s diagram



(b) h–s diagram

Fig. 12.22 *Reheating process on T–s and h–s diagrams*

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The amount of heat added during reheating

$$q_{reheat} = h_3 - h_2$$

The total heat supplied per kg in two stages

$$q_{in} = (h_1 - h_6) + (h_3 - h_2) \qquad ...(12.20)$$

For isentropic expansion in two stages, the total work done per kg of steam

$$w_T = (h_1 - h_2) + (h_3 - h_4)$$

The pump work per kg of steam;

$$w_p = h_6 - h_5 = -\int_{p_3}^{p_1} v\,dp$$

The net work done per kg of steam;

$$w_{net} = w_T - w_p$$
$$= (h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5) \qquad ...(12.21)$$

However, the pump work $-\int_{p_3}^{p_1} v\,dp$ is very small in comparasion with turbine work, thus it is neglected in most of the cases.

The heat rejected in the condenser per kg of steam;

$$q_{out} = h_4 - h_5 \qquad ...(12.22)$$

Then the efficiency of the turbine with reheating is given by

$$\eta_{reheat} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$
$$= 1 - \frac{h_4 - h_5}{(h_1 - h_6) + (h_3 - h_2)} \qquad ...(12.23)$$

It is evident from the *T*–*s* diagram Fig. 12.22(a) that there is very less gain in thermal efficiency by reheating the steam, only the quality of exhausted steam is improved. However, the mean temperature of heat addition can be increased by including the number of expansion and reheating processes. Thus, the thermal efficiency of the cycle would further increase.

12.9.2 Reheat Factor

In actual, when steam expands through the turbine, a considerable friction is always involved when steam glides over the blades. This friction resists the flow of steam. The isentropic enthalpy drop is not fully converted into kinetic energy but some of its part is utilized to overcome the frictional resistances. Thus, the kinetic energy produced is less than that corresponding to theoretical isentropic enthalpy drop. Further, this friction is converted into heat, consequently, the steam becomes dry and saturated, even superheated. This frictional heating causes an increase in entropy and hence actual enthalpy drop is always less than the isentropic enthalpy drop.

Let us consider the expansion of steam in three-stage turbine as shown in Fig. 12.23 on the *h* –*s* diagram. The superheated steam initially at pressure p_1 expands through three stages to exhaust pressure p_4. The isentropic expansion from pressure p_1 to p_2 in the first stage of expansion is represented by a vertical line 1–2*s*. Due to friction and irreversibilities, the actual state after expansion is 2 instead of 2*s*. Therefore, the actual enthalpy drop $(h_1 - h_2)$ is less than the isentropic enthalpy drop $(h_1 - h_{2s})$ and the difference between them $(h_2 - h_{2s})$ is the loss due to irreversibilities, and thus the first stage efficiency of the turbine can be expressed as

$$\eta_{stage\ I} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$
$$= \frac{\text{Actual enthalpy drop}}{\text{Isentropic enthalpy drop}} \qquad ...(12.24)$$

The stage efficiency is identical to isentropic efficiency. If stage efficiency is known, then

$$h_2 = h_1 - \eta_{stage\ I}\,(h_1 - h_{2s}) \qquad ...(12.25)$$

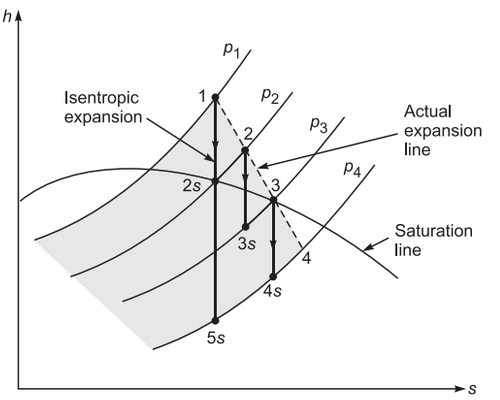

Fig. 12.23 *Three stage expansion h-s diagram*

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The locus of h_2 at pressure line p_2 can be marked at state 2 on the h–s diagram as shown in Fig. 12.23.

The second stage isentropic expansion takes place from the state 2 to 3s and actual enthalpy drop is

$$h_2 - h_3 = \eta_{stage\ II}\,(h_2 - h_{3s})$$

or $$h_3 = h_2 - \eta_{stage\ II}\,(h_2 - h_{3s}) \quad ...(12.26)$$

The locus of h_3 can also be marked in a similar way on the pressure line p_3. Then steam expands from state 3 to 4, and its actual enthalpy drop is

$$h_3 - h_4 = \eta_{stage\ III}\,(h_3 - h_{4s})$$

or $$h_4 = h_3 - \eta_{stage\ III}\,(h_3 - h_{4s}) \quad ...(12.27)$$

The sum of all isentropic enthalpy drops is referred as *cumulative isentropic enthalpy drop* and is designated as h_{cum}.

$$h_{cum} = (h_1 - h_{2s}) + (h_2 - h_{3s}) + (h_3 - h_{4s}) \quad ...(12.28)$$

If there was no irreversibility present during expansion of steam, then steam would expand isentropically through all stages from pressure p_1 to pressure p_4 as shown by the line 1–5s. The enthalpy drop during this expansion is $h_1 - h_{5s}$ referred as *isentropic enthalpy drop*.

The ratio of cumulative isentropic enthalpy drop to isentropic enthalpy drop from initial pressure to final pressure is called the *reheat factor* and is given by

$$RF = \frac{h_{cum}}{h_1 - h_{5s}} = \frac{h_1 - h_{2s} + h_2 - h_{3s} + h_3 - h_{4s}}{h_1 - h_{5s}} \quad ...(12.29)$$

The value of the reheat factor is always greater than unity and its effect is to increase the final enthalpy drop through friction heating, so the turbine efficiency improves in the same ratio. In actual case, the efficiency gain due to friction is very less. The turbine efficiency can be thus given by

$$\eta_{turbine} = \eta_{stage} \times RF \quad ...(12.30)$$

12.10 SUPER CRITICAL RANKINE CYCLE

The temperature of superheated steam entering the turbine is restricted by metallurgical constraints imposed by the materials used to fabricate the superheater, reheater and turbine. High pressure in the boiler also requires the piping that can withstand the stresses at elevated temperatures, Although these factors limit the gain that can be achieved by superheating and reheating.

The operating pressures in the boilers have gradually increased with improved materials and methods of fabrication over the years. The higher operating pressures and maximum allowed cycle temperatures have permitted a significant increase in thermal efficiency of the cycle.

If a vapour power plant is designed to operate with boiler pressures exceeding the critical pressure of water (22.1 MPa) and turbine inlet temperature exceeding 600°C, such cycle is referred as the *super critical Rankine cycle* as shown in Fig. 12.24. The Fig. 12.25 shows an ideal reheat cycle with super critical steam pressure. At super critical pressure

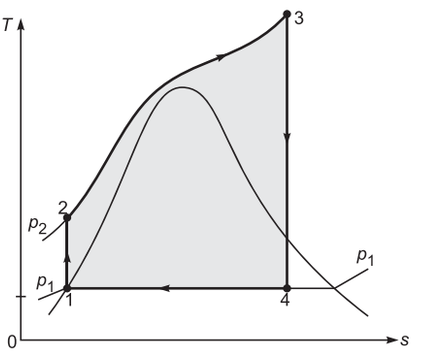

Fig. 12.24 *Super critical ideal Rankine cycle*

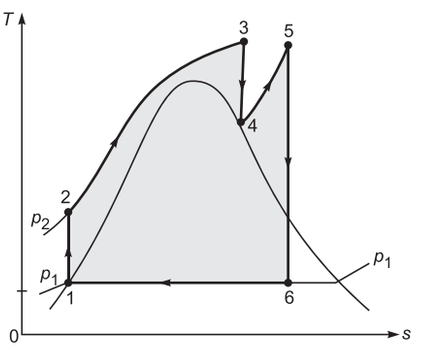

Fig. 12.25 *Super critical ideal Reheat Rankine cycle*

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(greater than 22.1 MPa), no phase change occurs during heat addition process 2–3.

Example 12.14 *An steam power plant operates on a theoretical reheat cycle. The steam from boiler at 150 bar and 550°C expands through the high-pressure turbine. It is reheated at constant pressure of 40 bar to 550°C and expands through the low pressure turbine to a condenser pressure of 0.1 bar. Draw T–s and h –s diagrams and find*

(a) quality of steam at turbine exhaust,

(b) Thermal efficiency of the cycle,

(c) Steam rate in kg/kWh.

Solution

Given A reheat Rankine cycle with

$p_1 = 150$ bar $\quad T_1 = 550°C$

$p_2 = 40$ bar $\quad T_3 = 550°C$

$p_3 = 0.1$ bar

To Find

(i) Quality of steam at turbine exhaust,

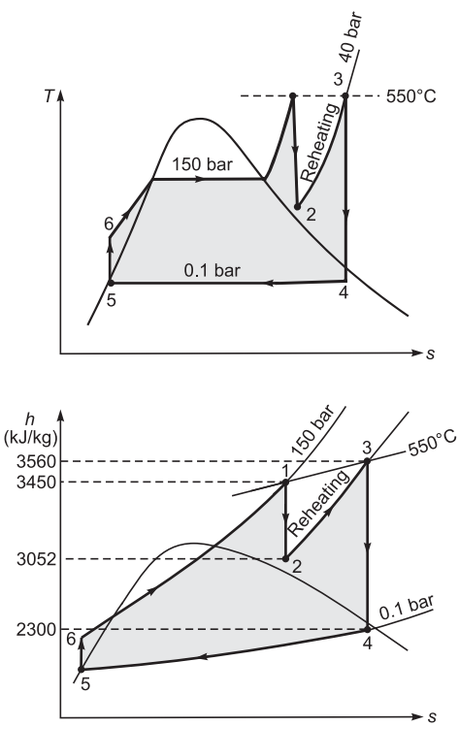

Fig. 12.26 *T-s and h-s diagrams for Example 12.14*

(ii) Thermal efficiency of the cycle, and

(iii) Steam rate in kg/kWh.

Properties of steam From Mollier chart

At state 1: Superheated steam

$p_1 = 150$ bar

$T_1 = 550°C$

$h_1 = 3450$ kJ/kg

State 2: After isentropic expansion, the steam is also superheated

$p_2 = 40$ bar

$T_2 = 335°C$

$h_2 = 3052$ kJ/kg

State 3: Superheated steam

$p_2 = 40$ bar

$T_3 = 550°C$

$h_3 = 3560$ kJ/kg

State 4: Wet steam

$p_3 = 10$ kPa

$x_4 = 0.88$

$h_4 = 2300$ kJ/kg

State 5: Saturated liquid (from saturated steam table)

$p_3 = 10$ kPa

$v_f = 0.001010$ m^3/kg

$h_f = 191.83$ kJ/kg

State 6: Compressed liquid $p_1 = 150$ bar

Analysis

(i) *Quality of steam at turbine exhaust*

Using Mollier (h–s) chart; draw a vertical straight line 3–4 from point 3 (coordinates: 40 bar and 550°C) to a pressure line of 0.1 bar as shown in h–s diagram of Fig. 12.26. The quality of steam at intersection point 4 as

$x = \mathbf{0.88}$

(ii) *Thermal efficiency of the cycle*

The pump work;

$$w_p = v_f (p_1 - p_3)$$
$$= 0.001010 \times (150 \times 10^2 - 10)$$
$$= 15.14 \text{ kJ/kg}$$

Enthalpy at the state 6;

$$h_6 = h_5 + w_p$$
$$= 191.93 + 15.14 = 206.97 \text{ kJ/kg}$$

The heat supplied per kg of steam

$$q_{in} = h_1 - h_6 + h_3 - h_2$$
$$= 3450 - 206.97 + 3560 - 3052$$
$$= 3751.03 \text{ kJ/kg}$$

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Turbine work per kg of steam;

$$w_T = h_1 - h_2 + h_3 - h_4$$
$$= 3450 - 3052 + 3560 - 2300$$
$$= 1658 \text{ kJ/kg}$$
$$w_{net} = w_T - w_p = 1658 - 15.14$$
$$= 1642.86 \text{ kJ/kg}$$

Thermal efficiency;

$$\eta_{cycle} = \frac{w_{net}}{q_{in}} = \frac{1642.86}{3751.03} = 0.438 \text{ or } \mathbf{43.8\%}$$

(iii) *Steam rate in kg/kWh*

$$ssc = \frac{3600}{w_{net}} = \frac{3600}{1642.86}$$
$$= \mathbf{2.19 \text{ kg/kWh}}$$

Example 12.15 *The steam is supplied to a turbine at a pressure of 32 bar and a temperature of 410°C. The steam then expands isentropically to a pressure of 0.08 bar. Find the dryness fraction of steam at the end of expansion and thermal efficiency of the cycle.*

If the steam is reheated at 5.5 bar to a temperaure of 395°C and then expands isentropically to 0.08 bar, what will be the dryness fraction and thermal efficiency of the cycle?

Solution

Given A steam power plant with superheated steam with and without reheating.

Turbine entry: p_1 = 32 bar (3200 kPa) and 410°C as superheated steam

Reheating; p_2 = 5.5 bar (550 kPa) and 395°C

Condenser exit; p_3 = 0.08 bar (8 kPa) as a saturated liquid

To find For simple Rankine cycle;

(i) Dryness fraction at the turbine exit,

(ii) Thermal efficiency of cycle,

For reheat Rankine cycle,

(iii) Dryness fraction at the turbine exit, and

(iv) Thermal efficiency of cycle.

Assumptions

(i) Each component of the cycle is in steady state.

(ii) All processes of working fluid are internally reversible.

(iii) The turbine and pump operate isentropically ($q = 0$).

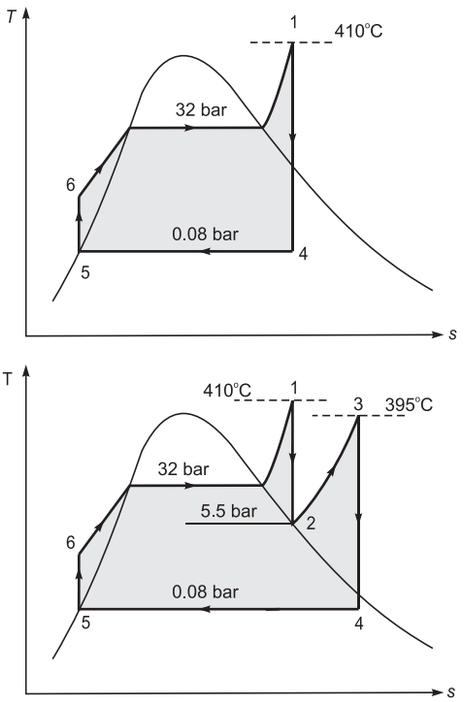

Fig. 12.27 *T-s diagram for (a) an ideal Rankine cycle, (b) reheat Rankine cycle*

(iv) The kinetic and potential energy effects are negligible.

Properties of steam From Mollier chart

At state 1: Superheated steam

p_1 = 32 bar

T_1 = 410°C

h_1 = 3250 kJ/kg

State 3: Superheated steam after reheating

p_2 = 5.5 bar

T_3 = 395°C

h_3 = 3263 kJ/kg

State 5: Saturated liquid

p_3 = 8 kPa

v_{f_3} = 0.001008 m^3/kg

$h_5 = h_{f_3}$ = 173.94 kJ/kg

State 4: Compressed liquid p_1 = 8000 kPa

Analysis

(i) **For simple Rankine cycle**: Fig. 12.27(a) and cycle 1–4–5–6;

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(a) *Dryness fraction* after isentropic expansion in a single stage:

Using Mollier (h–s) chart; draw a vertical straight line 1–4 from the point 1 (coordinates: 32 bar and 410°C) to a pressure line of 0.08 bar as shown in h–s diagram of Fig. 12.27(a). The quality of steam at intersection point 4 as

x_4 = **0.83**

and enthalpy at the state 4

h_4 = 2170 kJ/kg

(b) *Thermal efficiency of the cycle*

The pump work;

$$w_p = v_f (p_1 - p_3)$$
$$= 0.001010 \times (32 - 0.08) \times 10^2$$
$$= 3.22 \text{ kJ/kg}$$

Enthalpy at the state 6;

$$h_6 = h_5 + w_p = 173.94 + 3.22$$
$$= 177.16 \text{ kJ/kg}$$

The heat supplied per kg of steam

$$q_{in} = h_1 - h_6$$
$$= 3250 - 177.16 = 3072.84 \text{ kJ/kg}$$

The heat rejected per kg of steam

$$q_{in} = h_4 - h_5$$
$$= 2170 - 173.94 \approx 1996 \text{ kJ/kg}$$

Thermal efficiency;

$$\eta_{cycle} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{1996}{3072.84}$$
$$= 0.350 \quad \text{or} \quad \mathbf{35.0\%}$$

(ii) **For reheat Rankine cycle**:

Cycle 1–2–3–4–5–6; Fig. 12.27(b)

(c) *Dryness fraction* after isentropic expansion in low pressure turbine

Using Mollier (h–s) chart; draw a vertical straight line 3–4 from point 3 (coordinates: 5.5 bar and 395°C) to a pressure 0.08 bar as shown in h–s diagram of Fig. 12.27 (b). We get quality of steam at state 4 as

x_4 = **0.933**

and enthalpy at state 4

h_4 = 2426 kJ/kg

(d) *Thermal efficiency of the cycle*

The heat supplied per kg of steam

$$q_{in} = h_1 - h_6 + h_3 - h_2$$
$$= 3250 - 177.16 + 3263 - 2426$$
$$= 3528.88 \text{ kJ/kg}$$

The heat rejected per kg of steam

$$q_{in} = h_4 - h_5$$
$$= 2426 - 173.94 \approx 2252 \text{ kJ/kg}$$

Thermal efficiency;

$$\eta_{cycle} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{2252}{3528.88}$$
$$= 0.3618 \text{ or } \mathbf{36.18\%}$$

Example 12.16 *A steam power plant operates on ideal reheat cycle. The steam enters the high pressure turbine at 150 bar and 600°C and after expansion in two stages, is exhausted at 10 kPa. If the moisture content of the exhausted steam should not exceed 10.4 percent, determine*

(a) the pressure at which the steam should be reheated

(b) thermal efficiency of the cycle

Assume the steam is reheated to high pressure turbine inlet temperature.

Solution

Given A reheat Rankine cycle with

HP turbine inlet: p_1 = 150 bar

T_1 = 600°C

LP turbine inlet: T_3 = 600°C

Condenser inlet: p_3 = 10 kPa

x_4 = 1 – 0.104 = 0.896

To Find

(i) Intermediate pressure of reheat.

(ii) Thermal efficiency of the reheat cycle.

Assumptions

(i) Each component of the cycle is in steady state.

(ii) All processes of working fluid are internally reversible.

(iii) The turbine and pump operate isentropically (q = 0).

(iv) The kinetic and potential energy effects are negligible.

Properties of steam

At state 1: Superheated steam: (from Mollier chart)

p_1 = 150 bar

T_1 = 600°C

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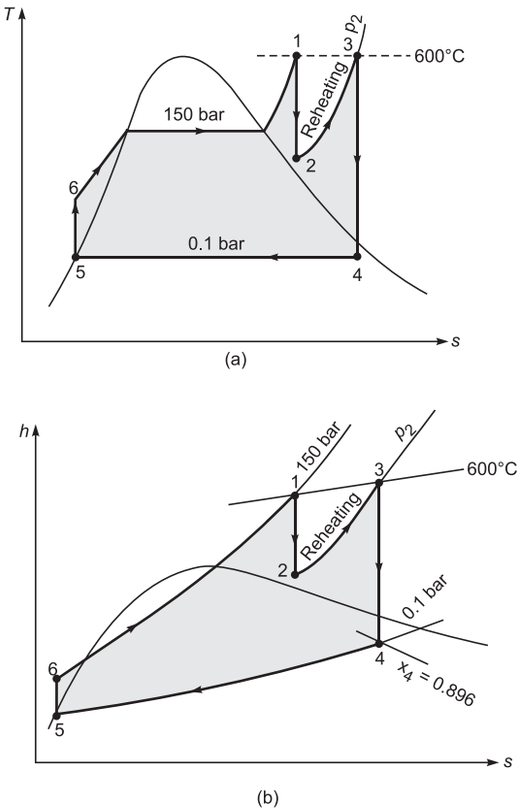

Fig. 12.28 *(a) T-s diagram and (b) h-s diagram for reheat Rankine cycle of Ex. 12.16*

$h_1 = 3580$ kJ/kg

$s_1 = 6.6776$ kJ/kg·K

State 2: Quality of steam is unknown

$s_2 = s_1 = 6.6776$ kJ/kg·K

State 3: Superheated steam $T_3 = 600°C$

State 4: Wet steam; (from steam Table A-13)

$p_3 = 10$ kPa

$h_f = 191.83$ kJ/kg

$h_{fg} = 2392.82$ kJ/kg

$s_f = 0.6492$ kJ/kg·K

$s_{fg} = 7.5010$ kJ/kg·K

State 5: Saturated liquid

$p_3 = 10$ kPa

$v_f = 0.00101$ m^3/kg

$h_f = 191.83$ kJ/kg

State 6: Compressed liquid

$p_3 = 150$ bar

Analysis

(i) *The intermediate pressure p_2*

Since the process 3–4 is an isentropic process, thus

$s_3 = s_4$

where $s_4 = (s_f + x_4 s_{fg})_{@10\text{ kPa}}$

$= 0.6492 + 0.896 \times 7.5010$

$= 7.370$ kJ/kg·K

Hence $s_3 = 7.370$ kJ/kg·K

Using Mollier (h–s) chart; draw a vertical straight line 3–4 from the point 4 (0.1 bar and $s_3 = 7.370$ kJ/kg·K) to a temperature line of $T_3 = 600°C$, as shown in h–s diagram of Fig. 12.28(b). At intersection point, the pressure and enthalpy found to be

$p_2 = p_3 =$ **40 bar**

$h_3 = 3675$ kJ/kg

Therefore, the steam should be reheated at 40 bar.

(ii) *Thermal efficiency of the cycle*

The pump work;

$w_p = v_f (p_1 - p_3)$

$= 0.00101 \times (150 \times 10^2 - 10)$

$= 15.14$ kJ/kg

Enthalpy at the state 6;

$h_6 = h_5 + w_p = 191.93 + 15.14$

$= 206.97$ kJ/kg

Specific enthalpy at the state 4

$h_4 = (h_f + x_4\, h_{fg})_{@\ 10\text{ kPa}}$

$= 191.83 + 0.896 \times 2392.82$

$= 2335.8$ kJ/kg

At the state 2, the pressure $p_2 = 40$ bar

$s_{g_2} = 6.070$ kJ/kg·K

and $T_{sat} = 250.4°C$

At this state $s_2 = s_1 = 6.6776$ kJ/kg·K $> s_{g\,@\,40\text{ bar}}$

Thus, steam is superheated at the state 2 and from Mollier diagram, we get

$h_2 = 3150$ kJ/kg, and $T_2 = 375°C$

The heat supplied per kg of steam

$q_{in} = h_1 - h_6 + h_3 - h_2$

$= 3580 - 206.97 + 3675 - 3150$

≈ 3898 kJ/kg

The heat rejected per kg of steam;

$q_{out} = h_4 - h_5 = 2335.8 - 191.83$

$= 2143.97$ kJ/kg

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and thermal efficiency

$$\eta_{cycle} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{2143.97}{3898} = 0.4496$$

or $= \mathbf{44.96\%}$

Example 12.17 *A steam power plant operates on an ideal reheat Rankine cycle between the pressure limits of 9 MPa and 10 kPa. The mass flow rate of steam through the cycle is 25 kg/s. Steam enters both stages of the turbine at 500°C. If the moisture content of the steam exiting the low-pressure turbine should not to exceed 10%; determine*

(a) the reheat pressure,
(b) total rate of heat input in the boiler,
(c) the thermal efficiency of the cycle.

Solution

Given A reheat Rankine cycle with

HP turbine inlet: $p_1 = 9$ MPa,
$T_1 = 500°C$
$\dot{m}_s = 25$ kg/s
LP turbine inlet: $T_3 = 500°C$
Condenser inlet: $p_3 = 10$ kPa,
$x_4 = 1 - 0.10 = 0.9$

To Find

(i) The reheat pressure,
(ii) Total rate of heat input in the boiler, and
(iii) The thermal efficiency of the cycle.

Properties of steam

At state 1: Superheated steam (from Mollier Chart)

$p_1 = 9$ MPa,
$T_1 = 500°C$
$h_1 = 3386$ kJ/kg,
$s_1 = 6.6575$ kJ/kg·K

State 2: Quality of steam is unknown

$s_2 = s_1 = 6.6575$ kJ/kg·K

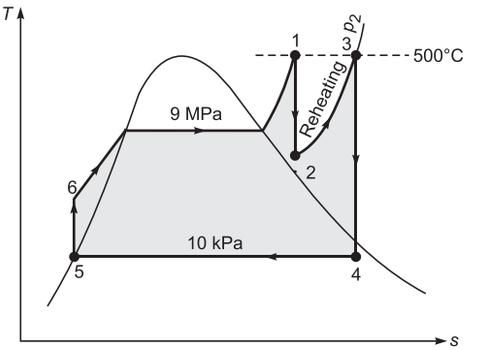
State 3: Superheated steam; $T_3 = 500°C$

State 4: Wet steam; (from steam table A-13)

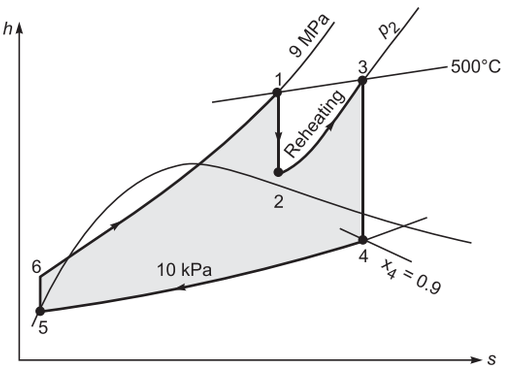
$p_3 = 10$ kPa
$h_f = 191.83$ kJ/kg,
$h_{fg} = 2392.82$ kJ/kg
$s_f = 0.6492$ kJ/kg·K
$s_{fg} = 7.5010$ kJ/kg·K

State 5: Saturated liquid

$p_3 = 10$ kPa



(a)



(b)

Fig. 12.29 *(a) T–s diagram, and (b) h–s diagram for reheat Rankine cycle of Ex. 12.17*

$v_f = 0.00101$ m^3/kg
$h_f = 191.83$ kJ/kg

State 6: Compressed liquid

$p_1 = 9$ MPa

Analysis

(i) *The intermediate pressure p_2*

Since the process 3–4 is an isentropic process, thus

$$s_3 = s_4$$

Specific entropy at the state 4;

$$s_4 = (s_f + x_4 s_{fg})_{@10\text{ kPa}}$$
$$= 0.6492 + 0.9 \times 7.5010$$
$$= 7.40 \text{ kJ/kg·K}$$

Hence $s_3 = 7.40$ kJ/kg·K

Using Mollier (h–s) chart; draw a vertical straight line 3–4 from the point 4 (10 kPa and s_3 = 7.40 kJ/kg·K) to a temperature line of $T_3 = 500°C$,

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as shown in the *h–s* diagram of Fig. 12.29(b). At intersection point, the pressure and enthalpy found to be

$$p_2 = p_3 = \mathbf{20\ bar}$$
$$h_3 = 3468\ \text{kJ/kg}$$

Therefore, the steam should be reheated at 20 bar.

Similarly, the state 2 can be defined as

$$s_2 = s_1 = 6.6575\ \text{kJ/kg·K}$$

Using Mollier (*h–s*) chart; draw a vertical straight line 1–2 from the point 1 (90 bar and s_1= 6.6575 kJ/kg·K) to a pressure line of 20 bar, we get

$$T_2 = 300°\text{C},$$
$$h_2 = 3015\ \text{kJ/kg}$$

The steam is superheated at the state 2.

The pump work;

$$w_p = v_f (p_1 - p_3)$$
$$= 0.00101 \times (9 \times 10^3 - 10)$$
$$= 9.08\ \text{kJ/kg}$$

Enthalpy at the state 6;

$$h_6 = h_5 + w_p = 191.93 + 9.08$$
$$= 201\ \text{kJ/kg}$$

Specific enthalpy at the state 4;

$$h_4 = (h_f + x_4\, h_{fg})_{@\ 10\ \text{kPa}}$$
$$= 191.83 + 0.9 \times 2392.82$$
$$= 2345.35\ \text{kJ/kg}$$

Total heat supplied per kg of steam

$$q_{in} = h_1 - h_6 + h_3 - h_2$$
$$= 3386 - 201 + 3468 - 3015$$
$$= 3638\ \text{kJ/kg}$$

(ii) *Rate of heat input in the boiler*

$$\dot{Q}_{in} = \dot{m}_s\, q_{in} = (25\ \text{kg/s}) \times (3638\ \text{kJ/kg})$$
$$= \mathbf{90950\ kW}$$

The heat rejected per kg of steam;

$$q_{out} = h_4 - h_5 = 2345.35 - 191.83$$
$$= 2153.52\ \text{kJ/kg}$$

(iii) *Thermal efficiency of the cycle*

$$\eta_{cycle} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{2153.52}{3638.0}$$
$$= 0.408 \quad \text{or} \quad \mathbf{40.8\%}$$

Example 12.18 *The steam is the working fluid in an ideal Rankine cycle with superheat and reheat. The steam enters the first stage turbine at 8.0 MPa, 480°C and expands to 0.7 MPa. It is then reheated to 440°C before entering the second stage turbine, where it expands to the condenser pressure of 0.008 MPa. The net power output of the cycle is 100 MW. Determine*

(a) thermal efficiency of the cycle,
(b) mass flow rate of steam in kg/h,
(c) specific steam consumption in kg/kWh,
(d) the rate of heat transfer from the condensing steam as it passes through the condenser in MW.

Discuss the effect of reheating on the vapour power cycle

Solution

Given An ideal reheat Rankine cycle operates with steam as working fluid.

HP turbine inlet; $p_1 = 8.0$ MPa
$T_1 = 480°\text{C}$
LP turbine inlet; $p_3 = 0.7$ MPa
$T_3 = 440°\text{C}$
Condenser pressure; $p_4 = 0.008$ MPa
Net Power output $P = 100$ MW

To find

(i) Thermal efficiency of the cycle.
(ii) Mass flow rate of steam in kg/h.
(iii) Specific steam consumption in kg/kWh.
(iv) Rate of heat transfer from the condensing steam as it passes through the condenser, in MW.

Assumptions

(i) Each component in the cycle as control volume at steady state.
(ii) All process are internally reversible.
(iii) Turbine and pump operate isentropically.
(iv) Kinetic and potential energy effects are negligible.

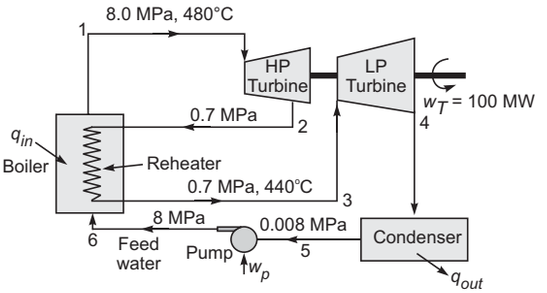
Properties of steam at each principal states from Mollier chart;

State 1: Superheated steam at

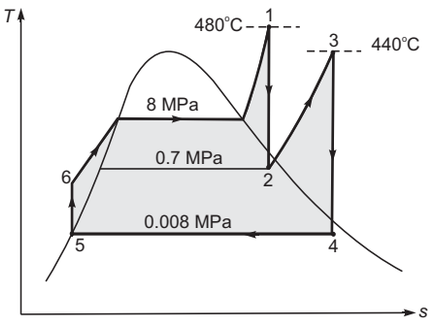
$$p_1 = 8.0\ \text{MPa}$$
$$T_1 = 480°\text{C}$$
$$h_1 = 3380\ \text{kJ/kg}$$

State 2: Wet steam at

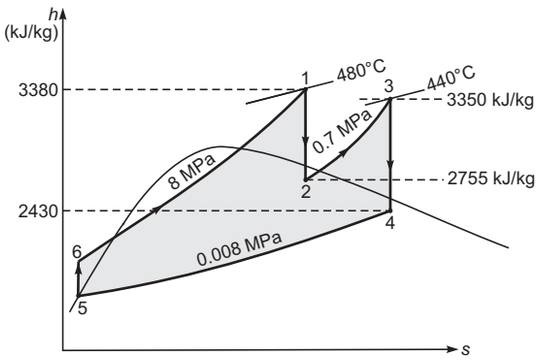
$$p_2 = 0.7\ \text{MPa}$$
$$s_2 = s_1$$
$$h_2 = 2755\ \text{kJ/kg} \quad x_2 = 0.98$$



(a) Schematic



(b) T–s diagram



(c) h–s diagram

Fig. 12.30 *Schematic with given data*

State 3: Superheated steam at

$p_2 = 0.7$ MPa

$T_3 = 440°C$

$h_3 = 3350$ kJ/kg

State 4: Wet steam at

$p_3 = 0.008$ MPa

$s_4 = s_3$

$h_4 = 2410$ kJ/kg $\quad x_4 = 0.94$

State 5: Saturated liquid at

$p_3 = 0.008$ MPa

$h_5 = 173.93$ kJ/kg·K

$v_f = 0.001008$ m^3/kg

State 6: Compressed liquid at

$p_6 = 8.0$ MPa

Analysis

(i) *Thermal efficiency of the cycle*

The pump work;

$$w_p = v_f (p_1 - p_3)$$
$$= 0.001008 \times (8 - 0.008) \times 10^3$$
$$= 8.06 \text{ kJ/kg}$$

Enthalpy at the state 6;

$$h_6 = h_5 + w_p = 173.93 + 8.06$$
$$= 182 \text{ kJ/kg}$$

The total turbine work per kg of steam;

$$w_T = h_1 - h_2 + h_3 - h_4$$
$$= 3380 - 2755 + 3350 - 2410$$
$$= 1565 \text{ kJ/kg}$$

Net work done per kg of steam

$$w_{net} = w_T - w_p = 1565 - 8.06$$
$$\approx 1557 \text{ kJ/kg}$$

Total heat supplied per kg of steam

$$q_{in} = h_1 - h_6 + h_3 - h_2$$
$$= 3380 - 182 + 3350 - 2755$$
$$= 3793 \text{ kJ/kg}$$

Thermal efficiency;

$$\eta_{cycle} = \frac{w_{net}}{q_{in}} = \frac{1557}{3793} = 0.410 \text{ or } \mathbf{41.0\%}$$

(ii) *Mass flow rate of steam*

$$\dot{m}_s = \frac{P}{w_{net}} = \frac{100 \times 10^3 \text{ kW}}{1557 \text{ kJ/kg}}$$
$$= 64.22 \text{ kg/s} = \mathbf{231.21 \times 10^3 \text{ kg/h}}$$

(iii) *Specific steam consumption*

$$ssc = \frac{3600}{w_{net}} = \frac{3600}{1557} = \mathbf{2.31 \text{ kg/kWh}}$$

(iv) *The rate of heat rejection in the condenser*

$$\dot{Q}_{in} = \dot{m}_s (h_4 - h_5)$$
$$= 64.22 \times (2410 - 182)$$
$$= 143.4 \times 10^3 \text{ kW}$$
$$= \mathbf{143.4 \text{ MW}}$$

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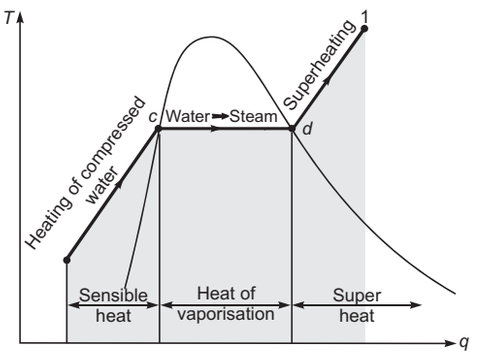
Comment With superheating and reheating of steam in the Rankine cycle, the effects observed are (in comparison with Example 12.8 given earlier)

(i) The quality of steam leaving the turbine is improved from $x = 0.674$ to 0.94.

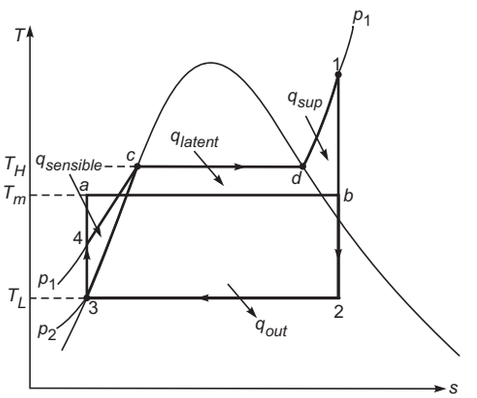
(ii) The efficiency of the cycle is increased by 41.0 – 37.1 = 3.9%.

12.11 MEAN TEMPERATURE OF HEAT ADDITION

The heat-addition pattern of the simple Rankine cycle is shown in Fig. 12.31(a). Initially, the heat added as sensible heat to compressed liquid coming out the pump is at much lower average temperature, the latent heat for vapourisation at constant temperature and then the superheat at higher average temperature.



(a) Heat additional pattern to working fluid



(b) Mean temperature of heat addition

Fig. 12.31 *Heat addition and meam temperature of working fluid in Rankine cycle*

If the mean temperature T_m of heat addition as shown in Fig. 12.31(b) is assumed in such a way that the area under curve 4–*c*–*d*–1 is equal to the area under the curve *a*–*b on T*–*s* diagram (Fig. 12.31), then

$$q_{in} = h_1 - h_4 = T_m(s_1 - s_4)$$

Mean temperature of heat addition;

$$T_m = \frac{h_1 - h_4}{s_1 - s_4}$$

$$= \frac{\text{Amount of heat supplied in the boiler}}{\text{Change in entropy during heat addition}} \quad ...(12.31)$$

Heat rejected per kg of steam;

$$q_{out} = h_2 - h_3 = T_L(s_1 - s_4)$$

Then Rankine cycle efficiency;

$$\eta_{Rankine} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_L(s_1 - s_4)}{T_m(s_1 - s_4)}$$

$$\therefore \quad \eta_{Rankine} = 1 - \frac{T_L}{T_m} \quad ...(12.32)$$

where T_L is temperature of heat rejection. The lower value of temperature T_L for given T_m will increase the thermal efficiency of the Rankine cycle. But the temperature of heat rejection cannot be lower than the temperature of surroundings. Therefore,

$$\eta_{Rankine} = f(T_m) \text{ only} \quad ...(12.33)$$

Thus, the higher the mean temperature of heat addition, the higher will be thermal efficiency of Rankine cycle.

12.12 REGENERATIVE RANKINE CYCLE

In a simple Rankine cycle, a significant amount of heat is added for sensible heating of compressed liquid coming out the pump. The mean temperature at which sensible heat added is much lower than the source temperature. Thus, the efficiency of the Rankine cycle is much lower than that of Carnot vapour power cycle. The efficiency of the Rankine cycle can be improved by heating the feed water regeneratively.

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12.12.1 Principle of Regeneration: Ideal Regeneration

The mean temperature of heat addition in the Rankine cycle can be improved by increasing the heat supplied at high temperature such as increasing superheat, increasing boiler pressure and using reheat. The mean temperature of heat addition can also be increased by decreasing the amount of heat supplied at lower temperatures.

In saturated steam Rankine cycle shown in Fig. 12.32, if the feed water enters the boiler at the state 4′ and all heat is supplied at constant temperatrure T_H, then the cycle is called an ideal regenerative cycle, shown schematically in Fig. 12.33(a) and corresponding T–s diagram in Fig. 12.33(b).

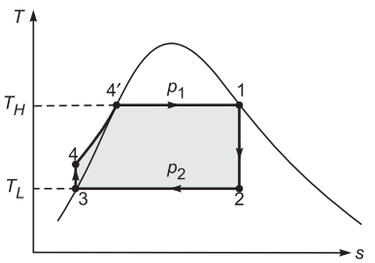


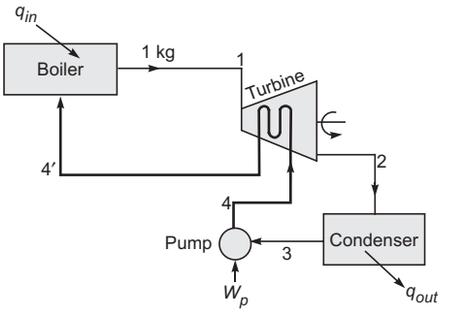
Fig. 12.32 *Saturated steam Rankine cycle*

In ideal regenerative cycle, the condensate leaving the pump enters the turbine at the state 4 and flows in counterflow direction to the steam flow. Thus, it is possible to heat the feed water to steam temperature at inlet to turbine. If at all points the temperature difference between steam and feed water is negligibly small then the heat transfer takes place in reversible manner. For such a process;

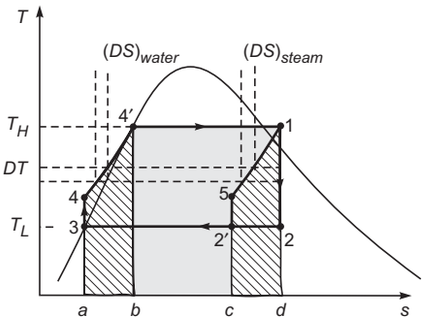
$$(\Delta T)_{Water} = -(\Delta T)_{Steam}$$

and $$(\Delta s)_{Water} = -(\Delta s)_{Steam}$$

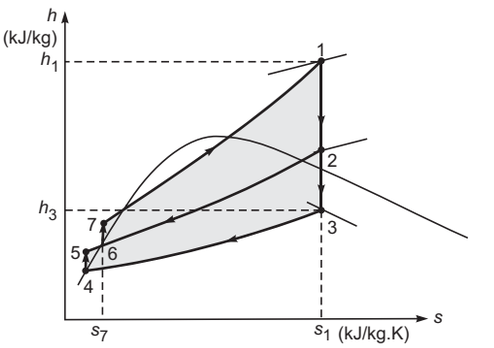
During an ideal regeneration, the steam cools first to the state 5 and then expands to the state 2′. The slope of line 1–5 and 4′–4 in Fig. 12.33(*b*) will be identical at every point. The area a–4–4′–b–a and area c–5–1–d–c are equal and congruous. Therefore, all the heat in the boiler is supplied at constant temperature T_H and all the heat is rejected at constant temperature T_L, both being reversible.



(a) Schematic of ideal regenerative cycle



(b) Ideal regenerative cycle on T–s plot



(c) h–s diagram for regenerative cycle

Fig. 12.33 *Ideal regenerative cycle*

Then $$q_{in} = h_1 - h_{4'} = T_H (s_1 - s_{4'})$$

and $$q_{out} = h_{2'} - h_3 = T_L (s_{2'} - s_3)$$

Since $s_1 - s_{4'} = s_{2'} - s_3$

$$\therefore \quad \eta_{Reg} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_L}{T_H} \qquad ...(12.34)$$

The efficiency of an ideal regenerative cycle is thus equal to eficiency of the Carnot cycle. Since

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steam is first used to heat the feed water from 1–5 and then allows to expand from state 5 to state 2′, and therefore, the net work output of an ideal regenerative cycle is less and steam rate will be more, although it is more efficient as compared with simple Rankine cycle. However, such a proposition is not practical for the following reasons:

1. The transfer of heat in reversible manner takes place very slowly.
2. Heat exchange in the turbine is not mechanically feasible.
3. The moisture content of steam in the turbine will be very high.

In actual practice, advantage of regenerative heating principle is used by extracting a part of the steam from turbine at a certain stage of the expansion and it is used for heating of feed water in separate feed-water heaters. This arrangement does not reduce the dryness fraction of remaining steam passing through the turbine.

If there were a large number of extraction stages of steam for feed water heating, then the resulting cycle would approach to be a Carnot cycle.

12.12.2 Regeneration with Open Feed-water Heater

The regenerative cycle with open feed-water heater is shown in Fig. 12.34. A part of superheated steam which enters the turbine at the state 1, is extracted from the turbine at the intermediate state 2 of turbine- expansion process. The extracted steam is supplied to a heat exchanger known as *feed water heater*. The remaining amount of steam in the turbine expands completely to condenser pressure (state 3). The condensate, a saturate liquid at state 4 is pumped isentropically by low pressure (LP) pump to the pressure of extracted steam. The compressed liquid at the state 5 enters the feed water heater and it mixes with steam extracted from the turbine. Due to direct mixing process, the feed water heater is called open or *direct-contact type feed-water heater*. The portion of steam extracted is so adjusted to make the mixture leaving the feed water

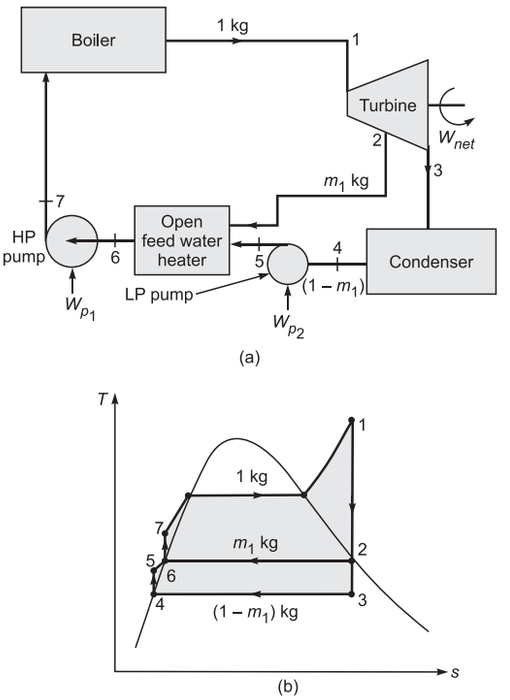

Fig. 12.34 *(a) Schematic of regenerative cycle with open feed-water heater (b) T-s diagram for regenerative cycle*

to be saturated at the state 6. Now this saturated water is pumped by high pressure (HP) pump to the boiler pressure state 7. With the regeneration, the average temperature at which heat is supplied has been increased, therefore, Rankine cycle efficiency improves.

Analysis Since the masses of steam flowing through the various components of the cycle vary, thus the analysis of this cycle differs from previous one.

Let 1 kg of steam be leaving the boiler and entering the turbine.

m_1 kg of steam per kg, is extracted at the state 2 from the turbine at intermediate pressure p_2.

$(1 - m_1)$ kg of steam per kg flow through the remaining part of the turbine during expansion from 2–3, condensation from 3–4 and pumping from 4–5.

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$(1 - m_1)$ kg of steam enters in open feed water heater and mixed with m_1 kg of steam blown from the turbine at the state 2. After mixing, the mass of saturated liquid becomes 1 kg at the state 6 and it is pumped to boiler pressure at the state 7.

Applying steady flow energy equation to mixing process 2–6;

$$(1 - m_1)h_5 + m_1 h_2 = h_6$$

or $$h_5 - m_1 h_5 + m_1 h_2 = h_6$$

which gives, $$m_1 = \frac{h_6 - h_5}{h_2 - h_5} \quad ...(12.35)$$

The heat supplied in the boiler

$$q_{in} = h_1 - h_7$$

Heat rejected in the condenser

$$q_{out} = (1 - m_1)(h_3 - h_4)$$

Turbine work,

$$w_T = (h_1 - h_2) + (1 - m_1)(h_2 - h_3) \quad ...(12.36)$$

Pump work

$$w_p = (h_7 - h_6) + (1 - m_1)(h_5 - h_4) \quad ...(12.37)$$

Net work done per kg of steam,

$$w_{net} = w_T - w_p$$
$$= (h_1 - h_2) + (1 - m_1)(h_2 - h_3) - [(h_7 - h_6) + (1 - m_1)(h_5 - h_4)] \quad ...(12.38)$$

Thermal efficiency of regenerative cycle;

$$\eta_{reg} = 1 - \frac{q_{out}}{q_{in}}$$
$$= 1 - \frac{(1 - m_1)(h_3 - h_4)}{h_1 - h_7} \quad ...(12.39)$$

In the regenerative cycle, the feed water enters the boiler at temperature T_7 and its mean temperature of heat addition is

$$T_{m,\,Reg} = \frac{q_{in}}{s_1 - s_7} = \frac{h_1 - h_7}{s_1 - s_7} \quad ...(12.40)$$

The mean temperature of heat addition without heat generation for simple Rankine cycle operating between same pressures p_1 and p_3 would be

$$T_{m,\,Rankine} = \frac{q_{in}}{s_1 - s_5} = \frac{h_1 - h_7}{s_1 - s_5} \quad ...(12.41)$$

Since $T_{m,Reg} > T_{m,Rankine}$

Thus the amount of heat supplied is decreased and efficiency of regenerative cycle will be higher than that of Rankine cycle. But at the same time, the turbine work and heat rejection are also reduced due to extraction of steam at intermediate pressure.

Advantages of Regeneration

1. It raises the temperature of feed water to saturation temperature, and thus the amount of heat addition in the boiler reduces.
2. The heat is added in the boiler at a higher average temperaure.
3. Open feed water heater serves as a *deaerator* to remove the air and other non-condensable gases from the feed water, otherwise they would cause corrosion.

12.12.3 Carnotisation of Rankine Cycle

Consider the Rankine cycle 1–2–3–4–5–6–7–8–9–10–1 with two stage feed-water heating operating between turbine inlet and condenser inlet as shown on the *T–s* diagram in Fig. 12.35(a).

Applying steady-flow energy equation for the mixing process 2–9;

$$(1 - m_1)\, h_8 + m_1 h_2 = h_9$$

which gives, $$m_1 = \frac{h_9 - h_8}{h_2 - h_8}$$

Similarly, the steady-flow energy equation for the mixing process 3–7;

$$(1 - m_1 - m_2)\, h_6 + m_2 h_3 = (1 - m_1) h_7$$

which gives, $$m_2 = (1 - m_1)\frac{h_7 - h_6}{h_3 - h_6}$$

The path 1–2–3–4 in Fig. 12.35(a) represents the states of decreasing mass of fluid during isentropic expansion. If the same mass of steam (1 kg) undergoes throughout regeneration and expansion, the states would be represented by 1–2′–3″–4′ as shown in Fig. 12.35(b). The turbine work with decreasing mass and unit mass for above two types of cycles would be;

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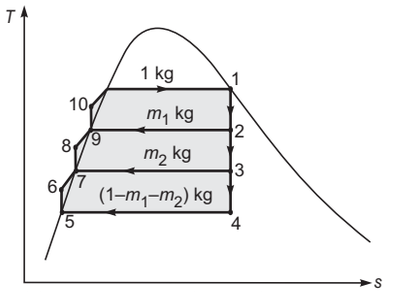
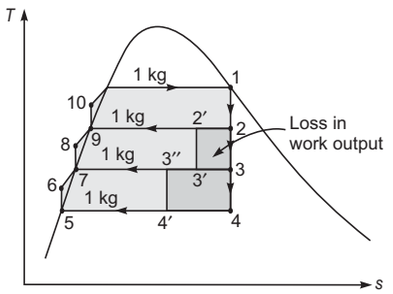


(a) Regenerative cycle with decreasing mass of steam during its expansion



(b) Regenerative cycle for unit mass of fluid

Fig. 12.35 *Carnotisation of Rankine cycle*

$$w_T = (h_1 - h_2) + (1 - m_1)(h_2 - h_3) + (1 - m_1 - m_2)(h_3 - h_4) \quad ...(12.42)$$

$$= (h_1 - h_2) + (h_{2'} - h_{3'}) + (h_{3''} - h_{4'}) \quad ...(12.43)$$

where $(1\,\text{kg}) \times (h_{2'} - h_{3'}) = (1 - m_1)(\text{kg}) \times (h_2 - h_3)$ ...(12.44)

and $(1\,\text{kg}) \times (h_{3''} - h_{4'}) = (1 - m_1 - m_2)(\text{kg}) \times (h_3 - h_4)$ ...(12.45)

The cycle 1–2′–3′–3″–4′–5–6–7–8–9–10–1 using 1 kg working fluid throughout. The heat released by steam from 2–2′ is used to heat compressed water from the states 8 to 9.

$$(1\,\text{kg}) \times (h_2 - h_{2'}) = (1\,\text{kg})\,(h_9 - h_8) \quad ...(12.46)$$

Similarly,

$$(1\,\text{kg}) \times (h_{3'} - h_{3''}) = (1\,\text{kg})\,(h_7 - h_6) \quad ...(12.47)$$

Rearranging Eq. (12.43)

$$w_T = (h_1 - h_{4'}) - (h_2 - h_{2'}) - (h_{3'} - h_{3''}) \quad ...(12.48)$$

Using Eqs. (12.46) and (12.47) in Eq. (12.48), the turbine work is

$$w_T = (h_1 - h_{4'}) - (h_9 - h_8) - (h_7 - h_6) \quad ...(12.49)$$

It is observed that the stepped cycle 1–2′–3′–3″–4′–5–6–7–8–9–10–1 approximates the ideal regeneration cycle in Fig. 12.33. By addition of the more number of feed water heaters, by extracting the steam in number of stages from turbine, the feed-water temperature could be raised, and it would give a closure approximation (Fig. 12.36) to Carnot Cycle.

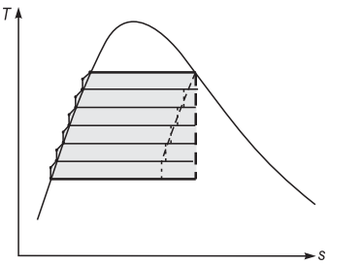


Fig. 12.36 *Many stages of feed-water heating approaches Carnot cycle*

By addition of feed-water heaters, the the heat rejection in the cycle decreases from $(h_4 - h_5)$ to $(h_{4'} - h_5)$, the turbine work output decreases (area under 2–2′ + area under 3–3′) as shown by shaded area in Fig. 12.35(b). The steam rate also increases with regeneration. But thermal efficiency of the cycle increases.

Example 12.19 *An ideal regenerative steam cycle operates with the steam entering the turbine at 30 bar and 500°C and is exhausted at 0.1 bar. A feed water heater is used, which operates at 5 bar. Calculate*

(a) the thermal efficiency,

(b) steam rate of the cycle, and

(c) increase in average temperature of heat addition, efficiency, and steam as compare to an ideal Rankine cycle operates between same conditions.

Solution

Given An ideal regenerative steam cycle with

$p_1 = p_7 = 30$ bar $\quad T_1 = 500°\text{C}$

$p_3 = p_4 = 0.1$ bar $\quad p_2 = p_6 = 5$ bar

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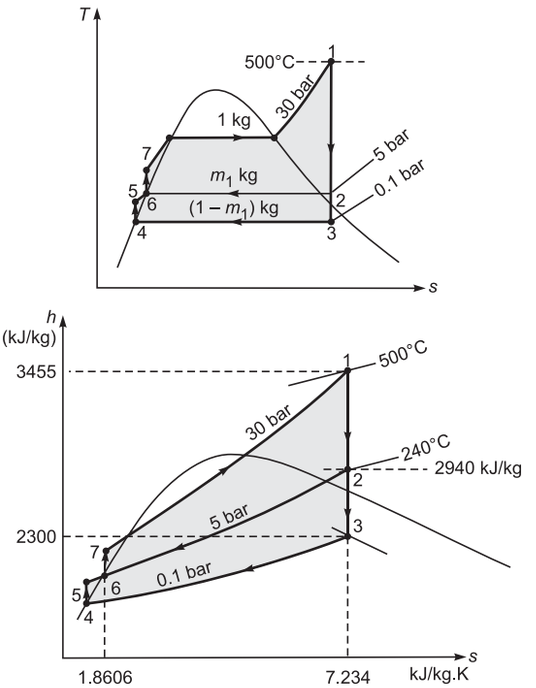

Fig. 12.37 *(a) T–s and h–s diagram for Regenerative Rankine cycle of Example 12.19*

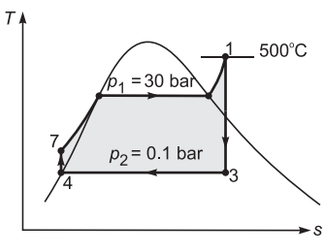

Fig. 12.38 *(b) T-s diagram for an ideal Rankine cycle*

To Find

(i) The thermal efficiency.
(ii) Steam rate of the cycle.
(iii) Increase in average temperature of heat addition, efficiency, and steam as compared to an ideal Rankine cycle operates between same conditions.

Properties of steam From Mollier chart

State 1: Superheated steam at turbine inlet

p_1 = 30 bar,
T_1 = 500°C
h_1 = 3455 kJ/kg,
s_1 = 7.234 kJ/kg·K

State 2: Superheated steam;

p_2 = 5 bar,
T_2 = 240°C
h_2 = 2940 kJ/kg
s_2 = 7.234 kJ/kg·K

State 3: Wet steam;

p_3 = 0.1 bar = 10 kPa,
x_3 = 0.88
h_3 = 2300 kJ/kg

State 4: Saturated liquid; (from steam tables);

p_4 = 0.1 bar
v_{f_4} = 0.001010 m^3/kg
$h_4 = h_{f_4}$ = 191.83 kJ/kg

State 5: Compressed liquid;

p_5 = 5 bar = 500 kPa

State 6: Saturated liquid (from steam tables);

p_6 = 5 bar;
$h_6 = h_{f_6}$ = 641.21 kJ/kg
v_{f_6} = 0.001094 m^3/kg
s_6 = 1.8606 kJ/kg·K

State 7: Compressed liquid;

p_7 = 30 bar = 3000 kPa

Analysis

(i) Regenerative cycle

The high-pressure pump work input per kg of steam,

$$w_{p_1} = v_{f@5\,\text{bar}} \times (p_1 - p_6)$$
$$= 0.001094\,(30 - 5) \times 10^2$$
$$= 2.735 \text{ kJ/kg}$$

Specific enthalpy at the state 7;

$$h_7 = h_6 + w_{p_2} = 641.21 + 2.73$$
$$= 643.94 \text{ kJ/kg}$$

The low pressure pump work input per kg of steam

$$w_{p_2} = v_{f@\,0.1\,\text{bar}} \times (p_5 - p_4)$$
$$= 0.001010 \times (5 - 0.1) \times 10^2$$
$$= 0.5 \text{ kJ/kg}$$

Specific enthalpy at the state 5;

$$h_5 = h_4 + w_{p_2}$$
$$= 191.83 + 0.5 = 192.32 \text{ kJ/kg}$$

The mass of steam m_1 extracted from turbine at 5 bar can be obtained by using Eq. (12.35)

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$$m_1 = \frac{h_6 - h_5}{h_2 - h_5} = \frac{641.21 - 192.32}{2940 - 192.32}$$
$$= \mathbf{0.163\ kg}$$

Therefore, 0.163 kg of steam is extracted from the turbine for each kg of steam entering the turbine.

Turbine work,

$$w_T = h_1 - h_2 + (1 - m_1)(h_2 - h_3)$$
$$= 3455 - 2940 + (1 - 0.163)(2940 - 2300)$$
$$= 1050.68\ \text{kJ/kg}$$

The total pump work

$$w_p = w_{p_1} + (1 - m_1) w_{p_2}$$
$$= 2.73 + (1 - 0.163) \times 0.5$$
$$= 3.1\ \text{kJ/kg}$$

Net work done per kg of steam

$$w_{net} = w_T - w_p = 1050.68 - 3.1$$
$$= 1047.58\ \text{kJ/kg}$$

The heat supplied in the boiler per kg of steam

$$q_{in} = h_1 - h_7 = 3455 - 643.94$$
$$= 2811\ \text{kJ/kg}$$

(a) *Thermal efficiency* of the cycle

$$\eta_{th,\ Reg} = \frac{w_{net}}{q_{in}} = \frac{1047.58}{2811}$$
$$= 0.3725 \quad \text{or} \quad \mathbf{37.26\%}$$

(b) *Steam rate* of the cycle;

$$ssc = \frac{3600}{w_{net}} = \frac{3600}{1047.58}$$
$$= \mathbf{3.436\ kg/kWh}$$

(c) Average temperature of heat addition, Eq. (12.31);

$$T_{m,\ Reg} = \frac{q_{in}}{s_1 - s_6} = \frac{2811}{7.234 - 1.8606}$$
$$= 523.13\ \text{K}$$

(ii) *Ideal Rankine cycle*

We have the properties of steam as

State 1: Superheated steam at turbine inlet

$$h_1 = 3455\ \text{kJ/kg},$$
$$s_1 = 7.234\ \text{kJ/kg·K}$$

State 3: Wet steam;

$$h_3 = 2300\ \text{kJ/kg}$$
$$x_3 = 0.88$$

State 4: Saturated liquid; (from steam tables);

$$p_4 = 0.1\ \text{bar}$$
$$h_4 = h_{f_4} = 191.83\ \text{kJ/kg}$$
$$v_{f_4} = 0.001010\ \text{m}^3\text{/kg}$$
$$s_4 = 0.6492\ \text{kJ/kg·K}$$

State 7: Compressed liquid;

$$p_7 = 30\ \text{bar}$$

The pump work input per kg of steam

$$w_p = v_{f\,@\,0.1\,\text{bar}} \times (p_7 - p_4)$$
$$= 0.001010 \times (30 - 0.1) \times 10^2$$
$$= 3.01\ \text{kJ/kg}$$

Specific enthalpy at the state 7;

$$h_7 = h_4 + w_p = 191.83 + 3.01$$
$$= 194.85\ \text{kJ/kg}$$

Turbine work, $w_T = h_1 - h_3 = 3455 - 2300$

$$= 1155\ \text{kJ/kg}$$

Net work done per kg of steam

$$w_{net} = w_T - w_p = 1155 - 3.01$$
$$= 1152\ \text{kJ/kg}$$

The heat supplied in the boiler per kg of steam

$$q_{in} = h_1 - h_7 = 3455 - 194.85$$
$$= 3260.15\ \text{kJ/kg}$$

(d) *Thermal efficiency* of the cycle;

$$\eta_{th,\ ideal} = \frac{w_{net}}{q_{in}} = \frac{1152}{3260.15} = 0.3533$$

or **35.33%**

(e) *Steam rate* of the cycle;

$$ssc = \frac{3600}{w_{net}} = \frac{3600}{1152}$$
$$= \mathbf{3.125\ kg/kWh}$$

(f) Average temperature of heat addition, Eq. (12.31);

$$T_{m,\ ideal} = \frac{q_{in}}{s_1 - s_4} = \frac{3260.15}{7.234 - 0.6492}$$
$$= 494.1\ \text{K}$$

Changes compared to regenerative cycle

Increase in efficiency with regeneration

$$= \eta_{th,\ Reg} - \eta_{th,\ ideal}$$
$$= 37.26 - 35.33 = \mathbf{1.93\%}$$

Increase in average temperature of heat addition with regeneration

$$= T_{m,\ Reg} - T_{m,\ ideal}$$
$$= 523.13\ \text{K} - 494.1\ \text{K}$$
$$= \mathbf{29.0\ K \quad or \quad 29°C}$$

Increase in steam rate with regeneration

$$= 3.436 - 3.125 = \mathbf{0.311\ kg/kWh}$$

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Example 12.20 *Consider a regenerative vapour power cycle with a feed-water heater. The steam enters the first-stage turbine at 8.0 MPa, 500°C and expands to 0.7 MPa, where some of the steam is extracted and diverted to feedwater heater operating at 0.7 MPa. The remaining steam expands through the second stage turbine to a condenser pressure of 0.008 MPa. The saturated liquid exits the feed-water heater at 0.7 MPa. The isentropic efficiency of each turbine is 85%, while each pump operates isentropically. If the net power output of the cycle is 105 MW, determine*

(a) thermal efficiency of the cycle,

(b) the mass flow rate of steam entering the first turbine stage.

Solution

Given A regenerative vapour power cycle with feed-water heater:

First stage turbine inlet, $p_1 = 8.0$ MPa,

$T_1 = 500°C$

Regeneration pressure, $p_2 = p_5 = p_6 = 0.7$ MPa

Condenser pressure, $p_3 = p_4 = 0.008$ MPa

Isentropic efficiency, $\eta_T = 0.85$

Power output plant, $P = 105$ MW

To find

(i) Thermal efficiency of the cycle.

(ii) Mass flow rate of steam entering the first turbine stage.

Assumptions

(i) Each component in the cycle is analysed as a control volume at steady state.

(ii) All processes of the working fluid are internally reversible except expansion through the turbines.

(iii) The turbines, pumps and feed-water heater operate adiabatically.

(iv) Kinetic and potential energy effects are negligible.

(v) Saturated liquid leave the feed water heater and condenser at their pressures.

Properties of steam From Mollier chart

State 1: Superheated steam at turbine inlet

$p_1 = 8$ MPa

$T_1 = 500°C$

$h_1 = 3400$ kJ/kg

State 2: Saturated steam;

$p_2 = 0.7$ MPa

$h_{2s} = 2750$ kJ/kg

State 3: Wet steam;

$p_3 = 0.008$ MPa $= 8$ kPa

$h_{3s} = 2150$ kJ/kg

State 4: Saturated liquid; (from steam tables);

$p_4 = 0.008$ MPa $= 8$ kPa

$h_4 = h_{f_4} = 173.88$ kJ/kg

$v_{f_4} = 0.001008$ m^3/kg

State 5: Compressed liquid;

$p_5 = 0.7$ MPa $= 700$ kPa

State 6: Saturated liquid (from steam tables);

$p_6 = 0.7$ MPa;

$v_{f_6} = 0.001080$ m^3/kg

$h_6 = h_{f_6} = 697.23$ kJ/kg

State 7: Compressed liquid

$p_7 = 8$ MPa $= 8000$ kPa

Schematic with given data

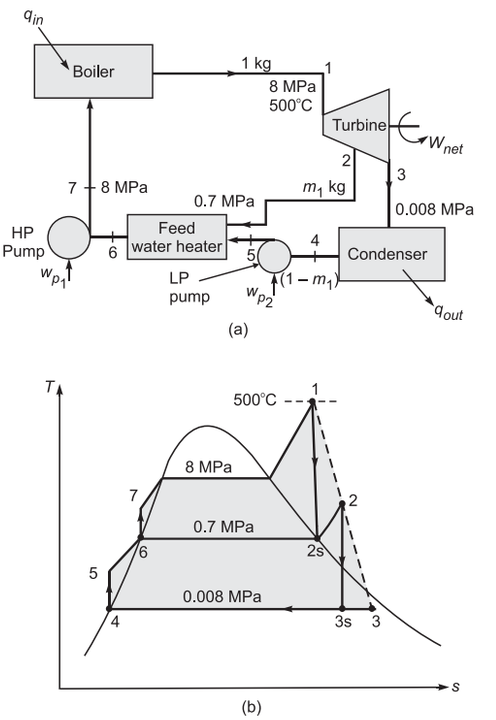

Fig. 12.39 *Schematic and T–s diagram for Regenerative Rankine cycle of Ex. 12.20*

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Analysis The high-pressure pump work input per kg of steam

$$w_{p_1} = v_{f@\,0.7\,\text{MPa}} \times (p_1 - p_6)$$
$$= 0.001080\,(8 - 0.7) \times 10^3$$
$$= 7.884\ \text{kJ/kg}$$

Specific enthalpy at the state 7;

$$h_7 = h_6 + w_{p_2} = 697.22 + 7.884$$
$$= 705.1\ \text{kJ/kg}$$

The low-pressure pump work input per kg of steam

$$w_{p_2} = v_{f@\,0.008\,\text{MPa}} \times (p_5 - p_4)$$
$$= 0.001008 \times (0.7 - 0.008) \times 10^3$$
$$= 0.697\ \text{kJ/kg}$$

Specific enthalpy at the state 5;

$$h_5 = h_4 + w_{p_2} = 173.88 + 0.697$$
$$= 174.6\ \text{kJ/kg}$$

The isentropic efficiency of first stage turbine

$$\eta_{T_1} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

or $$h_2 = h_1 - \eta_{T_1}(h_1 - h_{2s})$$
$$= 3400 - 0.85 \times (3400 - 2750)$$
$$= 2847.5\ \text{kJ/kg}$$

The isentropic efficiency of second stage turbine

$$\eta_{T_2} = \frac{h_2 - h_3}{h_2 - h_{3s}}$$

or $$h_3 = h_2 - \eta_{T_2}(h_2 - h_{3s})$$
$$= 2847.5 - 0.85 \times (2847.5 - 2150)$$
$$= 2254.62\ \text{kJ/kg}$$

The mass of steam m_1 extracted from turbine at 5 bar can be obtained by using Eq. (12.35)

$$m_1 = \frac{h_6 - h_5}{h_2 - h_5} = \frac{697.22 - 174.1}{2847.5 - 174.1}$$
$$= 0.195\,\text{kg}$$

Work done by first-stage turbine

$$w_{T_1} = h_1 - h_2 = 3400 - 2847.5$$
$$= 552.5\ \text{kJ/kg}$$

Work done by second-stage turbine

$$w_{T_2} = (1 - m_1)\,(h_2 - h_3)$$
$$= (1 - 0.195) \times (2845.5 - 2254.62)$$
$$= 479\ \text{kJ/kg}$$

Total turbine work per kg of steam

$$w_T = w_{T_1} + w_{T_2} = 552.5 + 479$$
$$= 1029.77\ \text{kJ/kg}$$

Total pump work input per kg of steam

$$w_p = w_{p_1} + (1 - m_1)\,w_{p_2}$$
$$= 7.884 + (1 - 0.195) \times 0.697$$
$$= 8.4\ \text{kJ/kg}$$

Net work developed in the cycle

$$w_{net} = w_T - w_p = 1029.77 - 8.4$$
$$= 1021.37\ \text{kJ/kg}$$

Heat supplied to the steam in the boiler

$$q_{in} = h_1 - h_7 = 3400 - 705.1$$
$$= 2694.9\ \text{kJ/kg}$$

(i) Thermal efficiency of the regenerative cycle:

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{1021.37}{2694.9} = 0.378 \quad \text{or} \quad \mathbf{37.8\%}$$

(ii) The mass flow rate of steam entering the first turbine:

$$\dot{m}_s = \frac{\text{Net power output}}{\text{Net workdone per kg of steam}} = \frac{P}{w_{net}}$$
$$= \frac{105 \times 10^3}{1021.37} = 102.82\ \text{kg/s}$$

or $\mathbf{3.70 \times 10^5\,kg/h}$

Example 12.21 *In a steam power plant, the condition of steam at inlet to turbine is 20 bar and 300°C and the condenser pressure is 10 kPa. Two feed water heaters operate at 5 bar and 1 bar. By neglecting the pump work, determine*

(a) The quality of steam at turbine exhaust,
(b) Masses of steam bled off at each pressure per kg of steam entering the turbine,
(c) Net work done per kg of steam flow,
(d) Thermal efficiency of the cycle, and
(e) Specific steam consumption.

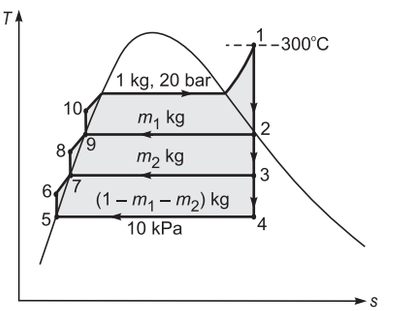

Fig. 12.40 *T-s diagram for Ex. 12.21*

Solution

Given A regenerative vapour power cycle with feed-water heater:

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First stage turbine inlet, $p_1 = 20$ bar
$T_1 = 300°C$
No. of feed water heaters $= 2$
Feed-water heater 1 pressure, $p_2 = p_8 = p_9 = 5$ bar
Feed-water heater 2 pressure, $p_3 = p_6 = p_7 = 1$ bar
Condenser pressure, $p_4 = p_5 = 10$ kPa

To find

(i) The quality of steam at turbine exhaust.
(ii) Masses of steam bled off at each pressure per kg of steam entering the turbine.
(iii) Net work done per kg of steam flow.
(iv) Thermal efficiency of the cycle.
(v) Specific steam consumption.

Properties at principal states From Mollier diagram:

State 1: Superheated steam at turbine inlet

$p_1 = 20$ bar
$T_1 = 300°C$
$h_1 = 3025$ kJ/kg
$s_1 = 6.7663$ kJ/kg·K

State 2: Wet steam

$p_2 = 5$ bar
$x_2 = 0.99$
$h_2 = 2725$ kJ/kg
$s_2 = s_1$

State 3: Wet steam;

$p_3 = 1$ bar
$x_3 = 0.92$
$h_3 = 2460$ kJ/kg
$s_3 = s_1$

State 4: Wet steam;

$p_4 = 0.1$ bar
$x_4 = 0.815$
$h_4 = 2140$ kJ/kg
$s_4 = s_1$

State 5: Saturated liquid; (from steam tables);

$p_5 = 0.1$ bar
$h_5 = h_{f_5} = 191.83$ kJ/kg

State 6: Compressed liquid;

$p_6 = 1$ bar $= 100$ kPa
$h_6 = h_5 = 191.83$ kJ/kg
(without pump work)

State 7: Saturated liquid (from steam tables);

$p_7 = 1$ bar
$h_7 = h_{f_7} = 417.5$ kJ/kg

State 8: Compressed liquid;

$p_8 = 5$ bar $= 500$ kPa,
$h_8 = h_7 = 417.5$ kJ/kg
(without pump work)

State 9: Saturated liquid (from steam tables);

$p_9 = 5$ bar
$h_9 = h_{f_9} = 641.21$ kJ/kg

State 10: Compressed liquid;

$p_{10} = 20$ bar $= 2000$ kPa
$h_{10} = h_9 = 641.21$ kJ/kg
(without pump work)

Analysis

(i) *The quality of steam* at the state 4 (turbine exhaust) is

$$x_4 = \mathbf{0.815}$$

(ii) *Masses of steam bled off at each pressure per kg of steam entering the turbine*

The masses of steam m_1 and m_2 extracted from turbine at pressures 5 bar and 1 bar, respectively;

$$m_1 = \frac{h_9 - h_8}{h_2 - h_8} = \frac{641.21 - 417.5}{2725 - 417.5}$$

$$= \mathbf{0.097\ kg/kg\ of\ steam}$$

and $$m_2 = (1 - m_1)\frac{h_7 - h_6}{h_3 - h_6}$$

$$= (1 - 0.097) \times \left(\frac{417.5 - 191.83}{2460 - 191.83}\right)$$

$$= \mathbf{0.09\ kg/kg\ of\ steam}$$

(iii) *Net work done per kg of steam flow*

The work done by first-stage turbine

$$w_{T_1} = h_1 - h_2 = 3025 - 2725$$
$$= 300 \text{ kJ/kg}$$

The work done by second-stage turbine

$$w_{T_2} = (1 - m_1)(h_2 - h_3)$$
$$= (1 - 0.097) \times (2725 - 2460)$$
$$= 239.19 \text{ kJ/kg}$$

The work done by third-stage turbine

$$w_{T_3} = (1 - m_1 - m_2)(h_3 - h_4)$$
$$= (1 - 0.097 - 0.09) \times (2460 - 2140)$$
$$= 260 \text{ kJ/kg}$$

Total turbine work per kg of steam

$$w_T = w_{T_1} + w_{T_2} + w_{T_2}$$

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$= 300 + 239.19 + 260$

$= 799.22$ kJ/kg

In absence of pump work, the net work developed in the cycle

$$w_{net} = w_T = \mathbf{799.22\ kJ/kg}$$

The heat supplied to the steam in the boiler

$$q_{in} = h_1 - h_{10}$$

$= 3025 - 641.21 = 2381.75$ kJ/kg

(iv) Thermal efficiency of the cycle,

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{799.22}{2381.75}$$

$= 0.3354$ or **33.54%**

(v) Specific steam consumption;

$$ssc = \frac{3600}{w_{net}} = \frac{3600}{799.22} = \mathbf{4.5\ kg/kWh}$$

Example 12.22 *Consider a reheat regenerative vapour power cycle with a feed-water heater. The steam enter the first turbine at 15 MPa, 600°C and expands to 2 MPa. Then the steam is reheated to 600°C at the same pressure. The steam for feed-water heater is extracted from the low-pressure turbine at the pressure of 0.5 MPa and the remaining steam is further expanded to a condenser pressure of 10 kPa. The working fluid experiences no irreversibities, passes through turbines, pumps, boiler, reheater condenser. Determine*

(a) The fraction of steam extracted from the turbine for feed-water heater,

(b) Thermal efficiency of the cycle, and

(c) Mass flow rate of steam in kg/h, if cycle produces 120 MW.

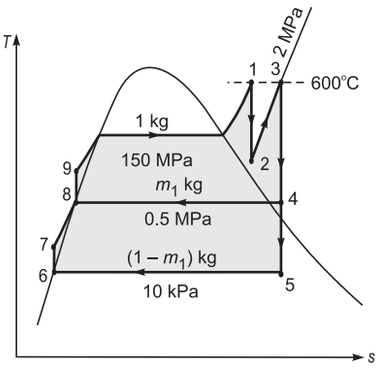

Fig. 12.41 *T-s diagram for Ex. 12.22*

Solution

Given A reheat–regenerative ideal vapour power cycle with feed-water heater with

First-stage turbine inlet = 15 MPa, 600°C

Intermediate pressures = 2 MPa and 0.5 MPa

Reheating = 2 MPa, 600°C

Condenser pressure = 10 kPa

Power output, P = 120 MW

To find

(i) Fraction of steam extracted from the turbine for feed water heater,

(ii) Thermal efficiency of the cycle,

(iii) Mass flow rate of steam in the cycle for network output of 120 MW.

Assumptions

(i) Each component in the cycle is analysed as a control volume at steady state.

(ii) All processes of the working fluid are reversible.

(iii) Turbines, pumps and feedwater heaters operate adiabatically.

(iv) No pressure drop in boiler, condenser and feed water heaters.

(v) Feed water heater is direct contact type.

(vi) The working fluid leaves the condenser and feedwater heaters as saturate liquid.

(vii) Kinetic and potential energy effects are negligible.

(viii) 1 kg of steam flow through the boiler and first-stage turbine.

Properties at principal states From Mollier diagram

State 1: Superheated steam at turbine inlet

$p_1 = 15$ MPa $= 150$ bar

$T_1 = 600$°C

$h_1 = 3580$ kJ/kg

State 2: Superheated steam

$p_2 = 2$ MPa $= 20$ bar

$s_2 = s_1$

$h_2 = 2985$ kJ/kg

State 3: Superheated steam

$p_3 = 2$ MPa $= 20$ bar

$T_3 = 600$°C

$h_3 = 3695$ kJ/kg

State 4: Superheated steam;

$p_4 = 0.5$ MPa $= 5$ bar

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$s_4 = s_3$

$h_4 = 3225$ kJ/kg

State 5: Wet steam;

$p_5 = 10$ kPa $= 0.1$ bar

$x_5 = 0.94$

$h_5 = 2440$ kJ/kg

State 6: Saturated liquid; (from steam tables);

$p_6 = 0.1$ bar

$v_{f_6} = 0.001010$ m^3/kg

$h_6 = h_{f_6} = 191.83$ kJ/kg

State 7: Compressed liquid;

$p_7 = 5$ bar $= 500$ kPa

State 8: Saturated liquid (from steam tables);

$p_8 = 5$ bar

$v_{f_8} = 0.001093$ m^3/kg,

$h_8 = h_{f_8} = 641.21$ kJ/kg

State 9: Compressed liquid;

$p_9 = 15$ MPa

Analysis The high-pressure pump work input per kg of steam

$$w_{p_1} = v_{f\,@\,5\,\text{bar}} \times (p_9 - p_8)$$
$$= 0.001094\,(15 - 0.5) \times 10^3$$
$$= 15.86 \text{ kJ/kg}$$

Specific enthalpy at the state 9;

$$h_9 = h_8 + w_{p_2} = 641.21 + 15.86$$
$$= 657 \text{ kJ/kg}$$

The low pressure pump work input per kg of steam

$$w_{p_2} = v_{f\,@\,10\,\text{kPa}} \times (p_5 - p_4)$$
$$= 0.001010 \times (5 - 0.1) \times 10^2$$
$$= 0.5 \text{ kJ/kg}$$

Specific enthalpy at the state 7;

$$h_7 = h_6 + w_{p_2}$$
$$= 191.83 + 0.5 = 192.32 \text{ kJ/kg}$$

(i) *The fraction of steam extracted from the turbine for feed-water heater*

The mass of steam m_1 extracted from the turbine at 5 bar for regeneration can be obtained by making mass balance on feed-water heater;

$(1 - m_1)\, h_7 + m_1\, h_4 = h_8$

$$m_1 = \frac{h_8 - h_7}{h_4 - h_7} = \frac{641.21 - 192.32}{3225 - 192.32}$$
$$= \mathbf{0.148 \text{ kg}}$$

Therefore, 0.148 kg of steam is extracted from the turbine for each kg of steam entering the turbine.

The total pump work input

$$w_p = w_{p_1} + (1 - m_1)\, w_{p_2}$$
$$= 15.86 + (1 - 0.148) \times 0.5$$
$$= 16.28 \text{ kJ/kg}$$

Turbine work output,

$$w_T = h_1 - h_2 + h_3 - h_4 + (1 - m_1)\,(h_4 - h_5)$$
$$= 3580 - 2985 + 3695 - 3225$$
$$+ (1 - 0.148) \times (3225 - 2440)$$
$$= 1733.82 \text{ kJ/kg}$$

Net work done per kg of steam

$$w_{net} = w_T - w_p = 1733.82 - 16.28$$
$$= 1717.54 \text{ kJ/kg}$$

The heat supplied in the boiler per kg of steam

$$q_{in} = h_1 - h_9 + h_3 - h_2$$
$$= 3580 - 647 + 3695 - 2985$$
$$= 3643 \text{ kJ/kg}$$

(ii) *Thermal efficiency* of the cycle

$$\eta_{th,\,reg} = \frac{w_{net}}{q_{in}} = \frac{1717.54}{3643}$$
$$= 0.471 \quad \text{or} \quad \mathbf{47.1\,\%}$$

(iii) *Mass flow rate of steam in kg/h*

$$\dot{m}_s = \frac{P}{w_{net}} = \frac{120 \times 10^3}{1717.54}$$
$$= 69.867 \text{ kg/s} \quad \text{or} \quad \mathbf{251.52 \text{ kg/h}}$$

12.12.4 Regeneration with Closed Feedwater Heater

A closed feed-water heater is an *indirect contact-type* feed water heater, usually a shell and tube type heat exchanger, in which heat is transferred from extracted steam to feed-water without mixing of two fluid streams. Thus the two fluid streams can be kept at different pressures. Figure 12.42 shows two arrangement for removing the condensate from the closed feedwater heater. In Fig. 12.42(a), the condensate is pumped forward to a feed-water line. In Fig. 12.42(b), the condensate is allowed to pass another feed-water heater or condenser through a device called *a steam trap*. A trap is a type of valve that allows only liquid to be throttled to a lower pressure, but it traps the vapour.

A regenerative vapour power cycle with one closed feed-water heater with the condensate

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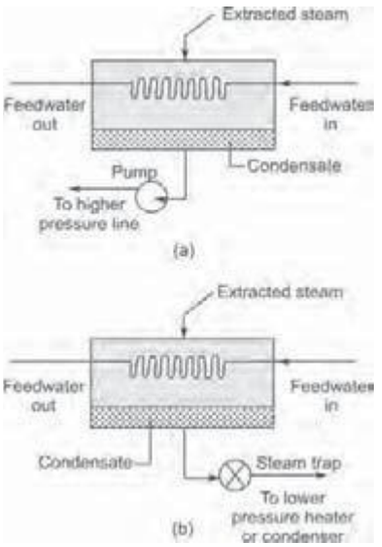

Fig. 12.42 *Arrangements of closed feed-water heaters*

trapped into the condenser is shown is Fig. 12.43. In this cycle, the working fluid passes isentropically through the turbine stages and pumps, and without any pressure drop in the components. As shown with the help of a *T–s* diagram, the total steam expands through first stage turbine from state 1 to state 2. At state 2, a part of steam (m_1, kg) is extracted and is supplied to closed feed-water heater, where it condenses, on outside of tubes, carrying the feed-water. The saturated liquid at extraction pressure exits the feed-water heater at state 5 and is routed to the condenser through a trap, where it is mixed with the condensate of steam passing the second-stage turbine. In the trap, the steam is throttled from state 5 to state 6. It is an irreversible process, and thus shown by dotted line. Then total condensate as saturate liquid at state 7 is pumped to boiler pressure and enters the feedwater at state 8. The temperature of feedwater in the heater is raised to state 4. The cycle completes as working fluid enters the boiler and heated at constant pressure from state 4 to state 1.

Cycle Analysis Let 1 kg of steam leaving the boiler enters the first stage turbine. Figure 12.43. is labelled with the fraction of the steam at various states. The mass fraction m_1, extracted from the

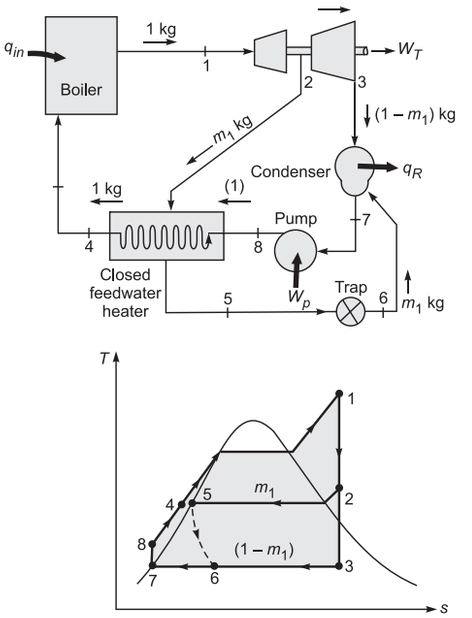

Fig. 12.43 *Regenerative vapour power cycle with one closed feedwater heater*

turbine at intermediate pressure can be determined by applying mass and energy balance to the closed water heater.

$$(m_1 \text{ kg})h_2 + (1 \text{ kg})h_8 = (1 \text{ kg})h_4 + (m_1 \text{ kg})h_5$$

or
$$m_1 = \frac{h_4 - h_8}{h_2 - h_5} \text{ kg/kg of steam} \qquad ...(12.50)$$

Remark With the use of closed feed water heater, separate pumps to handle the feedwater for each heater are not required. But the main drawback of such heaters, that they do not heat the feedwater very close to the saturation temperature as open heaters do.

Example 12.23 *The steam at 30 bar, 400°C is supplied to a turbine and is expanded isentropically to a pressure of 3 bar, where some steam is extracted for a surface heater, in which the feed-water is heated to 130°C. The remaining steam from feed-water is cooled in a drain cooler to 27°C. The feed-water before entering the feed heater is used as coolant in the drain cooler. The cooled drain water is then mixed with condensate at 0.04 bar.*

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Determine

(a) Mass of steam used for feedwater heating per kg of steam entering the turbine.

(b) Thermal efficiency of the cycle.

Solution

Given A regenerative vapour power cycle with one closed feed-water heater with operating conditions

First stage turbine inlet = 30 bar, 400°C

Extracted steam pressure = 3 bar

Feedwater temperature leaving the heater = 130°C

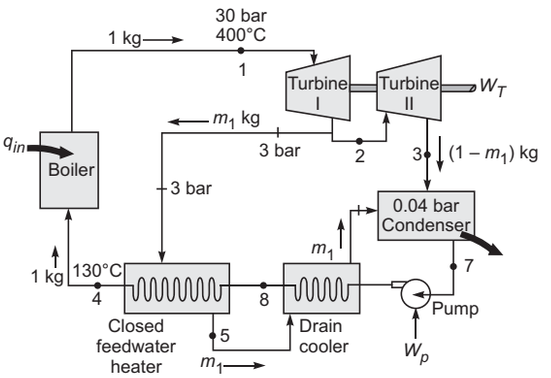
The temperature of feedwater in drain cooler = 27°C

To find

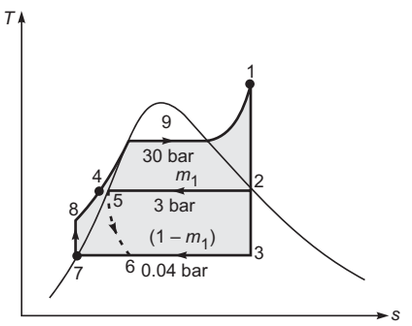
(i) Mass of steam extracted for feed-water heating per kg of steam.

(ii) Thermal efficiency of the cycle.

Schematic with given data



(a) Schematic



(b) T–s diagram

Fig. 12.44

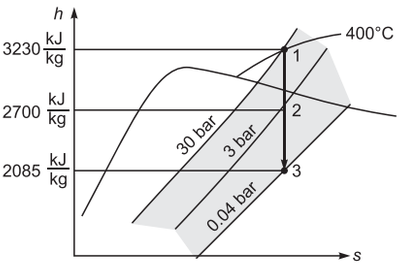


Fig 12.45 *h–s diagram to the obtain the specific enthalpies of steam during isentropic expansion through turbines*

Assumptions

(i) Each component in the cycle is analysis as a control volume at steady state.

(ii) Turbines, pump and feed-water heaters operate adiabatically.

(iii) All processes of working fluid are reversible.

(iv) The extracted steam leaving the feed-water heater as a saturate liquid.

(v) In drain cooler, both fluid get equilibrium temperature of 27°C.

(vi) 1 kg mass of steam flowing through the boiler and first stage turbine.

(vii) Kinetic and potential energy effects are negligible.

(viii) The specific heat of water as 4.187 kJ/kg·K.

Analysis Referring schematic and *T–s* diagram of Fig. 12.44.

State 7: Saturated liquid at

$$p_7 = 0.04 \text{ bar (steam table)}$$
$$v_{f_7} = 0.001004 \text{ m}^3/\text{kg}$$
$$h_7 = 121.44 \text{ kJ/kg}$$

State 8: Compressed liquid at

$$p_8 = 30 \text{ bar}, T_8 = 27°\text{C}$$

Work input to pump

$$w_p = v_{f_7}(p_8 - p_7)$$
$$= 0.001004 \times (30 \times 10^2 - 0.04 \times 10^2)$$
$$= 3.0 \text{ kJ/kg}$$

The specific enthalpy of water at state 8

$$h_8 = h_7 + w_p = 121.44 + 3.0$$
$$= 124.44 \text{ kJ/kg}$$

State 1: Superheated steam at

$$p_1 = 30 \text{ bar}, T_1 = 400°\text{C}$$
(From Mollier diagram)
$$h_1 = 3230 \text{ kJ/kg}, s_1 = 6.868 \text{ kJ/kg·K}$$

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State 2: Wet steam at

$p_2 = 3$ bar

$h_2 = 2700$ kJ/kg, $s_2 = 6.868$ kJ/kg·K

State 3: Wet steam at

$p_3 = 0.04$ bar

$h_3 = 2085$ kJ/kg, $s_3 = 6.868$ kJ/kg·K

State 4: Feed water at

$p_4 = 30$ bar, $T_4 = 130°C$

State 5: Saturated liquid,

$p_5 = p_4 = 3$ bar,

$h_5 = h_{f_5} = 561.45$ kJ/kg

(i) *Mass of steam m_1 extracted after first stage turbine expansion*

Energy balance on closed feedwater heated and drain cooler combined;

Heat lost by steam = Heat gain by feedwater

$(m_1 \text{ kg})(h_2 - h_5) = (1 \text{ kg})\, C_{pw}\, (T_4 - T_8)$

or $$m_1 = \frac{1 \times 4.187 \times (130 - 27)}{2700 - 561.45}$$

$= 0.20$ kg/kg of steam

(ii) *Thermal efficiency of the cycle*

Total work done per kg of steam turbines

$w_T = (h_1 - h_2) + (1 - m_1)(h_2 - h_3)$

$= (3230 - 2700) + (1 - 0.02) \times (2700 - 2085)$

$= 530 + 492 = 1022$ kJ/kg

Net work per kg of steam in the cycle

$w_{net} = w_T - w_P = 1022 - 3.0 = 1019$ kJ/kg

Heat supplied per kg of steam in the cycle

$q_{in} = h_1 - h_4 = 3230 - 4.187 \times (130.0)$

$= 2685.7$ kJ/kg

Thermal efficiency $\eta_{th} = \dfrac{w_{net}}{q_{in}} = \dfrac{1019}{2685.7}$

$= 0.379 =$ **37.9%**

Example 12.24 *A regenerative vapour power cycle uses two closed feed-water heaters. The steam is supplied to the turbine at 40 bar, 500°C and is exhausted to condenser at 0.035 bar. The intermediate bleed pressures are obtained such that the saturation temperature intervals are approximately equal, giving pressures of 10 and 1.1 bar.*

Determine the amount of steam extracted at each stage, the work output of the plant per kilogram of the boiler stream and the cycle efficiency of the plant. Assume that the condensed steam from first feed-water heater is throttled to second water heater and from second water heater to condenser.

Solution

Given A regenerative vapour power cycle with two closed feed-water heaters with operating parameters.

First stage turbine inlet = 40 bar, 500°C

Intermediate pressures 10 bar and 1.1 bar

Condenser pressure 0.035 bar

To find

(i) The amount of steam extracted per kg of steam for two closed water heaters.

(ii) The work output of the plant per kg of steam.

(iii) The thermal efficiency of the cycle.

Assumptions

(i) Each component in the cycle is analysed as a control volume at steady state.

(ii) Turbines, pump and feedwater heaters operate adiabatically.

(iii) All processes of working fluid are reversible.

(iv) The extracted steam leaving the feedwater heater as a saturate liquid.

(v) 1 kg mass of steam flowing through the boiler and first stage turbine.

(vi) Kinetic and potential energy effects are negligible.

Analysis Referring the Mollier diagram, we get specific enthalpies of steam:

At State 1: Superheated steam

$p_1 = 40$ bar, $T_1 = 500°C$

$h_1 = 3450$ kJ/kg, $s_1 = 7.08$ kJ/kg·K

State 2: Superheated steam

$p_2 = 10$ bar

$h_2 = 3034$ kJ/kg, $s_2 = s_1$

State 3: Wet steam

$p_3 = 1.1$ bar

$h_3 = 2588$ kJ/kg, $s_3 = s_1$

State 4: Wet steam

$p_4 = 0.035$ bar

$h_4 = 2122$ kJ/kg, $s_4 = s_1$

State 5: Saturated liquid at $p_5 = 0.035$ bar (From steam table)

$h_{f_5} = 112$ kJ/kg,

$v_{f_5} = 0.0010035$ m^3/kg

Pump work input

$w_p = v_{f_5}(p_6 - p_5)$

$= 0.0010035 \times (40 - 0.035) \times 10^2$

$= 4$ kJ/kg

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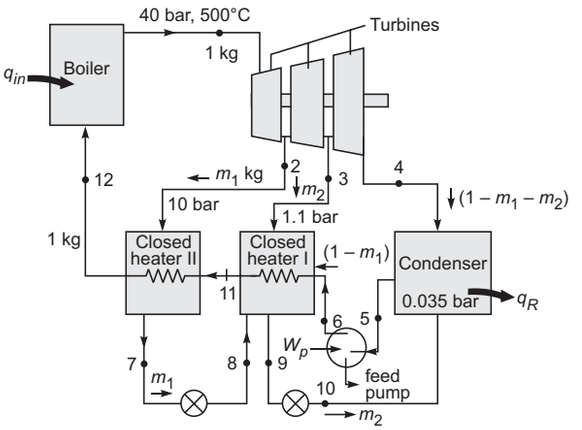
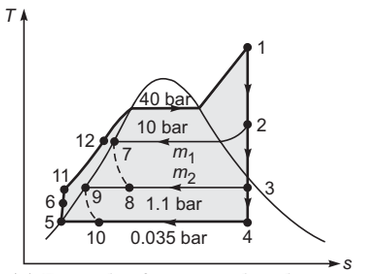
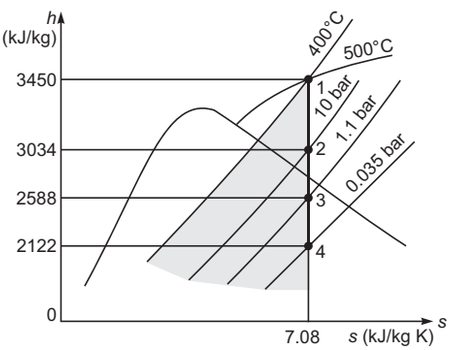




(a) Example of a power plant layout.



(b) h–s diagram for determination of specific entgalpies durning isentropic expansions.

Fig. 12.46 *Schematic with given data*

State 6: Compressed liquid

$p_6 = 40$ bar

$h_6 = h_5 + w_p$

$= 112 + 4 = 116$ kJ/kg

State 7: Saturated liquid,

$p_7 = 10$ bar

$h_7 = 762.79$ kJ/kg

State 8: Liquid at $p_8 = 1.1$ bar, throttled from state 7, thus,

$h_8 = h_7 = 762.79$ kJ/kg

State 9: Saturated liquid,

$p_9 = 1.1$ bar

$h_9 = 428.2$ kJ/kg

State 10: Liquid at $p_{10} = 0.035$ bar, throttled from state 9, thus,

$h_{10} = h_9 = 428.2$ kJ/kg

State 11: Heated liquid at $p_{11} = 40$ bar, with steam at 1.1 bar, thus

$h_{11} = h_9 = 428.2$ kJ/kg

State 12: Heated liquid at $p_{12} = 40$ bar with steam at 10 bar

$h_{12} = h_7 = 762.79$ kJ/kg

(i) Applying mass and energy balance to second closed feed-water heater II,

$\Sigma m_i h_i = \Sigma m_e h_e$

$h_{11} + m_1 h_2 = h_{12} + m_1 h_7$

$$m_1 = \frac{h_{12} - h_{11}}{h_2 - h_7} = \frac{762.79 - 428.2}{3034 - 762.79}$$

$= \mathbf{0.147\ kg/kg\ of\ steam}$

Applying mass and energy balance to, first closed feed-water heated,

$\Sigma m_i h_i = \Sigma m_e h_e$

$m_2 h_3 + (1 - m_1)\, h_6 + m_1 h_8 = h_{11} + m_2 h_9$

$2588\, m_2 + (1 - 0.147) \times 116 + 0147 \times 762.79$

$= 428.2 + 428.2\, m_2$

It gives $m_2 = \mathbf{0.10\ kg/kg\ of\ steam}$

(ii) Heat supplied to 1 kg steam in cycle

$q_{in} = h_1 - h_{12} = 3450 - 762.79$

$= 2687.21$ kJ/kg

Net work done per kg of steam in cycle

$w_{net} = w_T - w_P$

$= (h_1 - h_2) + (1 - m_1)\,(h_2 - h_3)$

$+ (1 - m_1 - m_2)\,(h_3 - h_4) - w_p$

$= (3450 - 3034) + (1 - 0.147)$

$(3034 - 2588) + (1 - 0.147 - 0.1)$

$\times (2588 - 2122) - 4$

$= \mathbf{1143.32\ kJ/kg}$

(iii) Thermal efficiency of the cycle

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{1143.32}{2687.21}$$

$= 0.4254 = \mathbf{42.54\%}$

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Example 12.25 *Consider a reheat-regenerative vapour power cycle with two feed-water heaters. The steam enters the first turbine at 15 MPa, 600°C and expands to 4 MPa. Then some steam is extracted from the turbine at this pressure for closed feed-water heater and the remaining steam is reheated to 600°C at the same pressure. The extracted steam is completely condensed in the heater and is then pumped to 15 MPa before it mixes with the feed-water at 260°C at the same pressure. The steam for open feed-water heater is extracted from low pressure turbine at the pressure of 0.5 MPa and the remaining steam is further expanded to condenser pressure of 10 KPa. If the working fluid experiences no irreversibities, as it passes thought turbines, pumps, boiler, reheater condenser, determine*

(a) The fraction of steam extracted from the turbines each time.

(b) Thermal efficiency of the cycle.

(c) Mass flow rate of steam in kg/h, if cycle produces 120 MW.

Solution

Given A reheat-regenerative ideal vapour power cycle with open and closed feed-water heater with

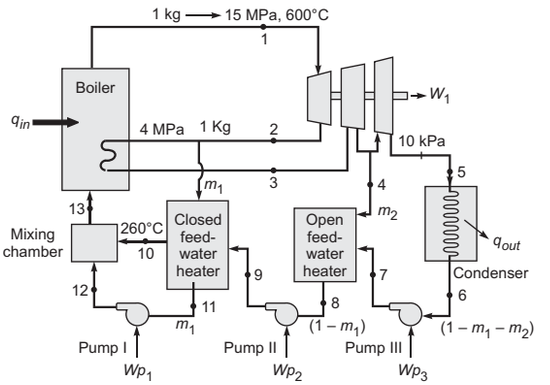
First stage turbine inlet = 15 MPa, 600°C
Intermediate pressures = 4 MPa and 0.5 MPa
Reheating = 4 MPa, 600°C
Condenser Pressure = 10 kPa.

To find

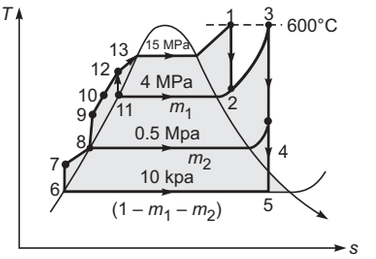
(i) Fraction of steam extracted after first and second stage expansion in the turbine.
(ii) Thermal efficiency of the cycle.
(iii) Mass flow rate of steam in the cycle for network output of 120 MW.

Assumptions

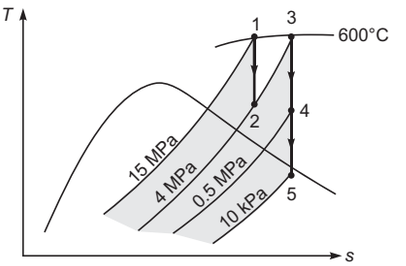
(i) Each component in the cycle is analysed as a control volume at steady state.
(ii) All process of the working fluid are reversible.
(iii) Turbines, pumps and feedwater heaters operate adiabatically.
(iv) No pressure drop in boiler, condenser and feedwater heaters.
(v) The working fluid leaves the condenser and feedwater heaters as saturated liquid.
(vi) Kinetic and potential energy effects are negligible.



(a) Schematic



(b) T–s diagram



(c) h–s diagram for determination of specific enthalpies

Fig. 12.47

(vii) 1 kg of steam flow through the boiler and first stage turbine.

Analysis

The specific enthalpies at various states from steam tables, and Mollier diagram.

h_{11} = 1087.31 kJ/kg $v_{f_{11}}$ = 0.001252 m^3/kg
h_1 = 3582.3 kJ/kg h_2 = 3154.3 kJ/kg
h_3 = 3674.4 kJ/kg h_4 = 3014.3 kJ/kg
h_5 = 2335.8 kJ/kg h_6 = 191.83 kJ/kg
h_8 = 640.23kJ/kg v_{f_8} = 0.001093 kJ/kg

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$h_{10} = C_{pw}\, T_{10} = 1134.35$ kJ/kg (at 260°C)

$v_{f_6} = 0.001080$.kJ/kg

The work input per kg of steam in pump 1

$$w_{p_1} = v_{f_{12}} (p_{13} - p_{12})$$
$$= 1.001252 \times (15 \times 10^3 - 4 \times 10^3)$$
$$= 13.772 \text{ kJ/kg}$$

The specific enthalpy of water at state 12

$$h_{12} = h_{11} + w_{p_1} = 1087.31 + 13.772$$
$$= 1101.08 \text{ kJ/kg}$$

The pump 2, work input $w_{p_2} = v_{f_8} (p_{13} - p_8)$

$$= 0.001093 \times (15 \times 10^3 - 0.5 \times 10^3)$$
$$= 15.85 \text{ kJ/kg}$$

The specific enthalpy of water at state 9

$$h_9 = h_8 + w_{p_2} = 640.23 + 15.85$$
$$= 656.08 \text{ kJ/kg}$$

The pump III. work input

$$w_{p_3} = v_{f_6} (p_8 - p_6)$$
$$= 0.001080 \times (0.5 \times 10^3 - 10)$$
$$= 0.53 \text{ kJ/kg}$$

The specific enthalpy of water at state 7,

$$h_7 = h_6 + w_{p_3}$$
$$= 191.83 + 0.53 = 192.36 \text{ kJ/kg}$$

(i) Let m_1 fraction of steam extracted at 4 MPa and m_2 fraction of steam extracted at 0.5 MPa.

The mass and energy balance on the closed feedwater heater

$$\Sigma m_i h_i = \Sigma m_e h_e$$
$$m_1 h_2 + (1 - m_1)\, h_9 = m_1 h_{11} + (1 - m_1)\, h_{10}$$

or $m_1 (h_2 - h_9 - h_{11} + h_{10}) = h_{10} - h_9$

With numerical values;

$$m_1 = \frac{1134.35 - 656.08}{3154.3 - 656.08 - 1087.31 + 1134.35}$$

or $m_1 =$ **0.188 kg/kg of steam**

(ii) Energy and mass balance on open feedwater heater

$$\Sigma m_i h_i = \Sigma m_e h_e$$
$$m_2 h_4 + (1 - m_1 - m_2)\, h_7 = (1 - m_1)\, h_8$$
$$3014.3\, m_2 + (1 - 0.188 - m_2) \times 192.36 = (1 - 0.188) \times 640.23$$

or $3014.3\, m_2 + 156.09 - 192.36\, m_2 = 519.47$

or $m_2 = \dfrac{363.66}{2821.94} =$ **0.129 kg/kg of steam**

The specific enthalpy of water at state 13 is determined by mass and energy balance in mixing chamber

$$m_1 h_{12} + (1 - m_1)\, h_{10} = h_{13}$$

or $h_{13} = 0.188 \times 1101.08 + (1 - 0.188) \times 1134.35$

$$= 1128.1 \text{ kJ/kg}$$

(iii) Thermal Efficiency of the cycle:

The heat supplied in the boiler

$$q_{in} = h_1 - h_{13} + (1 - m_1)\, (h_3 - h_2)$$
$$= 3582.3 - 1128.1 + (1 - 0.188) \times (3674.4 - 3154.3)$$
$$= 2876.5 \text{ kJ/kg}$$

Heat rejected in the condenser

$$q_{out} = (1 - m_1 - m_2)\, (h_5 - h_6)$$
$$= (1 - 0.188 - 0.129) \times (2335.8 - 191.83)$$
$$= 1464.33 \text{ kJ/kg}$$

Thermal efficiency of the cycle

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{1464.33}{2876.5}$$
$$= 0.491 \text{ or } \mathbf{49.1\%}$$

(iv) The mass flow rate in the cycle

Net work done per kg of steam

$$w_{net} = \eta_{th}\, q_{in} = 0.491 \times 2876.5$$
$$= 1412.17 \text{ kJ/kg}$$

The mass flow rate of steam in the cycle

$$\dot{m} = \frac{\text{Power output}}{\text{Work done/kg}} = \frac{120 \times 10^3 \text{ kJ/s}}{1412.17 \text{ kJ/kg}}$$
$$= 84.97 \text{ kg/s}$$
$$= \mathbf{30.59 \times 10^4 \text{ kg/h}}$$

12.12.5 Regeneration with Multiple Feedwater Heaters

The thermal efficiency of the regenerative vapour power cycle can be increased by incorporating several feedwater heaters at suitably intermediate pressures. However, the capital cost of plant also increases due to addition of heaters, piping, pumps and valves. Therefore, the incremental increase in thermal efficiency achieved with each additional heater must justify its added cost.

Figure 12.48 shows a schematic of vapour power cycle plant cycle with reheating and three closed feedwater hearers and one open feed-water heater. The power plants with multiple feed-water heaters must have at least one open feed-water heater, operating above atmospheric pressure for proper

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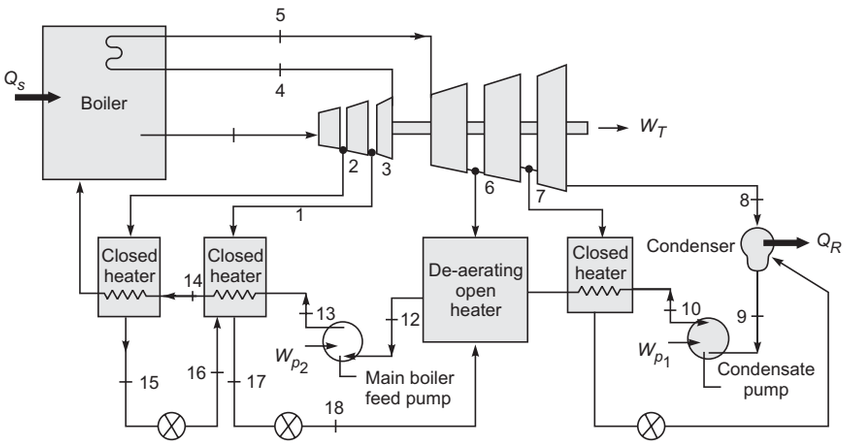

Fig. 12.48 *A power plant with reheating regeneration.*

de-aeration of air and other gases from the feed-water in order to minimise the corrosion.

Analysis of regeneration cycle with multiple feed-water heater is similar to previous article. Let 1 kg of steam enters the boiler and first stage turbine and then quantify the fraction passing through the various components. The fractions of steam extracted are determined by mass and energy balance principles.

12.12.6 Advantages and Disadvantages of Regenerative Cycle over Simple Rankine Cycle

Advantages

1. The heat is added in the boiler at a higher average temperature, thus, the heating process, tends to be reversible.
2. The variation in temperature of working fluid in the boiler is minimised, thus thermal stresses setup in the boiler are also reduced.
3. Amount of heat addition is substantial reduced, thus the efficiency of the cycle is improved.
4. The size of stage turbine is reduced, due to the reduced amount of steam passes through these turbines.
5. Due to many extractions, the turbine drainage is improved and erosion due to moisture is reduced.
6. The size of condenser is reduced.
7. In open feedwater heater, de-aeration takes place to remove the air and non-condensable-gases from the feedwater, otherwise they would lead to corrosion in the system.

Disadvantages

1. The vapour power plant becomes complicated.
2. Addition of heaters increase the cost of plant and maintenance.
3. In feedwater heaters, the length of fluid flow increases, which require more pump work.
4. The specific steam consumption increases with regeneration.

12.13 MODIFIED RANKINE CYCLE

Figure 12.49 shows the indicator diagram for modified Rankine cycle for steam engine. High-pressure steam enters the cylinder at constant pressure, it expands isentropically in the cylinder up to a lower pressure, which is called steam release point and then the steam is released at constant volume to a condenser or into atmosphere.

The expansion of steam is not taken to toe of the curve, i.e., the back (condenser) pressure or else the stroke of the engine will be very long , but the work obtained in this part is very small. Thus, the length of stroke reduces without appreciable reduction in

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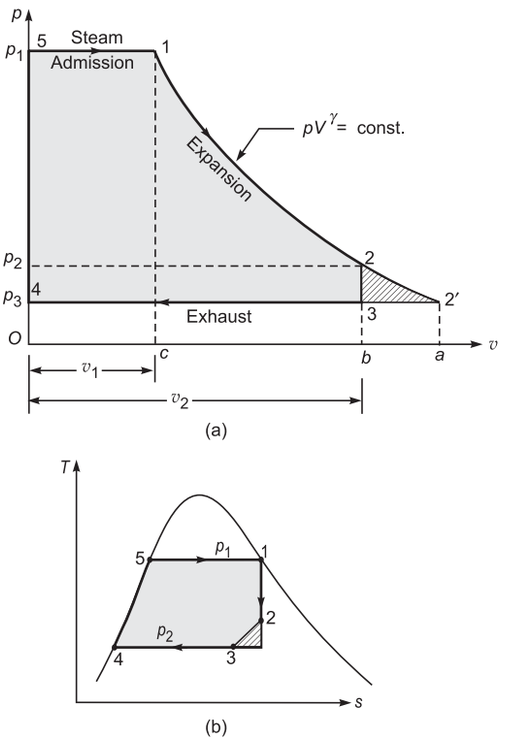

Fig. 12.49 *Modified Rankine cycle on p–V and T–s diagram*

power output of the engine. It is the modification in Rankine cycle used for steam engines.

Cycle 5–1–2–3–4–5 is modified Rankine cycle.

The sequence of operations are shown in Fig. 12.49(a).

Work done Work done by expansion of steam per cycle in modified Rankine cycle.

$$w = \text{Area of indicator diagram}$$
$$= \text{Area } 5\text{–}1\text{–}c\text{–}0\text{–}5 + \text{Area } 1\text{–}2\text{–}3\text{–}b\text{–}c\text{–}1 - \text{Area } 3\text{–}4\text{–}0\text{–}b\text{–}3$$
$$= p_1v_1 + \int pdv - p_3v_2$$

or $$w = p_1v_1 + u_1 - u_2 - p_3v_2$$

or $$w = (p_1v_1 + u_1) - (u_2 + p_2v_2) + p_2v_2 - p_3v_2$$
$$= h_1 - h_2 + v_2(p_2 - p_3)$$

Heat supplied to cycle

$$Q = h_1 - h_{f_4}$$

The efficiency of modified Rankine cycle

$$\eta = \frac{h_1 - h_2 + v_2(p_2 - p_3)}{h_1 - h_{f_4}} \quad \ldots(12.50)$$

12.14 CHARACTERISTICS OF THE WORKING FLUID IN VAPOUR POWER CYCLE

Water is the best suitable working fluid for majority of vapour power plants. Because it has following favourable characteristics:

1. It is easily available in large quantity at very low cost.
2. It is non-toxic, chemically stable and relatively poor corrosive.
3. It has a large change in specific enthalpy, when it vapourises at ordinary pressures, thus it limits the mass flow rate for a desired power output.
4. Due to very low specific volume of liquid water, the pumping work required is very less, thus, the work ratio of the cycle is quite high.
5. At low triple point temperature there is no chance of solidification of water in the cycle.
6. Good heat transfer characteristics.

Apart from certain desirable characteristics of water as a working fluid in vapour power cycle, it does not fare well for high critical temperature, controlled condenser pressure, and dry expansion, because.

(i) Low critical temperature restricts the addition of heat isothermally during vapourisation at maximum temperature.

(ii) Very high pressure at high temperature, which requires high strength of materials of components used in the plant.

(iii) During condensation of steam a large change in specific volume experiences a very low pressure in the condenser. The air leakage

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problem at the joints in the condenser are very commonly encountered.

(iv) The excessive formation of moisture during expansion in the turbine is undesirable. The moisture removal requires reheating of wet steam at number of stages in the turbine. Thus the cost of heat addition increases.

12.15 COGENERATION

In all the thermal cycles discussed so far, the effective conversion of heat energy into network is considered valuable. Therefore, the various methods are used to improve the thermal efficiency of the system. Typically, the thermal systems are used by almost every industry, consist of *a power generating unit* and *a heat recovery unit*. The power generating unit develops electrical power, while the heat recovery unit utilizes the remaining heat effectively as a *process heat*. Some industries heavily rely on process heat are chemical, paper, pulp, oil production, refineries, food processing, textile and steel making, etc. The process heat in these industries is usually supplied by steam at 5 to 7 bar and 150 to 200°C.

Cogeneration is a method, which is used to improve the energy resource utilization. The cogeneration method, produces the power as well as the required heat transfer (process heat) for some devices.

The main aim of the cogeneration is to produce power and process heat simultaneously, using an integrated system at a lower cost than that would be required to develop them individually.

Either a steam power cycle or gas power cycle or even a combined cycle can be used as a power cycle for cogeneration.

A schematic of a cogeneration system using vapour power cycle is shown in Fig. 12.50. The m_1 fraction of the steam supplied to turbine is extracted at an intermediate point 2 and diverted to some process that requires the steam at this condition. The remaining fraction $(1 - m_1)$ of the steam in the turbine expands to condenser pressure

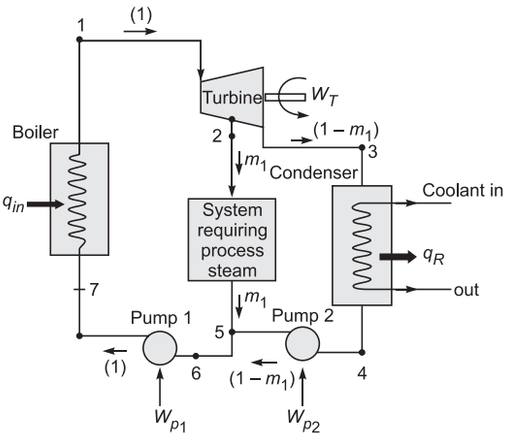

Fig. 12.50 *Schematic of a cogeneration system in which process steam is bled from the turbine*

at state 3. The fraction m_1 of steam is adjusted such that working fluid leaves the process heating section as a saturated liquid. This fraction rejoins the condensed fraction $(1 - m_1)$ and the combined steam is returned to the boiler through pump for next cycle. When no process steam is required, all of the steam generated by the boiler is allowed to expand through the turbine.

With the cogeneration, a large fraction of the energy transferred to steam in the boiler is utilized as either process heat and or electric power. The utilization fraction ε_u for a cogeneration plant is

$$\varepsilon_u = \frac{\text{Network output + Process heat diverted}}{\text{Total heat input}}$$

$$= \frac{w_{net} + q_{process}}{q_{in}}$$

or $$\varepsilon_u = 1 - \frac{q_{out}}{q_{in}} \qquad ...(12.51)$$

where q_{out} includes heat rejected in the condenser and all undesirable heat losses during process heat. The best designed cogeneration plant may have utilization factor as high as 70%.

Example 12.26 *Consider the cogeneration plant, shown in Fig. Ex. 12.51. Steam enters the turbine at 7 MPa and 500°C. Some steam is extracted from the turbine at 500 kPa for process heating. The remaining*

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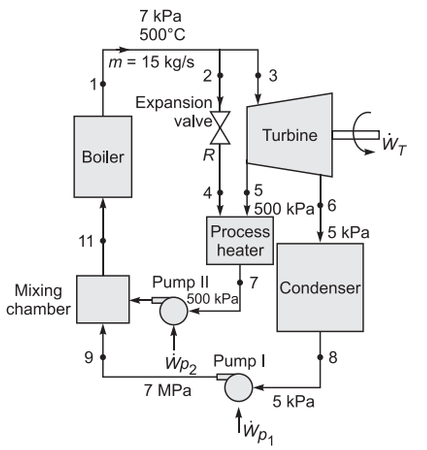
steam continues to expand to 5 kPa. Steam is then condensed at constant pressure and pumped to the boiler pressure of 7 MPa. At times of high demand for process heat, some steam leaving the boiler is throttled at 500 kPa and is routed to the process heater. The extraction fractions are adjusted so that steam leaves the process heater as a saturated liquid at 500 kPa. It is subsequently pumped to 7 MPa. The mass flow rate of steam through the boiler is 15 kg/s. Disregarding any pressure drops and heat losses in the piping and assuming the turbine and the pump to be isentropic, determine

(a) the maximum rate at which process heat can be supplied,

(b) the power produced and the utilization factor when no process heat is supplied, and

(c) the rate of process heat supply, when 10 percent of the steam is extracted before it enters turbine and 70 percent of the steam is extracted from turbine at 500 kPa for process heating,

(d) utilization factor for case (iii).



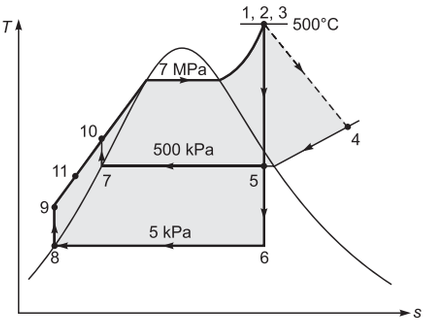


Fig. 12.51 *Schematic and T–s diagram*

Solution

Given The schematic of a cogeneration plant with data.

To find

(i) Maximum rate of process heat.
(ii) Power produced and utilization factor, when no process heat is supplied.
(iii) The rate of process heat supply, when 10% of steam is extracted, before it enters the turbine and 70% of steam is extracted from turbine at 500 kPa for process heat.
(iv) Utilization factor for case (iii).

Assumptions

(i) Each component in the cycle is analysis as a control volume at steady state.
(ii) The steam leaving the condenser and process heat as a saturated liquid.
(iii) All process of working fluid are internally reversible, except throttling.
(iv) Kinetic and potential energy effects are negligible.

Analysis The properties of steam at various states are taken from steam tables and Mollier diagram.

State 1, 2, 3: Superheated steam

$$p = 7 \text{ MPa}, T = 500°\text{C}$$
$$h_1 = h_2 = h_3 = 3410 \text{ kJ/kg}$$

State 4: Superheated region after throttling

$$h_4 = h_3 = 3410 \text{ kJ/kg}$$

State 5: Steam at $p = 500$ kPa, after first stage isentropic expansion

$$s_1 = s_5$$
$$h_5 = 2740 \text{ kJ/kg}$$
$$x_5 = 0.995$$

State 6: Steam after second stage expansion,

$$p = 5 \text{ kPa}$$
$$s_1 = s_6$$
$$h_6 = 2072 \text{ kJ/kg}$$
$$x_6 = 0.8$$

State 7: Saturated liquid at 500 kPa

$$h_7 = h_{f_7} = 640.23 \text{ kJ/kg}$$
$$v_7 = v_{f_7} = 0.001095 \text{ m}^3/\text{kg}$$

State 8: Saturated liquid at $p = 5$ kPa

$$h_8 = h_{f_8} = 137.82 \text{ kJ/kg}$$
$$v_8 = v_{f_8} = 0.001005 \text{ m}^3/\text{kg}$$

Work input to pump I,

$$w_{p_1} = v_8(p_9 - p_8)$$
$$= 0.001005 \times (7 \times 10^3 - 5)$$
$$= 7.03 \text{ kJ/kg}$$

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Work input to pump II,

$$w_{p_2} = 0.001095\,(7 \times 10^3 - 500)$$
$$= 7.11 \text{ kJ/kg}$$

State 9: Compressed liquid at $p = 7$ Mpa

$$h_9 = h_8 + w_{p_1} = 144.85 \text{ kJ/kg}$$

State 10: Compressed liquid at $p = 7$ Mpa

$$h_{10} = h_7 + w_{p_2} = 647.35 \text{ kJ/kg}$$

(i) *Maximum rate of Process heat*

Heat supply to turbine = 0

Heat supply to process heat is 100% (via throttle value)

$$\dot{Q}_{Process,max} = \dot{m}\,(h_4 - h_7)$$
$$= (15 \text{ kg/s})\,(3410 - 640.23)\,(\text{kJ/kg})$$
$$= \mathbf{4.155 \times 10^4\ kW}$$

Utilization factor is 100% since no heat is rejected of lost.

(ii) *Power produced and utilization factor, when no process heat is supplied*

Steam passes through turbine, $\dot{m} = 15$ kg/s

Turbine work

$$\dot{W}_T = \dot{m}\,(h_1 - h_6)$$
$$= (15 \text{ kg/s})\,(3410 - 2072)\,(\text{kJ/kg})$$
$$= 20070 \text{ kW}$$

Work input to pump

$$\dot{W}_p = \dot{m}\,w_{p_1} = (15 \text{ kg/s}) \times (7.03 \text{ kJ/kg})$$
$$= 105.45 \text{ kW}$$

Net work output of the plant

$$\dot{W}_{net} = \dot{W}_T - \dot{W}_p$$
$$= 20070 - 105.45 = \mathbf{19964.55\ kW}$$

Heat supplied in the boiler,

$$\dot{Q}_{in} = \dot{m}\,(h_1 - h_9)$$
$$= (15 \text{ kg/s}) \times (3410 - 144.35)\,(\text{kJ/kg})$$
$$= 48977.25 \text{ kW}$$

Utilization factor,

$$\varepsilon_u = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{19964.55}{48977.25}$$
$$= 0.408 = \mathbf{40.8\%}$$

Thus 40.8 percent of heat supplied is converted to useful work.

(iii) *Rate of process heat supply*

Mass of steam throttled,

$$\dot{m}_4 = 0.1 \times \dot{m} = 0.1 \times 15 = 1.5 \text{ kg/s}$$

Mass of steam extracted from turbine,

$$\dot{m}_5 = 0.7 \times 15 = 10.5 \text{ kg/s}$$

Total mass of steam for precess heat

$$\dot{m}_7 = 10.5 + 1.5 = 12 \text{ kg}$$

Applying mass and energy balance on the process heater

$$\Sigma m_i h_i = \Sigma \dot{m}_e h_e + \dot{Q}_p$$
$$\dot{m}_4 h_4 + \dot{m}_5 h_5 = \dot{m}_7 h_7 + \dot{Q}_p$$
$$1.5 \times 3410 + 10.5 \times 2740$$
$$= 12 \times 640.23 + \dot{Q}_p$$

or $\dot{Q}_p = 26202.24 \text{ kW} = \mathbf{26.2\ MW}$

(iv) *The rate of work produced in the turbine*

$$\dot{W}_T = \dot{m}_3\,(h_3 - h_5) + (\dot{m}_6)\,(h_5 - h_6)$$

where $\dot{m}_3$ = mass rate of steam entering the first stage turbine

$$= \dot{m} - \dot{m}_4 = 15 - 1.5 = 13.5 \text{ kg/s}$$

$\dot{m}_6$ = mass rate of steam entering second stage turbine

$$= \dot{m}_3 - \dot{m}_5 = 13.5 - 10.5 = 3 \text{ kg/s}$$

Thus $\dot{W}_T = 13.5 \times (3410 - 2740) + 3 \times (2740 - 2072) = 11049$ kW

Rate of work input to pump

$$\dot{W}_p = (\dot{m}_6)\,w_{p_1} + (\dot{m}_7)\,w_{p_2}$$
$$= 3 \times 7.03 + 12 \times 7.11 = 106.41 \text{ kW}$$

Net power output from the plant

$$\dot{W}_{net} = \dot{W}_T - \dot{W}_p$$
$$= 11049 - 106.41 = 10942.6 \text{ kW}$$

The rate heat supplied to working fluid in the boiler

$$\dot{Q}_{s,3} = \dot{m}_7\,(h_3 - h_{10}) + \dot{m}_6\,(h_3 - h_9)$$
$$= (12 \text{ kg/s})(3410 - 647.35)\,(\text{kJ/kg})$$
$$+ (3 \text{ kg/s}) \times (3410 - 144.85)(\text{kJ/kg})$$
$$= 33151.8 + 9795.45$$
$$= 42947.25 \text{ kW}$$

The utilization factor

$$\varepsilon_u = \frac{\dot{W}_{net} + \dot{Q}_p}{\dot{Q}_{s,3}}$$
$$= \frac{10942.6 + 26202.24}{42947.25}$$
$$= \mathbf{0.865 \quad or \quad 85\%}$$

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12.16 BINARY VAPOUR CYCLE

The *binary vapour* cycle is the combination of two cycles. One operates in the high temperature region, while other operates in low temperature region. It uses two working fluids one with good high temperature characteristics and another with good operating characteristics at lower temperature. In the binary vapour cycle, the two ideal Ranking cycles are combined such that the condenser of the high temperature cycle (also called *topping cycle*) serves as the boiler for the lower temperature cycle (*bottoming cycle*). Thus the heat rejection of the topping cycle is used as a heat input to the bottoming cycle.

Some common working fluids used in topping cycle are mercury, sodium, potassium, and sodium-potassium mixture. The mercury is a good working fluid for binary vapour cycle. Its critical temperature is 898°C, which is more than metallurgical limit, thus heat can be supplied to mercury isothermally at higher temperature, its critical pressure is only about 18 MPa, thus safer in the cycle. Although mercury cannot be used as sole working fluid for entire cycle, since at atmospheric temperature, 32°C, its saturation pressure is too low 0.07 Pa, which creates air leakage problems. At an acceptable condenser pressure of 7 kPa, its saturation temperature, 237°C, is high as minimum temperature in the cycle. Further due to its low specific enthalpy, the mass flow rate of mercury in the binary cycle is several times higher that of water. Therefore, the use of the mercury as a working fluid is limited to high temperature cycle only.

The schematic and *T–s* diagram of a binary vapour cycle mercury and water are shown in Fig. 12.52. The heat transfer between two cycles is accomplished in an interconnecting heat exchanger, which serves as a condenser for the mercury cycle and the boiler for steam cycle. It is evident from *T–s* diagram that the binary vapour cycle approximate the Carnot cycle more closely than a cycle for the same temperature limits. Therefore, the efficiency of such cycle is more than steam cycle.

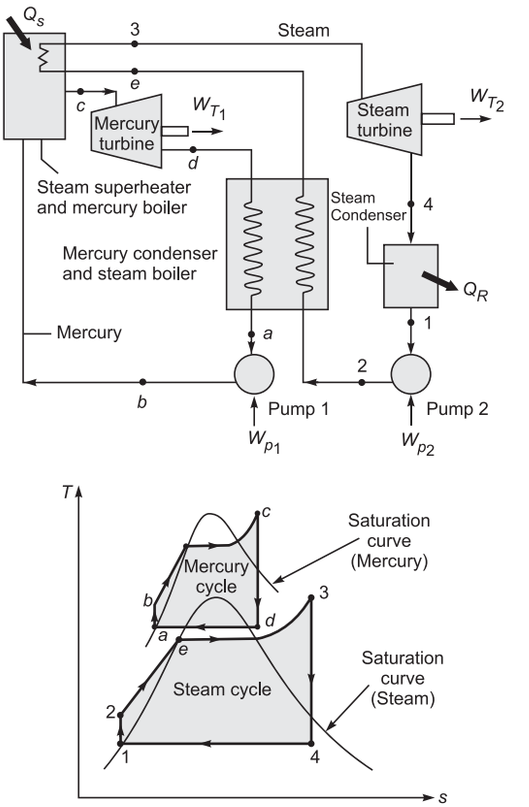

Fig. 12.52 *Mercury-water binary vapour cycle*

12.17 COMBINED GAS-VAPOUR POWER CYCLE

In order to improve the thermal efficiency of the power plant, the certain innovative modification are made to the conventional power plants. The binary vapour power cycle introduced above is an example of such modification. A more popular modifications is a gas power cycle topping the vapour power cycle, which is known as the *combined gas-vapour cycle*.

The gas turbine cycles typically operate at considerable higher temperature than vapour power cycle. The gases leaving the gas turbine cycle are at very high temperature (usually above 500°C). One way to utilise some of heat of high temperature

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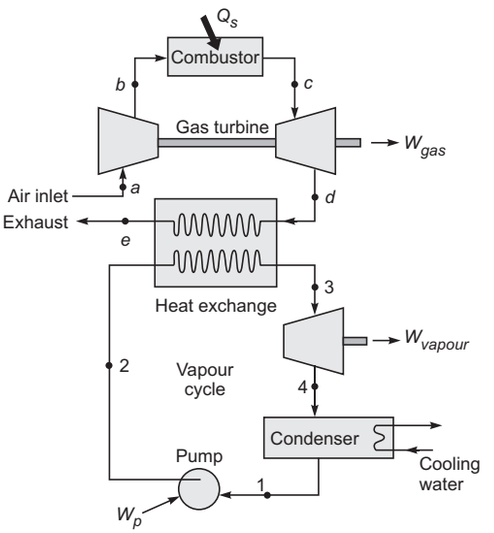
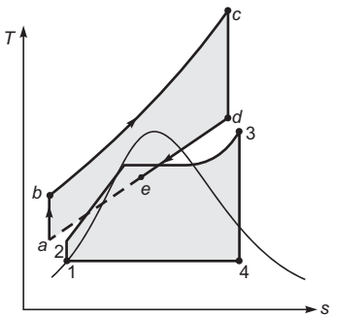
gases for regeneration, thereby improving overall fuel utilization. The combined cycle is shown in Fig. 12.53. The gases leaving the turbine of gas power cycle transfer the heat to vapour power cycle through a heat exchanger.

The combined cycle has high average temperature of heat addition for gas turbine cycle and low average temperature of heat rejection from vapour cycle, and thus an increased thermal efficiency. The thermal efficiency of the combined cycle is expressed as

$$\eta_{combined} = \frac{\dot{W}_{gas} + \dot{W}_{vapour}}{\dot{Q}_s} \qquad ...(12.52)$$



(a) Schematic



(b) T-s diagram

Fig. 12.53 *Combined gas-vapour cycle*

where $\dot{W}_{gas}$ = net power developed by the gas turbine plant,

$\dot{W}_{vapour}$ = net power developed by the steam cycle, and

$\dot{Q}_s$ = total rate of heat transfer to the combined cycle.

These energy balance on the heat exchange gives a relation.

$$\dot{m}_{steam}\,(h_3 - h_2) = \dot{m}_{gas}\,(h_d - h_e) \qquad ...(12.53)$$

For many applications, the combined cycle is economical and they are increasingly being used world wide for electric power generation.

Example 12.27 *A combined gas turbine vapour plant has a net power output of 45 MW. Air enters the compressor of the gas turbine at 100 kPa, 300 K, and is compressed to 1200 kPa. The isentropic efficiency of the compressor is 84%. The condition at the inlet to the turbine is 1200 kPa, 1400 K. Air expands through the turbine, which has an isentropic efficiency of 88%, to a pressure of 100 kPa. The air then passes through the interconnecting heat exchanger and is finally discharged at 400 K. steam enters the turbine of the vapour power cycle at 8 MPa, 400°C, and expands to the condenser pressure of 8 kPa. Water enters the pump as saturated liquid at 8 kPa. The turbine and pump of the vapour cycle have isentropic efficiencies of 90% and 80%, respectively.*

Determine

(a) The mass flow rates of the air and the steam, each in kg/s, and the net power developed by the gas turbine and vapour power cycle, each in MW.

(b) Thermal efficiency of the combined cycle.

Solution

Given A combined gas-vapour power plant operates at steady state with

Output power, P = 45 MW

Gas compressor inlet = 100 kPa, 300 K

Compressor exit = 1200 kPa

η_{comp} = 0.84

Gas turbine inlet = 1200 kPa, 1400 K

η_T = 0.88

Turbine exit = 100 kPa

Heat exchanger exit = 400 K gas

Inlet to steam turbine = 8 MPa, 400°C

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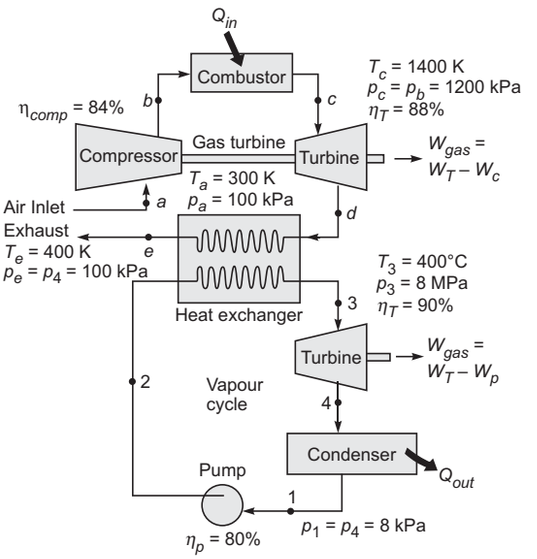
Condenser pressure = 8 kPa
Pump inlet = Saturated water at 8 kPa
For steam cycle $\eta_T = 0.9$
$\eta_P = 0.8$

To find

(i) Mass flow rate of air and vapour in kg/s.
(ii) Net power developed by gas turbine cycle and vapour power cycle, each in MW.
(iii) Thermal efficiency of the combined cycle.
(iv) Full accounting of a availability increase of air passing through the gas turbine combustor and discuss the results.

Schematic with given data



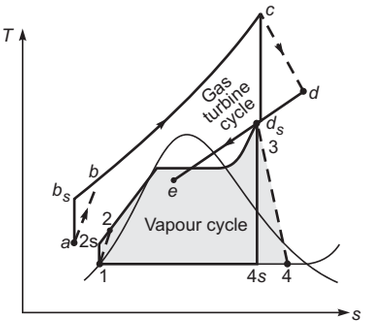


Fig. 12.54

Assumptions

(i) Each component is analyzed as a control volume at steady state.
(ii) The turbines, compressor, pump, and interconnecting heat exchanger operate adiabatically.
(iii) Kinetic and potential energy effects are negligible.
(iv) There are no pressure drops for flow through the combustor, interconnecting heat exchanger, and condenser.
(v) An air-standard analysis is used for the turbine.

The specific heat of air as $C_p = 1.005$ kJ/kg, $\gamma = 1.4$

Analysis The properties of steam at various states

State 1: Saturated liquid at

$p_1 = 8$ kPa
$h_1 = h_{f_1} = 173.88$ kJ/kg, $v_f = 0.0010084$ m^3/kg

State 2: Compressed water at

$p_2 = 8$ MPa
$w_{ps} = v_{f_1}(p_2 - p_1)$
$= 0.0010084 \times (8 \times 10^3 - 8) = 8.06$ kJ/kg
$h_{2s} = h_1 + w_{ps} = 173.88 + 8.06 = 181.94$ kJ/kg

Further, $\eta_p = \dfrac{w_{ps}}{w_p}$

or $w_p = \dfrac{w_{ps}}{\eta_p} = \dfrac{8.06}{0.8} = 10.075$ kJ/kg

or $h_2 = 173.88 + 10.075 = 183.96$ kJ/kg

State 3: Superheated steam

$p_3 = 8$ MPa, $T_3 = 400$°C
$h_3 = 3138.30$ kJ/kg, $s_3 = 6.3634$ kJ/kg·K

State 4: Wet steam at

$p_4 = 8$ kPa
$h_{f_4} = 173.88$ kJ/kg, $h_{fg_4} = 2403.1$ kJ/kg
$s_{f_4} = 0.5926$
$s_{fg_4} = 8.2287$ kJ/kg·K, $s_{g_4} = 8.8213$ kJ/kg·K
$s_{g_4} > s_3$, the steam is wet at state 4

Dryness fraction,

$$x_{4s} = \frac{s_3 - s_{f_4}}{s_{fg_4}} = \frac{6.364 - 0.5926}{8.2287}$$
$$= 0.701$$
$$h_{4s} = h_{f_4} + x_4 h_{fg_4}$$
$$= 173.88 + 0.701 \times 2403.1$$
$$= 1859.17 \text{ kJ/kg}$$

(or from Mollier diagram $h_{4s} = 1860$ kJ/kg)

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The isentropic efficiency of the vapour cycle turbine

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

or $h_4 = h_3 - \eta_T (h_3 - h_{4s})$
$= 3138.30 - 0.9 \times (3138.30 - 1859.17)$
$= 1987$ kJ/kg
$s_4 = 6.7282$ kJ/kg·K (From Mollier diagram)

Work done by the steam turbine

$w_{T,\,steam} = h_3 - h_4$
$= 3138.30 - 1987 = 1151.3$ kJ/kg

Net work done per kg of steam in the vapour power cycle

$w_{steam} = w_{T,steam} - w_p = 1151.3 - 10.075$
$= 1141.24$ kJ/kg

Analysis the gas power cycle.

State a: Atmospheric air $p_a = 100$ kPa, $T_a = 300$ K

State b: Compressed air, $p_b = 1200$ kPa

$$T_{bs} = T_a \left(\frac{p_b}{p_a}\right)^{\frac{\gamma-1}{\gamma}} = 300 \times \left(\frac{1200}{100}\right)^{\frac{1.4-1}{1.4}}$$

$= 610.18$ K

State c: Heated compressed air

$p_c = 1200$ kPa
$T_c = 1400$ K

State d: Exhaust gas $p_d = 100$ kPa

$$T_{ds} = \frac{T_c}{\left(\frac{p_c}{p_d}\right)^{\frac{\gamma-1}{\gamma}}} = \frac{1400}{\left(\frac{1200}{100}\right)^{\frac{1.4-1}{1.4}}}$$

$= 688.32$ K

Using compressor and turbine efficiency

$$\eta_{comp} = \frac{T_{bs} - T_a}{T_b - T_a}$$

or $T_b = T_a + \dfrac{T_{bs} - T_a}{\eta_{comp}} = 300 + \dfrac{610.18 - 300}{0.84}$
$= 699.26$ K

Similarly $T_d = T_c - \eta_T (T_c - T_{ds})$
$= 1400 - 0.88 \times (1400 - 688.32)$
$= 773.72$ K

Applying mass and energy balance to interconnecting heat exchanger

$\Sigma \dot{m}_i h_i = \Sigma \dot{m}_e h_e$
$\dot{m}_{air} (h_d - h_e) = \dot{m}_{steam} (h_3 - h_2)$

or $\dot{m}_{air} C_{p\,air} (T_d - T_e) = \dot{m}_{steam} (h_3 - h_2)$
$\dot{m}_{air} \times 1.005 (773.72 - 400)$
$= \dot{m}_{steam} (3138.30 - 183.96)$

or $\dfrac{\dot{m}_{air}}{\dot{m}_{steam}} = 7.865$

Net work done/kg of air in the gas power cycle

$w_{gas} = w_T - w_{comp}$
$= (h_c - h_d) - (h_b - h_a)$
$= C_p (T_c - T_d) - C_p (T_b - T_a)$
$= 1.005 \times [1400 - 773.72 - 699.26 + 300]$
$= 228.15$ kJ/kg

Net work output from combined cycle
= Steam cycle work + Gas cycle work

45×10^3 kJ/s $= \dot{m}_{steam} \times 1141.24 + \dot{m}_{air} \times 228.15$

Using $\dot{m}_{air} = 7.865\ \dot{m}_{steam}$, we get mass flow rates of steam and air

$\dot{m}_{steam} =$ **15.31 kg/s**

and $\dot{m}_{air} = 15.31 \times 7.865 =$ **120.48 kg/s**

(ii) Net work developed in gas power cycle

$\dot{m}_{air} \times w_{gas} = 120.48 \times 228.15$
$= 27.59 \times 10^3$ kW
$=$ **27.50 MW**

Net work done in vapour power cycle

$\dot{m}_{steam} \times w_{steam} = 15.31 \times 1141.24$
$= 17.50 \times 10^3$ kW
$=$ **17.5 MW**

Summary

- A thermodynamic cycle producing net amount of work as output is called power cycle. The vapour power cycle is a cycle with a working substance which alternatively vapourises and condenses.
- The most efficient cycle operating between a source temperature T_H and sink temperature T_L is the Carnot vapour power cycle and its thermal efficiency is given by

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$$\eta_{Carnot} = 1 - \frac{T_L}{T_H}$$

- The Rankine cycle is an ideal cycle for vapour power plants. Its thermal efficiency is function of heat supply and heat rejection and is given by

$$\eta = \frac{h_1 - h_2}{h_1 - h_4} = 1 - \frac{h_2 - h_3}{h_1 - h_4}$$

- The Rankine cycle uses complete condensation of steam and only liquid water is pumped back to the boiler. The Rankine cycle requires very less pumping work (back work), therefore, it has practically higher efficiency than that of Carnot vapour power cycle.
- *Back work ratio* is defined as the ratio of pump work input to the work developed by the turbine. It is denoted by *bwr,* and given as

$$r_{bw} = \frac{\text{Pump work}}{\text{Turbine work}} = \frac{w_p}{w_T} = \frac{h_4 - h_3}{h_1 - h_2}$$

- The *specific steam consumption* and *steam rate* is defined as the amount of steam required to produce 1 kWh (3600 kJ) of power. It is denoted by *ssc* and is expressed as

$$ssc = \frac{3600 \text{ kJ/kWh}}{w_{net} \text{ (kJ/kg)}}$$

- Thermal efficiency of the Rankine cycle can be improved by increasing the mean temperature of heat addition. The mean temperature of heat addition can be increasing by using superheat, reheat and regeneration.
- Superheat and reheat have an added advantage of reducing moisture content of steam at the turbine exit.
- The irreversibilities during expansion and pumping in the Rankine cycle reduce its thermal efficiency. The isentropic efficiency of turbine is defined as

$$\eta_T = \frac{\text{Actual enthalpy drop}}{\text{Isentropic enthalpy drop}}$$

The isentropic efficiency of the pump is given as

$$\eta_p = \frac{\text{Isentropic enthalpy drop}}{\text{Actual enthalpy drop}}$$

Glossary

Back work Work input to feed pump

Back work ratio Ratio of pump work input to the work developed by the turbine

Work ratio Ratio of the net work output to the work developed by the turbine

Steam rate Amount of steam required to produce 1 kWh (3600 kJ) of power

Heat Rate Amount of heat required by a power plant to produce 1 kWh of power

Reheating Heating of wet steam after its expansion in one stage for next stage

Reheat factor Ratio of cummulative isentropic enthalpy drop to isentropic enthalpy from initial pressure to final pressure

Review Questions

1. What are four basic components of a steam power plant? Write their function in brief.
2. What do you understand by steam rate and heat rate? What are their units?
3. Why Carnot cycle is not practical for a steam power plant?
4. Explain how the quality of steam at turbine exit gets restricted?
5. Draw the schematic for an ideal Rankine cycle. Draw p–v, T–s and h–s diagrams for this cycle.
6. What are methods which can lead to increase in thermal efficiency of Rankine cycle?
7. What are the irreversibilities in a steam power plant, which make its thermal efficiency less than that of Rankine cycle?

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8. What is reheating? What are the advantage of reheat Rankine cycle?
9. What is the effect of reheating of steam on (a) specific power output, (b) cycle efficiency, and (c) steam rate?
10. What is regeneration? Draw schematic and *T*–*s* diagram for an ideal regenerative cycle.
11. Why ideal regeneration is not possible? Explain in brief.
12. What is the effect of regeneration on (a) specific power output, (b) cycle efficiency, and (c) steam rate?
13. How is the regeneration of steam done in Carnotisation of Rankine cycle?
14. Explain the working and analysis of the regenerative Rankine cycle with one feed-water heater.

Problems

1. In a steam power cycle, the steam supply is at 15 bar, dry and saturated. The condenser pressure is 0.4 bar. Calculate the Carnot and Rankine efficiencies of the cycle. Neglect pump work.

[25.9%, 23.54%]

2. A steam power plant operates between a boiler pressure of 42 bar and a condenser pressure of 0.035 bar. The steam enters the turbine just dry and saturated. Calculate for these limits the cycle efficiency, work ratio, and specific steam consumption for

(a) Carnot cycle
(b) Ideal Rankine cycle
(c) Rankine cycle when expansion process has an isentropic efficiency of 80%

[(a) 42.2%,0.739, 4.91 kg/kWh, (b) 36.8%, 0.996, 3.64 kg/kWh, (c) 29.4%, 0.995, 4.56 kg/kWh]

3. A simple Rankine cycle works between pressures of 28 bar and 0.06 bar, the initial condition of steam being dry saturated. Calculate the cyclic efficiency, work ratio and specific steam consumption rate. *[33.57%, 0.997, 4.049 kg/kWh]*

4. An ideal Rankine cycle uses superheated steam at 50 bar and 500°C. The condenser pressure is 0.05 bar. Calculate cycle efficiency and specific steam consumption. *[39.6%, 2.75 kg/kWh]*

5. A steam turbine receives steam at 100 bar and 600°C. It is exhausted steam at 2 bar. For the ideal Rankine cycle, calculate net work, specific steam consumption, cycle efficiency and mean effective pressure.

[995.5 kJ/kg, 3.62 kg/kWh, 32.03%, 11.71 bar]

6. A steam power plant operates on ideal Rankine cycle, receives steam at 20 bar and 300°C at a rate of 3 kg/s and it is exhausted at 0.1 bar. Calculate the followings

(a) Net power output
(b) Steam rate
(c) Heat rejected in the condenser in kW
(d) Rankine cycle efficiency
(e) Actual thermal efficiency of the plant, if the boiler efficiency is 90%

[(a) 2636.4 kW, (b) 4.1 kg/kWh, (c) 5857.2 kW, (d) 31.04%, (e) 27.94%]

7. Steam at 20 bar, 360°C is expanded in a turbine to 0.08 bar. It then enters a condenser where it is condensed to saturated liquid water. The feed pump supplies saturated water back to the boiler.

(a) Calculate the net work per kg of steam and cycle efficiency for ideal Rankine cycle.
(b) If the turbine and pump have each 80% isentropic efficiency, calculate percentage reduction in net work and cycle efficiency.

[(a) 2983.41 kJ/kg, 32.55, (b) 20.1%, 20.1%]

8. A steam power plant operating on Rankine cycle is supplied with steam at 10 bar and 200°C. The condenser vacuum is 600 mm of mercury. Barometer pressure is 750 mm of Hg. If the efficiency of the plant relative to Rankine cycle is 65%, calculate (a) specific steam consumption, and (b) work ratio.

[(a) 8.888 kg/kWh, (b) 0.998]

9. A turbine used in a simple Rankine cycle receives steam at 15 bar with 5% moisture. The steam enters the condenser at a temperature of 29°C. Calculate Rankine cycle efficiency.

[31.77%]

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10. A steam power plant operates on the Rankine cycle. The steam enters the turbine at a flow rate of 500 kg/s, a pressure of 50 bar, a temperature of 600°C and a velocity of 30 m/s. The steam leaves the turbine at a pressure of 7.5 bar and a velocity of 100 m/s. Calculate the cycle efficiency and net power produced. *[39.87%, 696.05 MW]*

11. A steam turbine develops 5 MW, operating on Rankine cycle receives steam at 3.0 MPa and 300°C and exhausting it to condenser at a vacuum of 685 mm of Hg. The barometer reads 760 mm of Hg. Calculate

(a) Rankine cycle efficiency (b) Dryness fraction of steam enetering the condenser (c) Mass flow rate of steam required per hour.

[(a) 31.5%, (b) 0.785, (c) 20764 kg/h]

12. Two steam turbines *A* and *B* operate with same initial pressure of 15 bar and same degree of superheat 101.7°C. In turbine *A*, the exhaust pressure is 0.1 bar, whereas in turbine *B*, the exhaust pressure is 0.03 bar, Calculate

(a) Steam rate of turbines *A* and *B* in kg/kWh

(b) Percentage reduction in steam consumption due to lower exhaust pressure in the turbine *B*

(c) Percentage increase in thermal efficiency due to lower exhaust pressure in the turbine *B*

[(a) 4.33 kg/kWh and 3.712 kg/kWh, (b) 14.22 %, (c) 12.94%]

13. Steam expands in a turbine from 25 bar and 300°C to a condenser pressure of 20 kPa. Calculate Rankine cycle efficiency.

(a) What would be the efficiency if the initial temperature of steam be 500°C instead of 300°C?

(b) If the boiler pressure is increased to 60 bar maintaining the steam temperature at 500°C, calculate the cycle efficiency. Assume condenser pressure remains constant in all cases.

[27.64%, (a) 32.26%, (b) 36.63%]

14. A steam turbine has a specific steam consumption of 9 kg/kWh. It is supplied with steam at 10 bar. The steam is exhausted at 0.1 bar and 0.902 dry to a condenser. Calculate (a) quality of steam supplied to turbine, (b) Rankine cycle efficiency, (c) relative efficiency, and (d) steam consumption for a power output of 100 kW.

[(a) 400°C, (b) 29.28%, (c) 44.44%, (d) 900 kg/h]

15. The steam is supplied to a two-stage turbine at 40 bar and 350°C. It expands in the first stage turbine until it is just dry and saturated, then it is reheated to 350°C and expanded through the second-stage turbine. The condenser pressure is 0.035 bar. Calculate the work output and heat supplied per kg of steam for plant. Assuming ideal processes and neglecting feed-pump work, calculate the specific steam consumption and cycle efficiency.

[1290 kJ, 3362 kJ, 2.79 kJ/kWh, 38.4%]

16. A reheat Rankine cycle operates between the pressure limits of 26 bar and 0.04 bar. The steam entering the HP turbine and LP turbine has a temperature of 400°C. The steam leaves the HP turbine as dry saturated. Compare thermal efficiency and steam rate of Rankine cycle without and with reheating. Neglect the feed-pump work.

[(a) 36.02 %, and 36.94%, (b) 3.2 and 2.66 kg/kWh]

17. A reheat Rankine cycle receives steam at 3.5 MPa and 350°C to a HP turbine, where it expands isentropically to 0.8 MPa. The steam is then passed through the boiler and it is reheated to 350°C at 0.8 MPa. The steam is then expanded isentropically to 10 kPa in a LP turbine. Calculate (a) thermal efficiency, (b) quality of steam leaving the LP turbine, (c) power developed for a steam flow rate of 6 kg/min, and (d) thermal efficiency of the cycle without reheating, if it operates between 3.5 MPa, 350°C and 10 kPa.

[(a) 34.7%, (b) 0.895 dry, (c) 114.71 kW, (d) 33.63%]

18. In a single-heater regenerative cycle, the steam enters the turbine at 30 bar, 400°C and the exhaust pressure is 0.10 bar. The feed-water heater is a direct contact type which operates at 5 bar. Find

(a) the efficiency and the steam rate of the cycle

(b) the increase in mean temperature of heat addition, efficiency and steam rate as compared to the Rankine cycle (without regeneration). Pump work may be neglected

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[(a) 36.08%, 43.85 kg/kWh, (b) 27.4°C, 34.18%, 0.39 kg/kWh]

19. A steam turbine operates on simple regenerative Rankine cycle. The steam is supplied dry and saturated at 40 bar and is exhausted to condenser at 0.07 bar. The condensate is pumped at a pressure of 3.5 bar at which it is mixed with bled steam from the turbine at 3.5 bar. The resulting mixture is saturated water at 3.5 bar is then pumped to the boiler. Neglecting the pump work and for ideal cycle, calculate

(a) amount of bleed steam per kg of steam supplied,

(b) efficiency of the cycle,

(c) the specific steam consumption.

[(a) 0.19 kg/kg, (b)37%, (c) 4.39 kg/kWh]

20. A regenerative Rankine cycle includes open feed-water heater. The steam enters the turbine at 4 MPa and 400°C. The steam expands up to 400 kPa and then some quantity of steam is extracted from the turbine and is supplied to feed-water heater. The water leaves the feed-water heater as saturated liquid at 400 kPa. The remaining steam completes its expansion to 10 kPa. Calculate the cycle efficiency and steam rate.

[37.44%, 3.69 kg/kWh]

Objective Questions

1. Which one of the following is most popular vapour power cycle?
   (a) Carnot cycle (b) Rankine cycle
   (c) Joule cycle (d) Binary cycle
2. Which one of the following has the maximum thermal efficiency for a given range of temperatures?
   (a) Carnot cycle (b) Rankine cycle
   (c) Joule cycle (d) Binary cycle
3. Why Carnot vapour power cycle is considered non-practical cycle?
   (a) Saturate steam enters the turbine
   (b) Pump handles water-vapour mixture
   (c) Incompletely condensation of steam in the condenser
   (d) All of the above
4. Why is Rankine cycle considered as most practical cycle?
   (a) Saturate steam enters the turbine
   (b) Pump handles water-vapour mixture
   (c) Complete condensation of steam in the condenser
   (d) None of the above
5. The back work ratio is defined as
   (a) $\dfrac{\text{Net work done in the cycle}}{\text{Heat supplied in the cycle}}$
   (b) $\dfrac{\text{Pump work}}{\text{Turbine work}}$
   (c) $\dfrac{\text{Net work output}}{\text{Turbine work}}$ (d) None of above.
6. The back work ratio of Rankine cycle in comparison with Carnot vapour cycle is
   (a) less (b) more
   (c) same (d) none of the above
7. In the Rankine cycle, the heat is added
   (a) isothermally
   (b) at constant volume
   (c) at constant pressure
   (d) adiabatically
8. Heat rate of a power plant is expressed as
   (a) $\dfrac{\text{Pump work}}{\text{Turbine work}}$ (b) $\dfrac{3600\ \text{kJ}}{w_{net}\ (\text{kJ/kg})}$
   (c) $\dfrac{3600\ (\text{kJ})}{\eta_{th}(\text{kWh})}$ (d) none of the above
9. Specific steam consumption of a power plant is expressed as
   (a) $\dfrac{\text{Pump work}}{\text{Turbine work}}$ (b) $\dfrac{3600\ \text{kJ}}{w_{net}\ (\text{kJ/kg})}$
   (c) $\dfrac{3600\ (\text{kJ})}{\eta_{th}(\text{kWh})}$ (d) none of the above

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10. Thermal efficiency of Rankine cycle can be improved by steam
    (a) superheating (b) reheating
    (c) regeneration (d) none of the above
11. Specific steam consumption of Rankine cycle can be reduced by steam
    (a) superheating (b) reheating
    (c) regeneration (d) none of the above
12. Rankine cycle efficiency of well maintained steam power plant is in the range of
    (a) 25 to 35% (b) 10 to 20%
    (c) 35 to 45% (d) 50 to 60%
13. With reheat Rankine cycle
    (a) quality of exhausted steam is improved
    (b) net work out put of cycle increases
    (c) specific steam consumption decreases
    (d) all of the above
14. Rankine cycle with multiple regeneration can be approximated as
    (a) ideal Brayton cycle
    (b) Carnot vapour cycle
    (c) ideal Stirling cycle
    (d) none of the above
15. In regenerative Rankine cycle, the feed water is heated by
    (a) exhaust steam (b) condensed steam
    (c) bled-off steam (d) fresh steam
16. The steam engines operate on
    (a) simple Rankine cycle
    (b) ideal reheat Rankine cycle
    (c) Modified Rankine cycle
    (d) ideal regenerative cycle

Answers

1. (b) 2. (a) 3. (d) 4. (c) 5. (b) 6. (a) 7. (c) 8. (c)
9. (b) 10. (a) 11. (a) 12. (c) 13. (a) 14. (b) 15. (c) 16. (c)

13

Refrigeration

Introduction

A refrigerator and a heat pump are both heat engines operating in reverse direction. In operation, the reversed heat engine transfers heat energy from a low-temperature source to a high-temperature sink. The work input is required to transfer the heat from cooler to hotter region. A refrigerator or air conditioner removes heat energy from a low-temperature region while a heat pump delivers heat to a high-temperature region. The gas refrigeration cycle, vapour compression refrigeration cycle, vapour absorption cycle, heat pump cycle and refrigerant properties are explained in this chapter.

13.1 REFRIGERATION

Refrigeration is a branch of science which deals with the transfer of heat from a low temperature region to a high-temperature region, in order to maintain a desired region at a temperature below than that of surroundings.

In the refrigeration process, heat is continuously removed from a low temperature region to a high-temperature medium by using a low boiling point refrigerant. External power is required to carry out this process. Therefore, the refrigeration systems are power-absorbing devices.

13.1.1 Unit of Refrigeration

The capacity of refrigerating machines is measured often in terms of tonnes of refrigeration. *One tonne of refrigeration* is the amount of *refrigerating effect* (heat removed) produced by uniform melting of 1 tonne (1000 kg) of ice from and at 0°C in 24 hours. Using specific enthalpy of fusion of ice as 333.43 kJ/kg, then

$$1\,TR = \frac{(1000\text{ kg}) \times (333.43\text{ kJ/kg})}{(24\text{ h}) \times (60\text{ min/h})}$$

$$= 231.5\text{ kJ/min}$$

In practical calculations, 1 *TR* is taken as 210 kJ/min or 3.5 kW.

13.1.2 Applications of Refrigeration

1. Refrigeration is used for preservation of food items, fruits, vegetables, dairy products, fish and meats, etc.
2. Refrigeration is used for preservation of life-saving drugs, vaccines, etc., in hospitals and medical stores.
3. It is used in operation theaters and intensive care units (ICU) of hospitals.
4. It is used for making ice and preservation of ice-creams.

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5. It is used for providing comfort air-conditioning in offices, houses, restaurants, theatres, hotels, etc.
6. It is used in industries for improving working environment for their employees.
7. It is used for providing suitable working environment for some precision machines and precision measurement.
8. It is used in cold storages for preservation of seasonal vegetables, fruits, etc.
9. It is used for preservation of photographic goods, archeological and important documents.
10. It is used for chilling beverages (soft drinks).
11. It is used for cooling of drinking water.
12. It is used for liquefaction of gases.
13. It is used for processing of textiles, printing work, precision articles, photographic materials, etc.
14. It is used in control rooms and air-crafts.

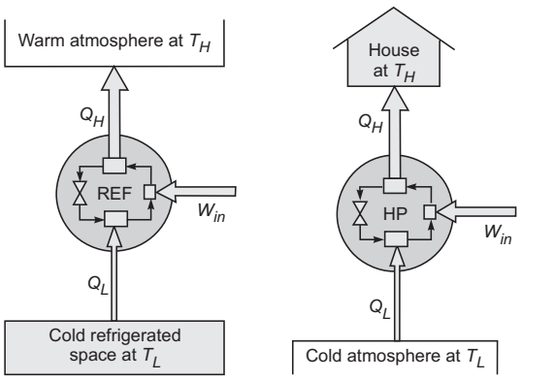
13.2 REFRIGERATORS AND HEAT PUMPS

The devices that produces *refrigeration* effect are called *refrigerators* and the cycles on which they operate are called the *refrigeration cycles.* The working fluids used in the refrigeration cycles are called the *refrigerants.*

Another device that also transfers heat from a low-temperature region to a high temperature region is called *heat pump*. The refrigerators and heat pumps operate on the same cycle, but they differ in their objectives. A heat pump maintains a region at a higher temperature than that of its surroundings. It absorbs heat from a low-temperature region and supplies it to a warmer region. Schematics of a refrigerator and heat pump are shown in Fig. 13.1.

The refrigerator removes heat Q_L from the low-temperature region. While a heat pump supplies heat Q_H to a high-temperature region.

The performance of a refrigerator and a heat pump is expressed in terms of coefficient of performance (COP), defined as



(a) Schematic of refrigerator *(b) Schematic of heat pump*

Fig. 13.1

$$(COP) = \frac{\text{Desired output}}{\text{Work input}}$$

For a refrigerator, it is the ratio of refrigerating (cooling) effect to the work input required to achieve that effect. Thus

$$(COP)_R = \frac{\text{Refrigerating effect}}{\text{Work input}} = \frac{\text{Cooling effect}}{\text{Work input}}$$

Referring Fig. 13.1(a)

$$(COP)_R = \frac{Q_L}{W_{in}} \qquad ...(13.1)$$

Similarly, for a heat pump, supplying heat Q_H to a high-temperature region at the cost of work input W_{in}, Fig. 13.1(b), the coefficient of performance

$$(COP)_{HP} = \frac{\text{Heating effect}}{\text{Work input}} = \frac{Q_H}{W_{in}} = \frac{W_{in} + Q_L}{W_{in}} = 1 + \frac{Q_L}{W_{in}} \qquad ...(13.2)$$

where quantities

Q_L = quantity of heat removed from a low-temperature region,

Q_H = quantity of heat supplied to a high-temperature region,

W_{in} = work input to cycle.

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From Eqs. (13.1) and (13.2), it is revealed that

$$(COP)_{HP} = (COP)_R + 1$$

Example 13.1 *A household refrigerator with a COP of 1.8 removes heat from a refrigerated space at a rate of 90 kJ/min. Determine*

(a) Electrical power consumed; and

(b) Heat rejected to surroundings

Solution

Given A household refrigerator

$(COP)_R = 1.8$ $\quad RE = 90$ kJ/min

To find

(i) Electrical power consumption, and

(ii) Heat rejected to surroundings.

Analysis

(i) The coefficient of performance of a refrigerator is given as

$$(COP)_R = \frac{\text{Refrigerating Effect}}{\text{Work input}} = \frac{RE}{W_{in}}$$

Using the given values

$$1.8 = \frac{90 \text{ kJ/min}}{W_{in}}$$

or $W_{in} = 50$ kJ/min = **0.833 kW**

The electrical power consumption is 0.833 kW.

(ii) Heat rejected to surroundings

$$Q_H = RE + W_{in}$$
$$= 90 \text{ kJ/min} + 50 \text{ kJ/min} = \textbf{140 kJ/min.}$$

Example 13.2 *An ice plant produces 10×10^3 kg of ice per day at 0°C using water at a temperature of 23°C. Estimate the power required by the compressor motor, if the COP of the plants is 3.5 and the transmission efficiency is 85%. Also find the amount of heat transferred from the system per minute.*

Take C_p (water) = 4.1868 kJ/kg·K, and

$h_{fg\,(ice)}$ = 334.5 kJ/kg

Solution

Given Production of ice at 0°C

$\dot{m}_{ice} = 10 \times 10^3$ kg per day

$T_L = 0°C = 273$ K

$T_i = 23°C = 296$ K

$(COP)_R = 3.5$

$\eta_{transmission} = 0.85$

$C_{pw} = 4.1868$ kJ/kg·K

$h_{fg} = 334.5$ kJ/kg

To find

(i) Power required by compressor motor,

(ii) Amount of heat transferred by the system per minute.

Analysis The amount of heat removed by the system per day

Q_L = Heat removed from water + Heat absorbed during phase change of water to ice

$$= \dot{m} C_{pw} (T_i - 0°C) + \dot{m} h_{fg}$$
$$= 10 \times 10^3 \times 4.1868 \times (23 - 0) + 10 \times 10^3 \times 334.5$$
$$= 430.8 \times 10^4 \text{ kJ per day}$$

Rate of heat removal per minute

$$RE = \frac{430.8 \times 10^4 (\text{kJ/day})}{(24\text{h} \times 60 \text{ min/h})} = \textbf{2991.67 kJ/min}$$

The *COP* of a refrigerator is given as

$$(COP)_R = \frac{RE}{\dot{W}_{in}}$$

or $$\dot{W}_{in} = \frac{RE}{(COP)_R} = \frac{2991.67}{3.5} = 854.76 \text{ kJ/min}$$
$$= 14.25 \text{ kW}$$

Power required by compressor motor

$$P = \frac{\dot{W}_{in}}{\eta_{transmission}} = \frac{14.25}{0.85} = \textbf{16.76 kW}$$

13.3 REFRIGERATION TERMINOLOGY

1. Refrigeration Load It is the amount of heat which must be removed per unit time from the cold region. It is also known as the *refrigeration capacity*. It is measured in Tonnes of Refrigeration (*TR*) and is designated as RE. Sometimes, it is also referred as refrigeraton effect.

2. Mass Flow Rate of Refrigerant The refrigeration capacity of a system decides the mass flow rate of a given refrigerant, when working under specified conditions, i.e.,

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Mass flow rate of refrigerant

$$\dot{m} = \frac{\text{Refrigeration capacity}}{\text{Refrigerating effect per kg of refrigerant}} \quad ...(13.3)$$

3. Evaporation Capacity It is the refrigeration effect produced at the evaporator of a refrigeration system, thus equivalent to refrigeration capacity.

The *evaporator* is the device that consists of the long thin tubing where the refrigerant evaporates by absorbing its heat of evaporation from the surrounding medium.

4. Freezer It is the term used in a household refrigerator. It is a place very near to evaporator coils, so the liquid can easily be freezed to solid state. It is also called *chiller*.

5. Frosting It is the deposition of ice on the evaporating coil due to its very low temperature. When air passes over the extremely cold coils, the moisture in the air separates and deposits on coils and gets solidified to form frosting. This frosting (ice) is a very bad conductor of heat, and reduces the heat transfer from cold region to refrigerant. Thus the refrigeration load increases. Therefore, it is recommended that this frost (ice) must be removed from the evaporator coils regularly. The *removal of ice from the evaporator coil* is called the *defrostation*.

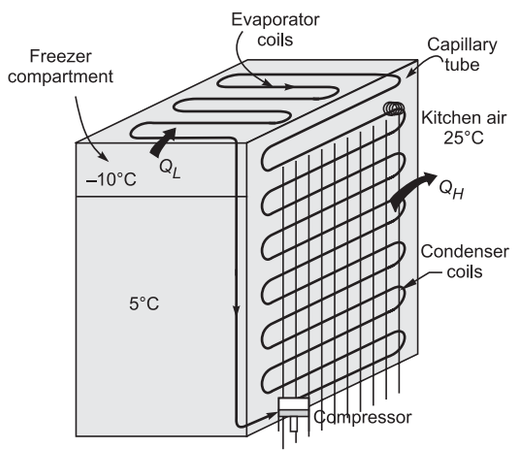

Fig. 13.2 *Domestic household refrigerator*

6. Capillary Tube The capillary tube is a long and narrow tube connecting the condenser directly to the evaporator. The refrigerant passes through this tube by capillary action (drop by drop), thus the fluid friction and flashing of liquid refrigerant into vapour due to heat transfer, cause the pressure drop in the tube. For a given state of the refrigerant, the pressure drop is directly proportional to length of the tube and inversely proportional to diameter of the tube.

13.4 TYPES OF REFRIGERATION SYSTEMS

The commonly used refrigeration systems are listed below:

(i) Gas refrigeration system
(ii) Vapour compression refrigeration system
(iii) Vapour absorption refrigeration system
(iv) Steam refrigeration system

13.5 GAS REFRIGERATION SYSTEMS

In a gas refrigeration cycle, the gas such as air is used as a refrigerant. It transfer only its sensible heat and does not undergo a change of phase. The gas refrigeration systems have a number of applications. They are used to achieve very low temperature for liquefaction of gases and for aircraft cooling. The gas power cycles used earlier can be used as gas refrigeration by simply reversing the direction of process involved in these cycles. The reversed Carnot cycle, reversed Brayton cycle and reversed Stirling cycle are some gas refrigeration cycles.

The main advantages of air refrigeration is its free availability, light weight and eco-friendly.

13.5.1 Reversed Carnot Cycle

As studied earlier in Section 6.22, if the direction of Carnot engine is reversed, then the cycle works as a *refrigeration cycle.* The reversed Carnot cycle will theoretically have a maximum possible coefficient of performance, but it is not possible to construct

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a refrigerating machine which will work on the reversed Carnot cycle.

A reversed Carnot cycle using air as working medium is shown on *p–V* and *T–S* diagrams in Fig. 13.3(a) and (b), respectively.

The cycle consists of four reversible processes in sequence.

Process 1–2 Isentropic expansion of air from higher temperature T_H to lower temperature T_L

Process 2–3 Heat removal from cold space in isothermal manner at temperature T_L

Process 3–4 Isentropic compression of air from low temperature T_L to high temperature T_H

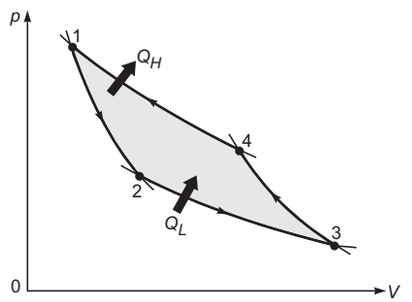
Process 4–1 Heat rejection isothermally to a medium at temperature T_H

The refrigeration effect = Heat absorbed by air during isothermal process 2–3 at temperature T_L

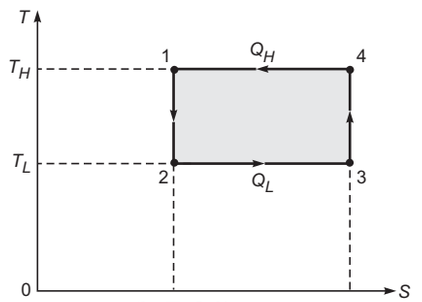
$$RE = T_L(S_3 - S_2) \qquad ...(13.4)$$

Heat rejected at temperature T_H

$$Q_H = T_H(S_3 - S_2) \qquad ...(13.5)$$



(a) p–V diagram



(b) T–S diagram

Fig. 13.3 *Reversed Carnot Cycle*

Net work input to cycle,

$$W_{in} = Q_H - Q_L$$
$$= T_H(S_3 - S_2) - T_L(S_3 - S_2)$$
$$= (T_H - T_L)(S_3 - S_2) \qquad ...(13.6)$$

The coefficient of performance of the reversed Carnot cycle operation as refrigerator

$$(COP)_{R,\,rev} = \frac{RE}{W_{in}} = \frac{T_L(S_3 - S_2)}{(T_H - T_L)(S_3 - S_2)}$$
$$= \frac{T_L}{T_H - T_L} \qquad ...(13.7)$$

The coefficient performance of reversed Carnot cycle operating as heat pump

$$(COP)_{HP,\,rev} = \frac{\text{Heat rejected at } T_H}{\text{Work input}} = \frac{Q_H}{W_{in}}$$
$$= \frac{T_H(S_3 - S_2)}{(T_H - T_L)(S_3 - S_2)} = \frac{T_H}{T_H - T_L} \qquad ...(13.8)$$

13.5.2 Practical Limitations of Carnot Refrigeration Cycle

Although the reversed Carnot cycle gives maximum *COP* between two fixed temperatures and is useful as a criterion of perfection, it has inherent drawbacks, when a gas is used as a refrigerant.

1. It is almost impractical to transfer heat at constant temperature during processes 2–3 and 4–1. Such heat transfer is only possible with a very slow process.
2. Secondly, with the irreversibilities of the compressor and expander, the isentropic compression and expansion cannot be achieved
3. Further, isothermal processes are extremely slow processes while isentropic processes are extremely fast processes. Therefore, combination of these two processes within a cycle is impossible.

Example 13.3 *A refrigerator has a working temperature in the evaporation and condenser coils of – 30°C and 30°C, respectively. What is the maximum possible COP of the refrigerator?*

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Solution

Given A refrigerator operating between

$T_L = -30°C = 243$ K

$T_H = 30°C = 303$ K

To find Maximum possible *COP* of a refrigerator.

Analysis The reversed Carnot cycle can give maximum $(COP)_R$, thus

$$(COP)_{R,\,rev} = \frac{T_L}{T_H - T_L} = \frac{243}{303 - 243} = \mathbf{4.05}$$

Example 13.4 *A reversed Carnot cycle is used for making ice at –5°C from water at 25°C. The temperature of the brine is –10°C. Calculate the quantity of ice formed per kWh of work input. Assume the specific heat of ice as 2 kJ/kg·K, latent heat of ice as 335 kJ/kg and specific heat of water as 4.18 kJ/kg·K.*

Solution

Given Formation of ice at –5°C from water at 25°C

$T_L = -10°C = 263$ K

$T_H = 25°C = 298$ K

$T_{ice} = -5°C$

$W_{in} = 1$ kWh $= 3600$ kJ

$C_{p,\,ice} = 2$ kJ/kg·K

$C_{pw} = 4.18$ kJ/kg·K

$h_{fg} = 335$ kJ/kg

To find The mass of ice formed.

Assumption The ambient temperature to be 25°C

Analysis The schematic is shown in Fig. 13.4.

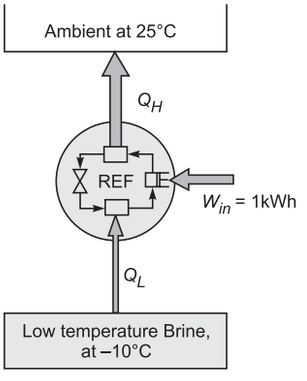

Fig. 13.4

Amount of heat removed from water–ice system per kg.

Q_L = Heat removed in cooling of water from 25°C to 0°C + Heat removed during formation of ice at 0°C + Heat removed from ice during its cooling from 0°C to –5°C.

$= C_{pw}(T_H - 0°C) + h_{fg} + C_{p,\,ice}\,(0 - T_{ice})$

$= 4.18 \times (25 - 0) + 355 + 2.0 \times [0 - (-5)]$

$= 449.5$ kJ/kg

The *COP* of a Carnot refrigerator

$$(COP)_{R,\,rev} = \frac{T_L}{T_H - T_L} = \frac{263}{298 - 263} = 7.514$$

Further, the *COP* can also be expressed as

$$COP = \frac{RE}{W_{in}}$$

or $$7.514 = \frac{RE}{3600\text{ kJ}}$$

or $$RE = 27051.43\text{ kJ}$$

Thus, the mass of ice formed with this refrigerating effect

$$\dot{m}_{ice} = \frac{RE}{Q_L} = \frac{27051.43}{449.5} = \mathbf{60.18\ kg/kWh}$$

13.6 BRAYTON REFRIGERATION CYCLE: BELL COLEMAN CYCLE

The Bell Coleman refrigeration cycle is the reverse of the closed Brayton power cycle. The schematic and *T–s* diagrams of the reverse Brayton cycle are shown in Fig. 13.5.

The refrigerant gas (may be air) enters the compressor at the state 1 and is compressed to the state 2. The gas is then cooled at constant pressure in a heat exchanger to the state 3. During cooling, the gas rejects heat to the surroundings and approaches the temperature of the warm environment. The gas is then expanded in an expander to the state 4, where it attains a temperature, that is well below the temperature of the cold region. The refrigeration effect is achieved through the heat transfer from the cold region to gas as it passes from state 4 to 1 in a heat exchanger and the cycle completes.

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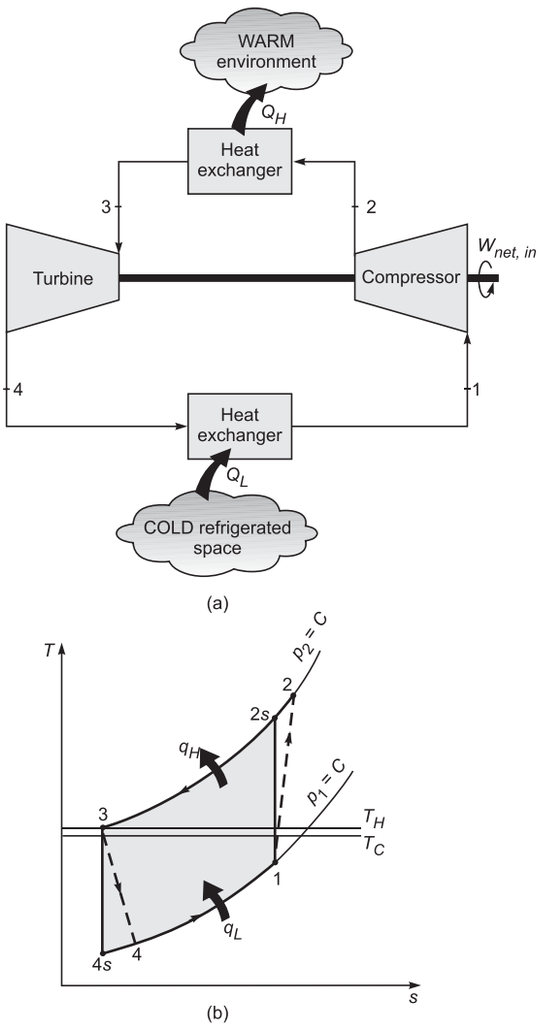

Fig. 13.5 *Brayton refrigeration cycle*

The *T–s* diagram for an ideal Brayton cycle is represented by the cycle 1–2*s*–3–4*s*–1, in which all processes are internally reversible, and compression and expansion are isentropic. The cycle 1–2–3–4–1 includes the effect of irreversibilities during adiabatic compression and expansion. The frictional pressure drops have been ignored.

Cycle Analysis At steady state, the compression and expansion work per kg of gas flow in the system are,

$$w_c = h_2 - h_1 \quad \text{and} \quad w_T = h_3 - h_4$$

The net work input to the cycle is

$$w_{net} = w_c - w_T$$
$$= (h_2 - h_1) - (h_3 - h_4) \qquad ...(13.9)$$

The refrigeration effect produced in the cycle

RE = Heat transfer from the cold region to the refrigerant gas

$$= h_1 - h_4 \qquad ...(13.10)$$

The coefficient of performance of the cycle is the ratio of the refrigeration effect to the net work input

$$(COP)_R = \frac{RE}{w_{net}} = \frac{h_1 - h_4}{(h_2 - h_1) - (h_3 - h_4)} \qquad ...(13.11)$$

The gas refrigeration cycle deviates from the reversed Carnot cycle because heat transfer processes are not isothermal. In fact, the gas temperature varies considerably during the heat-transfer process. Figure 13.6 shows a *T–s* diagram which compares the reversed Carnot cycle 1–2′–3–4′–1 and reversed Brayton cycle 1–2–3–4–1. It reveals the following facts:

1. The net work (area 1–2′–3–4′–1) required by the reverse Carnot cycle is a fraction of that required by the reverse Brayton cycle.
2. The reverse Carnot cycle produces greater refrigeration effect (area under line 4′–1) as compare to reverse Brayton cycle (area under curve 4–1).
3. The mean temperature of heat rejection is much greater and that of heat rejection is much lower in reverse Brayton.

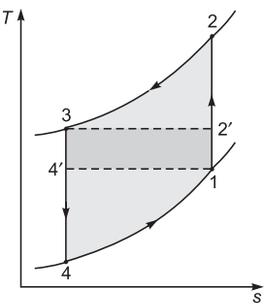

Fig. 13.6 *Comparison of reverse Brayton cycle with reversed Carnot cycle*

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Therefore, the *COP* of the reverse Brayton cycle is much lower than that of the vapour compression cycle or the reversed Carnot cycle.

Advantages

1. Air is a freely and easily available fluid.
2. There is no danger of fire and toxic effects, if it leaks.
3. The weight of air refrigeration system per tonne of refrigeration is less compared with other refrigeration systems.
4. An air cycle can work as an open or closed cycle system.
5. It is eco-friendly.

Disadvantages

1. The main drawback of an air refrigeration system is its very low value of *COP*.
2. Air has only sensible heat and cannot transfer heat at constant temperature across heat exchangers. Thus, as the temperature difference decreases, its heat transfer capacity decreases.
3. The quantity of air required per tonne of refrigeration capacity is much larger than the liquid refrigerants.
4. A turbine is required for the expansion of air instead of a throtting device. Since air contains some water vapour, thus danger of frosting during expansion is more possible.
5. It has more running cost than other systems.

Example 13.5 *Air enters the compressor of an ideal Brayton refrigeration cycle at 1 atm and 270 K with a volumetric flow rate of 1.5 m^3/s. If the compressor pressure ratio is 3 and the turbine inlet temperature is 300 K, determine*

(a) the net power input,
(b) the refrigeration capacity, and
(c) coefficient of performance.

Solution

Given An ideal Brayton refrigeration cycle operates with air.

Compressor inlet, T_1 : 1 atm, 270 K, 1.5 m^3/s

Pressure ratio, p_2/p_1 : 3

Turbine inlet, T_3 : 300 K

To find

(i) Net power input in the cycle,
(ii) Refrigeration capacity, and
(iii) The *COP* of the cycle.

Schematic with given data

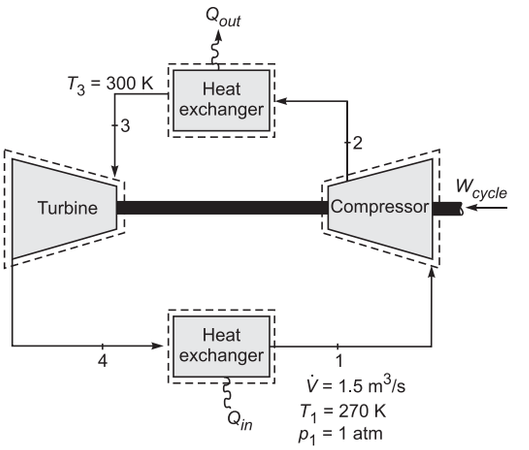

Fig. 13.7

Assumptions

(i) Each component of the cycle is analyzed as a control volume at steady state.
(ii) The turbine and compressor processes are isentropic.
(iii) There are no pressure drop through the heat exchangers.
(iv) Kinetic and potential energy effects are negligible.
(v) The working fluid is modelled as an ideal gas.
(vi) Cold air standard assumptions with C_p = 1.005 kJ/kg·K and γ = 1.4.

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Analysis The specific enthalpy at each state of the cycle.

State 1: $T_1 = 270$ K, $p_1 = 1$ atm

$h_1 = C_p T_1 = 1.005 \times 270 = 271.35$ kJ/kg

State 2: $p_2 = 3p_1 = 3$ atm

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 270 \times (3)^{\frac{1.4-1}{1.4}} = 370 \text{ K}$$

$h_2 = C_p T_2 = 1.005 \times 370 = 371.85$ kJ/kg

State 3: $T_3 = 300$ K, $p_3 = 3$ atm,

$h_3 = C_p T_3 = 1.005 \times 300 = 301.5$ kJ/kg

State 4: $p_4 = p_1 = 1$ atm

$$T_4 = \frac{T_3}{\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}} = \frac{300}{\left(\frac{3}{1}\right)^{\frac{1.4-1}{1.4}}} = 219.35 \text{ K},$$

$h_4 = C_p T_4 = 1.005 \times 219.35 = 220.44$ kJ/kg

The mass flow rate of air

$$\dot{m} = \frac{\dot{V}}{v_1}$$

where $$v_1 = \frac{RT_1}{p_1} = \frac{(0.287 \text{ kJ/kg·K}) \times (270 \text{ K})}{(101.325 \text{ kPa})}$$

$= 0.765$ m^3/kg

Given $\dot{V} = 1.5$ m^3/s

Then $$\dot{m} = \frac{\dot{V}}{v} = \frac{1.5}{0.765} = 1.96 \text{ kg/s}$$

(i) Net power input in the cycle

$$\dot{W}_{net,in} = \dot{m}[(h_2 - h_1) - (h_3 - h_4)]$$
$$= 1.96 \times [(371.85 - 271.35) - (301.5 - 220.44)]$$
$$= \mathbf{38.13 \text{ kW}}$$

(ii) The refrigeration capacity

$$RE = \dot{m}(h_1 - h_4)$$
$$= 1.96 \times [271.35 - 220.44] = \mathbf{99.78 \text{ kW}}$$

(iii) The coefficient of performance

$$(COP)_{R,\,Brayton} = \frac{RE}{\dot{W}_{net,in}} = \frac{99.78}{38.13} = \mathbf{2.62}$$

Example 13.6 *Air enters the compressor of an aircraft cooling system at 100 kPa, and 283 K. Air is now compressed to 2.5 bar with an isentropic efficiency of 72%. After being cooled to 320 K at constant pressure in a heat exchanger, the air then expands in a turbine to 1 bar with an isentropic efficiency of 75%. The cooling load of the system is 3 tonnes of refrigeration. After absorbing heat at constant pressure, the air re-enters the compressor, which is driven by the turbine. Find the COP of the refrigerator, driving power required and air mass flow rate.*

Solution

Given An aircraft cooling system

Compressor inlet, $p_1 = 100$ kPa, $T_1 = 283$ K

$\eta_C = 0.72$

Pressure ratio, r_p : 2.5

$\eta_T = 0.75$

Turbine inlet, $T_3 = 320$ K

$RE = 3$ TR $= 10.53$ kW

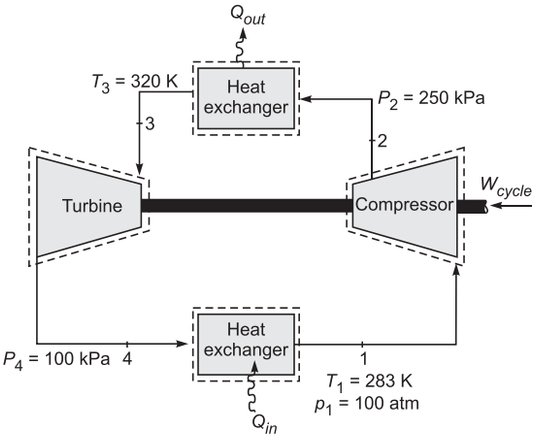
To find

(i) *COP* of refrigerator,

(ii) Mass flow rate of air, and

(iii) Power input.

Schematic with given data



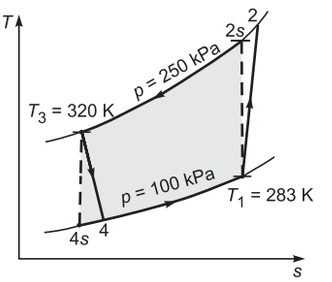


Fig. 13.8

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Assumptions

(i) Each component of the cycle is analyzed as a control volume at steady state.
(ii) There are no pressure drops through the heat exchangers.
(iii) Kinetic and potential energy effects are negligible.
(iv) The working fluid is modelled as an ideal gas.
(v) Cold air standard assumptions with C_p = 1.005 kJ/kg·K and γ = 1.4.

Analysis Analysing each component separately;

Compressor The temperature after isentropic compression

$$T_{2s} = T_1\left(r_p\right)^{\frac{\gamma-1}{\gamma}} = 283 \times \left(2.5\right)^{\frac{1.4-1}{1.4}} = 367.7 \text{ K}$$

The isentropic efficiency of the compressor is given by

$$\eta_C = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

or $$T_2 = 283 + \frac{367.7 - 283}{0.72} = 400.62 \text{ K}$$

The actual work input to compressor w_{in}

$$w_{in} = h_2 - h_1 = C_p(T_2 - T_1)$$
$$= 1.005 \times (400.62 - 283) = 118.21 \text{ kJ/kg}$$

Turbine The temperature T_{4s} after isentropic expansion;

$$T_{4s} = \frac{T_3}{\left(r_p\right)^{\frac{\gamma-1}{\gamma}}} = \frac{320}{\left(2.5\right)^{\frac{1.4-1}{1.4}}} = 246.3 \text{ K}$$

The isentropic efficiency of the turbine is given as

$$\eta_T = \frac{\text{Actual work output}}{\text{Isentropic work output}} = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

Actual temperature after expansion in turbine,

$$T_4 = 320 - 0.75 \times (320 - 246.3) = 264.47 \text{ K}$$

Turbine work output per kg of air

$$w_{out} = h_3 - h_4 = C_p(T_3 - T_4)$$
$$= 1.005 \times (320 - 264.72) = 55.556 \text{ kJ/kg}$$

Net work input per kg to the plant

$$w_{net} = w_{in} - w_{out}$$
$$= 118.21 - 55.556 = 62.65 \text{ kJ/kg of air}$$

Heat absorbed per kg of air

$$q_{in} = h_1 - h_4 = C_p(T_1 - T_4)$$
$$= 1.005 \times (283 - 264.72) = 18.37 \text{ kJ/kg}$$

1. *COP of refrigerator*

$$(COP)_R = \frac{\text{Heat absorbed per kg of air}}{\text{Net work input per kg of air}}$$
$$= \frac{q_{in}}{w_{net}} = \frac{18.37}{62.65} = \mathbf{0.293}$$

2. *Mass flow rate of air in the system*

$$\dot{m}_a = \frac{\text{Refrigerating effect}}{\text{Heat removed per kg of air}}$$
$$= \frac{RE}{q_{out}} = \frac{10.53 \text{ kJ/s}}{18.37 \text{ kJ/kg}} = \mathbf{0.573 \text{ kg/s}}$$

3. *Power input to refrigerator*

$$\dot{W}_{input} = \dot{m}_a \, w_{net}$$
$$= (0.573 \text{ kg/s}) \times (62.65 \text{ kJ/kg})$$
$$= \mathbf{35.91 \text{ kW}}$$

13.7 IDEAL VAPOUR COMPRESSION REFRIGERATION CYCLE

A schematic of an ideal vapour compression refrigeration cycle and its *T–s* diagram are shown in Fig. 13.9.

The vapour compression refrigeration cycle consists of four processes discussed below:

Process 1–2 Isentropic compression of saturated vapour in the compressor,

Process 2–3 Constant pressure heat rejection in the condenser,

Process 3–4 Throttling of refrigerant in an expansion device, and

Process 4–1 Constant pressure heat absorption in evaporator.

In an ideal vapour compression cycle, the refrigerant enters the compressor at the state 1, as dry and saturated vapour, where it is compressed to a relatively high pressure and temperature, the state 2. The refrigerant in the superheated state 2, enters the condenser and leaves as saturated liquid at the state 3, as a result of heat rejection to the surroundings. The saturate liquid refrigerant at the state 3 is throttled to the evaporator pressure by passing through an expansion valve or a capillary tube. The temperature of refrigerant at the state 4 drops below the temperature of the refrigerated space.

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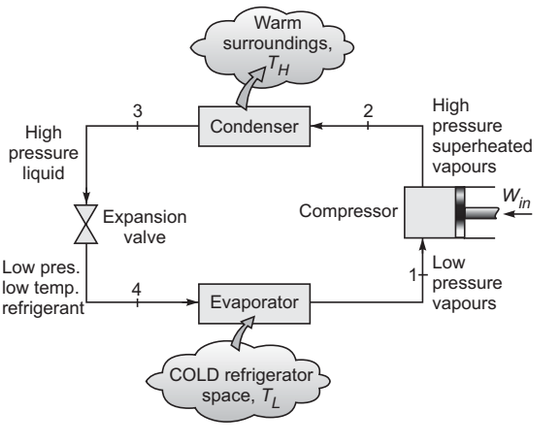
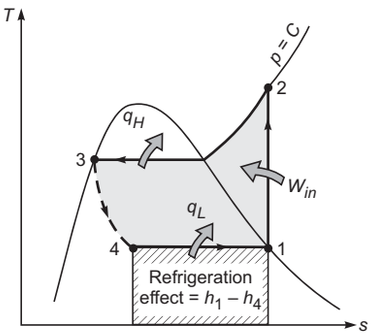


(a) Basic components of a refrigerator



(b) T–s diagram for ideal vapour compression cycle.

Fig. 13.9

At the state 4, the refrigerant as wet mixture, passes through an evaporator at constant pressure. The refrigerant is now completely evaporated by absorbing its latent heat from cold temperature (refrigerated) space. The heat absorbed during evaporation of refrigerant is the latent heat and its magnitude is higher at low pressures.

The only throttling process 3–4 is an irreversible process and hence it is shown by dotted lines. The remaining three processes are considered reversible. Therefore, the vapour compression cycle is not a reversible cycle.

13.7.1 Vapour Compression Cycle on Pressure–Enthalpy Diagram

The *pressure–enthalpy diagram* is more convenient to represent refrigeration cycles, since the enthalpy

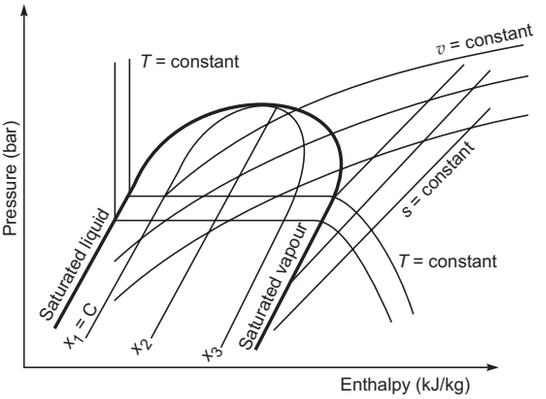


Fig. 13.10 *Schematic of pressure enthalpy chart for a refrigerant*

required for the calculation can directly be read off. The essential features of the diagram are given in Fig. 13.10 and a typical refrigeration cycle is shown in Fig. 13.11. The points 1, 2, 3 and 4 represent the same state of refrigerant as on *T–s* diagram in Fig. 13.9(b).

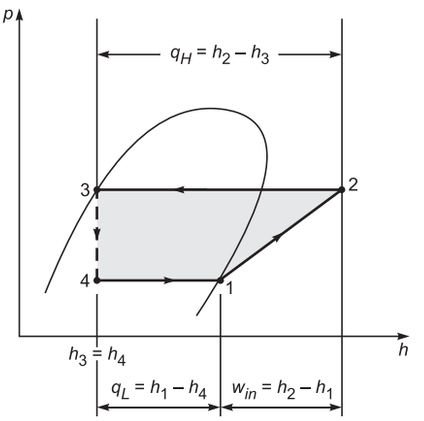


Fig. 13.11 *Vapour compression cycle on a p-h diagram*

13.7.2 Analysis of Vapour Compression Cycle

All four components in a vapour compression cycle are steady flow devices. Thus processes they perform are steady flow processes. In absence of any kinetic and potential energy changes, the steady flow energy equation on unit mass basis reduces to

$$q - w = \Delta h$$

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1. Evaporator ($w = 0$)
$$q_L = h_1 - h_4 \quad ...(13.12)$$
2. Compressor ($q = 0$)
$$w_{in} = h_2 - h_1 \quad ...(13.13)$$
3. Condenser ($w = 0$)
$$q_H = h_2 - h_3 \quad ...(13.14)$$
4. Expansion valve ($q = 0$, $w = 0$)
$$h_4 = h_3 \quad ...(13.15)$$

The coefficient of performance of refrigerator and heat pump can be expressed as

$$(COP)_R = \frac{q_L}{w_{in}} = \frac{h_1 - h_4}{h_2 - h_1} \quad ...(13.16)$$

$$(COP)_{HP} = \frac{q_H}{w_{in}} = \frac{h_2 - h_3}{h_2 - h_1} \quad ...(13.17)$$

where $h_1 = h_g @ p_1$
$h_3 = h_f @ p_2$

Example 13.7 *A vapour compression refrigerator cycle works between temperature limits of 25°C and –10°C. The vapour at the end of isentropic compression is just dry. Assuming there is no subcooling, find the COP of the system. Also find the capacity of refrigerator,if mass flow rate of refrigerant is 5 kg/min.The properties of refrigerant are tabulated below.*

Temperature	*Enthalpy, kJ/kg*		*Entropy of liquid,*
T, K	h_f	h_{fg}	*kJ/kg·K*
298 K	298.9	1465.84	1.1242
263	135.37	1433.05	0.5443

Solution

Given A vapour compression cycle as shown on *p–h* diagram in Fig. 13.12.

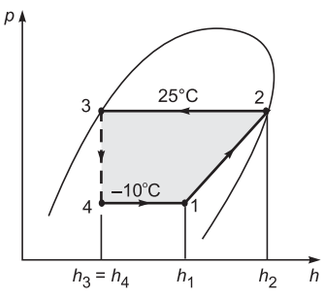

Fig. 13.12

$\dot{m}_R = 5$ kg/min.
$T_1 = -10°C = 263$ K
$T_3 = 25°C$
and tabulated properties of refrigerant.

To find

(i) $(COP)_R$,
(ii) Capacity of refrigerator.

Analysis Properties of refrigerant at end states:
$h_2 = 1465.84$ kJ/kg
$h_3 = 298.9$ kJ/kg
$h_4 = 298.9$ kJ/kg

Using the Clapeyron equation for calculating entropy change s_{fg_2} during evaporation

$$s_{fg_2} = \frac{h_{fg_2}}{T_{sat_2}} = \frac{h_{g_2} - h_{f_2}}{T_{sat_2}}$$
$$= \frac{1465.84 - 298.9}{298} = 3.9159$$

Entropy at the state 2
$$s_2 = s_{f_2} + s_{fg_2} = 1.1242 + 3.9159$$
$$= 5.0401 \text{ kJ/kg·K}$$

Similarly, $s_{fg_1} = \frac{h_{fg_1}}{T_{sat_1}} = \frac{h_{g_1} - h_{f_1}}{T_{sat_1}}$
$$= \frac{1433.05 - 135.37}{263} = 4.9341$$

Entropy at the state 1
$$s_1 = s_{f_1} + x_1 s_{fg_1} = 0.5443 + 4.9341 x_1$$

For isentropic compression,
$$s_1 = s_2 = 5.0401 \text{ kJ/kg·K}$$
or $5.0401 = 0.5443 + 4.9341 x_1$
or $x_1 = 0.911$

Enthalpy at the state 1
$$h_1 = h_{f_1} + x_1 h_{fg_1}$$
$$= 135.37 + 0.911 \times (1433.05 - 135.37)$$
$$= 1317.55 \text{ kJ/kg}$$

(i) *COP of refrigerator*

The compression work
$$w_{in} = h_2 - h_1 = 1465.84 - 1317.55$$
$$= 148.28 \text{ kJ/kg}$$

Refrigeration effect
$$q_{in} = h_1 - h_4$$
$$= 1317.55 - 298.9 = 1018.65 \text{ kJ/kg}$$

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Therefore, $(COP)_R = \dfrac{q_{in}}{w_{in}} = \dfrac{1018.65}{148.28} = \mathbf{6.87}$

(ii) *Capacity of refrigerator*

$RE = \dot{m}_R \times q_{in}$

$= 5 \times 1018.65$

$= 5093.25$ kJ/min or **84.88 kW**

Example 13.8 *A refrigerator used R-12 as a working fluid and it operates on an ideal vapour compression cycle. The temperature of refrigerant in the evaporator is –20°C and in the condenser is 40°C. The refrigerant is circulated at the rate of 0.03 kg/s. Determine the coefficient of performance and capacity of refrigeration plant in the TR.*

Solution

Given A vapour compression cycle as shown on *T–s* diagram

$T_1 = -20°C = 253$ K

$T_3 = 40°C$

$\dot{m}_R = 0.03$ kg/s

To find

(i) $(COP)_R$, and

(ii) Capacity in *TR*.

Analysis From *R*-12 table, we have saturation states

At –20°C $h_1 = 178.61$ kJ/kg,

$s_1 = 0.7082$ kJ/kg

At 40°C $h_3 = h_{f_3} = 74.53$ kJ/kg

For process 1 – 2;

$s_1 = s_2 = 0.7082$ kJ/kg·K

It gives $h_2 = 211.38$ kJ/kg

The compression work

$w_{in} = h_2 - h_1 = 211.38 - 178.61$

$= 32.77$ kJ/kg

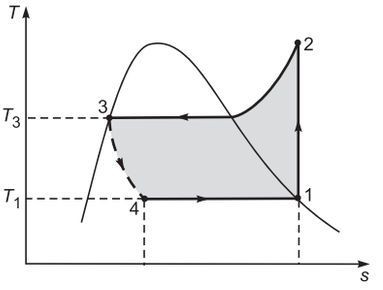

Fig. 13.13

For the process 3 – 4

$h_4 = h_3 = 74.53$ kJ/kg

Heat absorbed per kg of refrigerant

$q_{in} = h_1 - h_4$

$= 178.61 - 74.53 = 104.08$ kJ/kg

Therefore, $(COP)_R = \dfrac{q_{in}}{w_{in}} = \dfrac{104.08}{32.77} = \mathbf{3.176}$

Refrigerating Capacity,

$RE = \dot{m}_R \times q_{in} = (0.03 \text{ kg/s}) \times (104.08 \text{ kJ/kg})$

$= 3.12$ kJ/s $= 3.12$ kW

$= \dfrac{3.12}{3.5} = \mathbf{0.89\ TR}$

Example 13.9 *An ideal vapour compression system uses R-12 as the refrigerant. The system uses an evaporation temperature of 0°C and a condenser temperature of 40°C. The capacity of the system is 7 TR. Determine*

(a) The mass flow rate of refrigerant,

(b) Power required to run the compressor,

(c) Heat rejected in the condenser,

(d) COP of the system.

Use the properties of R-12 from the table given below:

Temp. °C	*Pressure* bar	h_f kJ/kg	h_g kJ/kg	s_f kJ/kg	s_g kJ/kg·K
0	3.087	36.05	187.53	0.142	0.696
40	9.609	74.59	203.2	0.727	0.682

Take C_p for superheated vapour as 0.6 kJ/kg·K.

Solution

Given An ideal vapour compression system as shown in Fig. 13.14.

$h_1 = 187.53$ kJ/kg.

$s_1 = 0.696$ kJ/kg·K

$h_3 = h_4 = 74.59$ kJ/kg·K

$s_{g_2} = 0.682$ kJ/kg·K

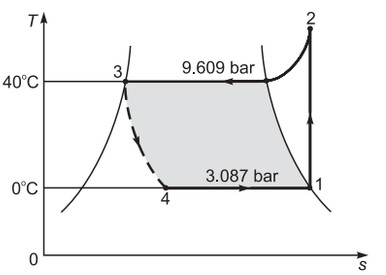

Fig. 13.14

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To find

(i) Mass flow rate of refrigerant,
(ii) Power input to compressor,
(iii) Heat rejected in the condenser,
(iv) *COP* of the system.

Assumptions

(i) Refrigerant leaving the evaporator as saturated vapour.
(ii) Compression is isentropic.
(iii) No pressure drop in condenser and evaporator.

Analysis The state 2 can be obtained by equating entropy at two states during compression.

$$s_1 = s_2 = s_g + C_{ps} \ln\left(\frac{T_{sup}}{T_{sat}}\right)$$

$$0.696 = 0.682 + 0.6 \times \ln\left(\frac{T_{sup}}{40 + 273}\right)$$

or $$\ln\left(\frac{T_{sup}}{313}\right) = 0.0233$$

or $$T_{sup} = 320.4 \text{ K}$$

The enthalpy after isentropic compression

$$h_2 = h_g + C_{ps}\,(T_{sup} - T_{sat})$$
$$= 203.2 + 0.6 \times (320.4 - 313)$$
$$= 207.63 \text{ kJ/kg}$$

(i) *Mass flow rate of refrigerant*

$$RE = 7\ TR$$
$$= 7 \times 210 \text{ kJ/min} = 1470 \text{ kJ/min}$$

The refrigerating effect per kg of refrigerant.

$$q_{in} = h_1 - h_4$$
$$= 187.53 - 74.59 = 112.94 \text{ kJ/kg}$$

The mass flow rate of refrigerant

$$\dot{m}_R = \frac{RE}{q_{in}} = \frac{1470}{112.94} = \mathbf{13.01 \text{ kg/min}}$$

(ii) *Power input to compressor*

$$P = \dot{m}_R(h_2 - h_1)$$
$$= \left(\frac{13.01}{60}\text{kg/s}\right) \times (207.63 - 187.53)\ (\text{kJ/kg})$$
$$= \mathbf{4.35 \text{ kW}}$$

(iii) *Heat rejected in the condenser*

$$= \dot{m}_R(h_2 - h_3)$$
$$= 13.01 \times (207.63 - 74.59)$$
$$= 1730.85 \text{ kJ/min}$$

(iv) *COP* of the system

$$(COP)_R = \frac{\text{Refrigerating effect}}{\text{Work input}} = \frac{h_1 - h_4}{h_2 - h_1}$$
$$= \frac{187.53 - 74.59}{207.63 - 187.53} = \mathbf{5.62}$$

Example 13.10 *A refrigerator operates between temperature limits of 30°C and –5°C. The refrigerant is 0.97 dry before leaving the evaporator coil. Find the condition of refrigerant entering the evaporator and COP of system. If the temperature rise of water circulating through the condenser is limited to 20°C, calculate mass flow rate of the coolant.*

Use the properties of refrigerant from table given below:

Temp.	*Enthalpy, kJ/kg*		*Entropym kJ/kg·K*		*Specific heat, kJ/kg·K*	
°C	h_f	h_g	s_f	s_g	$C_{p,L}$	$C_{p,g}$
30	323.22	1465.38	1.2037	4.9839	5.024	3.35
–5	158.26	1431.89	0.63	5.4072	–	–

Take C_p for superheated vapour as 3.35 kJ/kg·K.

Solution An ideal vapour compression system as shown in Fig. 13.15.

$$T_1 = -5°\text{C} = 268 \text{ K}$$
$$T_2 = 30°\text{C} = 303 \text{ K}$$
$$x_1 = 0.97$$
$$(\Delta T)_w = 20°\text{C},$$
$$C_{ps} = 3.5 \text{ kJ/kg·K}$$

To find

(i) Dryness fraction of refrigerant, x_4
(ii) *COP* of the system,
(iii) Coolant rate in the condenser.

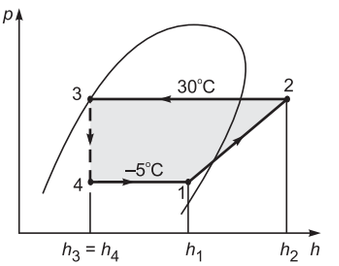

Fig. 13.15

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Assumptions

(i) Refrigerant leaving the evaporator as saturated vapour.
(ii) Compression is isentropic.
(iii) No pressure drop in condenser and evaporator.
(iv) Specific heat of water as $C_{pw} = 4.187$ kJ/kg·K.

Analysis The enthalpy at the state 1

$$h_1 = h_{f_1} + x_1(h_{fg_1} - h_{f_1})$$
$$= 158.26 + 0.97 \times (1431.89 - 158.26)$$
$$= 1393.68 \text{ kJ/kg}$$

The entropy at the state 1

$$s_1 = s_{f_1} + x_1(s_{fg_1} - s_{f_1})$$
$$= 0.63 + 0.97 \times (5.4072 - 0.63)$$
$$= 5.2639 \text{ kJ/kg·K}$$

The state 2 can be obtained by equating entropy at the two states during compression.

$$s_1 = s_2 = s_g + C_{ps} \ln\left(\frac{T_{sup}}{T_{sat}}\right)$$

$$5.2639 = 4.9839 + 3.35 \times \ln\left(\frac{T_2}{30 + 273}\right)$$

or $$\ln\left(\frac{T_2}{303}\right) = 0.0836$$

or $$T_2 = 329.41 \text{ K}$$

The enthalpy after isentropic compression

$$h_2 = h_g + C_{ps}(T_{sup} - T_{sat})$$
$$= 1465.38 + 3.35 \times (329.41 - 303)$$
$$= 1553.86 \text{ kJ/kg}$$

Enthalpy at the end of condensation at the state 3;

$$h_3 = h_{f@\,30°C} = 323.22 \text{ kJ/kg·K}$$

Enthalpy after throttling, at the state 4;

$$h_4 = h_3 = 323.22 \text{ kJ/kg·K}$$

(i) *State of refrigerant after throttling*

Enthalpy at state 4 can be expressed as;

$$h_4 = h_{f_4} + x_4(h_{fg_4} - h_{f_4})$$
$$323.22 = 158.26 + x_4 \times (1431.89 - 158.26)$$
or $$x_4 = \mathbf{0.13}$$

(ii) *COP of refrigerator*

The refrigerating effect per kg of refrigerant.

$$q_{in} = h_1 - h_4$$
$$= 1393.68 - 323.22 = 1070.46 \text{ kJ/kg}$$

Work input per kg of refrigerant

$$w_{in} = h_2 - h_1$$
$$= 1553.86 - 1393.68 = 160.18 \text{ kJ}$$

$$(COP)_R = \frac{q_{in}}{w_{in}} = \frac{1070.46}{160.18} = \mathbf{6.685}$$

(iii) *Mass flow rate of water in condenser*

Heat rejected per kg of refrigerant

$$q_{out} = h_2 - h_3 = 1553.86 - 323.22$$
$$= 1230.64 \text{ kJ/kg}$$

Water flow rate per kg of refrigerant

$$q_{out} = \dot{m}_w C_{pw} (\Delta T)_w$$
$$1230.64 = \dot{m}_w \times 4.187 \times (20°C)$$
or $$\dot{m}_w = \mathbf{14.7 \text{ kg/kg of refrigerant}}$$

Example 13.11 *A simple R-12 plant is to develop 4 tonnes of refrigeration. The condenser and evaporator temperatures are 35°C and –15°C, respectively. Determine*

(a) The mass flow rate of refrigerant in kg/s,
(b) Volume flow rate handled by compressor in m^3/s,
(c) The compressor discharge temperature,
(d) The pressure ratio,
(e) Heat rejected to condenser in kW,
(f) Flash gas percentage after throttling,
(g) The COP, and
(e) Power required to drive the compressor.

Compare this COP with COP of Carnot refrigerator operating between temperatures of 35°C and –15°C.

Solution

Given A vapour compression cycle as shown on the *p–h* diagram in Fig. 13.16.

$$T_1 = -15°C = 258 \text{ K}$$
$$T_3 = 35°C = 308 \text{ K}$$
$$RE = 4 \text{ TR} = 4 \times 3.51 = 14.04 \text{ kW}$$

To find

(i) The mass flow rate of refrigerant in kg/s,
(ii) Volume flow rate handled by compressor in m^3/s,
(iii) The compressor discharge temperaturem T_2,
(iv) The pressure ratio p_2/p_1,
(v) Heat rejected to condenser in kW,
(vi) Flash gas percentage after throttling,
(vii) The *COP*,
(viii) Power required to drive the compressor,
(ix) *COP* of Carnot refrigerator, and its comparison with *COP* of VCC.

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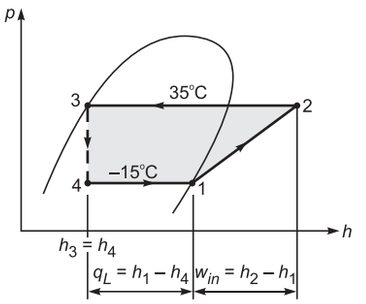

Fig. 13.16

Analysis From Table B-4, and *p–h* diagram, the properties at various states.

At –15°C $h_1 = h_{g_1} = 345$ kJ/kg,

$s_1 = 1.563$ kJ/kg

$v_{g_1} = 0.091$ m^3/kg

$p_1 = 1.8$ bar

$h_{f_1} = 186.28$ kJ/kg

We have $s_1 = s_2 = 1.563$ kJ/kg, from *p–h* chart, we get

$h_2 = 373$ kJ/kg

$T_2 = 46°C$

$p_2 = 9$ bar

At 35°C $h_3 = h_{f_3} = 233.5$ kJ/kg

For process 3 – 4

$h_4 = h_3 = 233.5$ kJ/kg

(i) *The mass flow rate of refrigerant in kg/s*

Heat removed per kg of refrigerant

$$q_{in} = h_1 - h_4 = 345 - 233.5 = 111.5 \text{ kJ/kg}$$

The mass flow rate of refrigerant

$$\dot{m}_R = \frac{\text{Refrigerating effect}}{\text{Heat removed per kg of air}} = \frac{RE}{q_{in}} = \frac{14.04 \text{ kJ/s}}{111.5 \text{ kJ/kg}} = \mathbf{0.126 \text{ kg/s}}$$

(ii) The volume flow rate of refrigerant

$$\dot{V}_R = \dot{m}_R v_{g_1} = 0.126 \times 0.091 = \mathbf{0.01145 \text{ m}^3\text{/s}}$$

(iii) *The compressor discharge temperature* $T_2 = \mathbf{46°C}$

(iv) *The pressure ratio* p_2/p_1

$$\frac{p_2}{p_1} = \frac{9 \text{ bar}}{\text{al.8 b r}} = \mathbf{5}$$

(v) *Heat rejected to condenser*

Heat rejected per kg of refrigerant

$$q_{out} = h_2 - h_3 = 373 - 233.5 = 139.5 \text{ kJ/kg}$$

Heat removal rate

$$\dot{Q}_{out} = \dot{m}_R q_{out} = 0.126 \times 139.5 = \mathbf{17.5 \text{ kW}}$$

(vi) *Flash gas percentage after throttling*

$$h_3 = h_{f_1} + x\,(h_{g_1} - h_{f_1})$$

$$233.5 = 186.28 + x\,(345 - 186.28)$$

or $$x = \frac{233.5 - 186.28}{345 - 186.28} = \mathbf{0.297}$$

(vii) *COP*

The compression work

$$w_{in} = h_2 - h_1 = 373 - 345 = 28 \text{ kJ/kg}$$

$$(COP)_R = \frac{q_{in}}{w_{in}} = \frac{111.5 \text{ kJ/kg}}{28 \text{ kJ/kg}} = \mathbf{3.982}$$

(viii) *Power input to refrigerator*

$$\dot{W}_{input} = \dot{m}_R w_{in} = (0.126 \text{ kg/s}) \times (28 \text{ kJ/kg}) = \mathbf{3.528 \text{ kW}}$$

(ix) *COP of Carnot refrigerator*

$$(COP)_{R,Carnot} = \frac{T_1}{T_2 - T_1} = \frac{258 \text{ K}}{(308 - 258)\,(\text{K})} = \mathbf{5.16}$$

$$\text{Relative } COP = \frac{(COP)_R}{(COP)_{R,Carnot}} = \frac{3.982}{5.16} = \mathbf{0.771}$$

13.7.3 Effect of Operating Conditions on Vapour Compression Cycle

The following analysis will show the effect of change in operating conditions on the performance of the vapour compression cycle.

(a) Effect of Evaporator Pressure

The evaporator pressure is reduced to $p_{1'}$ from the existing pressure p_1. The effect on the cycle is shown on *T–s* and *p–h* diagrams of Fig. 13.17. By reducing the evaporator pressure, the evaporator temperature decreases (favourable) but the refrigeration effect ($h_1' - h_4'$) (area under the curve on *T–s* diagram) decreases. The work input to the compressor ($h_2' - h_1'$) increases. The specific volume of vapour at low pressure is large, thus the volumetric efficiency will also decrease. Therefore, it is not desirable to decrease the evaporator pressure.

(b) Effect of Condenser Pressure

It is evident from *T–s* and *p–h* diagrams of

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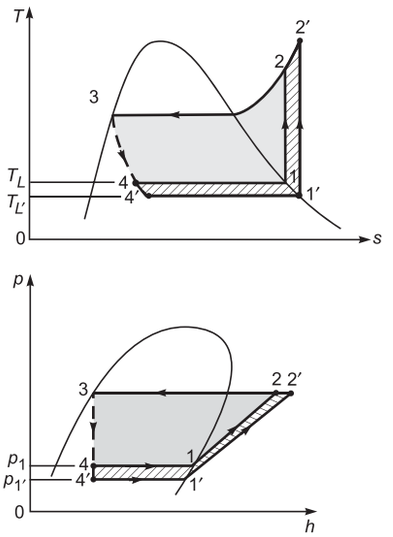

Fig. 13.17 *Effect of evaporator pressure*

Fig. 13.18, that by increasing the condenser pressure

(i) The work input to the compressor increases from 1–2 to 1–2′.

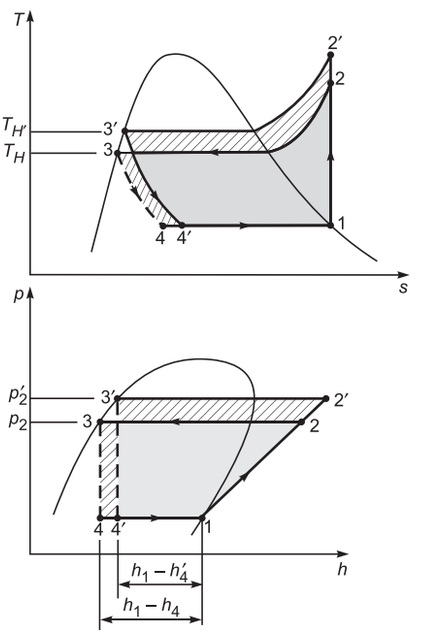

Fig. 13.18 *Effect of condenser pressure*

(ii) Refrigerant effect decreases from 4–1 to 4′–1.

(iii) The condenser temperature increases, thereby increasing heat rejection.

(iv) The *COP* of the system decreases.

(*c*) *Effect of Suction Vapour Superheat*

The effect of superheated vapour at the compression suction is shown with the help of *T–s* and *p–h* diagrams of Fig. 13.19.

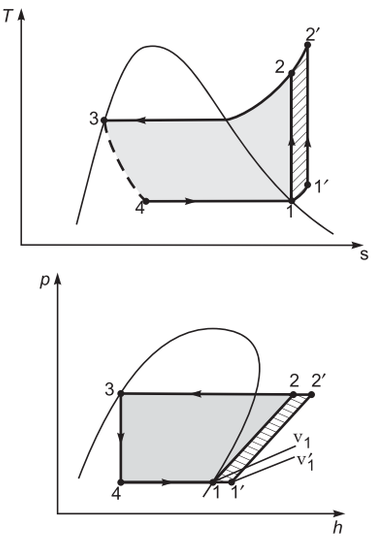

Fig. 13.19 *Effect of suction vapour superheat*

- It increases the specific volume of refrigerant vapour at suction from v_1 to v_1', thus the compression work input increases as shown by hatched area.
- It increases refrigeration effect from $(h_1 - h_4)$ to $(h_1' - h_4)$.
- The *COP* of the new system is given by

$$COP' = \frac{h_1' - h_4}{h_2' - h_1'}$$

$$= \frac{(h_1 - h_4) + (h_1' - h_1)}{(h_2 - h_1) + [(h_2' - h_1') - (h_2 - h_1)]} \quad ...(13.18)$$

Both the numerator and denominator increase, thus the *COP* of the new system may increase, decrease or remain the same.

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It is the practical necessity to allow the refrigerant vapour to become slightly superheated at the suction of the compressor in order to avoid any carry over of liquid refrigerant into the compressor. The amount of superheat should be kept minimum in order to keep the compression work minimum.

(*d*) *Effect of Liquid Subcooling*

The condensed liquid can be cooled to a temperature below the saturation temperature corresponding to the condenser pressure. This effect is shown with the help of *T–s* and *p–h* diagrams of Fig. 13.20 in which the constant pressure line is shown left to the saturated liquid line. The effect of *subcooling* (also called *undercooling*) is to move the line 3-4 (throttling process) to the left of the diagrams. It increases the refrigeration effect without increase in compression work. Thus the *COP* of the system increases. Therefore, the subcooling of condensed liquid is desirable.

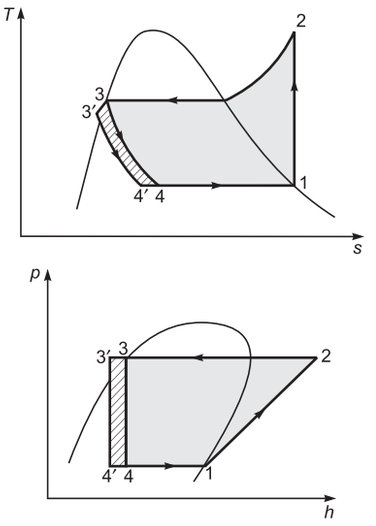

Fig. 13.20 *Effect of subcooling of the condensed liquid*

Example 13.12 *The pressure in the evaporator of an ammonia refrigerator is 1.902 bar and the pressure in the condenser is 12.37 bar. Calculate the refrigeration effect per unit mass of the refrigerant and $(COP)_R$ for the following cycles:*

(a) The dry saturated vapour delivered to the condenser after isentropic compression and no undercooling of the condensed liquid and then throttling of refrigerant to evaporator pressure.

(b) The dry saturated vapour delivered to the compressor, where it is compressed isentropically to the condenser pressure.

(c) The dry saturated vapour delivered to the compressor and liquid after condensation is undercooled by 10°C.

Solution

Given A refrigerator operating between a condenser pressure of 12.37 bar and evaporator pressure of 1.902 bar.

To find

(i) Refrigerating effect/ kg of refrigerant.

(ii) *COP* of the system.

Assumptions

(i) Each component in the all cycles is analysed as a control volume at steady state.

(ii) Compression, condensation and evaporation processes are internally reversible.

(iii) The kinetic and potential energy changes are negligible.

Analysis

(i) *The dry saturated vapour delivered to the condenser and after condensation throttled without sub-cooling*

At 12.37 bar from properties of ammonia; Table B-10,

$h_3 = h_f = 332.8$ kJ/kg·K

$h_2 = h_g = 1469.9$ kJ/kg·K

$s_2 = 4.962$ kJ/kg·K

At 1.902 bar

$h_{f_1} = 89.8$ kJ/kg,

$h_g = 1420$ kJ/kg

$s_{f_1} = 0.368$ kJ/kg·K,

$s_g = 5.623$ kJ/kg·K

(a) The quality of refrigerant at the state 1;

$$s_2 = s_1 = s_{f_1} + x_1 s_{fg_1} = s_{f_1} + x_1 (s_{g_1} - s_{f_1})$$

$$4.962 = 0.368 + x_1 (5.623 - 0.368)$$

or $x_1 = 0.874$

The specific enthalpy of refrigerant at the state 1;

$$h_1 = h_{f_1} + x_1 (h_{g_1} - h_{f_1})$$

$$= 89.8 + 0.874 \times (1420 - 89.8)$$

$$= \mathbf{1251.8\ kJ/kg}$$

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The refrigerating effect

$$q_{in} = h_1 - h_4 = h_1 - h_3 \quad (\because h_4 = h_3)$$
$$= 1251.8 - 332.8 = \mathbf{919\ kJ/kg}$$

(b) The *COP* of the system;

$$(COP)_R = \frac{\text{Refrigeration effect}}{\text{Work input}} = \frac{q_{in}}{h_2 - h_1}$$
$$= \frac{919}{1469.9 - 1251.8} = \mathbf{4.21}$$

(ii) *The dry saturated liquid delivered to the compressor*

The properties of the refrigerant, Table B-10, 11

$h_1 = h_f$ @ 1.902 bar = 1420 kJ/kg
$h_3 = h_f$ @ 12.37 bar = 332.8 kJ/kg
$h_4 = h_3$ = 332.8 kJ/kg
$s_1 = s_g$ = 5.623 kJ/kg·K @ 1.902 bar
s_{g_2} = 4.962 kJ/kg·K

s_{g_2} @ 12.37 < s_2 = 5.623

Thus state 2 is superheated.

Using *p-h* chart

h_2 = 1698.5 kJ/kg

(a) The refrigeration effect;

$$q_{in} = h_1 - h_4 = 1420 - 332.8$$
$$= \mathbf{1087.2\ kJ/kg}$$

(b) The coefficient of performance;

$$COP = \frac{\text{Refrigeation effect}}{\text{Work input}} = \frac{q_{in}}{h_2 - h_1}$$
$$= \frac{1087.2}{1698.5 - 1420} = \mathbf{3.9}$$

(iii) *Dry saturated refrigerant vapour delivered to compressor and liquid after condensation is undercooled by 10°C*

h_1 = 1420 kJ/kg
h_2 = 1698.5 kJ/kg
$h_3 = h_4$
$h_3 = h_f$ @ T_3
$T_3 = T_3' - 10 = 32 - 10 = 22°C$

h_3 @ 22°C = 284.6 kJ/kg

(a) Refrigeration effect

$$q_{in} = h_1 - h_4$$
$$= 1420 - 284.6 = \mathbf{1135.4}\ \text{kJ/kg}$$

(b) *COP* of the system

$$(COP)_R = \frac{q_{in}}{h_2 - h_1} = \frac{1135.4}{1698.5 - 1420} = \mathbf{4.08}$$

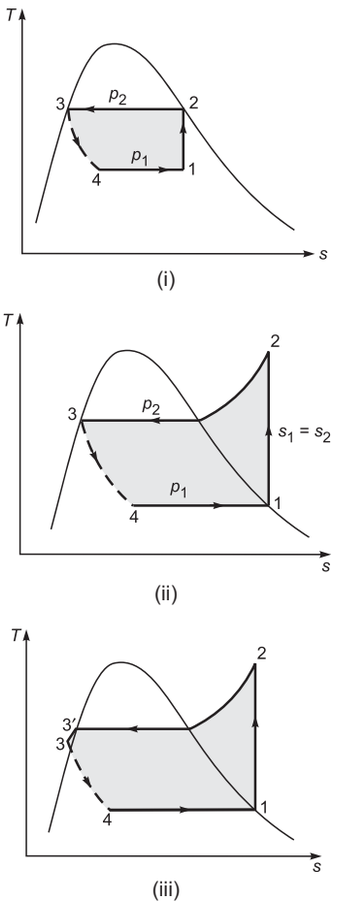

Fig. 13.21

Example 13.13 *Consider a vapour compression refrigeration system using R-12 as a refrigerant. The maximum and minimum pressure of the cycle are 8 bar and 1.2 bar respectively. At the compressor inlet , the vapour temperature is –12°C and the temperature of liquid at the condenser outlet is 30°C. The required refrigerating load is 2.2 kW. The compressor runs to 600 rpm and has a volumetric efficiency of 75%. Find COP and swept volume of compressor.*

Solution

Given A vapor compression refrigeration system:

p_1 = 1.2 kPa, p_2 = 8 bar
T_1 = –12°C = 285 K, T_3 = 30°C
RE = 2.2 kW, η_{vol} = 0.75, N = 600 rpm

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To find

(i) $(COP)_R$, and

(ii) Swept volume of compressor.

Assumptions

(i) Each component in the all cycle is analysed as a control volume at steady state.

(ii) Compression, condensation and evaporation processes are internally reversible.

(iii) The kinetic and potential energy changes are negligible.

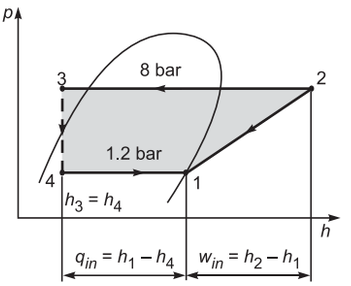

Fig. 13.22

Analysis Properties of refrigerant *R*-12 at end state:

$h_1 = 354$ kJ/kg, $v_{g_1} = 0.135$ m^3/kg

$h_2 = 388$ kJ/kg, $h_3 = h_4 = 226$ kJ/kg,

(i) *COP of the system*

Heat removed per kg of refrigerant

$$q_{in} = h_1 - h_4 = 354 - 226 = 128 \text{ kJ/kg}$$

The compression work

$$w_{in} = h_2 - h_1 = 388 - 354 = 34 \text{ kJ/kg}$$

$$(COP)_R = \frac{q_{in}}{w_{in}} = \frac{128 \text{ kJ/kg}}{28 \text{ kJ/kg}} = \mathbf{3.764}$$

(ii) *Swept volume of compressor*

The mass flow rate of refrigerant

$$\dot{m}_R = \frac{\text{Refrigerating effect}}{\text{Heat removed per kg of air}}$$

$$= \frac{RE}{q_{in}} = \frac{2.2 \text{ kJ/s}}{128 \text{ kJ/kg}} = 0.01718 \text{ kg/s}$$

Volume flow rate through compressor

$$\dot{V}_R = \dot{m}_R v_{g_1} = 0.01718 \times 0.135$$

$$= 0.00232 \text{ m}^3/\text{s}$$

The volumetric efficiency is given by

$$\eta_{vol} = \frac{\text{Actual volume rate}}{\text{Swept volume rate}} = \frac{\dot{V}_R}{\dot{V}_s}$$

or $$\dot{V}_s = \frac{\dot{V}_R}{\eta_{vol}} = \frac{0.00232}{0.75} = 0.00309 \text{ m}^3/\text{s}$$

Swept volume rate can be expressed as

$$\dot{V}_s = V_s \frac{N}{60}$$

or $$V_s = 0.00309 \times \frac{60}{600}$$

$$= 0.000309 \text{ m}^3 \text{ or } \mathbf{309 \text{ mm}^3}$$

13.7.4 Actual Vapour Compression Cycle: Deviation From Ideal Cycle

The actual refrigeration cycle deviates from the ideal vapour compression cycle, because of pressure drop in tubes associated with fluid friction and heat transfer to or from the surroundings. Further, the compression will be polytropic involving friction and heat transfer, thus migrating from an isentropic one. The actual vapour compression cycle may have some or all the processes deviate from an ideal ones. The actual cycle might be approached one shown in Fig. 13.23 with the help of (b) *T–s* diagram and (c) *p–h* diagram.

The refrigerant leaving the evaporator at the state 7 is slightly superheated in order to ensure completely dry refrigerant vapour entry to the compressor. In tube line connecting the evaporator with the compressor, the fluid friction causes a pressure drop of Δp_{ev} and temperature difference between refrigerant and surroundings causes a heat transfer to the refrigerant from the state 7 to 8. As a result, the refrigerant vapour at the state 8 gets superheated further, causing an increase in its specific volume, and thus increase in compression work.

Due to suction effect of the compressor, the sudden pressure drop from the state 8 to the state 1 takes place. The compression begins from the state 1 and continues to the state 2. Actual compression is polytropic with friction and heat transfer, and thus departs from isentropic compression. At the compressor discharge, a sudden pressure drop is observed from the state 2 to the state 3.

During the process 3–4, the refrigerant is first desuperheated to the state 3′ and then condensed during the process 3′-4. A pressure drop further

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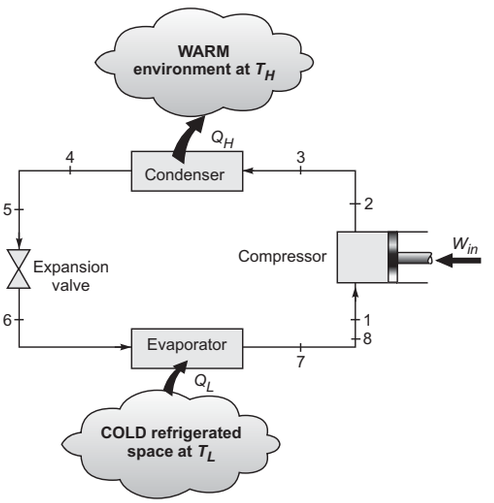
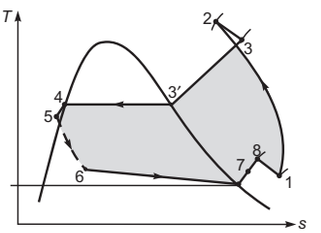
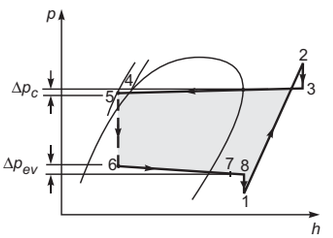


(a) Schematic arrangement of actual vapour-compression cycle



(b) Actual VCC on T–s diagram



(c) Actual VCC on p-h diagram

Fig. 13.23 *Actual vapour compression cycle*

takes place in the tube line connecting the condenser and throttle valve. The liquid refrigerant is slightly subcooled in the condenser from the state 4 to 5 in order to have at least saturated liquid before throttling at the state 5.

The throttling process is represented by a dotted line 5–6 and the refrigeration effect starts from the state 6 to 7. A slight pressure drop due to friction during evaporation makes the process 6–7 irreversible.

Further, in order to have the effective heat transfer between the refrigerant and hot or cold region, the refrigerant temperature in the evaporator is less than the cold region and that in the condenser is higher than the warm atmospheric temperature. It makes the actual cycle further irreversible.

Example 13.14 *Refrigerant 134a is the working fluid in an ideal vapour compression refrigeration cycle, that operated between a cold region at 0°C and a warm region at 26°C. The saturate a vapour enters the compressor at –10°C and the saturate liquid leaves the condenser at a pressure of 9 bar. Determine for $\dot{m}$ = 0.08 kg/s (a) compression power in kW, (b) refrigeration capacity in tonnes and (c) coefficient of performance.*

Solution

Given An ideal vapour compression refrigeration cycle operates with refrigerant 134*a* as the working fluid

Mass flow rate, $\dot{m}$ = 0.08 kg/s

Cold region temperature = 0°C

Warm region temperature = 26°C

Temperature of refrigerant vapour at compressor inlet, $T_1 = -10°C$

Pressure of saturate liquid leaving the condenser = 9 bar

To find

(i) Power input in compressor in kW,

(ii) Refrigerating capacity in tonnes,

(iii) *COP* of the system.

Schematic with given data

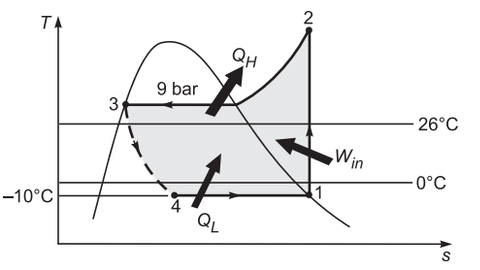


Fig. 13.24

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Assumptions

(i) All the components of the cycle are analysed as a control volume at steady state.
(ii) All processes except throttling process are internally reversible.
(iii) Kinetic and potential energy effects are negligible.
(iv) Saturate vapour leaves the evaporator and saturate liquid leaves the condenser.

Analysis The properties of refrigerant 134*a*

State 1: Saturate refrigerant vapour $T_1 = -10°C$

$h_1 = 241.35$ kJ/kg $\quad s_1 = 0.9253$ kJ/kg·K

State 2: Superheated refrigerant vapour $p_2 = 9$ bar

$s_2 = s_1 = 0.9253$

$h_2 = 272.4$ kJ/kg (by interpolation)

State 3: Saturate liquid at 9 bar

$h_3 = 99.56$ kJ/kg

State 4: Liquid vapour mixture after throttling

$h_4 = h_3 = 99.56$ kJ/kg

(i) Compressor work input;

$$\dot{W}_{comp} = \dot{m}(h_2 - h_1)$$
$$= 0.08 \times (272.4 - 241.35)$$
$$= \mathbf{2.484\ kW}$$

(ii) Refrigerating capacity;

$$RE = \dot{m}(h_1 - h_4)$$
$$= 0.08\ (241.35 - 99.56)$$
$$= 11.343\ \text{kW}$$
$$= \frac{(11.34\,\text{kW})}{(3.5\,\text{kW/ton})}$$
$$= \mathbf{3.24}\ \text{ton of Refrigeration}$$

(iii) Coefficient of performance;

$$(COP)_R = \frac{\text{Refrigerating effect}}{\text{Work input}}$$
$$= \frac{11.343}{2.484} = \mathbf{4.567}$$

Example 13.15 *The refrigerant R-12 enters the compressor of a refrigerator as a superheated vapour at 0.14 MPa and –20°C at a rate of 0.05 kg/s and leaves at 0.8 MPa and 50°C. The refrigerant is cooled in the condenser to 26°C and 0.72 MPa and is throttled to 0.15 MPa. Neglecting any heat transfer and pressure drop in the connecting line between the components, determine*

(a) rate of heat removal from the refrigerated space,
(b) power input to the compressor,
(c) the isentropic efficiency of the compressor,
(d) coefficient of performance of the refrigerator.

Solution

Given A refrigerator operates with *R*-12 refrigerant with the operating conditions

$\dot{m}_R = 0.05$ kg/s

To find

(i) Refrigeration effect,
(ii) Work input to the compressor,
(iii) Isentropic efficiency of the compressor,
(iv) *COP* of the system.

Assumptions

(i) All the components of the cycle are analysed as a control volume in steady state.
(ii) Neglect kinetic and potential energy effects.

Schematic with given data

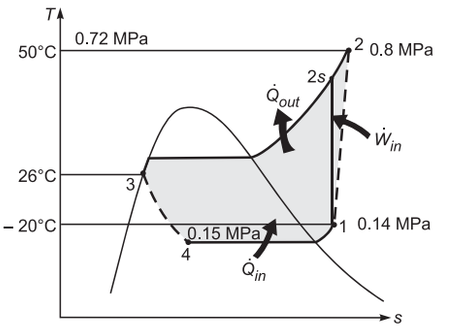

Fig. 13.25

Analysis The properties of *R*-12 from Table B-5

State 1: Superheated vapour,

$p_1 = 0.14$ MPa,

$T_1 = -20°C$

$h_1 = 179.01$ kJ/kg, $\quad s_1 = 0.7147$ kJ/kg.

State 2: Superheated compressed vapour at

$p_2 = 0.8$ MPa, $\quad T_2 = 50°C$

$h_2 = 213.45$ kJ/kg

$h_{2s} = 210.08$ kJ/kg $\quad s_2 = s_1 = 0.7147$ kJ/kg·K

State 3: Undercooled liquid refrigerant from Table B-4

$p_3 = 0.72$ MPa, $T_3 = 26°C$

$h_3 = h_f @\ 26°C = 60.68$ kJ/kg

State 4: Liquid-vapour mixture of refrigerant after throttling

$h_4 = h_3 = 60.68$ kJ/kg

(i) Refrigerating effect

$$RE = \dot{Q}_{in} = \dot{m}_R\,(h_1 - h_4)$$

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$= 0.05 \times (179.01 - 60.68)$
$= \mathbf{5.92\ kW}$ or $\mathbf{1.69\ TR}$

(ii) Work input in compressor

$$\dot{W}_{in} = \dot{m}_r(h_2 - h_1)$$
$$= 0.05 \times (213.45 - 179.01)$$
$$= \mathbf{1.722\ kW}$$

(iii) Isentropic efficiency η_C of the compressor

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{210.08 - 179.01}{213.45 - 179.01}$$
$$= 0.902 = \mathbf{90.2\%}$$

(iv) Coefficient of performance

$$(COP)_R = \frac{RE}{\dot{W}_{in}} = \frac{5.92\ \text{kW}}{1.722\ \text{kW}} = \mathbf{3.43}$$

13.7.5 Advantages and Disadvantages of Vapour Compression Refrigeration System over Air-Refrigeration System

The followings are the advantages and disadvantages of vapour compression refrigeration system over air refrigeration system:

Advantages

1. The vapour compression refrigeration system has a high value of coefficient of performance.
2. The vapour compression refrigeration system approaches reversed Carnot cycle except for expansion of refrigerant in expansion device.
3. The size of refrigeration system per tonne of refrigeration is smaller.
4. It has less running cost.
5. Since refrigerant has latent heat while air has sensible heat only.
6. The refrigerant has large value of latent heat at lower pressure, and heat is absorbed by evaporation of low pressure liquid refrigerant.
7. The temperature of refrigerant in the evaporator can easily be controlled by regulating the expansion valve.

Disadvantages

1. It has high initial cost.
2. The leakage of refrigerant may cause harmful effect.
3. The production of some refrigerant may be hazardous to the environment.

13.8 VAPOUR ABSORPTION REFRIGERATION CYCLE

In a vapour compression refrigeration cycle, the temperature of saturated vapour leaving the evaporator is increased by a compression process. Since the specific volume of vapour is relatively large, therefore, the input work to the compressor is also large. The input work to the compressor can be reduced significantly, if the refrigerant is compressed in liquid state. The absorption refrigeration is based on this approach.

A vapour absorption system operates with a condenser, a throttle valve and an evaporator in the same way as vapour compression system, but the compressor is replaced by an absorber, pump and generator units as shown in Fig. 13.26.

The low-pressure refrigerant vapour leaving the evaporator is absorbed by a secondary substance, called an *absorbent* to form a strong liquid solution. This liquid solution is then pumped at a higher pressure to the generator. The specific volume of the liquid solution is much less than that of a refrigerant vapour, and thus significant less work

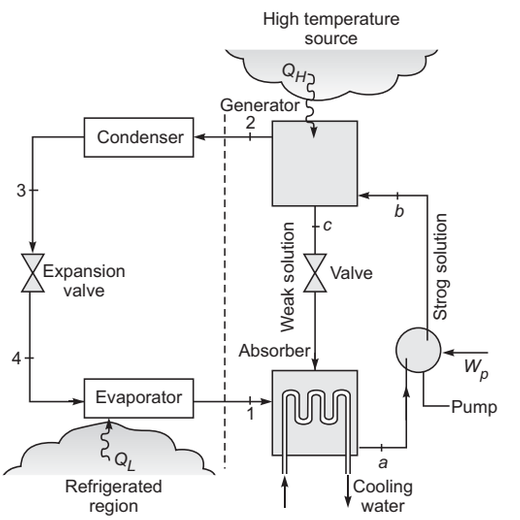

Fig. 13.26 *Schematic a vapour absorption refrigeration cycle*

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is required in the pump. The heat is supplied in the generator, where the refrigerant vapourises from the solution and leaves weak solution in the generator. The refrigerant vapour enters the condenser and the weak solution is again sent back to the absorber through a pressure relief valve.

The coefficient of vapour absorption refrigeration system can be expressed as

$$(COP)_R = \frac{\text{Heat absorbed in evaporator}}{\text{Heat input in generator + pump work}} = \frac{Q_L}{Q_H + W_p}$$

13.8.1 Ammonia–water Absorption Refrigeration System

Figure 13.27 shows a schematic arrangement of ammonia–water vapour absorption system using the solar energy for generator heating. The ammonia is used as refrigerant and water is used as absorbent. In the absorber, the ammonia vapour coming out the evaporator at the state 1 is absorbed by liquid water. The formation of this liquid solution is exothermic, thus heat is released. The solvency of ammonia in the water decreases as temperature increases. Thus the cooling is required in the absorber to absorb the energy released due to absorption of ammonia in the water. The strong ammonia–water solution is pumped to the generator through a heat exchanger, where it is preheated with the help of hot weak solution returning to absorber, thereby, reducing the heat supply in the generator. In the generator, the heat transfer from the source (solar energy) drives the ammonia vapour out of the solution (endothermic process), leaving a weak ammonia–water solution in the generator.

The ammonia vapour liberated passes to the condenser at the state 2 through a *rectifier*. The rectifier removes the traces of water from the refrigerant if any, before it enters the condenser. It avoids the formation of ice in the system. The remaining weak solution returns back to the absorbent through heat exchanger and valve. The condensed ammonia is expanded through an expansion valve and then enters the evaporator, where it absorbs heat from the low-temperature region.

13.8.2 Electrolux Refrigeration System

The electrolux absorption principle works on a three-fluid system. It uses natural circulation

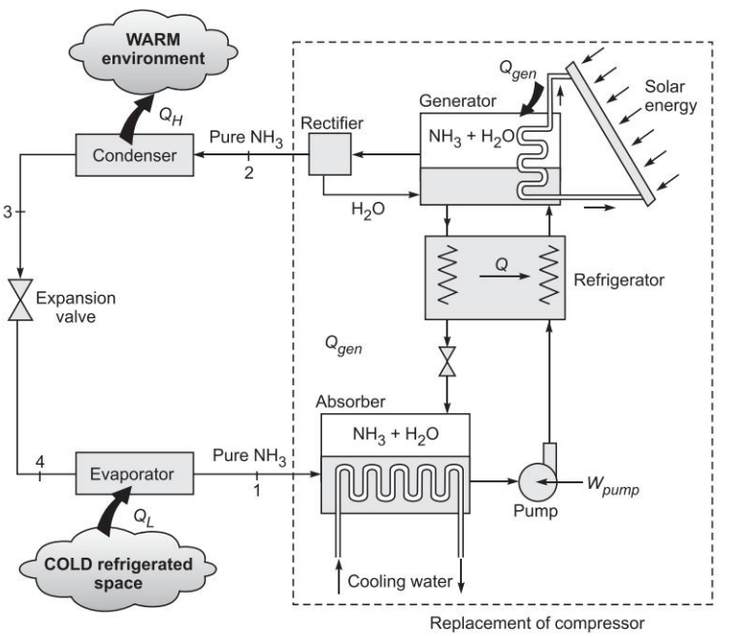

Fig. 13.27 *Ammonia-absorption refrigeration cycle*

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in absence of any pump. Figure 13.28 shows a schematic of an electrolux refrigeration system. The refrigerant used in the system is ammonia, the absorbent is water and the third fluid hydrogen remains mainly in the evaporator for reducing the partial pressure of the refrigerant to enable it to evaporate at low pressure and low temperature. The total pressure is constant throughout the system.

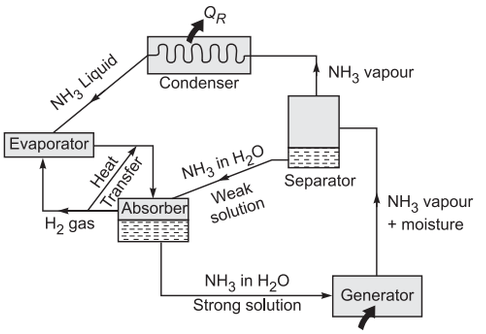

Fig. 13.28 *Electrolux refrigeration system*

The ammonia liquid leaving the condenser enters the evaporator, where it evaporates in the presence of H_2 gas at low temperature corresponding to its partial pressure. The ammonia and H_2 gas mixture passes to the absorber, where it mixes with weak ammonia–water solution coming from the separator. The water absorbs the ammonia vapour and the solution becomes strong. The H_2 gas separates from the solution and returns to the evaporator. The strong ammonia solution passes the generator, where it is heated externally and ammonia vapour drives out and rises to the separator. The moisture from the vapour is separated out and a weak ammonia solution returns back to the absorber. The ammonia vapour from the separator enters the condenser, where it is condensed and then returns to the evaporator.

With the certain modifications, the system can be used in places, where electricity is not available. It requires energy only in the form of heat (waste heat or solar energy) and no pump is necessary.

13.9 COMPARISON OF VAPOUR ABSORPTION SYSTEM WITH VAPOUR COMPRESSION SYSTEM

S.No.	*Aspect*	*Vapour Absorption System*	*Vapour Compression System*
1.	Energy input	The vapour absorption system takes in low-grade energy such as waste heat from furnace, exhaust steam or solar heat for its operation.	Vapour compression system takes in high grade energy such as electrical or mechanical energy for operation of compressor used in the cycle.
2.	Moving part	It uses a small pump as moving part, which is run by a small motor.	It uses a compressor driven by an electric motor or engine.
3.	Evaporator pressure	It can operate with reduced evaporator pressure, with little decrease in refrigeration capacity.	The refrigeration capacity decreases with lowered evaporator pressure.
4.	Load variation	The performance of vapour absorption system does not change with load variation.	The performance of vapour compression system is very poor at partial load.
5.	Evaporator exit	In vapour absorption system, the liquid refrigerant leaving the evaporator does not put any bad effect on the system except to reduce the refrigeration effect.	In a vapour compression system, it is desirable to superheat vapour before leaving the evaporator, so no liquid can enter the compressor.
6.	*COP*	The *COP* of the system is poor.	The *COP* of the system is excellent.
7.	Capacity	It can be built in capacities well above 1000 *TR*.	For a single compression system, it is not possible to have a system with more than 1000 *TR* capacity.
8.	Refrigerant	Water or ammonia is used as refrigerant.	Chlorofluorocarbon, hydrochlofluoro carbon and hydrofluorocarbon are used in most of the system.
9.	Lowest temperature	Since water is used as refrigerant, thus the lowest temperature attained is above 0°C.	With cascading, the temperature can be lowered upto −150°C or even less temperature.

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13.10 STEAM JET REFRIGERATION

It uses water as refrigerant, which is quite safe like air. If the pressure exerted on the surface of water is reduced then saturation temperature also lowers and water starts evaporating at lower temperature due to reduced pressure. It is the basic principle for steam jet refrigeration system.

The layout of steam jet refrigeration system is shown in Fig. 13.29. It consists of an evaporator (flash chamber), a steam nozzle, an ejector and a condenser. The steam expands through a nozzle to form a high speed, jet, which draws (sucks) the water from the flash chamber into the ejector. Therefore, the pressure in the flash chamber gets reduced, thereby results into further formation of vapour (evaporation) in the (flash chamber). This evaporation extracts heat (latent heat which is higher at lower pressures) for phase change, thus reducing the temperature of water in the flash chamber. The cold water is used for refrigeration. The mixture of steam and water vapour is diffused in the diverging part of the ventury tube to the exhaust pressure and fed to the condenser. After condenser some of water is returned to flash chamber as a make up water, and rest is used as feed water to the boiler.

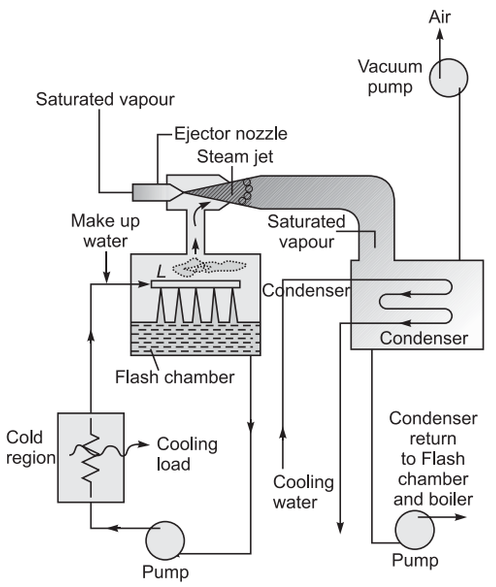

Fig. 13.29 *Steam jet refrigeration system*

The steam jet refrigeration system uses waste steam returning to condenser. Thus it is quite cheap, but the steam jet refrigeration systems are not used, when temperature below 5°C is required. It is widely used in food precessing plants for precooling of vegetables and concentration of fruit juices, gas plants, paper mills, breweries etc.

13.11 HEAT PUMP

A refrigerator maintains a region at low temperature by removing heat Q_L and it rejects heat Q_H to a high-temperature environment. However, the same basic cycle could maintain a region at higher temperature by supplying heat Q_H, while absorbing heat Q_L from low temperature medium such as atmosphere, lake, etc. Then the device, which maintains a region such as commercial building at higher temperature than that of its surroundings is called a *heat pump*. Figure 13.30 shows a schematic of a heat pump.

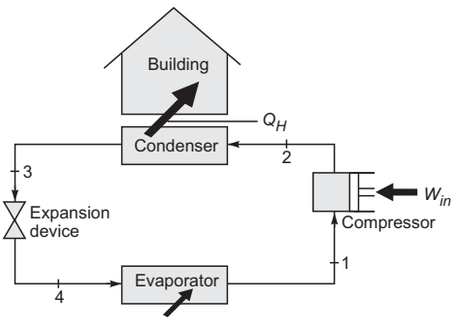

Fig. 13.30 *Schematic of a heat pump*

The modern air-conditioning unit combines both heating and cooling arrangement as shown in Fig. 13.31. When cooling is required, it is operated in the refrigeration mode and removes heat Q_L from living space and rejects heat Q_H outside the building environment. When heating of living space is required in the winter, it absorbs heat Q_L from the environment and supplies heat Q_H to the living space.

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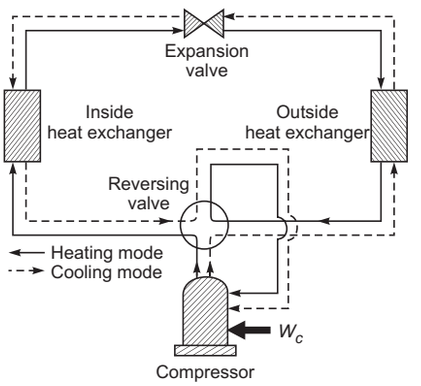

Fig. 13.31 *Example of an air-to-air reversing heat pump*

In the most common type of vapour-compression heat pump for space heating, the evaporator communicates thermally with the outside air. Such *air-source heat pumps* can also be used to provide cooling in the summer with the use of a reversing valve, as illustrated in Fig. 13.31. The solid lines show the flow path of the refrigerant in the heating mode as discussed above. To use the same components for cooling effect, the reverse valve is actuated and the refrigerant follows the path shown by the dashed line. In the cooling mode, the outside heat exchanger becomes the condenser and the inside heat exchanger becomes the evaporator.

Industrial Applications of Heat Pump The heat pump is used for warming of homes and offices in extreme cold climate. The heat pump also offers distinct opportunity for industrial applications. A heat pump can provide heating and cooling of two dissimilar fluids simultaneously. The different industrial applications of heat pump are for

1. Purification of salty water.
2. Concentration of juices, milk and syrups, dyes, chemicals, etc.
3. Preparation of powder milk and table salt.
4. For recovery of valuable solvents from different manufacturing processes.
5. For year round air-conditioning.

13.12 REFRIGERANT

The *refrigerant* is a heat-carrying medium, which undergoes the theromdynamics cycle of refrigeration (i.e., compression, condensation, expansion and evaporation). In a refrigeration system, it absorbs the heat from a low-temperature medium and discards the absorbed heat to a high-temperature environment.

13.12.1 Desirable Properties of a Refrigerant

A good refrigerant should have the following properties:

(i) The *saturation pressure* of the refrigerant at a desired low temperature should be above or equal to the atmospheric pressure in order to avoid leakage in the evaporator. The pressure at the condenser must not be excessively high for the same reasons.

(ii) The *latent heat of evaporation* at low temperature should be as high as possible to give a reasonably low mass flow rate of refrigerant for a given refrigeration capacity.

(iii) The size of the compressor depends on the *specific volume* of the refrigerant at evaporator pressure. Thus, the specific volume of the refrigerant at the compressor suction should not be high in order to avoid the large compressor for the required mass flow rate.

(iv) The refrigerant should be *chemically stable* and should not react with lubricant used in reciprocating compressor and should be miscible with oil.

(v) It should not be *non-flammable, non*-explosive.

(vi) It should be *non-toxic*. If toxic, then to a limit, below the acceptable level.

(vii) It should have low specific heat of liquid for better heat transfer in condenser.

(viii) The refrigerant should give high value of *COP* with low power input per tonne of refrigeration.

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(ix) The refrigerant should have good thermal conductivity for better heat transfer in the condenser and evaporator.

(x) The refrigerant must have freezing point temperature well below the lowest temperature in the cycle.

(xi) Other considerations are chemical stablility, non-corrosiveness, low cost and overall *eco-friendliness*.

13.12.2 Classification of Refrigerants

Refrigerants may be broadly classified into following groups:

1. Primary refrigerants, and
2. Secondary refrigerants.

The refrigerant which directly undergo the refrigeration cycle are called *primary refrigerants*, whereas the *secondary refrigerants* act only as heat carrier. They are first cooled by primary refrigerants, and then are circulated through other media to absorb heat.

The primary refrigerants are further classified into the following groups:

1. Halocarbons All the halocarbon refrigerants are divided into three subgroups according to their constituents.

(a) *Chlorofluorocarbons (CFCs)* composed of chlorine, fluorine and carbon atoms Examples are *R*-11, *R*-12, and *R*-114.

(b) *Hydro chlorofluorocarbons (HCFCs)* composed of hydrogen, chlorine, fluorine and carbon atoms. Examples are *R*-22 and *R*-123.

(c) *Hydro fluorocarbons (HFCs)* composed of hydrogen, fluorine and carbon. Examples are *R*-134*a* (HFC-134*a*), and *R*-125 (HFC-125).

2. Inorganic refrigerant such as air, water, ammonia and carbon dioxide.

3. Hydrocarbon refrigerants, such as ethane, propane, butane, etc.

There are also mixtures of above substances that are used as refrigerants. These are *azeotropes* and *zeotropes*.

Azeotropes are mixtures that behave as a single substance. All components of the mixture evaporate and condense at the same conditions. The frequently used azeotropes are *R*-500, a CFC/HFC mixtures, and *R*-502—a HCFC/CFC mixture.

Zeotropes, or *blends,* are mixtures that do not always behave as a single substance. For instance, they may not evaporate or condense at a constant temperature (called the *temperature glide.*)

13.12.3 Some Common Refrigerants

In early days, the commonly used refrigerants were ammonia, methyl chloride, sulphur dioxide, carbon dioxide, ethylchloride, propane, and butane. Some of them were toxic. Some had very high risk of explosion and were irritants. Thus they became obsolete except ammonia and carbon dioxide.

Apparently, the ideal refrigerants are in a chemical group called either fluorinated hydrocarbons or halocarbons (now known as *R*-12). They are being used since 1930, because of their excellent characteristics. They have good physical properties, are non-toxic, stable and inexpensive.

(*a*) *Air* It is designated as R-729. Dry air is used as a primary refrigerant in as air refrigeration system, mainly used for aircraft refrigeration. It is also used as a secondary refrigerant in domestic refrigerators.

(*b*) *Water* It is designated as R-118. It is used in its solid form as ice for chilling of beverages and other things. It is also used as refrigerant in vapour absorption systems. Its high freezing-point temperature limits its use in a vapour compression system.

(*c*) *Carbon dioxide* It is designated as R-744. It is a non-toxic, non-irritating and non-flammable gas. It has extremely low freezing temperature, but high operating pressure and temperatures in refrigeration system. Its specific volume is also very low. Therefore, it requires a very small compressor. It has low *COP*, and thus is not preferred for commercial use. It is suitable for manufacturing of dry ice and marine purposes.

(*d*) *Ammonia* It is designated as R-717. Its chemical formula is NH_3. It is a highly toxic gas, irritating to the eyes, nose and throat. It is non-corrosive

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to ferrous metals and attacks brass and bronze. Its boiling point at atmospheric pressure is –33.3°C and its freezing point is –78°C. Its low boiling-point temperature makes it favourable for use at well below 0°C without lowering its pressure below atmospheric. It is cheap and therefore, widely used in large and commercial cold storages and ice plants. It is also used in vapour absorption systems.

(*e*) *Freon-12* It is a very popular refrigerant and designated as *R*-12. It is colourless, almost odourless, non-toxic, non-flammable and non-irritating. It has a low boiling point temperature as –29°C at atmospheric pressure. It is non-reactive with lubricating oils. It is relative costly and has low latent heat. Thus, refrigeration system requires large mass flow rate. It is used on small refrigerating machines.

(*f*) *Freon-22* It is also a popular refrigerant and designated as *R*-22. It has a low boiling point temperature and is able to maintain a temperature up to – 40°C. It is widely used in air-conditioning units and in household refrigerators. It can be used with reciprocating and centrifugal compressors.

13.12.4 Selection of Refrigerant

The important parameters to be considered in selection of a refrigerant are temperature of refrigerated space and type of equipment to be used.

The temperature of the refrigerant in the evaporator and the condenser are governed by the temperatures of the cold and warm regions, respectively. In order to have as effective heat transfer rate, a temperature difference of at least 10°C should be maintained between the refrigerant and medium, with which the system interacts thermally. These temperatures also fix the operating pressures in the evaporator and condenser. Therefore, the selection of a refrigerant is based partly on the suitability of pressure–temperature relationship in the range of the particular application.

It is generally avoided to have a very low pressure in the evaporator and an excessive high pressure in the condenser in order to minimize the leakages to or from the system.

13.12.5 Ozone Depletion

Ozone (O_3) gas consists of three atoms of oxygen per molecule. It is a vigorous oxidising agent and it cleans the air in the atmosphere. Ozone is present in a layer in the earth's atmosphere, known as the *stratosphere*, about 11–50 km above the earth's surface. The ozone layer blocks out most of harmful ultraviolet (UV) radiation coming from the sun. The ozone layer has been progressively depleting. One chlorine atom can destroy 100,000 ozone molecules by a continuous chain reaction.

The effects of depletion of ozone layer over the earth include the following:

1. The ultraviolet rays in the solar radiation will reach the earth surface and cause an increase in skin cancer (most deadly form of cancer).
2. An increase in eye diseases.
3. Reduction in immunity against disease.
4. Harmful effects on crops, timber, wild life and marine life.

The relative ability of a substance to deplete the ozone layer is called the *ozone depletion potential (ODP)*. *R*-11 and *R*-12 have the highest value, *ODP* = 1.0. Table 13.1 lists some of the refrigerants and their *ODP* values. HCFCs have relatively low *ODP*, while HFCs do not cause any harm to the ozone layer (*ODP* = 0).

13.12.6 Eco-friendly Refrigerants

Refrigerants that are friendly to the ozone layer in the atmosphere that protects the earth from harmful ultraviolet rays are called *eco-friendly refrigerants*. For example, R-134*a* is a chlorine-free refrigerant and, thus it is an eco-friendly refrigerant.

The chlorine refrigerants react with the ozone layer and try to destroy the ozone layer in the upper atmosphere by continuous chain reaction. Hydrocarbons (HCs) and hydro fluorocarbon groups provide an alternative to chlorinated refrigerant. They contain no chlorine atom and therefore, have zero ozone depletion potential.

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Table 13.1 *Ozone Depletion Potential (ODP) of refrigerants*

Sub Group	*Refrigerant Designation*	*Chemical Formula*	*Refrigerant Name*	*ODP*
CFCs	*R*-11	CCl_3F	Trichlorofluoro methane	1.0
	R-12	CCl_2F_2	Dichlorodifluoro methane	1.0
	R-113	$CF_2Cl–CFCl_2$	Trifluoro-tirchlore ethane	0.8
	R-114	$CF_2Cl–CF_2Cl$	Tetrafluoro-dichloro ethane	1.0
HCFCs	*R*-22	$CHClF_2$	Chlorodifluoro methane	0.05
	R-123	$CHCl_2F_3$	Dichlorotrifluoro ethane	0.02
HFC Azeotrope	*R*-134a	CH_2FCF_3	Tetrafluoro ethane	0.0
	R-500	*R*-12/*R*-152	HCFC and CFC mixture	0.74
	R-717	NH_3	Ammonia	0

Two most commom refrigerants, Freon-11 and Freon-12, are highly chlorinated and are very harmful for ozone depletion. Freon-11 is replaced by HCFC-123, which has 98% less ozone deletion potential and Freon-12 is replaced by R-134a (Tetrafluoroethane) with zero ODP.

13.12.7 Montreal Protocol

The major sources of chlorine in the atmosphere are the production of CFCs and HCFCs. The major developed nations have agreed to control the use and manufacturing of CFCs and HCFCs and the United Nations through its environment programme has persuaded many nations to sign the *Vienna Convention 1987*, a treaty specifically intended to control the production of substances which cause ozone depletion. *The Montreal Protocol* of this treaty outlines the phase reduction of compounds. Production and use of some compounds have been banned and some will be banned in future.

The viable solutions include alternate halocarbon refrigerants

1. For CFC-12 (*R*-12) used in automobile air conditioning and household refrigerator; HFC-134*a* (*R*-134*a*) is permanent substitute (ODP = 0)
2. For CFC-11 (*R*-11) used in centrifugal compressor, an interior substitute is HCFC-123 until this group is phased out
3. HCFC-22 (*R*-22) used in window and commercial air-conditioning and refrigeration; possible substitute is HFC mixture *R*-407*c* and *R*-410*a*.

Summary

- The removal of heat from a lower temperature region to a higher temperature region is called refrigeration. Devices that produce refrigeration are called refrigerators and cycles at which they operate are called the refrigeration cycle. The working fluid used in the cycle is called the *refrigerant*. The refrigerators used for heating purpose of a space by transferring heat from a cold medium are called the *heat pumps.*
- The performance of refrigerators and heat pumps is measured in terms of the coefficient of performance (*COP*).

$$(COP)_R = \frac{\text{Desired cooling effect}}{\text{Net work input}} = \frac{Q_L}{W_{in}}$$

$$(COP)_{HP} = \frac{\text{Desired heating effect}}{\text{Net work input}} = \frac{Q_H}{W_{in}}$$

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- The most widely used refrigeration cycle is vapour compression cycle (VCC). In an ideal VCC, the refrigerant enters the compressor as a saturated vapour where it is compressed to high pressure. The superheated vapour is then cooled to saturate liquid state in the condenser before throttling to evaporator pressure, where the refrigerant evaporates, absorbs its enthalpy of evaporation (latent heat) from the refrigerated space.
- The power cycles can be used as refrigeration cycles by reversing the directions of processes involved. One of these is the reversed Brayton cycle, used for gas refrigeration systems. It is extremely used for aircraft cooling and liquefaction of gases. The *COP* of a gas refrigeration cycle is

$$(COP)_R = \frac{q_{in}}{w_{net,\,in}} = \frac{q_{in}}{w_{comp} - w_{Turbine}}$$

- One more attractive method of refrigeration is the vapour absorption refrigeration system, where the refrigeration is absorbed by a transport medium and is compressed in liquid form, the reduced compressor work to a pump work only. The used source energy is waste heat, solar energy, etc. The most commonly used vapour absorption refrigeration system is the ammonia–water system with ammonia as refrigerant and water as absorbent. The *COP* of such a system is

$$(COP)_R = \frac{\text{Refrigeration effect}}{\text{Heat input + Pump work}} = \frac{Q_L}{Q_H + W_p}$$

- The selection criteria of refrigerant for a refrigeration cycle involves normal operating pressure at very low and high temperatures, low specific volume at evaporator pressure, a high latent heat of evaporation at low temperature, chemical stability, non-toxic, non-corrosive and overall eco-friendly. CFCs and HCFCs contain large chlorine lebels and thus cause depletion to the ozone layer in the stratosphere of the earth's atmosphere. Thus, their use and production are gradually phased out.

Glossary

Air refrigeration A refrigeration system in which air is the working medium.

Heat pump A machine which keeps a space at higher temperature than its surroundings

Primary refrigerant A substance which undergoes refrigeration cycle

Refrigerant Working substance used in the refrigeration system

Refrigeration effect Cooling effect

Refrigeration Maintaining a space at low temperature than its surroundings

Refrigerator A machine which produces refrigeration effect

Secondary refrigerant A substance which acts as a heat carrier and exchanges heat with primary refrigerant only

Vapour absorption–Refrigeration A refrigeration system in which refrigerant and its absorbent are used for creating refrigeration effect without compressor

Vapour compression–Refrigeration A refrigeration system in which liquid and its vapour undergo refrigeration cycle

Review Questions

1. What are temperatures inside the fresh food and freezer compartments of a household refrigerator?
2. What is frosting? How can excessive frosting harm the performance of a refrigerator?
3. Define refrigerating effect. What is one tonne of refrigeration? What is the basic formula for calculating the tonnage of refrigeration?
4. What are the commonly used refrigerants for vapour compression refrigeration systems? Would water be a suitable refrigerant in a refrigerator?

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5. What do you understand by primary and secondary refrigerants? Explain in brief.
6. How is the heat absorbed or removed from a low-temperature source and transferred to high-temperature source in a vapour compression system?
7. What is the difference between a refrigerator and a heat pump?
8. Why is the reversed Carnot cycle executed within a saturation curve and not a realistic model for refrigeration cycles?
9. Prove that $(COP)_{HP} = (COP)_R + 1$
10. Sketch the vapour compression cycle on a *T–s* diagram and derive an expression for its *COP*.
11. What is the function of a condenser in a refrigeration cycle?
12. What are the causes of irreversibilities in an actual refrigeration cycle ? Explain with the help of a *T–s* diagram.
13. What is the effect of lower evaporator pressure in a VCC on its performance?
14. Consider two vapour compression refrigeration cycles. The refrigerant enters the throttle valve as a saturated liquid at 30°C in one cycle and as a subcooled liquid at 30°C in another cycle. The evaporator pressure for both cycles is same. Which cycle do you think will have higher *COP*?
15. Why are CFC phased out? Which are the alternatives to CFCs?
16. What is ozone depletion? What are the remedies to save ozone in atmosphere?

Problems

1. If the power consumed by a reversible refrigeration compressor is 2 kW per tonne of refrigeration, what is it *COP*? *[1.75]*

2. What is the refrigeration load in *TR*, when 20 m^3/min of water is cooled from 13°C to 8°C? *[33.0 TR]*

3. Calculate the power required by a reversible refrigerator to produce 400 kg of ice per hour at –20°C from feed water at 25°C. Take latent heat of ice formation as 335 kJ/kg · K and specific heat of ice as 2.09 kJ/kg · K. *[9.92 kW]*

4. A refrigerator working on Bell Coleman cycle operates between atmospheric pressure and 7 bar. Air is drawn from a cold chamber at 13°C. Before air expander, it is cooled to 28°C. Calculate *COP*, if index of expansion and compression is 1.35 *[1.53]*

5. The *COP* of a vapour compression refrigeration system is 3.0. If the compressor motor draws a power of 10.5 kW at 91% motor efficiency, what is the refrigeration effect in *TR* of system? *[9.84 TR]*

6. (a) Ice is formed at 0°C from water at 30°C. In the refrigeration system the same water is used for condenser cooling and the temperature of brine is –15°C at evaporator. Considering the system operates on Carnot refrigeration system, calculate the *COP* of the system. *[5.7]*

7. Refrigerant 134*a* enters the compressor of an ideal vapour compression refrigeration system as saturated vapour at –16°C with a volumetric flow rate of 1 m^3/min. The refrigerant leaves the condenser at 36°C, 10 bar. Determine
   (a) the compressor power, in kW
   (b) the refrigerating capacity, in tonnes
   (c) the coefficient of performance

8. The R-12 refrigerant is used in a refrigerator operating between 36°C and –7°C. The refrigerant is dry and saturated at the beginning of compression. If the actual *COP* is 0.65 times the theoretical *COP*, calculate the net cooling produced per hour for a flow rate of 5 kg/min. The properties of R-12 are given:

Temp. °C	h_f	h_{fg}	s_f	s_{fg}
36	456.4	585.3	4.74	0.67
–7	412.4	570.3	4.76	—

[1.77 TR]

9. The condenser and evaporator of a 15 *TR* ammonia refrigerating plant are maintained at 25°C and –10°C. The refrigerant is subcooled by 5°C in the condenser. The vapour leaving the evaporator is

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0.97 dry. Calculate *COP* and power required. Use the following properties:

Temp. °C	h_f	h_{fg}	s_f	s_{fg}
25	317.687	1483.18	1.4084	5.3175
–10	154.056	1450.22	.8296	5.755

[6.86, 7.65 kW]

10. A refrigeration system of 15 tonnes capacity operates on a vapour compression cycle using R-22 at an evaporator temperature of 5°C and condensing temperature of 50°C. Calculate the refrigerant mass flow rate and compressor intake volume flow rate, if the volumetric efficiency is 72%. Use following properties:

Pressure	Temp.	h_f	h_{fg}	v_f	v_g
bar	°C	kJ/kg	kJ/kg	lit/kg	lit/kg
5.836	5	205.9	407.1	0.791	0.0404
19.423	50	263.3	417.7	.922	.0117

[0.365 kg/s, 0.01475 m^3/kg]

11. A 20 tonnes vapour compression refrigeration system using Freon-12 operates between an evaporator pressure of 1.004 bar and a condenser pressure of 13.663 bar. The system uses 10°C superheating. Calculate the mass flow rate, *COP*, degree of sub cooling and power input. The refrigerant leaving the condenser is dry saturated liquid and leaving the evaporator is dry saturated vapour. The compression is isentropic. The properties of Freon -12 are

Pressure bar	Temp. °C	h_f kJ/kg	h_{fg} kJ/kg	s_f kJ/kg·K	s_g kJ/kg·K	$C_{p,f}$ kJ/kg·K	$C_{p,v}$ kJ/kg·K
1.004	–30	8.854	174.076	0.371	0.7165	–	0.579
13.663	55	90.201	207.766	0.3194	0.6777	1.074	–

[0.8346 kg/s, 1.8, 0, 38.9 kW]

12. In an aircraft cooling system, air enters the compressor at 0.1 MPa, 4°C and is compressed to 6 MPa with an isentropic efficiency of 72%. After being cooled to 55°C at constant pressure in a heat exchanger, the air then expands in a turbine to 0.1 MPa with an isentropic efficiency of 78%. The cooling load of the system is 3 tonnes of refrigeration at constant pressure before re-entering the compressor, which is driven by the turbine. Assuming that air is an ideal gas, determine the *COP* of refrigerator, driving power required and the mass flow rate of air.

Objective Questions

1. *COP* of a Carnot refrigeration cycle is greater than
   (a) vapour compression cycle
   (b) reversed Brayton cycle
   (c) vapour absorption cycle
   (d) all the above
2. In an ideal vapour compression refrigeration cycle, which process is irreversible?
   (a) Compression (b) Heat ejection
   (c) Throttling (d) Heat absorption
3. Subcooling of refrigerant in vapour compression refrigeration cycle
   (a) decreases *COP*
   (b) increases *COP*
   (c) decrease refrigerating effect
   (d) increases work input
4. Superheating of vapour refrigerant at evaporator in vapour compression cycle
   (a) decreases *COP*
   (b) increases *COP*
   (c) decrease refrigerating effect
   (d) increases work input
5. Heat is absorbed by a refrigerant during a refrigerant cycle
   (a) condenser (b) throttle valve
   (c) evaporator (d) compressor
6. Heat is rejected by a refrigerant during a refrigerant cycle
   (a) condenser (b) throttle valve
   (c) evaporator (d) compressor
7. In an ideal vapour compression refrigeration cycle, the refrigerant is in the form of superheated vapour before entering into
   (a) condenser (b) throttle valve
   (c) evaporator (d) compressor

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8. In an ideal vapour compression refrigeration cycle, the refrigerant is in form of dry saturated vapour before entering into
   (a) condenser (b) throttle valve
   (c) evaporator (d) compressor
9. The refrigerant used in vapour absorption refrigeration system is
   (a) Freon-12 (b) CO_2
   (c) ammonia (d) R-134*a*
10. The *COP* of Carnot cycle when working as heat pump is
   (a) less than when it works as refrigerator
   (b) more than when it works as refrigerator
   (c) less than when it works as heat engine
   (d) equal to when it work as refrigerator
11. The secondary refrigerant acts to
   (a) undergo the refrigeration cycle
   (b) protect the ozone depletion
   (c) carry the heat within refrigerated space
   (d) cool the water
12. A zeotropes are mixture of
   (a) primary and secondary refrigerant
   (b) Ammonia and water
   (c) CFCs and HFCs
   (d) HCFCs and HFCs
13. Ozone depletion is caused due to
   (a) use of HFCs
   (b) use of HCs
   (c) use of CFC
   (d) use of Ammonia
14. Refrigerant R-134a is
   (a) an ecofriendly refrigerant
   (b) mixture of hydrofluoro carbon
   (c) Halo carbone refrigerant
   (d) all of above

Answers

1. (d) 2. (c) 3. (b) 4. (d) 5. (c) 6. (a) 7. (a) 8. (d)
9. (c) 10. (b) 11. (c) 12. (c) 13. (c) 14. (d)

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14

Ideal Gas Mixtures

Introduction

A pure substance has been defined as a substance that is homogeneous and of constant chemical composition throughout the mass. The homogeneous mixture of gases, if they do not react each other, are therefore, also considered pure substances, for example, dry air, which is the mixture of oxygen, nitrogen, argon, and traces of other gases. The properties of such mixtures can be obtained, correlated and tabulated or fitted by equations like properties of any pure substance. It is therefore, important to calculate the properties of any mixture from the properties of its constituents.

The mixtures considered in this chapter are composed of perfect gases and vapours. The properties of such mixtures are important in combustion and air-conditioning calculations.

14.1 MASS AND MOLE FRACTIONS

Consider a closed system of gas, which is composed of k components, each a pure substance. The mass of the mixture is equal to the sum of masses of its components.

$$m = m_1 + m_2 + m_3 + m_4 + \ldots + m_k = \sum_{i=1}^{i=k} m_i \qquad \ldots(14.1)$$

where m_i refers to mass of the ith component.

The total number of moles of a mixture is the sum of the number of moles of its components.

$$n = n_1 + n_2 + n_3 + n_4 + \ldots + n_k = \sum_{i=1}^{i=k} n_i \qquad \ldots(14.2)$$

The ratio of the mass of a component to the mass of the mixture is referred as *mass fraction (x)*, expressed as

$$x_i = \frac{m_i}{m} = \frac{m_i}{\sum_{i=1}^{i=k} m_i} \qquad \ldots(14.3)$$

The ratio of mole number of a component to the total mole numbers of the mixture is referred as *mole fraction (y)* and

$$y_i = \frac{n_i}{n} = \frac{n_i}{\sum_{i=1}^{i=k} n_i} \qquad \ldots(14.4)$$

Dividing Eq. (14.1) by m (mixture mass) and Eq. (14.2) by n (mixture moles), we can easily obtain that the sum of mass fraction or mole fraction is equal to 1,

$$\text{or} \quad \sum_{i=1}^{i=k} x_i = 1 \quad \text{and} \quad \sum_{i=1}^{i=k} y_i = 1 \qquad \ldots(14.5)$$

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The mass of each component can be expressed in terms of mole number n and molecular weight $\mathcal{M}$.

$$m_i = n_i \mathcal{M}_i$$

and total mass $$m = \sum_{i=1}^{i=k} n_i \mathcal{M}_i \quad \ldots(14.6)$$

The *apparent (or average) molecular weight* of the mixture, $\mathcal{M}$, is defined as the ratio of the total mass of the mixture, m, to the total number of moles of the mixture, n

$$\mathcal{M} = \frac{m}{n}$$

Inserting Eq. (14.6), the average molecular weight of a gas mixture

$$\mathcal{M} = \frac{m}{n} = \frac{\sum_{i=1}^{i=k} m_i}{n} = \frac{\sum_{i=1}^{i=k} n_i \mathcal{M}_i}{n}$$

$$= \sum_{i=1}^{i=k} y_i \mathcal{M}_i \quad \ldots(14.7)$$

The gas constant for a gas mixture can be evaluated as

$$R = \frac{R_u}{\mathcal{M}} \quad \ldots(14.8)$$

Where R_u is universal gas constant (= 8314 J/kg mol · K)

14.2 DALTON'S LAW OF PARTIAL PRESSURES

In the absence of intermolecular forces, each component in the gas mixture behaves as an ideal gas, if it were alone at the temperature T and volume V of the mixture. It follows that the individual components would not exert the mixture pressure, but rather a partial pressure.

The partial pressure p_i of i^{th} component in a gas mixture is defined as

$$p_i = \frac{n_i R_u T}{V} \quad \ldots(14.9)$$

Using an ideal gas relation $V = \frac{n R_u T}{p}$ in Eq. (14.9)

$$p_i = \frac{n_i R_u T}{n R_u T}\ p = \frac{n_i}{n}\ p = y_i p \quad \ldots(14.10)$$

The *partial pressure* of each constituent is that pressure which the gas would exert, if it existed alone at the temperature and volume of the mixture. The total pressure of the mixture is defined as

$$p = \frac{(n_1 + n_2 + n_3 + \ldots + n_k) R_u T}{V}$$

$$= \frac{n_i R_u T}{V} + \frac{n_2 R_u T}{V} + \ldots$$

Thus $$p = p_1 + p_2 + p_3 + \ldots + p_k$$

$$= \Sigma p_i \quad \ldots(14.11)$$

The sum of partial pressures of the constituents in a gas mixture is equal to the total pressure of the mixture. It is known as *Dalton's law of partial pressure* or *law of additive pressures*.

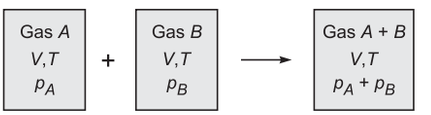

Fig. 14.1 *Dalton's model*

14.3 AMAGAT–LEDUCE LAW OF ADDITIVE VOLUMES

It is based on the assumption that the each mixture component behaves as an ideal gas, if it exists alone at the pressure p and temperature T of the mixture.

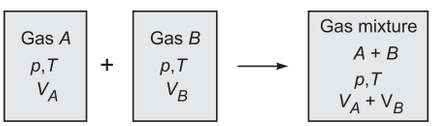

Fig. 14.2 *Amagat–Leduce model*

The partial volume V_i of i^{th} component in a gas mixture is defined as

$$V_i = \frac{n_i R_u T}{p} = y_i V \quad \ldots(14.12)$$

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The *partial volume* of a gas component in a gas mixture is the volume of that gas would occupy, if it existed alone at the mixture pressure and temperature.

The total volume of a gas mixture is expressed as

$$V = (n_1 + n_2 + n_3 + n_4 + \ldots + n_k)\ \frac{R_u T}{p}$$

$$= n_1 \frac{R_u T}{p} + n_2 \frac{R_u T}{p} + n_3 \frac{R_u T}{p} + \ldots + n_k \frac{R_u T}{p}$$

$$= V_1 + V_2 + V_3 + \ldots + V_k = \sum_{i=1}^{i=k} V_i \qquad \ldots(14.13)$$

Thus, sum of partial volumes of the constituents in a gas mixture is equal to the total volume of the mixture. It is known as *Amagat–Leduce law* of additive volumes.

14.4 DENSITY AND GAS CONSTANT OF GAS MIXTURE

The specific volume of the gas mixture v is defined as

$$v = \frac{V}{m} = \frac{V}{m_1 + m_2 + m_3 + \ldots + m_k}$$

or
$$\frac{1}{v} = \frac{m_1}{V} + \frac{m_2}{V} + \frac{m_3}{V} + \frac{m_4}{V} + \ldots + \frac{m_k}{V}$$

$$= \frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3} + \ldots + \frac{1}{v_k}$$

We have $\rho = \dfrac{1}{v}$, therefore, density of a gas mixture,

$$\rho = \rho_1 + \rho_2 + \rho_3 + \rho_4 + \ldots \rho_k = \Sigma \rho_i \qquad \ldots(14.14)$$

The characteristic gas equation for i^{th} component in a gas mixture can be expressed as

$$p_i V = n_i R_u T$$

where
$$n_i = \frac{m_i}{\mathcal{M}_i}$$

$$p_i V = \frac{m_i}{\mathcal{M}_i} R_u T$$

But
$$R_i = \frac{R_u}{\mathcal{M}_i}$$

Therefore,
$$p_i V = m_i R_i T \qquad \ldots(14.15)$$

For each constituent in the gas mixture

$$p_1 V = m_1 R_1 T$$
$$p_2 V = m_2 R_2 T$$
$$p_3 V = m_3 R_3 T$$
$$p_k V = m_k R_k T$$

Adding and using Dalton's low of partial pressures

$$pV = (m_1 R_1 + m_2 R_2 + m_3 R_3 + \ldots + m_k R_k) T$$

comparing it with characteristic gas equation

$$pV = mRT$$

we get apparent gas constant,

$$R = \frac{m_1 R_1 + m_2 R_2 + m_3 R_3 + \ldots + m_k R_k}{m}$$

$$= \Sigma x_i R_i \qquad \ldots(14.16)$$

Example 14.1 *A vessel of volume 0.4 m^3 contains 0.45 kg of carbon monoxide and 1 kg air, at 15°C. Calculate the partial pressure of each constituents and the total pressure in the vessel. The air contains 23.3% oxygen and 76.7% nitrogen by mass. Take the molar masses of carbon monoxide, oxygen and nitrogen as 28, 32 and 28 kg/k mol, respectively.*

Solution

Given A mixture of carbon monoxide and air in a vessel

$V = 0.4\ m^3$, $m_{air} = 1$ kg,
$T_{air} = 15°C = 288$ K, $m_{CO} = 0.45$ kg,
$\mathcal{M}_{CO} = 28$, $\mathcal{M}_{O_2} = 32$,
$\mathcal{M}_{N_2} = 28$

To find

(i) Partial pressure of CO, O_2 and N_2 in the mixture,
(ii) Total pressure of mixture.

Assumptions

(i) Each component in the mixture behaves like an ideal gas.
(ii) Mixture composition remains invariable.

Analysis Calculating the mass of each component of air

Mass of oxygen $m_{O_2} = 0.233\, m_{air}$
$= 0.233 \times 1 = 0.233$ kg

Mass of nitrogen, $m_{N_2} = 0.767 m_{air} = 0.767$ kg

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The specific gas constant of gas is given by

$$R = \frac{R_u}{\mathcal{M}} = \frac{8.314 \text{ kJ/kmol·K}}{\mathcal{M} \text{ (kg/kmol)}}$$

Thus $$R_{CO} = \frac{8.314}{28} = 0.297 \text{ kJ/kg·K}$$

$$R_{O_2} = \frac{8.314}{32} = 0.259 \text{ kJ/kg·K}$$

$$R_{N_2} = \frac{8.314}{28} = 0.297 \text{ kJ/kg·K}$$

(i) The partial pressure of a gas is given by

$$p_i = \frac{m_i R_i T}{V}$$

Since volume $V = 0.4$ m^3 and $T = 15 + 273 = 288$ K, remain constant, the partial pressure of gas constituents are

$$p_{CO} = \frac{m_{CO} R_{CO} T}{V}$$
$$= \frac{0.45 \times 0.297 \times 288}{0.4}$$
$$= 96.23 \text{ kPa} = \mathbf{0.9623 \text{ bar}}$$

$$p_{O_2} = \frac{m_{O_2} R_{O_2} T}{V}$$
$$= \frac{0.233 \times 0.259 \times 288}{0.4}$$
$$= 43.35 \text{ kPa} = \mathbf{0.4345 \text{ bar}}$$

$$p_{N_2} = \frac{m_{N_2} R_{N_2} T}{V}$$
$$= \frac{0.767 \times 0.297 \times 288}{0.4}$$
$$= 164.01 \text{ kPa} \approx \mathbf{1.64 \text{ bar}}$$

(ii) The total pressure of the gas mixture

$$p = \Sigma p_i$$
$$= p_{CO} + p_{O_2} + p_{N_2}$$
$$= 0.9623 + 0.4345 + 1.64$$
$$= \mathbf{3.03 \text{ bar}}$$

Example 14.2 *A volumetric analysis of a gaseous mixture gives the following results:*

$CO_2 = 12.0\%$ $O_2 = 4.0\%$

$N_2 = 82.0\%$ $CO = 2.0\%$

Determine the analysis of gas mixture on the mass basis, the molecular weight and the gas constant on the mass basis for the mixture. Assume ideal gas behaviour.

Solution

Given The components of gas mixture as

$CO_2 = 12\%$, $O_2 = 4\%$,

$N_2 = 82\%$ $CO = 2\%$

To find

(i) Analysis of gas in mixture on mass basis,
(ii) Molecular weight of mixture, and
(iii) Gas constant of mixture.

Analysis

(i) Analysis of component gas in mixture on mass basis is represented in Table 14.1.
(ii) Molecular weight of mixture $\mathcal{M} = 30.08$ kg/kmol
(iii) Gas constant for mixture

$$R = \frac{R_u}{\mathcal{M}} = \frac{8.314}{30.08} = \mathbf{0.276 \text{ kJ/kg·K}}$$

Example 14.3 *The gravitational analysis of air is 23.14% oxygen, 75.53% nitrogen, 1.28% argon and 0.05% carbon dioxide. Calculate the volumetric analysis and the partial pressure of each constituent in the mixture, when the total pressure is 1 bar.*

Table 14.1

Component gas	*% volume*	*Mole fraction* y_i	*Molecular Weight* $\mathcal{M}_i$	*Mass kg/kmol of mixture,* $m_i = y_i \mathcal{M}_i$	*Mass basis percentage* $= \frac{m_i}{m} \times 100$
CO_2	12.0	0.12	44	5.28	**17.55%**
O_2	4.0	0.04	32.0	1.28	**4.26%**
N_2	82.0	0.82	28.0	22.96	**76.33%**
CO	2.0	0.02	28.0	0.56	**1.86%**
		1.00		$m = 30.08$	**100%**

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Solution

Given Gravitational analysis of air as mixture of O_2, N_2, argon and CO_2.

To find

(i) Volumetric analysis,

(ii) Partial pressure of each constituent in air at $p = 1$ bar.

Analysis For 1 kg of air as mass and molecular weight of constituents.

$$m_{O_2} = 0.2314 \text{ kg} \qquad \mathcal{M}_{O_2} = 32 \text{ kg/kmol}$$
$$m_{N_2} = 0.7553 \text{ kg} \qquad \mathcal{M}_{N_2} = 28 \text{ kg/kmol}$$
$$m_{argon} = 0.0128 \text{ kg} \qquad \mathcal{M}_{argon} = 40 \text{ kg/kmol}$$
$$m_{CO_2} = 0.0005 \text{ kg} \qquad \mathcal{M}_{CO_2} = 44 \text{ kg/kmol}$$

Number moles of each gas constituents in the mixture

$$n_i = \frac{m_i}{\mathcal{M}_i}$$

$$n_{O_2} = \frac{0.2314}{32} = 0.00723$$

$$n_{N_2} = \frac{0.7553}{28} = 0.02697$$

$$n_{argon} = \frac{0.0128}{40} = 0.00032$$

$$n_{CO_2} = \frac{0.0005}{44} = 0.00001$$

Total number of moles

$$n = \Sigma n_i = 0.03453$$

The mole fraction is equal to volume fraction as well as pressure fraction, thus

$$y_i = \frac{n_i}{n} = \frac{p_i}{p} = \frac{V_i}{V}$$

$$x_{O_2} = \frac{V_{O_2}}{V} = \frac{0.00723}{0.03453} = 0.2093 \text{ or } \mathbf{20.93\%}$$

$$x_{N_2} = \frac{V_{N_2}}{V} = \frac{0.02697}{0.03453} = 0.781 \text{ or } \mathbf{78.1\%}$$

$$x_{argon} = \frac{V_{argon}}{V} = \frac{0.00032}{0.03453} = 0.00092 \text{ or } \mathbf{0.92\%}$$

$$x_{CO_2} = \frac{V_{CO_2}}{V} = \frac{0.00001}{0.03453} = 0.00028 \text{ or } \mathbf{0.028\%}$$

(ii) Using mole fractions to obtain partial pressures.

$$p_i = y_i p$$
$$p_{O_2} = 0.2093 \times 1 = \mathbf{0.2093 \text{ bar}}$$
$$p_{N_2} = 0.781 \times 1 = \mathbf{0.781 \text{ bar}}$$
$$p_{argon} = 0.00092 \times 1 = \mathbf{0.00092 \text{ bar}}$$
$$p_{CO_2} = 0.00028 \times 1 = \mathbf{0.00028 \text{ bar}}$$

Example 14.4 *A vessel contains a gaseous mixture of composition by volume 80% H_2, and 20% CO. It is designed that the mixture should be made in the proportion 50% H_2 and 50% CO by removing some of the mixture and adding some CO. Calculate per kmol of mixture the mass of mixture to be removed and the mass of CO to be added. The pressure and temperature in the vessel remain constant during the procedure.*

Take the molar mass of hydrogen and CO as 2 kg/kmol and 28 kg/kmol, respectively.

Solution

Given A mixture of H_2 and CO in a vessel

$$y_{H_2} = 0.8, \qquad y_{CO} = 0.2$$
$$\mathcal{M}_{H_2} = 2, \qquad \mathcal{M}_{CO} = 28$$

To find

(i) The mass of the mixture to be removed per kmol of mixture, and

(ii) The mass of CO to be added per kmol of the mixture.

Analysis

(i) Since the pressure and temperature remain constant, thus the mass of gas in the vessel remains constant. Therefore, the mass of mixture removed = mass of CO added.

Let x kg of mixture to be removed and y kg CO be added.

For a mixture the molecular weight

$$\mathcal{M} = \Sigma y_i \mathcal{M}_i$$
$$= 0.8 \times 2 + 0.2 \times 28 = 7.2 \text{ kg/kmol}$$

The number of moles of mixture

$$n = \frac{m}{\mathcal{M}}$$

The moles of mixture to be removed

$$= \frac{x \text{ kg}}{7.2 \text{ kg/kmol}} = \frac{x}{7.2} \text{ kmol}$$

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The moles of CO to be added = $\frac{y}{28}$ kmol and for given conditions

$$\frac{x}{7.2} = \frac{y}{28}$$

No. of moles of H_2 in the mixture to be removed

$$= y_{H_2} \frac{x}{7.2} = 0.8 \times \frac{x}{7.2} = \frac{x}{9} \text{ kmol}$$

The moles of H_2 in the vessel = $\left(0.8 - \frac{x}{9}\right)$ kmol

1 kg of new mixture contains 50% H_2 and 50% CO, therefore

$$0.8 - \frac{x}{9} = 0.5$$

or $x = 9(0.8 - 0.5) =$ **2.7 kg/kmol**

Thus the mass of mixture to be removed is 2.7 kg per kmol.

(ii) The mass of CO to be added

We have $\frac{x}{7.2} = \frac{y}{28}$

or $y = \frac{28x}{7.2} = \frac{28}{7.2} \times 2.7$

$=$ **10.5 kg/kmol**

Thus the mass of CO to be added = 10.5 kg/kmol.

Example 14.5 *A mixture of gases contains 1.2 kg of oxygen and 1.8 kg of nitrogen. The pressure and temperature of the mixture are 350 kPa and 300 K. Determine for mixture.*

(a) mass and mole fraction of each constituent gas,

(b) average molecular weight, (c) the partial pressures,

(d) the specific gas constant, (e) the volume and (f) the density.

Solution A number of gases with

$m_{O_2} = 1.2$ kg $\quad m_{N_2} = 1.8$ kg

$p = 350$ kPa $\quad T = 300$ K

To find

(i) Mass and mole fraction of each gas constituents,
(ii) Average molecular weight,
(iii) The partial pressures,
(iv) The specific gas constant,
(v) The volume, and
(vi) The density.

Analysis The total mass of the mixture can be calculated as

$$m = m_{O_2} + m_{N_2} = 1.2 \text{ kg} + 1.8 \text{ kg} = 3 \text{ kg}$$

(i) *The mass fraction of oxygen*

$$x_{O_2} = \frac{m_{O_2}}{m} = \frac{1.8}{3} = \mathbf{0.4}$$

The mass fraction of nitrogen

$$x_{N_2} = \frac{m_{N_2}}{m} = \frac{1.2}{3} = \mathbf{0.6}$$

The mass of the gas can also be expressed in terms of number of moles and molecular weight;

$$m_i = n_i \mathcal{M}_i$$

$$\therefore \quad n_{O_2} = \frac{m_{O_2}}{\mathcal{M}_{O_2}} = \frac{1.2}{32} = 0.0375$$

$$n_{N_2} = \frac{m_{N_2}}{\mathcal{M}_{N_2}} = \frac{1.8}{28} = 0.06428$$

Hence, the total number of mole in the gas

$$n = n_{O_2} + n_{N_2} = 0.0375 + 0.0642$$
$$= 0.1017 \text{ moles}$$

Thus, the mole fractions

$$y_{O_2} = \frac{0.0375}{0.1017} = \mathbf{0.368}$$

$$y_{N_2} = \frac{0.0642}{0.1017} = \mathbf{0.633}$$

(ii) *Apparent molecular weight*

$$\mathcal{M} = \sum_{i=1}^{i=k} y_i \mathcal{M}_i = x_{O_2} \mathcal{M}_{O_2} + x_{N_2} \mathcal{M}_{N_2}$$
$$= 0.368 \times 32 + 0.633 \times 28$$
$$= \mathbf{29.5 \text{ kg/kmol}}$$

(iii) *The partial pressures*

The partial pressure of oxygen

$$p_{O_2} = y_{O_2} p = 0.368 \times 350 = \mathbf{128.9 \text{ kPa}}$$

$$p_{N_2} = y_{N_2} p = 0.633 \times 350 = \mathbf{221.55 \text{ kPa}}$$

(iv) *The Apparent gas constant of mixture*

$$R = \frac{R_u}{\mathcal{M}} = \frac{8.314}{29.5} = \mathbf{0.2812 \text{ kJ/kg} \cdot \text{K}}$$

(v) *The volume of gas mixture*

$$V = V_{O_2} + V_{N_2} = (n_{O_2} + n_{N_2}) \frac{R_u T}{p}$$

$$(0.0375 + 0.064) \times \frac{8.314 \times 300}{350} = \mathbf{0.742 \text{ m}^3}$$

(vi) *The density of mixture*

$$\rho = \frac{m}{V} = \frac{3 \text{ kg}}{0.742 \text{ m}^3} = \mathbf{4.13 \text{ kg/m}^3}$$

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14.5 PROPERTIES OF GAS MIXTURE—GIBBS THEOREM

The extensive properties of an deal gas mixture can be obtained by an extension of Dalton's law of partial pressure. Dalton's law was reformulated by Gibbs. The combined statement is known as the *Gibbs–Dalton law*. According to this theorem, the internal energies, enthalpies, specific heats etc. of a gas mixture are respectively equal to sum of internal energies, enthalpies, specific heats etc. of individual components, each at temperature and volume of mixture. Thus

$$U = U_1 + U_2 + U_3 + U_4 + \ldots + U_K$$
$$= \Sigma U_i \qquad \ldots(14.17)$$

or $$mu = m_1 u_1 + m_2 u_2 + m_3 u_3 + \ldots + m_k u_k$$
$$= \Sigma m_i u_i$$

The specific internal energy of the mixture

$$u = \frac{m_1 u_1 + m_2 u_2 + m_3 u_3 + \ldots + m_k u_k}{m_1 + m_2 + m_3 + \ldots + m_k}$$

$$= \frac{U_m}{m} = \Sigma x_i u_i \qquad \ldots(14.18)$$

On the molal basis

$$U = n\,\overline{u}_m = \Sigma n_i \overline{u}_i$$

where $$\overline{u} = \frac{1}{n}\Sigma n_i \overline{u}_i = \Sigma y_i \overline{u}_i \qquad \ldots(14.19)$$

where $\overline{u}_i$ = internal energy per mole for ith component of gas mixture.

Similarly, the total enthalpy of a gas mixture

$$H = H_1 + H_2 + H_3 \ldots + H_K$$
$$= \Sigma H_i \qquad \ldots(14.20)$$

or $$mh = m_1 h_1 + m_2 h_2 + m_3 h_3 + \ldots + m_k h_k$$
$$= \Sigma m_i h_i$$

and $$h = \frac{m_1 h_1 + m_2 h_2 + m_3 h_3 + \ldots + m_k h_k}{m_1 + m_2 + m_3 + \ldots + m_k}$$

$$= \frac{\Sigma m_i h_i}{m} = \Sigma x_i h_i \qquad \ldots(14.21)$$

On the molal basis

$$H = n\overline{h}_m = \Sigma n_i \overline{h}_i$$

and $$\overline{h} = \frac{1}{n}\Sigma n_i \overline{h}_i = \Sigma y_i \overline{h}_i \quad \ldots(14.22)$$

where $\overline{h}_i$ is enthalpy per mole for ith component of gas mixture.

The specific heats

$$mC_v = m_1 C_{v_1} + m_2 C_{v_2} + m_3 C_{v_3} + \ldots + m_k C_{vk}$$
$$= \Sigma m_i C_{vi}$$

or $$C_v = \frac{m_1 C_{v_1} + m_2 C_{v_2} + m_3 C_{v_3} + \ldots + m_k C_{vk}}{m_1 + m_2 + m_3 + \ldots + m_k}$$

$$= \frac{\Sigma m_i C_{vi}}{m} = \Sigma x_i C_{vi} \qquad \ldots(14.23)$$

On the molal basis

$$\overline{C}_v = \Sigma y_i \overline{C}_{vi} \qquad \ldots(14.24)$$

and $$C_p = \frac{m_1 C_{p_1} + m_2 C_{p_2} + m_3 C_{p_3} + \ldots m_k C_{pk}}{m_1 + m_2 + m_3 + \ldots + m_k}$$

$$= \frac{\Sigma m_i C_{pi}}{m} = \Sigma x_i C_{pi} \qquad \ldots(14.25)$$

On molal basis

$$C_p = \Sigma y_i \overline{C}_{pi} \qquad \ldots(14.26)$$

The entropy of the gas mixture can be expressed as

$$S = S_1 + S_2 + S_3 + \ldots + S_k = \Sigma S_i$$
$$= \Sigma m_i s_i \qquad \ldots(14.27)$$

The specific entropy of gas mixture

$$s = \frac{m_i s_i + m_2 s_2 + m_3 s_3 + \ldots m_k s_k}{m_1 + m_2 + m_3 + \ldots + m_k}$$

$$= \frac{\Sigma m_i s_i}{m} = \Sigma x_i s_i \qquad \ldots(14.28)$$

Further on molal basis $S = \Sigma n_i \overline{s}_i$

and $$\overline{s} = \frac{1}{n}\Sigma n_i s_i = \Sigma x_i \overline{s}_i \qquad \ldots(14.29)$$

Example 14.6 *A mixture of ideal gases consists of 5 kg of nitrogen and 6 kg of carbon dioxide at a pressure of 4 bar and a temperature of 27°C. Find*

(a) The mole fraction of each constituent,
(b) The equivalent molecular weight of the mixture,
(c) The equivalent gas constant of the mixture,
(d) The partial pressures and partial volumes,
(e) The volume and density of the mixture, and
(f) The C_p and C_v of the mixture.

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If the mixture is heated at constant volume to 60°C, find the change in internal energy, enthalpy, and entropy of the mixture.

Take γ *for* $CO_2 = 1.286$ *and for* $N_2 = 1.4$

Solution

Given A mixture of ideal gases

$m_{N_2} = 5$ kg $\quad m_{CO_2} = 6$ kg

$p = 4$ bar $\quad T_1 = 27°C + 273°C = 300$ K

$V_2 = V_1 \quad T_2 = 60°C = 333$ K

$\gamma_{CO_2} = 1.28 \quad \gamma_{N_2} = 1.4$

To find

(i) Mole fraction of each constituent,
(ii) The equivalent molecular weight of the mixture,
(iii) The equivalent gas constant of the mixture,
(iv) The partial pressures and partial volumes,
(v) The volume and density of the mixture,
(vi) The C_p and C_v of the mixture,
(vii) Change in internal energy of mixture,
(viii) Change of entropy of the mixture.

Analysis

(i) For calculation of number of moles, using relation

$$n_i = \frac{m_i}{\mathcal{M}_i}$$

Thus $\quad n_{N_2} = \dfrac{5}{28} = 0.178$

$$n_{CO_2} = \frac{6}{44} = 0.136$$

The total number of moles in the mixture:

$$n = n_{N_2} + n_{CO_2} = 0.178 + 0.136 = 0.312$$

Therefore, mole fractions

$$y_{N_2} = \frac{n_{N_2}}{n} = \frac{0.178}{0.312} = \mathbf{0.570}$$

$$y_{CO_2} = \frac{n_{CO_2}}{n} = \frac{0.136}{0.312} = \mathbf{0.435}$$

(ii) Apparent molecular weight of the mixture, $\mathcal{M}$

$$\mathcal{M} = y_{N_2}\mathcal{M}_{N_2} + y_{CO_2}\mathcal{M}_{CO_2} = 0.570 \times 28 + 0.435 \times 44 = \mathbf{35.1\ kg/kg \cdot mole}$$

(iii) Apparent gas constant of mixture, R:

$$R = \frac{R_u}{\mathcal{M}} = \frac{8.314}{35.1} = \mathbf{0.237\ kJ/kg \cdot K}$$

(iv) The partial pressures and partial volumes:

$$p_{N_2} = y_{N_2} \times p = 0.570 \times 4 = \mathbf{2.28\ bar}$$

$$p_{CO_2} = y_{CO_2} \times p = 0.435 \times 4 = \mathbf{1.72\ bar}$$

$$V_{N_2} = \frac{m_{N_2} R_{N_2} T}{p} = \frac{5 \times \dfrac{8.314}{28} \times 300}{(4 \times 100\ \text{kPa})} = \mathbf{1.113\ m^3}$$

$$V_{CO_2} = \frac{m_{CO_2} R_{CO_2} T}{p} = \frac{6 \times \dfrac{8.314}{44} \times 300}{(4 \times 100\ \text{kPa})} = \mathbf{0.85\ m^3}$$

(v) The volume and density of the gas mixture:

The total volume

$$V = \frac{mRT_1}{p} = \frac{(5+6) \times 0.237 \times 300}{400} = \mathbf{1.954\ m^3}$$

The density of the gas mixture:

$$\rho = \frac{m}{V} = \frac{11\ \text{kg}}{1.954\ \text{m}^3} = \mathbf{5.63\ kg/m^3}$$

(vi) C_p and C_v of the mixture

$$C_{vN_2} = \frac{R_u}{\mathcal{M}_{N_2}(\gamma_{N_2} - 1)} = \frac{8.314}{28 \times (1.4 - 1)} = 0.742\ \text{kJ/kg} \cdot \text{K}$$

$$C_{pN_2} = \gamma_{N_2} \times C_{vN_2} = 1.4 \times 0.742 = 1.039\ \text{kJ/kg} \cdot \text{K}$$

$$C_{vCO_2} = \frac{R_u}{\mathcal{M}_{CO_2}(\gamma_{CO_2} - 1)} = \frac{8.314}{44 \times (1.286 - 1)} = 0.6606\ \text{kJ/kg} \cdot \text{K}$$

$$C_{pCO_2} = \gamma_{CO_2} \times C_{vCO_2} = 1.286 \times 0.6606 = 0.85\ \text{kJ/kg} \cdot \text{K}$$

For mixture,

$$C_p = \frac{m_{N_2} C_{pN_2} + m_{CO_2} C_{pCO_2}}{m_{N_2} + m_{CO_2}} = \frac{5 \times 1.039 + 6 \times 0.85}{5 + 6} = \mathbf{0.936\ kJ/kg \cdot K}$$

$$C_v = \frac{m_{N_2} C_{vN_2} + m_{CO_2} C_{vCO_2}}{m_{N_2} + m_{CO_2}} = \frac{5 \times 0.742 + 6 \times 0.6606}{5 + 6} = \mathbf{0.697\ kJ/kg \cdot K}$$

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When mixture is heated at constant volume

(vii) Change in internal energy of the mixture,

$$U_2 - U_1 = \Delta U = mC_v(\Delta T)$$
$$= 11 \times 0.697 \times (333 - 300)$$
$$= \mathbf{253.22\ kJ}$$

(viii) Change in enthalpy of the mixture,

$$\Delta H = H_2 - H_1 = mC_p(\Delta T)$$
$$= 11 \times 0.936 \times (333 - 300)$$
$$= \mathbf{339.76\ kJ}$$

(xi) Change in entropy of the mixture,

$$\Delta S = S_2 - S_1 = mC_v \ln\left(\frac{T_2}{T_1}\right) + mR \ln\left(\frac{v_2}{v_1}\right)$$
$$= 11 \times 0.697 \ln\left(\frac{333}{300}\right) + 0$$
$$= \mathbf{0.80\ kJ/K}$$

Example 14.7 *In an engine cylinder, a gas has volumetric analysis of 13% CO_2, 12.5% O_2 and 74.5% N_2. The temperature at the beginning of expansion is 1050°C and gas mixture expands reversibly through a volume ratio 8: 1 according to the law $pv^{1.2} = C$.*

Calculate per kg of gas:

(a) The work done

(b) The heat flow

(c) The change of entropy per kg of mixture

The value of C_p for constituents CO_2, O_2 and N_2 are 1.235 kJ/kg·K, 1.088 kJ/kg·K and 1.172 kJ/kg·K respectively.

Solution

Given The volumetric analysis of a gas mixture:

$CO_2 = 13\%$ $O_2 = 12.5\%$

$N_2 = 74.5\%$

$T_1 = 1050°C + 273°C = 1323\ K$

$\frac{v_2}{v_1} = 8$ $pv^{1.2} = C$

$C_{p,CO_2} = 1.235\ kJ/kg \cdot K$ $C_{p,O_2} = 1.088\ kJ/kg \cdot K$,

$C_{p,N_2} = 1.172\ kJ/kg \cdot K$

To find

(i) The work done,

(ii) The heat flow, and

(iii) The change of entropy per kg of mixture.

Assumptions

(i) The gas mixture in the cylinder is analysed as closed system at steady state.

(ii) Each constituent of the gas mixture behaves as an ideal gas.

(iii) The kinetic and potential energy effects are negligible.

(iv) Polytropic expansion is internally reversible.

Analysis Using $m_i = n_i \mathcal{M}_i$ for conversion of volume fraction to mass fraction for 1 mole of the mixture.

Constituents	n_i	$\mathcal{M}_i$	$m_i = n_i \mathcal{M}_i$	$x_i = m_i/m$ = *mass fraction*
CO_2	0.13	44	5.72	0.187
O_2	0.125	32	4.00	0.130
N_2	0.745	28	20.86	0.682
			$m = 30.58$	

The specific heat of the mixture

$$C_p = \frac{\Sigma m_i C_{pi}}{m} = \Sigma x_i C_{p_i}$$
$$= 0.187 \times 1.235 + 0.131 \times 1.088 + 0.682 \times 1.172$$
$$= 0.231 + 0.1425 + 0.799 = 1.1725\ kJ/kg \cdot K$$

The gas constant

$$R = \frac{\Sigma m_i R_1}{m} = \Sigma x_i R_i = \Sigma x_i \frac{R_u}{\mathcal{M}_i}$$

Thus $$R = 0.187 \times \frac{8.314}{44} + 0.131 \times \frac{8.314}{32} + 0.682 \times \frac{8.314}{28}$$
$$= 0.0353 + 0.0340 + 0.2025$$
$$= 0.2718\ kJ/kg \cdot K$$

and $$C_v = C_p - R = 1.1725 - 0.2718$$
$$= 0.90\ kJ/kg \cdot K$$

The temperature after expansion

$$\frac{T_2}{T_1} = \left(\frac{v_2}{v_1}\right)^{1-n} = \left(\frac{v_1}{v_2}\right)^{n-1}$$

or $$T_2 = (1323\ K) \times \left(\frac{1}{8}\right)^{1.2-1} = 872.85\ K$$

(i) The work done by 1 kg of gas mixture,

$$w = \frac{R(T_1 - T_2)}{n-1}$$
$$= \frac{0.2718 \times (1323 - 872.85)}{1.2 - 1}$$
$$= \mathbf{611.74\ kJ/kg}$$

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(ii) The heat flow per kg of gas mixture,

The change of internal energy

$$\Delta u = C_v(T_2 - T_1)$$
$$= 0.90 \times (872.85 - 1323)$$
$$= -405.13 \text{ kJ/kg.}$$

The heat flow

$$q = \Delta u + w = -405.13 + 611.74$$
$$= \mathbf{206.60 \text{ kJ/kg}}$$

(iii) The change of entropy per kg of mixture:

$$\Delta s = s_2 - s_1 = C_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)$$
$$= 0.90 \times \ln\left(\frac{872.85}{1325}\right) + 0.2713 \times \ln(8)$$
$$= -0.3756 + 0.5652 = \mathbf{0.1896 \text{ kJ/kg} \cdot \text{K}}$$

Example 14.8 *A mixture consisting of 1 kg of He and 2.5 kg of N_2 at 25°C and 10 N/cm^2, compressed in a reversible adiabatic process to 70 N/cm^2. Calculate*

(a) The final partial pressure of the constituents

(b) The final temperature and the change in internal energy of the mixture during the process.

Molecular weight of He = 4 kg

C_v *for* N_2 *= 0.743 kJ/kgK*

C_v *for He = 3.14 kJ/kgK*

C_p *for* N_2 *= 1.04 kJ/kgK*

C_p *for He = 5.23 kJ/kgK*

Solution

Given A mixture of He and N_2

$m_{He} = 1$ kg, $m_{N_2} = 2.5$ kg

$T_1 = 25°C = 298$ k, $p_1 = 10 \text{ N/cm}^2 = 1.0$ bar

$p_2 = 70 \text{ N/cm}^2 = 7.0$ bar, $\mathcal{M}_{He} = 4$

$C_{v,N_2} = 0.743 \text{ kJ/kg} \cdot \text{K}$, $C_{p,N_2} = 1.04 \text{ kJ/kg} \cdot \text{K}$

$C_{v,He} = 3.14 \text{ kJ/kg} \cdot \text{K}$, $C_{p,He} = 5.23 \text{ kJ/kg} \cdot \text{K}$

To find

(i) Final partial pressures of constituents,

(ii) Final temperature of mixture, and

(iii) Change in internal energy of mixture during process,

Analysis The number of moles of constituents

$$n_{He} = \frac{m_{He}}{\mathcal{M}_{He}} = \frac{1}{4} = 0.25$$

$$n_{N_2} = \frac{m_{N_2}}{\mathcal{M}_{N_2}} = \frac{2.5}{28} = 0.0893$$

Total moles, $n = n_{He} + n_{N_2} = 0.25 + 0.0893 = 0.3393$

Mole fractions:

$$y_{N_2} = \frac{n_{N_2}}{n} = \frac{0.0893}{0.3393} = 0.263$$

$$y_{He} = \frac{n_{He}}{n} = \frac{0.25}{0.3393} = 0.7368$$

(i) *Final partial pressures of constituents:*

(a) Final partial pressure of N_2

$$p_{2,N_2} = y_{N_2} p_2 = 0.263 \times (7 \text{ bar})$$
$$= \mathbf{1.84 \text{ bar}}$$

(b) Final partial pressure of He

$$p_{2,He} = y_{He} p_2 = 0.7368 \times (7 \text{ bar})$$
$$= \mathbf{5.1576 \text{ bar}}$$

(ii) *Final temperatures of mixture*

Total mass of mixture

$$m = m_{He} + m_{N_2}$$
$$= 1 + 2.5 = 3.5 \text{ kg}$$

Specific heat of mixture

$$C_p = \frac{m_{N_2} C_{p,N_2} + m_{He} C_{p,He}}{m}$$
$$= \frac{2.5 \times 1.0\ 4 + 1 \times 5.2\ 3}{3.5}$$
$$= 2.237 \text{ kJ/kg} \cdot \text{K}$$

$$C_v = \frac{m_{N_2} C_{v,N_2} + m_{He} C_{v,He}}{m}$$
$$= \frac{2.5 \times 0.743 + 1 \times 3.14}{3.5}$$
$$= 1.4278 \text{ kJ/kg} \cdot \text{K}$$

Ratio of two specific heats for mixture

$$\gamma = \frac{C_p}{C_v} = \frac{2.237}{1.4278} = 1.5667$$

Temperature after isentropic compression process

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$
$$= 298 \times \left(\frac{7}{1}\right)^{\frac{1.5667 - 9}{1.5667}}$$
$$= 602.42 \text{ K or } \mathbf{329.42°C}$$

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(iii) *Change in internal energy of mixture*

$$\Delta U = mC_v(T_2 - T_1)$$
$$= 3.5 \times 1.4278 \times (602.42 - 298)$$
$$= \mathbf{1521.27\ kJ}$$

14.6 MIXING OF IDEAL GASES

In the proceeding section, we have considered the mixture of ideal gases that have already been formed, and we related the properties of mixture to the properties of the gas components. Now we take up the *process of mixing* of ideal gases, that are initially separated. Such mixing of ideal gases is an irreversible process since the gases can mix spontaneously, but cannot separate to their initial states, without external work input. In this section, we relate the states of gas components before mixing to the state of the mixture.

14.6.1 Adiabatic Mixing at Constant Volume

Consider the adiabatic mixing of two gases A and B at different pressures and temperatures in a closed vessel by a thin diaphram as shown in Fig. 14.3(a). The diaphram is ruptured, and the gases mix each other and form a mixture as shown in Fig. 14.3(b).

The total volume and mass of the mixture are the sum of individual volumes and masses, receptively. The mixing process can be viewed as free expansion of each gas and is thus irreversible. Since the mixing process is adiabatic and there is no work done, thus the internal energy of the system remains constant, and

$$U = U_A + U_B$$

But the internal energy of the each component is generally not same before and after mixing, but change in internal energy of the system is zero.

$$\Delta U = \Delta U_A + \Delta U_B = 0$$

or $$\Delta U = m_A u_A + m_B u_B = 0$$

For any ideal gas, the internal energy is the function of absolute temperature and is expressed as

$$U = m\,C_v\,T$$

Thus, for a gas mixture before and after mixing

$$m\,C_v\,T = m_A\,C_{vA}\,T_A + m_B\,C_{vB}\,T_B$$

or
$$T = \frac{m_A C_{vA} T_A + m_B C_{vB} T_B}{mC_v}$$
$$= \frac{\Sigma m_i C_{vi} T_t}{mC_v} \qquad \ldots(14.30)$$

where C_v is the specific heat at constant volume of the mixture. The change in internal energy ΔU can also be calculated as

$$(n_A\bar{C}_{vA} + n_B\bar{C}_{vB})\,T = n_A\,\bar{C}_{vA}T_A + n_B\,\bar{C}_{vB}T_B$$

or
$$T = \frac{n_A\bar{C}_{vA}T_A + n_B\,\bar{C}_{vB}\,T_B}{n_A\bar{C}_{vA} + n_B\,\bar{C}_{vB}}$$
$$= \frac{\Sigma n_i \bar{C}_{vi}\,T_i}{\Sigma n_i\,\bar{C}_{vi}} \qquad \ldots(14.31)$$

Applying the equation of state, the final pressure of the mixture

$$p = \frac{mRT}{V} \qquad \ldots(14.32)$$

Where R is gas constant of mixture, m is mass of the mixture. On molar basis, the pressure of mixture

$$p = \frac{n\,R_u T}{V} \qquad \ldots(14.33)$$

Since mixing is an irreversible process, thus the entropy must increase during the process. The increase in entropy of mixture equal to sum of increase in entropies

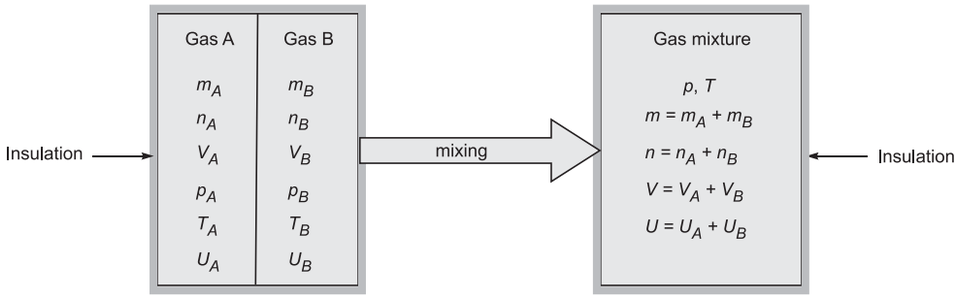

Fig. 14.3 *Adiabatic mixing of two gases, initially separate*

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of individual gas components, i.e.,

$$\Delta S = (S_2 - S_1)_A + (S_2 - S_1)_B > 0 \qquad \ldots(14.34)$$

where $(S_2 - S_1)_A$

$$= m_A \left[C_{pA} \ln\left(\frac{T}{T_A}\right) - R \ln\left(\frac{p_{A,2}}{p_{A,1}}\right) \right] \ldots(14.35)$$

where $p_{A,1}$ is initial partial pressure of gas A, $p_{A,2}$ is the final partial pressure of gas A in the mixture.

Similarly,

$(S_2 - S_1)_B$

$$= m_B \left[C_{pB} \ln\left(\frac{T}{T_B}\right) - R \ln\left(\frac{p_{B,2}}{p_{B,1}}\right) \right] \qquad \ldots(14.36)$$

On molal basis, the change in entropy of ith component in the gas mixture;

$$\Delta S_i = (S_2 - S_1)_i$$

$$= n_i \left[\bar{C}_{pi} \ln\left(\frac{T}{T_i}\right) - R_u \ln\left(\frac{p_{i,2}}{p_{i,1}}\right) \right] \qquad \ldots(14.37)$$

or $$\Delta S_i = n_i \left[\bar{C}_{vi} \ln\left(\frac{T}{T_i}\right) + R_u \ln\left(\frac{V}{V_i}\right) \right] \qquad \ldots(14.38)$$

Example 14.9 *Two vessels A and B, both containing oxygen, are connected by a valve which open to allow the mixing of two streams. Before mixing the following properties are known*

Vessel A	*Vessel B*
p_A = 15 bar	p_B = 6 bar
T_A = 50°C	T_B = 20°C
Contents = 0.5 kg mol	Contents = 2.5 kg

Temperature of oxygen after mixing reaches to 27°C Calculate final equilibrium pressure and amount of heat transferred to surroundings.

Take $\gamma = 1.4$

Given Oxygen in vessels A and B

$\gamma = 1.4$

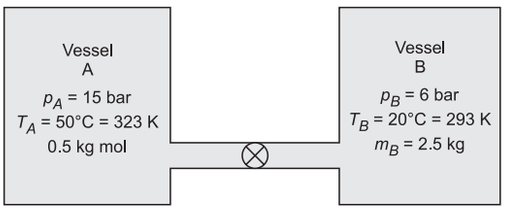

Fig. 14.4

To find

(i) The pressure of oxygen after mixing,

(ii) Heat transferred to surroundings.

Analysis For the oxygen in vessel A

$$p_A V_A = n_A R_u T_A$$

$$15 \times 10^2 \times V_A = 0.5 \times 8.314 \times 323$$

$$V_A = 0.895 \text{ m}^3$$

The mass of oxygen in vessel A

$$m_A = n_A \mathcal{M}_A$$

$$= 0.5 \times 32 = 16 \text{ kg}$$

The characteristic gas constant R for oxygen

$$R = \frac{R_u}{\mathcal{M}} = \frac{8.314}{32} = 0.259 \text{ kJ/kg·K}$$

For vessel B.

$$p_B V_B = m_B R T_B$$

$$6 \times 10^2 \times V_B = 2.5 \times 0.259 \times 293$$

$$V_B = 0.316 \text{ m}^3$$

Total volume of vessel A and B.

$$V = V_A + V_B$$

$$= 0.895 + 0.316 = 1.211 \text{ m}^3$$

Total mass of oxygen after mixing

$$m = m_A + m_B = 16 + 2.5 = 18.5 \text{ kg}$$

Final temperature after mixing = 27°C = 300 K

Final condition after mixing

$$pV = mRT$$

Using numerical values

$$p \times 1.211 = 18.5 \times 0.259 \times 300$$

or $$p = 1186.8 \text{ kPa} \approx \mathbf{11.86 \text{ bar}}$$

Heat transferred to the surroundings

During mixing of two streams of oxygen

$$W = 0$$

$$Q = U_2 - U_1$$

Where

U_1 = Internal energy of two stream of O_2 before mixing

U_2 = Internal energy of mixture

we have $U = m C_v T$

Where $C_v = \dfrac{R}{\gamma - 1} = \dfrac{0.259}{1.4 - 1} = 0.6475 \text{ kJ/kg·K}$

Then $U_1 = m_A C_v T_A + m_B C_v T_B$

$$= 16 \times 0.6475 \times 323 + 2.5 \times 0.6475 \times 293$$

$$= 3820.57 \text{ kJ}$$

and $U_2 = m C_v T = 18.5 \times 0.6475 \times 300 = 3593.62 \text{ kJ}$

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Therefore,

$$Q = 3593.62 - 3820.57 = \mathbf{-226.94\ kJ}$$

Example 14.10 *A vessel of 1.5 m^3 capacity contains oxygen at 7 bar and 40°C. The vessel is connected to another vessel of 3-m^3 capacity containing carbon monoxide at 1 bar and 15°C. A connecting valve is opened and the gases mix adiabatically. Calculate*

(a) Final temperature and pressure of the mixture

(b) Change in entropy of the system

For oxygen $\bar{C}_v = 21.07$ kJ/kmol·K,

for carbon monoxide $\bar{C}_v = 20.86$ kJ/kmol·K.

Solution

Given Adiabatic mixing of two gases at constant total volume. The initial state of two gases are shown in two separate boxes:

Oxygen	*Carbon monoxide*
$V = 1.5\ m^3$	$V = 3\ m^3$
$p = 7$ bar	$p = 1$ bar
$T = 40°C = 313$ K	$T = 15°C = 288$ K
$\bar{C}_v = 21.07$ kJ/kmol·K	$\bar{C}_v = 20.86$ kJ/kmol·K

To find

(i) The pressure and temperature of the mixture,

(ii) Change in entropy of the system.

Assumptions $\mathcal{M}_{O_2} = 32$, $\mathcal{M}_{CO} = 28$

Analysis

(i) The number of moles of each gas constituent are

$$n_{O_2} = \left(\frac{pV}{R_u T}\right)_{O_2} = \frac{(7 \times 100\ \text{kPa}) \times (1.5\ \text{m}^3)}{(8.314\ \text{kJ/kmol·K}) \times (313\ \text{K})} = 0.4035\ \text{kmol}$$

and $$n_{CO} = \frac{(1 \times 100\ \text{kPa}) \times (3\ \text{m}^3)}{(8.314\ \text{kJ/kmol·K}) \times (288\ \text{K})} = 0.1253\ \text{kmol}$$

The total internal energy of two gases before mixing

$$U_1 = \Sigma n_i \bar{C}_{vi} T_i = (0.4035 \times 21.07 \times 313) + (0.1253 \times 20.86 \times 288) = 3413.8\ \text{kJ}$$

The internal energy after mixing

$$U_2 = \Sigma(n_i \bar{C}_{vi})T = T[(0.4035 \times 21.07) + (0.1253 \times 20.86)] = 11.118\,T$$

For adiabatic mixing

$$U_2 = U_1$$

or $$11.118T = 3413.8\ \text{kJ}$$

or $$T = 307\ \text{K} = \mathbf{34°C}$$

The pressure of the mixture is

$$p = \frac{n R_u T}{V} = \frac{(0.4035 + 0.1253) \times 8.314 \times 307}{(1.5 + 3)} = 300\ \text{kPa} = \mathbf{3\ bar}$$

(ii) Change in entropy of the mixture:

$$\Delta S = \Delta S_{O_2} + \Delta S_{CO}$$

$$\Delta S_{O_2} = n_{O_2}\left[\bar{C}_{v_{O_2}} \ln\left(\frac{T}{T_{O_2}}\right) + R_u \ln\left(\frac{V}{V_{O_2}}\right)\right]$$

$$= 0.4035 \times \left[21.07 \times \ln\left(\frac{307}{313}\right) + (8.314) \times \ln\left(\frac{4.5}{1.5}\right)\right]$$

$$= 3.52\ \text{kJ/K}$$

$$\Delta S_{CO} = n_{CO}\left[\bar{C}_{v_{CO}} \ln\left(\frac{T}{T_{CO}}\right) + R_u \ln\left(\frac{V}{V_{CO}}\right)\right]$$

$$= 0.1253 \times \left[20.86 \times \ln\left(\frac{307}{288}\right) + 8.314 \times \ln\left(\frac{4.5}{3}\right)\right]$$

$$= 0.589\ \text{kJ/K}$$

and $$\Delta S = 3.52 + 0.589 = \mathbf{4.109\ kJ/K}$$

14.6.2 Adiabatic Mixing of Two Gas Streams

Another form of adiabatic mixing is the mixing of two streams of gases and to form a common gas stream in steady state. It is shown schematically in Fig. 14.5. In absence of any changes in kinetic and potential energy the steady flow energy equation yields to

$$\dot{m} = \dot{m}_A + \dot{m}_B$$

and $$\dot{Q} - \dot{W} = \dot{m}\,h - (\dot{m}_A h_A + \dot{m}_B h_B)$$

For adiabatic mixing, $\dot{Q} = 0$

No work transfer, thus $\dot{W} = 0$

Therefore, $$\dot{m}h = \dot{m}_A h_A + \dot{m}_B h_B$$

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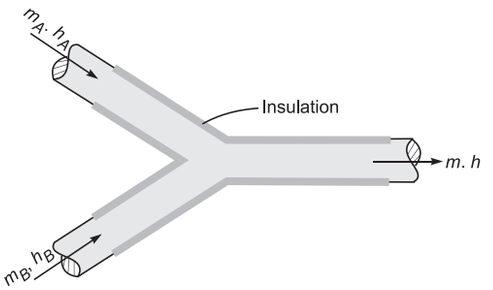

Fig. 14.5 *Adiabatic mixing of two gas streams*

Using $h = C_p T$,

hence $\dot{m} C_p T = \dot{m}_A C_{pA} T_A + \dot{m}_B C_{pB} T_B$

For more number of gas

$$\dot{m} C_p T = \Sigma \dot{m}_i C_{pi} T_i$$

or
$$T = \frac{\Sigma \dot{m}_i C_{pi} T_i}{\dot{m} C_p} \quad \ldots(14.39)$$

On the molal basis

$$T = \frac{\Sigma n_i \bar{C}_{pi} T_i}{\Sigma n_i \bar{C}_{pi}} \quad \ldots(14.40)$$

For change of entropy during mixing, the Eqs. (14.35) and (14.37) may be used, by replacing $p_{i,2}$ by mixture pressure p.

Example 14.11 *Methane at 100 kPa, 15°C enters an insulated mixing chamber, at a rate of 1.08 kg/s. It is mixed adiabatically with air at 100 kPa, 160°C in an air/methane mass ratio of 17.0. The flow is steady and kinetic and potential energy changes are negligible. The ambient conditions are 100 kPa, 15°C. Determine (a) the temperature of the mixture leaving the chamber, and (b) the irreversibility of the mixing per kg of methane.*

Solution

Given The adiabatic mixing of air and methane in proportion of 17.0

To find

(i) Temperature of the mixture leaving the chamber.

(ii) Irreversibility of mixing per kg of methane.

Assumptions

(i) The mixing chamber as a control volume.

(ii) The entering gases and exit mixture are regarded as ideal gases with mean values of specific heats as

$$C_{p,air} = 1.02 \text{ kJ/kg·K}, C_{p,CH_4} = 2.37 \text{ kJ/kg·K}$$

(iii) Dry air is treated as pure substance.

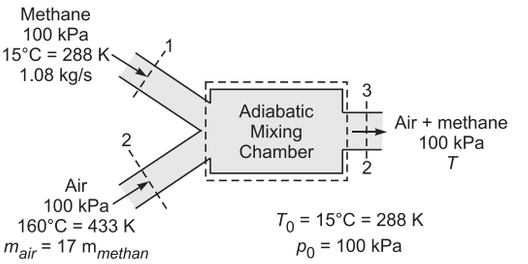

Fig. 14.6

Analysis

Mass flow rate of air $\dot{m}_{air} = 17\ \dot{m}_{methane}$

$= 17 \times 1.08 = 18.36$ kg/s

Specific heat of mixture

$$C = \frac{\Sigma \dot{m}_i C_{pi}}{\dot{m}} = \frac{1.08 \times 2.37 + 18.36 \times 1.02}{19.44}$$

$= 1.095$ kJ/kg K

(i) Applying the mass and energy balance on the control volume.

$$\dot{m} = \dot{m}_{methane} + \dot{m}_{air}$$
$$= 1.08 + 18.36 = 19.44 \text{ kg/s}$$

and $\dot{m}_{air} h_{air} + \dot{m}_{CH_4} h_{CH_4} = \dot{m} h$

or $18.36 \times 1.02 \times 433 + 1.08 \times 2.37 \times 288$

$= 19.44 \times 1.095\ T$

or $T = 415.56$ K = **142.56°C**

(ii) Irreversibility during the mixing process is given by

$$I = T_0 \Delta S$$
$$= T_0 [\dot{m}_{CH_4} \Delta s_{CH_4} + \dot{m}_{air} \Delta s_{air}]$$

For 1 kg of the methane gas

$$i = T_0 \left[\Delta s_{CH_4} + \frac{\dot{m}_{air}}{\dot{m}_{CH_4}} \Delta s_{air} \right]$$

For each component gas, Δs can be obtained as

$$\Delta s_i = C_p \ln \left(\frac{T}{T_i} \right) - R \ln \left(\frac{p_{i,1}}{p} \right)$$

Where p is the pressure of gas mixture, and $p_{i,1}$ is the partial pressure of gas in the mixture

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The number of moles for given mass can be calculated as

$$n_i = \frac{m_i}{\mathcal{M}_i}$$

$$n_{CH_4} = \frac{m_{CH_4}}{\mathcal{M}_{CH_4}} = \frac{1.08\ \text{kg}}{16} = 0.0675$$

$$n_{air} = \frac{m_{air}}{\mathcal{M}_{air}} = \frac{18.36}{28.97} = 0.633$$

Total moles, $n = 0.0675 + 0.633 = 0.701$ kmol

The mole fraction of methane

$$y_{CH_4} = \frac{n_{CH_4}}{n} = \frac{0.0675}{0.701} = 0.0962$$

and $y_{air} = 1 - x_{CH_4} = 0.9037$

Then $\frac{p_{i,1}}{p} = y_i$

$$\Delta s_{CH_4} = 2.37 \times \ln\left(\frac{415.56}{288}\right) - \left(\frac{8.134}{16}\right) \times \ln(0.0962) = 2.085\ \text{kJ/kg·K}$$

$$\Delta s_{air} = 1.02 \times \ln\left(\frac{415.56}{433}\right) - \left(\frac{8.134}{28.97}\right) \times \ln(0.9037) = -0.012\ \text{kJ/kg}$$

and $i = 288 \times \left[2.085 + 17 \times (-0.012)\right]$

$$= \mathbf{537.45\ kJ/kg}$$

Example 14.12 *At steady state, 100 m^3/min of dry air at 32°C and 1 bar is mixed adiabatically with a stream of oxygen (O_2)at 127°C and 1 bar to form a mixed stream at 47°C and 1 bar. The kinetic and potential energy effects are negligible. Determine (a) mass flow rates of dry air and oxygen in kg/min, (b) the mole of fraction of dry air and oxygen in the existing mixture, and (c) time rate of entropy production, in kJ/K.min.*

Solution

Given Adiabatic mixing of dry air and oxygen as

To find

(i) Mass flow rates of dry air and oxygen in kg/min.
(ii) Mole fractions of dry air and oxygen in mixture.
(iii) Time rate of entropy production in kJ/K·min.

Assumptions

$R_{air} = 0.287$ kJ/kg·K,
$R_{O_2} = 0.259$ kJ/kg·K, $C_{p,air} = 1.005$ kJ/kg·K,
$C_{p,O_2} = 0.928$ kJ/kg·K

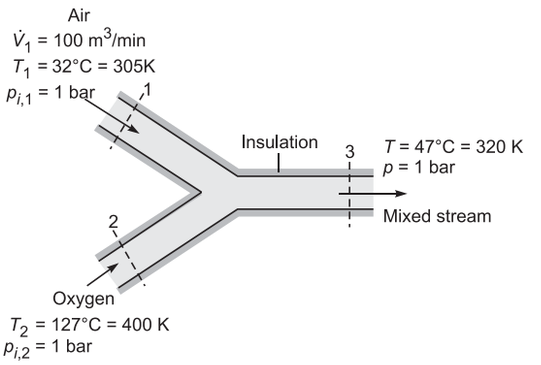

Fig. 14.7

Analysis

The specific volume of air at state 1 is calculated as

$$v_1 = \frac{RT_1}{p_1} = \frac{0.287 \times 305}{100} = 0.875\ \text{m}^3/\text{kg}$$

(i) The mass flow rate of dry air

$$\dot{m}_{air} = \frac{\dot{V}_1}{v_1}$$

or $\dot{m}_{air} = \frac{100\ \text{m}^3/\text{min}}{0.875\ \text{m}^3/\text{min}} = \mathbf{114.25\ kg/min}$

The mass flow rate of oxygen can be obtained by mass and energy balance at steady state

$\dot{m}_{air} h_{air} + \dot{m}_{O_2} h_{O_2} = (\dot{m}_{air} h_{air} + \dot{m}_{O_2} h_{O_2})_{mixture}$

or $\dot{m}_{air} h_{air}(T_1) + \dot{m}_{O_2} h_{O_2}(T_2) = \dot{m}_{air} h_{air}(T_3) + \dot{m}_{O_2} h_{O_2}(T_3)$

Thus $\dot{m}_{O_2} = \dot{m}_{air} \frac{[h_{air}(T_3) - h_{air}(T_1)]}{h_{O_2}(T_2) - h_{O_2}(T_3)}$

The specific enthalpies at given temperature can be obtained from properties of ideal gases; Tables A-8 and A-9, thus

$$\dot{m}_{O_2} = (114.25\ \text{kg/min}) \times \frac{(320.29 - 305.22)\ (\text{kJ/kg})}{\left(\frac{1}{32\ \text{kg/kmol}}\right)(11711 - 9325)\ (\text{kJ/kmol})}$$

$$= \mathbf{23.09\ kg/min}$$

The moles of gas components can be expressed as

$$n = \frac{\dot{m}}{\mathcal{M}}$$

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$$n_{air} = \frac{\dot{m}_{air}}{\mathcal{M}_{air}} = \frac{114.25}{28.97}$$
$$= 3.94 \text{ kmol/min}$$
$$n_{O_2} = \frac{\dot{m}_{O_2}}{\mathcal{M}_{O_2}} = \frac{23.09}{32}$$
$$= 0.721 \text{ kmol/min}$$

Total number of moles.

$$n = n_{air} + n_{O_2} = 3.94 + 0.721$$
$$= 4.66 \text{ kmol/min}$$

(ii) The *mole fraction*

air $y_{air} = \frac{n_{air}}{n} = \frac{3.94}{4.66} = \mathbf{0.845}$

oxygen $y_{O_2} = \frac{n_{O_2}}{n} = \frac{0.721}{4.66} = \mathbf{0.155}$

(iii) The entropy production during mixing is equal to rate of entropy increase during mixing

$$S_{gen} = \Delta S_{air} + \Delta S_{O_2}$$
$$\Delta S_{air} = \dot{m}_{air}\left[C_p \ln\left(\frac{T}{T_1}\right) - R\ln\left(\frac{p_{i,1}}{p}\right)\right]$$
$$= 114.25 \times \left[1.005 \times \ln\left(\frac{320}{305}\right) - 0.287 \times \ln(0.845)\right]$$
$$= 11.034 \text{ kJ/k} \cdot \text{min}$$

where,

$p_{i,1} = y_{air}\, p = 0.845$ bar

and $p_{i,2} = y_{O_2}\, p = 0.115$ bar

$$\Delta S_{O_2} = 23.09 \times \left[0.928 \times \ln\left(\frac{320}{400}\right) - 0.259 \times \ln(0.115)\right]$$
$$= 8.153 \text{ kJ/min}$$
$$S_{gen} = 11.034 + 8.153 = \mathbf{19.19\ kJ/K \cdot min}$$

14.7 MIXTURE OF REAL GASES

When the components of a gas mixture deviate considerably from an ideal gas behaviour, then the relations given above for ideal gas mixture cannot be used. However, the approximation can be considered by

(1) use of Dalton's law of partial pressure in conjunction with equation of state of component gases

(2) use of Amagat's law of partial volumes in conjunction with equation of state of components gases

(3) use of component compressibility factors.

14.7.1 Law of Additive Pressures

Dalton's law of partial pressures can be used with reasonable accuracy in some range of pressure and temperature. However, the component pressures should be evaluated from equation of state for real gases such as van der Waals equation, Beattie Bridgeman equation etc. we write

$$p = p_1 + p_2 + p_3 + \ldots = \sum_{i=1}^{i=k} p_i$$

where p_1, p_2 … denote the pressures that would exert by individual component gas, if they would be existed alone at temperature and volume of mixture. By selecting suitable equation of state, the pressure of each component gas in the mixture can be calculated such as using Van der Waal's equation.

$$p_i = \frac{R_i T}{v_i - b_i} - \frac{a_i}{v_i^2} \qquad \ldots(14.41)$$

where a_i and b_i are constants.

14.7.2 Law of Additive Volumes

The Amagat's law of additive volumes can also be used for gas mixture with considerable accuracy in some ranges of pressure and temperature.

$$V = V_1 + V_2 + V_3 + \ldots = \Sigma V_i$$

Where V_1, V_2, V_3 denote volumes of the individual components in the gas mixture, if they would be existed alone at pressure and temperature of mixture.

In terms of molal volume.

$$\overline{v} = \frac{V}{n} = \frac{V_1}{n_1} + \frac{V_2}{n_2} + \ldots$$

Using $y_i = \frac{n_i}{n}$,

we get $\overline{v} = \frac{y_1 V_1}{n_1} + \frac{y_2 V_2}{n_2} + \ldots$

$$= y_1 \overline{v}_1 + y_2 \overline{v}_2 + \ldots \qquad \ldots(14.42)$$

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By selecting a suitable equation of state, the molal specific volume of each component in gas mixture can be determined at pressure and temperature of mixture.

14.8 COMPRESSIBILITY FACTOR

The compressibility factors for various components of gas mixture are available. Therefore, the value of compressibility factor Z for real gas mixture can be obtained as

$$Z = y_1 Z_1 + y_2 Z_2 + y_3 Z_3 + \ldots$$
$$= \Sigma y_i Z_i \qquad \ldots(14.43)$$

If the compressibility factors Z_1, Z_2 etc. are evaluated at temperature and pressure of gas mixture, the above equation reduces to law of additive volumes.

The compressibility factor approach gives more accurate result when Z_i's are obtained by using Amagat's law of additive volumes. For a real gas mixture

$$pV = Z n R_u T \qquad \ldots(14.44)$$

The change in enthalpy and entropy of real gas mixture can be calculated as

$$\Delta h = x_1 \Delta h_1 + x_2 \Delta h_2 + \ldots = \Sigma x_i \Delta h_i$$

or $$\Delta \bar{h} = y_i \Delta \bar{h}_1 + y_2 \Delta \bar{h}_2 + \ldots = \Sigma y_i \Delta \bar{h}_i$$

and $$\Delta S = x_1 \Delta s_1 + x_2 \Delta s_2 + \ldots = \Sigma x_i \Delta s_i$$

or $$\Delta \bar{S} = y_i \Delta \bar{s}_1 + y_2 \Delta \bar{s}_2 + \ldots = \Sigma y_i \Delta \bar{s}_i$$

Summary

- The mass of the mixture is equal to sum of masses of its components:
$$m = m_1 + m_2 + m_3 + \ldots + m_k = \sum_{i=1}^{i=k} m_i$$
- The total number of moles
$$n = n_1 + n_2 + n_3 + n_4 + \ldots + n_k = \sum_{i=1}^{i=k} n_i$$
- The mass fraction and mole fraction are defined as
$$x_i = \frac{m_i}{m} \quad \text{and} \quad y_i = \frac{n_i}{n}$$
- The partial pressure p_i of ith competent in a gas mixture is given as
$$p_i = y_i p = \frac{n_i R_u T}{V}$$
- The partial volume V_i of ith component in a gas mixture
$$V_i = y_i V = \frac{n_i R_u T}{p}$$
- The *Dalton's law* of additive pressure states that the total pressure of mixture of ideal gases is sum of pressures of its components gases, if each existed alone at the temperature and volume of the mixture.
- *Amagat–Leduce law* states that the total volume of a mixture of ideal gases equals the sum of volumes of its components, if each existed alone at the pressure and temperature of the mixture.
- When the components of a gas mixture deviates from an ideal gas behaviour, then relation stated above cannot be used. However, an approximation is carried out by use of compressibility chart and viral expression, etc.
- The *compressibility factor Z* for a real gas mixture can be obtained as
$$Z = y_1 Z_1 + y_2 Z_2 + y_3 Z_3 + \ldots = \Sigma y_i Z_i$$
where Z_1, Z_2, etc. are compressibility factor of real gas components evaluated at mixture pressure and temperature. The equation of state for real gas mixture can be stated as
$$pV = Z n R_u T$$

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Glossary

Mass fraction Mass of a component divided by total mass of mixture.

Mole fraction Mole of a component divided by total moles of mixture.

Partial pressure Pressure of a component divided by total pressure of mixture at same temperature and volume.

Partial volume Volume of a component divided by total volume of mixture at same temperature and volume.

Apparent Molecular mass It is the ratio of mass and number of moles of a gas mixture.

Apparent Gas Constant It is the ratio of the universal gas constant to apparent molecular mass of gas mixture

Review Questions

1. What are the mass fraction and mole faction?
2. Using definition of mass and mole fractions, derive a relation between them.
3. What is an apparent molecular weight for a gas mixture? Does the mass of every molecule in the mixture equal to apparent molecular weight?
4. What is an apparent gas constant for a gas mixture? Can it be larger than the largest gas constant in the mixture?
5. Express Dalton's law of partial pressures. Does this law hold exactly for ideal gas mixtures?
6. What is the difference between the total pressure and the partial pressure?

Problems

1. A vessel of 0.35 m^3 capacity contains 0.4 kg of carbon monoxide and 1 kg of air at 20°C. Calculate:

(a) Partial pressure of each constituents,

(b) The total pressure in the vessel, and

The gravitational analysis of air is to be taken as 23.3% oxygen and 76.7% nitrogen.

[(a) 0.5068 bar, 1.9065 bar, 0.9943 bar, (b) 3.4076 bar]

2. A vessel contains a mixture of 1 mole of CO_2 and 4 moles of air at 1 bar and 20°C. Calculate for mixture:

(a) Mass of CO_2, O_2 and N_2 and total mass,

(b) The percentage carbon content by mass,

(c) The equivalent molecular weight and gas constant for the mixture,

(d) The specific volume of mixture.

The volumetric analysis of air can be taken as 21% oxygen and 79% nitrogen.

[(a) 44 kg, 26.88 kg, 88.48 kg, 159.36 kg, (b) 7.53%, (c) 31.872, 0.2608 kJ/kg·K, (d) 0.7641 m^3]

3. A gaseous mixture, by its volumetric composition, 78% H_2, and 22% CO is contained in a vessel. It is desired that the mixture should be made in proportion 52% H_2 and 48% CO by removing some of the mixture and adding some CO. Calculate per mole of the mixture, the mass of mixture to be removed, and mass of CO to be added.

Assume that the pressure and temperature in the vessel remains constant during the procedure.

[2.57 kg to be removed, 9.32 kg CO to be added]

4. A 0.825 m^3 tank contains 0.95 kg oxygen, 0.82 kg of nitrogen, 1.32 kg of carbon dioxide, and 0.091 kg of carbon monoxide at a temperature of 27°C. Calculate the total pressure of the mixture.

5. Carbon dioxide gas at 320 K is mixed with nitrogen at 280 K in a steady flow apparatus, adia-

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batically. Both gases flow at 100 kPa and have the mole ratio of 2 : 1. Find the exit temperature and total entropy generation per mole of the exit mixture.

Take C_p of CO_2 = 0.85 and
for N_2; = 1.04 kJ/kg·K.

[308.7 K, 5.35 kJ/kmol·K]

6. A gas mixture has volumetric analysis shows that it consists of 20% carbon dioxide, 5% carbon monoxide and remaining is nitrogen. For mixture temperature and pressure of 45% C and 500 kPa. Calculate (a) the partial pressures of the constituents, (b) equivalent molecular weight, and (c) the equivalent gas constant.

7. Flue gases consist of 81% nitrogen, 14% CO_2, 4.2% O_2 and 0.8% CO on volumetric basis. Calculate the density of flue gas mixture at 101.2 kPa and 47°C.

8. Air at 420 kPa, 80°C is introduced at the rate of 7.8 kg/min into a flowing air stream having a pressure of 102.2 kPa at 34°C. The pressure down stream from the point of mixing is 101.8 kPa and temperature is 42°C. Determine the mass flow rate of air at 34°C.

9. A vessel of 1.8 m^3 capacity contains oxygen at 8 bar and 50°C. The vessel is connected with another vessel of 3.6 m^3 capacity containing carbon monoxide at 1 bar and 20°C. The connecting valve is opened and the gases mixed adiabatically.

Calculate

(a) final temperature and pressure of the mixture;

(b) the change of entropy of the system

Take C_{vO_2} = 21.07 kJ/mol·K
C_{vCO} = 20.86 kJ/mol·K.

[(a) 437°C, 3.33 bar, (b) 5.411 kJ/K]

10. A vessel of 6 m^3 capacity contains two gases *A* and *B* in proportion of 45% and 55%, respectively at 30°C. If the gas constant *R* for these gases is 0.288 kJ/kg·K and 0.295 kJ/kg·K and total weight of the mixture is 2 kg. Calculate

(a) The partial pressure,

(b) The total pressure, and

(c) The mean value of *R* for the mixture.

[(a) 0.13 bar, 0.164 bar, (b) 0.294 bar, (c) 0.292 kJ/kg·K]

11. 4 kg of carbon dioxide at 40°C and 1.4 bar is mixed with 7 kg of nitrogen at 160°C and 1.0 bar form a mixture at a final pressure 0.7 bar. The process of mixing is adiabatic in a steady flow manner. Calculate

(a) final temperature of the mixture,

(b) the change in entropy.

Take C_p for CO_2 = 0.85 kJ/kg·K,
and N_2 = 1.04 kJ/kg·K

12. A mixture consists of 60% nitrogen and 40% CO_2 by mass. If the mixture is compressed from 2 bar, 5°C to 5 bar, 170°C, what is the entropy change of the mixture?

13. Two insulated tank *A* and *B* are connected by a valve. Tank *A* has a volume of 1 m^3 and initially contains Argon at 300 kPa, 10°C. Tank *B* has a volume of 2 m^3 and initially contains ethane at 200 kPa, 50°C. The valve is opened and remains open, until the resulting gas mixture reaches to an equilibrium state. Determine final pressure and temperature.

Take C_p of argon = 0.520 kJ/kg·K,
ethane C_p = 1.766 kJ/kg·K.

[242 kPa, 315.5 K]

14. A tank of capacity 0.45 m^3 is insulated and divided into two sections through a partition. One section contains hydrogen at 3 bar and 130°C and has volume of 0.3 m^3 and other section contains nitrogen at 6 bar and 30°C. The partition is then removed and gases are allowed to mix till they reach an equilibrium state. Calculate:

(a) Temperature of mixture,

(b) Pressure of mixture,

(c) The change in entropy for each component and total values.

Assume

C_p for N_2 = 0.744 kJ/kg·K,
C_v for H_2 = 10.35 kJ/K
$C_p(N_2)$ = 1.041 kJ/kg·K,
$C_p(H_2)$ = 14.47 kJ/kg

[(a) 345.7 K, (b) 3.99 bar, (c) 0.00626 kJ/kg·K, 0.424 kJ/K, 0.43026 kJ/K]

15. A mixture is made up of 25% N_2, 35% O_2, 20% CO_2 and 20% CO by volume. Calculate:

(a) the molar mass of the mixture;

(b) $\bar{C}_p$ and $\bar{C}_v$ for the mixture;

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(c) γ for the mixture;
(d) the partial pressure of each constituent when the total pressure is 1.5 bar;
(e) the density of the mixture at 1.5 bar and 15°C.

[(a) 32.6 kg/kmol; (b) 30.84, 22.53 kJ/kmol·K; (c) 1.37; (d) 0.375, 0.525, 0.3, 0.3 bar; (e) 2.04 kg/m^3]

16. Two vessels are connected by a pipe in which there is a valve. One vessel of 0.3 m^3 contains air at 7 bar and 32°C, and the other of 0.03 m^3 contains oxygen at 21 bar and 15°C. The valve is opened and the two gases are allowed to mix. Assuming that the system is well insulated, calculate:
(a) the final temperature of the mixture;
(b) the final pressure of the mixture;
(c) the partial pressure of each constituent;
(d) the volumetric analysis of the mixture;
(e) the values of C_p, C_v, R, $\mathcal{M}$ and γ for the mixture;
(f) the increase of entropy of the system per kilogram of mixture;
(g) the change in internal energy and enthalpy of the mixture per kilogram if the vessel is cooled to 10°C.

Assume that air consists only of oxygen and nitrogen.

[(a) 27.9°C; (b) 8.27 bar; (c) 3.31, 4.96 bar; (d) 60% N_2, 40% O_2; (e) 0.987, 0.709 kJ/kg·K; 0.278 kJ/kg·K; 29.91 kg/mol; 1.392; (f) 0.183 kJ/kg·K; (g) 12.69, 17.67 kJ/kg]

17. A rigid insulated tank has two compartments: Initially one contain 0.5 kmol of carbon dioxide (CO_2) at 27°C, 2 bar and the other contains 1 kmol of oxygen (O_2) at 152°C, 5 bar. The gases are allowed to mix while 500 kJ of energy are added by electrical work. Determine
(a) the final temperature, in°C
(b) the final pressure, in bar
(c) the change in energy, in kJ. for $T_0 = 20$°C
(d) the energy destruction, in kJ.

18. Using the ideal gas model with constant specific heats, determine the mixture temperature, in K, for each of two cases:
(a) Initially, 0.6 kmol of O_2 at 500 K is separated by a partition from 0.4 kmol of H_2 at 300 K in a rigid insulated vessel. The partition is removed and the gases mix to obtain a final equilibrium state.
(b) Oxygen (O_2) at 500 K and a molar flow rate of 0.6 kmol/s enters an insulated control volume operating at steady state and mixes with H_2 entering as a separate stream at 300 K and a molar flow rate of 0.4 kmol/s. A single mixed stream exits. Kinetic and potential energy effects can be ignored.

19. An insulated tank has two compartments connected by a valve. Initially, one compartment contains 0.7 kg of CO_2 at 500 K, 6.0 bar and the other contains 0.3 kg of N_2 at 300 K, 6.0 bar. The valve is opened and the gases are allowed to mix until equilibrium is achieved. Determine
(a) the final temperature, in K,
(b) the final pressure, in bar,
(c) the amount of entropy produced, in kJ/K.

Objective Questions

1. In a mixture of gases, the mass fraction, x_i is defined as

(a) $\frac{m_i}{m}$ (b) $\frac{n_i}{n}$
(c) $\frac{m_i}{n}$ (d) $\frac{n_i}{m}$

2. In a mixture of gases, the mole fraction, y_i in a mixture of

(a) $\frac{m_i}{m}$ (b) $\frac{n_i}{n}$
(c) $\frac{m_i}{n}$ (d) $\frac{n_i}{m}$

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3. The apparent molecular mass $\mathcal{M}$ of a gas mixture is given by
   (a) $\frac{m}{n}$ (b) $\Sigma x_i \mathcal{M}_i$
   (c) $\frac{R_u}{R}$ (d) all the above
4. Total pressure of a gas mixture is given by
   (a) $\frac{n R_u T}{V}$ (b) $\frac{p_i}{x_i}$
   (c) Σp_i (d) all the above
5. Which one of the following is true statement
   (a) $m = n\mathcal{M}$ (b) $m = \Sigma x_i n_i$
   (c) $m = \frac{n}{\mathcal{M}}$ (d) none of the above
6. According to Amagat–Leduce law, the partial volume of a gas in a gas mixture
   (a) volume occupied at partial pressure of gas
   (b) volume occupied at total pressure of the gas mixture
   (c) volume occupied by a gas at mixture pressure and temperature
   (d) none of the above
7. Partial pressure of a gas in a gas mixture is defined as
   (a) pressure exerted by a gas at mixture volume
   (b) pressure exerted by a gas at mixture temperature
   (c) pressure exerted by a gas at mixture volume and temperature
   (d) none of the above
8. Which of the following equations represents specific internal energy of a gas mixture (x_i = mass fraction and y_i = mole fraction)?
   (a) $\Sigma x_i u_i$ (b) $\Sigma y_i u_i$
   (c) $\Sigma x_i C_{vi}$ (d) None of the above
9. Which of the following equations represents specific enthalpy of a gas mixture (x_i = mass fraction and y_i = mole fraction)?
   (a) $\Sigma x_i h_i$ (b) $\Sigma y_i h_i$
   (c) $\Sigma x_i C_{pi}$ (d) none of the above
10. The specific heat of a gas mixture is given by
   (a) $\Sigma x_i C_{pi}$, (b) $\frac{\Sigma m_i C_{pi}}{m}$
   (c) $\Sigma n_i \bar{C}_{pi}$ (d) all the above

Answers

1. (a) 2. (b) 3. (d) 4. (d) 5. (a) 6. (c) 7. (c) 8. (a)
9. (a) 10. (d)

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15

Psychrometry

Introduction

The *Psychrometry* is the study of the properties of moist air and is useful to engineers concerned with heating, cooling, and ventilation of buildings, ships and aircrafts.

In most of air-conditioning applications, the atmospheric air is used at low temperatures (below 40°C). The water vapour present in atmospheric air is at very low partial pressure and it behaves as a perfect gas. Thus, the air-vapour mixture at low pressure and atmospheric temperature can be modeled as an ideal gas. That is,

(i) the equation of state is $pv = RT$,
(ii) the enthalpy of the gas mixture is the function of temperature only, and
(iii) the vapour behaves in all respects as it existed alone at its partial pressure and temperature of the mixture.

15.1 PSYCHROMETER

The psychrometer is an equipment used to measure dry-bulb and wet-bulb temperatures in the laboratory. It is used in the sling method, and therefore, the equipment is called a *sling psychrometer* as shown in Fig. 15.1. It consists of a dry-bulb thermometer and a wet-bulb thermometer mounted side by side in a protective case that is attached to a handle by a swivel connection, so that the case can be rotated. The sling thermometer is rotated for one minute in air and then readings are taken from the thermometers.

The dry-bulb thermometer is directly exposed to the atmospheric air and measures the actual temperature of air. The bulb of a wet-bulb thermometer is covered with water soaked cotton wick. The temperature measured by a wick-covered bulb is the temperature of the liquid water in the wick and thus is called the *wet bulb temperature*.

15.2 DRY, MOIST AND SATURATED AIR

1. Dry Air The *dry air* includes all components of atmospheric air except water vapour (moisture). Thus, the dry air is the mixture of oxygen, nitrogen, carbon dioxide, argon, etc. At low temperature (cold conditions), its specific heat remains almost constant and is taken as 1.005 kJ/kg·K.

2. Moist Air The term *moist air* is the mixture of the dry air and water vapour, and each component of the mixture behaves as an ideal gas at states under considerations.

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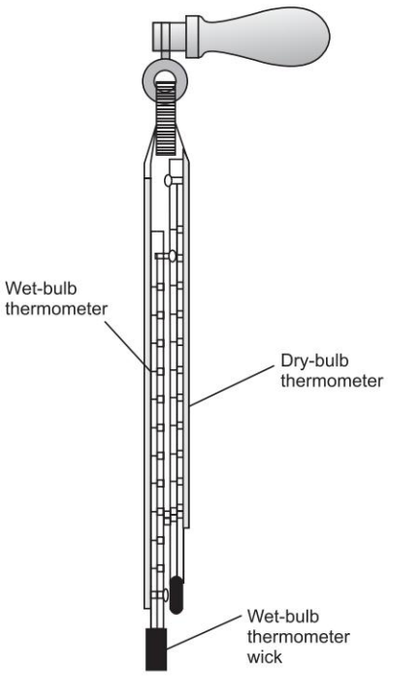

Fig. 15.1 *Sling psychrometer*

3. Unsaturated Air In atmospheric air, the moisture appears in the form of superheated vapour as invisible gas at atmospheric pressure. The mixture of superheated vapour and dry air is called *unsaturated air*.

4. Saturated Air The *saturated air* is moist air which contains maximum possible water vapour. The partial pressure of water vapour in saturated air is equal to the saturated pressure of steam corresponding to the temperature of moist air. Cooling of saturated air causes the separation of moisture and formation of fog.

Figure 15.2 illustrates the states of saturated and unsaturated air. At the state 1, the partial pressure p_v of water vapour is lower than the saturation pressure; thus the water vapour is in superheated state, the air is unsaturated at the mixture temperature T and total pressure p. At this state, partial pressure of air is $p_a = p - p_v$.

The state 2 is after isothermal cooling, when saturated water vapour is at the temperature T of the mixture. The corresponding partial pressure of air becomes p_{sat} equal to saturation pressure at temperature T and the partial pressure of air would be $p_a = p - p_{sat}$.

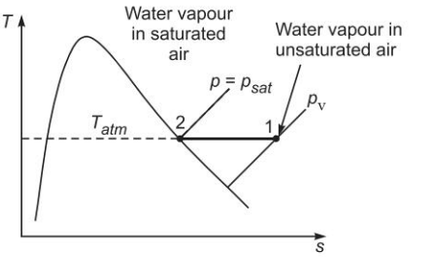

Fig. 15.2 *T–s diagram for unsaturated and saturated air*

15.3 PROPERTIES OF MOIST AIR

Some of the important psychrometric properties of moist are defined below:

15.3.1 Dry-bulb Temperature

The *dry-bulb temperature* (DBT), designated as T (or sometimes T_{db}), of the mixture is the temperature measured by an ordinary thermometer placed in the air-vapour mixture.

15.3.2 Wet-bulb Temperature

The *wet-bulb temperature* (WBT), T_{wb}, is the temperature measured by a thermometer whose bulb is covered by a thoroughly wetted cotton wick.

When air passes over the wetted cotton wick, some of the water evaporates, cooling effect is produced at the bulb and the temperature recorded is lower than that of the air stream.

Figure 15.3 shows the method of measurement of wet and dry bulb temperatures. The two thermometers are located in the stream of unsaturated air. As the air stream passes the wet cotton wick, some water evaporates and produces a cooling effect in the wick. The heat is transferred from air to wick with corresponding cooling. An equilibrium condition is reached at which the wet bulb thermometer indicates a temperature, that is lower than the dry bulb temperature.

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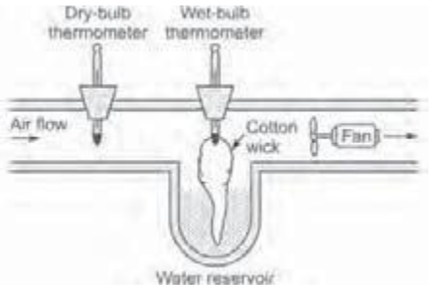

Fig. 15.3 *Steady-flow apparatus for measuring wet and dry-bulb temperature*

15.3.3 Wet-bulb Depression

The difference between the dry-bulb temperature and wet-bulb temperature is called as *wet-bulb depression*.

$$\text{Wet-bulb depression} = T - T_{wb} \qquad ...(15.1)$$

The amount of wet-bulb depression depends on moisture content of air. For completely dry air, the wet-bulb depression is maximum, because dry air is capable of absorbing maximum amount of moisture. On the other hand, when air is saturated, it cannot absorb moisture any more and the wet bulb temperature reading equals the dry bulb temperature readings. It means, wet bulb depression approaches zero for saturated air.

15.3.4 Dew-point Temperature

The *dew-point temperature,* T_{dp}, of an air–vapour mixture is a temperature at which the vapour starts to condense, when it is cooled at constant pressure. The partial condensation of water vapour on window panes in winter, or on pipes carrying cold water encounters dew most often. The formation of dew on the grass is the best example. It occurs due to cooling of water vapour at constant pressure to its saturation temperature.

Figure 15.4 illustrates the dew point on a T–s diagram. The air–vapour mixture at the state 1 is superheated. It is cooled at constant pressure p_v to the state 2, where the saturation temperature reaches and condensation of vapour begins. The temperature at state 2 is dew point temperature.

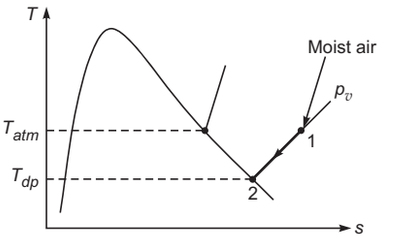

Fig. 15.4 *Cooling of vapour at constant pressure brings dew point temperature*

15.3.5 Dew-point Depression

The difference between the dry-bulb temperature and dew-point temperature is called as *dew-point depression*.

$$\text{Dew-point depression} = T - T_{dp} \qquad ...(15.2)$$

15.3.6 Relative Humidity

It is defined as the *ratio of actual mass of water vapour to the mass of saturated vapour* produced in a mixture of air and water vapour at the same temperature and pressure. It is denoted by ϕ(phi). Since water vapour behaves like an ideal gas,

$$\text{thus} \qquad \phi = \frac{m_v}{m_{sat}} = \frac{p_v V/R_v T}{p_{sat} V/R_v T} = \frac{p_v}{p_{sat}} \qquad ...(15.3)$$

where p_v = partial pressure of vapour in the mixture

p_{sat} = saturation pressure of vapour at temperature of mixture

T = mixture temperature

It can also be expressed as

$$\phi = \frac{p_v}{p_{sat}} = \frac{\rho_{fg}}{\rho_g} = \frac{v_g}{v_{fg}} \qquad ...(15.4)$$

15.3.7 Specific Humidity

The *specific humidity* is also called *humidity ratio* or *absolute humidity*. It is defined as *ratio of mass of water vapour to the mass of dry air* in a given volume of the mixture. It is denoted by ω (omega)

$$\omega = \frac{m_v}{m_a} \qquad ...(15.5)$$

where the subscript a refers to dry air and v refers to vapour. Since both the vapour and mixture are

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considered as ideal gases, thus

$$m_v = \frac{p_v V}{R_v T} = \frac{p_v V \mathcal{M}_v}{R_u T}$$

and $$m_a = \frac{p_a V}{R_a T} = \frac{p_a V \mathcal{M}_a}{R_u T}$$

Therefore, $$\omega = \frac{p_v \mathcal{M}_v}{p_a \mathcal{M}_a} \quad ...(15.6)$$

For an air–water vapour mixture($\mathcal{M}_v$ = 18 and $\mathcal{M}_a$ = 28.97), it reduces to

$$\omega = 0.622 \frac{p_v}{p_a} = 0.622 \frac{p_v}{p - p_v} \quad ...(15.7)$$

where p is the total pressure of the mixture (p_a + p_v), thus $p_a = p - p_v$.

15.3.8 Degree of Saturation

It is defined as the ratio of *actual humidity ratio* to the *humidity ratio of saturated air* at the same temperature and total pressure. It is designated as μ and is expressed as

$$\mu = \frac{\omega}{\omega_{sat}} \quad ...(15.8)$$

Using $\omega = 0.622 \frac{p_v}{p - p_v}$ from Eq. (15.7), and at saturation state

$$\omega_{sat} = 0.622 \frac{p_{sat}}{p - p_{sat}}$$

then $$\mu = \frac{p_v (p - p_{sat})}{p_{sat} (p - p_v)} = \phi \frac{p - p_{sat}}{p - p_v} \quad ...(15.9)$$

$$\mu p - \mu p_v = \phi (p - p_{sat})$$

Using $p_v = \phi p_{sat}$ from Eq. (15.3), then

or $$\mu p = \phi (p - p_{sat} + \mu p_{sat})$$
$$= \phi [p - (1 - \mu) p_{sat}] \approx \phi p$$

Since $(1 - \mu) p_{sat} \approx 0$

$\therefore$ $$\mu = \phi \quad ...(15.10)$$

15.3.9 Enthalpy of Moist Air

The enthalpy of moist air is the sum of enthalpy of dry air and that of water vapour. Thus, the enthalpy h of moist air per kg of dry air is expressed as

or $$m_a h = m_a h_a + m_v h_g$$

or $$h = h_a + \frac{m_v}{m_a} h_g$$
$$= (h_a + \omega h_g) \text{ kJ/kg of dry air} \quad ...(15.11)$$

where h_a = enthalpy of dry air
$= C_p T_{db} = 1.005 T$
h_g = enthalpy of water vapour calculated as
$= 2500 + 1.88 T$ kJ/kg of water vapour
T = dry-bulb temperature in °C

15.4 PARTIAL PRESSURE OF AIR AND VAPOUR

Figure 15.5 shows a closed system consisting of atmospheric air, a mixture of water vapour and dry air occupying volume V, at mixture pressure p and mixture temperature T. The mixture is assumed to follow ideal gas equation. Thus, the mixture pressure

$$p = \frac{n R_u T}{V}$$

Introducing $$n = \frac{m}{\mathcal{M}};$$

$$p = \frac{m R_u T}{\mathcal{M} V} \quad ...(15.12)$$

where n, m, $\mathcal{M}$ and R_u denote the number of moles, mass, molecular weight and universal gas constant, respectively.

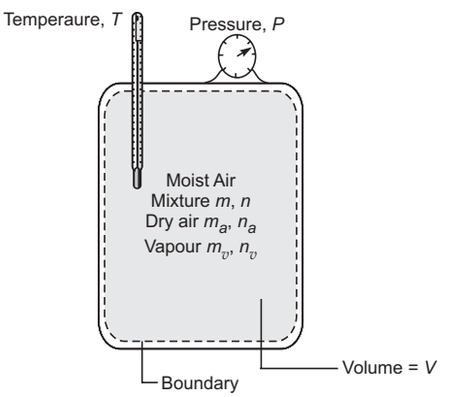

Fig. 15.5 *Mixture of moist air in a closed vessel*

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The partial pressure p_a of dry air in the mixture;

$$p_a = \frac{m_a R_u T}{\mathcal{M}_a V} \quad ...(15.13)$$

The partial pressure p_v of water vapour in the mixture

$$p_v = \frac{m_v R_u T}{\mathcal{M}_v V} \quad ...(15.14)$$

where m_a, m_v denote the mass of dry air and water vapour, respectively and $\mathcal{M}_a$ and $\mathcal{M}_v$ are respective molecular weights in the mixture.

The total pressure of the mixture is the sum of partial pressure of water vapour and dry air present in the mixture.

$$p = p_a + p_v \quad ...(15.15)$$

The saturation pressure of water corresponding to dew point can be obtained from steam tables, which is equal to the partial pressure of water vapour in air. But it is difficult to measure the dew-point temperature accurately.

The wet-bulb temperature of the mixture is easily measured with a wetted wick thermometer. The partial pressure p_v of vapour in the mixture can be calculated by using Dr Willis H *Carrier's equation*

$$p_v = p'_v - \frac{(p - p'_v)(T - T_{wb}) \times 1.8}{2800 - 1.3 \times (1.8T + 32)} \quad ...(15.16)$$

where p_v = partial pressure of water vapour at dry bulb temperatutre in mixture,
p'_v = saturation water pressure at wet bulb temperature,
p = total pressure of moist air,
T = dry-bulb temperature,
T_{wb} = wet-bulb temperature.

Example 15.1 *Atmospheric air at 95 kPa, 30°C has a relative humidity of 70 per cent. Determine humidity ratio.*

Solution

Given Atmospheric air

$p = p_{atm} = 95$ kPa $\quad T = 30°C = 303$ K
$\phi = 0.7$

To find Humidity ratio

Analysis From steam tables, the saturation pressure of water at 30°C

$p_{sat} = 4.246$ kPa

The partial pressure of water vapour in mixture

$p_v = \phi p_{sat}$
$= 0.7 \times 4.246 = 2.97$ kPa

The partial pressure of air,

$p_a = p - p_v = 95 - 2.97 = 92.03$ kPa

The specific humidity by using Eq. (15.7)

$$\omega = 0.622 \frac{p_v}{p_a}$$
$$= 0.622 \times \frac{2.97}{92.03}$$
$$= \mathbf{0.02007\ kg/kg/of\ dry\ air}$$

Example 15.2 *A sample of 450 gram of moist air at 22°C, 101 kPa and 70% relative humidity is cooled to 5°C, while keeping the pressure constant. Determine (a) the initial humidty ratio (b) dew point temperature, and (c) amount of water vapour that condenses.*

Solution

Given A sample of moist air is cooled at constant pressure

$m = 0.45$ kg $\quad T_1 = 22°C = 295$ K
$p = 101$ kPa $\quad p =$ constant
$\phi = 70\%$ $\quad T_2 = 5°C$

To find

(i) Initial humidity ratio,
(ii) Dew point temperature, and
(iii) Amount of water vapour that condenses.

Schematic with given data

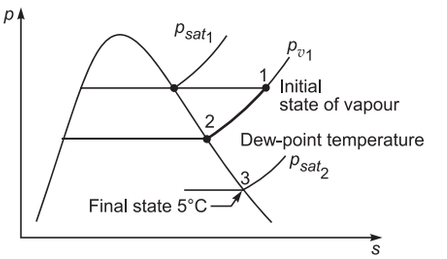

Fig. 15.6

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Assumptions

(i) 1 kg of moist air sample as a closed system.
(ii) The gas phase of air and vapour can be treated as an ideal gas mixture.

Analysis From steam tables at 22°C;

$$p_{sat_1} = p_{sat} @ 22°C = 0.02645 \text{ bar}$$

The partial pressure of water vapour p_{v_1}

$$p_{v_1} = \phi\, p_{sat_1}$$
$$= 0.7 \times 0.02645 = 0.0185 \text{ bar} = 1.85 \text{ kPa}$$

(i) The humidity ratio at initial state

$$\omega_1 = 0.622 \frac{p_{v_1}}{p - p_{v_1}}$$
$$= 0.622 \times \frac{1.85}{101 - 1.85}$$
$$= \mathbf{0.011 \text{ kg/kg of dry air}}$$

(ii) Dew-point temperature is the saturation temperature corresponding to partial pressure p_{v_1}. Interpolating between 16°C and 17°C, we get

$$T_{dp} = \mathbf{16.267°C}$$

(iii) The amount of condensate,

m_w = Initial amount of water vapour – Final amount of water vapour

$$= m_{v_1} - m_{v_2}$$

For 1 kg of dry air and moisture,

$$1 \text{ kg} = m_a + m_{v_1}$$

Using $\omega_1 = \dfrac{m_{v_1}}{m_a}$;

$$1 \text{ kg} = \frac{m_{v_1}}{\omega_1} + m_{v_1} = m_{v_1}\left(\frac{1}{\omega} + 1\right)$$

or $$m_{v_1} = \frac{\omega_1}{\omega_1 + 1}$$
$$= \frac{0.011}{0.011 + 1}$$
$$= 0.0109 \text{ kg/kg of dry air}$$

The mass of dry air/kg,

$$m_a = 1 - 0.0109 = 0.9891 \text{ kg}$$

Further, humidity ratio ω_2 at state 3, *i.e.,* at 5°C

$$p_{sat} = 0.00872 \text{ bar} = 0.872 \text{ kPa}$$

$$\omega_2 = \frac{0.622 \times 0.872}{101 - 0.872}$$
$$= 0.0054 \text{ kg/kg of dry air}$$

The mass of water vapour

$$m_{v_2} = \omega_2\, m_a = 0.0054 \times 0.9891$$
$$= 0.00535 \text{ kg/kg of dry air}$$

The amount of water vapour condensed

$$m_w = m_{v_1} - m_{v_2} = 0.0109 - 0.00535$$
$$= 0.0055 \text{ kg/kg of dry air}$$

In the sample of 0.450 kg

$$m_w = 0.45 \times 0.0055 = \mathbf{0.0025 \text{ kg}}$$

Example 15.3 *An air–water vapour mixture is contained in a rigid closed vessel with a volume of 35 m^3 at 1.5 bar, 120°C, and ϕ = 10%. The mixture is cooled at constant volume with its temperature decrease to 22°C. Determine (a) the dew point temperature corresponding to initial state, (b) the temperature at which the condensation actually begins, and (c) amount of water condensed.*

Solution

Given An air–vapour mixture in a closed vessel,

$V = 35 \text{ m}^3$ $\quad p_1 = 1.5 \text{ bar}$
$T_1 = 120°C = 393 \text{ K}$ $\quad \phi_1 = 0.1$
$V_1 = V_2$ $\quad T_2 = 22°C$

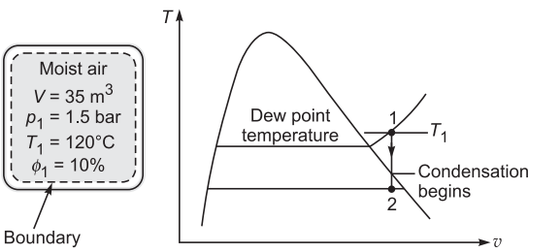

Fig. 15.7 *Schematic with given data and T–s diagram*

To find

(i) Dew point temperature,
(ii) The temperature, at which condensation starts,
(iii) Amount of water condensed.

Assumptions

(i) The contents of vessel are taken as closed system.
(ii) The gas phase of air and vapour can be treated as ideal gases.
(iii) When liquid water is present, the vapour exists at saturation temperature.

Analysis

(i) The partial pressure of vapour at the state 1;

$$p_{v_1} = \phi_1\, p_{sat_1}$$
$$p_{sat_1} = p_{sat} @ 120°C = 1.985 \text{ bar}$$

Thus $$p_{v_1} = 0.1 \times 1.985 = 0.1985 \text{ bar}$$

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The saturation temperature corresponds to 0.1985 bar

$$T_{sat} = \mathbf{60°C}$$

It is the dew point temperature

(ii) The specific volume of vapour in the mixture

$$v_{v_1} = \frac{R_u T}{\mathcal{M}_{v_1} p_{v_1}}$$

$$= \left(\frac{8.314}{18}\right) \times \left(\frac{393 \text{ K}}{0.1985 \times 100 \text{ kPa}}\right)$$

$$= 9.145 \text{ m}^3/\text{kg}$$

Interpolating, $v_{v_1} = v_g$, gives $T = \mathbf{56°C}$

Thus the condensation will begin at 56°C.

(iii) The mass of water initially present in moist air

$$m_{v_1} = \frac{V}{v_{v_1}} = \frac{35}{9.145} = 3.827 \text{ kg}$$

The propetrties of steam at $T = 56°C$ at the state 2,

$$v_{f_2} = 1.0022 \times 10^{-3} \text{ m}^3/\text{kg},$$

$$v_{g_2} = 15.445 \text{ m}^3/\text{kg}$$

The vapour has a two-phase mixture having specific volume of 9.145 m^3/kg

$$v_{v_1} = v_{f_2} + x_2 (v_{g_2} - v_{f_2})$$

$$9.145 = 1.0022 \times 10^{-3} + x_2 (15.445 - 1.0022 \times 10^{-3})$$

or $x_2 = 0.178$

The mass of water vapour at state 2

$$m_{v_2} = x_2 \, m_{v_1} = 0.178 \times 3.827 = 0.681 \text{ kg}$$

The mass of water vapour condensed

$$m_w = m_{v_1} - m_{v_2} = 3.827 - 0.681$$

$$= \mathbf{3.146 \text{ kg}}$$

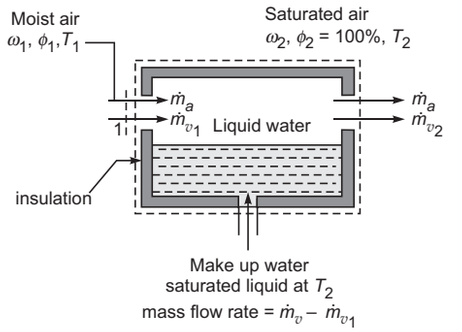
15.5 ADIABATIC SATURATION TEMPERATURE

In atmospheric air, the relative humidity is always less than 100%. The water-vapour pressure is lower than the saturation pressure. If this air is exposed to liquid water, some liquid evaporates and mixes with the air. The specific humidity of air increases. If such a process is carried in an insulated duct then the air temperature will also decrease due to latent heat of absorption during evaporation of water.

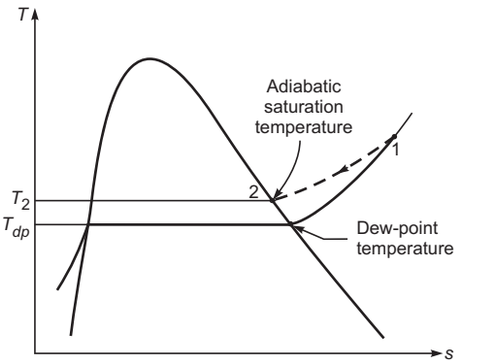
The *adiabatic saturation temperature* of atmospheric air is defined as the temperature which results from adding water vapour adiabatically to the atmospheric air in a steady flow manner until it becomes completely saturated.

A steady stream of unsaturated air with specific humidity ω_1 and temperature T_1 is passed through an insulated duct containing a pool of water as show in Fig. 15.8(a). As air flows over the water, some water evaporates and its vapour mixes with the air stream. The moisture content of air increases and its temperature decreases. If the duct is long enough, the air stream will come out completely saturated ($\phi = 100\%$) at the temperature T_2 which is called *adiabatic saturation temperature*.

If make up water is supplied to the duct at the rate at which it evaporates then the above process can be treated as a steady flow process. Figure 15.8(b) illustrates the adiabatic saturation process on the *T–s* diagram.



(a) Adiabatic saturator



(b) Process representation of T–s diagram

Fig. 15.8 *The adiabatic saturation process*

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Analysis At steady state, the mass-flow rate of dry air entering the saturator $\dot{m}_a$ must be equal to the mass-flow rate of dry air leaving the saturator. The mass flow rate of make up water is the difference between the exiting and entering vapour flow rates denoted by $\dot{m}_{v_2} - \dot{m}_{v_1}$, respectively. These flow rates are labelled on Fig. 15.8(a). In absence of kinetic and potential energy changes, the steady-flow energy equation reduces to

Enthalpy of mixture in + enthalpy of liquid water added = Enthalpy of mixture out.

$$\dot{m}_a h_{a_1} + \dot{m}_{v_1} h_{g_1} + (\dot{m}_{v_2} - \dot{m}_{v_1})\ h_{f_2} = \dot{m}_a h_{a_2} + \dot{m}_{v_2} h_{g_2} \quad \ldots(15.17)$$

Dividing both sides by $\dot{m}_a$,

$$h_{a_1} + \omega_1 h_{g_1} + (\omega_2 - \omega_1) h_{f_2} = h_{a_2} + \omega_2 h_{g_2}$$

or
$$\omega_1 (h_{g_1} - h_{f_2}) = h_{a_2} - h_{a_1} + \omega_2 (h_{g_2} - h_{f_2})$$

For ideal gas behaviour

$$\omega_1 (h_{g_1} - h_{f_2}) = C_{pa} (T_2 - T_1) + \omega_2 h_{fg_2}$$

or
$$\omega_1 = \frac{C_{pa} (T_2 - T_1) + \omega_2 h_{fg_2}}{h_{g_1} - h_{f_2}} \quad \ldots(15.18)$$

where
$$\omega_2 = 0.622 \frac{p_{sat_2}}{p_2 - p_{sat_2}}$$

and
$$h_{fg_2} = h_{g_2} - h_{f_2}$$

The temperature of completely saturated air is called *wet bulb temperature* (*wbt*).

Example 15.4 *The air at 28°C and 1 bar has a specific humidity of 0.016 kg per kg of dry air. Determine (a) partial pressure of water vapour, (b) relative humidity, (c) dew point temperature, and (d) specific enthalpy.*

Solution The moist air with

p = 1 bar = 100 kPa
T = 28°C = 301 K
ω = 0.016 kg/kg of dry air

To find

(i) Partial pressure of water vapour, p_{v_1},
(ii) Relative humidity ϕ,
(iii) Dew point temperature, T_{dp},
(iv) Specific enthalpy.

Analysis

(i) Partial pressure of water vapour

The specific humidity is expressed as

$$\omega = 0.622 \frac{p_v}{p - p_v}$$

using numerical values

$$0.016 = 0.622 \times \frac{p_v}{100 - p_v}$$

or $100 - p_v = 38.875\ p_v$

or p_v = **2.5 kPa**

(ii) Relative humidity ϕ;

From steam table, the saturation pressure p_{sat} corresponding to dry-bulb temperature of 28°C;

$$p_{sat} = 0.03778 \text{ bar} = 3.778 \text{ kPa}$$

$$\phi = \frac{p_v}{p_{sat}} = \frac{2.5}{3.778} = 0.661$$

= **66.1%**

(iii) Dew-point temperature

The dew-point temperature is the saturation temperature corresponding to partial pressure of vapour. Thus, saturation temperature corresponds to 2.5 kPa = **21.1°C**

(iv) Specific enthalpy

$$h = h_a + \omega h_g$$

where $h_a = C_p T_{db} = 1.005 \times (28°C)$
$= 28.14$ kJ/kg.
$h_g = 2500 + 1.88\ T_{db} = 2500 + 1.88 \times 28$
$= 2552.64$ kJ/kg
$h = 28.14 + 0.016 \times 2552.64$
= **6898 kJ/kg**

Example 15.5 *The pressure of the air entering and leaving the adiabaltic saturator is 1 bar. The air enters at 30°C and leaves as saturated air at 20°C. Calculate the specific humidity, relative humidity of the air–vapour mixture entering.*

Solution

Given Adiabatic saturator with

$p_1 = p_2$ = 1 bar = 100 kPa
T_1 = 30°C
T_2 = 20°C
ϕ_2 = 100%

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To find At entrance of adiabatic saturator

(i) Specific humidity, and

(ii) Relative humidity.

Assumptions

(i) Ideal gas mixture;

(ii) Constant specific heat of air; as C_{pa} = 1.0035 kJ/kg·K.

Analysis Properties of steam at 20°C (Table A-12)

At 30°C h_{g_1} = 2556.3 kJ/kg (From Table A-12)

p_{sat_1} = 4.246 kPa

p_{sat_2} = 2.339 kPa

h_{fg_2} = 2454.1 kJ/kg

h_{f_2} = 83.96 kJ/kg

(i) Since the water vapour leaving the saturator is completely saturated (ϕ_2 = 100%), thus

$$p_{v_2} = p_{sat_2}$$

The specific humidity of air leaving the saturator

$$\omega_2 = 0.622 \frac{p_{v_2}}{p - p_{v_2}}$$

$$= 0.622 \times \frac{2.339}{100 - 2.339}$$

$$= 0.0149 \text{ kg/kg of dry air}$$

The specific humidity of air entering is calculated as

$$\omega_1 = \frac{C_{pa}(T_2 - T_1) + \omega_2 h_{fg_2}}{(h_{g_1} - h_{f_2})}$$

Using these values,

$$\omega_1 = \frac{1.0035 \times (20 - 30) + 0.0149 \times 2454.1}{2556.3 - 83.96}$$

$$= \mathbf{0.0107 \text{ kg/kg of dry air}}$$

(ii) The specific humidity of air entering can also be expressed as

$$\omega_1 = 0.622 \frac{p_{v_1}}{p - p_{v_1}}$$

$$\Rightarrow \quad 0.0107 = 0.622 \times \frac{p_{v_1}}{100 - p_{v_1}}$$

or $100 - p_{v_1} = 58.13\, p_{v_1}$

or $p_{v_1} = 1.69$ kPa

Thus $\phi_1 = \dfrac{p_{v_1}}{p_{sat_1}} = \dfrac{1.69}{4.246} = 0.398 = \mathbf{39.8\%}$

15.6 PSYCHROMETRIC CHART

The psychrometric chart is a graphical presentation of properties of air–water vapour mixture. These charts are available in a number of different forms but the most commonly used chart is the ω–DBT chart. The typical layout of the chart is shown in Fig. 15.9. It has the following features:

1. The chart is based on the standard atmospheric pressure, i.e., 760 mm of Hg or 101.325 kPa.
2. The chart is a plot of *humidity ratio* and *vapour pressure*, on the vertical axis and *dry-bulb temperature* on the horizontal axis.
3. On the left end of the chart, the *saturation curve* represents the state of saturated air at different temperatures.
4. *Saturation curve* is a curve of 100% relative humidity. Other constant relative humidity curves also have the same shape.
5. The constant wet-bulb temperature lines have a downhill appearance to the right.
6. The constant specific-volume lines are steeper than constant wet-bulb temperature lines.
7. The constant-enthalpy lines are parallel to the constant wet-bulb temperature lines. Therefore, constant enthalpy and constant

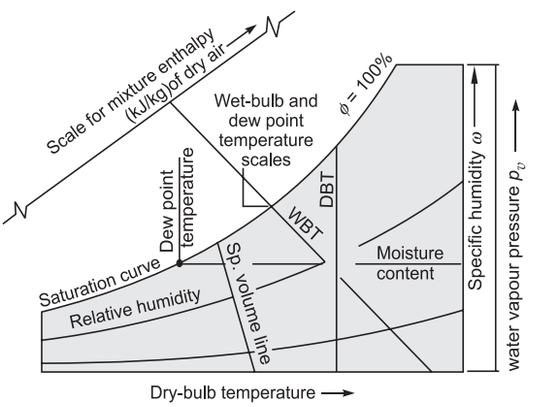

Fig. 15.9 *Psychrometric chart*

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wet bulb temperature are represented on the same line.

8. As shown in Fig. 15.10, on the saturation line, the dry-bulb temperature T, wet-bulb temperature T_{wb} and dew-point temperature T_{dp} are identical.

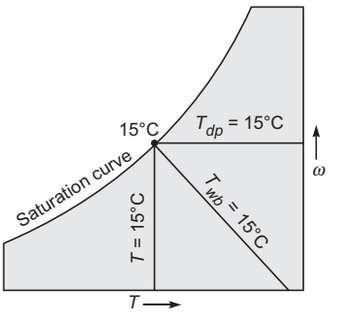

Fig. 15.10 *For saturated air, the dry-bulb, wet bulb and dew-point temperatures are identical*

The psychrometric chart is used extensively in air-conditioning applications. An ordinary cooling or heating process of air without change in its moisture content (ω = constant) is represented by the horizontal line. Any deviation from the horizontal line indicates that the moisture content of air is changed during the process.

Example 15.6 *The ambient conditions of air are 40°C DBT and 20°C WBT. Find the*

(a) Relative humidity,
(b) Specific humidity.

Solution

Given Ambient air with

T = 40°C and T_{wb} = 20°C

To find

(i) Relative humidity, and
(ii) Specific humidity.

Analysis From the psychrometric chart at intersection point of 40°C DBT and 20°C WBT,

Relative humidity = **14.5%**

Specific humidity = **0.0066 kg/kg of dry air**

Example 15.7 *For the moist air at 30°C DBT and 15°C WBT, calculate*

(a) Specific humidity,
(b) Enthalpy,
(c) Relative humidity,
(d) Dew-point temperature.

Solution

Given The moist air

T = 30°C T_{wb} = 15°C

To find

(i) Specific humidity (ω),
(ii) Enthalpy (h),
(iii) Relative humidity (ϕ),
(iv) DPT (T_{dp}).

Analysis From psychrometric chart at intersection point of T = 30°C and T_{wb} = 15°C

(i) Specific humidity ω = **0.00422 kg of dry air**
(ii) Enthalpy h = **42.2 kJ/kg of dry air**
(iii) Relative humidity ϕ = **17.5%**
(iv) Dew point temperature T_{dp} = **2.3°C**

Example 15.8 *Air at 40°C DBT and 25°C WBT is cooled down in an air-conditioning plant to 25°C DBT and 60% RH.*

Calculate the heat to be removed per kg of air if the COP of the unit is 3.5. Also, find the work required to cool 3 kg of air.

Solution

Given Moist air

State 1:	40°C DBT	and	25°C WBT
State 2:	25°C DBT	and	ϕ = 0.6
System	COP = 3.5	and	m = 3 kg

To find

(i) Heat removed per kg of air, and
(ii) Work input to system to cool 3 kg of air.

Analysis

(i) From the psychrometric chart,
At 40°C DBT and 25°C WBT, enthalpy of air;

h_1 = 76.5 kJ/kg of dry air

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At 25°C DBT and 60% RH, enthalpy of air;

h_2 = 55.2 kJ/kg of dry air

Heat removed per kg of dry air

$q_L = h_1 - h_2$

= 76.5 – 55.2 = **21.3 kJ/kg**

(ii) The coefficient of performance of a refrigerator is given by

$$(COP)_R = \frac{RE}{W_{in}} = \frac{q_L}{w_{in}}$$

or $$w_{in} = \frac{q_L}{(COP)_R} = \frac{21.3}{3.5} = 6.085 \text{ kJ/kg}$$

Work input for 3 kg of air

$W_{in} = m\, w_{in} = 3 \times 6.085$

= **18.26 kJ**

15.7 AIR-CONDITIONING PROCESS

The basic thermodynamic processes of moist air are illustrated on the psychrometric chart in Fig. 15.11. These processes are

1. Sensible heating and cooling
2. Humidification and dehumidification
3. Humidification with heating/cooling
4. Dehumidification with heating/cooling
5. Mixing of two air streams.

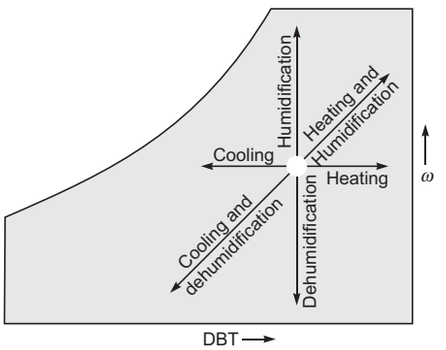
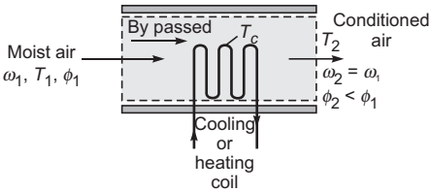


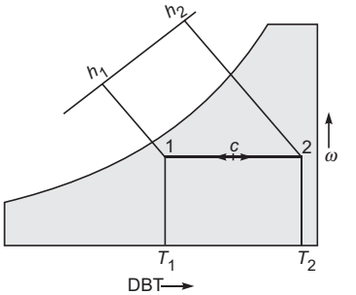
Fig. 15.11 *Basic air-conditioning processes*

15.7.1 Sensible Heating and Cooling

The sensible heating or sensible cooling process involves only change in dry bulb temperature, while keeping specific humidity ω constant. Therefore, simple heating and cooling processes appear as a horizontal line on the psychrometric chart as shown in Fig. 15.12. The process 1–2 represents a sensible heating process while the process 2–1 represents a sensible cooling process.



(a) Schematic



(b) Representation on psychrometric chart

Fig. 15.12 *Sensible heating and cooling processes*

The sensible heat-transfer rate is given by

$$\dot{Q} = \dot{m}_a(h_2 - h_1) = \dot{m}_a C_p (T_2 - T_1)$$

$$= \dot{m}_a C_{pa} (T_2 - T_1) + \dot{m}_a \omega C_{pv} (T_2 - T_1)$$

or $$\dot{Q} = \dot{m}_a (1.005 + 1.88\,\omega)(T_2 - T_1) \quad ...(15.19)$$

$\dot{m}_a$ = mass flow rate of air, kg/s

Where C_{pa} = specific heat of air, 1.005 kJ/kg·K

C_{pv} = specific heat of vapour, 1.88 kJ/kg·K

By-Pass Factor of Heating and Cooling Coils

As shown in Fig. 15.12(a), the air flows over the coil, and some part of the air directly passes the duct without coming in contact of cooling or heating coil. Such a stream is referred as the *by-passed stream*. Therefore, the temperature of air coming out of the duct is not equal to the coil temperature. Thus, the by-pass factor (*BPF*) is defined as

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$$BPF = \frac{\text{Temperature difference between coil and exit air}}{\text{Temperature difference between coil and entering air}}$$

$$= \frac{T_c - T_{exit}}{T_c - T_{inlet}} = \frac{T_c - T_2}{T_c - T_1} \qquad ...(15.20)$$

where T_c is the temperature of the coil, and T_1 and T_2 are inlet and exit temperatures of air.

Efficiency of Heating and Cooling Coils

The efficiency of heating and cooling coils is the ratio between actual temperature change to the maximum possible temperature change.

For heating or cooling coil

$$\eta_h = \frac{\text{Actual temperature change}}{\text{Maximum possible temperature change}}$$

$$= \frac{T_2 - T_1}{T_c - T_1} \qquad ...(15.21)$$

$$= \frac{T_2 - T_1 - T_c + T_c}{T_c - T_1}$$

$$= \frac{(T_c - T_1) - (T_c - T_2)}{T_c - T_1}$$

or $$\eta_h = 1 - \frac{(T_c - T_2)}{T_c - T_1} = 1 - BPF \qquad ...(15.22)$$

Example 15.9 *Moist air enters a duct at 10 °C, 80% relative humidty, and a volumetric flow rate of 150 m^3/min. The mixture is heated as it flows through the duct and leaves at 30°C. No moisture is added or removed and the mixture pressure remains approximately constant at 1 bar. For steady-state operation, determine (a) the rate of heat transfer, and (b) relative humidity at exit.*

Changes in kinetic and potential energy can be ignored and use a psychrometric chart for the solution.

Solution

Given Moist air enters a duct and is heated

At inlet: $T_1 = T_{db_1} = 10°C = 283\,K$

$\phi_1 = 80\% = 0.8$

$\dot{V} = 150\ m^3/min$

At exit: $T_2 = T_{db_2} = 30°C = 303\,K$

$\omega_1 = \omega_2$

$p_1 = p_2 = 1$ bar

To find

(i) The rate of heat transfer, and

(ii) Relative humidity at the exit.

Schematic

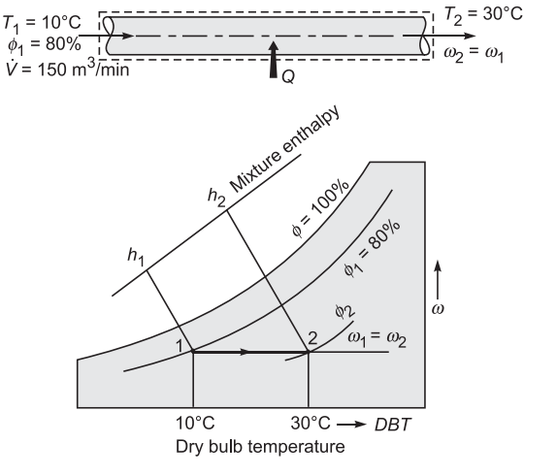

Fig. 15.13 *Sensible heating of moist air*

Assumption Specific gas constant for dry air as $R = 0.287$ kJ/kg·K

Analysis

(i) *Heat transfer rate*

From psychrometric chart

State 1: 10°C DBT and $\phi = 80\%$,

$h_1 = 25.7$ kJ/kg

State 2: $\omega_1 = \omega_2$ and 30°C DBT,

$h_2 = 45.9$ kJ/kg

The heat transfer per kg of dry air

$q = h_2 - h_1$

$= 45.9 - 25.7 = 20.2$ kJ/kg

The mass-flow rate of moist air can be obtained by using perfect gas relation

$$\dot{m} = \frac{p_1 \dot{V}_1}{R T_1}$$

$$= \frac{(100\ kPa) \times (150\ m^3/min)}{(0.287\ kJ/kg \cdot K)\,(283\ K)}$$

$= 185$ kg/min

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Then the total heat transfer rate to air in the duct

$$\dot{Q} = \dot{m}\,q$$
$$= (185 \text{ kg/min}) \times (20.2 \text{ kJ/kg})$$
$$= \mathbf{3737 \text{ kJ/min}}$$

(ii) Relative humidity at exit

From psychrometric chart, at the state 2, 30° DBT, $\omega_2 = \omega_1$

$$\phi_2 = \mathbf{23.1\%}$$

Example 15.10 *A quantity of air having a volume of 300 m^3 at 30 °C DBT and 25°C WBT is heated to 40 °C DBT. Calculate the amount of heat added, and relative humidity at both the states.*

Solution

Given A quantity of air is heated sensibly;

At inlet: $T_1 = 30°C$

$T_{wb_1} = 25°C$

$V = 300 \text{ m}^3$

At exit: $T_2 = 40°C = 313 \text{ K}$

$\omega_1 = \omega_2$

To find

(i) The rate of heat transfer, and

(ii) Relative humidity at inlet and exit.

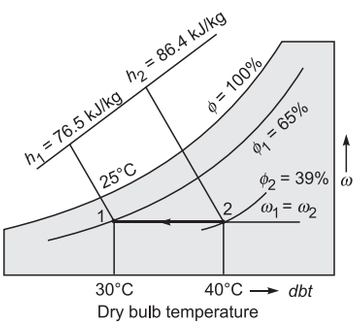

Fig. 15.14 *Sensible heating process*

Analysis From the psychrometric chart

State 1: 30°C *DBT* and 25°C *WBT*

$h_1 = 76.5 \text{ kJ/kg}$

$v_g = 0.885 \text{ m}^3/\text{kg}$

$\phi_1 = \mathbf{65\%}$

State 2: $\omega_1 = \omega_2$ and 40°C *DBT*,

$h_2 = 86.4 \text{ kJ/kg}$

$\phi_1 = \mathbf{39\%}$

Mass of moist air

$$m = \frac{V}{v_g} = \frac{300 \text{ m}^3}{0.885 \text{ m}^3/\text{kg}} = 339 \text{ kg}$$

The heat transfer to air

$$Q = m(h_2 - h_1)$$
$$= 339 \times (86.4 - 76.5) = \mathbf{3356 \text{ kJ}}$$

Example 15.11 *Air is cooled from 39°C DBT and 29% RH to 24°C at the rate of 5 m^3/s. Calculate the capacity of the cooling coil if the surface of the cooling coil is 20 °C. Also, calculate the by pass factor.*

Solution

Given A quantity of air is cooled sensibly;

At inlet: $T_1 = 39°C$

$\phi_1 = 29\%$

$\dot{V} = 5 \text{ m}^3/\text{s}$

At exit: $T_2 = 24°C$

$\omega_1 = \omega_2$

$T_{coil} = 20°C$

To find

(i) Cooling capacity, and

(ii) By-pass factor.

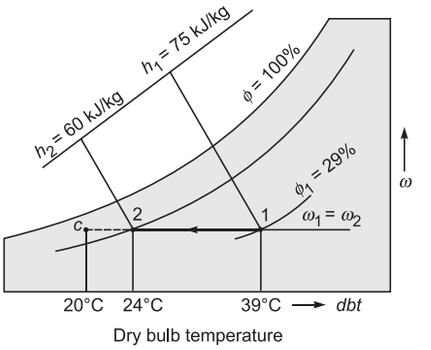

Fig. 15.15 *Sensible cooling process*

Analysis From the psychrometric chart

State 1: 39°C *DBT* and $\phi_1 = 29\%$

$h_1 = 75 \text{ kJ/kg}$

$v_g = 0.9 \text{ m}^3/\text{kg}$

State2: $\omega_1 = \omega_2$ and 24°C *DBT*

$h_2 = 60 \text{ kJ/kg}$

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Mass flow rate of air,

$$\dot{m}_a = \frac{\dot{V}}{v_g} = \frac{5 \text{ m}^3/\text{kg}}{0.9 \text{ m}^3/\text{kg}} = 5.56 \text{ kg/s}$$

(i) Capacity of cooling coil

$$\dot{Q} = \dot{m}_a (h_1 - h_2)$$
$$= 5.56 \times (75 - 60)$$
$$= \mathbf{83.4 \quad kW \text{ or } \quad 23.83 \ TR}$$

(ii) By-pass factor

$$BPF = \frac{T_c - T_{exit}}{T_c - T_{inlet}} = \frac{T_c - T_2}{T_c - T_1}$$
$$= \frac{20 - 24}{20 - 39} = \mathbf{0.21}$$

Example 15.12 *Atmospheric air at 760 mm of Hg, 15 °C DBT and 11°C WBT enters a heating coil, whose temperature is 41°C. The by-pass factor of the coil is 0.5. Calculate DBT, WBT and the relative humidity of air leaving the coil. Also, calculate the sensible heat added to air per kg of dry air.*

Solution

Given A quantity of air is heated sensibly;

At inlet: $T_1 = 15°C$

$T_{wb_1} = 11°C$

$p_1 = 760$ mm of Hg = 101.3 kPa

Heating coil $T_c = 41°C$

BPF = 0.5

and $\omega_1 = \omega_2$

To find

(i) T_2, T_{wb_2}, ϕ_2 of air leaving the coil,

(ii) The rate of heat transfer.

Analysis The sensible heating process is shown schematically and on a psychrometric chart in Fig. 15.16.

(i) T_2, T_{wb_2}, ϕ_2 of air leaving the coil;

From psychrometric chart

State 1: 15°C *DBT* and 11°C *WBT*

$T_1 = 15°C \quad T_{wb_1} = 11°C$

$h_1 = 31.8$ kJ/kg and $\phi_1 = 62\%$

The by-pass factor is given by Eq. (15.20)

$$BPF = \frac{T_c - T_{exit}}{T_c - T_{inlet}} = \frac{T_c - T_2}{T_c - T_1}$$

$$0.5 = \frac{41 - T_2}{41 - 15}$$

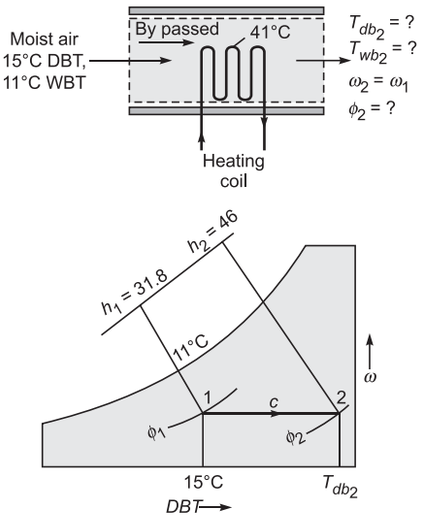

Fig. 15.16

$\Rightarrow \quad T_2 = \mathbf{28°C}$

State 2: Using psychrometric chart at coordinates of $\omega_1 = \omega_2$ and 28°C *DBT*, and wet bulb temperature and relative humidity are

$T_{wb_2} = \mathbf{16°C} \quad \phi_2 = \mathbf{29\%}$

and $h_2 = 46$ kJ/kg

Sensible heat transfer to air

$q = h_2 - h_1$

$= 46 - 31.8 = \mathbf{14.2 \text{ kJ/kg}}$

15.7.2 Humidification and Dehumidification

When the moisture content of air is increased at constant temperature, the process is referred as *humidification* [process (1 – 2)], and when moisture is reduced from the moist air at constant temperature the process is referred as *dehumidification* [process (2 – 1)].

During humidification and dehumidification, the specific humidity ω and humidity ratio ϕ, both change. The processes are shown in Fig. 15.17.

Actually, there is no method by which one can obtain simply humidification or dehumidification of air. These processes are accomplished by either simultaneous heating or cooling.

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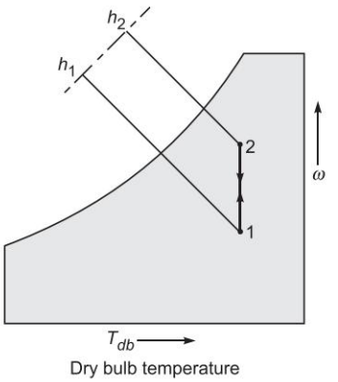

Fig. 15.17 *Humidification and dehumidification*

15.7.3 Heating and Humidification

This process is generally used in winter air-conditioning. It involves warming and humidifying of air simultaneously. In this process, the air passes through a humidifier where the hot water (or steam) is injected, both the humidity ratio and dry-bulb temperature increase. The process of heating and humidification is shown by the line 1–2 on the psychometric chart in Fig. 15.18.

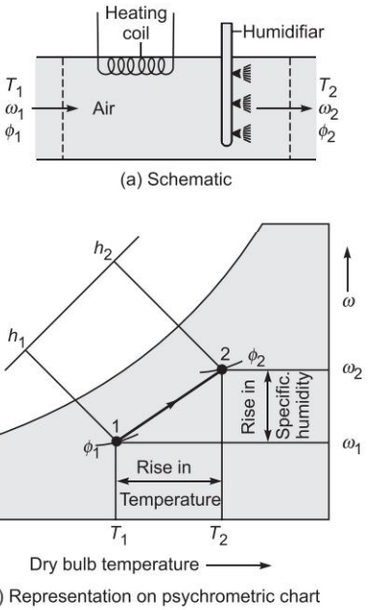

Fig. 15.18 *Heating with humidification*

Example 15.13 *Moist air at 22°C and a wet-bulb temperature of 9°C enters a steam spray humidifier. The mass flow rate of the dry air is 90 kg/min. Saturated water vapour at 110°C is injected into the mixture at a rate of 52 kg/h. There is no heat transfer with the surroundings and pressure is constant throughout at 1 bar. Determine (a) the humidity ratio, and (b) temperature of air leaving the humidifier.*

Solution

Given Humidification of moist air with the help of steam injection

$T_1 = 22°C$ $\quad T_{wb_1} = 9°C$
$\dot{m}_a = 90$ kg/min $\quad T_{sat} = 110°C$
$\dot{m}_s = 52$ kg/h $= 0.867$ kg/min
$p = 1$ bar $= 100$ kPa

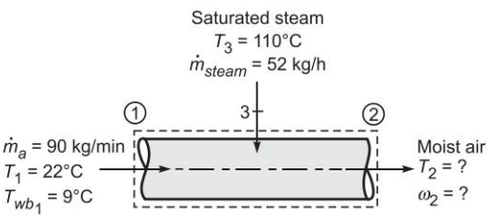

Fig. 15.19 *Schematic*

To find

(i) The specific humidity of air leaving the humidifier,
(ii) Exit temperature of moist air.

Analysis

(i) *The humidity ratio at the exit*

The mass flow rate balance during the flow through control volume between states 1–2

$$\dot{m}_{a_1} = \dot{m}_{a_2} \quad \text{(dry air)}$$
$$\dot{m}_{v_1} + \dot{m}_s = \dot{m}_{v_2} \quad \text{(water vapour)}$$

Dividing both sides by $\dot{m}_a$

$$\frac{\dot{m}_{v_1}}{\dot{m}_{a_1}} + \frac{\dot{m}_s}{\dot{m}_a} = \frac{\dot{m}_{v_2}}{\dot{m}_a}$$

or
$$\omega_2 = \omega_1 + \frac{\dot{m}_s}{\dot{m}_a}$$

From pychrometric chart against coordinates of 22°C *DBT* and 9°C *WBT*

$\omega_1 = 0.002$ kg/kg of dry air
$h_1 = 27.2$ kJ/kg of dry air

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Thus, the specific humidity at the exit

$$\omega_2 = 0.002 + \frac{(0.867 \text{ kg/min})}{(90 \text{ kg/min})}$$

$$= \mathbf{0.0116 \text{ kg/kg of dry air}}$$

(ii) The exit temperature of moist air

From steam table, A-12

$$h_s = h_g @ 110°C = 2691.47 \text{ kJ/kg}$$

The enthalpy of leaving air

$$h_2 = h_1 + \frac{\dot{m}_s}{\dot{m}_a} h_s$$

$$= (27.2 \text{ kJ/kg}) + \frac{0.867 \text{ kg/min}}{90 \text{ kg/min}} \times (2691.47 \text{ kJ/kg})$$

or $h_2 = 53.11$ kJ/kg of dry air

Using psychrometric chart at coordinates of h_2 = 53.11 kJ/kg and ω_2 = 0.0116 kg/kg, the DBT and relative humidity are

$$T_2 = \mathbf{24°C}$$

$$\phi_2 = 61\%$$

Example 15.14 *The atmospheric air at 25°C DBT and 12°C WBT is flowing at a rate of 100 m³/min through a duct. The dry saturated steam at 100°C is injected into the air stream at a rate of 72 kg/h. Calculate the specific humidity, DBT, WBT, relative humidity and enthalpy of air leaving the duct.*

Solution

Given Humidification of atmospheric air with the help of steam injection

$T_1 = 25°C$ $\quad T_{wb_1} = 12°C$

$\dot{V}_a = 100 \text{ m}^3/\text{min}$ $\quad T_{sat} = 100°C$

$\dot{m}_s = 72 \text{ kg/h} = 1.2 \text{ kg/min}$

To find

(i) The specific humidity, DBT, WBT, ϕ_2 of air leaving the humidifier, and

(ii) Specific enthalpy of air leaving.

Analysis From pychrometric chart against coordinate of 25°C *DBT* and 12°C *WBT*

$\omega_1 = 0.0034$ kg/kg of dry air

$h_1 = 33.2$ kJ/kg of dry air

$v_1 = 0.844 \text{ m}^3/\text{kg}$

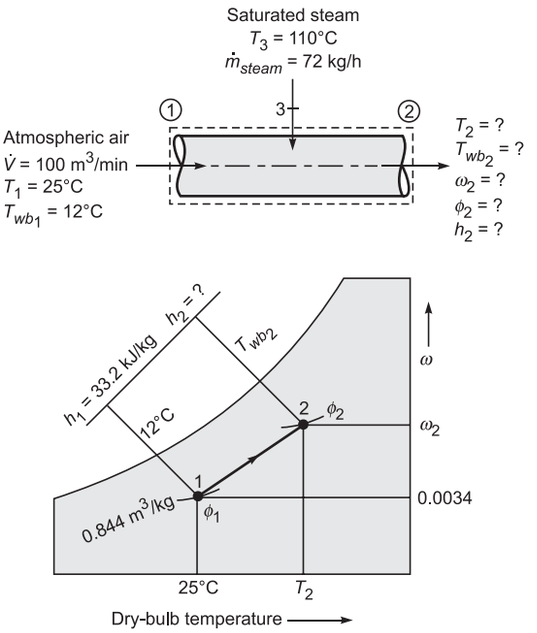

Fig. 15.20 *Process on psychrometric chart*

The mass flow rate of atmospheric air

$$\dot{m}_a = \frac{\dot{V}_a}{v_1} = \frac{100 \text{ m}^3/\text{min}}{0.844 \text{ m}^3/\text{kg}} = 118.48 \text{ kg/min}$$

(i) The specific humidity at the exit

$$\omega_2 = \omega_1 + \frac{\dot{m}_s}{\dot{m}_a} = 0.0034 + \frac{1.2}{118.48}$$

$$= \mathbf{0.0135 \text{ kg/kg of dry air}}$$

(ii) The exit temperature of moist air

From steam tables

$$h_s = h_g @ 100°C = 2676 \text{ kJ/kg}$$

The enthalpy of leaving air

$$h_2 = h_1 + \frac{\dot{m}_s}{\dot{m}_a} h_s$$

$$= 33.2 + \frac{1.2}{118.48} \times 2676$$

$$= \mathbf{60.3 \text{ kJ/kg of dry air}}$$

Using psychrometric chart at coordinates of h_2 = 60.3 kJ/kg and ω_2 = 0.0135 kg/kg, and locate point 2. The properties of moist air at state 2 are

DBT $\quad T_2 = \mathbf{26°C}$

WBT $\quad T_{wb_2} = \mathbf{21°C}$

RH $\quad \phi_2 = \mathbf{65\%}$

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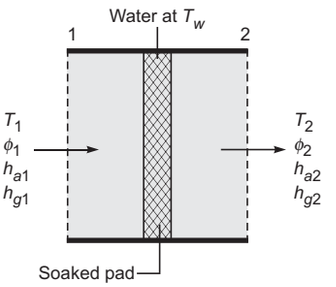
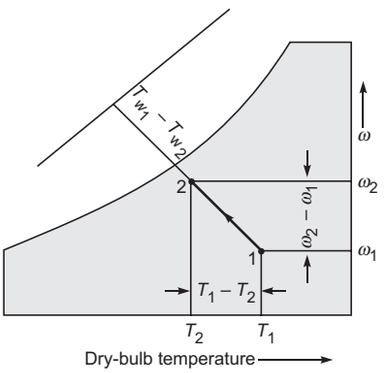
15.7.4 Cooling and Humidification: Evaporative Cooling

Cooling and humidification, simultaneously can be accomplished by evaporative cooling. It involves either spraying of liquid water into air or forcing the air through a soaked pad that is kept saturated with water as in desert coolers, also known as evaporative coolers. A schematic of an evaporative cooler is shown in Fig. 15.21(a).



(a) Evaporative cooler



(b) Representation of procession psychrometric chart

Fig. 15.21 *Cooling and humidification process*

The low humidity air enters the humidifier at the state 1, and when it passes through a water soaked pad a part of water evaporates and gets mixed with air. The energy for evaporation is extracted from the water body and air stream, thus both water and air get cooled. Therefore, the stream of air leaving the humidifier at the state 2 is rich in moisture and low in temperature.

For negligible heat transfer to surroundings and in absence of work transfer, kinetic and potential energy losses, the steady flow energy equation yields to

$$(h_{a_2} + \omega_2 h_{g_2}) = (\omega_2 - \omega_1) h_{f_1} + (h_{a_1} + \omega_1 h_{g_1}) \quad ...(15.23)$$

where h_{f_1} is the specific enthalpy of the water entering the control volume.

The evaporative cooling follows a line of constant wet-bulb temperature on the psychrometric chart as shown in Fig. 15.21(b). Thus

$$T_{wb} = \text{constant} \quad ...(15.24)$$

$$h \approx \text{constant} \quad ...(15.25)$$

The evaporative cooling (humidification and cooling) takes place in cooling towers, evaporative condensers and desert coolers.

Example 15.15 *Air at 37°C and 10% relative humidity enters an evaporative cooler with a volumetric flow rate of 140 m^3/min. Moist air leaves the cooler at 20°C. The water is added to the soaked pad of the cooler at 20°C and evaporates fully into the moist air. There is no heat transfer with the surroundings and the pressure is constant throughout the process at 1 atm. Determine (a) the mass flow rate of water to the soaked pad, and (b) relative humidity of the moist air at the exit of the evaporative cooler.*

Solution

Given Humidification of moist air with the help of evaporative cooling.

$T_1 = 37°C$ $\quad \phi_1 = 10\% = 0.1$

$\dot{V} = 140\ m^3/min$ $\quad T_2 = 20°C$

$T_3 = 20°C$ $\quad p = 1\ atm = 101.3\ kPa$

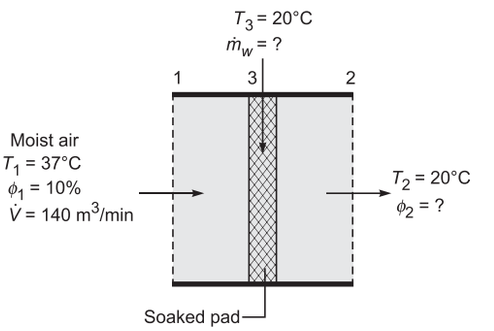


Fig. 15.22

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To find

(i) The mass flow rate of water to the soaked pad.
(ii) The relative humidity of moist air leaving the evaporative cooler.

Assumptions

(i) Specific gas constant of air, $R = 0.287$ kJ/kg·K.
(ii) Specific heat at constant pressure of air, $C_p = 1.005$ kJ/kg·K.

Analysis

(i) The mass flow rate of water to the soaked pad

The mass conservation gives

$$\dot{m}_w = \dot{m}_{v_2} - \dot{m}_{v_1}$$
$$= \dot{m}_a(\omega_2 - \omega_1) \quad ...(i)$$

The saturation pressure p_{sat_1} at 37°C is 6.289 kPa

$$p_{v_1} = \phi_1 p_{sat_1} = 0.1 \times 6.289 \text{ kPa}$$
$$= 0.6289 \text{ kPa}$$

The specific humidity at the state 1

$$\omega_1 = 0.622 \frac{p_{v_1}}{p - p_{v_1}}$$
$$= 0.622 \times \frac{0.6289 \text{ kPa}}{101.3 - 0.6289 \text{ kPa}}$$
$$= 0.00388 \text{ kg/kg of dry air}$$

The mass flow rate of dry air at the state 1

$$\dot{m}_a = \frac{p\dot{V}}{RT} = \frac{101.3 \times 140}{0.287 \times 310} = 159.4 \text{ kg}$$

The specific humidity ω_2 can be obtained by using steady-flow energy Eq. (15.23)

$$h_{a_2} + \omega_2 h_{g_2} = (\omega_2 - \omega_1) h_{f_1} + h_{a_1} + \omega_1 h_{g_1}$$

or $$\omega_2(h_{g_2} - h_{f_1}) = h_{a_1} - h_{a_2} + \omega_1(h_{g_1} - h_{f_1})$$

or $$\omega_2 = \frac{C_p(T_1 - T_2) + \omega_1(h_{g_1} - h_{f_1})}{h_{g_2} - h_{f_1}} \quad ...(ii)$$

From steam table A-12

$$h_{g_1} = h_g \text{ @ } 37°C = 2568.9 \text{ kJ/kg}$$
$$h_{g_2} = h_g \text{ @ } 20°C = 2538.1 \text{ kJ/kg}$$
$$h_{f_1} = h_f \text{ @ } 20°C = 83.96 \text{ kJ/kg}$$

Thus from Eq. (ii)

$$\omega_2 = \frac{1.005 \times (37 - 20) + 0.00388 \times (2568.9 - 83.96)}{2538.1 - 83.96}$$
$$= 0.0109 \text{ kg/kg of dry air}$$

Substituting in Eq. (i)

$$\dot{m}_w = 159.4 \times (0.0109 - 0.00388)$$
$$= \mathbf{1.12 \text{ kg/min}}$$

(ii) The relative humidity of the moist air

The partial pressure of water vapour in air

$$\omega_2 = 0.622 \frac{p_{v_2}}{p - p_{v_2}}$$

or $$p_{v_2} = \frac{\omega_2 p}{\omega_2 + 0.622} = \frac{0.0109 \times 101.3}{0.0109 + 0.622}$$
$$= 1.744 \text{ kPa}$$

At 20°C, the saturation pressure of vapour,

$$p_{sat_2} = 0.02339 \text{ bar} = 2.339 \text{ kPa}$$

Thus $$\phi_2 = \frac{p_{v_2}}{p_{sat_2}} = \frac{1.744}{2.339} = 0.745 = \mathbf{74.5\%}$$

Alternatively, the relative humidity ϕ_2 can be obtained from psychrometric chart with $\omega_2 = 0.0109$ and $T_2 = 20°C$, the intersecting point gives $\phi_2 = 75\%$.

15.7.5 Dehumidification with Heating

It is based on the principle of adsorption, i.e., capillary action. Thermodynamically, an adsorption process is the nerve of the *adiabatic saturation process*. In this process, the air is passed over the adsorbing surface (chemicals are silica gel, activited alumina, etc. which have affinity for moisture). As the water vapour comes in contact of these substances, the moisture gets condensed out of air and gives up its latent heat, resulting into rise in temperature of air. The dehumidification and heating process is shown in Fig. 15.23.

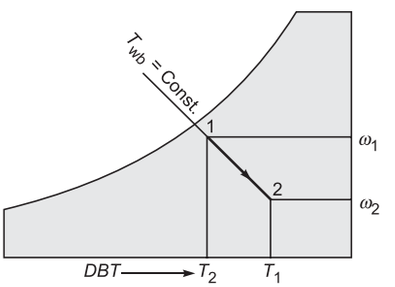

Fig. 15.23 *Adsorption dehumidification process*

15.7.6 Dehumidification with Cooling

When a moist air stream is cooled at constant pressure to a temperature below its dew point temperature, some of the water vapour initially present would condense. Fig. 15.24 shows the schematic of a dehumidifier and cooling process on psychrometric chart.

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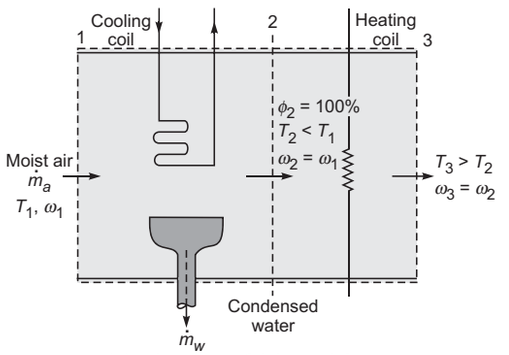
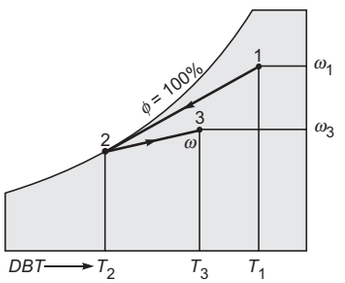


(a) schematic of dehumidifier



(b) Representation of dehumidification and cooling on psychrometic chart

Fig. 15.24

The moist air enters the state 1, flows across a cooling coil, some of the water vapour initially present in the moist air condenses and a saturated moist air mixture leaves the dehumidifier section at the state 2. Since the moist air leaving the cooling coil is saturated at temperature below the temperature of the moist air entering, the moist air stream might be unsuitable for direct use in occupied space. Thus this stream passes the heating section and is brough to the comfort condition at the state 3.

Example 15.16 *Air at 40°C DBT and 60% RH is cooled to 25°C DBT. It is achieved by cooling and dehumidification. Air flow rate is 40 m³/min. Using psychrometric chart, calculate:*

(a) the dew point temperature,

(b) mass of water drained out per hour,

(c) Capacity of cooling coil.

If the apparatus dew-point temperature (ADPT) is 20°C, find the by pass factor of coil.

Solution

Given Dehumidification and cooling of moist air:

$T_1 = 40°C$ $\phi_1 = 60\%$

$T_2 = 25°C$ $\dot{V}_a = 40\ m^3/min$

$T_{ADP} = T_O = 20°C$

To find

(i) Dew-point temperature,

(ii) Mass of water drained out per hour,

(iii) Capacity of cooling coil, and

(iv) By pass factor of coil.

Analysis The process of cooling and dehumidification is shown on psychrometric chart in Fig. 15.25.

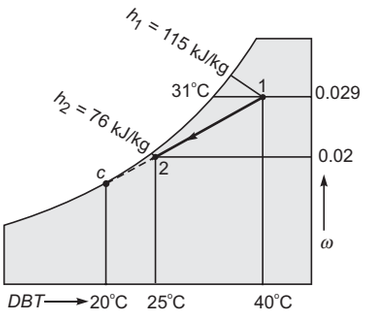


Fig. 15.25

Inlet conditions: $\phi_1 = 60\%$, $T_1 = 40°C$

From psychrometric chart;

$v_1 = 0.925\ m^3/kg$

$\omega_1 = 0.029$ kg/kg of air

$h_1 = 115$ kJ/kg

Exit conditions: $T_2 = 25°C$

$T_C = 20°C$

$\omega_2 = 0.02$ kg/kg of air

$h_2 = 76$ kJ/kg

(i) Dew-point temperature corresponding to the state 1 is **31°C**.

The mass flow rate of dry air,

$$\dot{m}_a = \frac{\dot{V}}{v_{a_1}} = \frac{40}{0.925}$$

$= 43.24$ kg/min or 2595.6 kg/h

(ii) Mass of moisture removed

$$\dot{m}_w = \dot{m}_a(\omega_1 - \omega_2)$$

$= 2595.6 \times (0.029 - 0.02)$

$=$ **23.36 kg/h**

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(iii) Capacity of cooling coil

The heat transfer;

$$\dot{Q} = \dot{m}_a (h_1 - h_2) = 43.24 \times (115 - 76)$$
$$= 1686.36 \text{ kJ/min}$$

$$\text{Coilc apacity} = \frac{\dot{Q} \text{ (kJ/min)}}{(210 \text{ kJ/min/Ton})} = \frac{1686.36}{210}$$
$$= \mathbf{8.03 \text{ Ton}}$$

(iv) By pass factor

The by-pass factor is given by Eq. (15.33)

$$BPF = \frac{T_c - T_{exit}}{T_c - T_{inlet}} = \frac{T_c - T_2}{T_c - T_1}$$
$$= \frac{20 - 25}{20 - 40} = \mathbf{0.25}$$

Example 15.17 *Air enters a window air conditioner at 1 atm, 30°C and 80% RH at a rate of 10 m^3/min and it leaves at saturated at 14°C. A part of moisture, which condenses during the process is also removed at 14°C. Determine the rate of heat and moisture removal from air.*

Solution

Given Dehumidification and cooling of moist air:

$p_1 = 1$ atm $\quad T_1 = 30°C$

$\phi_1 = 80\%$ $\quad \dot{V}_a = 10$ m^3/min

$T_2 = 14°C$ $\quad \phi_2 = 100\%$

To find

(i) Rate of moisture removal,

(ii) Rate of heat transfer.

Analysis The process of cooling and dehumidification is shown on the psychrometric chart in Fig. 15.26.

Inlet conditions: $\phi_1 = 80\%$ $\quad T_1 = 30°C$

From psychrometric chart;

$v_1 = 0.86$ m^3/kg

$\omega_1 = 0.022$ kg/kg of air

$h_1 = 85.5$ kJ/kg

Exit conditions: $T_2 = 24°C$ $\quad \phi_2 = 100\%$

$\omega_2 = 0.0102$ kg/kg of air

$h_2 = 39.5$ kJ/kg

The mass flow rate of dry air

$$\dot{m}_a = \frac{\dot{V}_a}{v_{a_1}} = \frac{10}{0.86} = 11.63 \text{ kg/min}$$

(i) Removal rate of moisture from air

$$\dot{m}_w = \dot{m}_a (\omega_1 - \omega_2)$$
$$= 11.63 \times (0.022 - 0.0102)$$
$$= 0.1372 \text{ kg/min or } \mathbf{8.23 \text{ kg/h}}$$

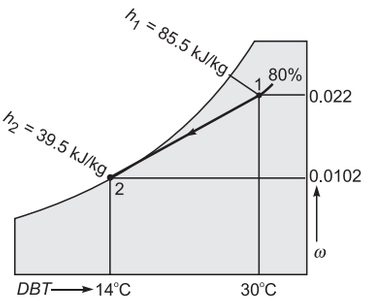

Fig. 15.26

(ii) Heat removal rate

$$\dot{Q} = \dot{m}_a (h_1 - h_2)$$
$$= 11.63 \times (85.5 - 39.5)$$
$$= 534.98 \text{ kJ/min or } \mathbf{8.916 \text{ kW}}$$

Example 15.18 *Moist air at 30°C and 50% relative humidity enters a dehumidifier operating at steady state with a volumetric flow rate of 280 m^3/min. The moist air passes over a cooling coil and water vapour condenses. The condensate leaves the dehumidifier saturated at 10°C. Saturated moist air leaves in a separate stream at the same temperature. During the process, the pressure remains constant at 1.013 bar and heat transfer to the surroundings is negligble. Determine (a) the mass flow rate of dry air, (b) rate of water condensation in the dehumidifier in kg/kg of dry air flowing through the control volume, and (c) the required refrigerating capacity in tonnes.*

Solution

Given Dehumidification of moist air over a cooling coil.

$\dot{V} = 280$ m^3/min $\quad T_1 = 30°C = 303$ K

$\phi_1 = 50\% = 0.5$ $\quad T_2 = 10°C$

$p = 1.013$ bar $= 101.3$ kPa

To find

(i) The mass flow rate of dry air,

(ii) The rate of condensation of water vapour in dehumidifier in kg/kg of dry air,

(iii) Required refrigerating capacity in tonnes.

Assumptions

(i) The control volume is shown in Fig. 15.27.

(ii) Moist air leaves the cooling coil as saturated at 10°C.

(iii) Specific gas constant for dry air as $R = 0.287$ kJ/kg·K.

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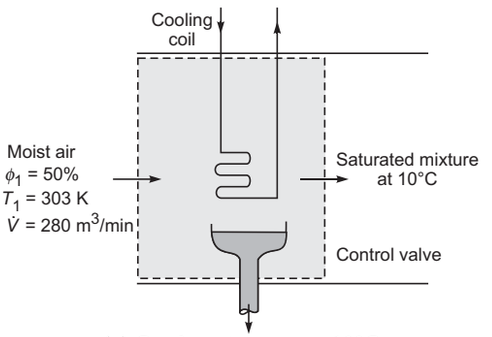
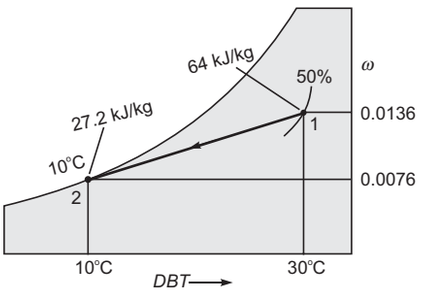


(a) Condensate water at 10°C



(b) Process on psychrometric chart

Fig. 15.27

Analysis

(i) The mass-flow rate of dry air

From psychrometric chart

The inlet conditions: p_1 = 101.3 kPa

ϕ_1 = 50% T_1 = 303 K

ω_1 = 0.0136 kg/kg of air

v_{a_1} = 0.875 m^3/kg

h_1 = 64 kJ/kg

Exit conditions

T_2 = 10°C, ϕ_2 = 100%

h_2 = 27.2 kJ/kg

The mass flow rate of dry air

$$\dot{m}_a = \frac{\dot{V}}{v_{a_1}} = \frac{280}{0.875} = \mathbf{320\ kg/min}$$

(ii) Rate of moisture removal from air

$$\frac{\dot{m}_w}{\dot{m}_a} = (\omega_1 - \omega_2) = 0.0136 - 0.0076$$

$$= 0.006 \text{ kg/kg of dry air}$$

(iii) Refrigeration effect

Heat removal rate

$$\dot{Q} = \dot{m}_a (h_1 - h_2)$$

$$= 320 \times (64 - 27.2) = 11776 \text{ kJ/min}$$

The refrigeration effect in Tons of refrigeration (1 *TR* = 210 kJ/min)

$$TR = \frac{11776}{210} = \mathbf{56.07\ Tons}$$

15.8 ADIABATIC MIXING OF TWO MOIST AIR STREAMS

It is a common practice in air conditioning that the fresh and recirculated moist air streams are mixed as shown in Fig. 15.28. During such mixing, normally no heat is transferred to the surroundings, and thus mixing is almost adiabatic.

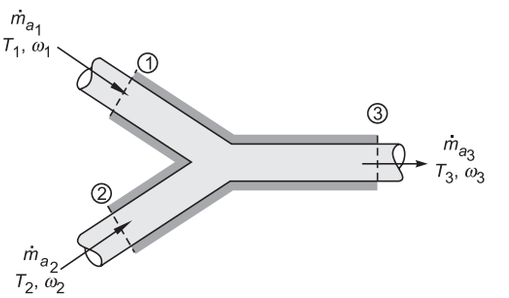


Fig. 15.28 *Adiabatic mixing of two streams of moist air*

Applying mass and energy balance on the control volume

Mass balance

$$\dot{m}_{a_1} + \dot{m}_{a_2} = \dot{m}_{a_3} \text{ (dry air)} \quad ...(15.26)$$

$$\dot{m}_{v_1} + \dot{m}_{v_2} = \dot{m}_{v_3} \text{ (water vapour)}$$

Using $\dot{m}_v = \omega \dot{m}_a$, we get

$$\omega_1 \dot{m}_{a_1} + \omega_2 \dot{m}_{a_2} = \omega_3 \dot{m}_{a_3} \text{ (water vapour)}$$

or $$\omega_3 = \frac{\omega_1 \dot{m}_{a_1} + \omega_2 \dot{m}_{a_2}}{\dot{m}_{a_3}} = \frac{\omega_1 \dot{m}_{a_1} + \omega_2 \dot{m}_{a_2}}{\dot{m}_{a_1} + \dot{m}_{a_2}} \quad ...(15.27)$$

Energy balance

$$\dot{m}_{a_1} h_1 + \dot{m}_{a_2} h_2 = \dot{m}_{a_3} h_3$$

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or $$h_3 = \frac{m_{a_1} h_1 + m_{a_2} h_2}{m_{a_3}} \quad ...(15.28)$$

where $h = h_a + \omega h_g$; the specific enthalpy of entering and exiting water vapour are evaluated at saturated vapour values at the respective dry-bulb temperatures.

Example 15.19 *25 kg of air at 25°C DBT and 61% RH is mixed with 5 kg of air at 5°C DBT and 30% RH. Calculate the condition of mixed air.*

Solution

Given Mixing of two streams of air

$m_1 = 25$ kg, $T_1 = 25°C$ $\phi_1 = 61\%$

$m_2 = 5$ kg $T_2 = 5°C$ $\phi_2 = 30\%$

To find Condition of mixed stream of air.

Assumptions

(i) There is no heat transfer with the surroundings.

(ii) The pressure remains constant throughout at 1 atm.

Schematic Mixing of two streams is shown on psychrometric chart in Fig. 15.29.

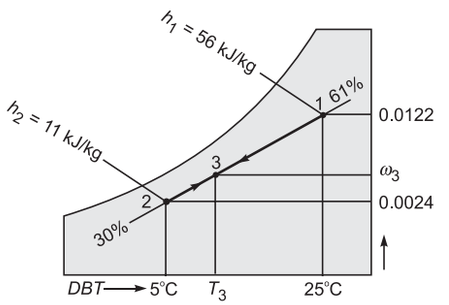

Fig. 15.29 *Mixing of two streams of air*

Analysis From psychometric chart;

Coordinates: 25°C DBT and 61% RH

$\omega_1 = 0.0122$ kg/kg of dry air

$h_1 = 56$ kJ/kg

Coordinates: 5°C DBT and 30% RH

$\omega_2 = 0.0024$ kg/kg of dry air

$h_2 = 11$ kJ/kg

The mass balance

$$\dot{m}_{a_1} + \dot{m}_{a_2} = \dot{m}_{a_3}$$

or $25 \text{ kg} + 5 \text{ kg} = 30 \text{ kg}$

Humidity of mixed stream

$$\omega_3 = \frac{\omega_1 \dot{m}_{a_1} + \omega_2 \dot{m}_{a_2}}{\dot{m}_{a_3}} = \frac{0.0122 \times 25 + 0.0024 \times 5}{30}$$

$= 0.01056$ kg/kg of air

Specific enthalpy of mixed stream

$$h_3 = \frac{m_{a_1} h_1 + m_{a_2} h_2}{m_{a_3}} = \frac{25 \times 56 + 5 \times 11}{30} = 48.5 \text{ kJ/kg}$$

From psychrometric chart; using coordinates ω_3 and h_3, temperature and humidity of mixed stream

$T_3 = \mathbf{22.5°C}$, $\phi_3 = \mathbf{60\%}$

Example 15.20 *142 m^3/min moist air at 5°C with specific humidity of 0.002 kg/kg of dry air is mixed adiabatically with 425 m^3/min of moist air stream at 24°C and 50% relative humidity. If the pressure is constant throughout at 1 bar. Determine (a) the humidity ratio, and (b) the temperature of the mixed stream.*

Solution

Given Adiabatic mixing of two moist air streams as shown on Fig. 15.30.

To find

(i) Humidity ratio ω_3 of mixed stream, and

(ii) Temperature of the mixed stream.

Analysis From psychrometic chart

Coordinates: $\omega_1 = 0.002$ and $T_1 = 5°C$

$v_1 = 0.79$ m^3/kg

$h_1 = 10$ kJ/kg

Coordinates: $\phi_2 = 50\%$ and $24°C$

$v_2 = 0.855$ m^3/kg $\omega_2 = 0.0094$ kg/kg

$h_2 = 47.8$ kJ/kg

Mass of inlet stream 1;

$$\dot{m}_{a_1} = \frac{\dot{V}_1}{v_1} = \frac{142}{0.79} = 179.746 \text{ kg/min}$$

Mass of inlet stream 2;

$$\dot{m}_{a_2} = \frac{\dot{V}_2}{v_2} = \frac{425}{0.855} = 497.07 \text{ kg/min}$$

(i) Specific humidity of mixture can be obtained by Eq. (15.27)

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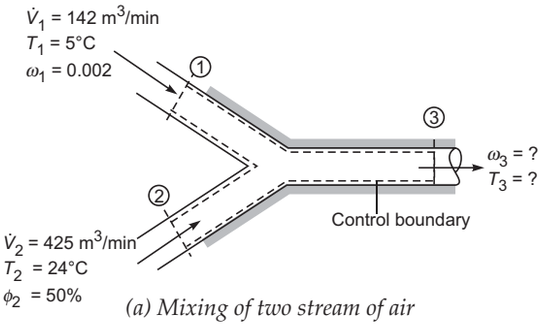
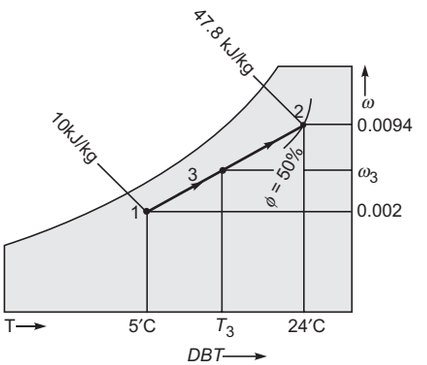


(a) Mixing of two stream of air



(b) Mixing procession psychrometric chart

Fig. 15.30 *Mixing of two stream of air*

$$\omega_3 = \frac{\omega_1 \dot{m}_{a_1} + \omega_2 \dot{m}_{a_2}}{\dot{m}_{a_3}}$$

Using numerical values

$$\omega_3 = \frac{0.002 \times 179.746 + 0.0094 \times 497.07}{179.746 + 497.07}$$

$$= \mathbf{0.0077\ kg/kg\ of\ dry\ air}$$

(ii) Temperature of the mixed stream

Specific enthalpy of mixture, Eq. (15.28);

$$h_3 = \frac{m_{a_1} h_1 + m_{a_2} h_2}{m_{a_3}}$$

Using numerical values

$$h_3 = \frac{179.746 \times 10 + 497.07 \times 47.8}{179.746 + 497.07}$$

$$= 37.7\ \text{kJ/kg}$$

From psychometric chart at coordinates

$\omega_3 = 0.0077$ and $h_3 = 37.7$ kJ/kg

$T_3 = \mathbf{19°C}$

Example 15.21 *1 kg air at 40°C dry-bulb temperature and 50% relative humidity is mixed with 2 kg of air at 20°C dry-bulb temperature and 20°C dew-point temperature. Determine for the mixed stream (b) temperature, and (c) specific humidity.*

Solution

Given Mixing of two streams of air

$m_1 = 1$ kg $\quad T_1 = 40°C \quad \phi_1 = 50\%$

$m_2 = 2$ kg $\quad T_2 = 20°C \quad T_{dp} = 20°C$

To find

(i) Specific humidity, and

(ii) Temperature of the mixture.

Assumptions

(i) There is no heat transfer with the surroundings.

(ii) The pressure remains constant throughout at 1 atm.

Schematic Mixing of two stream is shown on psychrometric chart in Fig. 15.31.

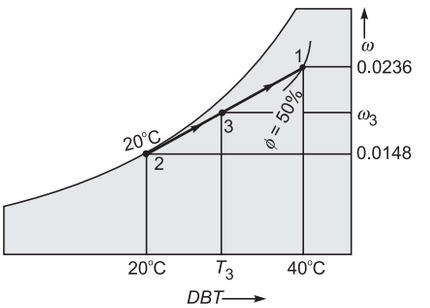


Fig. 15.31

Analysis From psychometric chart;

Coordinates: 40°C DBT and 50% RH,

$\omega_1 = 0.0236$ kg/kg of dry air

Coordinates: 20°C *DBT* and 20°C *DPT*,

$\phi_2 = 100\%$

and $\omega_2 = 0.0148$ kg/kg of dry air

(i) Specific humidity of mixture can be obtained by Eq. (15.38)

$$\omega_3 = \frac{\omega_1 \dot{m}_{a_1} + \omega_2 \dot{m}_{a_2}}{\dot{m}_{a_3}} = \frac{\omega_1 \dot{m}_{a_1} + \omega_2 \dot{m}_{a_2}}{\dot{m}_{a_1} + \dot{m}_{a_2}}$$

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Using numerical values

$$\omega_3 = \frac{0.0236 \times 1 + 0.0148 \times 2}{1 + 2}$$

$$= \mathbf{0.0177\ kg/kg\ of\ dry\ air}$$

(ii) Temperature of mixed stream

Draw a straight line between points 1 and 2 on the psychrometric chart and locate the point 3 corresponding to ω_3 = 0.0177, we get the dry-bulb temperature.

$$T_3 = \mathbf{26.8°C}$$

15.9 AIR WASHER

An air washer is a system in which the air is passed through a spray of water. The condition of air leaving the washer depends on the mean surface temperature of water droplets.

According to the requirement of conditioned air, the water used for spray is externally heated or cooled or simply re-circulated by a pump as shown in Fig. 15.32. The make-up water is added to maintain the water level in the washer constant. The louvers are provided to the loss of water droplets.

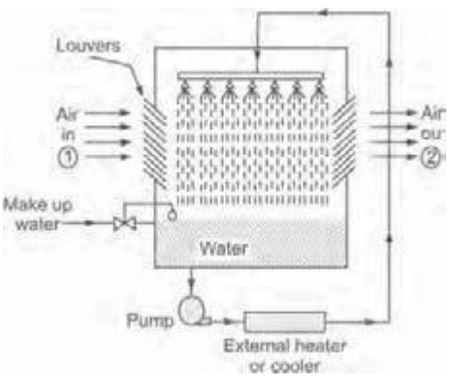

Fig. 15.32 *Schematic of air washer*

The droplets of water in the washer act as wetted surface. When air passes over it, its treatment depends on vapour pressure and temperature of water. Figure 15.33 shows a range of psychrometric

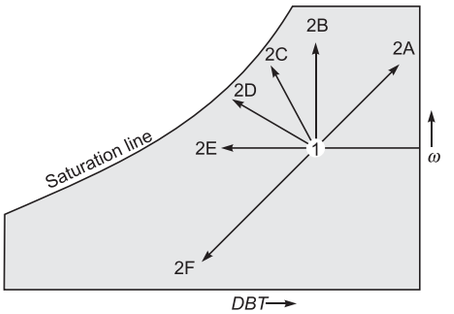

Fig. 15.33 *Range of psychrometric process*

processes that can take place between air entry at the state 1 and air exit at the state 2. These are the following:

1. When the mean temperature of water droplets is greater than the dry-bulb temperature of air, the air is *heated and humidified,* process 1–2A.
2. When the mean temperature of water droplets is equal to the dry-bulb temperature of air, the air is *humidified*, with an increase in its enthalpy, process 1–2B.
3. When the mean temperature of water droplets is less than the dry-bulb temperature of air but greater than the wet-bulb temperature of air, the air is *cooled and humidified* with increase in its enthalpy, process 1–2C.
4. When the mean temperature of water droplets is equal to the wet-bulb temperature of air, the air is *saturated adiabatically*, process 1–2D.
5. When the mean temperature of water droplets is equal to the dew-point temperature of air, the air is *cooled* with decrease in its enthalpy, process 1–2E.
6. When the mean temperature of water droplets is less than the dew-point temperature of air, the air is *cooled and dehumidified*, process 1–2F.

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Summary

- The science of measuring properties of moist air is psychrometry.
- The temperature measured by an ordinary thermometer is termed as *dry-bulb temperature.*
- The *dew-point temperature* of dry air and water vapour mixture is defined as the saturation temperature corresponding to the partial pressure of the vapour in the mixture.
- The *adiabatic saturation temperature* of atmospheric air is defined as the temperature which results from adding water vapour adiabatically to the atmospheric air in steady flow, until it becomes saturated.
- The *wet-bulb temperature* is the temperature measured by a thermometer whose bulb is covered with water soaked cotton wick and placed in a stream of air.
- *The relative humidity* ϕ is defined as the ratio of actual mass of vapour to mass of vapour required to produce a saturated mixture of air at the same pressure and temperature:

$$\phi = \frac{m_v}{m_{sat}} = \frac{p_v}{p_{sat}}$$

- *The humidity ratio* ω is defined as the ratio of the mass of vapour in the atmospheric air to the mass of dry air:

$$\omega = \frac{m_v}{m_a} = 0.622 \frac{p_v}{p - p_v}$$

where $p = p_a + p_v$ = total pressure of mixture.

- *The degree of saturation* μ is defined as the ratio of actual humidity ratio to the humidity ratio of saturated air at the same temperature and pressure:

$$\mu = \frac{\omega}{\omega_{sat}}$$

- The enthalpy of moist air is the sum of enthalpy of dry air and water vapour:

$$h = h_a + \omega h_g$$
$$= 2500 + 1.88\,T \text{ kJ/kg of water vapour.}$$

- The psychrometric chart is a graphical representation of properties of atmospheric air.
- Air washer is a system in which the air is passed through a spray of water.

Glossary

DBT Temperature indicated by an ordinary thermometer

Dehumidification Reduction of water vapour from moist air

DPT Temperature at which vapour starts to condense

Humidification Addition of water vapour in air

Psychrometry Study of moist air

Relative humidity Ratio of partial pressure of vapour to total pressure of mixture

Sensible heating/cooling Heat transfer with only change in DBT

Specific humidity Mass of water vapour per kg of dry air

WBT Temperature indicated by thermometer, when its bulb is covered by a wick thoroughly wetted.

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Review Questions

1. What is the dew-point temperature?
2. List the properties of moist air that a psychrometric chart provides for an air-conditioning engineer.
3. Sketch a psychrometric chart and indicate the lines of constant wet-bulb temperature and constant relative humidity.
4. What is the difference between dry air and atmospheric air?
5. Define (a) specific humidity, and (b) relative humidity.
6. The moist air is passed through a cooling section where it is cooled and dehumidified. How do the specific humidity and relative humidity of air change during this process?
7. In the summer, the outer surface of a glass filled with iced water frequently 'sweats'. How can you explain this sweating?
8. When are the dry-bulb and dew-point temperatures identical?
9. What is an adiabatic saturation? When does the wet-bulb temperature equal the saturation temperature?
10. How is the enthalpy of air–water vapour mixture defined?
11. What do you understand by evaporative cooling? List three important characteristics of evaporative cooling systems.
12. What is sensible heating or cooling?
13. Explain the process of cooling and dehumidification.
14. Explain the process of heating and humidification.
15. Prove that the specific humidity is given by

$$\omega = 0.622 \frac{p_v}{p - p_v}$$

where p_v = partial pressure of water vapour
p = total pressure of air

Problems

1. The moist air at 1 atm has 32°C DBT and 26°C WBT. Calculate
   (a) Partial pressure of water vapour,
   (b) Specific humidity
   (c) Dew point temperature,
   (d) Relative humidity
   (e) Degree of saturation,
   (f) Enthalpy of mixture.

[(a) 0.03 bar (b) 0.0186 kg/kg of dry air, (c) 24.1°C, (d) 62.5% (e) 0.614, (f) 80.55 kJ/kg]

2. A sling psychrometer reads 40°C DBT and 36°C WBT. Calculate (a) specific humidity, (b) relative humidity, and (c) dew point.

[0.0238, 50%, 28.2°C]

3. Calculate the amount of heat removed per kg of dry air, if the initial condition of air is 35°C DBT, 70% RH and final condition is 25°C DBT and 60% RH. *[46 kJ/kg]*

4. The moist air has 60% RH at 1 atm and 30°C. Calculate the specific humidity, dew point temperature, mass of water vapour and mass of dry air for 100 m^3 of moist air and partial pressure of water vapour and air.

[0.016 kg/kg of dry air, 21.25°C, 1.817 kg, 113.54 kg, 2.544 kPa, 98.78 kPa]

5. On a hot summer day, the ambient conditions are 40°C DBT, 20% RH. A desert cooler is used to increase the RH to 80%. Show the process on a psychrometric chart and by using it, calculate the temperature of exit air and minimum temperature to which the air can be cooled by a well-designed desert cooler. *[24.8°C, 22°C]*

6. DBT and WBT of atmospheric air at 1 atm are 25 and 15°C, respectively. Determine (a) specific humidity, (b) relative humidity, and (c) enthalpy of air.

[(a) 0.0065 kg/kg of dry air, (b) 33.2%, (c) 41.8 kJ/kg]

7. The air at 50°C has a relative humidity of 100%. What is its dew-point temperature? What mass

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of liquid water per kg of dry air will result if the mixture is cooled to 10°C at constant pressure of 90 kPa? *[50°C]*

8. Consider 100 m^3 of atmospheric air at 100 kPa, 15°C and 40% relative humidity. Find mass of water and humidity ratio. What is the dew point of the mixture? *[0.51 kg, 0.0043 kg, 1.4°C]*

9. Air at 10°C, 90% relative humidity and air at 30°C, 90% relative humidity are mixed steadily and adiabatically. The mass-flow rate of the colder stream is twice that of the other stream. What would be the state after mixing?

10. 10 m^3/min of air at 1 atm and 20°C with 90% RH is mixed with 20 m^3/min of air at 1 atm and 40°C with 20% RH. Calculate the resulting state of the mixture. *[h_3 = 61 kJ/kg, T_3 = 35.5°C]*

11. Consider a 500-litre rigid tank containing air–water vapour mixture at 100 kPa, 35°C with a 70%, RH. The system is cooled until the water just begins to condense. Determine the final temperature in the tank and the heat transfer for the process. *[28.2°C, –2.77 kJ]*

12. A stream of air at 1 atm, 30°C, 80% relative humidity flows at a rate of 28.0 kg/min. It is proposed to mix air at 1 atm, 15°C with a stream, steadily in an insulated mixing chamber so that the resulting temperature is 25°C. What is the rate of mass flow of the cooler air required and what would be the relative humidity of air leaving?

13. Air enters a dehumidifier at 34°C, 101 kPa and 60% RH. After being dehumidified, the air is heated to 20°C, with RH being 48%. Determine the temperature of air leaving the demudifier, if the air is saturated.

14. Saturated air at 20°C at a rate of 70 m^3/min is mixed adiabatically with the outside air at 35°C and 50% RH at a rate of 30 m^3/min. Assuming that the mixing process occurs at a pressure of 1 atm, determine the specific humidity, the relative humidity, and dry bulb temperature and volume flow rate of the mixture.

[0.0157 kg/kg of dry air, 80%, 26° C, 100.13 m^3/min]

15. Air at 1°C DBT and 80% relative humidity mixes adiabatically with an air stream at 18°C DBT, 40% relative humidity in the ratio of 1 to 3 by volume. Calculate the temperature and per cent saturation of the mixture. Take the barometric pressure 1.013 bar throughout constant.

[13.6°C and 48%]

Objective Questions

1. The psychrometry is
   (a) study of cooled air (b) study of hot air
   (c) study of moist air (d) none of the above
2. A psychrometer is an instrument, which measures
   (a) wet-bulb temperature
   (b) dry-bulb temperature
   (c) dew-point temperature
   (d) dry-bulb and wet-bulb temperatures
3. Temperature recorded by an ordinary thermocouple is known as
   (a) wet-bulb temperature
   (b) dry-bulb temperature
   (c) dew-point temperature
   (d) saturation temperature
4. Saturated air is
   (a) mixture of moisture and vapour
   (b) mixture of superheated vapour and air
   (c) moist air containing maximum possible moisture
   (d) none of the above
5. Temperature at which condensation of vapour begin is called
   (a) boiling temperature
   (b) saturation temperature
   (c) dew-point temperature
   (d) all of the above
6. Humidity ratio is defined as ratio of
   (a) mass of water vapour to mass of dry air in the mixture
   (b) mass of water vapour to mass of total mixture
   (c) mass of dry air to mass of water vapour
   (d) none of the above

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7. The degree of saturation is defined as
   (a) mass of water vapour to mass of dry air in the mixture
   (b) ratio of actual humidity ratio to humidity ratio of saturated air
   (c) ratio of actual mass of water vapour to mass of saturated vapour
   (d) none of the above
8. Humidity ratio is called
   (a) relative humidity (b) specific humidity
   (c) degree of humidity (d) none of the above
9. Wet-bulb depression is equal to
   (a) sum of dry-bulb temperature and wet bulb temperature
   (b) average of dry-bulb temperature and wet-bulb temperature
   (c) difference of dry-bulb temperature and wet-bulb temperature
   (d) product of dry-bulb temperature and wet-bulb temperature, one of the above
10. On a psychrometric chart, the horizontal scale shows
    (a) wet-bulb temperature
    (b) dry-bulb temperature
    (c) dew-point temperature
    (d) absolute humidity
11. On a psychrometric chart, the vertical scale shows
    (a) wet-bulb temperature
    (b) dry-bulb temperature
    (c) dew-point temperature
    (d) absolute humidity
12. On a psychrometric chart, the wet-bulb temperature lines are
    (a) curved
    (b) horizontal but not uniform
    (c) vertical
    (d) straight and inclined
13. On a psychrometric chart, the relative humidity lines are
    (a) curved
    (b) horizontal and uniform
    (c) vertical
    (d) straight and inclined
14. On a psychrometric chart, the dew point temperature lines are
    (a) curved
    (b) horizontal and uniform
    (c) vertical
    (d) straight and inclined
15. On a psychrometric chart, the dry-bulb temperature lines are
    (a) curved
    (b) horizontal and uniform
    (c) vertical
    (d) straight and inclined
16. For a moist air, if DBT is 15°C, WBT is 15°C and DPT is also 15°C, the saturation temperature will be
    (a) 15°C (b) 25°C
    (c) 10°C (d) 35°C
17. The process of cooling of air at same humidity ratio is known as
    (a) sensible heating (b) sensible cooling
    (c) humidification (d) dehumidification
18. The process of adding moisture to air at constant dry-bulb temperature is known as
    (a) sensible heating (b) sensible cooling
    (c) humidification (d) dehumidification
19. The process of removing moisture from air at constant dry-bulb temperature is known as
    (a) sensible heating (b) sensible cooling
    (c) humidification (d) dehumidification
20. During the process of adiabatic saturation,
    (a) DBT decreases
    (b) specific humidity increase
    (c) RH increases
    (d) all of the above

Answers

1. (c)	2. (d)	3. (b)	4. (c)	5. (d)
6. (a)	7. (b)	8. (b)	9. (c)	10. (b)
11. (d)	12. (d)	13. (a)	14. (b)	15. (c)
16. (a)	17. (b)	18. (c)	19. (d)	20. (d)

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16

Fuels and Combustion

Introduction

In the preceding chapters, our discussion was limited to non-reactive systems, in which chemical composition remains unchanged during a process with transfer of sensible internal energy and latent internal energy. In reactive systems (fuels), the chemical internal energy associated with destruction and formation of chemical bonds between atoms is transferred during a reaction. Such a chemical reaction is called *combustion*.

In this chapter, focus is put on types of fuels and combustion reactions, amount of air required for complete combustion of fuel, air–fuel ratio, enthalpy of reaction and calorific values of fuels.

16.1 FUELS

A fuel is simply a combustible substance. It burns in the presence of oxygen and releases heat energy. The various types of fuels, like liquid, solid and gaseous fuels, are available for burning in boilers, furnaces and other combustion equipment.

Most fuels contain carbon and hydrogen as main constituent; thus they are called *hydrocarbon fuels,* and designated with the chemical formula C_n H_m. The hydrocarbon fuels consist of all phases such as coal (a solid fuel), petroleum (a liquid fuel), and natural gas (a gaseous fuel). These fuels are available for burning in boilers, furnaces and other combustion equipments.

Each fuel consists of a certain amount of bonded energy in between its constituent elements. This energy is called the *chemical energy* and it is in the form of internal energy. In combustion, the interactive bonds of the molecules of the fuel and oxygen are broken and rearrangement of atoms in new molecular combination takes place. During this process, the energy is released in the form of heat. Therefore, the combustion is an exothermic chemical reaction. Though, during a combustion process, the reacting molecules rearrange themselves after the combustion but the total number of atoms of each element remain unchanged following the law of conservation of mass.

16.2 CHARACTERISTIC OF AN IDEAL FUEL

The selection of the right type of fuel depends on various factors such as availability, storage, handling, pollution and landed cost. The knowledge of fuel properties helps in selecting the right fuel for the right purpose and efficient use of fuel.

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A good fuel should have the following qualities:

(i) It should have a high heating value.
(ii) It should be free from moisture and non-combustible matter, i.e., ash, etc.
(iii) Its products of combustion should not be harmful and pollutant.
(iv) It should have moderate ignition temperature. High ignition temperature may cause difficulty in combustion and low ignition temperature may cause fire hazards.
(v) It should be easy to transport and store in minimum space.
(vi) It should have moderate rate of combustion and controlled combustion.
(vii) It should have high combustion efficiency.
(viii) It should be readily available at low cost.

16.3 COAL

Coal is the most important solid fuel and its main constituent is carbon. It also contains some amount of oxygen, hydrogen, nitrogen, sulphur, moisture and ash. Coal is a mineral substance of vegetable origin. It is the result of decay of vegetable matter which accumulated under earth millions of years ago and got transformed by the action of heat and pressure. Its transformation occurs progressively.

16.3.1 Classification of Coal

The coal passes through different stages during its formation from vegetation. These stages are listed and discussed below:

Peat It is a spongy humidified substance and the first stage at which the fuel is derived from the wood and vegetable matter. Thus, it possesses constituents very nearest to those of wood with 30% moisture. It has lowest calorific value about 14,500 kJ/kg. It is mainly used as a fuel in gas-producer plants. Other varieties of coal are derived from peat.

Lignite Lignite is found next to peat. It is a soft coal, composed mainly of volatile matter and moisture content with low fixed carbon of about 60%. Its appearance is brown and it contains high ash and low heating value. It is a low-grade fuel having a calorific value about 21, 000 kJ/kg. It is too brittle, and hence it is converted into briquetts, which can be easily handled. It burns with heavy smoke. There are comparatively large deposits of lignite in Kashmir, Neyvelli and Rajasthan. Neyvelli Power Plant uses this type of fuel.

Bituminous Coal Its formation is next to lignite. It is shining black in appearance. It contains about 70% carbon and the remaining as a large percentage of volatile matter. Thus, it is easy to ignite and it burns with yellow smoky flames. It has an average calorific value of 31, 500 kJ/kg. In India, its reserves are available in Bihar, MP, West Bengal and Orissa. It is used for producing producer gas.

Anthracite It is the next generation of bituminous coal, and has the highest rank. It is a hard coal composed of about 90% carbon with little volatile content and practically no moisture. It is brittle and has a shining black lusture. It burns without flame and smoke but is difficult to ignite. It is non-cracking and it has minimum ash, sulphur, volatile matter and moisture content. It has highest heating value, approximately 36,000 kJ/kg. In India, its reserves are in Kashmir and the eastern Himalayas. It is used as fuel in steam power plants.

Wood Charcoal It is obtained by heating of wood up to a temperature of 280°C in the absence of atmospheric air. During this process, the volatile matter and water are expelled and the residue is charcoal. It is an excellent fuel but costly. Its calorific value is about 28,000 kJ/kg.

Coke It is obtained from bituminous coal by the carbonization process, i.e., the bituminous coal is heated strongly in the absence of air for 48 hours. Thus, the volatile matter present in the coal is driven off. Coke is dull black in colour, hard, brittle, and porous. It consists of 90 to 95% carbon, small quantities of sulphur, hydrogen, nitrogen and phosphorus. It is a smokeless and clear fuel. Its average

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calorific value is 32,500 kJ/kg. It is used in steam power plants, blast furnaces and gas producers.

Briquettes It is produced from finely grounded low-grade coal by the moulding operation. The coal powder is mixed with a suitable binder and pressed together to form blocks or briquettes. The binding material may be pitch, coal tar, crude oil or clay. By this method, the heating value of low-quality fuel is improved.

Pulverised Coal Crushed coal in fine powder form is called *pulverized coal*. During combustion, it floats and thus comes in better contact of air, improving the combustion efficiency. Pulverised coal has flexibility of control, complete combustion with less excess air and high flame temperature. Pulverised coal is used in cement industries and in furnaces.

16.3.2 Analysis of Coal

It is difficult to give an exact mass analysis of coal, since its composition varies considerably from one geographical location to another. The following two type of analysis of coal are most popular:

(a) Proximate Analysis

Proximate analysis indicates the percentage by mass of the fixed carbon, volatiles, ash, and moisture content in coal. The amount of fixed carbon and volatile matter directly contribute to the heating value of coal.

Fixed carbon acts as a main heat generator during burning. It consists mostly of carbon but also contains some hydrogen, oxygen, sulphur and nitrogen. Fixed carbon gives a rough estimate of the heating value of coal.

Volatile matters are methane, hydrocarbons, hydrogen and carbon monoxide, and incombustible gases like carbon dioxide and nitrogen found in coal. High volatile matter content indicates easy ignition of fuel.

Ash is an impurity that will not burn. The ash content is important in the design of the furnace grate, combustion volume, pollution control equipment and ash-handling systems of a furnace.

Moisture in coal decreases the heat content per kg of coal. The *moisture* content in coal ranges from 0.5 to 10%. A typical proximate analysis of various types of coal is given in the Table 16.1.

Table 16.1 *Typical proximate analysis of various coals*

			Percentage by mass of fuel	
Fuel	*Inherent Moisture %*	*Volatile matter %*	*Fixed Carbon %*	*Ash %*
Anthracite	2	6	75	17
Bituminous coal	6	21	35	38
Indonesian coal	9	30	47	14

Table 16.2 *Ultimate Analysis of various coals*

	Percentage by mass of dry fuel						
Fuel	*Rank*	C	H_2	O_2	N_2	S	*Mineral matter*
Anthracite	101	88.2	2.7	1.7	1	1.2	5.2
Medium-rank coal	401	81.8	4.9	4.4	1.8	1.9	5.2
Low-rank coal	902	75.0	4.6	10.7	1.6	2.1	6.0
Coke	–	90.0	0.4	1.9	–	–	7.7

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(b) Ultimate Analysis

The ultimate analysis gives an accurate chemical analysis by mass of important elements of fuel such as carbon, hydrogen, oxygen, sulphur, etc. It is useful in determining the quantity of air required for combustion and the volume and composition of the combustion gases. This information is required for the calculation of flame temperature and the flue duct design, etc. Such analysis may vary from one sample to another within the group also and hence cannot be taken as a reference guide. Typical ultimate analyses of various coals are given in Table 16.2.

16.4 LIQUID FUELS

Most liquid hydrocarbon fuels are a mixture of hydrocarbons that are derived from *natural petroleum,* called the *crude oils.* Some of the these are gasolene, diesel, methanol, kerosene, and fuel oil. Within each classification, there is a variety of grades and each is the mixture of a large number of different hydrocarbons. For convenience, the composition of certain hydrocarbons in a fuel is represented by a single hydrocarbon. For an example, gasolene—a mixture of more than 40 hydrocarbons—is treated as *octane* C_8H_{18}, diesel as dodecane $C_{12}H_{26}$, methanol as CH_3OH.

The light diesel oil, furnace oil and LSHS (low sulphur heavy stock) are predominently used in industrial applications. The ultimate analysis of certain liquid fuels is given in Table 1.3.

Advantages of Liquids Fuel over Solid Fuels

1. The calorific value of liquid fuels is higher than solid fuels.
2. They can be stored in comparatively less space.
3. Their combustion rate can be easily controlled.
4. During its burning, ash formation is almost absent and its burning is clean.
5. They have higher thermal efficiency.
6. They are non-corrosive and light in weight.
7. They are easy to ignite and easy to shutting of operation.
8. Semi-skilled labour can be employed.

Disadvatages of Liquid Fuels

1. Liquid fuels are costly.
2. Their reserves are limited and will be depleted soon.
3. There is greater risk of fire hazard.
4. Costly containers are required for their transportation and storage.
5. They cannot be burn in the presence of insufficient air.

16.5 GASEOUS FUEL

Gaseous fuels are the chemically simplest among the three groups. The two primary sources of gaseous hydrocarbon fuels are natural gas wells and certain chemical manufacturing process.

Table 16.3 *Analysis of liquid fuels*

Fuel	*Carbon*	*Hydrogen*	*Sulphur*	*Ash, etc.*
Petrol (Iso-octane)	85.1	14.9	0.01	–
Commercial Petrol	85.5	14.4	0.1	–
Benzole	91.7	8.0	0.3	–
Kerosene	86.3	13.6	0.1	–
Diesel oil	86.3	12.8	0.15	–
Light diesel oil	86.2	12.4	1.5	–
Furnace oil	86.1	11.8	3	–
Residual fuel oil	88.3	9.5	1.2	1.0

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LPG (liquified petroleum gas) is mainly a mixture of propane and butane. LPG is a mixture of those hydrocarbons which are in gaseous phase at normal atmospheric pressure, but may be condensed to the liquid state at normal temperature, by the application of moderate pressures.

LPG vapour is denser than air, butane is about twice as heavy as air and propane about 1.5 times as heavy as air. Consequently, the vapour may flow along the ground and into drains sinking to the lowest level of the surroundings and be ignited at a considerable distance from the source of leakage.

Natural gas contains methane as the main constituent and other gases in small amounts. Natural gas has high calorific value and it burns without smoke. It is lighter than air and disperses into air easily in case of leak.

Coal gas is obtained by the carbonization process of coal. The quality of coal gas depends on the quality of coal used and the temperature of carbonization. It contains mainly H_2, CO and various hydrocarbons. Its calorific value is about 22, 500 kJ/m^3. It is used in towns for street lighting and domestic lighting purposes.

Producer gas is obtained by partial combustion of coal, coke and charcoal in a mixed air stream. Its manufacturing cost is low and it has a calorific value of about 6500 kJ/m^3. It is used in furnaces.

Water gas is a mixture of H_2 and CO and is produced by passing steam over incandescent coke.

Blast furnace gas is a by-product in the production of pig iron in the blast furnace. It is used to preheat the blast in the furnace. It is used as fuel in metallurgical furnaces. It has calorific value of the order of 3700 kJ/m^3.

Coke oven gas is obtained as a by-product during the formation of bituminous coal. It is used for industrial heating and power generation application. It has a calorific value of about 17,000 kJ/m^3.

Carbon monoxide, coal gas, biogas, methane, ethane, are some gaseous fuels. Table 16.4 shows a typical analysis of natural gas by volume at atmospheric pressure and temperature.

Advantages of Gaseous Fuel

1. The supply of fuel can be easily and accurately controlled.
2. The gaseous fuels are free from impurities.
3. The combustion of gaseous fuel is possible with very less excess air.
4. They produce relatively less smoke and pollution.
5. They are capable of attaining very high temperatures.
6. The gaseous fuels are light in weight.
7. Transportation of gaseous fuels is easy.

Disadvantages

1. They require larger storage capacity.
2. They are highly flammable and thus can cause fire hazards.

16.6 CONVERSION OF VOLUMETRIC ANALYSIS TO GRAVIMETRIC ANALYSIS

If the volumetric analysis of a mixture is known, it can easily be converted into gravimetric (mass) analysis by using the relation

$$m_i = n_i \mathcal{M}_i \qquad ...(16.1)$$

where m_i = mass of the i^{th} constituent
n_i = number of moles of the i^{th} constituent
$\mathcal{M}_i$ = molar mass (molecular weight) of the i^{th} constituent

Table 16.4 *Analysis by volume of a typical natural gas*

Methane	*Ethane*	*Propane*	*Butane*	*Nitrogen*	*Carbon dioxide*
CH_4	C_2H_6	C_3H_8	C_4H_{10}	N_2	CO_2
92.6%	3.6%	0.8%	0.3%	2.6%	0.1

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The volume percentage of a mixture constituent is considered as mole fraction and when the molar mass is multiplied to mole fraction, it results into mass of the constituent. The conversion of volume percentage to mass percentage is referred as *conversion of volumetric analysis to gravimetric analysis.* The following steps should be taken into consideration for such conversion.

1. Start with volume percentage of each constituent.
2. Multiply the volume percentage of each constituent by its molecular weight.
3. Take the sum of products of the step 2 above.
4. Find the percentage analysis by mass as $\frac{m_i}{\Sigma m_i} \times 100$.

The following example illustrates the conversion of volume percentage to mass percentage of constituents in the gas mixture.

Example 16.1 *The percentage analysis of a gas by volume is given as*

$CO_2 = 5.5\%$ $CO = 38.3\%$
$CH_4 = 0.4\%$ $O_2 = 0.1\%$
$H_2 = 52.8\%$ $N_2 = 2.9\%$

Obtain the percentage analysis by mass.

Solution The solution is presented in the following tabular form.

Constituent	*% Volume* n_i	*Molecular weight* $\mathcal{M}_i$	*Mass* $m_i = n_i \mathcal{M}_i$	*% Mass in mixture* $= \frac{m_i}{\Sigma m_i} \times 100$
CO_2	5.5	44	242.0	16.02%
CO	38.3	28	1072.4	70.98%
CH_4	0.4	16	6.4	0.423%
O_2	0.1	32	3.2	0.211%
H_2	52.8	2	105.6	6.99
N_2	2.9	28	81.2	5.37
			$\Sigma m_i = 1510.8$	100%

16.7 CONVERSION OF GRAVIMETRIC ANALYSIS TO VOLUMETRIC ANALYSIS

The reverse of the above procedure, which converts the mass fraction to mole fraction by using the same relation as $n_i = \frac{m_i}{\mathcal{M}_i}$ is referred as *conversion of gravimetric analysis to volumetric analysis*. It is generally applied to gaseous fuels. The following steps should be followed for such a conversion.

1. Start with the percentage of mass (mass fraction) of each constituent.
2. Divide mass fraction of each constituent by its molecular mass.
3. Take the sum of quotients of the step 2 above.
4. Obtain the percentage volume as $\frac{n_i}{\Sigma n_i} \times 100$.

The following example illustrates the conversion of gravimetric analysis to volumetric analysis.

Example 16.2 *A gaseous mixture contains CO_2 = 16%, CO= 1%, O_2 = 8% and N_2 = 75% by mass. Convert this gravimetric analysis to volumetric analysis.*

Solution According to the steps given above, the conversion is taken into tabular form, as

Constituent	*Mass fraction* m_i	*Molecular mass* $\mathcal{M}_i$	*Number of moles,* $n_i = \frac{m_i}{\mathcal{M}_i}$	*Volume percentage* $\frac{n_i}{\Sigma n_i}$
CO_2	16	44	0.364	10.93%
CO	1	28	0.035	1.040%
O_2	8	32	0.25	7.50%
N_2	75	28	2.68	80.53%
			$\Sigma n_i = 3.327$	100%

16.8 COMBUSTION

16.8.1 Combustion Terminology

(*i*) *Reactants* The components of fuel that exist before the combustion reaction are called the *reactants.*

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(*ii*) *Products* The components that exist after combustion reaction are called the *products*. Both the reactants and products are pure substances before and after the chemical reaction and we can calculate the properties change.

$$\text{Reactants} \xrightarrow[\text{Heat Release}]{\text{Chemical reaction}} \text{Products}$$

(*iii*) *Combustion* The combustion is an exothermic process in which the rapid oxidation of combustible elements of a fuel takes place with release of heat energy. The combustion process is a fuel-burning process in the presence of oxygen. The combustion is called the *complete combustion* when all the carbon present in the fuel is burned to carbon dioxide, all the hydrogen is burned to water vapour, all the sulphur is burned to sulphur dioxide and all other combustible elements are fully oxidised. When these conditions are not fulfilled or the constituents of fuel are not fully oxidised then combustion is called *incomplete combustion.*

(*iv*) *Ignition Temperature* The intimate contact of fuel and oxygen cannot initiate the combustion process. The fuel must be brought to its ignition temperature to start the combustion. The *ignition temperature* of a fuel element is the lowest temperature at which it starts burning. The minimum ignition temperature for gasolene is 260°C, for carbon it is 400°C, for hydrogen it is 580°C, for carbon monoxide it is 610°C and for methane it is 630°C.

16.8.2 Physical laws of Combustion

There are certain physical laws on which the analysis of combustion process is based.

(*i*) *Law of Conservation of Mass* During a chemical reaction, the mass is conserved, that is, the mass of the reactants equals the mass of the products. The total mass of each chemical element must be same on both sides of the equation even though the elements exist in different chemical compositions in reactants and products. However, the number of moles of products may differ from the number of moles of reactants. For example,

$$1H_2 + \frac{1}{2}O_2 \rightarrow H_2O \qquad ...(16.2)$$

Introducing the molar masses,

$$2 \text{ kg } H_2 + 16 \text{ kg } O_2 \rightarrow 18 \text{ kg } H_2O$$

Hence, the mass of the reactants is equal to the mass of products. When using the number of moles,

$$1 \text{ mole } H_2 + \frac{1}{2} \text{ mole } O_2 \rightarrow 1 \text{ mole of } H_2O$$

Thus the number of moles of the product differs from the number of moles of reactants.

(*ii*) *Ideal gas Laws* All the combustion equations follow the ideal gas equation and general gas equations:

$$pV = nR_uT$$

where n = number of moles of a gas
V = molar volume, m^3/kmol
R_u = Universal gas constant
= 8314.3 J/kmol·K

(*iii*) *Avagadro's Law* It states that molecular mass of all perfect gases occupy the same volume under similar conditions of pressure and temperature. At NTP, the molar volume

$$V_0 = \frac{nR_uT}{p}$$

$$= \frac{(1 \text{ kmol}) \times (8.314 \text{ kJ/kmol} \cdot \text{K}) \times (273 \text{ K})}{(101.325 \text{ kPa})}$$

$$= 22.41 \text{ m}^3\text{/kmol}$$

16.8.3 Basic Combustion Reactions

(*a*) *Combustion of Solid and Liquid Fuels*

The soilid and liquid fuels usually consist of carbon, hydrogen and sulphur. These constituents burn with rapid oxidation to produce heat. The products of combustion may be CO, CO_2, H_2O and SO_2. The basic combustion reactions for these three elements are the following:

(*i*) *Burning of carbon to carbon monoxide: incomplete combustion*

$$2C + O_2 \rightarrow 2CO$$

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Introducting the molar masses;

$$(2 \times 12) \text{ kg C} + 32 \text{ kg } O_2 \rightarrow 56 \text{ kg CO}$$

or $$1 \text{ kg C} + \frac{4}{3} \text{ kg } O_2 \rightarrow \frac{7}{3} \text{ kg CO} \quad ...(16.3)$$

It indicates that 1 kg of carbon requires 4/3 kg of oxygen to produce 7/3 kg of carbon monoxide.

(*ii*) *Burning of carbon to carbon dioxide: complete combustion*

$$C + O_2 \rightarrow CO_2$$

Using molar masses;

$$12 \text{ kg C} + 32 \text{ kg } O_2 \rightarrow 44 \text{ kg } CO_2 \quad ...(16.4)$$

or $$1 \text{ kg C} + \frac{8}{3} \text{ kg } O_2 \rightarrow \frac{11}{3} \text{ kg } CO_2$$

It means 1 kg of carbon requires $\frac{8}{3}$ kg of oxygen to produce $\frac{11}{3}$ kg of carbon dioxide.

(*iii*) *Burning of carbon monoxide to carbon dioxide: intermediate combustion*

$$2CO + O_2 \rightarrow 2CO_2$$

Using molar masses;

$$56 \text{ kg CO} + 32 \text{ kg } O_2 \rightarrow 88 \text{ kg CO}$$

or $$1 \text{ kg CO} + \frac{4}{7} \text{ kg } O_2 \rightarrow \frac{11}{7} \text{ kg } CO_2 \quad ...(16.5)$$

That is, 1 kg of carbon monoxide requires $\frac{4}{7}$ kg of O_2 to produce $\frac{11}{7}$ kg of carbon dioxide.

(*iv*) *Burning of sulphur to sulphur dioxide*

$$S + O_2 \rightarrow SO_2$$

Molar mass basis

$$32 \text{ kg S} + 32 \text{ kg } O_2 \rightarrow 64 \text{ kg } SO_2$$

or $$1 \text{ kg S} + 1 \text{ kg } O_2 \rightarrow 2 \text{ kg } SO_2 \quad ...(16.6)$$

Thus 1 kg sulphur requires 1 kg O_2 to produce 2 kg SO_2.

(*b*) *Combustion of Gaseous Fuel*

When dealing with gases, the volumes or number of moles may be substituted for gas elements.

(*i*) *Burning of carbon monoxide to carbon dioxide; Referring (Eq. 16.5)*

$$2 \text{ mole of CO} + 1 \text{ mole of } O_2 \rightarrow 2 \text{ mole of } CO_2$$

$$1 \text{ volume CO} + \frac{1}{2} \text{ volume } O_2 \rightarrow 1 \text{ volume of } CO_2 \quad ...(16.7)$$

It can be interpreted as one volume of carbon monoxide requires $\frac{1}{2}$ volume of oxygen to produce 1 volume of carbon dioxide.

(*ii*) *Burning of H_2 gas to water vapour*

$$H_2 + \frac{1}{2} O_2 \rightarrow H_2O \quad ...(16.8)$$

On mole basis

$$1 \text{ mole } H_2 + \frac{1}{2} \text{ mole } O_2 \rightarrow 1 \text{ mole } H_2O$$

or $$1 \text{ volume } H_2 + \frac{1}{2} \text{ volume } O_2 \rightarrow 1 \text{ volume } H_2O$$

Thus, 1 volume of H_2 combines with $\frac{1}{2}$ volume of O_2 to form 1 volume of H_2O.

On mass basis,

$$2 \text{ kg } H_2 + 16 \text{ kg } O_2 \rightarrow 18 \text{ kg } H_2O$$

$$1 \text{ kg } H_2 + 8 \text{ kg } O_2 \rightarrow 9 \text{ kg } H_2O \quad ...(16.9)$$

Thus, 1 kg H_2 requires 8 kg of O_2 to form 9 kg of water vapour.

(*iii*) *Burning of marsh gas (methane gas)*

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

Mole basis:

$$1 \text{ mole } CH_4 + 2 \text{ mole } O_2 \rightarrow 1 \text{ mole } CO_2 + 2 \text{ mole } H_2O \quad ...(16.10)$$

It means 1 mole of methane gas combines with 2 moles of oxygen gas to produce 1 mole of CO_2 and 2 moles of water vapour.

Mass basis:

$$16 \text{ kg } CH_4 + 64 \text{ kg } O_2 \rightarrow 44 \text{ kg } CO_2 + 36 \text{ kg } H_2O$$

$$1 \text{ kg } CH_4 + 4 \text{ kg } O_2 \rightarrow \frac{11}{4} \text{ kg } CO_2 + \frac{9}{4} \text{ kg } H_2O \quad ...(16.11)$$

That is, 1 kg of CH_4 gas reacts with 4 kg of O_2 to produce $\frac{11}{4}$ kg of CO_2 and $\frac{9}{4}$ kg of H_2O.

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(*iv*) *Burning of ethylene to carbon dioxide and water vapour*

$$C_2H_4 + 3O_2 \rightarrow 2CO_2 + 2H_2O$$

1 mole C_2H_4 + 3 mole O_2 → 2 mole CO_2 + 2 mole H_2O ...(16.12)

1 m^3 of ethylene completely burns with 3 m^3 oxygen to produce 2 m^3 of CO_2 and 2 m^3 of H_2O.

(*v*) *Burning of octane to carbon dioxide and water vapour*

$$C_8H_{18} + 12.5O_2 \rightarrow 8CO_2 + 9H_2O$$

1 mole C_8H_{18} + 12.5 mole O_2 → 8 mole CO_2 + 9 mole H_2O ...(16.13)

The above equations are helpful in calculating the amount of oxygen required to burn the fuel completely.

The combustion equation for any hydrocarbon (say, $C_{10}H_{22}$) can be written as

$$C_{10}H_{22} + a\,O_2 \rightarrow b\,CO_2 + d\,H_2O$$

Equating C, H, O on both sides, we get

$b = 10$

$2d = 22$ or $d = 11$

$2a = 2b + d$ or $a = 15.5$

Then the equation becomes

$$C_{10}H_{22} + 15.5\,O_2 \rightarrow 10\,CO_2 + 11\,H_2O$$

16.8.4 3 T's of Combustion

The objective of good combustion is to release all of the heat in the fuel. This is accomplished by controlling the three T's of combustion which are (1) temperature high enough to ignite and maintain ignition of the fuel, (2) turbulence for intimate mixing of the fuel and oxygen, and (3) time sufficient for complete combustion.

Too much or too little fuel with the available combustion air may potentially result in unburned fuel and carbon monoxide generation. A very specific amount of O_2 is needed for perfect combustion and some additional (excess) air is required for ensuring complete combustion. However, too much excess air will result in heat and efficiency losses. Figure 16.1 shows complete, good and incomplete combustion, respectively.

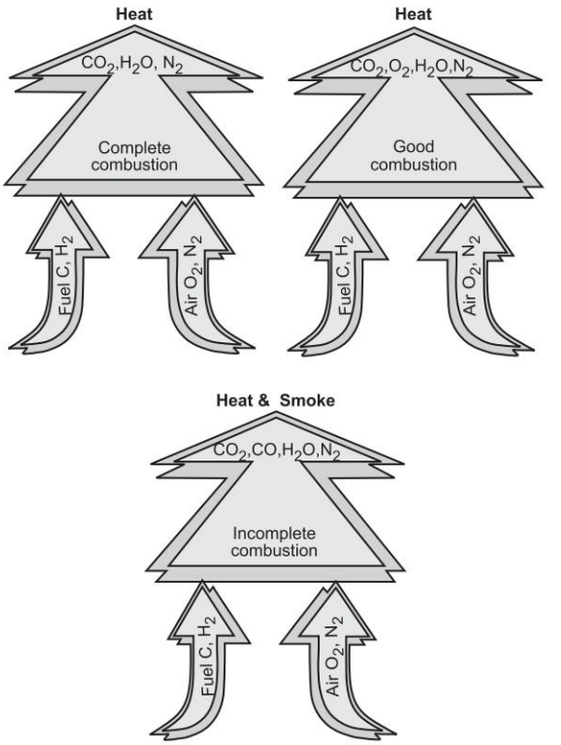

Fig. 16.1 *Complete, good and incomplete combustions*

16.9 COMPOSITION OF DRY AIR

Instead of supplying only oxygen for the chemical reactions, atmospheric air is supplied. Atmospheric air is the mixture of mainly O_2 and N_2 with small traces of argon, carbon dioxide, water vapour, etc. The composition of dry air on molar basis is given by 78.09% of N_2, 20.95% of O_2, 0.93% of argon and 0.03% of CO_2. The molecular weight of this mixture is 28.97 kg/kmol. For all calculations, argon and carbon dioxide are considered as an additional nitrogen, because they are also inert gases like N_2 and appear in very small quantities. Accordingly, air is considered to consist of 79% of nitrogen and 21% of oxygen by volume or on a molar basis.

With this assumption, the molar (volume) ratio of nitrogen to oxygen is $\frac{0.79}{0.21} = 3.76$, i.e., air supplied

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for combustion contains 3.76 mole of nitrogen with each mole of oxygen.

The mass composition can be derived from this molar composition by using relation m (mass) = n (moles) × $\mathcal{M}$ (mol. weight). Thus, it is found that air consists of 23% O_2 and 77% N_2 on mass basis. The composition of air taken for calculations is given in Table 16.5.

Table 16.5 *Composition of air*

Molar basis	*Mass basis*
0.21 mole O_2 + 0.79 mole N_2 = 1 mole of air	0.23 kg O_2 + 0.77 kg N_2 = 1 kg of air
1 mole O_2 + 3.76 mole N_2 = 4.76 mole of air	1 kg O_2 + 3.347 kg N_2 = 4.347 kg of air
Molecular weights $\mathcal{M}_{O_2} = 32$, $\mathcal{M}_{N_2} = 28$,	$\mathcal{M}_{air} = 28.97$ (kg/kmol)

16.10 AMOUNT OF AIR REQUIRED FOR COMBUSTION

16.10.1 Minimum Air Required for Complete Combustion of Solid Fuel

The minimum amount of air that supplies just sufficient oxygen for the complete combustion of all the carbon, hydrogen and sulphur present in the fuel is called the *theoretical amount of air* or *stoichiometric air*. It is also called the chemically correct amount of air required for complete combustion.

For 1 kg of fuel consisting of carbon, hydrogen and sulphur, the minimum amount of O_2 required can be obtained with the help of basic combustion. Equations (16.4), (16.6), (16.9) respectively as

$$1 \text{ kg of carbon requires } \frac{8}{3} \text{ kg of } O_2$$

$$\therefore \quad C \text{ kg of carbon requires } \left(\frac{8}{3}C\right) \text{ kg of } O_2$$

Similarly,

S kg of sulphur requires (S) kg of O_2.

H kg of hydrogen requires (8 H) kg of O_2.

$$\text{Total oxygen required} = \left(\frac{8}{3}C + 8H + S\right) \text{kg}$$

All the oxygen supplied with air must be consumed by the fuel elements to form the completely oxidised products. No free oxygen should appear in the products. If (O) kg of oxygen is already present in the fuel then minimum mass of oxygen required for complete combustion

$$= \frac{8}{3}C + 8H + S - O$$

$$= \frac{8}{3}C + 8\left(H - \frac{O}{8}\right) + S \qquad ...(16.14)$$

$\therefore$ minimum mass of air required becomes,

$$m_{th} = \frac{\mathbf{100}}{\mathbf{23}}\left[\frac{\mathbf{8}}{\mathbf{3}}\mathbf{C} + \mathbf{8}\left(\mathbf{H} - \frac{\mathbf{O}}{\mathbf{8}}\right) + \mathbf{S}\right] \qquad ...(16.15)$$

since air contains 23% of oxygen by mass.

16.10.2 Minimum Air Required for Complete Combustion of Gaseous Fuel

Consider 1 m^3 of gaseous fuel consisting of CO, H_2, CH_4, C_2H_4, etc. Let the volume of these gases in the mixture be CO m^3, H_2 m^3, CH_4 m^3 and C_2H_4 m^3, respectively. From Eqs. (16.7), (16.8), (16.10) and (16.12), respectively, the minimum volume of oxygen required for complete combustion

$$= 0.5\ CO + 0.5\ H_2 + 2\ CH_4 + 3\ C_2H_4\ (m^3) \qquad ...(16.16)$$

If fuel itself contains O_2 m^3 of oxygen then volume of oxygen required for complete combustion

$$= 0.5\ CO + 0.5\ H_2 + 2\ CH_4 + 3\ C_2H_4 - O_2\ (m^3) \qquad ...(16.17)$$

The atmospheric air contains 21% of oxygen by volume. Hence, the minimum volume of air required for complete combustion is

$$V_{th} = \frac{100}{21} \times [0.5\ CO + 0.5\ H_2 + 2\ CH_4 + 3\ C_2H_4 - O_2]\ (m^3) \qquad ...(16.18)$$

16.10.3 Excess Air

In actual combustion process, the amount of air supplied is either greater or less than the theoretical

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amount. The amount of air actually supplied is usually expressed in terms of *per cent of stoichiometric air.*

The amount of air in excess, which is necessary to burn fuel completely is called the *excess air.* This can be expressed as a per cent excess air in terms of stoichiometric or theoretical amount of air as

$$\text{Percent excess air} = \frac{\text{Actual air used} - \text{Stoichiometric air required}}{\text{Stoichiometric air required}} \qquad ...(16.19)$$

Similarly, if the actual amount of air supplied is less than the theoretical amount then it can be expressed as *per cent deficiency of air.* For an example, consider the complete combustion of methane with 150% of theoretical air (50% excess air). The balanced chemical equation is

$$CH_4 + (1.5) \times 2\,(O_2 + 3.76\,N_2) \rightarrow CO_2 + 2H_2O + O_2 + 11.28\,N_2 \qquad ...(16.20)$$

Here the amount of air per mole of fuel is 1.5 times the theoretical amount determined by Eq. (16.13).

16.10.4 Equivalence Ratio

It is the ratio of actual air–fuel ratio to the theoretical air–fuel ratio for complete combustion. The reactants form a rich mixture when the equivalence ratio is less than unity. Accordingly, for excess air supplied with fuel, the equivalence ratio is always greater than unity.

16.11 AIR–FUEL RATIO

A frequently used quantity in the analysis of the combustion processes is the *air–fuel ratio.* It is usually expressed on mass basis and is defined as the *ratio of mass of the air to the mass of the fuel* during combustion process. It is designated as A/F.

$$\text{A/F} = \frac{\text{Mass of air}}{\text{Mass of fuel}} \qquad ...(16.21)$$

Simply A/F ratio is the equivalent amount of air required for burning of 1 kg of fuel completely.

Air–fuel ratio can also be interpreted on mole basis as the ratio of number of moles of air to the number of moles of fuel. But the former definition is normally used.

Calculation for the air–fuel ratio for CH_4 on mass basis by Eq. (16.21);

Mass of air

$= \text{number of moles of air} \times \mathcal{M}_{air}$

$= 2 \times (1 \text{ mole of } O_2 + 3.76 \text{ mole } N_2) \times 28.97$

$= 9.52 \text{ mole air} \times 28.97 = 275.8 \text{ kg}$

The mass of fuel, $CH_4 = 1 \text{ mole} \times \mathcal{M}_{CH_4} = 16 \text{ kg}$

Thus $$\text{A/F} = \frac{278.8 \text{ kg}}{16 \text{ kg}} = 17.237$$

16.12 AIR–FUEL RATIO FROM ANALYSIS OF FLUE GASES

When the volumetric analysis of products of combustion is known. then the actual air–fuel ratio can be obtained by using the following methods:

1. Carbon balance method
2. Hydrogen balance method
3. Carbon- Hydrogen balance method

16.12.1 Carbon Balance Method

If all carbon present in the fuel burns completely without leaving traces in flue gases then carbon balance method is used to obtain the actual amount of air used to form the dry products of combustion. In this method, the mass of carbon present in the fuel is balanced with mass of carbon present in the dry flue gases. It is a quite accurate method.

Let C be the mass of carbon in 1 kg of fuel; and CO_2 and CO are the masses of carbon dioxide and carbon monooxide, respectively, formed per kg of flue gases. Then

1 kg of CO_2 contains $\frac{3}{11}$ kg of carbon ($\because$ 1 kg of carbon produces $\frac{11}{3}$ kg of CO_2), and

1 kg of CO contains $\frac{3}{7}$ kg of carbon ($\because$ 1 kg of carbon produces $\frac{7}{3}$ kg of CO);

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Then the *mass of Carbon per kg of dry flue gases*

$$= \left(\frac{3}{11}CO_2 + \frac{3}{7}CO\right) \text{ kg/kg of flue gases}$$

The *mass of dry flue gases per kg of fuel* burnt

$$m_{dry\ flue\ gases} = \frac{\text{Mass of carbon present in 1 kg of fuel}}{\text{Mass of carbon present in 1 kg of flue gases}} \quad ...(16.21)$$

The mass of vapour formed per kg of fuel during combustion

$$m_{H_2O} = 9 \times H_2 \text{ present per kg of fuel}$$

Then the total mass of dry flue gases per kg of fuel burnt

$$m_{flue\ gases} = m_{dry\ flue\ gases} + m_{H_2O} \quad ...(16.22)$$

16.12.2 Hydrogen Balance Method

When some solid carbon is suspected in combustion products, then the carbon-balance method cannot give an accurate mass of air used in combustion. In such a situation, the hydrogen method is used.

In the hydrogen-balance method, the mass of hydrogen in the fuel composition is balanced with the mass of hydrogen per kg of flue gases.

16.12.3 Carbon–Hydrogen Balance Method

If the Orasat apparatus cannot measure the percentage of nitrogen accurately, the masses of carbon and hydrogen are balanced in fuel composition and products of combustion. It is a very accurate method.

Example 16.3 *Calculate the theoretical air to fuel ratio on molar and mass basis for the following fuels:*

(a) Pure carbon, (b) pure hydrogen (c) petrol ($C_{8.5}H_{18.4}$) (d) Heptane (C_7H_{16}) (e) Methanol (CH_3OH)

Solution

Given The fuels:

(i) Pure carbon (ii) Pure hydrogen
(iii) Petrol ($C_{8.5}H_{18.4}$) (iv) Heptane (C_7H_{16})
(v) Methanol (CH_3OH)

To find

(i) Theoretical air–fuel ratio on molar basis,
(ii) Theoretical air–fuel ratio on mass basis.

Assumptions

(i) The air contains 23% O_2 and 77% N_2 by mass.
(ii) Complete combustion.

Analysis

(i) *Pure carbon*

Combustion reaction of carbon

$$C + O_2 \rightarrow CO_2$$

1 mole of C combines with 1 mole of O_2 to form 1 mole of CO_2. But one mole of O_2 associates with 3.76 moles of nitrogen and 4.76 moles of air. Thus

moles of air with 1 mole O_2

$$= 4.76 \text{ moles of air per mole of } O_2$$

A/F ratio on molar basis

$$= \frac{\text{Number of moles of air}}{\text{Number of moles of carbon}}$$

$$= \frac{4.76}{1} = \mathbf{4.76}$$

Minimum mass of air required for complete combustion of 1 kg of carbon

$$= \frac{100}{23} \times \left(\frac{8}{3} \text{ kg per kg of carbon}\right)$$

$$= \frac{100}{23} \times \frac{8}{3} \times 1 = 11.59 \text{ kg/kg of C}$$

Theoretical air–fuel ratio

$$= \frac{\text{Mass of air}}{\text{Mass of carbon}} = \frac{11.59 \text{ kg}}{1 \text{ kg}} = \mathbf{11.59}$$

(ii) *Pure hydrogen*

Combustion reaction of hydrogen

$$H_2 + \frac{1}{2}O_2 \rightarrow H_2O$$

1 mole of H_2 combines with $\frac{1}{2}$ mole of O_2 to form 1 mole of H_2O. Thus

moles of air = $\frac{1}{2}$ mole of $O_2 \times 4.76$ moles of air

$$= 2.38 \text{ moles of air/mole of } H_2$$

A/F ratio on molar basis

$$= \frac{\text{Number of moles of air}}{\text{Number of moles of hydrogen}}$$

$$= \frac{2.38}{1} = \mathbf{2.38}$$

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Minimum mass of air required for complete combustion of 1 kg of hydrogen

$$= \frac{100}{23} \times (8 \text{ kg per kg of } H_2)$$

$$= \frac{100}{23} \times 8 \times 1 = 34.78 \text{ kg/kg of } H_2$$

Theoretical air–fuel ratio

$$= \frac{\text{Mass of air}}{\text{Mass of hydrogen}} = \frac{34.78 \text{ kg}}{1 \text{ kg}}$$

$$= \mathbf{34.78}$$

(iii) *Petrol ($C_{8.5}\,H_{18.4}$)*

Combustion reaction for petrol

$$C_{8.5}H_{18.4} + 13.1\ O_2 \rightarrow 8.5\ CO_2 + 9.2\ H_2O$$

1 mole of petrol combines with 13.1 moles of O_2 to form 8.5 mole of CO_2 and 9.2 mole of H_2O. Thus

moles of air = 13.1 moles of $O_2 \times 4.76$ moles

= 62.356 moles of air/mole of petrol

A/F ratio on molar basis

$$= \frac{\text{Number of moles of air}}{\text{Number of moles of petrol}}$$

$$= \frac{62.356}{1} = \mathbf{62.356}$$

Mass of 1 mole of petrol ($C_{8.5}H_{18.4}$)

$= 8.5 \times 12 + 18.4 \times 1 = 120.4$ kg

Mass of 62.356 moles of air

$= 62.356 \times 28.97 = 1806.45$ kg

Theoretical air–fuel ratio

$$= \frac{\text{Mass of air}}{\text{Mass of petrol}} = \frac{1806.45 \text{ kg}}{120.4 \text{ kg}}$$

$$= \mathbf{15.00}$$

(iv) *Heptane (C_7H_{16})*

Combustion reaction for heptane

$$C_7H_{16} + 11\ O_2 \rightarrow 7\ CO_2 + 8\ H_2O$$

1 mole of heptane combines with 11 moles of O_2 to form 7 moles of CO_2 and 8 moles of H_2O. Thus,

moles of air = 11 moles of $O_2 \times 4.76$ moles

= 52.36 moles of air/mole of heptane

A/F ratio on molar basis

$$= \frac{\text{Number of moles of air}}{\text{Number of moles of heptane}}$$

$$= \frac{52.36}{1} = \mathbf{52.36}$$

Mass of 1 mole of heptane (C_7H_{16})

$= 7 \times 12 + 16 \times 1 = 100$ kg

Mass of 52.36 moles of air

$= 52.36 \times 28.97 = 1516.87$ kg

Theoretical air–fuel ratio

$$= \frac{\text{Mass of air}}{\text{Mass of Heptane}} = \frac{1516.87 \text{ kg}}{100 \text{ kg}}$$

$$= \mathbf{15.168}$$

(v) *Methanol ($CH_3\,OH$)*

Combustion reaction for methanol

$$CH_3OH + 1.5O_2 \rightarrow CO_2 + 2H_2O$$

1 mole of methanol combines with 1.5 moles of O_2 to form 1 moles of CO_2 and 2 mole of H_2O. Thus,

moles of air = 1.5 moles of $O_2 \times 4.76$ moles

= 7.14 moles of air/mole of methnol

A/F ratio on molar basis

$$= \frac{\text{Number of moles of air}}{\text{Number of moles of methnol}}$$

$$= \frac{7.14}{1} = \mathbf{7.14}$$

Mass of 1 mole of methanol (CH_3OH)

$= 1 \times 12 + 1 \times 16 + 4 \times 1 = 32$ kg

Mass of 7.14 moles of air

$= 7.14 \times 28.97 = 206.845$ kg

Theoretical air–fuel ratio

$$= \frac{\text{Mass of air}}{\text{Mass of methanol}} = \frac{206.845 \text{ kg}}{32 \text{ kg}}$$

$$= \mathbf{6.464}$$

Example 16.4 *Determine the air–fuel ratio and the theoretical amount of air required by mass for complete combustion of a fuel containing 85% of carbon, 8% of hydrogen, 3% of oxygen, 1% of sulphur and the remaining as ash. If 40% of excess air is used, what is the volume of air at 27°C and 1.05 bar pressure? Does this represent per kg of fuel?*

Solution

Given The composition of fuel by mass. For 1 kg of fuel

C = 0.85 kg H_2 = 0.8 kg

O_2 = 0.03 kg S = 0.01 kg

Ash = 0.03 kg

Excess air = 40% T_1 = 27°C = 300 K

p_1 = 1.05 bar = 105 kPa

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To find

(i) Minimum amount of air required,
(ii) Air–fuel ratio by mass, and
(iii) Volume of air required with 40% excess air.

Assumptions

(i) The air contains 23% O_2 and 77% N_2 by mass.
(ii) Complete combustion.
(iii) Specific gas constant for air as 0.287 kJ/kg·K.

Analysis

(i) Minimum mass of air required for complete combustion,

$$m_{th} = \frac{100}{23}\left[\frac{8}{3}\text{C} + 8\left(\text{H} - \frac{\text{O}}{8}\right) + \text{S}\right]$$

$$= \frac{100}{23} \times \left[\frac{8}{3} \times 0.85 + 8 \times \left(0.08 - \frac{0.03}{8}\right) + 0.01\right]$$

$$= \mathbf{12.55\ kg/kg\ of\ fuel}$$

(ii) Air–fuel ratio by mass = **12.55 : 1**

(iii) Since air supplied is 40% in excess, the actual amount of air supplied is equal to

$$m_a = 1.4 \times 12.55 = 17.57 \text{ kg.}$$

The volume occupied by air can be calculated as follows:

$$pV = m_a RT$$

$$105\,V = 17.57 \times 0.287 \times 300 \text{ K}$$

or, $$V = \mathbf{14.407\ m^3}$$

Example 16.5 *The gravimetric analysis of a sample of coal gives 80% of carbon, 12% of H_2 and 8% of ash. Calculate the theoretical air required and analysis of products by volume.*

Solution

Given Gravimetric analysis of a sample of coal

C = 0.8, H_2 = 0.12

ash = 0.08

To find

(i) Theoretical air required, and
(ii) Analysis of products by volume.

Analysis Considering 1 kg coal,

mass of carbon = 0.8 kg, and

mass of hydrogen = 0.12

(i) Theoretical air required for complete combustion

$$= \frac{100}{23}\left[\frac{8}{3}\text{C} + 8\left(\text{H} - \frac{\text{O}}{8}\right) + \text{S}\right]$$

$$= \frac{100}{23} \times \left[\frac{8}{3} \times 0.8 + 8 \times 0.12 + 0.0\right]$$

$$= \mathbf{13.45\ kg/kg\ of\ fuel}$$

(ii) Number of moles of C;

$$n_{\text{C}} = \frac{m}{\mathcal{M}} = \frac{0.8}{12} = 0.067 \text{ mole}$$

Number of moles of hydrogen;

$$n_{\text{H}_2} = \frac{0.12}{1} = 0.12 \text{ mole}$$

Combustion reaction with theoretical air

$$0.067\ \text{C} + 0.12\ \text{H} + x(\text{O}_2 + 3.76\ \text{N}_2) \rightarrow a\,\text{CO}_2 + b\text{H}_2\text{O} + 3.76\text{N}_2$$

The element balance yields to

a = 0.067 mole,

and b = 0.06 mole

and $2x = 2a + b$ or $x = 0.097$ mole

Total moles of products

$$= 0.067 + 0.06 + 3.76 \times 0.097$$

$$= 0.492 \text{ mole}$$

Hence volumetric analysis

$$\text{CO}_2 = \frac{0.067 \times 100}{0.492} = \mathbf{13.62\%}$$

$$\text{H}_2\text{O} = \frac{0.06 \times 100}{0.492} = \mathbf{12.2\%}$$

$$\text{N}_2 = \frac{3.76 \times 0.097 \times 100}{0.492} = \mathbf{74.13\%}$$

Example 16.6 *A producer gas has the following percentage composition by volume, H_2 =15%, CH_4 = 2%, CO = 20%, CO_2 = 6%, O_2 = 3% and N_2 = 54%. If 50% excess air is supplied for combustion, determine the volume of air supplied per m^3 of gas and volumetric analysis of the combustion products.*

Solution

Given A producer gas with following percentage composition

H_2 = 15% CH_4 = 2%

CO = 20% CO_2 = 6%

N_2 = 54% O_2 = 3%

Excess air = 50%

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To find

(i) Volume of air supplied per m^3 of producer gas.

(ii) Analysis of dry products of combustion by volume.

Analysis Minimum volume of air required, Eq. (16.18)

$$V_{th} = \frac{100}{21} \times [0.5\,CO + 0.5\,H_2 + 2\,CH_4 + 3\,C_2H_4 - O_2]\ (m^3)$$

$$= \frac{100}{21} \times [(0.5 \times 0.2 + 0.5 \times 0.15 + 2 \times 0.02) - 0.03]\ (m^3)$$

$$= 0.88095\ m^3$$

Actual volume of air supplied

$$V_{act} = 1.5 \times 0.88095 = \mathbf{1.3214\ m^3}$$

Excess air supplied

$$= V_{act} - V_{th} = 1.3214 - 0.88095$$

$$= 0.44045\ m^3$$

The constituents of flue gases are CO_2, O_2, and N_2

Excess oxygen supplied

$$= \frac{21}{100} \times 0.44045 = 0.0925\ m^3$$

Volume of O_2 in flue gases = 0.0925 m^3

Volume of N_2 in fuel = 0.54 m^3

Volume of N_2 in air supplied

$$= \frac{79}{100} \times \text{Volume of actual air supplied}$$

$$= \frac{79}{100} \times 1.3214 = 1.044\ m^3$$

Total volume of N_2 in flue gases

$$= 0.54 + 1.044 = 1.584\ m^3$$

Volume of CO_2 in the fuel = 0.06 m^3

Volume of CO_2 produced by combustion of 0.2 m^3 of CO = 0.2 m^3

Volume of CO_2 produced by combustion of 0.02 m^3 of CH_4 = 0.02 m^3

Total volume of CO_2 in flue gases

$$= 0.06 + 0.2 + 0.02 = 0.28\ m^3$$

Total volume of flue gases

= Volume of O_2 + volume of N_2 + volume of CO_2

$$= 0.0925 + 1.584 + 0.28 = 1.9565\ m^3$$

The analysis of dry products by volume

% volume of element A

$$= \frac{\text{Volume of element A}}{\text{Total volume of flue gases}} \times 100$$

$$CO_2 = \frac{0.28}{1.9565} \times 100 = 14.31\%$$

$$O_2 = \frac{0.0925}{1.9565} \times 100 = 4.73\%$$

$$N_2 = \frac{1.584}{1.9565} \times 100 = \mathbf{80.96\%}$$

Example 16.7 *A gas engine is supplied with natural gas of the following composition. CH_4 = 93%, C_2H_6 = 3%, N_2 = 3%, CO = 1%. If the A/F ratio is 30 by volume, calculate the analysis of the dry products of combustion. It can be assumed that the stoichiometric A/F ratio is less than 30.*

Solution

Given A natural gas with composition

CH_4 = 93% C_2H_6 = 3% N_2 = 3%

CO = 1% A/F = 30 by volume

To find Analysis of dry products of combustion by volume.

Analysis It is mentioned that the stoichiometric air–fuel ratio is less than 30. Thus the excess air is supplied with fuel. The combustion equation

$$0.93\,CH_4 + 0.03\,C_2H_6 + 0.01\,CO + 0.03\,N_2 + 30(0.21\,O_2 + 0.79\,N_2) \rightarrow a\,CO_2 + b\,H_2O + c\,O_2 + d\,N_2$$

The element balance yields

C: $a = 0.93 + 0.03 \times 2 + 0.01$ or $a = 1$

H_2: $2b = 0.93 \times 4 + 0.03 \times 6$,

or $b = 1.95$

O_2: $2a + b + 2c = 0.01 + 30 \times 0.21$

or $c = 1.18$

N_2: $2d = 0.03 \times 2 + 30 \times 0.79$

or $d = 23.73$

Total moles of dry products are 1 + 1.18 + 23.73

$$= 25.91\ \text{kmol}$$

The analysis of dry products by volume

$$CO_2 = \frac{1}{25.91} \times 100 = 3.86\%$$

$$O_2 = \frac{1.18}{25.91} \times 100 = 4.55\%$$

$$N_2 = \frac{23.73}{25.91} \times 100 = \mathbf{91.56\%}$$

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Example 16.8 *A sample of dry anthracite has the following composition by mass:*

C = 90% *H = 3%* *O = 2.5%*
N = 1% *S = 0.5%* *ash = 3%*

Calculate

(a) Stoichiometric air–fuel ratio,

(b) the actual air–fuel ratio and dry and wet analysis of products of combustion by mass and by volume when 20% excess air is supplied.

Solution

Given A sample of dry anthracite with mass composition

C = 90% H = 3% O = 2.5%
N = 1% S = 0.5% ash = 3%

To find

(i) Stoichiometric air–fuel ratio, and

(ii) Actual air–fuel ratio, dry and wet analysis of products of combustion by mass and volume when 20% excess air is supplied.

Assumptions

(i) Each mole of O_2 is accompanied by 3.347 kg N_2.

(ii) Nitrogen is an inert gas.

(iii) Combustion is complete.

Analysis

(i) For complete combustion of 1 kg of anthracite, the chemical equations are the following:

Carbon $C + O_2 \rightarrow CO_2$

$$1 \text{ kg C} + \frac{8}{3} \text{ kg } O_2 \rightarrow \frac{11}{3} \text{ kg } CO_2$$

$$0.9 \text{ kg C require } 0.9 \times \frac{8}{3} \ O_2 = 2.4 \text{ kg } O_2$$

$$\text{Carbon dioxide produced} = 0.9 \times \frac{11}{3} = 3.3 \text{ kg}$$

Hydrogen $H_2 + \frac{1}{2} O_2 \rightarrow H_2O$

$$1 \text{ kg } H_2 + 8 \text{ kg } O_2 \rightarrow 9 \text{ kg } H_2O$$

O_2 required for 0.03 kg of H_2 $= 0.03 \times 8 = 0.24$ kg

H_2O produced $= 0.03 \times 9 = 0.27$ kg

Sulphur $S + O_2 \rightarrow SO_2$

$$1 \text{ kg S} + 1 \text{ kg } O_2 \rightarrow 2 \text{ kg } SO_2$$

O_2 required by 0.005 kg of S $= 0.005 \times 1 = 0.005$ kg

SO_2 produced $= 0.005 \times 2 = 0.01$ kg

The total O_2 required $= 2.4 + 0.24 + 0.005 = 2.645$ kg

O_2 already present with fuel = 0.025 kg

Thus, net O_2 required for combustion $= 2.645 - 0.025 = 2.62$ kg

N_2 required $= 3.347 \times 2.62$ kg $O_2 = 8.77$ kg

Theoretical mass of air required $= 2.62 + 8.77 = 11.39$ kg

Stoichiometric A/F ratio

$$= \frac{\text{Mass of air}}{\text{Mass of fuel}} = \frac{11.39 \text{ kg}}{1 \text{ kg}}$$

$$= \mathbf{11.39}$$

(ii) Actual air–fuel ratio with 20% excess air

$= 1.2 \times$ stoichiometric A/F

$= 1.2 \times 11.39 = 13.67$

O_2 supplied $= 1.2 \times 2.62$ kg $= 3.144$ kg

Excess O_2 supplied $= 3.144 - 2.62 = 0.524$ kg

N_2 supplied $= 1.2 \times 8.77 = 10.524$ kg

Table 16.8.1 on next page shows dry and wet analysis of product of combustion.

Example 16.9 *Determine the air–fuel ratio on both mass and molar basis for the complete combustion of octane (C_8H_{18}), with (a) theoretical amount of air, (b) 150% theoretical air.*

Solution

Given Octane C_8H_{18} burns completely with

(i) theoretical

(ii) excess air.

To find Air–fuel ratio.

Assumptions

(i) Each mole of oxygen is accompanied with 3.76 moles of nitrogen.

(ii) Nitrogen is an inert gas.

(iii) Complete combustion of octane.

Analysis

(i) The chemical reaction for complete combustion

$C_8H_{18} + a\ O_2 \rightarrow b\ CO_2 + c\ H_2O$

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Table 16.8.1 *Dry and wet analysis of products of combustion by mass*

Products	*Mass m_i kg/kg*	*Wet analysis* $\frac{m_i}{m_1}$	*Dry analysis* $\frac{m_i}{m_2}$
CO_2	3.30	22.559%	22.98%
H_2O	0.27	1.846%	—
SO_2	0.01	0.068%	0.07%
O_2	0.524	3.582%	3.65%
N_2	10.524	71.944%	73.30%
	$m_1 = \sum_{\text{wet}} m_i = 14.628$ kg	100.00%	100.00%
	$m_2 = \sum_{\text{Dry}} m_i = 14.358$ kg		

Table 16.8.2 *Dry and wet analysis of products of combustion by volume*

Products	*Mass m_i kg/kg*	*Mol. weight $\mathcal{M}_i$ kg/kmol*	*Mole* $n_i = \frac{m_i}{\mathcal{M}_i}$	*Wet Analysis* $\frac{n_i}{n_2}$	*Dry Analysis* $\frac{n_i}{n_2}$
CO_2	3.3	44	0.075	15.55%	16.05%
H_2O	0.27	18	0.015	3.11%	–
SO_2	0.01	64	0.00015	0.032%	0.033%
O_2	0.524	32	0.0163	3.38%	3.50%
N_2	10.524	28	0.376	77.96%	80.43%
			$n_1 = \sum_{\text{Wet}} n_i = 0.4823$	100.00%	100.00%
			$n_2 = \sum_{\text{Dry}} n_i = 0.4673$		

Using mass conservation, we get

$a = 12.5$, $b = 8$, and $c = 9$, and

Thus, the chemical equation is

$$C_8H_{18} + 12.5\ O_2 \rightarrow 8\ CO_2 + 9\ H_2O$$

On the molar basis

1 kmol C_8H_{18} + (12.5 × 4.76) kmol of air →

8 kmol CO_2 + 9 kmol H_2O

The moles of air required,

$n_a = 12.5 \times 4.76 = 59.5$ kmol/ kmol of fuel

$$\overline{\text{A/F}} = \frac{\text{Number of kmol of air}}{\text{Number of kmol of fuel}} = \frac{59.5}{1}$$

$= \mathbf{59.5}$

On the mass basis, using, $m_i = n_i \mathcal{M}_i$

The mass of fuel,

$m_f = n_f \mathcal{M}_f = 1 \times 114$

$= 114$ kg

The mass of air,

$m_a = n_a \mathcal{M}_a$

$= 59.5$ kmol × 28.97 kg/kmol

$= 1723.715$ kg

The air–fuel ratio on the mass basis

$$\text{A/F} = \frac{\text{Mass of air}}{\text{Mass of fuel}} = \frac{1723.715}{1\ 14} = \mathbf{15.12}$$

(ii) 150% theoretical air

Actual moles of air required

= 1.5 × theoretical moles

= 1.5 × 59.5 = 89.5 kmol/ kmol of fuel

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Thus, $\overline{A/F}$ ratio on molar basis is **89.25.**

Similarly, on mass basis A/F

$= 1.5 \times 15.12 =$ **22.68**

Example 16.10 *A steam boiler uses pulverised coal in the furnace. The ultimate analysis of coal by mass is given as*

C = 78%	*H_2 = 3%*
O_2 = 3%,	*S = 1%*
Ash = 10%	*moisture = 5%*

Excess air supplied is 30%. Calculate the mass of air to be supplied and mass of products of combustion per kg of coal burnt.

Solution

Given The ultimate analysis of coal

C = 78%	H_2 = 3%
O_2 = 3%	S = 1%
Ash = 10%	moisture = 5%

Excess air supplied = 30% of theoretical air

To find

(i) Mass of air supplied for combustion

(ii) Mass of products of combustion per kg of fuel burnt

Analysis

(i) *Mass of air supplied for combustion*

The theoretical amount of air required for complete combustion of 1 kg of fuel

$$m_{th} = \frac{100}{23}\left[\frac{8}{3}\text{C} + 8\left(\text{H} - \frac{\text{O}}{8}\right) + \text{S}\right] \text{ kg/kg of fuel}$$

$$= \frac{100}{23}\left[\frac{8}{3} \times 0.78 + 8 \times \left(0.03 - \frac{0.03}{8}\right) + 0.01\right]$$

$= 13.435$ kg/kg of fuel

Excess amount of air supplied,

$$m_{ex} = 0.3 \times m_{th} = 0.3 \times 13.435$$
$$= 4.03 \text{ kg/kg of fuel}$$

Total amount of air supplied for combustion of fuel;

$$m_{total} = m_{th} + m_{ex} = 13.435 + 4.03$$
$$= \textbf{17.465 kg/kg of fuel}$$

(ii) *The mass of products of combustion*

Mass of CO_2 formed

$$= \frac{11}{3} \text{ kg per kg of C} \times 0.78 \text{ kg}$$
$$= \textbf{2.86 kg/kg of fuel}$$

Mass of H_2O formed

$= 9$ kg per kg of $H_2 = 9 \times 0.03$

$=$ **0.27 kg/kg of fuel**

Mass of SO_2 formed

$= 2$ kg per kg of $S = 2 \times 0.01$

$=$ **0.02 kg/kg of fuel**

Mass of N_2 associated with air

$= 0.77 \times m_{total} = 0.77 \times 17.465$

$=$ **13.448 kg/kg of fuel**

The mass of oxygen with combustion products

= Mass of oxygen with excess air – mass of air in fuel

$= 0.23 \times m_{ex} -$ 3% in fuel on mass basis

$= 0.23 \times 4.03 - 0.03$

$=$ **0.897 kg/kg of fuel**

Example 16.11 *The gravimetric analysis of coal gives 80% of carbon, 8% of hydrogen, 4% of moisture and 8% of ash. Actual air supplied is 18 kg per kg of coal. Calculate the theoretical amount of air required, if 80% of carbon is burned to CO_2 and the remaining to CO. Also determine the volumetric composition of dry products of combustion.*

Solution

Given The gravimetric analysis of coal as

C = 80%	H_2 = 8%	moisture = 4%
Ash = 8%	m_a = 18 kg/kg of coal	

To find

(i) Theoretical air required

(ii) Volumetric analysis of dry combustion products

Analysis Since 80% of carbon burns to CO_2, the mass of carbon burns to

$$CO_2 = 0.8 \times 0.8 = 0.64 \text{ kg}$$

Remainder carbon burns to

$$CO = 0.8 - 0.64 = 0.16 \text{ kg}$$

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The minimum oxygen required for formation of CO_2 and H_2O

$$= \frac{8}{3}C + 8H$$

$$= \frac{8}{3} \times 0.64 + 8 \times 0.08 = 2.346 \text{ kg}$$

The oxygen required for formation of CO

$$= \frac{4}{3} \times 0.16 = 0.213 \text{ kg}$$

Total oxygen required

$$= 2.346 + 0.213 = 2.56 \text{ kg}$$

The minimum mass of air required

$$= \frac{100}{23} \times 2.56 = 11.13 \text{ kg/kg of coal}$$

A/F ratio = **11.13:1**

The dry products of combustion per kg of coal burnt

$$\text{Mass of } CO_2 \text{ formed} = \frac{11}{3} \text{ kg per kg of C} \times 0.64$$

$$= 2.3467 \text{ kg}$$

$$\text{Mass of CO formed} = \frac{7}{3} \text{ kg per kg of C} \times 0.16$$

$$= 0.3733 \text{ kg}$$

$$\text{Mass of } N_2 \text{ with air} = 0.77 \times m_{total}$$

$$= 0.77 \times (18 \text{ kg air})$$

$$= 13.86 \text{ kg}$$

Mass of O_2 supplied with excess air

$$= O_2 \text{ in actual air} - O_2 \text{ used in combustion}$$

$$= 0.23 \times 18 - 2.56 = 1.58 \text{ kg}$$

The volumetric analysis of dry products of combustion is given in the table below.

Products	*Mass* m_i *kg*	*Molecular mass* $\mathcal{M}_i$	*No. of mole* $n_i = \frac{m_i}{\mathcal{M}_i}$	*% volume* $\frac{n_i}{\Sigma n_i} \times 100$
CO_2	2.3467	44	0.0533	8.728%
CO	0.3733	28	0.0133	2.181%
O_2	1.5800	32	0.0493	8.08%
N_2	13.8600	28	0.495	81.0%
Total			$\Sigma n_i = 0.611$	100%

Example 16.12 *If ethyl alcohol (C_2H_5OH) is burned in a petrol engine, calculate*

(a) the stoichiometric air fuel ratio.

(b) the A/F ratio, the wet and dry analysis by volume of exhaust gas for a mixture strength of 90%.

(c) the A/F ratio, wet and dry analysis by volume of the exhaust gas for a mixture strength of 120%.

Solution

Given The burning of ethyl alcohol (C_2H_5OH)

To find

(i) Stoichiometric air–fuel ratio,

(ii) A/F ratio, the dry and wet analysis for products of combustion for mixture strength of 90%,

(iii) A/F, ratio, the dry and wet analysis by volume of products of combustion for mixture strength of 120%.

Analysis

(i) *The combustion reaction with minimum air*

Combustion reaction for methanol;

$$C_2H_5OH + 3\ O_2 \rightarrow 2CO_2 + 3\ H_2O$$

1 mole of ethyl alcohol combines with 3 moles O_2 to form 2 moles of CO_2 and 3 moles of H_2O. Thus,

moles of air = 3 moles of $O_2 \times 4.76$ moles

= 14.28 mole/mole of ethanol

Mass of 1 mole of ethnol (C_2H_5OH)

$$= 2 \times 12 + 1 \times 16 + 6 \times 1 = 46 \text{ kg}$$

Mass of 14.28 moles of air

$$= 14.28 \times 28.97 = 413.69 \text{ kg}$$

Stoichiometric A/F ratio

$$= \frac{\text{Mass of air}}{\text{Mass of ethanol}} = \frac{413.69 \text{ kg}}{46 \text{ kg}}$$

$$= \mathbf{8.99}$$

(ii) When mixture strength becomes 90%, the actual air fuel ratio $= \frac{8.99}{0.9} = \mathbf{9.99}$

It means the amount of air supplied is $\frac{1}{0.9}$ times that of stoichiometric air. Then the combustion reaction,

$$C_2H_5OH + \frac{1}{0.9}[3O_2 + 3 \times 3.76\ N_2] \rightarrow 2CO_2 + 3H_2O + 0.33O_2 + 12.53\ N_2$$

The products of combustion are

2 mole CO_2 + 3 mole H_2O + 0.33 mole O_2 + 12.53 mole N_2

Total moles of products = 17.86 mole

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Wet analysis by volume

$$CO_2 = \frac{2}{17.86} \times 100 = \mathbf{11.19\%}$$

$$H_2O = \frac{3}{17.86} \times 100 = \mathbf{16.79\%}$$

$$O_2 = \frac{0.33}{17.86} \times 100 = \mathbf{1.85\%}$$

$$N_2 = \frac{12.53}{17.86} \times 100 = \mathbf{70.15\%}$$

For dry products analysis by volume

Total moles of dry products = 2 + 0.33 + 12.53
= 14.86 moles

$$CO_2 = \frac{2}{14.86} \times 100 = \mathbf{13.46\%}$$

$$O_2 = \frac{0.33}{14.86} \times 100 = \mathbf{2}$$

$$N_2 = \frac{12.53}{14.86} \times 100 = \mathbf{84.32\%}$$

(iii) For mixture strength of 120%.

actual air–fuel ratio = $\frac{8.99}{1.2} = \mathbf{7.49}$

Then combustion reaction is

$$C_2H_5OH + \frac{1}{1.2}[3O_2 + 3 \times 3.76\ N_2] \rightarrow a\,CO_2 + b\,CO + 3H_2O + 9.4\ N_2$$

The element balance on two sides of reaction

C: $a + b = 2$

O_2: $2a + b + 3 = 1 + \frac{3}{1.2} \times 2$

or $2a + b = 3$

Solution yields to

$a = 1$ and $b = 1$

Therefore, the products of combustion are

1 kmol CO_2 + 1 kmol CO + 3 kmol H_2O + 9.4 kmol N_2

Total moles = 14.4 kmol

Wet analysis by volume

$$CO_2 = \frac{1}{14.4} \times 100 = \mathbf{6.94\%}$$

$$CO = \frac{1}{14.4} \times 100 = \mathbf{6.94\%}$$

$$H_2O = \frac{3}{14.4} \times 100 = \mathbf{20.83\%}$$

$$N_2 = \frac{9.4}{14.4} \times 100 = \mathbf{65.29\%}$$

Total dry moles of products = 1 + 1 + 9.4
= 11.4 kmol

Dry products analysis by volume

$$CO_2 = \frac{1}{11.4} \times 100 = \mathbf{8.77\%}$$

$$CO = \frac{1}{11.4} \times 100 = \mathbf{8.77\%}$$

$$N_2 = \frac{9.4}{11.4} \times 100 = \mathbf{82.46\%}$$

Example 16.13 *A fuel having a chemical formula $C_{12}H_{26}$ is burnt with 50% excess air. Calculate the stoichiometric air required and percentage analysis of products of combustion including water vapour.*

Solution

Given A fuel with chemical formula $C_{12}H_{26}$

Excess air = 50%

To find

(i) Mass of stoichiometric air required,

(ii) Analysis of products of combustion.

Analysis

(i) For complete combustion of fuel with chemical formula $C_{12}H_{26}$, the chemical reaction is

$$C_{12}H_{26} + 18.5\ (O_2 + 3.\ 76\ N_2) \rightarrow 12\ CO_2 + 13\ H_2O + 69.56\ N_2$$

A minimum of 18.5 moles of O_2 is required for complete combustion

The mass of O_2 required
= 18.5 × 32 = 592 kg/kmol of fuel

Mass of minimum air required

$$= \frac{100}{23} \times 592 = 2574\ \text{kg/kmol}$$

Mass of one mole of fuel
= 12 × 12 + 26 × 1 = 170 kg/kmol

$$\text{A/F ratio} = \frac{\text{Mass of air per mole of fuel}}{\text{Mass of 1 mole of fuel}}$$

$$= \frac{2574}{170} = \mathbf{15.14}$$

(ii) When 50% excess air is supplied;

Mass of actual air supplied
= 15.14 × 1.5 = 22.71 kg/kmol

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Excess O_2 supplied
$= 18.5 \times 0.5 = 9.25$ kg/kmol of fuel
Actual O_2 supplied
$= 18.5 \times 1.5 = 27.75$ kg/kmol of fuel
Actual N_2 supplied
$= 3.76 \times 18.5 \times 1.5$
$= 104.34$ kg/kmol of fuel
Therefore, actual chemical reaction

$$C_{12}H_{26} + 27.75\,(O_2 + 3.76\,N_2) \rightarrow 12CO_2 + 13H_2O + 9.25\,O_2 + 104.34\,N_2$$

Products of combustion

Mass of CO_2 produced per kg of fuel

$$= \frac{\text{Mass of 12 moles of } CO_2}{\text{Mass of 1 mole of fuel}}$$

$$= \frac{12 \times 44}{170} = 3.1 \text{ kg/kg of fuel}$$

Mass of H_2O produced

$$= \frac{13 \times 18}{170} = 1.376 \text{ kg/kg of fuel}$$

Mass of excess O_2

$$= \frac{9.25 \times 32}{170} = 1.741 \text{ kg/kg of fuel}$$

Mass of N_2 accompanied with air

$$= \frac{104.34 \times 28}{170}$$

$= 17.185$ kg/kg of fuel

Percentage analysis of these products

Constituent	*Products* m_i	*Mass fraction* $= \frac{m_i}{\Sigma m_i} \times 100$
CO_2	3.1	13.25%
H_2O	1.376	5.88%
O_2	1.741	7.44%
N_2	17.185	73.43%
Total	$\Sigma m_i = 23.04$	100%

Percentage excess air supplied

$$= \frac{\text{Excess mass of air}}{\text{Theoretical mass of air}} \times 100 = \frac{5.94}{14.29} \times 100$$

$$= \frac{5.94}{14.29} \times 100 = \mathbf{41.17\%}$$

Example 16.14 *The dry exhaust gas from an oil engine has the following composition by volume:*

$CO_2 = 8.85\%$,
$CO = 1.2\%$,
$O_2 = 6.8\%$ *and* $N_2 = 83.15\%$

The fuel oil has a percentage composition by mass as $C = 84\%$ $H_2 = 14\%$ *and* $O_2 = 2\%$

Determine

(a) Mass of carbon per kg of dry flue gases

(b) The air–fuel ratio by mass

Solution

Given The composition of fuel oil by mass. For 1 kg of fuel

$C = 0.84$ kg $H_2 = 0.14$ kg $O_2 = 0.02$ kg

Composition of flue gases by volume:

$CO_2 = 8.85\%$ $CO = 1.2\%$ $O_2 = 6.8\%$
$N_2 = 83.15\%$

To find

(i) Mass of carbon per kg of dry flue gases,
(ii) The air–fuel ratio, and
(iii) Mass of excess air supplied.

Analysis

(i) *Mass of carbon supllied per kg of dry flue gases:* The conversion of volumetric analysis of exhaust gases in gravimetric analysis is tabulated below:

Constituent	*% Volume* n_i	*Molecular weight* $\mathcal{M}_i$	*Mass* $m_i = n_i \mathcal{M}_i$	*Mass fraction in flue gases* $= \frac{m_i}{\Sigma m_i}$
CO_2	0.0885	44	3.894	0.1311
CO	0.0120	28	0.336	0.0113
O_2	0.0680	32	2.176	0.0733
N_2	0.8315	28	23.282	0.7842
			$\Sigma m_i = 29.688$	1 kg

The mass of carbon in exhaust gases per kg of flue gases

$$= \frac{3}{11} \text{ kg } CO_2 + \frac{3}{7} \text{ kg CO}$$

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$$= \frac{3}{11} \times 0.1311 + \frac{3}{7} \times 0.0113$$

$$= \mathbf{0.0406\ kg/kg\ of\ dry\ flue\ gases}$$

(ii) *Air–fuel ratio by mass*

The mass of dry flue gases formed per kg of fuel

$$= \frac{\text{Mass of carbon in 1 kg of fuel}}{\text{Mass of Carbon in 1 kg of flue gases}}$$

$$= \frac{0.84\ \text{kg}}{0.0406\ \text{kg}} = 20.69\ \text{kg/kg of flue gases}$$

The mass of vapour (H_2O) formed from H_2 present in fuel

$$= 9\ H = 9 \times 0.14 = 1.26\ \text{kg}$$

Total mass of flue gases

= mass of dry flue gases + mass of vapour

$$= 20.69 + 1.26 = 21.95\ \text{kg/kg of flue gases}$$

Actual air supplied per kg of fuel for its combustion

= Total mass of flue gases – 1

$$= 21.95 - 1 = 20.95\ \text{kg}$$

$$\text{Air–fuel ratio} = \frac{\text{Mass of air uesd}}{\text{Mass of fuel}} = \frac{20.95\ \text{kg}}{1\ \text{kg}}$$

$$= \mathbf{20.95}$$

Example 16.15 *A fuel oil has mass composition as*

C = 85% H_2 = 13% O_2 = 2%

The dry exhaust gases have the following volumetric composition:

CO_2 = 9%, CO = 1.5%,

O_2 = 7%, N_2 = 82.5%

Determine the mass of air supplied per kg of fuel and percentage of excess air supplied.

Solution

Given The composition of fuel oil by mass. For 1 kg of fuel

C = 0.85 kg H_2 = 0.13 kg O_2 = 0.02 kg

Composition of flue gases by volume:

CO_2 = 9% CO = 1.5% O_2 = 7%

N_2 = 82.5%

To find

(i) Mass of air supplied per kg of fuel, and

(ii) The percentage of excess air supplied.

Analysis

(i) *Mass of air supllied per kg of fuel*

The conversion of volumetric analysis of exhaust gases in gravimetric analysis is tabualted below:

Constituent n_i	*% Volume weight*	*Molecular $\mathcal{M}_i$*	*Mass $m_i = n_i\mathcal{M}_i$*	*Mass fraction in flue gases $= \frac{m_i}{\Sigma m_i}$*
CO_2	0.090	44	3.960	0.1332
CO	0.015	28	0.42	0.0141
O_2	0.070	32	2.24	0.0753
N_2	0.825	28	23.1	0.7772
			$\Sigma m_i = 29.72$	1 kg

The mass of carbon in exhaust gases per kg of flue gases

$$= \frac{3}{11}\ \text{kg}\ CO_2 + \frac{3}{7}\ \text{kg CO}$$

$$= \frac{3}{11} \times 0.1332 + \frac{3}{7} \times 0.0141$$

$$= 0.04237\ \text{kg/kg of dry flue gases}$$

The mass of dry flue gases formed per kg of fuel

$$= \frac{\text{Mass of carbon in 1 kg of fuel}}{\text{Mass of carbon in 1 kg of flue gases}}$$

$$= \frac{0.85\ \text{kg}}{0.04237\ \text{kg}} = 20.06\ \text{kg/kg of flue gases}$$

The mass of vapour (H_2O) formed from H_2 present in fuel

$$= 9\ H = 9 \times 0.13 = 1.17\ \text{kg}$$

Total mass of flue gases

= mass of dry flue gases + mass of vapour

$$= 20.06 + 1.17 = 21.23\ \text{kg/kg of flue gases}$$

Actual air supplied per kg of fuel for its combustion

= Total mass of flue gases – 1

$$= 21.23 - 1 = \mathbf{20.23\ kg}$$

(ii) *Percentage of excess air supplied*

Theoretical mass of air required for complete combustion

$$m_a = \frac{100}{23}\left[\frac{8}{3}C + 8\left(H - \frac{O}{8}\right) + S\right]$$

$$= \frac{100}{23} \times \left[\frac{8}{3} \times 0.85 + 8 \times \left(0.13 - \frac{0.02}{8}\right) + 0.0\right]$$

$$= \mathbf{14.29\ kg/kg\ of\ fuel}$$

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Excess air supplied per kg of fuel,

= Actual air – Theoretical air

= 20.23 – 14.29 = 5.94 kg/kg of fuel

Percentage excess air supplied

$$= \frac{\text{Mass of excess air}}{\text{Theoretical mass of air}} \times 100 = \frac{5.94 \text{ kg}}{14.29 \text{ kg}} \times 100$$

= **41.56%**

Example 16.16 *The coal gas supplied to a gas engine has the following composition:*

H_2 = 50.5% CO = 10% CH_4 = 26%

C_4H_8 = 4% O_2 = 0.4% CO_2 = 3%

N_2 = 6%

The fuel burns with some excess air and its air–fuel ratio by volume is worked out to be 7. Calculate the analysis of dry products of combustion by volume.

Solution

Given The volumetric analysis of coal gas:

H_2 = 50.5% CO = 10% CH_4 = 26%

C_4H_8 = 4% O_2 = 0.4% CO_2 = 3%

N_2 = 6%

Air–fuel ratio with excess air = 7

To find The volumetric analysis of dry exhaust gases

Assumption 1 mole of air consists of 0.21 mole of O_2 and 0.79 mole of N_2 by volume.

Analysis The chemical reaction for coal gas

$$0.506\ H_2 + 0.1\ CO + 0.26\ CH_4 + 0.04\ C_4H_8 + 0.004\ O_2 + 0.03\ CO_2 + 0.06\ N_2 + 7 \times [0.21 O_2 + 0.79\ N_2] \rightarrow a\ CO_2 + b\ H_2O + c\ O_2 + d\ N_2$$

The values of *a, b c,* and *d* are to be worked out by balancing the elements of fuel. The element balance yields

C $0.1 + 0.26 + 0.04 \times 4 + 0.03 = a$ or $a = 0.55$

H_2 $0.506 + 0.26 \times 4 + 0.04 \times 8 = 2b$ or $b = 1.186$

O_2 $0.1 + 0.004 \times 2 + 0.03 \times 2 + 7 \times 0.21 \times 2 = 2c$

or $c = 0.411$

N_2 $0.06 \times 2 + 7 \times 0.79 \times 2 = 2d$ or $d = 5.59$

Summation of moles of dry products (excluding moles of H_2O)

$$\Sigma DP = a + c + d$$

$$= 0.55 + 0.411 + 5.59 = 6.55$$

The complete volumetric analysis of dry products

Constituent (a)	*Number of moles (n_i)*	*% volume of constituent* $= \frac{n_i}{\Sigma n_i}$
CO_2	0.55	8.39%
O_2	0.411	6.27%
N_2	5.59	85.34%
Total	$\Sigma n_i = 29.688$	100%

Example 16.17 *A fuel has ultimate analysis as 88% of carbon, 4.4% of hydrogen, and the remaining being incombustible.*

A partial volumetric analysis of dry products of combustion shows 13.2% of CO_2, 3.2% of O_2 and it is suspected that some CO is also present.

Calculate

(a) Complete volumetric analysis of dry exhaust gases

(b) Air to fuel ratio by mass

Assume all carbon and hydrogen is burned.

Solution

Given The ultimate analysis for 100 kg of fuel

C = 88 kg H_2 = 4.4 kg and remaining is incombustible

Composition of flue gases by volume:

CO_2 = 13.2% O_2 = 3.2%

To find

(i) Complete volumetric analysis of dry exhaust gases

(ii) Air–fuel ratio by mass

Analysis

(i) *Complete volumetric analysis of dry exhaust gases*

For 100 kg of fuel, there are $\frac{88}{12}$ moles of carbon and $\frac{4.4}{2}$ moles of H_2. The chemical reaction for the fuel is

$$\frac{88}{12} C + \frac{4.4}{2} H_2 + x(O_2 + 3.76\ N_2) \rightarrow a\ CO_2 + b\ CO + 2.2\ H_2O + c\ O_2 + 3.76x\ N_2$$

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The values of *a, b c,* and *x* are to be worked out by balancing the elements of fuel.

The element balance yields

C $\quad \frac{88}{12} = a + b \quad$ or $\quad b = 7.33 - a$

$O_2 \quad 2x = 2a + b + 2.2 + 2c$

or $\quad c = x - 0.5a - 4.765$

Summation of moles of dry products

$$\Sigma DP = a + b + c + 3.76\,x$$

The volume fraction of CO_2 is 0.132 and O_2 is 0.032. Thus,

$$0.132 = \frac{a}{\Sigma DP}$$

and $$0.032 = \frac{c}{\Sigma DP}$$

or $$a + b + c + 3.76\,x = \frac{a}{0.132}$$

and $$a + b + c + 3.76\,x = \frac{c}{0.032}$$

The solution of these two algebraic equations leads to

$a = 5.52$

$b = 1.81$

$c = 1.3$

$x = 8.82$

The moles of N_2, $\quad n_{N_2} = 3.76; \quad x = 33.16$

It results into the following chemical reaction:

$$7.33\ C + 2.2\ H_2 + 8.82\ (O_2 + 3.76\ N_2) \rightarrow$$
$$5.52\ CO_2 + 1.81CO + 2.2\ H_2O + 1.3\ O_2$$
$$+ 33.16\ N_2$$

The complete volumetric analysis

Constituent (a)	*Number of moles (n_i)*	*% Volume of constituent* $= \frac{n_i}{\Sigma n_i}$
CO_2	5.52	13.21
CO	1.81	4.33
O_2	1.30	3.11
N_2	33.16	79.35
Total	$\Sigma n_i = 29.688$	100

(ii) *Air–fuel ratio*

For 100 kg of fuel, mass of air supplied

$$= \frac{100}{23} \times (\text{Moles of } O_2 \times \text{Mol. mass of } O_2)$$

$$= \frac{100}{23} \times (8.82 \times 32) = 1227 \text{ kg}$$

The air–fuel ratio by mass

$$= \frac{\text{Mass of air uesd}}{\text{Mass of fuel}} = \frac{1227 \text{ kg}}{100 \text{ kg}}$$

$$= \mathbf{12.27}$$

16.13 FLUE GAS ANALYSIS—ORSAT APPARATUS

16.13.1 Principle

The Orsat analysis technique is a quick, simple and inexpensive method for volumetric analysis of the combustion products. An Orsat gas analyzer uses a chemical absorption technique to determine mole fractions of CO_2, CO and O_2 in an exhaust gas. Since it cannot measure H_2O content, the exhaust gas sample is cooled to atmospheric temperature, so most of the water vapour in the combustion products gets condensed out. Therefore, the Orsat analysis is treated as an analysis of the dry combustion products. The Orsat apparatus also does not detect unburnt hydrocarbons and free hydrogen in the products.

16.13.2 Construction

The Orsat apparatus is shown in Fig. 16.2. It consists of a burette, levelling bottle and three pipettes to absorb CO_2, O_2 and CO. These pipettes are interconnected by means of a manifold fitted with cocks. Each pipette is also fitted with a number of small glass tubes (not shown in Fig. 16.2) to increase the surface area within the pipette. The measuring burette is surrounded by a water jacket to keep the temperature and density of the gas constant. The first pipette contains *caustic soda* (*KOH*), which absorbs *carbon dioxide*, the second pipette contains *pyrogallic acid* to absorb *oxygen* content of the gas and third one contains *cuprous chloride* to absorb *carbon monoxide*. Further, the apparatus is also equipped with a three-way cock to connect the apparatus either to gases or to the atmosphere.

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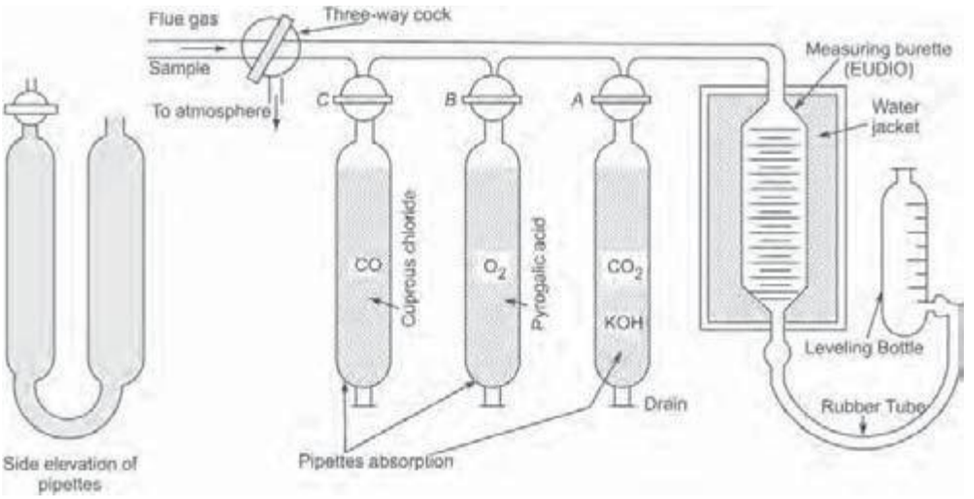

Fig. 16.2 *Orsat apparatus*

16.13.3 Procedure

A sample of gas is drawn in the measuring burette by lowering the levelling bottle. The sample of flue gas is taken as 100 cc, and any excess gas is expelled to the atmosphere via three-way cock. The cock *A* is opened and the flue gas is first passed through the first pipette containing KOH, where CO_2 component of the gas is absorbed. The process is repeated several times to ensure complete absorption of CO_2 in the KOH solution. The levelling bottom is then lowered to suck the gas back in the measuring burette. The difference in level of the gas gives the percentage of CO_2 by volume in the flue gases.

Now the cock *A* is closed, the procedure is repeated with the second and third pipettes by opening cocks *B* and *C*, respectively and the volumetric percentage analysis of O_2 and CO in the flue gases is measured.

When the percentage of CO_2, O_2 and CO have been determined then the remaining gas in the sample is assumed to be nitrogen, N_2.

The Orsat apparatus gives the volumetric analysis of dry flue gases, since steam is condensed at atmospheric temperature. In order to get good approximation, the flue gases must be passed through absorption pipettes in the same sequence as discussed above.

Example 16.18 *The volumetric analysis of a gas is as follows:*

$H_2 = 49.4\%$ $CO = 18\%$ $CH_4 = 20\%$
$C_4H_8 = 2\%$ $O_2 = 0.4\%$ $N_2 = 6.2\%$ $CO_2 = 4\%$

Calculate

(a) Stoichiometric air fuel ratio,

(b) Wet and dry analysis of products of combustion, if the actual mixture is 20% weak.

Solution

Given The volumetric analysis of a gas for 1 m^3 as

$H_2 = 0.494$ $CO = 0.18$ $CH_4 = 0.2$
$C_4H_8 = 0.02$
$O_2 = 0.004$ $N_2 = 0.062$ $CO_2 = 0.04$

To find

(i) Stoichiometric air–fuel ratio, and

(ii) The wet and dry analysis of products of combustion if the actual mixture is 20% weak.

Assumptions

(i) Each molecule of oxygen is accompanied by 3.76 moles of nitrogen.

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(ii) The nitrogen remains as inert gas.
(iii) Since it is a weak mixture, thus excess air is needed for complete combustion.

Analysis

(i) For complete combustion of each element, the O_2 required and products produced:

Hydrogen

$$H_2 + \frac{1}{2}O_2 \rightarrow H_2O$$

1 mole of H_2 requires $\frac{1}{2}$ mole of O_2 to produce 1 mole of H_2O, 1.88 mole of N_2. Thus, 0.494 mole of H_2

$$\text{requires} = 0.494 \times \frac{1}{2} = 0.247 \text{ mole of } O_2$$

$$\text{produces} = 0.494 \text{ of } H_2O,$$

CO: $$CO + \frac{1}{2}O_2 \rightarrow CO_2$$

0.18 mole of CO requires

$$= 0.18 \times \frac{1}{2} = 0.09 \text{ mole of } O_2$$

$$\text{produces} = 0.18 \text{ mole } CO_2$$

$$CH_4\ CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

$$0.2 \text{ mole } CH_4 + (0.2 \times 2 \text{ mole})\ (O_2) \rightarrow$$

$$0.2 \text{ mole } CO_2 + (0.2 \times 2 \text{ mole})\ H_2O$$

Thus, 0.2 mole of CH_4 requires

$$= 0.4 \text{ mole of } O_2$$

$$\text{produces} = 0.2 \text{ mole } CO_2 + 0.4 \text{ mole } H_2O$$

C_4H_8: $C_4H_8 + 6O_2 \rightarrow 4CO_2 + 4H_2O$

For 0.02 mole; $0.02\ C_4H_8 + (0.02 \times 6)\ O_2 \rightarrow (0.02 \times 4)\ CO_2 + (0.02 \times 4)\ H_2O$

0.02 mole of C_4H_8

$$\text{requires} = 0.12 \text{ mole } O_2$$

$$\text{produces} = 0.08 \text{ mole } CO_2 + 0.08 \text{ mole } H_2O$$

Thus, the O_2 mole requires for complete combustion

$$= 0.247 + 0.09 + 0.4 + 0.12$$
$$= 0.857 \text{ mole}$$

The fuel gas contains 0.004 mole of O_2; thus

$$\text{Net } O_2 \text{ mole requires} = 0.857 - 0.004$$
$$= 0.853 \text{ mole}$$

The moles of air associated with 0.853 mole of O_2;

$$= 0.853 \times 4.76 \text{ mole} = 4.060 \text{ mole}$$

Stoichiometric fuel ratio

$$= \frac{\text{mole of air}}{\text{mole of fuel gas}} = \frac{4.060}{1}$$
$$= \mathbf{4.060}$$

(ii) When mixture is 20% weak, excess air by 20% is needed.

Actual air–fuel ratio = $1.2 \times 4.060 = 4.872$

Oxygen associated with air supplied

$$= \frac{4.872}{4.76} = 1.023 \text{ moles}$$

Excess O_2 supplied

$$= 1.023 - 0.853 = 0.170 \text{ mole}$$

Nitrogen associated with O_2 supplied

$$= 3.76 \times 1.023 = 3.846 \text{ moles}$$

Total nitrogen in products

$$= 3.846 + 0.062 = 3.908 \text{ moles}$$

Analysis by volume of wet and dry products of combustion

Product	*Mole/mole of fuel*	*% by volume (dry products)*	*% by volume wet products*
CO_2	0.18 + 0.2 + 0.08 + 0.04 = 0.5	10.92	9.01%
			—
H_2O	0.494 + 0.4 + 0.08 = 0.974	—	17.54%
O_2	0.170	3.71	3.07%
N_2	3.908	85.36	70.39%
	Total dry = 4.578	100.00	100.00
	Total wet = 5.552		

Example 16.19 *A fuel having the formula C_7H_{16} is burnt with 10% excess air. Assuming 90% carbon is burnt to CO_2 and the remaining to CO, determine the volumetric analysis of dry flue gases and air to fuel ratio by mass.*

Solution

Given A fuel with chemical formula C_7H_{16}

Excess air supplied = 10% and 90% carbon to CO_2

To find

(i) Air fuel ratio by mass, and
(ii) Volumetric analysis of dry flue gases.

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Analysis

(i) *Air–fuel ratio*

The mass of 1 mole of fuel C_7H_{16}

$$= 7 \times 12 + 16 \times 1$$

$$= 100 \text{ kg/mole of fuel}$$

The mass fraction of carbon in fuel

$$= \frac{7 \times 12}{100} = 0.84$$

Mass fraction of hydrogen in the fuel

$$= \frac{16}{100} = 0.16$$

The minimum mass of air required to burn 1 kg of fuel

$$= \frac{100}{23}\left[\frac{8}{3}\text{C} + 8\text{H}\right]$$

$$= \frac{100}{23}\left[\frac{8}{3} \times 0.84 + 8 \times 0.16\right]$$

$$= 15.3 \text{ kg/kg of fuel}$$

Thus, the stoichiometric air fuel ratio

$$= \frac{15.3 \text{ kg air}}{1 \text{ kg fuel}} = 15.3$$

But 10% excess air is supplied, thus mass of actual air supplied per kg of fuel

$$= 15.3 \times 1.10 = 16.834 \text{ kg/kg of fuel}$$

Actual A/F ratio

$$= \mathbf{16.834}$$

(ii) *Volumetric analysis of combustion products*

The mass of carbon burnt to CO_2

$$m_{CO_2} = 0.84 \times 0.9 = 0.756 \text{ kg/kg of fuel}$$

The mass of carbon burnt to CO

$$m_{CO} = 0.84 \times 0.1 = 0.084 \text{ kg/kg of fuel}$$

The moles of elements in products coming from fuel;

$$n_{CO_2} = \frac{0.756}{12} = 0.063$$

$$n_{CO} = \frac{0.084}{12} = 0.007$$

$$n_{H_2} = \frac{0.16}{2} = 0.08$$

$$n_{O_2} = \left(\frac{23}{100}\right) \times \frac{16.834}{32} = 0.121$$

$$n_{N_2} = \left(\frac{77}{100}\right) \times \frac{16.834}{28} = 0.463$$

Total mole of carbon in the fuel

$$= 0.063 + 0.007 = 0.07$$

The chemical reaction on the molar basis.

$$0.07\text{C} + 0.08\text{H} + 0.121\text{O}_2 + 0.463\text{N}_2 \rightarrow$$

$$0.063\text{CO}_2 + 0.007\text{CO} + \frac{0.08}{2}\text{H}_2\text{O} + a\text{O}_2 + 0.463\text{N}_2$$

Equating the constants for O_2

$$0.121 = 0.063 + \frac{0.007}{2} + 0.04 + a$$

It gives $a = 0.0145$

Total moles of dry flue gases

$$= \text{moles of } [\text{CO}_2 + \text{CO} + \text{O}_2 + \text{N}_2]$$

Volumetric composition

$$= 0.063 + 0.007 + 0.0145 + 0.463$$

$$= 0.5475$$

$$\%\text{C O}_2 \text{ by volume} = \frac{0.063}{0.5475} \times 100 = 11.5\%$$

$$\%\text{ CO by volume} = \frac{0.007}{0.5475} \times 100 = 1.278\%$$

$$\%\text{ O}_2 \text{ by volume} = \frac{0.0145}{0.5475} \times 100 = 2.648\%$$

$$\%\text{ N}_2 \text{ by volume} = \frac{0.463}{0.5475} \times 100 = 84.56\%$$

$$\text{Total} = 100\%$$

Example 16.20 *The exhaust gas of a gasoline-fueled automobile engine was cooled to 20°C and then subjected to Orsat gas analysis. The results were*

CO_2 = 7.2% CO = 0.8% O_2 = 9.9% N_2 = 82.1%

Determine

(a) the hydrocarbon model of the fuel,

(b) the composition of fuel on mass basis,

(c) the air–fuel ratio on mass basis,

(d) the theoretical air used in the combustion process.

Solution

Given The exhaust gas analysis of a gasoline engine

To find

(i) The hydrocarbon model of the fuel.

(ii) The composition of fuel on the mass basis.

(iii) The air fuel ratio on mass basis.

(iv) The percentage theoretical air used in the combustion process.

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Assumptions

(i) Since the analysis of Orsat apparatus is carried out at 20°C, we consider that all the water in the combustion products has condensed out.
(ii) Steady flow chemical reaction.
(iii) The combustion reaction of 100 moles.
(iv) The each oxygen mole in the reaction is accompanied with 3.76 moles of nitrogen.

Analysis

(i) The chemical reaction for a hydrocarbon is

$$C_nH_m + a(O_2 + 3.76\,N_2) \rightarrow 7.2\,CO_2 + 0.8\,CO + 9.9\,O_2 + b\,(H_2O) + 82.1\,N_2$$

The balancing the elements of reaction

Carbon, C $\quad n = 7.2 + 0.8 = 8.0$

Hydrogen, H_2 $\quad m = 2b$

Nitrogen, N_2 $\quad 3.76\,a = 82.1 \quad$ or $\quad a = 21.9$

Oxygen, O_2 $\quad a = 21.9$

$$= 7.2 + \frac{0.8}{2} + 9.9 + \frac{b}{2}$$

or $\quad b = 8.8$

Thus, $\quad m = 2b = 17.6$

And the model of hydrocarbon becomes

$$C_n\,H_m = C_8H_{17.6}$$

(ii) Mass of 1 mole of fuel $= 12 \times 8 + 17.6 \times 1$

$= 113.6$ kg/mole of fuel

Mass of O_2 moles in the fuel

$= 21.9 \times 32 = 700.8$ kg

Mass of air associated with this O_2

$$= \frac{100}{23} \times 700.8$$

$= 3047$ kg/mole of fuel

$$\text{A/F ratio} = \frac{\text{Mass of air}}{\text{Mass of fuel}} = \frac{3047}{113.3}$$

$= \mathbf{26.82}$

(iii) Mass percentage of carbon in the fuel

$$= \frac{8 \times 12}{113.6} \times 100 = \mathbf{84.51\%}$$

Mass percentage of H_2 in the fuel

$$= \frac{17.6 \times 1}{113.6} \times 100 = \mathbf{15.5\%}$$

(iv) *Theoretical air–fuel ratio*

The mass of air required for complete combustion

$$= \frac{100}{23}\left[\frac{8}{3}C + 8H\right]$$

$$= \frac{100}{23} \times \left[\frac{8}{3} \times 0.845 + 8 \times 0.155\right]$$

$= \mathbf{15.18\ kg/kg\ of\ fuel}$

16.14 HEAT GENERATED BY COMBUSTION

During a chemical reaction, some chemical bonds in the fuel are broken and other new ones are formed. The chemical energy associated with these bonds is released or absorbed in the form of heat. That is,

Energy of reactants = Energy of products + Chemical energy (as heat)

In absence of any change in kinetic and potential energies, the energy change of a system during a chemical reaction will be due to change in state and a change in chemical composition. That is,

$$\Delta E_{sys} = \Delta E_{state} + \Delta E_{chemical} \qquad ...(16.23)$$

When products formed during a chemical reaction are at the inlet state of reactants then $\Delta E_{state} = 0$ and the energy change of the system is due to change in the chemical compostion only.

16.14.1 Standard Reference State

The heat generated during a reaction does not depend only on the chemical energy of reactants but also on pressure and temperature at which the reaction takes place. Further, after the chemical reaction, the composition of the system is also different from that at the beginning of the process. In such cases $p_{ref} = 1$ atm and $T_{ref} = 25°C$ (= 298 K) is known as the *standard reference state*. The property values at the standard reference state are designated by the superscript '°', such as $h°$, $u°$, etc.

At standard reference state, the enthalpy values can be assigned to elements for use in analysis of a reacting system. For some chemically stable elements, the property value (like enthalpy) at the standard reference state is assigned a zero value.

16.14.2 Enthalpy of Formation

The enthalpy of an element at the standard reference state is called the *enthalpy of formation.*

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The enthalpy of formation is the energy released or absorbed when the products are formed from its element, the products and elements both being T_{ref} and p_{ref}. It is denoted as $\bar{h}^{\circ}_{f,i}$, where the subscript f indicates formation, the subscript i means element (species) and superscript '°' means the standard reference state of 1 atm and 25°C. It is measured in kJ/kmol. **Table 16.6** presents the values of enthalpy of formation for several elements.

Consider the formation of CO_2 from its elements, carbon (graphite) and oxygen. Carbon and oxygen both enter the reaction chamber at 25°C and 1 atm and react completely at steady state to form CO_2 at 25°C and 1 atm.

$$C + O_2 \rightarrow CO_2 + Q \text{ (Heat)}$$

The formation of CO_2 from carbon is an *exothermic* reaction and therefore, chemical energy is relased in the form of heat and this heat must be rejected from the reaction chamber to surroundings during the process to keep the product CO_2 at 25°C and 1 atm.

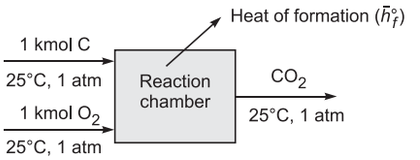

Fig. 16.3 *Formation of CO_2 at 25°C and 1 atm*

Since the process does not involve any work interaction, therefore, the steady-flow energy equation in absence of change in potential and kinetic energies yields to

$$Q = \Delta H = H_P - H_R \qquad ...(16.24)$$

where H_P = enthalpy of product

H_R = enthalpy of reactants, both are at the same state

Table 16.6 *Typical values of enthalpy of formation $\Delta\bar{h}^{\circ}_f$ of selected substances at 25°C and 1 atm*

Substance	*Formula*	*State*	*Molar mass (kg/kmol)*	*Enthalpy of formation $\Delta\bar{h}^{\circ}_f$ (kJ/kmol)*
Carbon	C	Graphite	12.01	0
Carbon	C	Diamond	12.01	1900
Hydrogen	H_2	Gas	2.016	0
Oxygen	O_2	Gas	32.00	0
Nitrogen	N_2	Gas	28.01	0
Carbon monoxide	CO	Gas	28.01	− 110 530
Carbon dioxide	CO_2	Gas	44.01	−393 520
Water	H_2O	Liquid	18.02	−285 830
Water	H_2O	Gas	18.02	−241 820
Methane	CH_4	Gas	16.04	−74 850
Ethylene	C_2H_2	Gas	28.05	52 280
Ethane	C_2H_6	Gas	30.07	− 84 680
Propylene	C_3H_6	Gas	42.08	20 410
Propane	C_3H_8	Gas	44.09	−103 850
Butane	C_4H_{10}	Gas	58.12	−126 150
Pentane	C_5H_{12}	Gas	72.15	−146 440
Octane	C_8H_{18}	Gas	114.22	−208 450
Benzene	C_6H_6	Gas	78.11	82 930
Methyle alcohol	CH_3OH	Gas	32.08	− 200 890
Methyle alcohol	CH_3OH	Liquid	32.08	−238 810
Ethyle alcohol	C_2H_5OH	Gas	46.07	−235 310
Ethyle alcohol	C_2H_5OH	Liquid	46.07	− 277 690

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The products and reactants both are at same state, and thus the enthalpy change during the process is only due to change in chemical composition of the system. This energy is referred as *enthalpy of formation.*

The *enthalpy of formation is defined as the enthalpy of reaction at a specified state for the formation of a substance from its elements in their most stable form.* The stable forms of elements means forms such as H_2 and O_2 for hydrogen and oxgyen instead of H and O, the monoatomic forms. The stable form of carbon is graphite instead of diamond, etc.

The enthalpy of formation for all stable elements such as O_2, N_2 and C is assigned as value zero at the standard reference state. That is, $\bar{h}_f^\circ = 0$, for all stable elements

Now reconsider the formation of CO_2 (product) from its elements carbon and oxygen at 25°C and 1 atm during a steady flow process. The enthalpy of formation of reactants C and O_2, $H_R = 0$ and

$$Q = H_P - H_R = -393,\ 520 \text{ kJ/kmol}$$

or $$\bar{h}^\circ_{f,\text{co}_2} = -393,\ 520 \text{ kJ/kmol}$$

The negative sign is used to account for the fact that the enthalpy of 1 kmol of CO_2 at 25°C and 1 atm is 393,520 kJ less than that of 1 kmol of C and 1 kmol of O_2 at the same state. In other words, the reaction is leaving the system. According to sign convention, this heat is *negative*. Thus the enthalpy of formation of substances produced by *exothermic reactions are considered negative.*

16.14.3 Enthalpy of Reaction

The enthalpy of formation concept can be used to obtain the change in chemical energy during a reaction.

Consider the formation of CO_2 from its elements, CO and oxygen, at standard reference state.

$$CO + (1/2)O_2 \rightarrow CO_2 + Q \text{ (Heat)}$$

In this case, the term 'enthalpy of formation' cannot be used since one of the reactants, CO, is not a stable element. In such a case, the change in enthapy is called the *enthalpy of reaction,* $\bar{h}_R$. In this case, the reaction is written as

$$CO + (1/2)O_2 \rightarrow CO_2 + \bar{h}_R$$

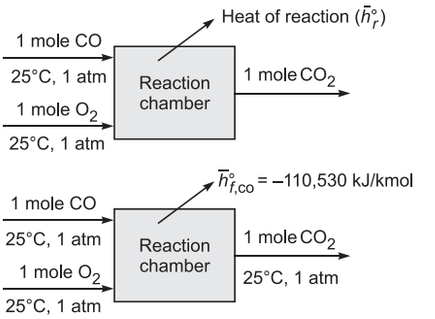

Fig. 16.4 *Formation of CO_2 from CO and O_2 at 25°C and 1 atm*

Thus, the *enthalpy of reaction* can be defined as the *difference between the enthalpy of products and enthalpy of reactants* at same specificed state for a chemical reaction.

The formation of CO_2 from reactants C and O_2 leads to

$$C + O_2 \rightarrow CO_2 + \bar{h}^\circ_{f,\text{co}_2} \qquad ...(16.25)$$

And formation of CO from reactants leads to

$$C + (1/2)O_2 \rightarrow CO + \bar{h}^\circ_{f,\text{co}} \qquad ...(16.26)$$

Subtracting Eq. (16.26) from (16.25) and transposing CO to the left, leads to

$$CO + (1/2)O_2 \rightarrow CO_2 + \bar{h}^\circ_{R,\text{co}} \qquad ...(16.27)$$

where $\bar{h}^\circ_{R,\text{co}} = \bar{h}^\circ_{f,\text{co}_2} - \bar{h}^\circ_{f,\text{co}}$

$= -393{,}520 - (-\ 110{,}530)$

$= -28{,}990$ kJ/kmol

For a combustion process, the enthalpy of reaction is referred as *enthalpy of combustion* $\bar{h}_c$, which can be interpreted as the amount of heat released during a steady-flow combustion process, when 1 kmol of (1 kg) of fuel burns completely at a specificed temperature and pressure. It is expressed as

$$\bar{h}_c = \sum_P n_e \bar{h}_e - \sum_R n_i \bar{h}_i \text{ (kJ/kmol)} \qquad ...(16.28)$$

where $n_i h_i = \bar{h}^\circ_{f,i} + \Delta h_i$ (for i^{th} species)

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where, n's represent the number of moles of reactants and products per mole of fuel and $\bar{h}$'s represent the enthalpy per mole.

When the enthalpy of combustion is expressed on a unit mass basis, it is designated as h_c and measured in kJ/kg. If the enthalpy of formation data are available for all reactants and products, the enthalpy of combustion can be calculated directly from Eq. (16.28) as illustrated in Examples 16.21 and 16.22.

Example 16.21 *Calculate the enthalpy of combustion at 25°C of ethyl alcohol, C_2H_5OH.*

Solution

Given Ethyl alcohol at 25°C

To find Enthalpy of combustion

Analysis The chemical reaction is

$$C_2H_5OH + 3O_2 \rightarrow 2CO_2 + 3H_2O$$

The enthalpy of reaction for 1 kmol of C_2H_5OH (fuel).

$$\bar{h}_c = \sum_P n_e h_e - \sum_R n_i h_i$$

Referring to Table 16.6,

$$\sum_R n_i h_i = 3\bar{h}^\circ_{f,O_2} + 1\bar{h}^\circ_{f,C_2H_5OH}$$

$$= (3\,\text{kmol}) \times 0 + (1\,\text{kmol})(-235{,}310\,\text{kJ/kmol})$$

$$= -235{,}310\ \text{kJ/kmol}$$

and $$\sum_P n_e h_e = 2\bar{h}^\circ_{f,CO_2} + 3\bar{h}^\circ_{f,H_2O}$$

$$= 2\ \text{kmol} \times (-393{,}520\ \text{kJ/kmol}) + (3\,\text{kmol}) \times (-241{,}820\ \text{kJ/kmol})$$

$$= -1512500\ \text{kJ/kmol}$$

$$\bar{h}_c = -1512500 - (-235{,}310)$$

$$= \mathbf{-12{,}77{,}190\ kJ/kmol}$$

Example 16.22 *Calculate the enthalpy of combustion of gaseous methane in kJ/kg of fuel*

(a) at 25°C, 1 atm with liquid water in the products

(b) at 25°C , 1 atm with water vapour in products

Solution

Given Fuel as gaeous methane at 25°C and 1 atm

To find Enthalpy of combustion.

Analysis The chemical reaction is

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

The enthalpy of reaction for 1 kmol of CH_4 (fuel).

$$\bar{h}_c = \sum_P n_e h_e - \sum_R n_i h_i$$

(i) *The liquid water in the products*

Using the values from Table 16.6;

$$\sum_R n_i h_i = 1\bar{h}^\circ_{f,CH_4(g)}$$

$$= (1\,\text{kmol}) \times (-74\,850\ \text{kJ/kmol})$$

$$= -74\,850\ \text{kJ/kmol}$$

and $$\sum_P n_e h_e = 1\bar{h}^\circ_{f,CO_2} + 2\bar{h}^\circ_{f,H_2O(l)}$$

$$= (1\,\text{kmol}) \times (-393{,}520\ \text{kJ/kmol}) + (2\ \text{kmol}) \times (-285\,830\ \text{kJ/kmol})$$

$$= -965\,180\ \text{kJ/kmol}$$

Then $$\bar{h}_c = -965\,180 - (-74\,850)$$

$$= -890\,330\ \text{kJ/kmol}$$

Dividing it by the molar mass of methane

$$h_c = -\frac{890330\ \text{kJ/kmol}}{16.04\ \text{kg/kmol}}$$

$$= \mathbf{55\,506.85\ kJ/kg}$$

(ii) *The water vapour in the products*

Using the values from Table 16.6;

$$\sum_R n_i h_i = 1\bar{h}^\circ_{f,CH_4(g)}$$

$$= -74\,850\ \text{kJ/kmol}$$

and $$\sum_P n_e h_e = 1\bar{h}^\circ_{f,CO_2} + 2\bar{h}^\circ_{f,H_2O(g)}$$

$$= (1\,\text{kmol}) \times (-393{,}520\ \text{kJ/kmol}) + (2\ \text{kmol}) \times (-241\,820\ \text{kJ/kmol})$$

$$= -877\,160\ \text{kJ/kmol}$$

Then $$\bar{h}_c = -877\,160 - (-74\,850)$$

$$= -802\,310\ \text{kJ/kmol}$$

Dividing it by the molar mass of methane

$$h_c = -\frac{802310\ \text{kJ/kmol}}{16.04\ \text{kg/kmol}}$$

$$= \mathbf{50\,019.32\ kJ/kg}$$

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16.15 CALORIFIC VALUE, OR HEATING VALUE OF FUEL

The term 'calorific value' is most commonly used in conjunction with the combustion of fuel. The calorific value of a fuel is defined as the amount of heat energy liberated by complete combustion of unit quantity of a fuel. It is also called heating value of the fuel and it can also be considered as an absolute value of enthalpy of combustion. That is,

$$\text{calorific (heating) value} = |h_c|$$

The calorific value is measured in kJ/kg or kJ/kmol for solid and liquid fuels and kJ/m^3 for gaseous fuels.

There are two aspects of relation for heat of formation (or reaction) and calorific value, which should be kept in mind.

1. The calorific value of the fuel is the absolute value of enthalpy of formation (or reaction), but expressed per unit quantity of fuel (i.e., reactants) rather than the products.
2. By convention, the calorific value is positive, it has opposite sign convention to that for enthalpy of formation (or reaction).

16.15.1 Higher and Lower Calorific Values of Fuels

Basically, all fuels are hydrocarbons, thus their main constituents are carbon and hydrogen. During the combustion process, carbon burns to carbon dioxide and hydrogen reacts with oxygen and forms water vapour. The magnitude of the calorific value depends on the phase of water vapour in the combustion products.

When combustion products are cooled to the reactant's temperature, the water vapour gets condensed and the heat of its vaporisation is recovered. The calorific value thus obtained is called the *higher calorific value (HCV)* or *gross calorific value.*

The *lower calorific value (LCV)* or *net calorific value* is the amount of heat released by complete combustion of unit quantity of fuel, when the vapour carries its heat of vaporisation. It is obtained by deducting the heat necessary to form the vapour from hydrogen. That is,

lower calorific value (LCV)

$$= \text{HCV} - m_v h_{fg} \qquad ...(16.29)$$

where m_v = mass of water vapour formed per kg of fuel,

h_{fg} = the heat of vaporisation of water at 25°C and 1 atm,

= 2441.5 kJ/kg·K

The two calorific values are also related as

$$\text{HCV} = \text{LCV} + (n\bar{h}_f^\circ)_{H_2O} \text{ kJ/kg of fuel} \qquad ...(16.30)$$

where n is number of kmol/kg of H_2O and $\bar{h}_{fg}$ is the enthalpy of vaporisation of water at 25°C.

The two units of calorific value can be correlated as

Calorific value of fuel in kJ/kg

$$= \frac{\text{Calorific value of fuel in kJ/kmol}}{\text{Molecular weight of fuel (kg/kmol)}} \qquad ...(16.31)$$

Thus, the calorific value in kJ/kg is obtaind by dividing the calorific value in kJ/kmol by the respective molecular weight.

The calorific values of some fuels in kJ/kmol are presented in Table 16.7.

16.15.2 Determination of Calorific Value of a Fuel

Normally, fuels contain carbon, hydrogen, oxygen, nitrogen, ash and sulphur. The nitrogen and ash are inert to the combustion. The carbon, hydrogen, sulphur and oxygen are participating in the combustion. Let C, H, O and S be the percentage of these constituents, respectively in the fuel.

The calorific value of these constituents can be obtained from Table 16.7 as

$$\text{For carbon (C)} = \frac{393520 \text{ kJ/kmol}}{12 \text{ kg/kmol}}$$

$$\approx 33{,}000 \text{ kJ/kg}$$

$$\text{For hydrogen (H)} = \frac{286043}{2}$$

$$= 1{,}43{,}000 \text{ kJ/kg (HCV)}$$

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Table 16.7 *HCV and LCV of fuels in kJ/kmol*

Substance	*Symbol*	*Approximate Molecular weight*	*HCV $H_2O(l)$*	*LCV $H_2O(g)$*
Acetylene	C_2H_2	26	1300566	1256520
Benzene	C_6H_6	78	3303878	3171736
n-Butane	C_4H_{10}	58	2879190	2658955
Isobutane	C_4H_{10}	58	2870817	2650580
Carbon (graphite)	C	12	393520	—
Carbon monoxide	CO	28	283192	—
Ethane	C_2H_6	30	1560997	1428855
Ethelene	C_2H_4	28	1411982	1323930
n-heptane	C_7H_{18}	100	4856962	4504584
n-hexane	C_6H_{14}	86	4197760	3889430
Methane	CH_4	16	890994	802900
n-octane	C_8H_{18}	114	5516163	5119738
n-pentane	C_5H_{12}	72	3538685	3274400
Propane	C_3H_8	44	2221622	2045475
Hydrogen	H_2	2	286043	242000
Sulphur	S	32	293120	—

$$= \frac{242000}{2}$$

$$= 1{,}21{,}000 \text{ kJ/kg (LCV)}$$

$$\text{For sulphur (S)} = \frac{293120}{32} = 9{,}160 \text{ kJ/kg}$$

For 100 kg of fuel, the total heat produced after complete combustion

$$= 33000\,C + 9160\,S + 143000\left(H - \frac{O}{8}\right) \text{ kJ/kg}$$

The higher calorific value (HCV) of fuel

$$= \frac{33000\,C + 9160\,S + 143000\left(H - \frac{O}{8}\right) \text{kJ}}{100 \text{ kg}}$$

$$= 330\,C + 91.6\,S + 1430\left(H - \frac{O}{8}\right) \text{kJ/kg} \quad \ldots(16.32)$$

It is also known as *Dulong's formula*, where C, S, H and O are percentage constituents, respectively in 1 kg of fuel. If the lower calorific value of hydrogen as 121000 kJ/kg is used then the obtained heating value is the lower calorific value of the fuel and can similarly be obtained as

$$\text{LCV} = 330\,C + 91.6\,S + 1210\left(H - \frac{O}{8}\right) \quad \ldots(16.33)$$

Example 16.23 *Calculate the calorific value of ethane.*

Solution

Given Ethane (C_2H_6) as fuel

To find Calorific value (absolute value of enthalpy of combustion) of ethane

Analysis The molecular weight of ethane is 30. Fraction of carbon content per kg of fuel

$$m_c = \frac{2 \times 12}{30} = 0.8 \text{ kg/kg of fuel}$$

Fraction of hydrogen content per kg of fuel

$$m_{H_2} = \frac{6 \times 1}{30} = 0.2 \text{ kg/kg of fuel}$$

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The heat released by complete combustion

$$Q = m_C (HCV)_c + m_{H_2}(HCV)_{H_2}$$

$$= 0.8 \times \left(\frac{393{,}520}{12}\ \text{kJ/kg}\right) + 0.2 \times \left(\frac{286\,043}{2}\ \text{kJ/kg}\right)$$

$$= \mathbf{54{,}839\ kJ/kg\ of\ fuel}$$

Example 16.24 *A sample of gobar gas contains 55% methane and the rest of CO_2. What is the calorific value ?*

Solution

Given A sample of gobar gas with 55% CH_4 and 45% CO_2

To find Calorific value of gobar gas.

Analysis Since CO_2 does not participale in combustion, it remains inert to combustion. And the mass fraction of CH_4 in gobar gas is 0.55 kmol/kmol of fuel.

For 1 kmol of CH_4 is

HCV = 890,994 kJ/kmol

Thus, for 0.55 kmol/kmol of CH_4

$= 0.55 \times 890\ 994 =$ **490,047 kJ/kmol**

Example 16.25 *The ultimate analysis of a sample of coal has the following analysis: carbon-84.4%, hydrogen-4% , oxygen-5.6% and the remainder is ash. Calculate the higher and lower calorific values of the fuel.*

Solution

Given Ultimate analysis of a sample of coal

C = 84.4% H = 4%

O = 5.6% Ash = 6% (by calculation)

To find

(i) Higher calorific value of fuel,

(ii) Lower calorific value of fuel.

Assumptions

(i) Complete combustion.

(ii) Ash as inert to combustion.

Analysis

(i) *Higher calorific value of a fuel* Using Eq. (16.32) in absence of percentage of sulphur (S = 0).

$$HCV = 330\,C + 1430\left(H - \frac{O}{8}\right)\ \text{kJ/kg}$$

$$= 330 \times 84.4 + 1430 \times \left(4 - \frac{5.6}{8}\right)$$

$$= \mathbf{32{,}571\ kJ/kg}$$

(ii) *Lower calorific value* Using Eq. (16.33) in absence of sulphur percentage,

$$LCV = 330\,C + 1210\left(H - \frac{O}{8}\right)$$

$$= 330 \times 84.4 + 1210\left(4 - \frac{5.6}{8}\right)$$

$$= \mathbf{31{,}845\ kJ/kg}$$

16.15.3 Experimental Determination of Calorific Value of Fuels

The apparatus used for determining the calorific value of fuels is known as a calorimeter. The basic principle used in determining the calorific value of fuels is that a known quantity of fuel is burned in controlled environment and the heat energy liberated is transferred to a medium of known mass and specific heat and the rise in temperature of the medium is measured. Though there are various types of calorimeters available, we shall only discuss Bomb calorimeter and Junker's gas calorimeters used for determining the calorific value of fuels.

16.16 BOMB CALORIMETER

The calorific value of powdered and liquid fuels is determined at constant volume in the bomb calorimeter. It resembles the shape of a bomb, and thus it is known as the bomb calorimeter. It is shown in Fig. 16.5.

16.16.1 Construction

The fuel is burnt in a strong steel chamber, known as *bomb,* which is immersed in a known mass of water. The fuel sample is placed in a crucible inside the bomb, which is filled with oxygen under a pressure above 25 atm. It is then ignited by an electrically heated platinum wire. The combustion thus takes place at constant volume, the fuel burns almost in a constant-pressure environment due to the high

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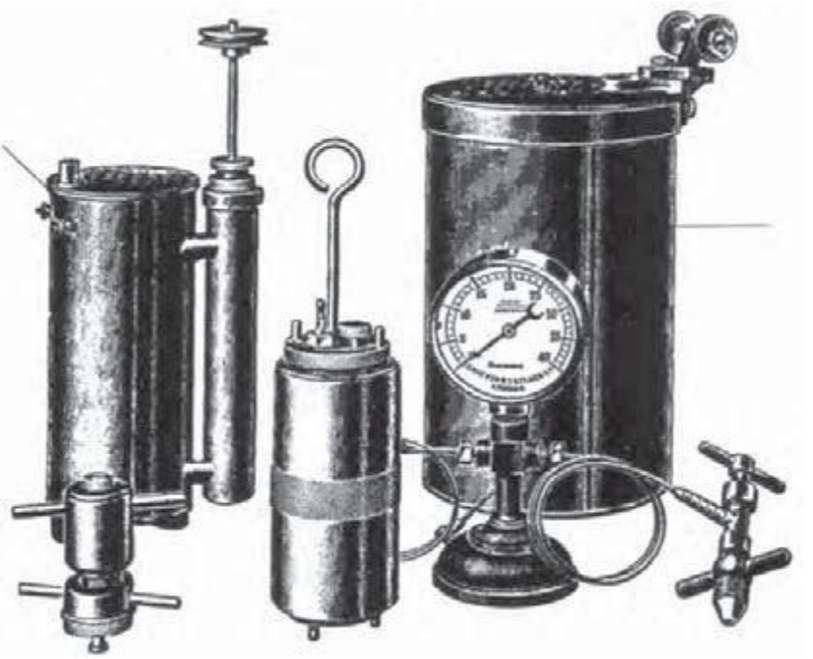

Fig. 16.5 *Parts of a bomb calorimeter*

pressure of oxygen. To reduce any losses of heat, the calorimeter is also provided with additional water jacket and air. A motor-driven stirrer is used to keep the water temperature uniform around the bomb and an accurate thermometer (Beckman type) is immersed in water to measure the temperature accurately.

16.16.2 Procedure

A known quantity of fuel sample as a briquette is placed into the crucible and a fuse wire is connected with the electrodes as shown in Fig. 16.6. The bomb is then placed in a calorimeter with a weighed quantity of water. After making necessary connections, the stirrer is started and temperature measurements are taken every minute. At the end of the fifth minute, a charge is fired and temperature readings are taken carefully every 10 seconds during this period. When the temperature readings begin to fall, the frequency of readings may be reduced to one every minute.

After experimentation, the bomb is taken out from its housing. The products of combustion are released through the release valve. Then it is opened, and the unburnt fuse wire, if any, is collected and weighed. A temperature–time curve is plotted.

The measured temperature rise is corrected for various losses. The allowance for combustion of fuse wire is determined from the weight of the fuse and its known calorific value. The water equivalence of a calorimeter must be used in calculation to accommodate its allowance.

The heat released by combustion of fuse is absorbed by water surrounding the bomb and calorimeter. Thus an energy balance yields to

$$\left.\begin{array}{l}\text{Mass of fuel} \times \text{calorific}\\ \text{value} + \text{mass of fuse}\\ \text{wire burn} \times \text{calorific}\\ \text{value of fuse wire}\end{array}\right\} = \left.\begin{array}{c}\text{(Mass of water equivaent}\\ \text{of calorimeter)} \times \text{specific}\\ \text{heat of water} \times \text{corrected}\\ \text{temperature rise}\end{array}\right.$$

$$m_f CV + m_{fuse} CV_1 = (m_w + m_e)\, C_{pw}\, [(T_2 - T_1) + T_c]$$

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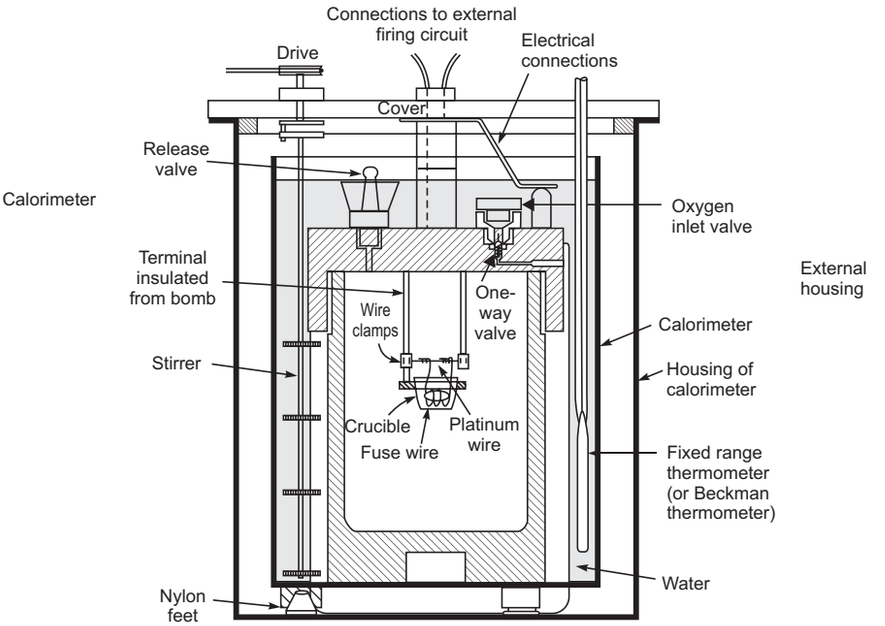

Fig. 16.6 *Sectional view of a bomb calorimeter*

$$CV = \frac{(m_w + m_e)\, C_{pw}\, [(T_2 - T_1) + T_c] - m_{fuse}\, CV_1}{m_f} \quad ...(16.34)$$

where

T_c = radiation correction to temperature, it is obtained from graphical presentation of observation before and after firing

m_f = mass of fuel

m_{fuse} = mass of fuse wire

m_w = mass of water filled in calorimeter

m_e = water equivalent of the calorimeter

CV_1 = Calorific value of fuse wire

$T_2 - T_1$ = Observed temperature difference

The bomb calorimeter measures a higher calorific value of fuel. If a liquid fuel is being tested, it is contained in a gelatin capsule and the firing may be assisted by paraffin of known calorific value in the crucible.

Example 16.26 *The table given below gives the results of a calorimeter test of a sample of coal. The mass of the coal burned was 0.825 gram and the total water equivalent of the apparatus was determined as 2500 g. Calculate the calorific value of the coal in kJ/kg.*

Pre firing Period time	*Temp. °C (min)*	*Heating time (min)*	*Temp. °C*	*Cooling period time (min)*	*Temp. °C*
0	25.730	6	27.840	10	27.880
1	25.732	7	27.880	11	27.878
2	25.734	8	27.883	12	27.876
3	25.736	9	27.885	13	27.874
4	25.738			14	27.872
5	25.740			15	27.870

Solution

Given

Mass of the coal, $m_f = 0.825$ g

Total water equivalent of apparatus = 2500 g

To find Calorific value of the fuel in kJ/kg.

Assumptions

(i) Negligible mass of fuse wire.

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(ii) Total water equivalent consists of mass of water and water equivalent of calorimeter.

(iii) Complete combustion of fuel.

Analysis The rate of cooling,

$$r = \frac{27.880 - 27.870}{15 - 10} = 0.002°C/min$$

Correction in the temperature,

$$T_c = \frac{r}{2} \cdot t_{firing} = \frac{0.002}{2} \times (4) = 0.004°C$$

Calorific value of the fuel,

$$CV = \frac{m_w C_{pw} [(T_2 - T_1) + T_c]}{m_f}$$

$$= \frac{(2500 \times 10^{-3} \text{ kg}) \times (4.187 \text{ kJ/kg.K}) \times [(27.885 - 25.740) + 0.004]}{(0.825 \times 10^{-3} \text{ kg})}$$

$$= \mathbf{27,202.8 \text{ kJ/kg}}$$

Example 16.27 *The following observations were taken during a bomb calorimeter test to determine the water equivalent of the calorimeter:*

Mass of the fuel burnt = 2.25 g
Calorific value of the fuel = 26,480 kJ/kg
Mass of the fuse wire burnt = 0.025 g
Calorific value of the fuse wire = 6700 kJ/kg
Quantity of water in the calorimeter = 2000 cc
Temperature rise during burning of fuel = 5.2°C in 5 minutes
Drop in temperature after burning of fuel = 0.2°C in 5 minutes
Specific heat of water = 4.187 kJ/kg·K

Solution

Given Data as above

To find Water equivalent of the calorimeter.

Analysis The rate of cooling,

$$r = \frac{0.2}{5} = 0.04°C/min$$

Correction in temperature

$$= \frac{r}{2} \cdot t_{firing} = \frac{0.04}{2} \times 5 = 0.1°C$$

Energy balance on the calorimeter yields to

Heat liberated by fuel and fuse wire = Heat absorbed by water and calorimeter

$$m_f CV + m_{fuse} CV_1 = (m_w + m_e) C_{pw} [(T_2 - T_1) + T_c]$$

Inserting numerical values

$$(2.25 \times 10^{-3} \times 26,480) + (0.025 \times 10^{-3} \times 6700) = (2000 \text{ cc} \times 10^{-3} \text{ kg/cc} + m_e) \times 4.187 \times (5.2 + 0.1)$$

or $22.19\, m_e + 44.3822 = 59.7475$

$m_e = 0.6924$ kg

$= \mathbf{692.4 \text{ gram}}$

16.17 JUNKER'S GAS CALORIMETER

The calorific value of a gaseous fuel is determined by Junker's or *Boy's gas calorimeter*. The metered gaseous fuel is continuously supplied to the calorimeter at constant pressure, where it burns in the presence of air as shown in Fig. 16.7. The products of combustion are cooled to the initial temperature of the reactants by continuous circulating water. The gas temperature and pressure are measured and the amount of the gas burned is referred to 25°C and 1.013 bar. The temperature rise of circulating water is measured and the condensate from the products of combustion is collected. This test is carried out for a fixed time period. The water flow rate and weight of condensate are measured. Thus, we have

(Volume of fuel at 1.013 bar, 25°C) × Calorific value = Mass of water circulated × specific heat × temperature rise of water

$$V \times CV = m_w C_{pw} \times (\Delta T)_w \qquad ...(16.35)$$

where $$V = \frac{p_{gas} V_{gas}}{p T_{gas}} \times T = \frac{298}{1.013 \times 100} \times \frac{p_{gas} V_{gas}}{T_{gas}}$$

where p = 1.013 bar, atmospheric pressure

T = 25°C, standard reference temperature

In both the bomb calorimeter test and Junker's calorimeter test, the steam formed on the combus-

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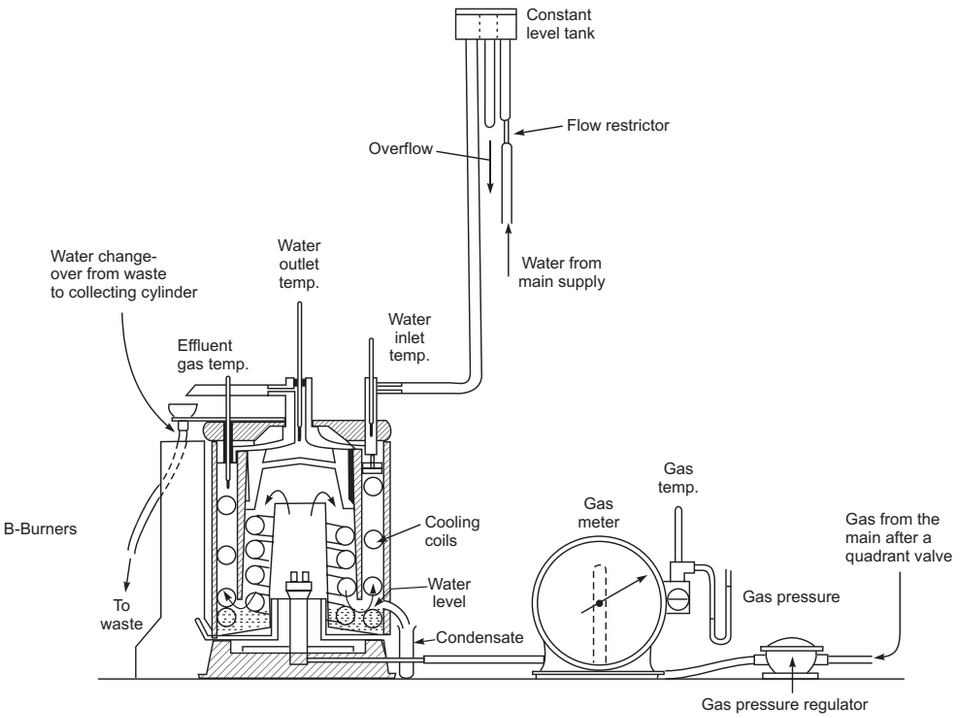

Fig. 16.7 *Boy's gas calorimeter*

tion is condensed and the heat of steam formation is recovered. Thus, they measure a higher calorific value of fuel.

Example 16.28 *The following results were obtained during the trail on the gas calorimeter: Gas supplied = 0.08 m^3 at 32°C; Pressure of gas = 40 mm of water, Barometric reading = 750 mm; Temperature of water at inlet and outlet 30°C and 38°C, respectively; Mass of cooling water circulated = 24 kg; Specific heat of water = 4.2 kJ/kgK; Steam condensed = 0.055 kg. Determine the higher and lower calorific value of the fuel at 25°C. Assume standard barometric pressure = 760 mm of Hg, Take h_{fg} = 2442.5 kJ/kg at 25°C.*

Solution

Given A test for determination of calorific value of a gas

$V_{gas} = 0.08\ m^3$ $\quad T_1 = 30°C$

$T_{gas} = 32°C = 305\ K$ $\quad T_2 = 38°C$

$p_{gas} = 40$ mm of water above 750 mm of Hg

$m_w = 24$ kg

$C_{pw} = 4.2$ kJ/kg·K

$T = 25°C$

$p = 760$ mm of Hg

$m_s = 0.055$ kg

$h_{fg} = 2442.5$ kJ/kg·K

To find

(i) Higher calorific value of the gas.

(ii) Lower calorific value of the gas.

Assumptions

(i) Density of water as 1000 kg/m^3.

(ii) Density of mercury as 13.6×10^3 kg/m^3.

(iii) Acceleration due to gravity $g = 9.81$ m/s^2.

Analysis

(i) *The absolute pressure of the gas supplied*

$$p_{gas} = p_{gage} + p_{atm}$$

$$= 40 \text{ mm of } H_2O + 750 \text{ mm of Hg}$$

$$= 1000 \times 9.81 \times \left(\frac{40}{1000}\right) + 13.6 \times 10^3 \times 9.81 \times \left(\frac{750}{1000}\right)$$

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$= 392.4 + 100062 = 1{,}00{,}454.4$ Pa

$= 100.45$ kPa

Standard barometric pressure

$$p = 760 \text{ mm of Hg}$$

$$= 13.6 \times 10^3 \times 9.81 \times \left(\frac{760}{1000}\right) \times 10^{-3}$$

$= 101.3$ kPa

The volume of gas at standard reference state

$$\frac{pV}{T} = \frac{p_{gas} V_{gas}}{T_{gas}}$$

or $$V = \frac{298}{101.3} \times \frac{100.45 \times 0.08}{305} = 0.0775 \text{ m}^3$$

$(\Delta T)_{water} = 38 - 30 = 8°C$

Then the higher calorific value of the gas

$$CV = \frac{m_w C_{pw} (\Delta T)_w}{V} = \frac{24 \times 4.2 \times 8}{0.0775}$$

$$= \mathbf{10{,}403.5 \text{ kJ/m}^3}$$

(ii) *The lower calorific value of the gas*

$$\text{LCV} = \text{HCV} - \text{Mass of steam/m}^3 \times h_{fg}$$

$$= 10403.5 - \frac{0.055}{0.0775} \times 2442.5$$

$$= \mathbf{8670 \text{ kJ/m}^3}$$

Summary

- Fuel is a combustible matter. It consists of bonded chemical energy between its constituent elements. The combustion is an *exothermic* process, in which the rapid oxidation of fuel takes place with a release of heat energy.
- The air has 23% of oxygen and 77% of nitrogen by mass and 21% of oxygen and 79% of nitrogen by volume. The theoretical amount of air that supplies just sufficient oxygen for complete combustion of all elements of the fuel is termed as *stoichiometric* air required. The *air–fuel ratio* is defined as the ratio of mass of air to the mass fuel during the combustion process.
- The Orsat apparatus is used for fuel gas analysis on the volume basis. It uses the chemical-absorption technique.
- The *enthalpy of formation* is defined as the enthalpy of a substance at a specified state due to its chemical composition. During a combustion process, the amount of heat released is called the *enthalpy of combustion*.
- The *calorific value* of the fuel is defined as the amount of heat energy liberated by complete combustion of a unit quantity of a fuel. It is also called *the heating value.* It is measured in kJ/kg or kJ/m^3.
- After the combustion, if the products are cooled to the reactant's temperature then the water vapour gets condensed and the heat of its vaporisation is recovered. Thus, the calorific value is called the higher calorific value or gross calorific value.
- The lower or net calorific value is the amount of heat released by complete combustion of unit quantity of fuel, when water vapour in the products is in gaseous form.
- The gross or higher calorific value of powdered and liquid fuels is determined at constant volume in the bomb calorimeter, while calorific value of a gaseous fuel is determined by Junker's or Boy's calorimeter at constant pressure.

Glossary

Fuel A combustible substance which burns in the presence of oxygen and releases heat energy

Briquettes Blocks, produced from finely grounded low-grade coal by moulding operation

Pulverised coal Crushed coal in fine power form

Reactants The components of fuel that exist before the combustion reaction

Products The components that exist after combustion reaction

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Combustion An exothermic reaction with oxygen, in which heat energy is released

Ignition temperature Lowest temperature at which a fuel starts burning

3 T's of Combustion Temperature, turbulence and time for complete combustion

Stoichiometric air Minimum amount for the complete combustion of fuel elements

Air fuel ratio Ratio of mass of the air to the mass of the fuel

Equivalence Ratio Ratio of actual air–fuel ratio to the theoretical air–fuel ratio for complete combustion

Standard reference state p_{ref} = 1 atm and T_{ref} = 25°C (= 298 K).

Enthalpy of Formation Enthalpy of an element at standard reference state

Enthalpy of reaction Difference between the enthalpy of products and enthalpy of reactants at the same specificed state

Calorific value Amount of heat energy liberated by complete combustion of unit quantity of a fuel

Review Questions

1. Define fuels and classify their various types.
2. Enumerate the advantages of liquid and gaseous fuels over solid fuels.
3. What do you understand by higher and lower calorific values of fuels?
4. Explain the procedure to determine the higher calorific value of a solid fuel.
5. Explain the working of Boy's gas calorimeter with the help of a neat sketch.
6. What is the use of Orsat apparatus ? Discuss its working with the help of a neat sketch.
7. Write the characteristic of an ideal fuel.
8. Explain the various stages of coal.
9. Write the steps involved in conversion of volumetric analysis to gravimetric analysis.
10. Write the steps involved in conversion of gravimetric analysis to volumetric analysis.
11. Define (a) reactants, (b) products, (c) combustion, and (d) ignition temperature.
12. Write the physical laws of combustion.
13. Write the basic combustion equations.
14. Write the composition of air on mass and molar basis.
15. Define (a) stoichiometric air–fuel ratio (b) Excess air (c) and equivalence ratio.
16. Define enthalpy of formation and enthalpy of combustion.
17. Explain the procedure to determine the calorific value of gaseous fuel.
18. Define calorific value, and differentiate between net and gross calorific values.
19. Explain the construction and working of bomb calorimeter.
20. Define minimum and excess air for combustion and air fuel ratio.

Problems

1. The volumetric analysis of a fuel is given as follows:

CO_2 = 14%, CO = 1%, O_2 = 5% and N_2 = 80%. Obtain the flue gas composition by mass.

[CO_2 = 20.23%, CO = 0.92%, O_2 = 5.26% and N_2 = 73.59%]

2. The ultimate analysis of a solid fuel by mass gives

C = 86%, H_2 = 12%, O_2 = 1%, and S = 1%.

Calculate the theoretical amount of air required for complete combustion at NTP and its volume occupied. Assume specific volume of air at NTP is 0.773 m^3/kg. *[14.1 kg, 10.9 m^3]*

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3. The mass analysis of coal is as follows:
C = 77.2%, H_2 = 5.2%, S = 1.25%, N_2 = 1.5%, O_2 = 8.65 and the remainder is ash. Determine the theoretical amount air required for complete combustion and the volumetric analysis of products of combustion with 25% excess air.
[10.33 kg/kg of fuel; CO_2 = 13.8%, O_2 = 4.1%, H_2O = 5.6%, SO_2 = 0.1% and N_2 = 76.4%]

4. The volumetric analysis of flue gases obtained from the combustion of an unknown hydrocarbon fuel is CO_2 = 12.1%, CO = 0.5%, O_2 = 3.2% and N_2 = 84.2%.
Determine the excess air supplied in percentage of theoretical air. *[15%]*

5. The analysis of coal used in a boiler is carbon-82%, hydrogen-10%, oxygen-4% and ash-4%.
The boiler consumes coal at the rate of 0.2 kg/s with 30% excess air. Calculate the
(a) Volume flow rate of air at intake—the intake conditions are 1 bar and 20°C. Take $R = 0.287\,kJ/kg \cdot K$.
(b) Percentage composition of dry flue gases by mass
[(a) 2.8 m^3/s, (b) CO_2 18%, O_2 = 5.3%, N_2 = 76.7%]

6. During a trial on a boiler, the coal sample has been tested and analysed as carbon-89%, hydrogen-4%, oxygen-3%, Sulphur-1% and the remainder being ash. Determine the minimum mass of air required for complete combustion of 1 kg of coal. If 60% excess air is supplied, estimate the percentage of dry flue gases by mass.
[11.53 kg/kg of coal, CO_2 = 17.14%, O_2 = 8.34%, N_2 = 74.52%]

7. A petrol sample contains 15% of hydrogen, 85% of carbon, and 50% excess air is supplied to ensure complete combustion. Determine the percentage analysis of dry products of combustion by mass.
[CO_2 = 14.1%, O_2 = 7.9%, N_2 = 78%]

8. A composition of petrol on mass basis was C = 85.5%, and H_2 = 14.5%. It burns with 20% excess air. Calculate the precentage analysis of dry products of combustion by mass and convert them to volume basis.

9. In the actual combustion of benzene in an engine, the A/F ratio was 12/1. Calculate the analysis of the wet products of combustion.
[CO_2 13.38%; CO 3.94%, H_2O 8.66%; N_2 74.03%]

10. The ultimate analysis of a sample of petrol was C-85.5% and H-14.5%. Calculate the
(a) stoichiometric A/F ratio
(b) A/F ratio when the mixture strength is 90%
(c) A/F ratio when the mixture strength is 120%
(d) analysis of the dry products for (ii) and (iii)
(e) the volume flow rate of the products through the engine exhaust per unit rate of fuel consumption for (iii) when the pressure is 1.013 bar and the temperature is 110°C
[14.76/1; 16.4/1; 12.3/1; CO_2 13.38%; O_2 2.24%; N_2 84.38%; CO_2 8.67%; CO 8.79%; N_2 82.54%; 15.11 m^3/s per kg/s]

11. The ultimate analysis of a sample of petrol was C-85.5% and H-14.5%. The analysis of the dry products gave 14% of CO_2 and some O_2. Calculate the A/F ratio supplied to the engine and the mixture strength. *[15.72/1; 94%]*

12. In an engine test, the dry product analysis was CO_2-15.5%; O_2-2.3% and the remainder was N_2. Assuming that the fuel burned was a pure hydrocarbon, calculate the ratio of carbon to hydrogen in the fuel, the A/F ratio used, and the mixture strength. *[11.5; 14.84/1; 89.5%]*

13. A quantity of coal used in a boiler had the following analysis : 82% of C; 5% of H; 6% of O; 2% of N, 5% of ash. The dry flue gas analysis showed 14% of CO_2 and some oxygen. Calculate the
(a) oxygen content of the dry flue gas,
(b) A/F ratio and the excess air supplied.
[5.52%; 14.29/1; 31.8%]

14. The products of combustion of a hydrocarbon fuel, carbon to hydrogen ratio 0.85:0.15, are found to be CO_2-8%, CO-1%, O_2-8.5%. Calculate the A/F ratio for the process by two methods and hence check the consistency of the data.
[23.70, 23.53]

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15. A bomb calorimeter is used to determine the calorific value of coal. Results obtained are as follows:

Mass of coal	= 1.5 g
Mass of water	= 3.75 kg
Water equivalent of calorimeter	= 1.0 kg
Temp. rise of cooling water	= 2.62°C
Cooling correction factor	= 0.018°C
Specific heat of water	= 4.2 kJ/kg·K

Determine the calorific value of coal.

[29458 kJ/kg]

16. The following data was obtained during an experimental determination of the calorific value of a sample of coal by a bomb calorimeter.

Carbon	= 90%
Hydrogen	= 5%
Mass of coal	= 0.8 g
Mass of fuse wire	= 0.03 g
Calorific value of fuse wire	= 7000 J/g
Mass of water in calorimeter	= 2000 g
Water equivalent of colorimeter	= 400 g
Temp. rise of cooling water	= 3.2°C
Cooling correction	= 0.02°C

Calculate higher and lower calorific values of the fuel. Take h_{fg} = 2465 kJ/kg.

[45815.5 kJ/kg, 44708 kJ/kg]

17. The following results were obtained during experimentation on a Boy's gas calorimeter:

Gas used	= 2×10^{-3} m^3 at 30°C
Water circulated	= 0.5 kg
Temperature rise of water	= 10.1°C
Manometer reading	= 60 mm of Hg
Barometer pressure	= 750 mm of Hg
Specific heat of water	= 4.2 kJ/kg·K
Steam condensed	= 0.0015 kg

Calculate higher and lower calorific values of the gas at 1 atmosphere and 25°C.

[10117.3 kJ/m^3, 8368 kJ/m^3]

18. The percentage analysis of the fuel used in a boiler by mass is as follows:

C = 90%, H_2 = 3.5%, O_2 = 3%, N_2 = 1%, S = 1% and the remainder is ash.

(a) Find theoretical amount of air required and the mass analysis of dry products of combustion.

(b) If 50% excess air is supplied, determine the volumetric analysis of dry products of combustion.

[(a) 11.583 kg/kg of fuel; CO_2 = 27.00%; N_2 = 73%, (b) CO_2 = 18.31%, O_2 = 7.4% and N_2 = 74.29%]

19. The ultimate analysis of coal is as follows:

C = 82%, H_2 = 6%, O_2 = 4% and remainder is ash. Determine the amount of theoretical air required for complete combustion. If the actual air supplied is 40% in excess and 80% of the given carbon is burnt to CO_2 and the remainder to CO, determine the volumetric analysis of dry products of combustion.

[11.41 kg/kg of fuel; O_2 = 7.22%, N_2 = 80.31%, CO_2 = 9.98% and CO = 2.49%]

20. A gaseous fuel has the following volumetric composition:

CO_2 = 5.5%, CO = 38.3%, CH_4 = 0.4%, O_2 = 0.1%, H_2 = 52.8% and N_2 = 2.9%.

Determine the air–fuel ratio by volume and the analysis of dry products of combustion, if 10% excess air is supplied.

[2.42:1; CO_2 = 18.2%, O_2 = 1.9%, N_2 = 79.9%]

21. A fuel mixture with volumetric analysis of 94.4% of CH_4, 3.4% of C_2H_6, 0.6% of C_3H_8, 0.5% of C_4H_{10} and 1.1% of N_2, burns completely with 20% excess air in a reactor operating steadily. The fuel flow rate is 0.1 kmol/h. Determine the volume flow rate of air in kmol/h.

22. A coal sample has gravimetric analysis of 77.39% of carbon, 4.1% of hydrogen, 5.31% of oxygen, 1.62% of nitrogen, 1.1% of sulphur and the rest is ash. For complete combustion, 110% of the theoretical amount of air is supplied. Calculate the air–fuel ratio on the mass basis.

23. Decane ($C_{10}H_{22}$) burns with 95% of theoretical air, producing a gaseous mixture of CO_2, CO, H_2O and N_2. Determine the

(a) air–fuel ratio on a molar basis,

(b) analysis of products on a dry molar basis.

24. A boiler uses coal which has the mass composition as: C = 84%, H_2 = 3% and the remainder as ash. The Orsat apparatus gave the volumetric analysis as follows: CO_2 = 11.5%, O_2 = 8.4% and N_2 =

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80.1%. Calculate the air–fuel ratio and the mass of excess.

[17.8:1; 7.0 kg]

25. The volumetric analysis of dry exhaust gases as determined by an Orsat apparatus is CO_2 = 10%, CO = 11.5% and O_2 = 8%.

The fuel used has the mass composition as C = 80%, H_2 = 6%, O_2 = 7% and the remainder being ash. Determine by weight the air supplied per kg of fuel and the excess air supplied.

[17 kg/kg of fuel; 5.95 kg/kg of fuel]

26. 100 kmol of propane (C_3H_8) together with 3572 kmol of air enters a furnace per unit time. The products are carbon dioxide, carbon monoxide and unburnt fuel. Determine the percentage excess or per cent deficiency of air, whichever is appropriate.

27. A certain kind of petrol consists of 86% of carbon and 14% of hydrogen by mass. If the fuel is burnt with 20% excess air and the combustion is complete, calculate volumetic composition of products of combustion.

[CO_2 = 10.97%, H_2O = 10.73%, O_2 = 3.27%, N_2 = 75%]

28. The enthalpy of combustion of propane gas, C_3H_8, at 25°C with H_2O in the products in the liquid phase is –50360 kJ/kg. Calculate the enthalpy of combustion with H_2O in the vapour phase per unit mass of fuel and per unit amount of substance of fuel.

[– 46364 kJ/kg; – 2040030 kJ/kmol]

Objective Questions

1. The main constituents of a fuel are
   (a) hydrogen and oxygen
   (b) carbon and hydrogen
   (c) sulphur and hydrogen
   (d) sulphur and oxygen
2. Which of the following elements is not a constituent of coal?
   (a) Hydrogen (b) Manganese
   (c) Nitrogen (d) Carbon
3. The amount of heat generated per kg fuel is known as
   (a) higher calorific value
   (b) lower calorific value
   (c) calorific value
   (d) none of the above
4. The variety of coal having lowest calorific value is
   (a) steam coal (b) lignite
   (c) anthracite, (d) bituminous coal
5. The variety of coal having highest calorific value
   (a) steam coal (b) lignite
   (c) anthracite, (d) bituminous coal
6. The fuel generally used in cement industries and metallurgical furnaces is
   (a) hard coke (b) lignite
   (c) anthracite (d) pulverised coal
7. Coking is
   (a) burning of coal in a furnace
   (b) formation of coke in a boiler furnace
   (c) heating of coal in absence of air
   (d) none of the above
8. The fuel produced when wood is heated with a limited supply of air at 280°C is
   (a) coke (b) wood charcoal
   (c) bituminous coal (d) briquetted coal
9. The ultimate analysis of coal is done to determine the percentage of
   (a) carbon (b) ash
   (c) sulphur (d) moisture
10. The symptom showing incomplete combustion of coal is
    (a) presence of free carbon in exhaust
    (b) presence of oxygen in exhaust
    (c) presence of free nitrogen in exhaust
    (d) presence of free CO in exhaust
11. Calorific value of a liquid or solid fuel is the amount of heat liberated in kJ by
    (a) complete combustion of 1 kg of fuel
    (b) complete combustion of 1 m^3 of fuel

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(c) temperature rise of fuel by 1°C
(d) none of the above

12. The calorific value of a gaseous fuel is the amount of heat liberated in kJ by
(a) complete combustion of 1 kg of fuel
(b) complete combustion of 1 m^3 of fuel
(c) temperature rise of fuel by 1°C
(d) none of the above

13. A good fuel has
(a) low ignition point and high calorific value
(b) low ignition point and low calorific value
(c) high ignition point and high calorific value
(d) high ignition point and low calorific value

14. The amount of heat obtained by complete combustion of 1 kg of fuel when the products of combustion are cooled to the temperature of supplied air is known as
(a) calorific value
(b) higher calorific value
(c) lower calorific value
(d) none of the above

15. One kg of carbon requires 4/3 kg of oxygen and the mass of carbon monoxide produced is

(a) $\frac{7}{3}$ kg (b) $\frac{8}{3}$ kg
(c) $\frac{11}{3}$ kg (d) $\frac{11}{7}$

16. One kg of carbon requires 8/3 kg of oxygen and the mass of carbon dioxide produced is

(a) $\frac{7}{3}$ kg (b) $\frac{8}{3}$ kg
(c) $\frac{11}{3}$ kg (d) $\frac{11}{7}$

17. One kg of CO requires 4/7 kg of oxygen and the mass of carbon dioxide produced is

(a) $\frac{7}{3}$ kg (b) $\frac{8}{3}$ kg
(c) $\frac{11}{3}$ kg (d) $\frac{11}{7}$

18. The mass of carbon per kg of flue gas is given by

(a) $\frac{11}{3}CO_2 + \frac{3}{7}CO$ (b) $\frac{3}{11}CO_2 + \frac{3}{7}CO$
(c) $\frac{3}{7}CO_2 + \frac{11}{3}CO$ (d) $\frac{7}{3}CO_2 + \frac{11}{3}CO$

19. Which of the follwing is not a petroleum product?
(a) Petrol (b) Kerosene
(c) Methylated sprit (d) Lubricating oil

20. Liquid fuels as compared to solid fuels have the calorific value.
(a) higher (b) lower
(c) same (d) none of the above

21. A bomb calorimeter is used to determine
(a) higher calorific value of solid or liquid fuel
(b) lower calorific value of solid or liquid fuel
(c) higher calorific value of gaseous fuel
(d) lower calorific value of gaseous fuel

22. In a bomb calorimeter, the fuel burns at constant
(a) volume (b) pressure
(c) temperature (d) entropy

23. The gas having higher calorific value is
(a) water gas (b) coke-oven gas
(c) blast-furnace gas (d) producer gas

24. The gas having lower calorific value is
(a) coal gas (b) coke-oven gas
(c) blast-furnace gas (d) producer gas

Answers

1. (b) 2. (b) 3. (c) 4. (b) 5. (c) 6. (d) 7. (c) 8. (b)
9. (a) 10. (d) 11. (a) 12. (b) 13. (a) 14. (c) 15. (a) 16. (c)
17. (d) 18. (b) 19. (c) 20. (a) 21. (a) 22. (a) 23. (b) 24. (c)

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17

Steam Generators

Introduction

A *steam boiler* or *steam generator* is a closed vessel in which water is heated, vaporised and converted into steam at a pressure higher than the atmospheric pressure. The heat energy required for steam generation is produced by burning fuel in the furnace. The steam produced in the boiler may be used for producing power, for industrial processes or for heating purposes. All power plants as well as industries are using steam.

The *advantages* of the use of steam are the following:

1. It is capable of supplying process heat at constant temperature while condensing.
2. It is cheap, and can be produced everywhere.
3. It is a clean, odourless and tasteless source of heat energy.
4. It can be used repeately again and again as well as first used for power generation and then for process heating.
5. Its flow rate can easily be controlled and readily distributed.

The *main applications* of steam are the following:

1. The heat content of steam is large and thus it is suitable for process heating (for curing, drying, etc.) in many indusrtries like sugar mills, textile mills, and chemical industries.
2. It is also used for power generation in thermal power plants.
3. Due to its large heat content, steam is used for cooking items like steamed rice, *idlies*, etc.
4. Steam can also be used for heating buildings and producing hot water in winter.
5. Steam is also used for creation of vacuum, ejection of gases and sterlisation.

17.1 INDIAN BOILER REGULATION (IBR)

An IBR Steam Boiler means any closed vessel exceeding 22.75 litres in capacity and which is used expressively for generating steam under pressure. It includes any mounting or other fitting attached to such a vessel, which is wholly or partly under pressure when the steam is shut off.

The Indian Boiler Regulations (IBR) Act 1923 by the Government of India, was enacted to consolidate and amend the law relating to steam boilers. The main features of Indian Boiler Regulations are as follows:

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1. The boiler should be operated after it has been registered with the chief inspector of boilers.
2. The maximum working pressure of the boiler has to be determined by the Boiler Inspector. In any circumstances, the operator should not run the boiler above this pressure.
3. In case of any accident, the boiler owner should submit a report containing full details of nature and cause of accident within 24 hours of occurrence of the accident.
4. The rules and regulation, procedure for registration, inspection, decision upon maximum working pressure and other conditions, etc., are subjected to revision by a Central Board under the control of the Government of India.
5. Violation of any law in the Boiler Act is liable to prosecution and punishment with fine.

17.2 BOILER SYSTEMS

The boiler system comprises of a feed-water system, steam system and fuel system. The *feed-water system* supplies treated water to the boiler and regulates it automatically to meet the steam demand. Various valves and controls are provided to access for maintenance and monitoring.

The *steam system* heats and vaporises the feed water and controls the steam produced in the boiler. Steam is directed through a piping system to the application. Throughout the system, steam pressure is regulated using valves and monitored with steam pressure gauges.

The *fuel system* consists of all equipment used to supply of fuel to generate the necessary heat. The equipment required in the fuel system depends on the type of fuel used in the system.

17.2.1 Classification of Boilers

There are a large number of boiler designs, but they may be classified according to the following criteria:

1. According to Relative Passage of Water and Hot Gases

(*a*) *Water Tube Boiler* A boiler in which the water flows through a number of small tubes which are surrounded by hot combustion gases, e.g., Babcock and Wilcox, Stirling, Benson boilers, etc.

(*b*) *Fire Tube Boiler* The hot combustion gases pass through the boiler tubes, which are surrounded by water, e.g., Lancashire, Cochran, locomotive boilers, etc.

2. According to Water Circulation Arrangement

(*a*) *Natural Circulation* Water circulates in the boiler due to density difference of hot and cold water, e.g., Babcock and Wilcox boilers, Lancashire boilers, locomotive boilers etc.

(*b*) *Forced Circulation* A water pump forces the water along its path, therefore, the steam generation

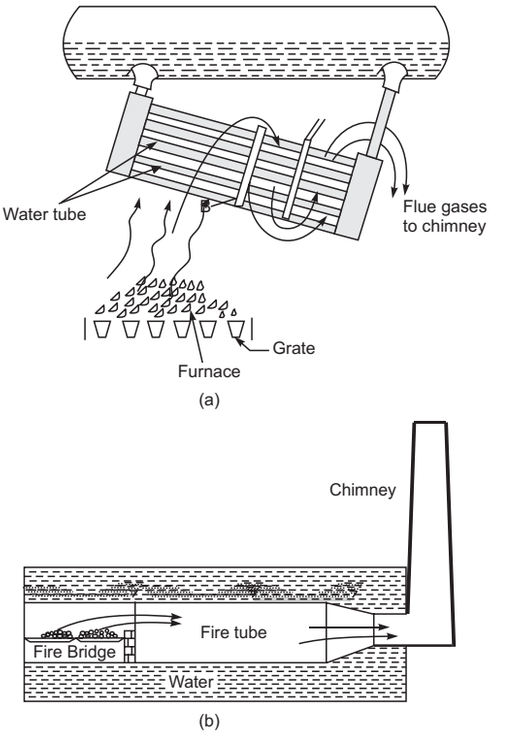

Fig. 17.1 *(a) Water tube boiler (b) Fire tube boiler*

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rate increases, e.g., Benson, La Mount, Velox boilers, etc.

3. According to the Use

(*a*) *Stationary Boiler* These boilers are used for power generation in thermal power plants or process steam in plants.

(*b*) *Portable Boiler* These are small units of mobile boilers and are used for temporary uses at the sites.

(*c*) *Locomotive* These are specially designed boilers. They produce steam to drive railway engines.

(*d*) *Marine Boiler* These are used on ships.

4. According to Position of the Boilers Horizontal, inclined or vertical boilers

5. According to Position of Furnace

(*a*) *Internally Fired* The furnace is located inside the shell, e.g., Cochran, Lancashire boilers, etc.

(*b*) *Externally Fired* The furnace is located outside the boiler shell, e.g., Babcock and Wilcox, Stirling boilers, etc.

6. According to Pressure of Steam Generated

Low-Pressure Boiler A boiler which produces steam at pressures of 15–20 bar is called a *low-pressure boiler*. This steam is used for process heating.

Medium-Pressure Boiler It has a working pressure of steam from 20 bar to 80 bar and is used for power generation or combined use of power generation and process heating.

High-Pressure Boiler It produces steam at a pressure of more than 80 bar.

(*b*) *Sub-critical Boiler* If a boiler produces steam at a pressure which is less than the critical pressure (221.1 bar), it is called sub-critical boiler.

(*c*) *Supercritical Boiler* These boilers produce steam at a pressure greater than critical pressure. These boilers do not have an evaporator and the water directly flashes into steam and thus they are called *once through boilers.*

7. According to Charge in the Furnace

(a) Pulverised fuel,
(b) Supercharged fuel, and
(c) Fluidised bed combustion boilers

17.2.2 Principal Parts and their Functions

Generally, a boilers consists of the following parts:

1. Drum The *boiler drum* consists of a shell and end heads. The shell of the boiler consists of one or more steel plates bent into the cylindrical form and riveted or welded together. The ends of the shell are closed by means of the flat or curved plates called *boiler head* and a boiler drum is formed.

2. Setting It is also called foundation and is constructed of bricks. It supports the boiler drum and other components. It forms the wall of the furnace, combustion chamber and passage to flue gases.

3. Grate It is the space located below the furnace and consists of cast-iron bars upon which the fuel is burned. The air can pass through the spaces between the bars and can support the combustion process, the ash can fall down through these spaces.

4. Furnace It is the space above the grate and below the boiler shell. It is the space where the volatile matter and combustible gases are burnt and flue gases are generated.

5. Flue Gases It is the hot mixture of products of combustion, generated in the furnace.

6. Flue The hot gas passage in the boiler is known as flue. It provides the direction to the hot gases to pass around the boiler.

7. Heating Surface It is the surface of the boiler which is exposed to hot flue gases on one side, water on other side.

8. Stocker It is a mechanical system for charging of coal to the furnace and keep the firing continued.

9. Water Wall The closely spaced water tubes arranged near the furnace wall form a layer like

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a wall and hence are called water wall. The tube surface in the water wall receives the heat by radiation.

10. Water Space The space of the boiler shell occupied by water is called *the water space*. The level of water in the boiler can be seen through the water level indicator.

11. Steam Space The entire space of boiler shell which is not occupied by the water is called *steam space*.

12. Feed Water The water supplied to the boiler is called *feed water*. The pump which supplies the water is called *feed pump*.

13. Working Pressure It is the pressure of steam generated in the boiler and superheater.

14. Economiser The feed water supplied by the feed pump is heated by the waste hot gases before they escape to the chimney. Therefore, some of the waste heat is recovered by feed water and plant efficiency improves.

15. Air Preheater Similar to feed water heating, the fresh air going to the furnace is also preheated to improve the combustion process. The air-heating system device is known as the air preheater.

16. Superheater This is the device which heats the saturated steam generated in the boiler. The superheaters are located above the furnace and they increase the heat content of steam without increasing its pressure.

17.2.3 Characteristics of a Good Boiler

A good boiler should have the following characteristics:

1. The boiler should have maximum steam-generation rate with minimum fuel consumption.
2. It can be started or stopped quickly.
3. Its initial cost, running and maintenance cost should not be high.
4. Its erection time should be less and its parts should be easily dismantable.
5. The boiler should have reliable controls and safety apparatus.
6. It should have high rate of heat transfer and better combustion efficiency.
7. It should be able to accomodate the load variation.
8. It should occupy less floor space.
9. It should be trouble free, and require less attention and less maintenance.
10. It should be free from manufacturing defects.
11. Mud should not get deposited on the heating surface. Soot or scale should not be deposited on the tubes.
12. All parts of the boiler should be accessible for cleaning and inspection.
13. It should conform to IBR acts.

17.2.4 Factors Affecting the Selection of Boilers

The following factors should be considered, while selecting a boiler:

1. The *steam-generation* rate of the boiler at working pressure,
2. The *type of fuel* used and its rate of burning,
3. *Availability of fuel* and water,
4. *Floor space* occupied by the boiler,
5. The *type of load*—steady, fluctuating, etc.
6. *Initial cost*, *running cost* and *maintenance cost* of the boiler.

17.3 COMPARISON BETWEEN FIRE TUBE AND WATER TUBE BOILERS

See Table 17.1

17.4 FIRE-TUBE BOILERS

17.4.1 Simple Vertical Boiler

It is the simplest form of an internal furnace, vertical fire-tube boiler as shown in Fig. 17.2. It is a portable boiler and it requires a small floor space.

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Table 17.1

Sr.	*Fire-tube boiler*	*Water-tube boiler*
1.	The hot flue gases pass through tubes and water surrounds them.	Water passes through tubes and hot flue gases surround them.
2.	These are operated at low pressures up to 20 bar.	The working pressure is high enough, up to 250 bar in supercritical boilers.
3.	The rate of steam generation and quality of steam are low, therefore, not suitable for power generation.	The rate of steam generation and quality of steam are better and suitable for power generation.
4.	Load fluctuations cannot be handled.	Load fluctuations are easily handled.
5.	It requires more floor area for a given output.	It requires less floor area for a given output.
6.	They are bulky and difficult to transport.	These are light in weight, hence transporation is not a problem.
7.	Overall efficiency is up to 75%.	Overall efficiency with an economiser is up to 90%.
8.	Water does not circulate in a definite direction.	Direction of water circulation is well defined.
9.	The drum size is large and damage caused by bursting is large.	If any water tube is damaged, it can be easily replaced or repaired.
10.	Less initial cost, but cost per unit is more.	Initial cost is very high, but cost per unit is low.
11.	Simple in design, easy to erect and low maintenance cost.	Complex design, difficult to erect and high maintenance cost.
12.	Less skill is required for efficient operation.	Skilled operators are required.
13.	The treatment of feed water is not very essential, as overheating due to scale formation cannot burst the thick shell.	Treatment of feed water is very essential as small scale deposits inside the tubes can cause overheating and bursting.
14.	Used in process industry.	Use in large power plants.

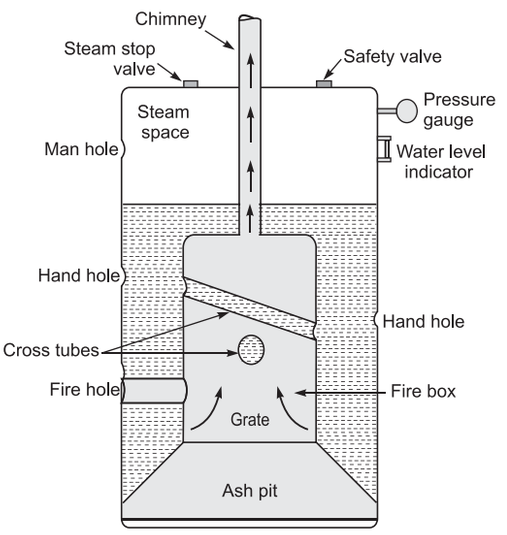

Fig. 17.2 *Simple vertical boiler*

The steam rating normally does not exceed 2500 kg per hour and pressure is limited to 10 bar.

Construction It consists of a vertical, cylindrical shell, equipped with a fire box in the bottom, water space in the middle and steam space in the upper portion. The grate is placed at the bottom of the fire box and coal is fired in the fire box. An ash pit is located at the bottom of the grate to collect the ash of burnt coal, which is periodically removed.

One or more cross tubes are either flanged or riveted to the water space and are located in the fire box to increase the heating surface area and to improve the water circulation. A short chimney is connected at the top of the fire box to discharge the waste flue gases at some greater height. Man hole and hand holes are provided for cleaning the interior of the boiler shell and cross tubes.

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The boiler consists of a pressure gauge, water-level indicator, safety valve, steam stop valve and a man hole as mountings to provide safety and ease of working.

Working Fuel burns on the grate in the fire box. The resulting hot flue gases are allowed to pass around the cross tubes. The water surrounding the cylindrical fire box also receives heat by convection and radiation. Thus steam is produced. The water circulation in the boiler depends on the density difference in the water, created by temperature difference in the water.

17.4.2 Cochran Boiler

Construction and Operation It is a vertical, coal- or oil-fired, fire-tube boiler. It is the modification of a simple vertical boiler with increase in the heating surface area. The flue gases from the furnace are passed through a number of small tubes surrounded by water.

As shown in Fig. 17.3, a Cochran boiler consists of a cylindrical shell with a hemispherical crown, grating, fire box, combustion chamber, number of smoke tubes, smoke box, chimney and various mountings. The grate is placed at the bottom of the hemispherical furnace. The coal is fed into the grate through the fire door and ash formed is collected in the ash pit located just below the grate, and then it is removed manually.

Working The fuel is burnt on grating. The hot-flue gases pass through a short flue to a combustion chamber, small horizontal smoke tubes and are then collected in the smoke box, from where they are discharged to the atmosphere through the chimney.

The heat is transferred to water by radiation through the dome of the fire place and by convection from the walls of the smoke tubes. On heating, the water is vaporised and converted into steam. The generated steam is collected in the steam space above the water. This steam is then taken for use through the main steam stop valve.

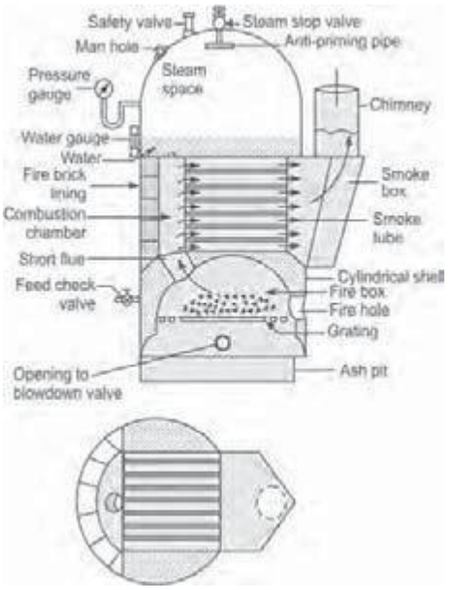

Fig. 17.3 *Cochran boiler*

The man hole is provided in the the crown of boiler for periodic cleaning and maintenance. A mud hole is provided at the bottom for draining out the muddy water from the boiler. The pressure gauge, water gauge, blow off cock, feed check valve, feed pump, fusible plug and chimney are provided for proper functioning of the boiler.

The Cochran boilers are made in sizes from 1 m to 3 m in diameter, 2 m to 6 m heigh. Its steam generation rate is approximately 3600 kg/h with working pressure limited to 11 bar.

Salient Features

1. The spherical crown and spherical shape of a fire box are the special features of this boiler. These shapes require least material for a given volume.
2. It is very compact and requires minimum floor area.
3. Any type of fuel can burn in the boiler.
4. It is well suited for small industries.
5. It gives about 70% thermal efficiency with coal firing.

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17.4.3 Lancashire Boiler

It is a horizontal, internally fired, fire tube, natural circulation, stationary boiler. It is a widely used boiler due to its good steam-generation capacity. This boiler can also be used for power generation at a moderate steam pressure of 15 bar.

Construction As shown in Fig. 17.4, the boiler consists of a large shell supported by refractory brick masonry. The cylindrical shell is usually 2 to 3 m in diameter and 7 to 9 m long. Two large, horizontal and parallel flue gas tubes pass through shell. The fire place is located in front of the flue tubes. In brick work, a flue passage *A* below the boiler shell, two flue passages *B* and *C* at the sides of boiler are formed. The flue passages *B* and *C* are connected to a chamber and then to the chimney.

The dampers in the form of sliding doors are located at the end of side flues to control the flow of gases. They regulate the combustion rate as well as steam-generation rate. These dampers are operated by a chain passing over a pulley at the front of the boiler.

The boiler is also provided with usual mountings like pressure gauge, water level indicator, steam stop valve, safety valve, low water and high steam safety valve, man hole on the top of the shell. The low water and high steam alarm gives an audio signal for low water level and high steam pressure. The blow off cock, and feed check valve are also provided in front of the boiler and the fusible plug is provided in the main flues just over the grates to prevent the overheating of boiler tubes by extinguishing the fire, when water level falls below a particular level.

Working The fuel is burnt at the grating and the hot gases travel along internal flue tubes followed by flue passage *A* and then in side passages *B* and *C*. The flue gases are then collected in the chamber before they lead to the atmosphere through a chimney. The hot flue gases transfer its maximum heat contents to water during its long passages. The water is converted into steam and collected in the steam space in the shell and it is then taken out through the steam stop valve for use.

Special Features

1. Its heating surface area per unit volume is considerably large.
2. Its maintenance is easy.
3. This boiler can easily handle the load fluctuation to large steam capacity.
4. It is highly suitable for process industries.

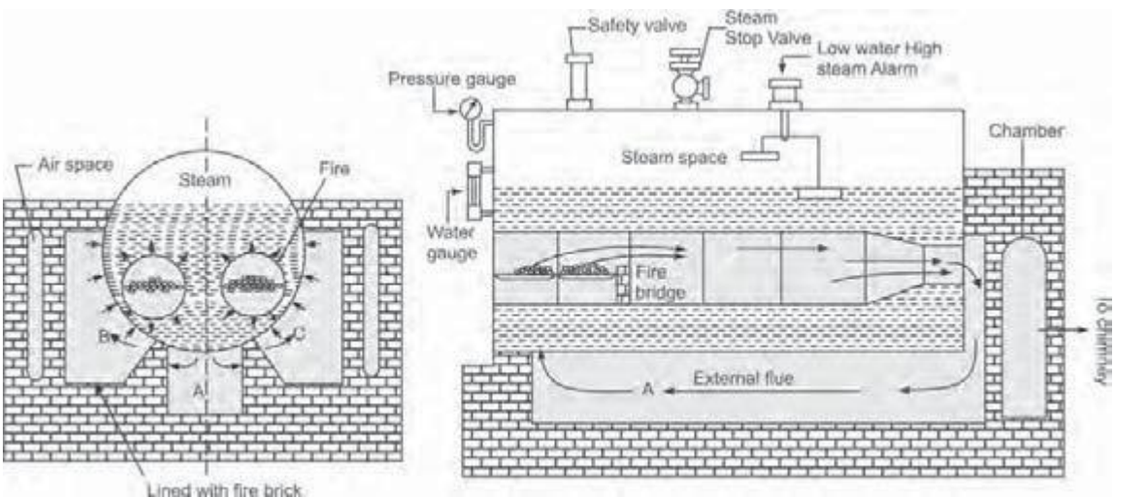

Fig. 17.4 *Lancashire boiler*

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17.4.4 Cornish Boiler

The Cornish boiler is very similar to a Lancashire boiler. It is also a horizontal, fire tube, internally fired, natural circulation, stationary boiler. However, it differs from a Lancashire boiler in two respects.

1. It is small in size.
2. It has only one flue tube.

The Cornish boiler as shown in Fig. 17.5 consists of a horizontal cylindrical shell of 1.25 m to 1.75 m diameter and 4 m to 7 m long. It has flat ends. The flue tubes containing the furnace are located in the centre of the boiler shell. The products of combustion from the fire grate first pass forward through the central flue tube and then return by the two side flues to the front of the boiler and again pass to the back end of the boiler along the bottom flue and finally get discharged through chimney.

The various mountings are provided on the boiler to provide safety. These are pressure gauge, water-level indicator, steam stop valve, fusible plug, blow off cock, high steam and low water safety valve, feed check valve and man hole.

It has a unique feature that any sediments in the feed water are deposited at the bottom of the shell, which is not in the zone of hottest flues gases. Thus, heating rate remains unaffected. The steam-generation capacity is 6500 kg/h at a pressure of 10.5 bar.

17.4.5 Locomotive Boiler

It is also an internally fired, horizontal, multi-tube, mobile, fire-tube boiler. The Locomotive boiler generates steam at a pressure of about 25 bar with a steam rate of 60–70 kg/h per square metre of the heating surface. This steam rate is quite high and thus it is mostly used on locomotives for generating steam to drive a steam engine train.

Construction A view of the locomotive boiler is shown in Fig 17.6. It consists of the following main parts:

1. Boiler shell (horizontal)
2. Fire box surrounded by water
3. Smoke tubes
4. Super-heater tubes
5. Smoke box
6. Chimney
7. Blast pipe
8. Damper
9. Steam Dome with regulator
10. Safety valve and other mountings

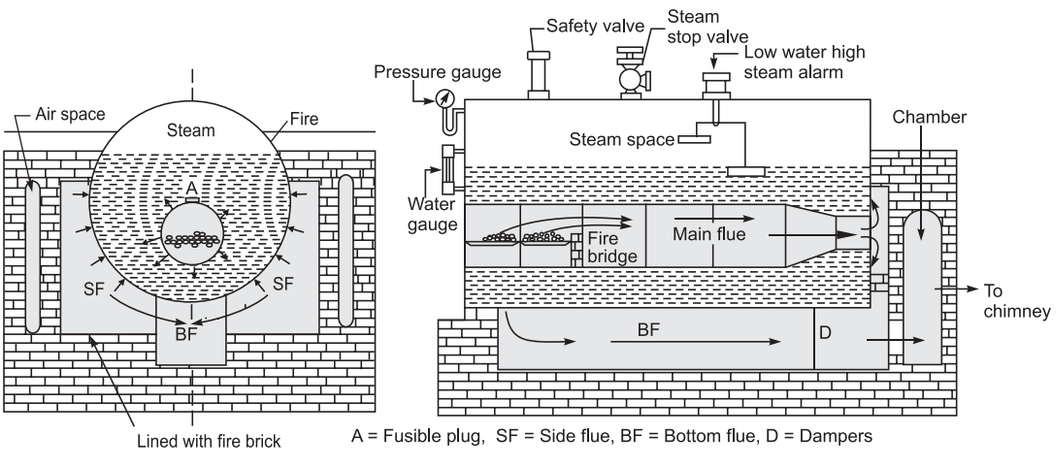

Fig. 17.5 *Cornish boiler*

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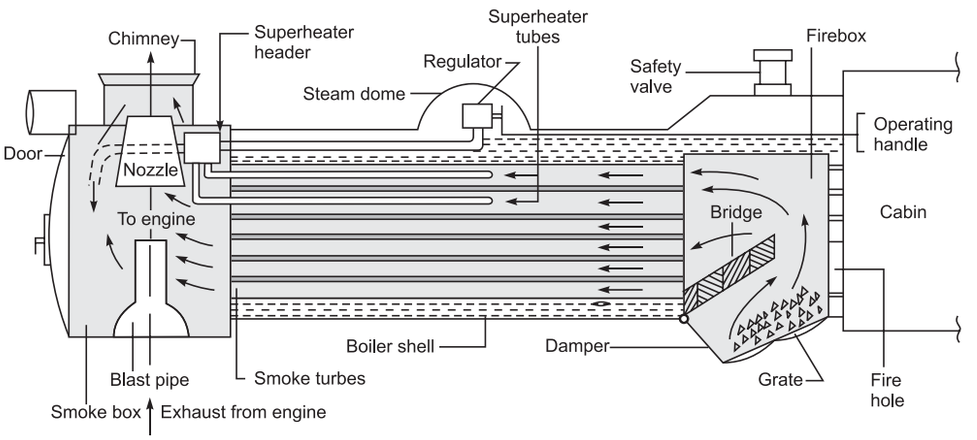

Fig. 17.6 *Locomotive boiler*

It has horizontal shell, 1.5 m is diameter and 4 m long. The coal is fed into the fire box through the fire door and burnt on the grate. The entire fire box is properly heated by deflecting the flue gases from the grate by a bridge. The heat is transferred to water through the walls of the fire box, smoke tubes and superheater tubes.

Working The flue gases are formed due to combustion of coal in presence of air on the grate. These gases rise up and are deflected by a brick arch for their proper distribution to pass through the smoke tubes and over superheater tubes and then finally get discharged into the atmosphere through a short chimney.

The steam generated is collected in the steam space above the water in the boiler drum. A steam regulator is located in the steam dome and is operated by a long regulator rod from the engine cabin by the driver. When this valve is opened, the wet steam passes through the superheater header and to the superheated tubes located in a smoke tube. As steam passes through superheated tubes, it picks up additional heat and becomes superheated. The superheated steam is then supplied to the steam engine.

Since the long chimney cannot be installed on a locomotive engine, thus sufficient suction effect (natural draught) cannot be created. Therefore, the exhausted steam from the engine is discharged through a nozzle at the top of the blast pipe (exhaust pipe). The jet of steam draws the flue gases to the atmosphere through the short chimney, and thus creates sufficient suction in the fire box to suck the fresh air.

17.4.6 Scotch Marine Boiler

The Scotch Marine boiler is the most commonly used fire-tube marine boiler. It has a large heating surface area for the space occupied. It has excellent steaming capacity of about 1000 kg/h at a pressure of 17 bar. It is compact in size and occupies a small floor space. It is a self-contained boiler and supported by a cradle, securely fastened to the frame of the ship. Adjustable stays hold the boiler in place in the cradle.

Construction Figure. 17.7 shows the schematic arrangement of a Scotch Marine boiler. The boiler consists of a cylindrical shell that houses one to four cylindrical, corrugated steel furnaces. These furnaces are internally fired and surrounded by water. A combustion chamber is located above the furnace and it is also surrounded by water. The boiler has a number of tubes passing from the front plate of combustion chamber to the front plate of the shell. These tubes are surrounded by water.

Working The fuel burns in the furnace on the grate. The hot flue gases resulting due to burning of fuel move to the combustion chamber. Then they

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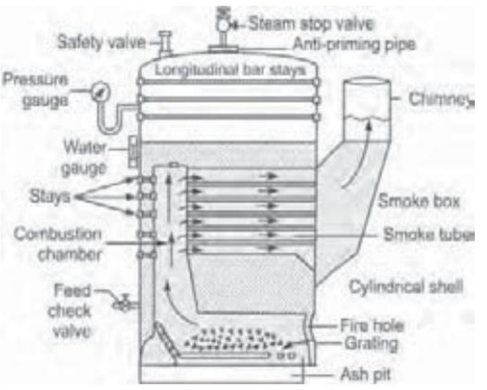

Fig. 17.7 *Scotch Marine boiler*

travel to the smoke box through the fire tubes and finally get discharged to the atmosphere through uptake and chimney. The heat is transferred to water around the furnace, combustion chamber, and fire tubes and steam is generated.

17.5 WATER TUBE BOILERS

17.5.1 Babcock and Wilcox Boiler

The babcock and Wilcox boiler is probably the first water tube boiler designed and widely used. A schematic of the Babcock and Wilcox boiler is shown in Fig. 17.8 and its construction and operation are stated below.

Construction

1. A horizontal Steam and Water Drum This is the main part of the boiler. It is supported by a steel structure at a certain height and is independent of brick works. The size of the boiler drum is small as compared with the boiler drum of a fire-tube boiler of same capacity. It contains water and steam. All safety and control devices are mounted over the boiler drum.

2. A Bundle of Steel Tubes The front end of the boiler drum is connected to the uptake header (water box) by a short tube and the rear end is connected to the downtake header (water box) by a long tube. In between the headers, a number of small-diameter steel tubes are fitted at an angle of 5° to 15° with the horizontal to promote the water circulation. These steel tubes are arranged in the combustion chamber in a zigzag manner so that more surface area of the tube is exposed to hot gases.

3. Combustion Chamber It is the space above the grate, below the front end of the drum where combustion of fuel takes place. This chamber is enclosed by brickwork and it is lined from

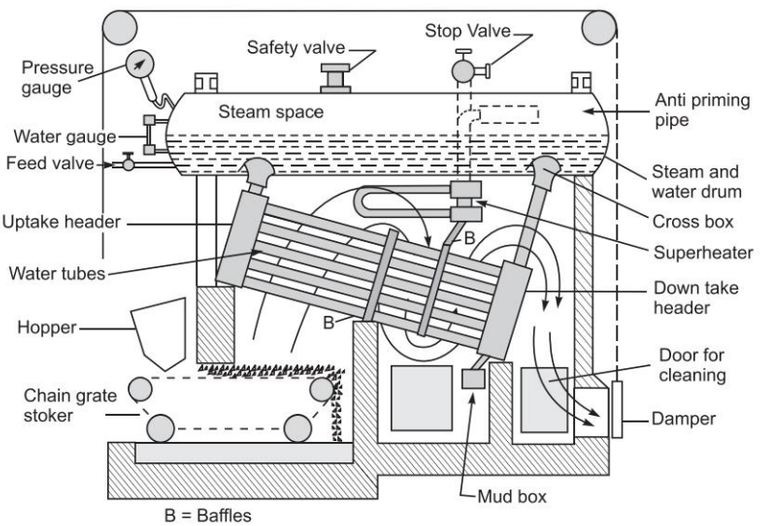

Fig. 17.8 *Babcock and Wilcox boiler*

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inside by fire bricks. Doors are provided to give access for cleaning, inspection and repairing. The Combustion chamber is divided into three separate compartments by baffles. Thus, the first compartment above the furnace is the hottest and the last chamber is of lowest temperature. This makes the path of hot gases longer before leaving the boiler through the chimney. The superheater is placed between the drum and water tubes. During the first turn of the hot gases, the gases are passed over superheater tubes. Dampers are provided at the rear end of the chamber to regulate the fresh air supply for maintaining proper combustion of fuel.

4. Safety and Control Devices Safety and control devices are called mountings, as basically these devices are mount over a boiler drum. These are the safety valve, pressure gauge, water-level indicator, feed check valve, steam-stop valve, blow of cock, fusible plug and man hole.

Operation The water is pumped by a feed pump and it enters the drum through the feed check valve up to the prespecified level so that the headers and tubes are flooded always. When the combustion takes place above the grate, the products of hot gases come out and rush through each compartment of the combustion chamber. Hence, the front portion of the tubes has highest temperature and the rear portion has the lowest. When water is heated inside the tubes, it becomes lighter and rises up in the tube. Due to continuous heat supply, some of the water gets vaporised into steam inside the tubes and the mixture of water and steam enters the boiler drum through the uptake header. The cold water from the boiler drum comes down through the downtake header and enters the lower end of the water tubes for getting heated further. This natural circulation of water remains continuous due to difference in temperature. Such a circulation is called ***thermosiphon system***.

The steam generated gets collected in the steam space above water space in the boiler drum. In order to remove all water particles from the steam, it is finally passed through the superheater tubes for its superheating. The superheated steam is then available for use.

Special Features

1. Its evaporating capacity is quite high compared with other boilers (20,000 to 40,000 kg/h). The operating pressure lies between 11.5 to 17.5 bar.
2. The draught losses are minimum.
3. The defective tubes can be replaced easily.
4. The entire boiler rests on an iron structure, independent of brick structure.

17.5.2 Stirling Boiler

It is a water-tube boiler and has two water steam drums at the top and a mud drum at the bottom. These drums are connected by a number of small bent tubes through which water flows. The use of bent tubes avoids the thermal stresses at the joints near the drum due to expansion and contraction.

Figure 17.9 shows the principle working of a Stirling boiler. The steam drums containing water and steam are connected in series by the tubes above and below the water level. The upper tubes are steam-circulating tubes and are used to equalise the pressure and the lower water circulating tubes maintain same level of water in all drums.

Water is supplied to the first drum, which then passes to the mud drum through the rear bank of the tubes. The suspended impurities are collected in the mud drum. The water from the mud drum is circulated to other upper drums.

The flue gases from the grate rise above and pass over the bent tubes and then are deflected by baffles for their proper distribution. Finally they are discharged to the atmosphere through the chimney.

17.6 SOME INDUSTRIAL BOILERS

17.6.1 Packaged Boiler

The packaged boiler is so called because it comes as a complete package. Once delivered to site, it

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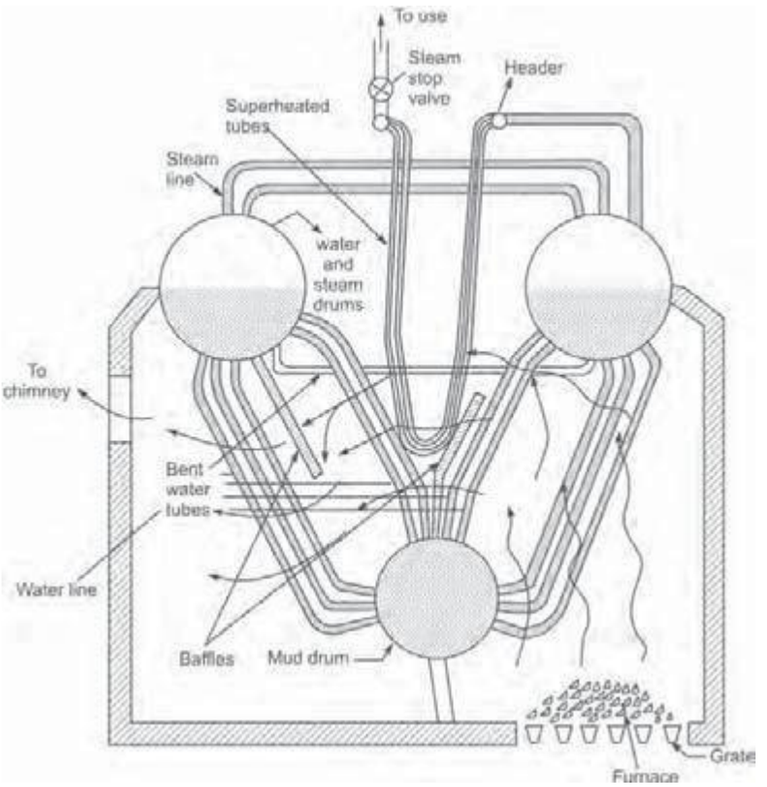

Fig. 17.9 *Schematic diagram of Stirling boiler*

requires only steam, water pipe work, fuel supply and electrical connections to be made for it to become operational. Package boilers are generally of a shell type with fire-tube design so as to achieve high heat transfer rates by both radiation and convection (refer Fig. 17.10).

Fig. 17.10 *Packaged boiler*

The salient features of package boilers are

(i) Small combustion space and high heat release rate resulting in faster evaporation,
(ii) Large number of small-diameter tubes leading to good convective heat transfer,
(iii) Forced or induced draft systems resulting in good combustion efficiency,
(iv) Number of passes resulting in better overall heat transfer,
(v) Higher thermal efficiency levels compared with other boilers.

These boilers are classified based on the number of passes—the number of the hot combustion gases passes through the boiler. The combustion chamber is taken as the first pass after which there may be one, two or three sets of fire tubes. The most common boiler of this class is a three-pass unit with

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two sets of fire-tubes and with the exhaust gases exiting through the rear of the boiler.

17.6.2 Pulverized Fuel Boiler

Most coal-fired power station boilers use pulverized coal, and many of the larger industrial water-tube boilers also use this pulverized fuel. This technology is well developed, and there are thousands of units around the world, accounting for well over 90% of coal-fired capacity. The coal is ground (pulverised) to a fine powder, so that less than 2% is 300 micro metre (μm) and 70–75% is below 75 microns, for bituminous coal.

The pulverised coal is blown with a part of the combustion air into the boiler plant through a series of burner nozzles. Secondary and tertiary air may also be added. Combustion takes place at temperatures from 1300–1700°C, depending largely on coal grade. Particle residence time in the boiler is typically 2 to 5 seconds, and the particles must be small enough for complete combustion to have taken place during this time.

This system has many advantages such as ability to fire varying quality of coal, quick responses to changes in load, use of high pre-heat air temperatures, etc. One of the most popular systems for firing pulverized coal is the tangential firing using four burners corner to corner to create a fireball at the centre of the furnace as shown in Fig. 17.11.

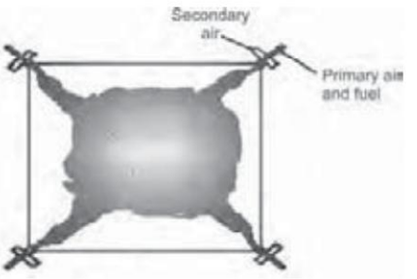

Fig. 17.11 *Pulverised fuel firing*

17.6.3 Fluidised Bed Combustion Boiler

In the fluidised bed combustion boiler, Fig. 17.12, the slightly compressed air is passed upward through a finely divided bed of solid particles such as sand

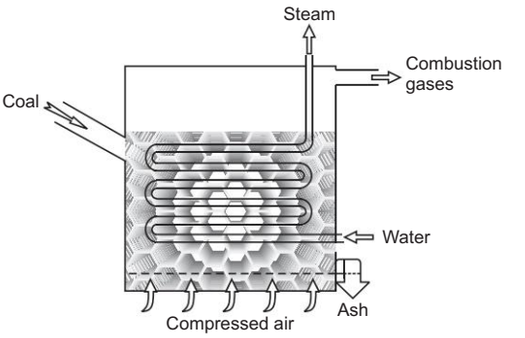

Fig. 17.12 *Fluidised bed combustion*

supported on a fine mesh. The fuel in powder form is fed downward. The air velocity supports the fuel in the air and creates bubble formation, vigorous turbulence and rapid mixing and combustion of fuel, and the bed is said to be fluidized.

If the sand in a fluidized state is heated to the ignition temperature of the coal and the coal is injected continuously in to the bed, the coal will burn rapidly, and the bed attains a uniform temperature due to effective mixing. Proper air distribution is very important for maintaining uniform fluidisation across the bed.

Fluidised bed combustion has significant advantages over conventional firing systems and offers multiple benefits, namely, fuel flexibility, reduced emission of noxious pollutants such as SO*x* and NO*x*, compact boiler design and higher combustion efficiency.

17.6.4 Supercharged Boiler

In a supercharged boiler, the combustion is carried under the pressure in the combustion chamber. More mass of air is supplied by increasing the density of air in an air compressor. Thus more fuel can burn in the presence of more oxygen. The exhaust gases from the combustion chamber are used to spin a gas turbine. The gas turbine drives a rotary compressor to supply compressed air to the combustion chamber. The supercharging boiler has certain advantages:

1. With the compressed air, the convection heat-transfer coefficient increases, which

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increases the heat-transfer rate. Thus, less combustion space is required in comparing to a conventional boiler.
2. A part of the gas turbine can be used to drive other auxiliaries.
3. Rapid start and efficient combustion.

17.7 HIGH-PRESSURE BOILERS

A boiler is called a high-pressure boiler when it operates with a steam pressure above 80 *bar.* The high-pressure boilers are widely used for power generation in thermal power plants.

If the feed-water pressure increases, the saturation temperature of water rises and the latent heat of vaporisation decreases. The feed water can be heated to saturation temperature in the economizer with the help of waste heat recovery from the exhaust gases escaping to the chimney. Then the boiler supplies only latent heat of vaporisation and superheat. Thus, a boiler operating at high pressure will require less heat addition for steam generation.

17.7.1 Features of High Pressure Boilers

High-pressure boilers have the following unique features:

1. Forced circulation of water
2. Large number of small-diameter tubes
3. Higher steam pressure and temperature
4. Improved mode of heat transfer
5. Improved method of heating
6. Pressurised combustion
7. Compactness
8. High efficiency
9. Intensive heating
10. Once through construction

1. Forced Circulation of Water In all modren high pressure boilers, the forced circulation of water is maintained with the help of pumps. It increases the mean temperature of heat addition and evaporation capacity of the boiler.

2. Large Number of Small-Diameter Tubes The surface-area-to-volume-ratio (area density) increases with the use of small-diameter tubes. It helps in high rate of heat transfer to water flowing inside the tubes. Therefore, a large number of small diameter tubes in a zigzag manner are used for water circulation in forced circulation. Further, use of a short tube reduces the pressure loss and gives better control over the quality of steam.

3. Higher Steam Pressure and Temperature The steam is generated at a pressure between 80 bar to 300 bar and temperature of 450°C to 585°C with two superheaters in series. The use of such steam is very suitable for power generation. It increases thermal efficiency of the plant and reduces the moisture contents in low pressure stages of expansion in the turbine.

4. Improved Mode of Heating Modren high-pressure boilers use the heat transfer by radiation along conduction and convection. The total heat-receiving equipment is divided into several parts, so they can easily be located in various zones of the furnace for most efficient heat transfer to the water circuit.

5. Improved Method of heating The high pressure boilers use the following methods of heating for improved heat transfer rate:

(i) *Evaporation of water* above critical pressure of steam
(ii) Heating of water by mixing superheated steam for high heat transfer rate
(iii) Increasing the combustion air velocity over the tube

6. Pressurised Combustion For increasing the combustion rate and thus heat-release rate, pressurised air is used in the furnace. It gives large amount of heat in a small space.

7. Compactness The high rate of heat transfer inside the boiler reduces the overall size of the boiler and the boiler becomes compact.

8. High Efficiency High-pressure boilers have better firing methods, monitoring, furnace conditions, control of flue gases and water velocity.

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Thus, the efficiency of such boilers ranges from 85 to 90%.

9. Intensive Heating The furnace temperature in high-pressure boilers is high enough, and therefore, 70% of heat is transferred to water by radiation, which is a faster way of heat transfer.

10. Once Through Construction In high-pressure boiler operating at and above the critical pressure, the water directly flashes into steam in the tube itself. It eliminates the need of a boiler drum.

17.7.2 Advantages of High-Pressure Boilers

1. High-pressure boilers use the forced circulation of water which ensures the positive circulation of water and increased evaporative capacity.
2. They require less heat for vaporisation.
3. They are compact, and thus require less floor space.
4. Due to high velocity of water, the tendency of scale formation is minimised.
5. All the parts are uniformly heated, and the danger of overheating is minimised.
6. The steam can be raised quickly to meet the variable load requirements without use of complicated control devices.
7. The plant efficiency is increased.
8. With the use of high pressure, the steam generation is economical.

17.7.3 La Mont Boiler

The La Mont boiler is a high-pressure, water-tube type boiler. It works on a forced-circulation principle. The water circulation is maintained by a centrifugal pump. Figure 17.13 shows the schematic of a La Mont boiler. The feed water is circulated through the water walls and drums continuously and prevents the tubes from being overheated.

The feed water first passes through the economiser. Most of the sensible heat is supplied to the feed water in the economiser. Then water enters the boiler drum. A water circulation pump draws

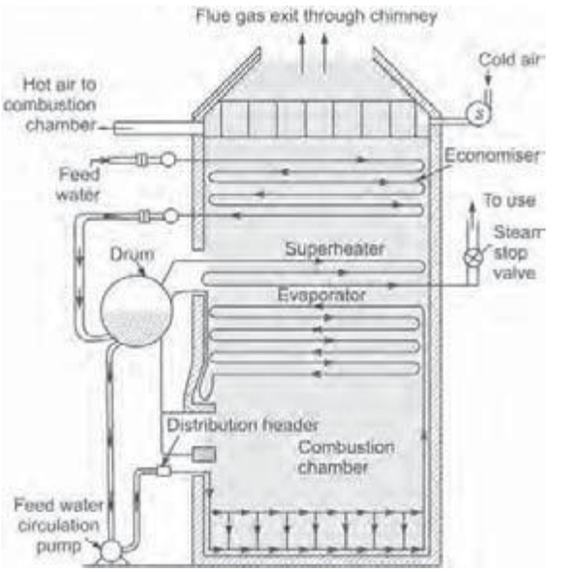

Fig. 17.13 *La Mont boiler*

water from the drum and delivers to the tubes of the evaporating section, where water is heated in a large number of small-diameter tubes and a mixture of steam and water is formed. This mixture is stored in the drum. The convective superheater draws the wet steam from the drum and heats the steam for its superheating. The superheated steam is supplied to a prime mover. The La Mont boiler generates approximately 50 tonnes of steam per hour at a pressure of 130 bar and a temperature of 500°C.

Advantages of a La Mont Boiler

1. With the use of small diameter tubes, the high heat-transfer rate is maintained.
2. The multiple-tube circuit gives flexibility for suitable location of heat transfer equipments.
3. With forced circulation of water through the tubes, a high evaporation rate is achieved.

17.7.4 Loeffler Boiler

This boiler is a water-tube boiler and also uses forced circulation of water. It uses superheated steam for vaporisation of feed water in the evaporator. The hot flue gases from the furnace are mainly used for superheating of steam.

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Figure. 17.14 shows the schematic of a Loeffler boiler. The high-pressure feed pump supplies the water to the economiser, where water is heated and then delivered to the evaporator. In the evaporator, the feed water is further heated with the help of superheated steam. The steam produced is then drawn from the evaporator drum by a steam-circulating pump and is then forced through tubes in the combustion chamber, where the radiant superheater is located before entering into the convective superheater. Thus, the steam becomes superheated. Approximately, two-thirds of the superheated steam is sent to the evaporator and the remaining is available for use. The flue gases from the combustion chamber move to the convection superheater and to the economiser before proceeding to the atmosphere through the chimney. The Loeffler boiler generates steam approximately at 100 tonne per hour at 140 bar.

Advantages of a Loeffler Boiler

1. This boiler uses forced circulation of water, and is thus capable to carry high salt concentrations than any other type.
2. It uses superheated steam in the evaporator for generation of steam from heated water. Thus the operation is salient and clean.
3. It is able to respond rapidly to sudden variations of load.
4. With the use of a radiant superheater, it makes better heat recovery from the boiler furnace.
5. The high circulation rate of steam over the tubes causes low temperature difference between the tube and steam.

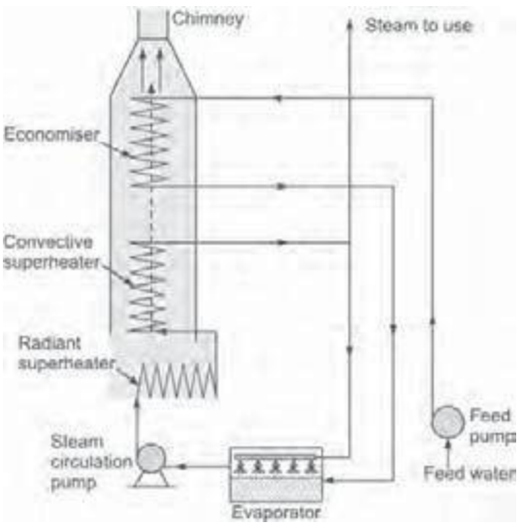

Fig. 17.14 *Loeffler boiler*

17.7.5 Velox Boiler

It is a subcritical boiler using supercharged furnace at 2–3 atmospheric pressure. The flue gas from the boiler drives the gas turbine, which in turn drives a rotary compressor to supply high-pressure air to the furnace.

The Velox boiler is operated on the principle that when the gas velocity exceeds the sonic velocity (velocity of sound), the heat transfer rate from the gas becomes much higher than that achieved with subsonic flow.

The schematic arrangement of Velox boiler is shown in Fig 17.15. The air is compressed in an axial flow compressor to a pressure of 2.5 bar. In presence of this pressurised air in the furnace, there is a high combustion rate, and thus a high rate of heat release. The generated flue gases pass through a nozzle section, where the velocity of flue gases is increased to sonic velocity.

The flue gases enter the evaporator section with sonic velocity and heat the water and steam. The flue gases coming out of the evaporator are further passed over a convective superheater, where a portion of their heat content is used to superheat the steam. The gases coming out of the superheater are used to drive a gas turbine. The gas turbine drives the air compressor. The flue gases coming out of the gas turbine pass through the economiser for heating the feed water.

The feed water after receiving heat in the economiser is pumped into the evaporator section tubes. The mixture of steam and water thus formed enters the separator with a spiral flow. The circular

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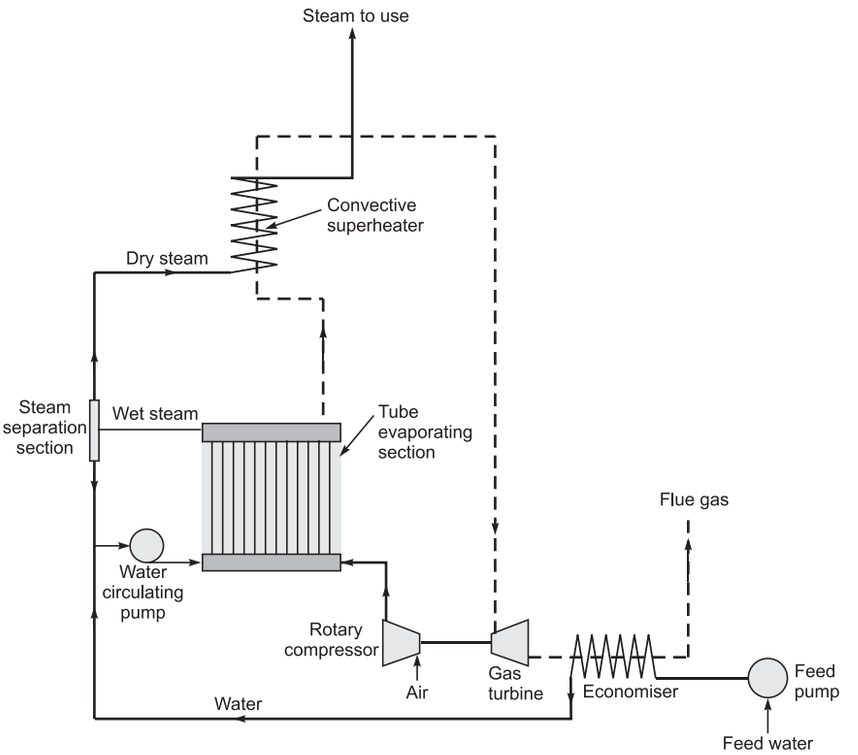

Fig. 17.15 *Velox boiler*

motion thus separates the heavier water particles by throwing them outward on the walls. This effect separates the steam from water. The separated steam then enters the convection superheater before going to use. The removed water from steam in the separator is again passed to the evaporator section tubes with the help of a pump.

17.7.6 Benson Boiler

The Benson boiler is a high-pressure (supercritical), drumless, once-through, water-tube boiler. The boiler uses forced-circulation heat-transfer mechanism and uses oil as fuel. It operates at a pressure of 250 bar, which is more than the critical pressure of water, and thus the latent heat of vaporisation becomes zero. This boiler as shown in Fig. 17.16 does not have any drum. The feed water enters one end of the tube and comes out as superheated steam from the other end. Thus, it is also called *a once-through boiler.*

The feed pump increases the water pressure to supercritical pressure and forces the water through tubes. It first passes through the economiser, where it is heated. Then it passes through the radiant water heater, where the water is further heated and its temperature increases to almost critical temperature. It then enetrs the transit heater, gets converted into steam and then passes through the convective superheater and finally becomes available for applications.

The thermal efficiency of a Benson boiler reaches up to 90% and it generates approximately 135 tonnes of steam per hour. It can be started within 15 minutes to produce the required flow rate of steam.

Advantages

1. It requires less floor space.
2. Its weight is 20% less than the other boilers.
3. It can be started or stopped very quickly.

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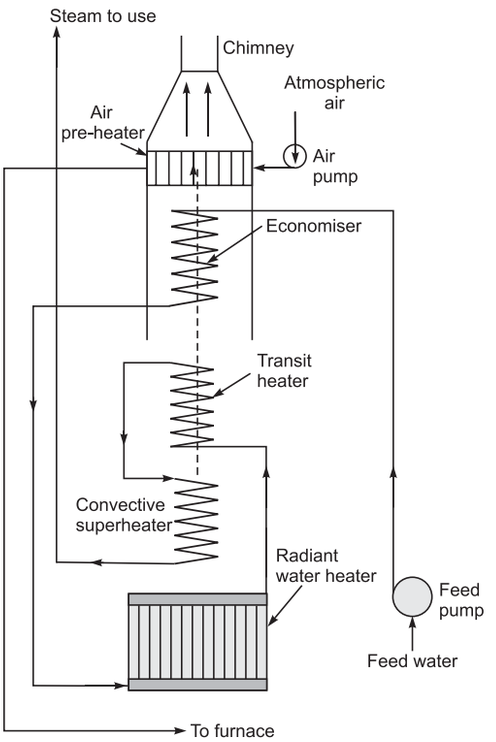

Fig. 17.16 *Benson boiler*

4. Parts can be transported very easily and can be assembled at the site because there is no drum.
5. It can be operated economically.
6. Explosion hazards are less severe because it consists of only small-diameter tubes and has very less storage capacity.

17.7.7 Ramsin Boiler

It is a supercritical boiler. It is very similar to the Benson boiler, except that its evaporator section consists of spiral tubes instead of straight tubes. Its water circuit is shown in Fig. 17.17.

Water is forced into the economiser by a feed pump, where it is heated and then it enters spiral evaporating tubes, where water flashes into steam. This steam then passes the superheater before going to some application.

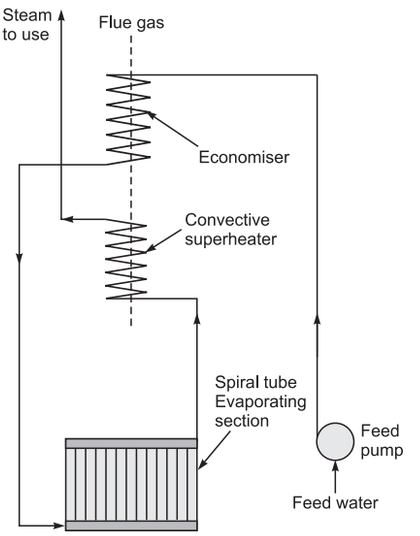

Fig. 17.17 *Ramsin boiler*

17.7.8 Supercritical Boiler

A large number of industrial boilers are designed to operate between working pressure ranges from 125 bar to 300 bar. When a boiler operates at a pressure below the critical pressure of water, i.e., 221 bar, then it is called a sub-critical boiler and it consists of an economiser, evaporator and superheater.

When a boiler operates at a pressure greater than 221 bar, it is termed as a supercritical boiler and it consists of an economiser and superheater only. It does not have an evaporator, because at critical pressure or above it, the enthalpy of evaporation becomes zero. Its special features are the following:

1. It has rapid heat-transfer rate.
2. The temperature of water can be raised to the critical temperature in the economiser, and thus the boiler has a very high thermal efficiency.
3. The pressure level is more stable and therefore, the boiler gives better response.
4. The boiler is more suitable with load fluctuations.
5. Due to absence of a two-phase mixture, the problem of corrosion and erosion is minimised.

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Summary

- The *steam boiler* or *steam generator* is a closed vessel in which water is heated, vaporised and converted into steam at a pressure higher than the atmospheric pressure.
- In a *water-tube boiler*, the water flows through a number of small tubes, which are surrounded by hot combustion gases, while in a *fire-tube boiler*, the hot combustion gases pass through the boiler tubes, and water surrounds them.
- *A boiler is called a high-pressure boiler*, when it operates with a steam pressure above 80 bar. The high-pressure boilers are widely used for power generation in thermal power plants.
- The *packaged boiler* comes as a complete package. It requires only the steam, water-pipe work, fuel supply and electrical connections to be made for it to become operational.
- In the *fluidised-bed combustion boiler*, the slightly compressed air is passed upward through a finely divided bed of solid particles, while the fuel in powder form is fed downward.

Glossary

Boiler The *steam boiler* is a closed vessel in which water is heated, vaporised and converted into steam

IBR Indian Boiler Regulation Act

Steam system It collects, controls and distributes the steam produced in the boiler

Fuel system It includes all equipment used to provide fuel to generate the necessary heat

Fire-tube boiler A boiler in which flue gases pass through large-diameter tubes and water surrounds them

Water-tube boiler A boiler in which water flows through a number of small-diameter tubes, which are surrounded by hot flue gases.

High-pressure boiler A boiler which produces steam at more than 80 bar pressure.

Packaged boiler A fire-tube boiler, which comes as a complete package. It requires only electricity, fuel, water and steam connection at sight.

Pulverised-fuel boiler A boiler which uses pulverised coal as fuel in the furnace

Fluidised-bed combustion boiler In this boiler, fuel in powder form is fed in the presence of slightly compressed air. The air velocity supports fuel in air and creates bubble formation, vigorous turbulence, rapid mixing and combustion of fuel.

Supercharged boiler In this boiler, the combustion is carried under pressure on the combustion chamber. More mass of air is supplied by increasing the density of air in an air compressor.

Review Questions

1. What is a boiler?
2. What is the definition of 'boiler' according to IBR?
3. What is the difference between a steam boiler and a steam generator?
4. What are the requirements of boilers?
5. Classify the different types of boilers.
6. Differentiate a water-tube boiler from a fire-tube boiler.
7. What are the factors affecting the selection of a boiler?
8. What are the characteristics of a good boiler?
9. Explain the construction and working of a simple vertical boiler with the help of a neat sketch.
10. Explain the working of a Cochran boiler with the help of a neat sketch.
11. Explain the working of a Lancashire boiler with the help of a neat sketch.

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12. Explain the working of a locomotive boiler with the help of a neat sketch.
13. Explain the working of Babcock and Wilcox boiler with the help of a neat sketch.
14. Explain the construction and working of a La Mont boiler with the help of a neat sketch.
15. Explain the working principle of a Velox boiler with the help of a neat sketch.
16. Explain the construction and working of a Loffler boiler with the help of a neat sketch.
17. What are the advantages of operating a boiler at or above the critical pressure of water?
18. Explain the working of a supercharged boiler.

Objective Questions

1. A closed vessel is termed as a boiler if
   (a) its pressure exceeds 10 bar
   (b) its volume exceeds 22.75 litres
   (b) it consists of mountings
   (d) it consists of accessories
2. Feed-water system of a boiler consists of
   (a) supply of fuel
   (b) supply of water and steam generation
   (b) supply of water to condenser
   (d) supply of steam to utilities
3. In a fire-tube boiler
   (a) water flows through the tubes
   (b) flue gas flows through the tubes
   (b) fire is produced in the tubes
   (d) flue gas surrounds the tube
4. In a water-tube boiler
   (a) water flows through the tubes
   (b) flue gas flows through the tubes
   (b) fire is produced in the tubes
   (d) flue gas surrounds the tube
5. A Cornish boiler is
   (a) multi-tubular boiler
   (b) a water-tube boiler
   (c) a fire tube boiler
   (d) flue gas surrounds the tube
6. Locomotive boiler is
   (a) non-portable boiler
   (b) an externally fired boiler
   (c) an internally fired boiler
   (d) a water-tube boiler
7. Which one of the following is a water-tube boiler?
   (a) Babcock and Wilcox boiler
   (b) Stirling boiler
   (c) La Mont boiler
   (d) All of the above
8. Which one of the following is a high-pressure boiler?
   (a) Babcock and Wilcox boiler
   (b) Stirling boiler
   (c) La Mont boiler
   (d) All of the above
9. Which one of the following is a once-through boiler?
   (a) Loeffler boiler
   (b) Benson
   (c) La Mont boiler
   (d) All of the above
10. Which one of the following is a bent-tube boiler?
    (a) Babcock and Wilcox boiler
    (b) Stirling boiler
    (c) La Mont boiler
    (d) All of the above
11. The rate of steam generation in water-tube boilers as compared to fire-tube boilers is
    (a) less (b) same
    (c) more (d) none of the above

Answers

1. (b) 2. (b) 3. (b) 4. (a) 5. (c) 6. (c) 7. (d) 8. (c)
9. (b) 10. (b) 11. (a)

18

Boiler Mountings and Accessories

Introduction

In accordance with the Indian Boiler Regulations (IBR), mountings are essential fittings for safe working and control of a boiler.

The boiler accessories are those devices which are fitted either inside or outside the boiler to improve the performance of a boiler or help in proper working of a boiler plant.

18.1 BOILER MOUNTINGS

The boiler mountings are the different fittings and devices which are mounted on a boiler shell for proper functioning and safety. These form an integral part of the boiler. These are in two groups:

(*a*) *Mountings for Safety*

1. *Safety valve* 2 numbers
2. *High pressure and low water safety valve on Lancashire and Cornish boiler* 1 number each
3. *Water-level indicator* 2 numbers
4. *Fusible plug* 1 number

(*b*) *Mountings for Control*

5. *Pressure gauge* 1 number
6. *Steam stop valve* 1 number
7. *Feed check valve* 1 number
8. *Blow off cock* 1 number
9. *Man hole* 1 number
10. *Mud box* 1 number

18.1.1 Safety Valve

Safety valves are located on the top of the boiler. They guard the boiler against the excessive high pressure of steam inside the drum. If the pressure of steam in the boiler drum exceeds the working pressure then the safety valve allows to blow-off a certain quantity of steam to the atmosphere, and thus the pressure of steam falls in the drum. The escape of steam makes an audible noise as alarm to warn the boiler attendant.

There are four types of safety valves.

1. Dead-weight safety valve
2. Spring-loaded safety valve
3. Level-loaded safety valve
4. High steam and low water safety valve

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(*i*) *Dead-Weight Safety Valve* Figure 18.1 shows the schematic of a dead-weight safety valve. It is very similar to the dead weight (whistle) loaded on a pressure cooker and functions in a similar way. A gunmetal valve rests on a gunmetal seat. The gunmetal seat is mounted on a steel steam pipe. The valve is fastened to a weight carrier. The dead weights in the form of cylindrical discs are placed on the carrier. Therefore, the total weight placed on the carrier acts downward. This is the weight of the cast-iron carrier and the valve itself. This weight W is calculated on the basis of the working pressure p and cross-sectional area a of the valve.

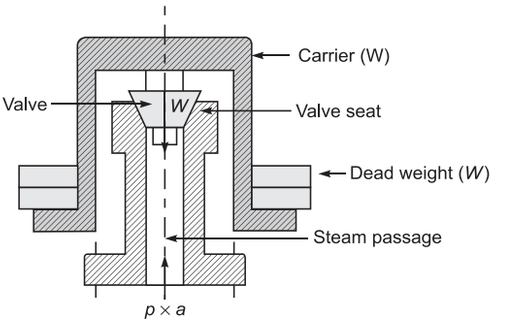

Fig. 18.1 *Dead-weight safety valve*

When the force due to steam pressure ($p \times a$) is less than the total dead weight on the valve, the valve remains closed. When the force due to steam pressure exceeds the total dead weight acting downward, the valve lifts up from the seat and some quantity of steam escapes to the atmosphere, thus reducing the steam pressure in the boiler shell, and the valve is again closed. The dead-weight safety valves are only used on stationary boilers.

(*ii*) *Spring-Loaded Safety Valve* The dead-weight safety valve cannot be used on locomotive and marine boilers. The jerks, pitching and rolling may change the load on the valve and it can open the valve frequently under working pressure.

The spring-loaded safety valve is used on locomotive marines and on high-pressure boilers.

Figure 18.2 shows the schematic of a spring-loaded safety valve. The device has two steam passages in the form of ⋃ and two valves. These valves close the steam passages under the action of a central helical spring. The operating pressure of the valves is adjusted by varying the tension in the spring. The extended lever is provided to check the function of the valve from time to time.

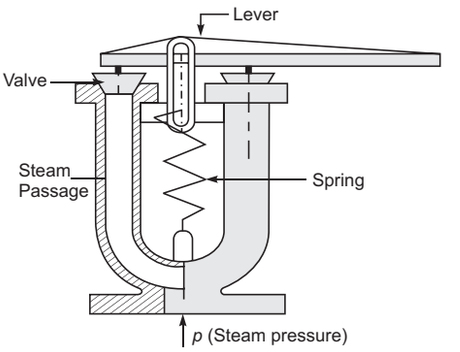

Fig. 18.2 *Spring-loaded safety valve*

When the upward force of steam exceeds the downward spring tension, the valves open and some steam escapes to the atmosphere. Thus lowers the steam pressure in the boiler and the valves are again closed under the spring force.

(*iii*) *Lever-Loaded Safety Valve* The schematic of a lever-loaded safety valve is shown in Fig. 18.3. The body of the valve is fastened on the top of the boiler shell. A gunmetal valve is placed on the steam passage formed in the casing. A cast-iron lever attached to a fulcrum on one end and loaded

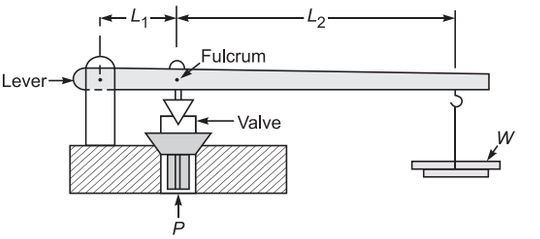

Fig. 18.3 *Lever-loaded safety valve*

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by weight on the other end keeps the valve on the seat in a closed position.

When the upward force due to steam pressure exceeds the load on the valve, the valve opens, and allows some quantity of steam to escape. The pressure of steam in the boiler falls and the valve again rests on the seat.

As the weight is acting through a long lever, a small weight can give a large thrust on the valve according to the equation

$$W(L_1 + L_2) = p\ a\ L_1.$$

where a is the cross-sectional area of steam passage.

(iv) High-Steam and Low Water Safety Valve This valve is a combination of two valves as shown in Fig. 18.4. It is used in Cornish and Lancashire boilers. One of the valves is lever loaded and is operated when steam pressure in the boiler exceeds the working pressure. The second valve operates and blows off steam with a louder noise, when water level in the boiler falls below the normal level.

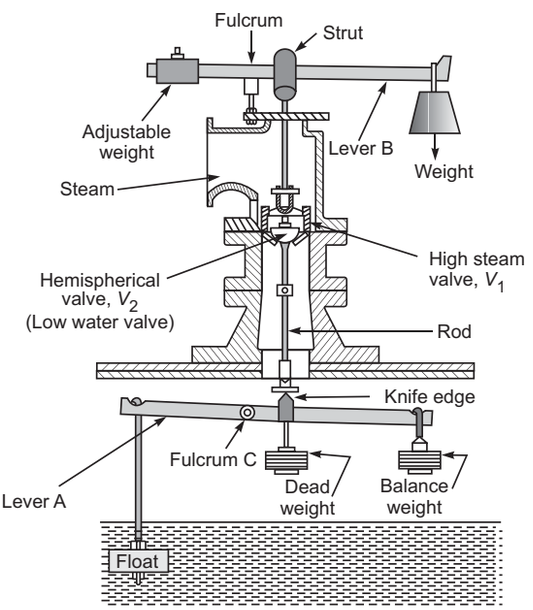

Fig. 18.4 *High-steam and low water safety valve*

It consists of a lever A hung inside the boiler shell and hinged at the fulcrum C. One side of the lever A is loaded by a balance weight and the other side carries a float immersed in water. The high-steam valve V_1 is loaded by a lever safety valve. In the centre of the valve V_1, a hemispherical valve V_2, is located. This hemispherical valve is connected with a valve rod and projects with a knife edge on the lever A. The hemispherical valve is loaded directly by central dead weights.

When the water level falls below the normal level in the boiler shell, the float end of the lever A lowers down and the valve rod is pushed up to open the hemispherical valve V_2. The steam leaks with a louder noise and sounds an alarm to the attendant.

When steam pressure exceeds the maximum working pressure, the steam valve V_1 lifts up with the valve V_2 and steam leaks out, thereby decreasing the steam pressure.

18.1.2 Water-Level Indicator

The water level indicator is located in front of the boiler in such a position that the level of water can easily be seen by the attendant. Two water-level indicators are used on all boilers.

A water-level indicator consists of a metal tube and a strong glass tube with markings. The upper and lower ends of these tubes are connected to two gunmetal hollow pipes. The upper pipe has a steam cock and the lower pipe has a water cock which are bolted to the boiler plate by two flanges. The upper pipe is opened to steam and the lower pipe is opened to water with the help of steam and water cocks, respectively. The drain cock is used frequently to ensure that the water and steam cocks are clear.

During the boiler operation, the steam cock and water cock remain opened while the drain cock is kept closed. During the normal operation, the two balls provided inside the gunmetal pipe remain in position as shown in Fig. 18.5. Hence, the water can reach the glass gauge and its level can be seen.

In case the glass gauge breaks accidently, the water and steam simultaneously rush out through

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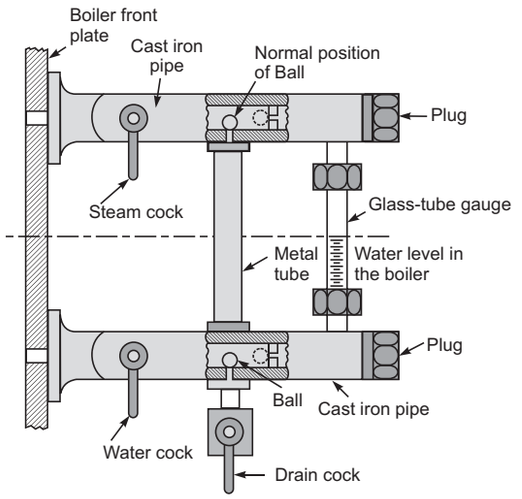


Fig. 18.5 *Water-level indicator*

the gunmetal pipes. The force is exerted on two balls and they are carried away by water and steam and the passages are closed. The water and steam cocks are then closed and the glass gauge is replaced.

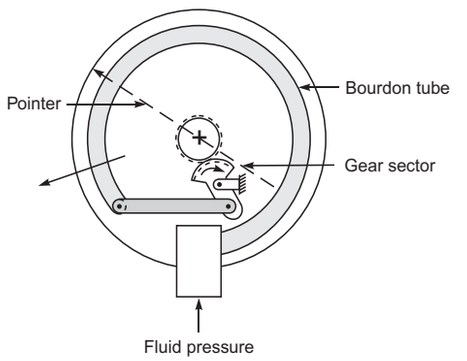
18.1.3 Pressure Gauge

A pressure gauge is fitted in front of the boiler in such a position that the operator can conveniently read it. It reads the pressure of steam in the boiler and is connected to the steam space by a siphon tube.

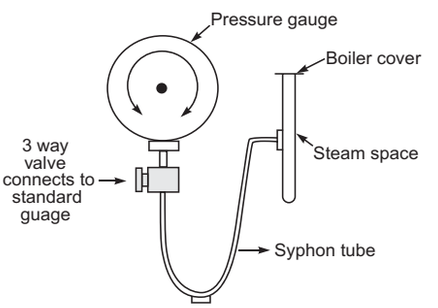
The most commonly used gauge is the Bourdon pressure gauge. Figure 18.6(a) illustrates the Bourdon pressure gauge. It consists of an elliptical spring Bourdon tube. One end of this tube is connected to the siphon tube and other end is connected by levers and gears to pointer.

When fluid pressure acts on the Bourdon tube, it tries to make its cross-section change from elliptical to circular. In this process, the lever end of the tube moves out as indicated by an arrow. The tube movement is magnified by the mechanism and given to pointer to move over a circular scale indicating the pressure.

The siphon tube, is shown in Fig. 18.6(b), it connects the steam space of the boiler to the Bourdon gauge is filled with water in order to avoid



(a) Bourdon pressure gauge



(b) Pressure gauge with syphon tube

Fig. 18.6

the effect of high temperature steam on the gauge components. The steam pressure is transferred by water to the Bourdon pressure gauge.

18.1.4 Fusible Plug

It is a very important safety device which protects the fire-tube boiler shell against overheating. It is located just above the furnace in the boiler. It consists of a gunmetal plug fixed in a gunmetal body with a fusible molten metal as shown in Fig. 18.7.

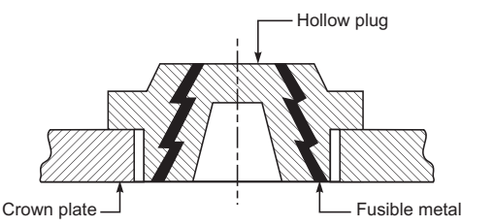


Fig. 18.7 *Fusible plug*

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During the normal boiler operation, the fusible plug is covered by water and its temperature does not rise to its melting state. But when the water level falls too low in the boiler, it uncovers the fusible plug. The furnace gases heat up the plug, the fusible metal of the plug melts, and the inner plug falls down. The water and steam then rush through the hole and extinguish the fire before any major damage occurs to the boiler due to overheating.

18.1.5 Blow-off Cock

The function of the blow-off cock is to discharge mud and other sediments deposited in the bottom-most part of the water space in the boiler, while the boiler is in operation. It can also be used to drain off the boiler water. Hence it is mounted at the lowest part of the boiler. When it is open, water under the pressure rushes out, thus carrying sediments and mud.

The blow-off cock as shown in Fig. 18.8 consists of a conical hollow gunmetal plug type of valve, which fits accurately into the corresponding hole in the casing. The plug valve has a hole, which when brought in line with the hole in the casing, by rotating the plug, causes the water to flow out of the boiler.

The flow of water through the cock can be stopped by rotating the plug valve in such a way that its solid side comes in the line of the hole in the casing. The stuffing box prevents the leakage of water at the valve shank. The guard provided

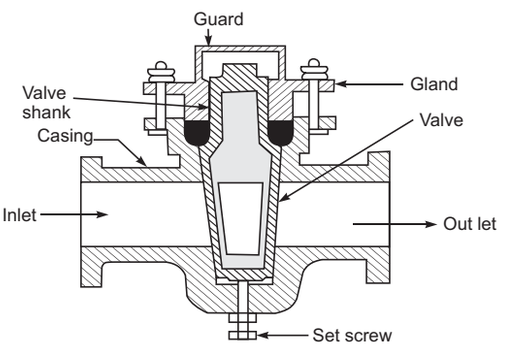

Fig. 18.8 *Blow-off cock*

over the spindle prevents the handle being removed unless the cock is closed.

18.1.6 Feed Check Valve

The feed check valve is fitted to the boiler, slightly below the working level in the boiler. It is used to supply high-pressure feed water to the boiler. It also prevents the returning of feed water from the boiler if the feed pump fails to work.

A feed check valve consists of two valves: feed valve and check valve as shown in Fig. 18.9. The feed valve is operated by a hand wheel for its opening or closing. The check valve operates automatically on its seat, up and down under the pressure difference of water.

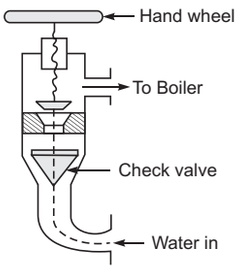

Fig. 18.9 *Feed check valve*

In normal working, the feed-water pressure is more than the boiler pressure, thus the check valve remains open. But in case of its failure, the boiler pressure becomes more than the feed water, the valve rests on its seat and closes the water passage and prevents its reverse flow.

18.1.7 Steam Stop Valve

The steam stop valve is located on the highest part of the steam space. It regulates the steam supply for use. The steam stop valve can be operated manually or automatically.

A hand-operated steam stop valve is shown in Fig. 18.10. It consists of a cast-iron body and two flanges at right angles. One flange is fastened to the boiler shell and the other is fastened to the steam pipe. A steel valve connects the hand wheel through

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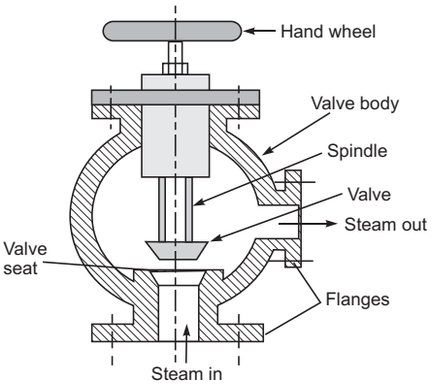

Fig. 18.10 *Steam stop valve*

the spindle. When the hand wheel is rotated, the spindle also rotates and carries the valve up or down to open or close the valve. The spindle passes through a stuffing box and glands in order to prevent the leakage.

When the spindle is rotated anticlockwise, the valve lifts up and steam is allowed to pass through the clearance between the valve and its seat. The amount of steam passing the valve is controlled by the valve lift. When the hand wheel is rotated clockwise, the valve rests on its seat and closes the steam passage.

18.1.8 Man Hole and Mud Box

The *man hole* is provided on the boiler shell at a convenient position so that a person can enter through it, inside the boiler for cleaning and inspection purposes.

The *mud box* is placed at the bottom of the boiler to collect mud discharged through the blow-off cock. Therefore, it is connected with the blow-off cock.

18.2 BOILER ACCESSORIES

The accessories are mounted on the boiler to increase its efficiency. These units are optional on an efficient boiler. The following accessories are normally used on a modern boiler:

(i) Superheater
(ii) Economiser
(iii) Air preheater
(iv) Feed-water pump
(v) Steam injector
(vi) Steam separator
(vii) Steam trap
(viii) Boiler draught equipments

18.2.1 Superheater

It is a heat exchanger in which products of heat of combustion are utilized to dry the wet steam and to make it superheated by increasing its temperature. During superheating of the steam, pressure remains constant, and its volume and temperature increase. A super-heater consists of a set of small-diameter U tubes in which steam flows and takes up the heat from hot flue gases.

The smaller diameter tubes have lower pressure stresses and withstand better. The tube material should be carefully selected, because the tubes are subjected to high temperature, pressure and thermal stresses. The maximum steam temperature at the superheater exit is about 540°C. The superheaters and re-heaters, which are operating at this temperature, are made of special high-strength alloy steels, which have high strength and corrosion resistance.

Superheaters are classified as convective, radiant and of combination types.

In the *convective superheater*, the heat of the hot flue gases is transferred to the surface of the superheater by convection. These are located in the path of hot flue gases.

In a *radiant superheater*, the heat of the combustion is transferred to the surface of the superheater by thermal radiation. These are located in one or more walls of the furnace. These are used in high-pressure boilers.

In a *combination type of superheater*, the heat is transferred to the surface of the tubes by both modes of heat transfer. The radiant superheaters are occasionally used in combination with convective superheaters and are arranged in series.

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When the boiler load increases, the rate of energy absorption by the furnace water walls (radiant superheater) does not increase as rapidly as the steam flow rate. Thus, a radiant superheater shows a decrease in steam temperature with an increase in boiler load and steam flow as shown in Fig. 18.11(a) and (b). A convective superheater shows an opposite variation in the steam temperature with change in the boiler load. These opposite characteristics can be utilised to control the final temperature of steam from the combination type of superheaters.

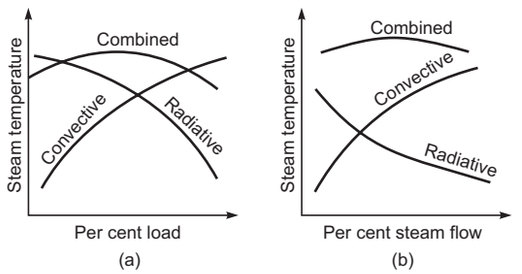

Fig. 18.11 *Superheater characteristics*

Figure 18.12 shows a schematic of *Surgden's superheater*. It consists of two mild steel headers. The ∪-shaped steel tubes are connected to these headers.

The steam generated into the boiler passes the valve *C* and enters the superheater (∪) tubes through the intake header. The steam is made dry and superheated in these tubes by supplying heat and then it is taken for use through the valve *B* via the uptake header. The superheating of steam is controlled by controlling the quantity of flue gases by operating the dampers manually.

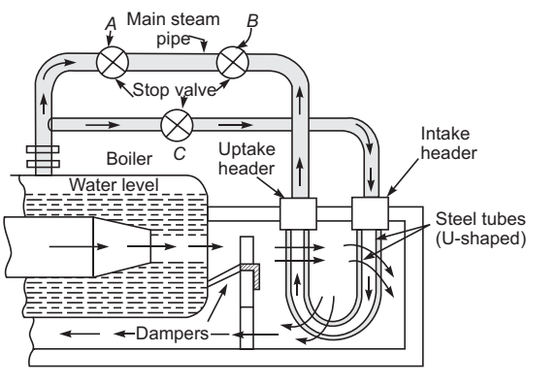

Fig. 18.12 *Superheater*

If superheated steam is not needed or the superheater is under maintenance, the valves *B* and *C* are closed and steam is then taken out through the valve *A*.

18.2.2 Economiser

An economiser is a heat exchanger used for heating the feed water before it enters the boiler. The economiser recovers some of waste heat of hot flue gases going to the chimney thus it helps in improving the boiler efficiency. It is placed in the path of flue gases at the rear end of the boiler just before the air preheater.

The most popular economiser is *Green's economiser* and it is shown in Fig. 18.13.

Green's economiser consists of a set of vertical cast-iron pipes joined with horizontal lower and upper headers. The cold feed water flows through the vertical pipes via the lower header. The hot flue gases pass over them transferring heat to the water. The heated water is supplied to the boiler via the upper header. The scrappers are provided on pipes, which move up and down slowly by means of chains and sprockets to avoid the soot depositon on the pipe surface. The soot collected in the soot chamber can be removed from the door.

Each economiser is equipped with a safety valve, a drain valve, a release valve, pressure gauge and thermometers.

18.2.3 Air Preheater

The function of an air preheater is similar to that of an economiser. It recovers some portion of the waste heat of hot flue gases going to the chimney, and transfers the same to the fresh air before it enters the combustion chamber. A tubular air preheater is shown in Fig. 18.14.

Due to preheating of air, the furnace temperature increases. It results in rapid combustion of fuel with less soot, smoke and ash. The high furnace

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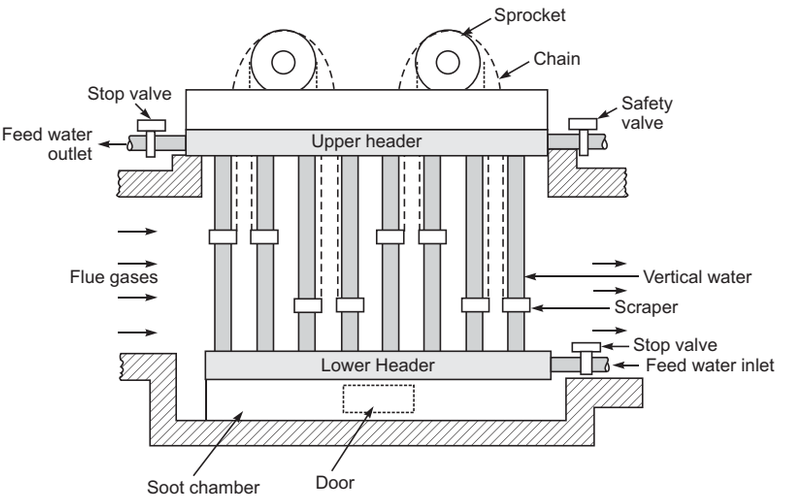

Fig. 18.13 *Green's economiser*

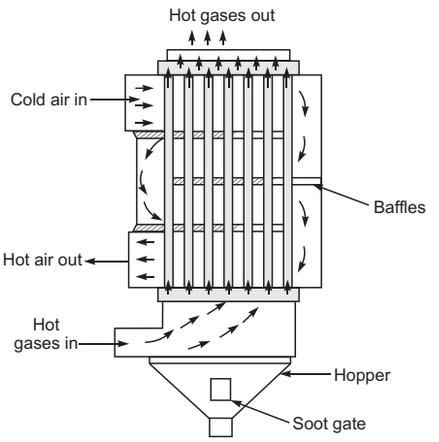

Fig. 18.14 *Tubular air preheater*

temperature can permit a low-grade fuel with less atmospheric pollution. The air preheater is placed between the economiser and chimney.

18.2.4 Steam Injector

A steam injector lifts and forces the feed water into the boiler. It is usually used for vertical and locomotive boilers and can be accomodated in a small space. It is less costly and does not have any moving parts. Thus, operation is silent.

In a steam injector, the water is delivered to the boiler by steam pressure. The kinetic energy of the expanding steam is given to feed water. Thus the pressure and velocity of water increase.

Figure 18.15 shows an injector. It consists of a set of nozzles—steam nozzle, suction jet, delivery nozzle, steam and water inlet passages, delivery passage, spindle handle and a overflow valve.

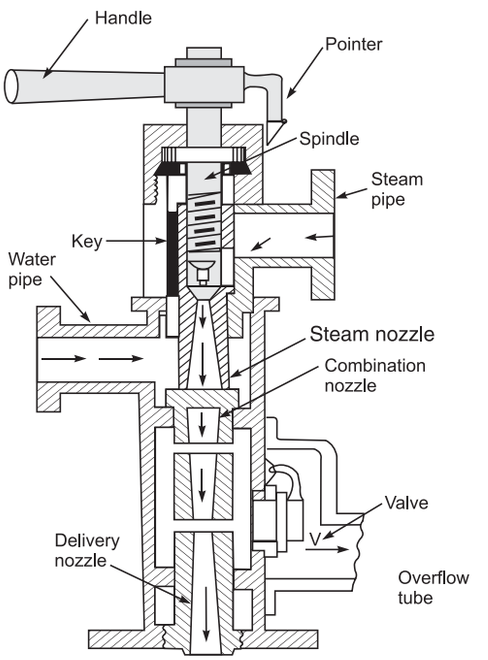

Fig. 18.15 *Steam injector*

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When the steam valve is opened with the help of a spindle and handle, the steam from the boiler enters the steam nozzles. The steam pressure drops in the nozzles and steam velocity increases. The high-velocity steam creates a partial vacuum in the steam jet. Thus, the water from the feed tank is sucked and mixed with steam and the steam condenses. This mixture then enters the delivery tube, where the kinetic energy of the mixture is converted to pressure energy.

When the pressure of water delivered by the nozzle is less than the boiler pressure, the water leaving the injector is passed through the overflow valve to the overflow pipe. But when sufficient pressure in the delivery tube is raised, the water is supplied to the boiler.

18.2.5 Feed Pump

The feed pump delivers feed water at a pressure higher than that in the boiler. The commonly used feed pumps are the following:

(*a*) *Rotary Pump* It is a high-speed centrifugal pump and is used to deliver a large quantity of water into the boiler. It consists of an impeller and a casing. The impeller shaft is driven by an electric motor or by a prime mover. It utilizes a centifugal force of the impeller for pumping water.

(*b*) *Reciprocating Pump* These are positive-displacement type pumps. The most popular type of reciprocating feed pump used in a boiler is a duplex feed pump. It consists of a steam engine and a water pump side by side.

The reciprocasting water pump is driven by the reciprocating steam engine as shown in Fig. 18.16. The piston rods of the water pump and steam engine are coupled together. So the steam engine and water pump operate simultaneously. The steam generated in the boiler is used to run the steam engine. The steam engine drives the water pump, Double acting pumps are commonly used for medium-size boilers.

18.2.6 Steam Traps

A *steam trap* is a valve device that drains away the condensed steam and air automatically from the steam pipe, steam jackets or steam separator without discharging the steam. The purpose of installing the steam traps in the process equipment is to obtain fast heating of the product and equipment by keeping the steam lines and equipment free of condensate, air and non-condensable gases.

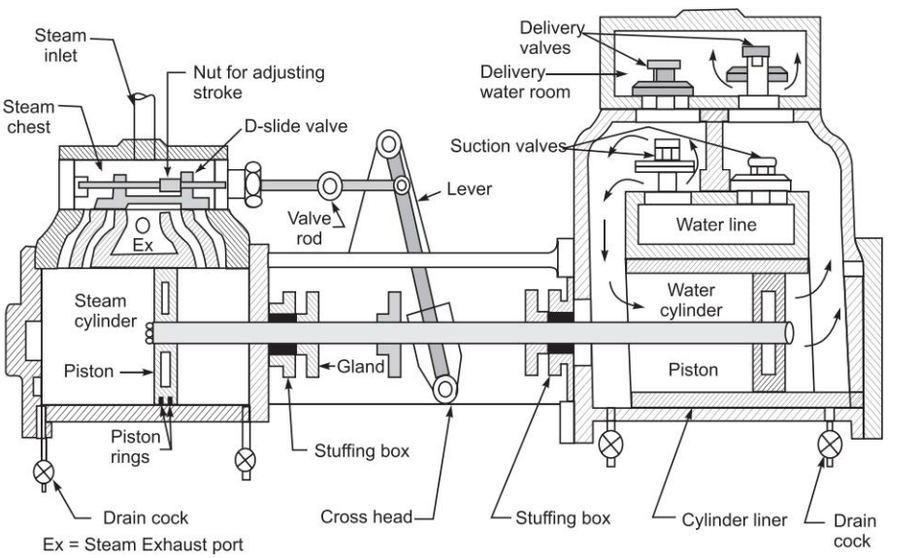

Fig. 18.16 *Feed pump*

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Functions of Steam Traps The three important functions of steam traps are

- To discharge condensed steam as soon as it is formed
- Not to allow dry steam to escape
- To be capable of discharging air and other incondensible gases

Types of Steam Traps

There are three basic types of steam traps discussed below:

1. *Thermostatic Steam Traps* operated by changes in fluid temperature.

In the steam space, condensed steam and dry steam are present. But as a result of any heat loss, the temperature of condensed steam falls. A thermostatic trap will pass the condensate when this lower temperature is sensed. As steam reaches the trap, the temperature increases and the trap closes.

2. *Mechanical Steam Traps* operated by changes in fluid density.

This range of steam traps operates by sensing the difference in density between the steam and condensate. These steam traps include 'ball float traps' and 'inverted bucket traps'. In the 'ball float trap', the float ball rises in the presence of condensed steam, opens a valve which passes the denser condensate out. A ball float steam trap is shown in Fig. 18.17.

3. *Thermodynamic Steam Traps* operated by changes in fluid dynamics.

Thermodynamic steam traps rely partly on the formation of flash steam from the condensate.

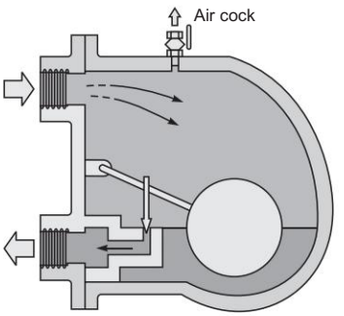

Fig. 18.17 *Ball float steam trap*

18.2.7 Steam Separator

A steam separator is installed on the steam main as well as on the branch lines to separate any water particles present from the steam and to improve the quality of the steam going to the units. It is installed very close to units on main steam pipes. By change of direction of steam, steam seperators cause the condensed water particles to be separated out and delivered to a point where they can be drained away as condensate through a conventional steam trap. A few types of separators are illustrated in Fig. 18.18.

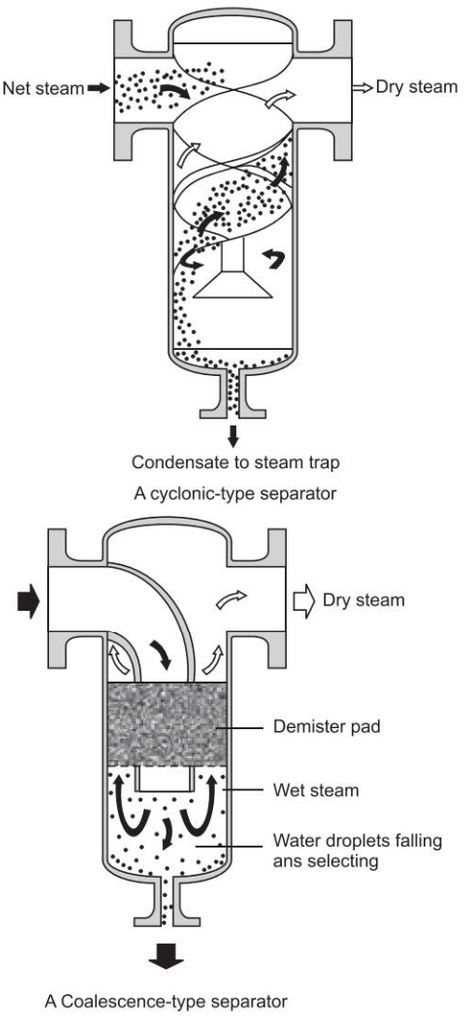

Fig. 18.18 *Steam separators*

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Summary

(a) Boiler Mountings

Name	*Function*
1. *Safety valves*	Do not allow boiler pressure to rise beyond its safe pressure
2. *Water-level indicator*	Shows working level of water in the boiler
3. *Pressure gauge*	Indicates working pressure of boiler
4. *Steam stop valve*	Regulates the amount of outgoing steam
5. *Feed check valve*	Checks the amount of feed water going to the boiler and does not allow its return
6. *Blow-off cock*	Allows to drain water from boiler
7. *Man hole*	Allows a person to go inside the boiler drum for repairs, etc.
8. *Mud hole* (*with door*)	Facilates removal of heavy impurities settled in the boiler drum
9. *Fusible plug*	Stops the boiler if its heating surface is overheated due to low water level
10. *Dual function safety valves*	
(a) *High pressure, low water safety valve*	Allows escape of steam in case of (i) unsafe higher pressure, or (ii) unsafe low water level
(b) *Low water, high water safety valve*	Whistles by blowing steam in case of (i) unsafe low or (ii) high water level in the drum
11. *Feed pipe*	Lead the feed water to the inside of the boiler

(b) Boiler Accessories

Name of the accessory	*Function*
1. *Economiser*	Preheating the feed water by utilising the heat of waste flue gases
2. *Superheater*	Increase the temperature of steam at constant pressure beyond saturation
3. *Anti-priming pipe* (*anti-priming devices*)	Filter out moisture from outgoing steam
4. *Air preheater*	Preheat the fresh air by using the heat of waste flue gases
5. *Steam injector*	Lift and force the feed water into the boiler

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Review Questions

1. List the boiler mountings and acccessories.
2. Discuss various types of safety valves.
3. Explain the high steam and low water safety valve.
4. Explain the working of a water-level indicator with a neat diagram.
5. Explain the working of Bourdon's pressure gauge.
6. Discuss in brief with their function
   (a) Fusible plug
   (b) Man hole
   (c) Blow-off cock
   (d) Feed check value
7. Explain the working of a steam stop valve.
8. Why are superheaters used on boilers? Explain their working.
9. Write the advantages of using feed water heaters on boilers.
10. Explain the function of an air preheater in a boiler.
11. Write the function of a steam separator and a steam trap.

Objective Questions

1. Which one of the following is essential for operation of a boiler?
   (a) Safety valve (b) Economiser
   (c) Superheater (d) Injector
2. Which one of the following is not boiler mounting?
   (a) Superheater (b) Feed check valve
   (c) Man hole (d) Fusible plug
3. Which one of the following is an accessory on a boiler?
   (a) Pressure gauge
   (b) Blow-off cock
   (c) Economiser
   (d) Feed check valve
4. Which one of the following must be used in pair?
   (a) Pressure gauge
   (b) Water level indicator
   (c) Fusible plug
   (d) All of the above
5. Which boiler generally uses the high steam and low water safety valve?
   (a) Cochran boiler
   (b) Cornish boiler
   (c) Lancashire boiler
   (d) Stirling boiler
6. Which one of the following equipment prevents the boiler against excessive pressure?
   (a) Pressure gauge
   (b) Steam stop valve
   (c) Fusible plug
   (d) Safety valve
7. Which one of the following safety valves is used on stationary boilers?
   (a) Dead-weight safety valve
   (b) Spring-loaded safety valve
   (c) High-steam and low water valve
   (d) Lever-loaded safety valve
8. Which one of the following safety valves is used on locomotive boilers?
   (a) Dead-weight safety valve
   (b) Spring-loaded safety valve
   (c) High-steam and low water valve
   (d) Lever-loaded safety valve
9. The function of a fusible plug is to
   (a) Superheat the steam
   (b) Extinguish the fire
   (c) Maintain constant temperature
   (d) Increase flue gas temperature
10. The function of superheater is to
   (a) Superheat the steam

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(b) Extinguish the fire
(c) Maintain constant temperature
(d) Preheat the feed water

11. The function of an air preheater is to
(a) Increase the fuel consumption
(b) Heat the fresh air
(c) Burn fuel efficiently
(d) None of the above

12. The function of an ijector in a boiler plant is to
(a) Lift the feed water
(b) Lift and force the feed water
(c) Inject the fuel
(d) Heat the feed water

13. Which one of the following is the correct sequence of accessories in a boiler plant?
(a) Boiler–economiser–superheater–chimney
(b) Economiser–boiler–superheater–chimney
(c) Economiser–air preheater–superheater–chimney
(d) Economiser–boiler–preheater–chimney

14. The function of a steam trap is to
(a) Lift the steam
(b) Separate the steam
(c) Discharge the condesate
(d) Heat the steam

Answers

1. (a) 2. (a) 3. (c) 4. (b) 5. (c) 6. (d) 7. (a) 8. (b)
9. (b) 10. (a) 11. (b) 12. (b) 13. (b) 14. (c)

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19

Boiler Draught and Performance

Introduction

In a steam-generating unit, the hot flue gases are to be moved from the furnace to the places where the heat can be transferred and these gases are discharged in the atmosphere at a certain height through the chimney. For continuous flow of gases, a certain pressure difference is to be maintained inside the boiler.

19.1 BOILER DRAUGHT (DRAFT)

The boiler draught may be defined as the small pressure difference which causes the continuous flow of gases inside the boiler. In other words, the draught is a small pressure difference between the air outside the boiler and gases within the furnace or chimney.

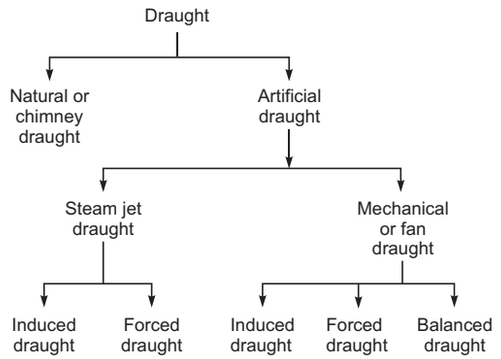
19.1.1 Function of Draught

The boiler draught performs the following functions:

1. It forces a sufficient quantity of air into the furnace for proper combustion of fuel.
2. It circulates the hot flue gases through the flue tubes, superheater, economiser, air preheater etc.
3. It discharges the hot flue gases to the atmosphere through the chimney.

19.1.2 Classification of Boiler Draught

The detailed classification of boiler draught is given below:



19.2 NATURAL, OR CHIMNEY, DRAUGHT

The draught obtained by use of a chimney is called *natural* or *chimney draught*. A chimney is a veritcal

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long cylindrical structure made of either brick masonary, reinforced concrete or steel. A chimney carries the products of combustion to such a height before discharging so that they will not be harmful to the surroundings. The draught produced by the chimney is due to the density difference between the column of hot flue gases inside the chimney and the equivalent column of cold air outside the chimney.

Figure 19.1 shows a schematic arrangement of a chimney of height H metres above the grate level.

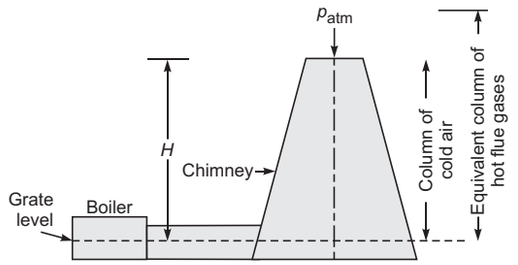

Fig. 19.1 *Chimney draught*

The pressure at grate level in the combustion chamber

p_1 = atmospheric pressure + pressure due to column of hot flue gases in the chimney.

$= p_a + \rho_g g H$

Similarily, pressure at grate level outside the chimney

p_2 = atmospheric pressure + pressure due to cold air column of height H

$= p_a + \rho_a g H$

where ρ_a and ρ_g represent the densities of cold air and hot flue gases, respectively.

Since the density of hot flue gases is less than that of cold air, thus pressure difference will act on the grate level, which will cause flow of fresh air in the combustion chamber. The net pressure difference

$$\Delta p = p_2 - p_1 = (\rho_a - \rho_g) g H \qquad ...(19.1)$$

The pressure difference causing the flow of gases is known as *static draught* and is generally very small. Thus, it is measured by water manometers.

Advantages

1. Easy to construct.
2. No power is required for producing the draught.
3. Long life of chimney.
4. No maintenance is required.

Disadvantages

1. Tall chimney is required.
2. Poor efficiency.
3. Efficiency decreases with increase in outside temperature.
4. No flexibility to create more draughts to take peak loads.

19.2.1 Height of Chimney

The amount of natural draught produced by a chimney mainly depends on the height of the chimney, the temperature of hot flue gases and atmospheric air. Refer to Fig. 19.2.

Let m_a = Mass of air supplied in kg/kg of fuel

$m_a + 1$ = Mass of flue gases, kg/kg of fuel

p_a = atmospheric pressue, N/m^2

T_o = Absolute temperature at N.T.P = 273 K

T_a = Temperature of atmospheric air, K

T_g = Average temperature of flue gases, K

H = Height of chimney, m

h = Draught required in mm of water

The specific volume of air at NTP

$$v_0 = \frac{RT_0}{p_0} = \frac{(0.287 \text{ kJ/kg·K}) \times (273 \text{ K})}{(101.325 \text{ kPa})}$$

$$= 0.7732 \text{ m}^3/\text{kg}$$

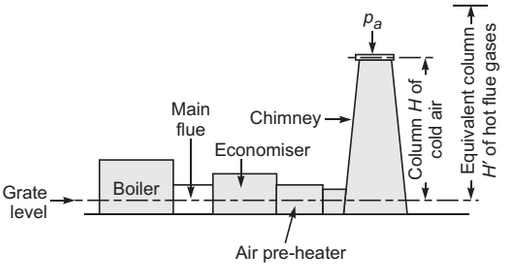

Fig. 19.2 *Height of chimney in natural draught*

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Since the volume of fuel is negligible as compared to the volume of air supplied per kg of fuel, therefore, the volume of flue gases can be taken equal to the volume of air.

The volume of atmospheric air outside the chimney

$$\frac{V_a}{T_a} = \frac{V_0}{T_0} = \frac{m_a v_0}{T_0}$$

or $$V_a = \frac{m_a \times 0.7732}{273} \times T_a \quad ...(19.2)$$

and density of air $$\rho_a = \frac{m_a}{V_a}$$

$$= \frac{273}{0.7732} \times \frac{1}{T_a} = \frac{353}{T_a} \quad ...(19.3)$$

Similarly, the volume of hot flue gases inside the chimney

$$V_g = \frac{V_0}{T_0} T_g = \frac{0.7732}{273} m_a T_g$$

and density of flue gas

$$\rho_g = \frac{m_a + 1}{V_g}$$

$$= \frac{(m_a + 1)}{0.7732\ m_a} \times \frac{273}{T_g} = \frac{(m_a + 1)}{m_a} \times \frac{353}{T_g} \quad ...(19.4)$$

The total static pressure difference Δp in N/m^2 can be obtained by using ρ_a and ρ_g in Eq. (19.1);

$$\Delta p = 353 g H \left[\frac{1}{T_a} - \left(\frac{m_a + 1}{m_a} \right) \times \frac{1}{T_g} \right] \text{ (N/m}^2\text{)} \quad ...(19.5)$$

This pressure difference can be expressed in terms of the water column (mm) as

$$\Delta p = (\rho g h)_w$$

Where $\rho_w = 1000$ kg/m^3

$$h_w = h \text{ (mm of water)} = \frac{h}{1000} \text{ (metre)}$$

$$\therefore \quad \Delta p = (1000 \text{ kg/m}^3) \times g \left(\frac{h}{1000} \text{ m} \right)$$

$$= gh \text{ (N/m}^2\text{)} \quad ...(19.6)$$

Equating (19.5) and (19.6), we get

$$h = 353 H \left[\frac{1}{T_a} - \left(\frac{m_a + 1}{m_a} \right) \times \frac{1}{T_g} \right] \text{ (mm of water)} \quad ...(19.7)$$

Equation (19.7) represents the theoretical value of natural draught which depends on chimney height and weather conditions. The draught is directly proportional to chimney height and it decreases with increase in temperature of atmospheric air or decrease in temperature of flue gases. The actual draught available would be less than the theoretical draught because of friction losses offered by the passages.

19.2.2 Draught in terms of Hot Gas Column

Let H' be the height of hot gases, equivalent to the column of cold outside air H and produce the same pressure difference, i.e.,

$$\Delta p = \rho_g g H'$$

Using $$\rho_g = \left(\frac{m + 1}{m} \right) \times \frac{353}{T_g}$$

Equating with Eq. (19.5),

$$\left(\frac{m_a + 1}{m_a} \right) \times \frac{353}{T_g} \times g H' = 353 g H \left[\frac{1}{T_a} - \left(\frac{m_a + 1}{m_a} \right) \times \frac{1}{T_g} \right]$$

or $$H' = H \left[\left(\frac{m_a}{m_a + 1} \right) \times \frac{T_g}{T_a} - 1 \right]$$

(metres of hot gases) ...(19.8)

19.2.3 Diameter of Chimney

The theoretical velocity of hot flue gas flowing through chimney

$$v_g = \sqrt{2gH'} \quad ...(19.9)$$

However, the actual velocity of the flue gases will be less than the theoretical velocity given by Eq. (19.9), due to frictional losses. If h_f be the frictional losses of height of column of flue gases

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then the velocity of flue gases in the chimney,

$$\mathcal{V}_g = \sqrt{2g(H' - h_f)} = 4.43\sqrt{(H' - h_f)} \quad ...(19.10)$$

It can be arranged as

$$\mathcal{V}_g = 4.43\sqrt{H'} \times \sqrt{1 - \frac{h_f}{H'}} = k\sqrt{H'} \quad ...(19.11)$$

where $$k = 4.43\sqrt{1 - \frac{h_f}{H'}} \quad ...(19.12)$$

The value of k has been found experimentally

$k = 0.825$ for brick chimney, and

$= 1.1$ for steel chimney.

The mass of flue gases flowing through a chimney of cross-sectional area A

$$\dot{m}_g = \rho_g A \mathcal{V}_g \text{ (kg/s)}$$

$$= \rho_g \left(\frac{\pi}{4} D^2\right) \mathcal{V}_g$$

or $$D^2 = \frac{4\dot{m}_g}{\pi \rho_g \mathcal{V}_g}$$

or $$D = 1.128\sqrt{\frac{\dot{m}_g}{\rho_g \mathcal{V}_g}} \quad ...(19.13)$$

19.2.4 Condition for Maximum Discharge through a Chimney

The velocity of flue gases through the chimney without any losses

$$\mathcal{V}_g = \sqrt{2gH'} \quad \text{when} \quad h_f = 0$$

Using H' from Eq. (19.8), we get

$$\mathcal{V}_g = \sqrt{2gH\left\{\left(\frac{m_a}{m_a + 1}\right) \times \frac{T_g}{T_a} - 1\right\}} \quad ...(19.14)$$

The density of flue gases is given by

$$\rho_g = \frac{p}{RT_g}$$

Therefore, the rate of mass of flue gases discharged

$$\dot{m}_g = A\mathcal{V}_g \rho_g$$

$$= A\left[\sqrt{2gH\left\{\left(\frac{m_a}{m_a + 1}\right) \times \frac{T_g}{T_a} - 1\right\}}\right] \times \frac{p}{RT_g}$$

or $$\dot{m}_g = C\sqrt{\left(\frac{m_a}{m_a + 1}\right) \times \frac{1}{T_a T_g} - \frac{1}{T_g^2}} \quad ...(19.15)$$

where constant $C = \frac{Ap}{R}\sqrt{2gH}$

For maximum discharge rate, differentiating Eq. (19.15) with respect to T_g and equating it to zero,

$$\frac{d\dot{m}_g}{dT_g} = C\frac{d}{dT_g}\left\{\sqrt{\left(\frac{m_a}{m_a + 1}\right) \times \frac{1}{T_a T_g} - \frac{1}{T_g^2}}\right\} = 0$$

$$\frac{d}{dT_g}\left\{\left(\frac{m_a}{m_a + 1}\right) \times \frac{1}{T_a T_g} - \frac{1}{T_g^2}\right\}^{\frac{1}{2}} = 0$$

$$0 = \frac{1}{2}\left\{\left(\frac{m_a}{m_a + 1}\right) \times \frac{1}{T_a T_g} - \frac{1}{T_g^2}\right\}^{-\frac{1}{2}} \times \left\{\left(\frac{m_a}{m_a + 1}\right) \times \frac{1}{T_a}\left(-\frac{1}{T_g^2}\right) + \frac{2}{T_g^3}\right\}$$

$$\Rightarrow -\left(\frac{m_a}{m_a + 1}\right) \times \frac{1}{T_a T_g^2} + \frac{2}{T_g^3} = 0$$

or $$\left(\frac{m_a}{m_a + 1}\right) \times \frac{1}{T_a} = \frac{2}{T_g}$$

or $$\frac{T_g}{T_a} = 2\left(\frac{m_a + 1}{m_a}\right) \quad ...(19.16)$$

Thus, for maximum discharge through the chimney, the absolute temperature of flue gases should be greater than twice the absolute atmospheric temperature.

Using the value of $\frac{T_g}{T_a}$ in Eq. (19.8), we get

$$H'_{max} = H\left[\left(\frac{m_a}{m_a + 1}\right) \times 2\left(\frac{m_a + 1}{m_a}\right) - 1\right]$$

$$= H \quad ... (19.17)$$

For maximum discharge, the height of the hot-gas column should be equal to the height of the

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chimney. Similarly, draught in mm of water to height of chimney,

$$h'_{max} = 353H\left[\frac{1}{T_a} - \frac{1}{2T_a}\right] = \frac{176.5H}{T_a} \quad ...(19.18)$$

19.2.5 Efficiency of Chimney

A certain minimum flue-gas temperature is required to produce a given draught with a given height of the chimney. The temperature of the flue gases leaving the chimney in case of natural draught has to be higher than that in an artificial draught. It leads to higher thermal losses through the flue gases and therefore, poor boiler efficiency in case of natural draught. In artificial fan draught, the temperature of the flue gases discharging through chimney can be lowered to any desired value and the recovered heat can be used in economiser and air preheater, etc., without change in draught developed.

The efficiency of the chimney is defined as the ratio of the energy equivalent of draught in metre head or in N-m per kg of gases, produced by artificial draught fan to the energy equivalent in N-m per kg of gases of the additional heat carried away by the flue gases to create natural draught.

$$\eta_{chimney} = \frac{\text{Energy equivalent of artificial draught}}{\text{Energy equivalent of hot flue gases required to create natural draught}}$$

$$= \frac{H'g}{C_p(T_g - T'_g)} \quad ...(19.19)$$

where

H' = Column of hot flue gases, equivalent to artificial draught in metre head

$$= H\left(\frac{m_a}{m_2 + 1} \times \frac{T_g}{T_a} - 1\right) \text{ (metres)}$$

C_p = Specific heat of hot flue gases in J/kg·K

T_g = Temperature of hot flue gases discharged through chimney in case of natural draught in K

T'_g = Temperature of hot flue gases discharged through chimney in case of artificial draught in K

T_a = Atmospheric temperature in K

g = Acceleration due to gravity as 9.81 m/s^2

The efficiency of the chimney is directly proportional to the height of the chimney. The efficiency of a chimney is less than 20%. It is obvious from the above statement that the chimney is very insufficient means of producing the draught.

19.2.6 Draught Losses

The actual natural draught is always less than the calculated value due to the following reasons:

1. The frictional resistance between chimney walls and flue gases (approx. 1mm of water).
2. Frictional losses during the flow through the bends in gas flow passages (approx. 1.25 mm of water).
3. Frictional resistance offered due to path of flue gases in furnace with baffles, grates, superheaters, economisers, and air preheater.

The total natural draught losses are nearly 20% of the total static draught produced by the chimney. Therefore, the available draught is approximately 80% of the total static draught calculated theoretically.

Example 19.1 *A thermal power plant has a chimney draught of 3.5 cm of H_2O column. The flue-gas temperature flowing through the chimney is 280°C and the ambient temperature is 15°C. The amount of air supplied/kg of the fuel is 20 kg. Calculate the height of the chimney.*

Solution

Given The chimney of a thermal power plant

h = 3.5 cm = 35 mm of water,

T_g = 280°C = 553 K

T_a = 15°C = 288 K,

m_a = 20 kg/kg of fuel

To find The height of the chimney.

Analysis The height of the chimney in terms of the water column

$$h = 353H\left[\frac{1}{T_a} - \left(\frac{m_a + 1}{m_a}\right) \times \frac{1}{T_g}\right] \text{ (mm of water)}$$

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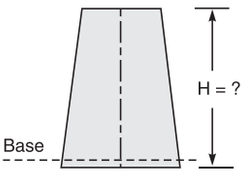

Fig. 19.3

$$35 = 353H\left[\frac{1}{288} - \left(\frac{20+1}{20}\right)\times\frac{1}{553}\right]$$

$$H = \frac{35}{353\times1.573\times10^{-3}} = \mathbf{63.0\ m}$$

Example 19.2 *Find the minimum height of the chimney required to produce a draught of 16 mm of H_2O, if 19 kg of air is required/kg of fuel burnt on the grate. The mean temperature of flue gases inside the chimney is 330°C and the atmpospheric temperature is 30°C.*

Solution

Given $H = ?$ $h = 16$ mm of H_2O
$m_a = 19$ kg/kg of fuel
$T_g = 330°C + 273 = 603$ K
$T_a = 30°C + 273 = 303$ K

To find Height of the chimney

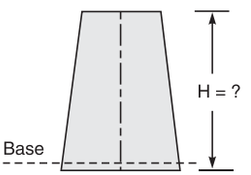

Fig. 19.4

Analysis The draught produced in mm of H_2O column is given by,

$$h = 353H\left[\frac{1}{T_a} - \left(\frac{m_a+1}{m_a}\right)\times\frac{1}{T_g}\right] \text{ (mm of water)}$$

Now substituting the values,

$$16 = 353H\left[\frac{1}{303} - \left(\frac{19+1}{19}\right)\times\frac{1}{603}\right] \text{ mm of } H_2O$$

$$= 353\times H\times1.55\times10^{-3} = 0.5487H$$

$$\therefore \quad H = \frac{16}{0.5487} = \mathbf{29.15\ m}$$

Example 19.3 *A thermal power station works on natural draught. The height of the chimney is restricted to 40 m. The ambient temperature of the air is 20°C and the temperature of the flue-gas passing through the chimney at its base is 300°C. The air–fuel ratio is 17:1. Calculate the diameter of the chimney at the base, if head lost due to friction is 25% of the ideal draught.*

Solution

Given The chimney of a thermal power plant

$H = 40$ m $T_a = 20°C = 293$ K
$T_g = 300°C = 573$ K $m_a = 17$ kg/kg of fuel
$h_f = 0.25\,H'$ $m_g = 17 + 1$
$= 18$ kg/kg of fuel

To find Diameter of the chimney.

Analysis The natural draught in terms of hot-gas column H' is given by

$$H' = H\left[\left(\frac{m_a}{m_a+1}\right)\times\frac{T_g}{T_a} - 1\right] \text{ (metres of hot gases)}$$

$$= 40\times\left[\left(\frac{17}{17+1}\right)\times\frac{573}{293} - 1\right] = 33.879\ \text{m}$$

Actual draught produced by chimney

$H_{act} = H' - h_f$

$= H' - 0.25\,H' = 0.75\times33.879 = 25.4$ m

Actual velocity of flue gases in the chimney

$$\mathcal{V}_g = \sqrt{2gH_{act}} = \sqrt{2\times9.81\times25.4} = 22.32\ \text{m}$$

Density of flue gases

$$\rho_g = \left(\frac{m_a+1}{m_a}\right)\times\frac{353}{T_g}$$

$$= \left(\frac{17+1}{17}\right)\times\frac{353}{573} = 0.65\ \text{kg/m}^3$$

The diameter of the chimney

$$D = 1.128\times\sqrt{\frac{\dot{m}_g}{\rho_g\mathcal{V}_g}}$$

$$= 1.128\times\sqrt{\frac{18}{0.65\times22.32}} = \mathbf{1.255\ m}$$

Example 19.4 *A boiler uses 16 kg of air per kg of fuel, when the fuel consumption is at the rate of 1800 kg/h. Actual draught required is 20 mm of water when all losses are considered. The surrounding air temperature is 27°C and the flue-gas temperature is 277°C.*

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Determine the chimney height and its diameter if actual velocity of the flue gases is 0.35 times the theoretical velocity due to roughness of interior surfaces of the chimney.

Solution

Given A boiler with a chimney

m_a = 16 kg/kg of fuel $\dot{m}_f$ = 1800 kg/h

h = 20 mm of water T_a = 27°C = 300 K

T_g = 277°C = 550 K $\mathcal{V}_{act} = 0.35\mathcal{V}_g$

To find

(i) Chimney height, and

(ii) Diameter of chimney.

Analysis The draught in mm of water is expressed

$$h = 353H\left[\frac{1}{T_a} - \left(\frac{m_a+1}{m_a}\right)\times\frac{1}{T_g}\right] \text{ (mm of water)}$$

$$20 = 353H\left[\frac{1}{300} - \left(\frac{16+1}{16}\right)\times\frac{1}{550}\right]$$

H = **40.43 m**

Density of flue gases

$$\rho_g = \frac{353}{T_g}\times\left(\frac{m_a+1}{m_a}\right)$$

$$= \frac{353}{550}\times\left(\frac{16+1}{16}\right) = 0.683 \text{ kg/m}^3$$

Mass-flow rate of flue gases,

$$\dot{m}_g = (m_a+1)\times\frac{\dot{m}_f}{3600} = (16+1)\times\frac{1800}{3600} = 8.5 \text{ kg/s}$$

Draught equivalent to hot-gas column

$$H' = H\left[\left(\frac{m_a}{m_a+1}\right)\times\frac{T_g}{T_a} - 1\right] \text{ (metres of hot gases)}$$

$$= 40.43\times\left[\left(\frac{16}{16+1}\right)\times\frac{550}{300} - 1\right] = 29.33 \text{ m}$$

Actual velocity of flue gases

$$\mathcal{V}_{act} = k\sqrt{2g\,H'} = 0.35\times\sqrt{2\times9.81\times29.33} = 8.396 \text{ m/s}$$

Further, mass-flow rate of flue gases

$$\dot{m}_g = \rho_g A\mathcal{V}_{act}$$

$$8.5 = 0.682\times\left(\frac{\pi}{4}D^2\right)\times8.396$$

$$\left(\frac{\pi}{4}D^2\right) = 1.4846$$

Diameter of chimney D = **1.375 m**

Example 19.5 *A boiler is equipped with a chimney of 30 m height. The ambient temperature is 25°C. The temperature of flue gases passing through the chimney is 300°C. If the air flow is 20 kg/kg of fuel burnt, find*

(a) draught produced

(b) the velocity of flue gases passing through chimney if 50% of the theoretical draught is lost in friction

Solution

Given A boiler with a chimney

H = 30 m

T_a = 25°C + 273 = 298 K

T_g = 300°C + 273 = 573 K

m_a = 20 kg/kg of fuel

h_f = 0.5 × theoretical draught = $0.5H'$

To find

(i) Draught produced, and

(ii) Velocity of flue gas.

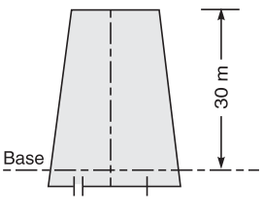

Fig. 19.5

Analysis

(i) The draught produced is given by Eq. (19.5)

$$\Delta p = 353gH\left[\frac{1}{T_a} - \left(\frac{m_a+1}{m_a}\right)\times\frac{1}{T_g}\right] \text{ (N/m}^2\text{)}$$

$$= 353\times9.81\times30\times\left[\frac{1}{298} - \left(\frac{20+1}{20}\right)\times\frac{1}{573}\right]$$

= **158.25 N/m^2**

(ii) The velocity of flue gases,

$$\mathcal{V}_g = 4.43\sqrt{H' - h_f}$$

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Height of hot-gas column

$$H' = H\left[\left(\frac{m_a}{m_a+1}\right)\times\frac{T_g}{T_a}-1\right]$$

$$= 30\times\left[\left(\frac{20}{20+1}\right)\times\frac{573}{298}-1\right] = 24.93 \text{ m}$$

$$H' - h_f = (1-0.5)H' = 0.5\times 24.93 = 12.4688 \text{ m}$$

$$\therefore \quad \mathcal{V}_g = 4.43\times\sqrt{12.4688} = \mathbf{15.64 \text{ m/s}}$$

Example 19.6 *Find the mass-flow rate of flue gases through the chimney when the draught produced is equal to 20 mm of water column. The temperature of gases is 300 °C and the ambient temperature is 30 °C. The mass of air used is 19 kg per kg of fuel burnt. Diameter of the chimney is 2 m. Neglect the losses.*

Solution

Given A boiler with a chimney

m_a = 19 kg/kg of fuel $\quad D$ = 2 m

h = 20 mm of water $\quad T_a$ = 30°C = 303 K

T_g = 300°C = 573 K

To find Mass-flow rate of flue gases

Analysis The draught in mm of water is expressed

$$h = 353H\left[\frac{1}{T_a}-\left(\frac{m_a+1}{m_a}\right)\times\frac{1}{T_g}\right] \quad \text{(mm of water)}$$

$$20 = 353H\left[\frac{1}{303}-\left(\frac{19+1}{19}\right)\times\frac{1}{573}\right]$$

H = 38.72 m

Height of hot-gas column

$$H' = H\left[\left(\frac{m_a}{m_a+1}\right)\times\frac{T_g}{T_a}-1\right]$$

$$= 38.72\times\left[\left(\frac{19}{19+1}\right)\times\frac{573}{303}-1\right] = 30.84 \text{ m}$$

Velocity of flue gases;

$$\mathcal{V}_g = \sqrt{2g\,H'} = \sqrt{2\times 9.81\times 30.84}$$

$$= 24.6 \text{ m/s}$$

Density of flue gases

$$\rho_g = \frac{353}{T_g}\times\left(\frac{m_a+1}{m_a}\right)$$

$$= \frac{353}{573}\times\left(\frac{19+1}{19}\right) = 0.648 \text{ kg/m}^3$$

Mass-flow rate of flue gases,

$$\dot{m}_g = \rho_g A \mathcal{V}_g$$

$$= 0.648\times\frac{\pi}{4}\times(2)^2\times 24.6 = \mathbf{50.11 \text{ kg/s}}$$

Example 19.7 *A boiler plant uses 1500 kg of coal per hour with the formation of 16 kg of dry flue gases per kg of coal. The probable draught losses are*

Through fuel bed = *4.5 mm of water column*

Boiler passages = *7.8 mm of water column*

Straight breeching = *3 mm of water column*

Velocity of gas flow = *0.5 mm of water column*

Bends in breeeching = *0.4 mm of water column*

The expected outside temperature is 27 °C and mean temperature of flue gases in the chimney is 250 °C. The actual draught may be taken as 0.8 times the theoretical draught and the velocity of flow coefficient as 0.65. Find the height of the chimney.

Solution

Given A boiler plant

$\dot{m}_f$ = 1500 kg/h

T_g = 250°C + 273 = 523 K

T_a = 27°C + 273 = 300 K

$m_a + 1$ = 16 kg/kg of fuel

h_{act} = 0.8 × theoretical draught

k = 0.65

Total draught losses = 4.5 + 7.8 + 3 + 0.5 + 0.4

= 16.2 mm

To find

(i) Height of chimney, and

(ii) Diameter of chimney.

Analysis Minimum theoretical draught required

$$= \frac{\text{Draught losses}}{0.8} = \frac{16.2}{0.8} = 20.25 \text{ mm of water}$$

The draught in mm of water is expressed as

$$h = 353H\left[\frac{1}{T_a}-\left(\frac{m_a+1}{m_a}\right)\times\frac{1}{T_g}\right] \quad \text{(mm of water)}$$

$$20.25 = 353H\left[\frac{1}{300}-\left(\frac{16}{15}\right)\times\frac{1}{523}\right]$$

Height of chimney; H = **44.338 m**

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Example 19.8 *How much air is used per kg of coal burnt in a boiler having a chimney of 35-m height to create a draught of 20 mm of water? The temperature of gases in the chimney is 370°C and the boiler-house temperature is 34°C. Does this chimney satisfy the condition for maximum discharge? Also, find the height of hot-gas column under maximum condition of discharge.*

Solution

Given The chimney of a boiler;

h = 20 mm of water $\quad T_g$ = 370°C = 643 K

T_a = 34°C = 307 K $\quad H$ = 35 m

To find

(i) The mass of air used per kg of coal,

(ii) Whether the chimney satisfy the condition for maximum discharge, and

(iii) Height of hot gas column for maximum discharge.

Analysis

(i) *The mass of air used per kg of coal*

The height of chimney in terms of water column

$$h = 353H\left[\frac{1}{T_a} - \left(\frac{m_a+1}{m_a}\right)\times\frac{1}{T_g}\right] \quad \text{(mm of water)}$$

$$20 = 353\times 35\times\left[\frac{1}{307} - \left(\frac{m_a+1}{m_a}\right)\times\frac{1}{643}\right]$$

$$\frac{m_a+1}{m_a} = 1.0536$$

or m_a = **18.66 kg of air/kg of coal**

(ii) *For maximum discharge*;

$$\frac{T_g}{T_a} = 2\left(\frac{m_a'+1}{m_a'}\right)$$

or $$\frac{643}{307} = 2\left(\frac{m_a'+1}{m_a'}\right)$$

or m_a' = 21.17 kg of air per kg of coal

Since $m_a' > m_a$, thus the chimney does not satisfy the condition of maximum discharge.

(iii) *Height of hot-gas column for maximum discharge*

For maximum discharge condition, the density of flue gases using Eq. (19.4);

$$\rho_g = \frac{(m_a'+1)}{m_a'}\times\frac{353}{T_g}$$

$$= \left(\frac{21.17+1}{21.17}\right)\times\frac{353}{643}$$

$$= 0.575 \text{ kg/m}^3$$

Under the maximum discharge condition, the draught produced in mm of water, Eq. (19.18);

$$h'_{max} = \frac{176.5H}{T_a} = \frac{176.5\times 35}{307}$$

$$= 20.122 \text{ mm of water}$$

The equivalent column of hot gases

$$H'_{max} = \frac{\rho_w \times h'_{max}}{\rho_g}$$

$$= \frac{(1000 \text{ kg/m}^3)\times(21.12\times10^{-3} \text{ m})}{(0.575 \text{ kg/m}^3)}$$

$$= \mathbf{36.73 \text{ m}}$$

Example 19.9 *A 40-m high chimney is discharging flue gases at 350°C, when the ambient temperature is 30°C. The quantity of air supplied is 18 kg per kg of fuel burnt. Determine*

(a) draught produced in mm of water,

(b) equivalent draught in metres of hot-gas column,

(c) efficiency of the chimney, if minimum temperature of artificial draught is 150°C; the mean specific heat of flue gases is 1.005 kJ/kg·K,

(d) the percentage of the heat spent in natural draught system, if the net calorific value of the fuel supplied be 30600 kJ/kg, and

(e) the temperature of chimney gases for maximum discharge in a given time and what would be the corresponding draught in mm of water produced.

Solution

Given The chimney;

H = 40 m $\quad T_g$ = 350°C or 623 K

T_a = 30°C = 303 K $\quad m_a$ = 18 kg/kg of fuel

T_g' = 150°C = 423 K $\quad C_p$ = 1.005 kJ/kg·K

CV = 30600 kJ/kg

To find

(i) Draught produced in mm of water,

(ii) Equivalent draught in metres of hot-gas column,

(iii) Efficiency of the chimney,

(iv) The percentage of the heat spent in natural draught system, and

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(v) The temperature of chimney gases for maximum discharge and corresponding draught in mm of water.

Analysis

(i) *Draught in mm of water*

$$h = 353H\left[\frac{1}{T_a} - \left(\frac{m_a + 1}{m_a}\right) \times \frac{1}{T_g}\right] \text{ (mm of water)}$$

$$h = 353 \times 40 \times \left[\frac{1}{303} - \left(\frac{18+1}{18}\right) \times \frac{1}{623}\right]$$

h = **22.676 mm**

(ii) *Draught equivalent to hot-gas column*

$$H' = H\left[\left(\frac{m_a}{m_a + 1}\right) \times \frac{T_g}{T_a} - 1\right] \text{ (metres of hot gases)}$$

$$= 40 \times \left[\left(\frac{18}{18+1}\right) \times \frac{623}{303} - 1\right] = \textbf{37.915 m}$$

(iii) *Efficiency of the chimney*

$$\eta_{chimney} = \frac{H'g}{C_p(T_g - T_g')}$$

$$= \frac{37.915 \times 9.81}{1005 \times (623 - 423)}$$

$$= 0.00185 \text{ or } 0.185\%$$

(iv) *The percentage of the heat spent in natural draught system*

Extra heat carried away by flue gases in the natural draught system per kg of fuel burnt

$$Q_{Ex} = m_g C_p (T_g - T_g')$$

$$= (18 + 1) \times 1.005 \times (623 - 423)$$

$$= 3819 \text{ kJ/kg of fuel}$$

Percentage heat spent

$$= \frac{Q_{Ex}}{CV} \times 100 = \frac{3819}{30600} \times 100$$

$$= \textbf{12.48\%}$$

(v) *Temperature of chimney gases for maximum discharge and corresponding draught in mm of water*

Using Eq. (19.16),

$$T_g = 2T_a\left(\frac{m_a + 1}{m_a}\right)$$

$$= 2 \times 303 \times \left(\frac{18+1}{18}\right) = \textbf{639.66 K}$$

Corresponding draught,

$$h = 353 \times 40 \times \left[\frac{1}{303} - \left(\frac{18+1}{18}\right) \times \frac{1}{639.66}\right]$$

$$= \textbf{23.3 mm of water}$$

19.3 ARTIFICIAL DRAUGHT

Usually, the static draught produced by the chimney is not sufficient to meet the requirement of draught of a boiler that varies from 30 to 350 mm of water column. When artificial draught is used on the boiler then the waste heat carried by the flue gases can be better utilized in an economiser, air preheater, etc.

An artificial draught produced by a fan or a blower is known as *mechanical draught* and that produced by steam jet is known as *steam jet draught*.

19.3.1 Mechanical Draught

Draught produced by a fan or blower may be of three types: (a) induced, (b) forced, and (c) balanced draught.

(*a*) *Induced Draught* The fan is placed near the base of the chimney as shown in Fig. 19.6. The fan draws the flue gases from the furnace. So the pressure above the fuel bed is reduced below the atmospheric pressure. The fresh air rushes to the furnace and after combustion, the flue gases get discharged through the chimney in the atmosphere.

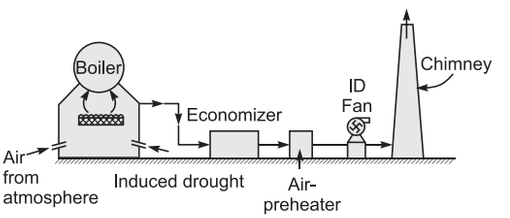

Fig. 19.6 *Induced draught*

(*b*) *Forced Draught* The fan or blower is located near or at the base of the boiler grate to force atmospheric air on to the furnace under pressure. This pressure helps in circulation of flue gases

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through components of the boiler and then through chimney to atmosphere. It is shown in Fig. 19.7.

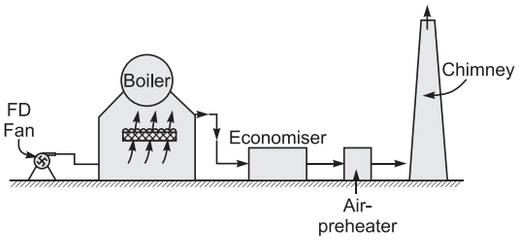

Fig. 19.7 *Forced draught*

(c) *Balanced Draught* A combination of induced and forced draught in a boiler is known as a balanced draught. A forced draught fan located near the grate supplies air under the pressure through the furnace and an induced draught fan located near the chimney base, draws in flue gases through the economiser, air preheater, etc., and discharges them into the atmosphere through a chimney. Figure 19.8 illustrates the balanced draught system.

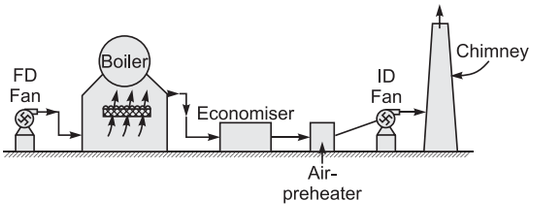

Fig. 19.8 *Balanced draught*

19.3.2 Advantages of Artificial Draught Over Natural Draught

(i) Forced draught puts better control on combustion.
(ii) Low-grade fuel can be efficiently burnt.
(iii) Overall efficiency of a thermal power plant is higher due to better heat recovery from the exhaust flue gases in economisers and preheaters.
(iv) The height of the chimney can be reduced than that required in natural draught.
(v) Higher the rate of fuel-burning capacity on the grate, higher the heat-supply rate at higher effective temperature.
(vi) Smoke formation is less.
(vii) Tendancy for air leakage in the furnace is less.

19.3.3 Power Required to Drive the Fan

Fans are used in forced and induced draught arrangements. The fan is driven by an electric motor. The power required to drive the fan can be calculated as under.

Let Δp = pressure difference caused by fan (draught) in kPa,

$\dot{V}$ = volume flow rate of combustion air/gases at fan conditions, m^3/h

η_f = fan efficiency

The power required to drive the fan

$$P = \frac{\Delta p \dot{V}}{\eta_f} \qquad ...(19.20)$$

Expressing draught in terms of water column

$$\Delta p = \rho_w g h$$

where $\rho_w = 1000\ kg/m^3$,

$g = 9.81\ m/s^2$ and h in mm, therefore,

$$\Delta p = (1000\ kg/m^3) \times (9.81\ m/s^2) \times \left(\frac{h}{1000} m\right)$$

$$= 9.81h\ (N/m^2)$$

Then $$P = \frac{9.81 h \dot{V}}{\eta_f}\ (watts)$$

or $$P = \frac{9.81 h \dot{V}}{1000 \times \eta_f}\ (kW) \qquad ...(19.21)$$

Forced Draught Power, P_{FD} In a forced draught system, the fan is located before the combustion chamber and it handles only atmospheric air.

Let v_0 = Specific volume of air at NTP

$$= \frac{RT_0}{p_0} = \frac{(0.287\ kJ/kg.K) \times (273\ K)}{(101.325\ kPa)}$$

$$= 0.7732\ m^3/kg$$

m_a = Mass of air supplied, kg/kg of fuel

$\dot{m}_f$ = Rate of fuel firing, kg/s

The volume of air supplied to furnace at NTP;

$$\dot{V}_0 = v_0 m_a \dot{m}_f = 0.7732\ m_a \dot{m}_f\ (m^3/s)$$

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The volume of air supplied to the furnace at fan conditions;

$$\dot{V}_a = \frac{\dot{V}_0}{T_0} T_a = \frac{0.7732 m_a \dot{m}_f}{273} T_a$$

or $$\dot{V}_a = \frac{m_a \dot{m}_f}{373} T_a \quad ...(19.22)$$

Using in Eq. (19.21), the power required to drive the forced-draught fan

$$P = \frac{9.81 h m_a \dot{m}_f}{1000 \times 353 \eta_f} T_a \text{ (kW)} \quad ...(19.23)$$

Induced Draught Power, P_{FD} In an induced draught system, the fan is located near the base of the chimney and it handles the products of combustion, i.e., volume of flue gases.

Let m_a = Mass of air supplied, kg/kg of fuel
$m_a + 1$ = Mass of flue gases, kg/kg of fuel
$\dot{m}_f$ = Rate of fuel firing, kg/s
$\dot{m}_f (m_a + 1)$ = Mass flow rate of flue gases across the induced draught fan, kg/s

The volume of air supplied to furnace at NTP;

$$\dot{V}_g = \frac{\text{Mass flow rate of flue gases}}{\text{Density of flue gases}} = \frac{\dot{m}_f (m_a + 1)}{\rho_g} \text{ (m}^3\text{/s)}$$

Using $$\rho_g = \frac{(m_a + 1)}{m_a} \times \frac{353}{T_g}$$ from Eq. (19.4)

then $$\dot{V}_g = \frac{\dot{m}_f m_a}{353} T_g \text{ (m}^3\text{/s)} \quad ...(19.24)$$

Using in Eq. (19.21), the power required to drive the forced-draught fan

$$P = \frac{9.81 h m_a \dot{m}_f}{1000 \times 353 \eta_f} T_g \text{ (kW)} \quad ...(19.25)$$

If induced-draught and forced-draught fans are equally efficient and produce same draught then

$$\frac{\text{Power of ID fan}}{\text{Power of FD fan}} = \frac{T_g}{T_a} \quad ...(19.26)$$

Example 19.10 *Calculate the motor power required to drive a fan which maintains a draught of 45 mm of water under the following conditions for (a) induced-draught fan, (b) forced-draught fan.*

The temperature of flue gases leaving the boiler in each case is 200°C and of air in the boiler house is at 33°C. The air supplied is 18.5 kg per kg of fuel and mass of coal fired per hour is 1820 kg. Assume a fan efficiency of 80% in both cases.

Solution

Given

h = 45 mm of water T_g = 200°C or 473 K
T_a = 33°C = 306 K m_a = 18.5 kg/kg of fuel
$\dot{m}_f$ = 1820 kg/h η_f = 0.8

To find

(i) Power required to drive induced-draught fan.
(ii) Power required to drive forced-draught fan.

Analysis

(i) *Power for Induced draught fan,* Eq. (19.25)

$$P = \frac{9.81 h m_a \dot{m}_f}{1000 \times 353 \eta_f} T_g = \frac{9.81 \times 45 \times 18.5 \times (1820/3600)}{1000 \times 353 \times 0.8} \times 473$$

= **6.91 kW**

(ii) *Power for forced draught fan,* Eq. (19.23)

$$P = \frac{9.81 h m_a \dot{m}_f}{1000 \times 353 \eta_f} T_a \text{ (kW)} = \frac{9.81 \times 45 \times 18.5 \times (1820/3600)}{1000 \times 353 \times 0.8} \times 306$$

= **4.47 kW**

Example 19.11 *Compare the fan power input for forced and induced draught fans and also compare the quantity of heat carried away by the flue gases per kg of fuel fired in each case. Assume specific heat of flue gases as 1.005 kJ/kg·K.*

Types of draught	*Outside air temperature °C*	*Flue gas leaving boiler Temperature, °C*	*Mass of air in kg/kg of fuel*
Forced draught	27	150	15
Induced draught	27	150	15
Chimney draught	27	370	20

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Solution

Given

$T_g' = 150°C$ or 423 K $T_g = 370°C$ or 643 K
$T_a = 27°C = 300$ K $m_{a_1} = 15$ kg/kg of fuel
$m_{a_2} = 20$ kg/kg of fuel $C_{pg} = 1.005$ kJ/kg·K

To find

(i) Comparision of power input to induced and forced draught fans, and
(ii) Comparision of heat carried away by flue gases.

Analysis

(i) *Comparision of power input to induced and forced draught fan*, Eq. (19.26)

$$\frac{\text{Power of ID fan}}{\text{Power of FD fan}} = \frac{T_g}{T_a} = \frac{423}{300} = \mathbf{1.41}$$

The heat carried away by flue gases in artificial draught

$$Q_1 = (m_{a_1} + 1)\, C_{pg}(T_g' - T_a)$$
$$= (15 + 1) \times 1.005 \times (423 - 300)$$
$$= \mathbf{1977.84\ kJ/kg\ of\ fuel}$$

The heat carried away by flue gases in natural draught

$$Q_2 = (m_{a_2} + 1) C_{pg}(T_g' - T_a)$$
$$= (20 + 1) \times 1.005 \times (643 - 300)$$
$$= \mathbf{7239\ kJ/kg\ of\ fuel}$$

Further, $\frac{Q_2}{Q_1} = \frac{7239}{1977.84} = 3.66$

In case of natural draught, the flue gases carry 3.66 times more heat than that in artificial draught.

19.3.4 Steam Jet Draught

Artificial draught can easily be produced by a steam jet of either induced type or forced type.

In an *induced steam-jet draught*, the exhausted steam from an steam engine or live steam from the boiler is introduced into the smoke box or in the chimney in the form of a jet. A partial vacuum is created, which draws in air through the ash pit, fuel bed and boiler flue tubes into the smoke box, from where they are discharged to the atmosphere through chimney. Such an arrangement is used on a locomotive boiler.

In a *forced steam-jet draught*, the jet of live steam from the boiler is introduced into the ash pit. The high velocity jet of steam draws in atmospheric air and forces it on to the furnace, fire tubes and finally discharges to the atmosphere through chimney.

The steam-jet draught system is economical, simple in operation and maintenance free and requires very little space for installation. But draught produced by a steam jet is very low and can be installed only on small boilers. It has also great disadvantage that it cannot be operated until proper steam pressure is available. The passing on to the furnace also carries away some quantity of heat along with flue gases.

19.3.5 Advantages of Steam-jet Draught

(i) It is a very simple and economical artificial draught.
(ii) Low-grade fuel can be burnt efficiently.
(iii) It is automatically adjusted to suit the boiler requirement, because the exhaust steam from the steam engine is used to produce draught.
(iv) It does not require any maintenance cost.
(v) In a forced steam-jet draught, the fire bars are kept cool by the steam, thus adhering of clinker to the fire bars are prevented.

19.4 PERFORMANCE EVALUATION OF BOILERS

The performance of boiler is calculated for comparing the boilers for its evaporating capacity, energy consumption and heat balance sheet. The quantity of heat supplied to vaporise water from and at 100°C and 1 atm is referred as evaporation unit.

19.4.1 Evaporative Capacity of Boilers

The quantity of steam generated by a boiler at full load is called the *evaporative capacity* of the boiler. The performance of boilers is often measured in terms of evaporative capacity. The evaporative

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capacity of a boiler can be expressed on the basis of time, heating area, or fuel burned. These are

Evaporation rate

$$= \frac{\text{Mass of steam generated}}{\text{Time period in hours}} \text{ (kg/h)} \quad ...(19.27)$$

or Evaporation rate in kg of steam/h/m^2

$$= \frac{\text{Mass of steam generated per hour}}{\text{Heating area of grate in m}^2} \quad ...(19.28)$$

or Evaporation rate in kg of steam/kg of fuel

$$= \frac{\text{Mass of steam generated per hour}}{\text{Mass of fuel burned per hour}} \quad ...(19.20)$$

19.4.2 Equivalent Evaporation

The quantity of steam generated by a boiler depends on the following four important parameters: (i) feed-water temperature, (ii) working pressure, (iii) fuel used, and (iv) quality of steam generated. In practical situations, the different boilers use feed water at different temperatures, operate at different pressures and generate different qualities of steam. Therefore, such boilers cannot be compared unless some standard parameters are adopted. Hence feed water is taken at 100 °C and working pressure is at 1.01325 bar. At this state, water evaporates at 100°C and requires 2257 kJ/kg of *latent heat* to convert it into steam at 100°C.

Equivalent evaporation is defined as the *amount of dry and saturated steam generated from feed water at 100°C at normal atmospheric pressure*. In short, it is defined as the amount of steam generated from and at 100°C.

Let $\dot{m}_s$ = Mass of steam generated at pressure p and temperature T per hour in the boiler

h = Enthalpy of steam

= $h_f + x h_{fg}$ for wet steam

= h_g for dry and saturated steam

= $h_g + C_{ps}(T_{sup} - T_{sat})$ for superheated steam

h_{f_1} = Enthalpy of water at feed temperature

$\dot{m}_f$ = Mass of fuel burned per hour

m_e = Equivalent mass of dry and saturated steam generated from and at 100°C

The evaporation per kg of fuel

$$m_a = \frac{\text{Mass of steam generated per hour}}{\text{Mass of fuel burned per hour}} = \frac{\dot{m}_s}{\dot{m}_f} \quad ...(19.30)$$

The total heat supplied to water in the boiler

$$= m_a(h - h_{f_1})$$

and the equivalent evaporation

$$m_e = \frac{m_a(h - h_{f_1})}{h_{fg}\,@100°C} = \frac{m_a(h - h_{f_1})}{2257}$$

(kg of steam/kg of fuel) ...(19.31)

19.4.3 Factor of Evaporation

It is defined as the ratio of heat received by 1 kg of feed water for evaporation under actual working conditions to that received by 1 kg of water evaporated from and at 100°C. It is denoted by *Fe* and expressed as

$$Fe = \frac{(h - h_{f_1})}{h_{fg}\,@100°C} = \frac{h - h_{f_1}}{2257} \quad ...(19.32)$$

It can also be expressed as

$$Fe = \frac{m_e}{m_a}$$

$$= \frac{\text{Equivalent evaporation from and at 100°C}}{\text{Evaporation per kg of fuel}} \quad ...(19.33)$$

19.4.4 Boiler Efficiency

Although the equivalent evaporation of steam in the boiler is a fair indication of the performance of the boiler, it does not take into account the quantity of fuel burned in the furnace of the boiler. Hence, the term boiler efficiency is used for comparison of the performance of boilers.

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The boiler efficiency is defined as the ratio of actual heat utilized in producing steam to the amount of heat librated by burning of fuel.

$$\eta_{boiler} = \frac{\text{Heat required to produce steam from feed water}}{\text{Heat librated by burning of fuel}} = \frac{\dot{m}_s (h - h_{f_1})}{\dot{m}_f \times CV} \quad ...(19.34)$$

where $\dot{m}_s$ = Rate of mass of steam generation, kg/h

$\dot{m}_f$ = Rate of mass of fuel firing, kg/h

h = Total enthalpy of steam, kJ/kg

h_{f_1} = Enthalpy of water at feed temperature

CV = Calorific value of fuel in kJ/kg

In Eq. (19.34), the quantity $\frac{\dot{m}_s}{\dot{m}_f} = m_a$ (evaporation per kg of fuel). Hence, the boiler efficiency can also be expressed as

$$\eta_{boiler} = \frac{m_a (h - h_{f_1})}{CV} \quad ...(19.35)$$

19.4.5 Economiser Efficiency

It is defined as the ratio of heat absorbed by the feed water in the economiser to the heat available to flue gases in the economiser.

$$\eta_{economiser} = \frac{\text{Heat utilised in economiser}}{\text{Heat available of flue gases}} \quad ...(19.36)$$

More often, it is necessary to obtain the percentage heat utilised in an economiser rather than the economiser efficiency. The percentage heat utilised in the economiser is defined as the ratio of heat absorbed by the feed water in the economiser to the heat liberated by combustion of fuel. Mathematically;

% heat utilised in economiser

$$= \frac{\text{Heat utilised in economiser}}{\text{Heat liberated by combustion of fuel}} = \frac{\dot{m}_s (\Delta h)_{feed\ water}}{\dot{m}_f\, CV} = \frac{\dot{m}_a (\Delta h)_{feed\ water}}{CV} \quad ...(19.37)$$

19.4.6 Percentage Heat Utilised in the Superheater

The percentage heat utilised in the superheater is defined as the ratio of heat absorbed by the steam in the superheater for its superheating to the heat liberated by combustion of fuel. Mathematically;

% heat utilised in superheater

$$= \frac{\text{Heat utilised in superheater}}{\text{Heat liberated by combustion of fuel}} = \frac{\dot{m}_s (h_{sup} - h)}{\dot{m}_f\, CV} = \frac{(h_{sup} - h)}{CV} \quad ...(19.38)$$

Where

$\dot{m}_s$ = Rate of mass of steam generation, kg/h

$\dot{m}_f$ = Rate of mass of fuel firing, kg/h

h = Total enthalpy of steam entering the superheater, kJ/kg

$= h_f + x\, h_{fg}$

h_{sup} = Enthalpy of superheated steam leaving the super-heater, kJ/kg;

$= h_g + C_{ps} (T_{sup} - T_{sat})$

CV = calorific value of fuel in kJ/kg.

19.4.7 Overall Efficiency of Boiler

It is expressed as

$$\eta_{overall} = \frac{\text{Heat utilised in boiler plant}}{\text{Heat produced by burning of fuel}} \quad ...(19.39)$$

19.4.8 Boiler Power

According to the American Society of Mechanical Engineers (ASME), the boiler power is expressed as

$$= \frac{\text{Equivalent evaporation from and at 100°C per hour}}{21.296} \quad ...(19.40)$$

Example 19.12 *Calculate the equivalent evaporation of a boiler per kg of coal fired, if the boiler produces 50,000 kg of wet steam per hour with a dryness fraction*

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of 0.95 and operating at 10 bar. The coal burnt per hour in the furnace is 5500 kg and feed-water temperature is 40°C.

Solution

Given Steam generation in a boiler

$p = 10$ bar $\quad x = 0.95$

$\dot{m}_s = 50{,}000$ kg/h $\quad \dot{m}_f = 5500$ kg/h

$T_{f_1} = 40°C$

To find Equivalent evaporation of the boiler per kg of coal fired

Analysis Properties of steam at 10 bar (1000 kPa)

$T_{sat} = 179.91°C$

$h_f = 762.79$ kJ/kg

$h_{fg} = 2015.29$ kJ/kg

Enthalpy of feed water at 40°C;

$h_{f_1} = 167.54$ kJ/kg

The mass of steam generated per kg of coal fired

$$m_a = \frac{\dot{m}_s}{\dot{m}_f} = \frac{50000}{5500} = 9.09 \text{ kg/kg of fuel}$$

The specific enthalpy of steam at 10 bar

$$h = h_f + x h_{fg}$$
$$= 762.79 + 0.95 \times 2015.29 = 2677.31 \text{ kJ/kg}$$

The equivalent evaporation

$$m_e = \frac{m_a (h - h_{f_1})}{2257}$$
$$= \frac{9.09 \times (2677.31 - 167.54)}{2257}$$
$$= \mathbf{10.1 \text{ kg/kg of fuel}}$$

Example 19.13 *In an experiment on a small oil-fired boiler, the steam produced is at 6 bar gauge. The quality of steam produced is found out to be 0.96 dry. 75 litres of water is converted into steam in 9.5 minutes. The fuel is a light diesel oil with specific gravity of 0.85 and calorific value of 43125 kJ/kg. 10 litres of oil is consumed in 11 minutes and 25 seconds. The feed-water temperature is 35°C. Determine the boiler efficiency. Take atmospheric pressure as 1 bar.*

Solution

Given Steam generation in a small oil-fired boiler

$p_g = 6$ bar $\quad p_{atm} = 1$ bar

$m_s = 75$ litres $= 75$ kg $\quad \Delta t_1 = 9.5$ min

$x = 9.6$

$m_f = 10 \text{ lit} = 10 \times 10^{-3} \text{ m}^3$

SG $= 0.85$

$\Delta t_2 = 11$ min., 25 s

$T_{f_1} = 35°C \quad CV = 43125$ kJ/kg

To find Thermal effciency of boiler, η_{boiler}.

Analysis The absolute pressure of generated steam

$p = p_{atm} + p_g = 6 + 1 = 7$ bar

Density of fuel

$\rho_f = \text{SG} \times \rho_{water} = 0.85 \times 1000$

$= 850 \text{ kg/m}^3$

Properties of steam at 7 bar

$h_f = 697.20$ kJ/kg, $\quad h_{fg} = 2064.85$ kJ/kg

Enthalpy of water at 35°C

$h_{f_1} = 146.66$ kJ/kg

The specific enthalpy of steam at 7 bar

$h = h_f + x h_{fg}$

$= 697.20 + 0.96 \times 2064.85$

$= 2763.01$ kJ/kg

Rate of steam generation

$$\dot{m}_s = \frac{m_s}{\Delta t_1} = \frac{75}{9.5 \times 60} = 0.1315 \text{ kg/s}$$

Rate of fuel consumption

$$\dot{m}_f = \frac{m_f}{\Delta t_2} = \frac{(10 \times 10^{-3} \text{ m}^3) \times (850 \text{ kg/m}^3)}{(11 \text{ min.} \times 60 \text{ s/min} + 25 \text{ s})}$$
$$= 0.0124 \text{ kg/s}$$

Thermal efficiency of boiler

$$\eta = \frac{\dot{m}_s (h - h_{f_1})}{\dot{m}_f \, CV}$$
$$= \frac{0.1315 \times (2763.01 - 146.66)}{0.0124 \times 43125}$$
$$= 0.6429 \text{ or } \mathbf{64.29\%}$$

Example 19.14 *Calculate the equivalent evaporation from and at 100°C for a boiler, which receives water at 60°C and produces steam at 1.5 MPa and 300°C. The steam-generation rate is 16000 kg/h. Coal is burnt at the rate of 1800 kg/h. The calorific value of coal is 34750 kJ/kg. Also calculate the thermal efficiency of the boiler.*

If the thermal efficiency of the boiler increases by 5% due to use of an economiser, find the saving in coal consumption per hour.

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Solution

Given Steam generation in a boiler

p = 1.5 MPa T_{sup} = 300°C

$\dot{m}_s$ = 16000 kg/h $\dot{m}_f$ = 1800 kg/h

T_{f_1} = 60°C CV = 34750 kJ/kg

$\eta_2 = \eta_1 + 0.05$

To find

(i) Equivalent evaporation from and at 100°C,

(ii) Thermal effciency of boiler, η_1,

(iii) Saving in fuel consumption, when boiler effciency increases by 5%.

Analysis Properties of superheated steam at 1.5 MPa (1500 kPa) and 300°C

h = 3038.9 kJ/kg

Enthalpy of water at 60°C

h_{f_1} = 251.1 kJ/kg

(i) The mass of steam generated per kg of coal fired

$$m_a = \frac{\dot{m}_s}{\dot{m}_f} = \frac{16000}{1800} = 8.89 \text{ kg/kg of fuel}$$

The equivalent evaporation from and at 100°C

$$m_e = \frac{m_a(h - h_{f_1})}{2257}$$

$$= \frac{8.89 \times (3038.9 - 251.1)}{2257}$$

$$= \mathbf{10.1 \text{ kg/kg of fuel}}$$

(ii) Thermal efficiency of boiler

$$\eta_1 = \frac{\dot{m}_s(h - h_{f_1})}{\dot{m}_f \, CV}$$

$$= \frac{16000 \times (3038.9 - 251.1)}{1800 \times 34750}$$

$$= 0.7131 \text{ or } \mathbf{71.31\%}$$

(iii) *Saving in fuel consumption, when boiler effciency increases by 5%*

With use of economiser, thermal effciency of boiler;

$$\eta_2 = \eta_1 + 0.05 = 0.7131 + 0.05 = 0.7631$$

Then $$\eta_2 = \frac{\dot{m}_s(h - h_{f_1})}{\dot{m}_{f_2} \, CV}$$

or $$\dot{m}_{f_2} = \frac{16000 \times (3038.9 - 251.1)}{0.7631 \times 34750}$$

$$= 1682 \text{ kg/h}$$

The saving in fuel consumption

$$= \dot{m}_f - \dot{m}_{f_1} = 1800 - 1682 = \mathbf{118 \text{ kg/h}}$$

Example 19.15 *A boiler generates 8 kg of steam per kg of fuel burnt at a pressure of 12 bar from feed water entering at 80°C. The boiler is 75% efficient and its factor of evaporation is 1.15. Calculate*

(a) Degree of superheat and temperature of steam generated,

(b) Calorific value of fuel in kJ/kg,

(c) Equivalent evaporation in kg of steam per kg of fuel.

Take specific heat of superheated steam as 2.3 kJ/kg·K.

Solution

Given A boiler with a superheater

m_a = 8 kg/kg of fuel C_{ps} = 2.3 kJ/kg·K

η_{boiler} = 0.75

p = 12 bar Fe = 1.15

T_{f_1} = 80°C

To find

(i) Degree of superheat and temperature of steam generated,

(ii) Calorific value of fuel in kJ/kg, and

(iii) Equivalent evaporation in kg of steam per kg of fuel.

Analysis The properties of steam

At 12 bar: T_{sat} = 188°C

h_g = 2784.83 kJ/kg

At 80°C: h_{f_1} = 334.88 kJ/kg

(i) *Degree of superheat and temperature of steam generated*

Factor of evaporation

$$Fe = \frac{h - h_{f_1}}{2257} = \frac{\left[h_g + C_{ps}(T_{sup} - T_{sat}) - h_{f_1}\right]}{2257}$$

$$1.15 = \frac{2784.83 + 2.3 \times (T_{sup} - T_{sat}) - 334.88}{2257}$$

$$2595.55 = 2452.25 + 2.3 \times (T_{sup} - T_{sat})$$

$$(T_{sup} - T_{sat}) = \mathbf{62.3°C}$$

The temperature of superheated steam is

T_{sup} = 188 + 62.3 = **250.3°C**

Specific enthalpy of superheated steam;

$$h = h_{sup} = h_g + C_{ps}(T_{sup} - T_{sat})$$

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$= 2784.83 + 2.3 \times (250.3 - 188)$

$= 2928.12$ kJ/kg·K

(ii) *Calorific value of fuel*

The boiler efficiency is given as

$$\eta_{boiler} = \frac{m_s(h - h_f)}{m_f \times CV} = \frac{m_a(h - h_{f_1})}{CV}$$

$$0.75 = \frac{8 \times (2928.12 - 334.88)}{CV}$$

Calorific value of fuel;

$$CV = \frac{20745.92}{0.75} = \textbf{27661.22 kJ/kg}$$

(iii) *Equivalent evaporation*

$$m_e = \frac{m_a(h - h_{f_1})}{2257}$$

$$= \frac{8 \times (2928.12 - 334.88)}{2257}$$

$= \textbf{9.19 kg/kg of fuel}$

Example 19.16 *Calculate the efficiency of (a) boiler, (b) economiser, and (c) whole plant having the following data:*

(a) Boiler

Mass of the feed water = 2060 kg/h
Mass of the coal burnt = 227 kg/h
Calorific value of coal = 30,000 kJ/kg
Enthalpy of steam produced = 2750 kJ/kg

(b) Economiser

Inlet temperature of feed water = 15°C
Exit temperature of feed water = 105°C
Atmospheric air temperature = 18°C
Temperature of flue gases entering = 370°C
Mass of flue gases = 4075 kg/h
Specific heat of flue gases = 1.3 kJ/kg·°C

Solution

Given A boiler plant with an economiser

$\dot{m}_f = 227$ kg/h, $\dot{m}_s = 2060$ kg/h
$CV = 30{,}000$ kJ/kg C, $h = 2750$ kJ/kg
$T_1 = 15$°C, $T_{f_1} = 105$°C
$T_g = 370$°C, $\dot{m}_g = 4075$ kg/h
$C_{pg} = 1.3$ kJ/kg,°C

To find

(i) Efficiency of boiler,
(ii) Efficiency of economiser, and
(iii) Efficiency of boiler plant.

Analysis

(i) Heat produced by complete burning of fuel

$$Q_1 = \dot{m}_f \times CV$$

$= 227 \times 30{,}000 = 681 \times 10^4$ kJ/h

Heat utilized in generation of steam

$$Q_2 = \dot{m}_s (h - h_{f_1})$$

$= 2060 \times (2750 - 4.187 \times 105)$

$= 475.93 \times 10^4$ kJ/h

The efficiency of boiler

$$\eta_{boiler} = \frac{\text{Heat utilised in generation of steam}}{\text{Heat produced by burning of fuel}}$$

$$\eta_{boiler} = \frac{475.93 \times 10^4 \text{ kJ/h}}{681 \times 10^4 \text{ kJ/h}}$$

$= 0.6988$ or **69.88%**

(ii) Heat utilized by economiser

= Heat used to raise the temp. of water from 15°C to 105°C.

$$Q_3 = \dot{m}_s C_{pw} (T_{f_1} - T_1)$$

$= 2060 \times 4.187 \times (105 - 15)$

$= 77.62 \times 10^4$ kJ/h

Heat available in flue gases passing to economiser

Q_4 = Heat of flue gases above atmospheric temperature

$= \dot{m}_g \times C_{pg} \times (T_g - T_a)$

$= 4075 \times 1.30 \times (370 - 18)$

$= 147.74 \times 10^4$ kJ/h

The efficiency of economiser

$$\eta_{economiser} = \frac{\text{Heat utilised in economiser}}{\text{Heat available of flue gases}}$$

$$= \frac{77.62 \times 10^4 \text{ kJ}}{147.74 \times 10^4 \text{ kJ}} = \textbf{52.53\%}$$

(iii) Overall efficiency of plant

$$\eta_{overall} = \frac{\text{Heat utilised in boiler plant}}{\text{Heat produced by burning of fuel}}$$

$$= \frac{475.93 \times 10^4 + 77.62 \times 10^4}{681 \times 10^4}$$

$= 0.8128$ or **81.28%**

Example 19.17 *A boiler generates steam at the rate of 6000 kg/h at a pressure of 800 kPa with a dryness fraction of 0.98. The feed water is supplied at 40°C. If the efficiency of the boiler is 75%, calculate the rate of coal consumption, which has a calorific value of 31000 kJ/kg. What is equivalent evaporation from this boiler?*

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If the superheater is used with the boiler and temperature of superheated steam reaches 250°C, then (a) what is the equivalent evaporation from the boiler, and (b) what is the thermal efficiency of boiler?

Take specific heat of superheated steam as C_{ps} = 2.27 kJ/kg · °C

Solution

Given A boiler plant with economiser

$\dot{m}_s$ = 6000 kg/h p = 800 kPa
x = 0.98 T_{f_1} = 40°C
η_1 = 0.75 CV = 31000 kJ/kg
T_{sup} = 250°C C_{ps} = 2.27 kJ/kg · K

To find

(i) Rate of coal consumption without superheater in the boiler,
(ii) Equivalent evaporation from boiler without superheater, and
(iii) Equivalent evaporation from boiler with superheater.
(iv) Efficiency of boiler plant.

Analysis The properties of steam:

At 800 kPa T_{sat} = 170.40°C
h_f = 720.9 kJ/kg
h_{fg} = 2047.5 kJ/kg
At 40°C h_{f_1} = 167.54 kJ/kg

(i) *Boiler without superheater*

The specific enthalpy of water

$$h = h_f + x\,h_{fg}$$
$$= 720.9 + 0.98 \times 2047.5$$
$$= 2727.45 \text{ kJ/kg}$$

(a) *Mass of coal fired per hour*

Thermal efficiency of boiler

$$\eta_{boiler} = \frac{\text{Heat used in generation of steam}}{\text{Heat produced by burning of fuel}}$$
$$= \frac{\dot{m}_s (h - h_{f_1})}{\dot{m}_f \times CV}$$
$$\dot{m}_f = \frac{6000 \times (2727.45 - 167.54)}{0.75 \times 31000}$$
$$= \mathbf{660.62 \text{ kg/h}}$$

(b) The evaporation rate per kg of coal fired

$$m_a = \frac{\dot{m}_s}{\dot{m}_f} = \frac{6000}{660.62}$$
$$= 9.08 \text{ kg/kg of fuel}$$

The equivalent evaporation

$$m_e = \frac{m_a (h - h_{f_1})}{2257}$$
$$= \frac{9.08 \times (2727.45 - 167.54)}{2257}$$
$$= \mathbf{10.30 \text{ kg/kg of fuel}}$$

(c) *Boiler with superheater*

The specific enthalpy of water

$$h_{sup} = h_g + C_{ps}(T_{sup} - T_{sat})$$
$$= h_f + h_{fg} + C_{ps}(T_{sup} - T_{sat})$$
$$= 720.9 + 2047.5 + 2.27 \times (250 - 170.40)$$
$$= 2949.09 \text{ kJ/kg}$$

Heat utilised by steam,

$$q_u = h_{sup} - h_{f_1} = 2949.09 - 167.54$$
$$= 2781.55 \text{ kJ/kg}$$

Then thermal efficiency of boiler

$$\eta_{boiler} = \frac{\text{Heat utilsed in generation of steam}}{\text{Heat produced by burning of fuel}}$$
$$= \frac{\dot{m}_s\, q_u}{\dot{m}_f \times CV}$$
$$= \frac{(6000 \text{ kg/h}) \times (2781.55 \text{ kJ/kg})}{(660.62 \text{ kg/h}) \times (31000 \text{ kJ/kg})}$$
$$= 0.8149 \text{ or } \mathbf{81.49\%}$$

Example 19.18 *In a boiler trial of one-hour duration, the following observations were made:*

Steam generated = 5250 kg
Fuel burnt = 695 kg
Calorific value of fuel = 30200 kJ/kg
Steam condition = 0.94
Boiler (steam) pressure = 11 bar (gauge)
Temperature of hot well = 47°C,
Temp. of steam leaving superheater = 240°C

Calculate:

(a) Equivalent evaporation per kg of coal without and with superheater
(b) Boiler efficiency with and without superheater

Assume atmospheric pressure as 100 kPa.

Solution

Given A trial on boiler with superheater

$\dot{m}_s$ = 5250 kg/h x = 0.94

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p_g = 11 bar $\quad p_{atm}$ = 100 kPa = 1 bar
$\dot{m}_f$ = 695 kg/h $\quad CV$ = 30200 kJ/kg
T_{f_1} = 47°C $\quad T_{sup}$ = 240°C

To find

(i) Equivalent evaporation without and with superheater,
(ii) Efficiency of boiler with and without superheater.

Assumption The specific heat for superheated steam as 2.1 kJ/kg·K.

Analysis The absolute pressure of steam in the boiler

$$p = p_g + p_{atm} = 11 + 1 = 12 \text{ bar}$$

The properties of steam:

At 12 bar (1200 kPa): T_{sat} = 188°C,
h_f = 798.64 kJ/kg,
h_{fg} = 1986.19 kJ/kg,
At 47°C: h_{f_1} = 196.8 kJ/kg.

(i) *Equivalent evaporation*:

Evaporation rate per kg of fuel;

$$m_a = \frac{\dot{m}_s}{\dot{m}_f} = \frac{5250}{695} = 7.554 \text{ kg/kg of fuel}$$

The specific enthalpy of steam without superheating

$$h_1 = h_f + x h_{fg} = 798.64 + 0.94 \times 1986.19$$
$$= 2665.65 \text{ kJ/kg}$$

The equivalent evaporation without superheater

$$m_e = \frac{m_a(h_1 - h_{f_1})}{2257}$$
$$= \frac{7.554 \times (2665.65 - 196.80)}{2257}$$
$$= \mathbf{8.26 \text{ kg/kg of fuel}}$$

The specific enthalpy of water with superheating

$$h_2 = h_f + h_{fg} + C_{ps}(T_{sup} - T_{sat})$$
$$= 798.64 + 1986.19 + 2.1 \times (240 - 188)$$
$$= 2894.03 \text{ kJ/kg}$$

The equivalent evaporation with superheater

$$m_e = \frac{m_a(h_2 - h_{f_1})}{2257}$$
$$= \frac{7.554 \times (2894.03 - 196.80)}{2257}$$
$$= \mathbf{9.02 \text{ kg/kg of fuel}}$$

(ii) *Thermal efficiency of boiler*

Without superheater;

$$\eta_{boiler} = \frac{\text{Heat used in generation of steam}}{\text{Heat produced by burning of fuel}}$$
$$= \frac{\dot{m}_s(h_1 - h_{f_1})}{\dot{m}_f \times CV}$$
$$= \frac{5250 \times (2665.65 - 196.80)}{695 \times 30200}$$
$$= \mathbf{0.6175 \text{ or } 61.75\%}$$

With superheater;

$$\eta_{boiler} = \frac{\dot{m}_s(h_2 - h_{f_1})}{\dot{m}_f \times CV}$$
$$= \frac{5250 \times (2894.03 - 196.80)}{695 \times 30200}$$
$$= \mathbf{0.6746 \text{ or } 67.46\%}$$

Example 19.19 *The following reading was recorded during a boiler trial:*

Feed water = *2400 kg/h*
Ambient temperature = *33°C*
Feed water temperature = *42°C,*
Fuel used = *205 kg/h*
Composition of fuel by mass:
Carbon = 84%, Hydrogen = 9.27%, Oxygen = 6.73%
Calorific value of fuel = *39500 kJ/kg*
Average chimney temp. = *307°C*
Height of chimney = *32 m*
Steam pressure = *11.054 bar (gauge)*
Barometric pressure = *710 mm of Hg*
Air supplied in excess = *50%*
Steam condition = *0.96*

Calculate

(a) Boiler efficiency,
(b) Equivalent evaporation from and at 100°C,
(c) Draught produced in mm of water.

Solution

Given A boiler plant with an economiser

$\dot{m}_s$ = 2400 kg/h $\quad x$ = 0.96
T_a = 33°C
p_g = 11.054 bar
p_{atm} = 710 mm of Hg
H = 32 m $\quad T_{f_1}$ = 42°C
$\dot{m}_f$ = 205 kg/h $\quad CV$ = 39500 kJ/kg
T_g = 307°C $\quad m_a = 1.5\, m_{th}$

Composition of fuel by mass:
Carbon = 84%

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Hydrogen = 9.27%
Oxygen = 6.73%

To find

(i) Efficiency of boiler,
(ii) Equivalent evaporation from and at 100°C,
(iii) Draught produced in mm of water.

Analysis The pressure in bar corresponding to 710 mm of Hg

$$p_{atm} = (1.0325 \text{ bar}) \times \frac{710 \text{ mm of Hg}}{760 \text{ mm of Hg}}$$
$$= 0.9466 \text{ bar}$$

The absolute pressure of steam in the boiler

$$p = p_g + p_{atm} = 11.054 + 0.9466 = 12 \text{ bar}$$

The properties of steam

At 12 bar (1200 kPa)

h_f = 798.64 kJ/kg
h_{fg} = 1986.19 kJ/kg

At 42°C: h_{f_1} = 175.92 kJ/kg

The specific enthalpy of steam;

$$h = h_f + x h_{fg} = 798.64 + 0.96 \times 1986.19$$
$$= 2705.38 \text{ kJ/kg}$$

(i) *Thermal efficiency of boiler*

$$\eta_{boiler} = \frac{\text{Heat used in generation of steam}}{\text{Heat produced by burningo f fuel}}$$
$$= \frac{\dot{m}_s (h - h_{f_1})}{\dot{m}_f \times CV}$$
$$= \frac{2400 \times (2705.38 - 175.92)}{205 \times 39500}$$
$$= \mathbf{0.7497 \text{ or } 74.97\%}$$

(ii) *Equivalent evaporation from and at 100°C*

Evaporation rate per kg of fuel;

$$m_a = \frac{2400}{205} = 11.7 \text{ kg/kg of fuel}$$

The equivalent evaporation

$$m_e = \frac{m_a (h - h_{f_1})}{2257}$$
$$= \frac{11.7 \times (2705.38 - 175.92)}{2257}$$
$$= \mathbf{13.12 \text{ kg/kg of fuel}}$$

(iii) *Draught produced in mm of water*

Theoretical mass of air required for complete combustion of fuel

$$m_{th} = \frac{100}{23}\left[\frac{8}{3}\text{C} + 8\left(\text{H} - \frac{\text{O}}{8}\right) + \text{S}\right]$$
$$= \frac{100}{23} \times \left[\frac{8}{3} \times 0.84 + 8\left(0.0927 - \frac{0.0673}{8}\right) + 0\right]$$
$$= 12.67 \text{ kg/kg of fuel}$$

The mass of actual air supplied;

$$m_a = 1.5\, m_{th} = 1.5 \times 12.67$$
$$= 19 \text{ kg/kg of fuel}$$

The draught produced in water column

$$h = 353H\left[\frac{1}{T_a} - \left(\frac{m_a + 1}{m_a}\right) \times \frac{1}{T_g}\right] \text{ (mm of water)}$$
$$= 353 \times 32 \times \left[\frac{1}{(33 + 273)} - \left(\frac{19 + 1}{19}\right) \times \frac{1}{(307 + 273)}\right]$$
$$= \mathbf{16.415 \text{ mm of water}}$$

Example 19.20 *The following readings were recorded during two hour trial on a boiler:*

Feed water supplied	*= 14000 kg*
Boiler working pressure	*= 10 bar*
Dryness fraction of steam	*= 0.96*
Temp. of feed water entering economiser	*= 35°C*
Temp. of feed water leaving economiser	*= 90°C*
Temperature of steam leaving superheater	*= 250°C*
Coal burnt	*= 1500 kg*
Calorific value of coal	*= 33500 kJ/kg*

Calculate:

(a) Enthalpy received by feed water in economiser, boiler and superheater,
(b) Percentage of heat utilized in economiser, boiler and superheater,
(c) Overall thermal efficiency of plant,
(d) Overall equivalent evaporation from and at 100°C.

Solution

Given A trial on boiler equipped with economiser and superheater:

Δt = 2 hours, m_s = 14000 kg
m_f = 1500 kg, p = 10 bar
x = 0.96, CV = 33500 kJ/kg
T_1 = 35°C, T_{f_1} = 90°C
T_{sup} = 250°C

To find

(i) Enthalpy received by feed water in economiser, boiler and superheater,

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(ii) Percentage of heat absorbed in economiser, boiler and superheater,

(iii) Overall thermal efficiency of plant,

(iv) Overall equivalent evaporation from and at 100°C.

Analysis The properties of steam:

At 10 bar: $T_{sat} = 179.91°C$

$h_f = 762.8$ kJ/kg

$h_{fg} = 2014.3$ kJ/kg

At 35°C: $h_1 = 146.66$ kJ/kg

At 90°C: $h_{f_1} = 381.04$ kJ/kg

At 10 bar and 250°C:

$h_{sup} = 2942.58$ kJ/kg

The steam generation per hour;

$$\dot{m}_s = \frac{m_s}{\Delta t} = \frac{14000 \text{ kg}}{2 \text{ hour}} = 7000 \text{ kg/h}$$

The coal supplied per hour,

$$\dot{m}_f = \frac{m_f}{\Delta t} = \frac{1500 \text{ kg}}{2 \text{ hour}} = 750 \text{ kg/h}$$

(i) *Enthalpy received by feed water in economiser, boiler and superheater:*

Heat supplied per kg of water in economiser;

$$q_1 = h_{f_1} - h_1$$
$$= 381.04 - 146.66 = \mathbf{234.38 \text{ kJ/kg}}$$

Heat supplied per kg of water in boiler;

$$q_2 = h_f + x\, h_{fg} - h_{f_1}$$
$$= 762.8 + 0.96 \times 2014.3 - 381.04$$
$$= \mathbf{2315.48 \text{ kJ/kg}}$$

Heat supplied per kg of steam in superheater;

$$q_3 = (1 - x) h_{fg} + C_{ps}(T_{sup} - T_{sat})$$
$$= h_{sup} - (h_f + x\, h_{fg})$$
$$= 2942.58 - (762.8 + 0.96 \times 2014.3)$$
$$= \mathbf{246.05 \text{ kJ/kg}}$$

(ii) *Percentage of heat absorbed in economiser, boiler and superheater*

Total heat utilised per kg of formation of superheated steam from water at 35°C

$$q_u = q_1 + q_2 + q_3$$
$$= 234.38 + 2315.48 + 246.05$$
$$= 2795.91 \text{ kJ/kg}$$

Percentage of heat absorbed in the economiser

$$= \frac{q_1}{q_u} \times 100 = \frac{234.38}{2795.91} \times 100$$
$$= \mathbf{8.38\%}$$

Percentage of heat absorbed in the boiler

$$= \frac{q_2}{q_u} \times 100 = \frac{2315.48}{2795.91} \times 100$$
$$= \mathbf{82.82\%}$$

Percentage of heat absorbed in superheater

$$= \frac{q_3}{q_u} \times 100 = \frac{246.05}{2795.91} \times 100$$
$$= \mathbf{8.80\%}$$

(iii) *Overall thermal efficiency of the plant*

$$\eta_{overall} = \frac{\text{Total heat utilised in boiler plant}}{\text{Heat supplied by burning of fuel}} \times 100$$
$$= \frac{\dot{m}_s\, q_u}{\dot{m}_f \times CV} \times 100$$
$$= \frac{(7000 \text{ kg/h}) \times 2795.91 \text{ kJ/kg})}{(750 \text{ kg/h}) \times (33500 \text{ kJ/kg})} \times 100$$
$$= \mathbf{77.9\%}$$

(iv) *Overall equivalent evaporation from and at 100°C*

Evaporation rate per kg of fuel,

$$m_a = \frac{\dot{m}_s}{\dot{m}_f} = \frac{7000 \text{ kg/h}}{750 \text{ kg/h}}$$
$$= 9.33 \text{ kg/kg of fuel}$$

$$\therefore \quad m_e = \frac{m_a(h_{sup} - h_{f_1})}{2257}$$
$$= \frac{9.33 \times (2942.58 - 381.04)}{2257}$$
$$= \mathbf{10.58 \text{ kg/kg of fuel}}$$

Example 19.21 *A trial on a water-tube boiler gave the following data:*

Boiler working pressure	*= 12 bar*
Degree of superheat	*= 77°C*
Temperature of feed water	*= 80°C*
Feed water supplied per hour	*= 4000 kg*
Coal fired per hour	*= 450 kg*
Calorific value of dry coal	*= 31000 kJ/kg*
Ash	*= 45 kg*
Percentage of combustible in ash	*= 10%*
Calorific value of combustible in ash	*= 30000 kJ/kg*
Moisture in coal	*= 4.5%*

Calculate

(a) Efficiency of the boiler plant including the superheater,

(b) Efficiency of boiler and furnace combined.

Take specific heat of superheated steam as 2.1 kJ/kg·K.

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Solution

Given A trial on a boiler equipped with a superheater:

$\dot{m}_s = 4000$ kg/h $\quad \dot{m}_f = 450$ kg/h
$CV_1 = 31000$ kJ/kg $\quad CV_2 = 30000$ kJ/kg
$p = 12$ bar $\quad T_{sup} - T_{sat} = 77°C$
Ash $= 45$ kg $\quad T_{f_1} = 80°C$
moisture in coal = 4.5%
Combustible in ash = 10%
Ash $= 45$ kg $\quad C_{ps} = 2.1$ kJ/kg·K

To find

(i) Efficiency of the boiler plant including the superheater, and
(ii) Efficiency of boiler and furnace combined.

Analysis The properties of steam:

At 12 bar $\quad T_{sat} = 188°C$
$h_g = 2784.83$ kJ/kg
At 80°C $\quad h_{f_1} = 334.88$ kJ/kg

(i) *Efficiency of the boiler plant including superheater*

Specific enthalpy of superheated steam;

$$h = h_{sup} = h_g + C_{ps}(T_{sup} - T_{sat})$$
$$= 2784.83 + 2.1 \times 77$$
$$= 2946.53 \text{ kJ/kg·K}$$

Heat utilised to produce steam;

$$Q_u = \dot{m}_s (h_{sup} - h_{f_1})$$
$$= 4000 \times (2946.53 - 334.88)$$
$$= 10446600 \text{ kJ/h}$$

Mass of dry coal;

$$\dot{m}_{df} = \dot{m}_f - \frac{4.5}{100} \times \dot{m}_f$$
$$= 450 - \frac{4.5}{100} \times 450 = 429.75 \text{ kg/h}$$

Heat produced by burning of coal;

$$Q_s = \dot{m}_{df}\, CV_1 = 429.75 \times 31000$$
$$= 13322250 \text{ kJ/h}$$

Boiler efficiency;

$$\eta_{boiler} = \frac{Q_u}{Q_s} \times 100 = \frac{10446600}{13322250} \times 100$$
$$= \mathbf{78.41\%}$$

(ii) *Efficiency of boiler and furnace combined*

Combustible matter in ash per hour

$$= 45 \times \frac{10}{100} = 4.5 \text{ kg}$$

Actual heat produced per hour,

$$Q_{act} = 429.75 \times 31000 - 4.5 \times 30000$$
$$= 13187250 \text{ kJ/kg}$$

Combined efficiency of boiler and furnace

$$\eta_{combined} = \frac{Q_u}{Q_{act}} \times 100 = \frac{10446600}{13187250} \times 100$$
$$= \mathbf{79.21\%}$$

Example 19.22 *A boiler is supplied with 15,900 kg of feed water at 15°C, during a trial period of 8 hours and 20 minutes. At the end of the trial, the weight of water was 680 kg less than that of commencement of trial. The steam produced at the pressure of 12.5 bar is 95% dry.*

The coal having the calorific value of 30,000 kJ/kg is fired at the rate of 275.8 kg/h. Calculate (a) actual evaporation rate, (b) equivalent evaporation, and (c) thermal efficiency of the boiler.

Solution

Given A trial on a boiler

$m_{water} = 15{,}900$ kg $\quad T_{f_1} = 15°C$
$\Delta t = 8$ h, 20 min = 500 min
$m_{make\ up} = 680$ kg $\quad \dot{m}_f = 275.8$ kg/h
$CV = 30{,}000$ kJ/kg
$x = 0.95 \quad p = 12.5$ bar

To find

(i) Actual evaproation,
(ii) Equivalent evaporation, and
(iii) Boiler efficiency.

Analysis The total water evaporated during the trial

$$m_s = 680 \text{ kg} + 15{,}900 \text{ kg} = 16{,}580 \text{ kg}$$

(i) *Actual evaporation rate*

$$\dot{m}_s = \frac{16580 \text{ kg}}{500 \text{ min}}$$
$$= 33.16 \text{ kg/min or } 1989.6 \text{ kg/h}$$

Actual evaporation in terms of evaporation per kg coal burnt

$$m_a = \frac{1989.6 \text{ kg/h}}{275.8 \text{ kg/h}} = \mathbf{7.214 \text{ kg/kg of coal}}$$

From steam table

At 12.5 bar
$h_f = 806.9$ kJ/kg,
$h_{fg} = 1977.5$ kJ/kg
At 15°C $\quad h_{f_1} = 62.98$ kJ/kg

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Enthalpy of steam produced;

$$h = h_f + x\, h_{fg}$$
$$= 806.9 + 0.95 \times 1977.5$$
$$= 2685.52 \text{ kJ/kg}$$

The amount of heat supplied to steam per kg

$$h - h_{f_1} = 2685.52 - 62.98$$
$$= 2622.54 \text{ kJ/kg}$$

(ii) *Equivalent evaporation*

$$m_e = \frac{m_a (h - h_{f_1})}{2257}$$
$$= \frac{7.214 \times 2622.54 \text{ kJ/kg}}{2257}$$
$$= \mathbf{8.38 \text{ kg/kg of fuel}}$$

(iii) *Boiler efficiency*

Heat utilised by water during formation of steam

$$Q_u = \dot{m}_s (h - h_{f_1})$$
$$= (1989.6 \text{ kg/h}) \times (2622.54 \text{ kJ/kg})$$
$$= 52.178 \times 10^5 \text{ kJ/h}$$

Heat supplied by fuel;

$$Q_s = \dot{m}_f \times CV = 275.8 \times 30{,}000$$
$$= 82.5 \times 10^5 \text{ kJ/h}$$

Boiler efficiency

$$\eta_{boiler} = \frac{\text{Heat utilised}}{\text{Heat supplied}} \times 100$$
$$= \frac{52.178 \times 10^5 \text{ kg/h}}{82.5 \times 10^5 \text{ kg/h}} \times 100 = \mathbf{63.24\%}$$

Example 19.23 *A steam generator evaporates 18000 kg/h of steam at 12.5 bar and a quality of 0.97 dry from feed water at 105°C, when coal is fired at 2040 kg/h. If the higher calorific value of coal is 27400 kJ/kg, find the*

(a) heat rate of the boiler in kJ/h,
(b) equivalent evaporation, and
(c) thermal effciency.

Solution

Given A steam generator;

$\dot{m}_s = 18000$ kg/h $p = 12.5$ bar
$x = 0.97$ $T_{f_1} = 105°C$
$\dot{m}_f = 2040$ kg/h $CV = 27400$ kJ/kg

To find

(i) The heat rate of the boiler,
(ii) Equivalent evaporation, and
(iii) Thermal efficiency of boiler.

Analysis The properties of steam:

From steam table;

At 12.5 bar $h_f = 806.9$ kJ/kg
$h_{fg} = 1977.5$ kJ/kg
At 105°C $h_{f_1} = 440.13$ kJ/kg

(i) *The heat rate of the boiler*

$Q_u =$ Heat utilised by water during formation of steam

Specific enthalpy of steam;

$$h = h_f + x\, h_{fg}$$
$$= 806.9 + 0.97 \times 1977.5$$
$$= 2725.07 \text{ kJ/kg}$$

Enthalpy increase of steam in the boiler;

$$= h - h_{f_1} = 2725.07 - 440.13$$
$$= 2284.94 \text{ kJ/kg}$$

Heat rate of the boiler;

$$Q_u = \dot{m}_s (h - h_{f_1})$$
$$= 18000 \times 2284.94$$
$$= \mathbf{411.289 \times 10^5 \text{ kJ/h}}$$

(ii) *Equivalent evapoartion*

Evapoartion rate per kg of fuel;

$$m_a = \frac{\dot{m}_s}{\dot{m}_f} = \frac{18000}{2040} = 8.823 \text{ kg/kg of fuel}$$

$$m_e = \frac{m_a (h - h_{f_1})}{2257}$$
$$= \frac{8.823 \times 2284.94}{2257}$$
$$= \mathbf{8.932 \text{ kg/kg of fuel}}$$

(iii) *Thermal efficiency*

Heat supplied by fuel;

$$Q_s = \dot{m}_f \times CV = 2040 \times 27400$$
$$= 558.96 \times 10^5 \text{ kJ/h}$$

Boiler efficiency

$$\eta_{boiler} = \frac{\text{Heat utilised}}{\text{Heat supplied}} \times 100$$
$$= \frac{411.289 \times 10^5 \text{ kg/h}}{558.96 \times 10^5 \text{ kg/h}} \times 100 = \mathbf{73.61\%}$$

19.5 ENERGY BALANCE IN A BOILER

In a boiler, the heat is produced by burning of fuel in the presence of atmospheric air. A part of this heat is used to generate the steam and the remaining

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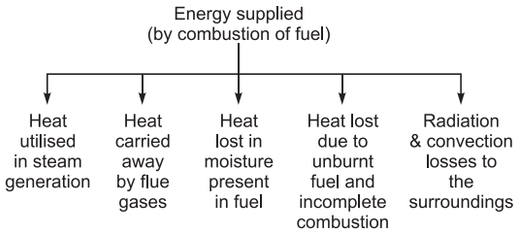
portion is lost. The energy diagram for a boiler may be given as follows:



We express each quantity per kg of fuel burnt in the following manner.

Heat generated by combustion of 1 kg (or 1 m^3 for gaseous) fuel in the furnace:

$$q_{in} = CV$$

where CV = Lower calorific value of fuel, kJ/kg.

(*a*) *Heat Absorbed by Water During its Heating and Evaporation*

The mass of steam generation per kg of fuel

$$m_a = \frac{\text{Mass of steam generated per hour}}{\text{Mass of fuel fired per hour}}$$

(kg of steam per kg of fuel)

$$q_1 = m_a(h - h_{f_1}) \text{ (kJ/kg of fuel)} \quad ...(19.41)$$

where m_a = Mass of steam generated per kg of fuel

h = Specific enthalpy of steam, kJ/kg

h_{f_1} = Initial specific enthalpy of feed water, kJ/kg

(*b*) *Heat Carried by Dry Flue Gases*

The flue gases carry the dry products of combustion as well as vapour generated due to combustion of hydrogen in fuel.

Actual air supplied per kg of fuel burnt,

$$m_{a_1} = \frac{N_2 \times C}{33 \times (CO + CO_2)} \text{ (kg/kg of fuel)} \quad ...(19.42)$$

where,

C = Mass fraction of carbon in fuel

N_2 = Volumetric percent of N_2 in flue gases

CO_2 = Volumetric percent of CO_2 in flue gases

CO = Volumetric percent of CO in flue gases

The mass of dry gases formed per kg of fuel,

m_g = Mass of air supplied + Mass of Carbon present in the fuel

$= m_{a_1} + m_C$ (kg/kg of fuel)

Heat Carried Away by dry Flue Gases

$$q_2 = m_g C_{pg}(T_g - T_a) \text{ (kJ/kg of fuel)} \quad ...(19.43)$$

where, C_{pg} = Specific heat of combustion gases

T_g = Temperature of hot flue gases

T_a = Atmospheric temperature

(*c*) *Heat Carried by Moisture per kg of Fuel*

The moisture formed per kg of fuel

= 9 H (kg/kg of fuel)

If some moisture is present in the fuel, total mass of moisture;

m_v = fraction mass of moisture in fuel + mass of vapour formed

Heat carried by moisture

$$q_3 = m_v[h_g + C_{ps}(T_g - T_{sat}) - C_{pw}T_a] \text{ (kJ/kg of fuel)} \quad ...(19.44)$$

where,

h_g = Total specific enthalpy of steam at partial pressure of steam in flue gases

C_{ps} = Specific heat of superheated steam ≈ 2.1 kJ/kg · K

C_{pw} = Specific heat of water ≈ 4.2 kJ/kg · K

T_a = Temperature of boiler house

T_g = Temperature of hot flue gases

T_{sat} = Saturation temperature at partial pressure of steam in flue gases

(*d*) *Heat Lost Due to Incomplete Combustion*

If carbon present in fuel is burnt to CO instead of CO_2, the combustion is said to be incomplete.

1 kg of carbon releases 33800 kJ of heat, if it burns to CO_2, and

1 kg of carbon releases 10120 kJ of heat, if it burns to CO.

∴ heat lost due to incomplete combustion

= 33800 − 10120 = 23680 kJ/kg of coal.

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If the percentages of CO and CO_2 in the gases by volume are known, the mass of carbon burnt to CO

$$= \frac{CO \times C}{CO_2 + CO}$$

and heat lost due to incomplete combustion

$$q_4 = \frac{CO \times C}{CO_2 + CO} \times 23680 \text{ (kJ/kg of fuel)} \quad ...(19.45)$$

The heat lost due to incomplete combustion can be reduced by supplying excess quantity of air in turbulent motion.

(e) *Heat Lost Due to Unburnt Fuel*

If m_{uf} is the mass of unburnt fuel per kg of fuel then

$$q_5 = m_{uf} \times CV \quad ...(19.46)$$

where

m_{uf} = Mass of unburnt fuel per kg of fuel,
CV = Calorific value.

(f) *Unaccounted Heat Losses*

The boiler furnace and hot surfaces are exposed to atmosphere, therefore, some heat is radiated and convected to the surroundings.

Unaccounted heat losses

$= q_{in} - (q_1 + q_2 + q_3 + q_4 + q_5)$ kJ/kg of fuel ...(19.47)

Heat Balance Sheet

Heat supplied/ kg of fuel	*%*	*Heat utilisation*	*kJ/kg of fuel*	*%*
$q_{in} = CV$	100	(a) Heat used to generate steam	$q_1 = m_a \times (h - h_{f_1})$	
		(b) Heat carried away by dry flue gases	$q_2 = m_g C_{pg} \times (T_g - T_a)$	
		(c) Heat carried away by steam in dry flue gases	$q_3 = m_v \times (h_{sup} - C_{pw} T_a)$	
		(d) Heat lost due to incomplete combustion	$q_4 = \frac{CO \times C}{CO_2 + CO} \times 23680$	
		(d) Heat lost due to incomplete unburnt fuel	$q_5 = m_{uf} \times CV$	
		(e) Unaccounted heat losses	$q_6 = q_{in} - q_1 - q_2 - \ldots$	
				100%

Example 19.24 *The following data were obtained during a boiler trial:*

Mass of steam = *700 kg/h*
Temperature of feed water = *60°C*
Steam pressure = *10 bar*
Oil consumption = *55 kg/h*
CV of oil = *44000 kJ/kg*
Dryness fraction of steam = *0.98*
Percentage composition of oil by mass:
C = 85%, H_2 = 14%, Ash = 1%
Analysis of dry flue gases by volume:
CO_2 = 12.5%, O_2 = 4.5%, N_2 = 83%
Temp. of flue gases leaving the boiler = *350°C*
Boiler room temperature = *25°C*
Specific heat of flue gases = *1.02 kJ/kg·K*
Partial pressure of steam = *0.08 bar*
Heating surface area = *21.4 m^2*

Find

(a) equivalent evaporation per kg of fuel from and at 100°C,
(b) equivalent evaporation per sq. m of heating area,
(c) thermal efficiency of the boiler,
(d) heat balance sheet on the basis of 1 kg of fuel and on the percentage basis.

Solution

Given A trial on a boiler

Mass of steam, $\dot{m}_s$ = 700 kg/h
Temp. of feed water, T_{f_1} = 60°C
Steam pressure = 10 bar
Oil consumption, $\dot{m}_f$ = 55 kg/h
CV of oil = 44000 kJ/kg
x = 0.98

Percentage composition of oil by mass:

C = 85% H_2 = 14% Ash = 1%

Analysis of dry flue gases by volume:

CO_2 = 12.5% O_2 = 4.5% N_2 = 83%

T_g = 350°C

Boiler room temperature,

T_a = 25°C
C_{pg} = 1.02 kJ/kg·K

Partial pressure of steam = 0.08 bar
Heating surface area = 21.4 m^2

To find

(i) Equivalent evaporation per kg of fuel from and at 100°C,

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(ii) Equivalent evaporation per sq. m of heating area,
(iii) Thermal efficiency of the boiler, and
(iv) Heat balance sheet on the basis of 1 kg of fuel and on percentage basis.

Assumption The specific heat of superheated steam, $C_{ps} = 2.1$ kJ/kg·K

Analysis The properties of steam:

At 10 bar (1000 kPa) pressure

$$h_f = 762.79 \text{ kJ/kg}$$
$$h_{fg} = 2015.29 \text{ kJ/kg}$$

At 60°C

$$h_{f_1} = 251.11 \text{ kJ/kg}$$

The specific enthalpy of steam,

$$h = h_f + x\, h_{fg}$$
$$= 762.79 + 0.98 \times 2015.29$$
$$= 2737.77 \text{ kJ/kg}$$

Heat supplied to steam in the boiler

$$= h - h_{f_1} = 2737.77 - 251.11$$
$$= 2486.66 \text{ kJ/kg}$$

(i) *Equivalent evaporation per kg of fuel from and at 100°C*

Mass of steam generated per kg of fuel

$$m_a = \frac{700 \text{ kg/h}}{55 \text{ kg/h}} = 12.72 \text{ kg/kg of fuel}$$

The equivalent evaporation,

$$m_e = \frac{m_a(h - h_{f_1})}{2257}$$
$$= \frac{(12.72 \text{ kg/kg of fuel}) \times (2486.66 \text{ kJ/kg})}{(2257 \text{ kJ/kg})}$$
$$= \mathbf{14 \text{ kg of steam/per kg of fuel}}$$

(ii) *Equivalent evaporation per sq.m of heating area*

Mass of steam generated per sq.m of heating surface

$$m_a = \frac{700 \text{ kg/h}}{21.4 \text{ m}^2}$$
$$= 32.71 \text{ kg/m}^2 \text{ of heaing area/h}$$

Equivalente vaporation,

$$m_e = \frac{m_a(h - h_{f_1})}{2257}$$
$$= \frac{(32.71 \text{ kg/m}^2 \text{ grate area}) \times (2486.66 \text{ kJ/kg})}{2257 \text{ kJ/kg}}$$
$$= \mathbf{36 \text{ kg of steam/m}^2 \text{ of heating area/h}}$$

(iii) *Thermal efficiency*

Heat utilised in boiler,

$$\dot{Q}_u = \dot{m}_s (h - h_{f_1})$$
$$= (700 \text{ kg/h}) \times (2486.66 \text{ kJ/kg})$$
$$= 1740662 \text{ kJ/h}$$

Heat supplied by fuel,

$$\dot{Q}_s = \dot{m}_f \, CV$$
$$= (55 \text{ kg/h}) \times (44000 \text{ kJ/kg})$$
$$= 2420000 \text{ kJ/h}$$

Efficiency,

$$\eta_{boiler} = \frac{\dot{Q}_u}{\dot{Q}_s} \times 100$$
$$= \frac{(1740662 \text{ kJ/h})}{(2420000 \text{ kJ/h})} \times 100 = \mathbf{71.93\%}$$

(iv) *Heat balance sheet*

(a) Heat supplied by 1 kg of fuel,

$$q_{in} = CV = 44000 \text{ kJ/kg}$$

(b) Heat absorbed by water per kg of fuel during its heating and evaporation,

$$q_1 = m_a(h - h_{f_1})$$
$$= (12.72 \text{ kg/kg of fuel}) \times (2486.66 \text{ kJ/kg})$$
$$= 31630.31 \text{ kJ/kg of fuel}$$

(c) Actual air required per kg of fuel burnt,

$$m_a = \frac{\text{N} \times \text{C}}{33 \times (\text{CO} + \text{CO}_2)} = \frac{83 \times 85}{33 \times (0 + 12.5)}$$
$$= 17.1 \text{ kg/kg of fuel}$$

Since 0.85 kg carbon is also present per kg of fuel, thus the mass of dry gases formed per kg of fuel,

$$m_g = m_a + m_C$$
$$= 17.1 + 0.85 = 17.95 \text{ kg/kg of fuel}$$

(d) Heat carried by dry flue gases,

$$q_2 = m_g C_{pg} (T_g - T_a)$$
$$= 17.95 \times 1.02 \times (350 - 25)$$
$$= 5944.8 \text{ kJ/kg of fuel}$$

(e) *Heat carried by moisture per kg of fuel*

The moisture formed per kg of fuel,

$$m_v = 9\text{H} = 9 \times 0.14 = 1.26 \text{ kg/kg of fuel}$$

Properties of steam at 0.08 bar (8 kPa)

$$h_g = 2577 \text{ kJ/kg}$$
$$T_{sat} = 41.54°\text{C}$$

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Heat carried by moisture

$$q_3 = m_v[h_g + C_{ps}(T_g - T_{sat}) - C_{pw}T_a]$$
$$= 1.26 \times [2577 + 2.1 \times (350 - 41.54) - 4.2 \times 25]$$
$$= 3931 \text{ kJ/kg of fuel}$$

Heat lost by radiation and convection, etc.

$$q_4 = q_{in} - q_1 - q_2 - q_3$$
$$= 44000 - 31630.31 - 5944.8 - 3931$$
$$= 2493.9 \text{ kJ/kg of fuel}$$

Heat Balance Sheet

Heat supplied/ kg of fuel	*%*	*Heat utilisation*	*kJ/kg of fuel*	*%*
$q_{in} = CV$ = 44000 kJ/kg	100	(a) Heat used to generate steam	$q_1 = 31648.4$	71.92%
		(b) Heat carried away by dry flue gases	$q_2 = 5944.8$	13.51%
		(c) Heat carried away by steam in dry flue gases	$q_3 = 3931$	8.93%
			$q_6 = 2493.9$	5.66%
		(d) Unaccounted heat losses		
				100%

Example 19.25 *The following data were recorded during a boiler trial:*

Boiler room temperature = *25°C*
Temperature of feed water = *55°C*
Mass of steam = *14000 kg/h*
Steam pressure = *128 N/cm²*
Steam temperature = *250°C*
Coal consumption = *1600 kg/h*
CV of coal = *35000 kJ/kg*
Analysis of flue gases by volume:
CO_2 = *9.4%* O_2 = *1.1%* N_2 = *79.5%*
Percentage composition of coal by mass:
C = 85.2% H_2 = *4.8%* *Ash = 10%*
Moisture = 1.8%
Temp. of flue gases leaving the boiler = *310°C*
Partial pressure of steam = *0.08 bar*
Specific heat of air and dry flue gases = *1.02 kJ/kg·K*
Calculate
(a) Heat carried by excess air,
(b) Make complete heat balance sheet on the basis of 1 kg of coal and on percentage basis.

Solution

Given A trial on a boiler

Mass of steam, $\dot{m}_s = 14000$ kg/h
Temp. of feed water, $T_{f_1} = 55°C$
Steam pressure = 128 N/cm²
$T_{sup} = 250°C$
Coal consumption, $\dot{m}_f = 1600$ kg/h
CV of coal = 35000 kJ/kg

Analysis of dry flue gases by volume

$CO_2 = 9.4\%$ $O_2 = 1.1\%$
$N_2 = 79.5\%$ $T_g = 310°C$
Boiler room temperature, $T_a = 25°C$
$C_{pg} = 1.02$ kJ/kg·K
Partial pressure of steam = 0.08 bar

Percentage composition of coal by mass

C = 85.2% H_2 = 4.8%
Ash = 10% Moisture = 1.8%

To find

(i) Heat carried away by excess air in kJ/kg of fuel,
(ii) Heat balance sheet on the basis of 1 kg of coal and on percentage basis.

Assumption The specific heat of superheated steam, $C_{ps} = 2.1$ kJ/kg·K.

Analysis

(i) *Heat carried away by excess air*

Flue gases produced per kg of fuel burnt,

$$m_{a_1} = \frac{N \times C}{33 \times (CO + CO_2)} = \frac{79.5 \times 85.2}{33 \times (0 + 9.4)}$$
$$= 21.83 \text{ kg/kg of fuel}$$

The theoretical amount of air required for complete combustion of 1 kg of fuel

$$m_{th} = \frac{100}{23}\left[\frac{8}{3}C + 8\left(H - \frac{O}{8}\right) + S\right] \text{kg/kg of fuel}$$
$$= \frac{100}{23} \times \left[\frac{8}{3} \times 0.852 + 8 \times (0.048) + 0\right]$$
$$= 11.55 \text{ kg/kg of fuel}$$

The mass of excess air supplied per kg of fuel

$$m_{ex} = 21.83 - 11.55$$
$$= 10.28 \text{ kg/kg of fuel}$$

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Heat carried by excess air

$= m_{ex} C_{pg} \times (T_g - T_a)$

$= 10.28 \times 1.02 \times (310 - 25)$

$=$ **2988.4 kJ/kg of fuel**

The properties of steam

At 128 N/cm² (1280 kPa) $h_{fg} = 2785$ kJ/kg

$T_{sat} = 190.5°C$

At 55°C $h_{f_1} = 230.3$ kJ/kg

The specific enthalpy of steam,

$h = h_g + C_{ps}(T_{sup} - T_{sat})$

$= 2785 + 2.1 \times (250 - 190.5)$

$= 2909.95$ kJ/kg

Heat supplied to steam per kg

$= h - h_{f_1} = 2905.95 - 230.3$

$= 2679.65$ kJ/kg

Mass of steam generated per kg of fuel

$$m_a = \frac{14000 \text{ kg/h}}{1600 \text{ kg/h}} = 8.75 \text{ kg/kg of fuel}$$

(ii) *Heat balance sheet*

(a) Heat supplied by 1 kg of fuel,

$q_{in} = CV = 35000$ kJ/kg

(b) Heat absorbed by water per kg of fuel during its heating and evaporation,

$q_1 = m_a(h - h_{f_1})$

$= 8.75 \times 2679.65$ kJ/kg

$= 23446.9$ kg/kg of fuel

(c) The mass of products formed per kg of fuel,

m_p = Mass of air + Mass of combustible matter in the fuel

$= m_{a_1} + m_C = 21.83 + 0.9$

$= 22.73$ kg/kg of fuel

(10% ash in fuel)

The moisture formed per kg of fuel,

$= 9\text{H} = 9 \times 0.048$

$= 0.432$ kg/kg of fuel

Fuel contains 0.018 kg moisture per kg of fuel, total mass of moisture

$m_v = 0.432 + 0.018 = 0.45$ kg/kg of fuel

Mass of dry flue gases

$m_g = m_p - m_v = 20.73 - 0.45$

$= 22.28$ kg/kg of fuel

Heat carried by dry flue gases,

$q_2 = m_g C_{pg}(T_g - T_a)$

$= 22.28 \times 1.02 \times (310 - 25)$

$= 6476.8$ kJ/kg of fuel

(d) *Heat carried by moisture per kg of fuel,*

Properties of steam at 0.08 bar (8 kPa)

$h_g = 2577$ kJ/kg

$T_{sat} = 41.54°C$

Heat carried by moisture

$q_3 = m_v[h_g + C_{ps}(T_g - T_{sat}) - C_{pw}T_a]$

$= 0.45 \times [2577 + 2.1 \times (310 - 41.54) - 4.2 \times 25]$

$= 1366.1$ kJ/kg

Heat lost by radiation and convection, etc.

$q_4 = q_{in} - q_1 - q_2 - q_3$

$= 35000 - 23446.9 - 6476.8 - 1366.1$

$= 3710.2$ kJ/kg of fuel

Heat Balance Sheet

Heat supplied/ kg of fuel	*%*	*Heat utilisation*	*kJ/kg of fuel*	*%*
$q_{in} = CV = 35000$ kJ/kg	100	(a) Heat used to generate steam	$q_1 = 23446.9$	67.00%
		(b) Heat carried away by dry flue gases	$q_2 = 6476.8$	18.50%
		(c) Heat carried away by steam in dry flue gases	$q_3 = 1366.1$	3.90%
		(d) Unaccounted heat losses	$q_6 = 3710.2$	10.60%
				100%

Example 19.26 *A boiler is fired with an oil having percentage analysis by mass: C = 84% and H_2 = 16%. The air initially at 15°C is first passed through the flue gas air preheater and then supplied to the boiler furnace at 260°C. The air supply is 25% in excess of theoretical air. Calculate the temperature of flue gas as it enters the heater, if the flue gas is leaving the heater at 137°C.*

Assume C_p for air and flue gas as 1.003 kJ/kg·K and for superheated steam as 2.1 kJ/kg·K

Solution

Given *Oil composition by mass*

C = 84% H_2 = 16%

$T_{a_1} = 15°C$ $T_{a_2} = 260°C$

$T_{g_2} = 137°C$ $m_{act} = 1.25\ m_{th}$

$C_{pg} = 1.003$ kJ/kg·K $C_{ps} = 2.1$ kJ/kg·K

To find Temperature of flue gas inlet to air preheater.

Analysis Theoretical air required for complete combustion

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$$m_{th} = \frac{100}{23}\left[\frac{8}{3}\text{C} + 8\text{H}\right]$$

$$= \frac{100}{23}\left[\frac{8}{3} \times 0.84 + 8 \times 0.16\right]$$

$$= 15.3 \text{ kg/kg of oil}$$

Actual air supplied

$$m_a = m_{th} + 0.25\, m_{th} = 1.25 \times 15.3$$

$$= 19.13 \text{ kg/kg of oil}$$

Mass of products formed

$$m_p = m_a + 1 = 19.13 + 1 = 20.13 \text{ kg/kg of oil}$$

Mass of vapour formed

$$m_v = 9\text{H} = 9 \times 0.16 = 1.4 \text{ kg/kg of oil}$$

Mass of dry flue gases

$$m_g = m_p - m_v = 20.13 - 1.44$$

$$= 18.69 \text{ kg/kg of oil}$$

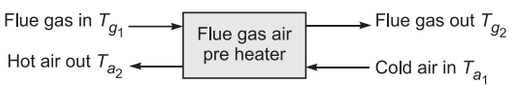

Fig. 19.9 *Schematic of air preheater*

Energy balance on air preheater

Heat gain by air

= Heat given by dry flue gases

+ heat given by water vapour

$$m_a C_{pa}(T_{a_2} - T_{a_1}) = m_g C_{pg}(T_{g_1} - T_{g_2}) + m_v C_{ps}(T_{g_1} - T_{g_2})$$

$$19.13 \times 1.003 \times (260 - 15) = 18.69 \times 1.003 \times (T_{g_1} - 137) + 1.44 \times 2.1 \times (T_{g_1} - 137)$$

or $21.77 \times (T_{g_1} - 137) = 4700.91$

Temperature of flue gas;

$T_{g_1} =$ **352.93°C**

Example 19.27 *During a trial on a boiler with economiser, the following results were obtained:*

	CO_2	O_2	N_2
% volumetric analysis of flue gas entering economiser	*8.3*	*11.4*	*80.3*
% volumetric analysis of flue gas leaving economiser	*7.9*	*11.5*	*80.6*

The fuel contains 80% carbon and it does not contain N_2. But the fuel contains 15% incombustible matter.

(a) Calculate the air leakage into the economiser per minute, if the fuel is used in the boiler is at 50 kg/min.

(b) Also, find the reduction in temperature of flue gas due to air leakage if the atmospheric air temperature is 15°C and flue-gas temperature is 350°C.

Assume C_p for air as 1 kJ/kg·K and flue gas as 1.1 kJ/kg·K.

Solution

Given

Fuel composition by mass: C = 80%

Incombustibles = 15%

$T_{a_1} = 15°\text{C}$ $T_{g_1} = 350°\text{C}$

$\dot{m}_f = 50$ kg/min

$C_{pa} = 1.0$ kJ/kg·K

$C_{pg} = 1.1$ kJ/kg·K

Volumetric composition of flue gas before and after economiser.

To find

(i) Mass of air leakage into the economiser per minute, and

(ii) Reduction in temperature of flue gas due to air leakage.

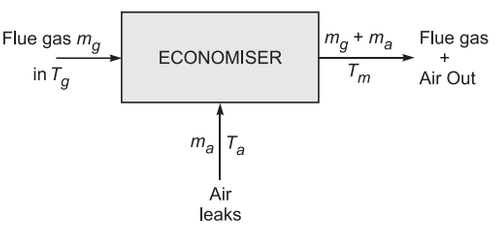

Fig. 19.10 *Energy and mass balance on economiser*

Analysis

(i) *Mass of air per kg of fuel entering economiser*

$$m_{a_1} = \frac{\text{N} \times \text{C}}{33 \times (\text{CO} + \text{CO}_2)} = \frac{80.3 \times 80}{33 \times (0 + 8.3)}$$

$$= 23.45 \text{ kg/kg of fuel}$$

Mass of air per kg of fuel leaving economiser;

$$m_{a_2} = \frac{\text{N} \times \text{C}}{33 \times (\text{CO} + \text{CO}_2)} = \frac{80.6 \times 80}{33 \times (0 + 7.93)}$$

$$= 24.73 \text{ kg/kg of fuel}$$

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Mass of air leakage into economiser;

$$= m_{a_2} - m_{a_1} = 24.73 - 23.45$$

$$= 1.28 \text{ kg/kg of fuel}$$

The air leaks into economiser per minute

$$= (50 \text{ kg fuel/min}) \times (1.28 \text{ kg of air/min})$$

$$= \mathbf{64 \text{ kg/min}}$$

(ii) *Reduction in temperature*

At enterance to the economiser, the mass of flue gas products formed per kg of fuel;

$$m_g = \text{Mass of combustible} + \text{Mass of air}$$

$$= (1 - 0.15) + 23.45$$

$$= 24.3 \text{ kg/kg of fuel}$$

Energy balance on economiser;

$$m_g C_{pg} T_g + m_a C_{pa} T_a = (m_a + m_a) C_{pg} T_m$$

where T_m is the temperature of mixture of exhaust gases leaving the economiser.

With numerical values;

$$24.3 \times 1.1 \times 350 + 1.28 \times 1 \times 15 = (24.3 + 1.28) \times 1.1 \times T_m$$

or $28.138\, T_m = 9355.5 + 19.2 = 9374.7$

or $T_m = 333.17°C$

Reduction in temperature of flue gas due to air leakage;

$$= T_g - T_m = 350 - 333.17 = \mathbf{16.83°C}$$

19.6 ENERGY-CONSERVATION OPPORTUNITIES

The various energy-saving opportunities in a boiler system can be related to combustion, heat transfer, avoidable losses, high auxiliary power consumption, water quality and blowdown. These opportunities are discussed below.

1. Stack (Chimney) Temperature

The flue gases temperature should be as low as possible. However, it should not be reduced to such value at which water vapour in the exhaust condenses on the chimney walls. The fuels containing significant sulphur at low temperature can lead to sulphur dew-point corrosion. Flue-gas temperatures above 200°C indicate potential for waste heat recovery.

2. Feed-Water Preheating using an Economiser

The flue gases leaving a boiler are at temperatures of 200 to 300°C. Thus, there is a potential to recover some of heat from these gases. The flue-gas exit temperature from a boiler is usually kept at 200°C, so that the sulphur oxides in the flue gas do not condense and cause corrosion in heat transfer surfaces. When a clean fuel such as natural gas, LPG or gas oil is used, the economy of heat recovery must be worked out, as the flue gas temperature may be well below 200°C. Thermal efficiency can be increased up to 5%.

3. Combustion Air Preheat

The preheating of combustion air also saves the fuel consumption. It improves thermal efficiency by 1%, if the combustion air temperature is raised by 20°C.

4. Incomplete Combustion

Incomplete combustion is result of a shortage of air or a surplus of fuel or poor distribution of fuel. It is from the colour or smoke, and must be corrected immediately.

In case of oil- and gas-fired systems, formation CO or smoke with normal or high excess air indicates burner system problems. A frequent cause of incomplete combustion is the poor mixing of fuel and air at the burner. With coal firing, unburned carbon can result into a big loss. Non-uniform fuel size could be one of the reasons for incomplete combustion.

5. Excess Air Control

Excess air is required to ensure complete combustion. The optimum excess air level for maximum boiler efficiency occurs when the sum of the losses due to incomplete combustion and loss due to heat in flue gases is minimum. This level varies with furnace design, type of burner, fuel and process variables. It can be determined by conducting tests with different air–fuel ratios.

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6. Radiation and Convection Heat Loss

The external surfaces of a boiler shell are always hotter than the surroundings. The surfaces thus lose heat to the surroundings depending on the surface area and the difference in temperature between the surface and the surroundings.

The heat loss from the boiler shell is normally a fixed energy loss, irrespective of the boiler output. With modern boiler designs, this may represent only 1.5% on the gross calorific value at full rating, but will increase to around 6%, if the boiler operates at only 25 per cent of capacity.

7. Automatic Blowdown Control

Uncontrolled continuous blowdown results into wastage of useful heat energy. Automatic blowdown controls can be installed that sense and respond to boiler water conductivity and pH value. A 10% blowdown in a 15 kg/cm^2 boiler results in 3% efficiency loss.

8. Reduction of Scaling and Soot Losses

In oil and coal-fired boilers, soot buildup on tubes acts as resistance to heat transfer. Any such deposits should be removed on a regular basis. Rise in stack temperatures indicates excessive soot build-up. Also, same result will occur due to scaling on the water side. High exit gas temperatures at normal excess air indicate poor heat transfer performance. This condition can result from a gradual build-up of gas-side or waterside deposits. Waterside deposits require a review of water-treatment procedures and tube cleaning to remove deposits. An estimated 1% efficiency loss occurs with every 22°C increase in stack temperature.

9. Variable Speed Control for Fans, Blowers and Pumps

Variable speed control is an important means of achieving energy savings. Generally, combustion air control is effected by throttling dampers fitted at forced and induced draft fans. Though dampers are simple means of control, they lack accuracy, giving poor control characteristics at the top and bottom of the operating range. In general, if the load characteristic of the boiler is variable, there is a possibility of replacing the dampers by a variable speed drive fan and blowers.

10. Effect of Boiler Loading on Efficiency

The maximum efficiency of the boiler does not occur at full load, but at about two-thirds of the full load. If the load on the boiler decreases further, efficiency also tends to decrease. At zero output, the efficiency of the boiler is zero, and any fuel fired is used only to supply the losses.

11. Proper Boiler Scheduling

Since the optimum efficiency of boilers occurs at 65–85% of full load. Therefore, a fewer number of boilers at higher loads, should be operated than to operate a large numbers at low loads.

12. Boiler Replacement

The potential savings from replacing a boiler depend on the expected increase in overall efficiency. A change in a boiler can be financially attractive, if the existing boiler is

(i) old and inefficient,
(ii) not capable of firing cheaper substitution fuel,
(iii) over or under-sized for present requirements,
(iv) not designed for ideal loading conditions.

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Summary

- The boiler draught may be defined as a small pressure difference which causes the flow of gases inside the boiler.

  The draught obtained by use of a chimney is called *natural* or *chimney draught*. But the static draught produced by the chimney is not sufficient to meet the requirement of the draught of a boiler. Most commonly, the artificial draught is used on the boilers. The waste heat carried by the flue gases can be better utilized in an economiser, air precheater, etc, with the help of artificial draught.
- An *artificial draught* produced by a fan or a blower is known as *mechanical draught* and that produced by a steam jet is known as a *steam jet draught*. In the induced draught system, the fan is placed near the base of the chimney. While in the forced draught, the fan or blower is located near or at the base of the boiler grate to force atmospheric air on to the furnace under pressure.
- A combination of induced and forced draught in a boiler is known as *balanced draught*.
- *Equivalent evaporation* is defined as the *amount of dry and saturated steam generated from feed water at 100°C at a normal atmospheric pressure*. It is expressed as

$$m_e = \frac{m_a (h - h_{f_1})}{h_{fg}@100°C} = \frac{m_a (h - h_{f_1})}{2257} \text{ (kg/kg of fuel)}$$

- The *boiler efficiency* is defined as the ratio of actual heat utilized in producing steam to amount of heat liberated by burning of fuel.

$$\eta_{boiler} = \frac{\text{Enthalpy rise of water in the boiler}}{\text{Heat librated by burning of fuel}}$$

Glossary

Boiler Draught A small pressure difference between air outside the boiler and gases within the furnace or chimney

Natural draught The draught obtained by use of a chimney

Artificial Draught The draught produced artificially by a fan or a blower

Steam jet draught Draught produced by steam jet

Evaporation rate Mass of steam generated per hour

Equivalent evaporation Amount of steam generated from and at 100°C.

Factor of evaporation Ratio of heat received by 1 kg of feed water for evaporation under actual working condition to that received by 1 kg of water evaporated from and at 100°C

Boiler Efficiency Fraction of heat utilized in producing steam of heat librated by burning of fuel

Economiser efficiency Fraction of heat absorbed by the feed water in the economiser of heat available of flue gases

$$\textbf{\textit{Boiler Power:}} = \frac{\text{Equivalent evaporation from and at 100°C per hour}}{21.296}$$

Review Questions

1. What is boiler draught and how it is produced by a chimney?
2. Compare natural and artificial draughts.
3. Explain a steam jet draught with its features.
4. Compare force and induced draught.
5. Derive an expression for maximum discharge through a chimney.
6. State briefly the difference between systems of producing draught in a boiler.
7. Derive an expression for draught produced in

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terms of height of chimney, ambient and flue gas temperatures. State clearly the assumptions made.

8. Establish a condition for maximum discharge of flue gases through a chimney of given height.
9. Define equivalent evaporation, factor of evaporation, boiler efficiency.
10. State the advantages of artifical draught over natural draught.
11. Define evaporative capacity and equivalent evaporation of the boiler.
12. What is the factor of evaporation?
13. Define boiler efficiency.
14. Establish the energy balance in a boiler. How can its performance be improved?

Problems

1. A chimney of 30-m height is discharging hot gases at 320°C, when outside temperature is 30°C. The air–fuel ratio is 20. Determine the

(a) draught produced in mm of water column,

(b) temperature of gases for maximum discharge in a given time and what would be the draught produced corresponding?

[(a) 16.2 mm (b) 362.3°C]

2. How much air is used per kg of coal in a boiler having a chimney of 30-m height to create a draught of 20 mm of water, when temperature of flue gases in the chimney is 350°C and temperature of the boiler house is 35°C? Does the chimney satisfy the condition of maximum discharge? *[18.3 kg, No]*

3. A 30-m high chimney is discharging flue gases at 561 K, when the ambient temperature is 294 K. The quantity of air supplied is 18 kg of air per kg of fuel burnt. Determine

(a) draught in mm of water column produced by chimney.

(b) equivalent draught in metre of hot gas column.

(c) velocity of flue gases in chimney, if 50% of draught is lost in friction at the grate and passages.

(d) draught produced in mm of water and temperature of flue gases under the condition of maximum discharge.

[(a) 16.1 mm, (b) 24.3 m, (c) 15.45 m/s, (d) 18 mm and 620K]

4. Find the minimum temperature of flue gases required to produce a draught of 15 mm of water by a chimney of 30-m height, when air–fuel ratio used in the combustion is 20. The atmospheric air temperature is 27°C. *[274.75°C]*

5. Find the draught produced in mm of water by a chimney of 40-m height. The mass of the flue gases is 20 kg/kg of fuel burnt in the combustion chamber. The temperature of flue gases and ambient are 270°C and 23°C, respectively.

Assuming diameter of the chimney is 150 cm and 30% of the theoretical draught is lost due to friction, find the mass of flue gases discharged through the chimney per minute.

[20.33 mm, and 1464.65 kg/min]

6. A boiler plant generates one tonne of steam per hour at a pressure of 12 bar and 50°C of superheat, the boiler details are:

Feed water temperature 50°C,

Grate area 0.6 m^2,

Heating surface = 12 m^2.

Determine actual and equivalent evaporation (a) per hour, (b) per m^2 of grate area per hour, and (c) per m^2 of heating surfcace area per hour.

7. A boiler generates steam at a rate of 40 tonnes/h consuming coal at 4000 kg/h of calorific value 35,000 kJ/kg. The steam pressure is 15 bar and dry saturated. Calculate (a) factor of evaporation, (b) equivalent evaporation from and at 100°C, and (c) efficiency of the boiler.

[(a) 1.237, (b) 12.37 kg/kg of fuel, (c) 79.77%]

8. A boiler generates steam at 18 bar and 325°C when feed water is supplied at 41.45°C. The thermal efficiency of the boiler is 80%. It uses

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furnace oil of CV 45500 kJ/kg. The steam is supplied to a turbine producing 500 kW power, having a specific steam consumption of 10 kg/kWh. Calculate the furnace-oil consumption in kg/h and equivalent evaporation.

[395.71 kg/h; 6382.55 kg/h]

9. A boiler generates 500 kg/h of steam at 16 bar and 300°C from feed water at 30°C. Coal used is 60 kg/h of CV 30,000 k/kg. Find

(a) equivalent of evaporation

(b) boiler efficiency

[(a) 10.75 kg/kg of coal, (b) 80.9%]

10. The equivalent of evaporation of a boiler from and at 100°C is 10.4 kg/kg of fuel. The CV of fuel is 29800 kJ/kg.

Determine efficiency of boiler. If the boiler produces 15000 kg of steam per hour at 20 bar from a feed water at 40°C and the fuel used is 1650 kg/h, determine the condition of steam produced.

[(a) 78.76% (b) x = 0.9734]

11. Calculate the thermal efficiency and equivalent evaporation from and at 100°C of a boiler from which the following data were obtained during a trial: steam pressure = 10.6 bar, steam temperature = 260°C, feed-water temperature = 38°C, water evaporated 10 kg per kg of coal of calorific value = 33450 kJ/kg.

[62.48%, 9.26 kg/kg of coal]

12. The following observations were taken during a test on a steam boiler:

Pressure of steam = 9.8 bar

Temp. of steam leaving the superheater = 250°C

Feed-water temp. entering the economiser = 25°C

Feed-water temp. leaving the economiser = 80°C

Dryness fraction of steam entering the superheater = 0.95

Calorific value of coal = 30,000 kJ/kg

Quantity of coal burned per hour = 750 kg

Quantity of steam generated per hour = 7000 kg

Determine thermal efficiency of the plant and percentage heat utilised in the economiser, boiler and superheater.

[92.84%, 7.16%, 72.85%, 12.84%]

13. The data were collected during a boiler trial for a duration of one hour:

Pressure of steam = 9.8 bar

Temp. of steam leaving the superheater = 250°C

Feed-water temperature = 25°C

Calorific value of coal = 30,000 kJ/kg

Quantity of coal burned per hour = 95 kg

Quantity of steam generated per hour = 650 kg

Ash collected = 9 kg

CV of ash = 2500 kJ/kg

Quantity of air used per kg of fuel = 19 kg

Flue-gas temperature = 350°C

Boiler-room temperature = 30°C

Mean sp. heat of flue gases = 1.005 kJ/kg·K

Determine equivalent evaporation, boiler efficiency, % heat losses in flue gases, ash and radiation.

[8.55 kg/kg of coal, 64.35%, 21.44%, 0.79%, 13.42%]

14. The observation recorded during a boiler trial are as given below:

Duration of trial = 60 min

CV of coal burnt = 30, 200 kJ/kg

Steam generated = 5250 kg

Coal burnt = 695 kg

Boiler pressure = 12 bar

Dryness fraction of steam = 0.94

Temp of steam leaving superheater = 250°C

Temperature of hot well = 45°C

Calculate with and without superheater:

(i) Equivalent evaporation from and at 100°C

(ii) Boiler efficiency

(iii) Heat supplied by superheater

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Objective Questions

1. The draught in a boiler is provided to
   (a) force the air on the furnace
   (b) force the hot gases on superheater
   (c) discharge the flue gases through chimney
   (d) all of the above
2. The boiler draught is defined as
   (a) pressure difference between furnace bed and chimney base
   (b) pressure difference between feed pump and injector
   (c) temperature difference between economiser and air preheater
   (d) all of the above
3. The natural draught is produced by
   (a) fan before the furnace
   (b) fan after the fan
   (c) chimney height
   (d) none of the above
4. The actual velocity of flue gases through the chimney is given by
   (a) ρgh
   (b) $\sqrt{\rho gh}$
   (c) $\sqrt{2gH'}$
   (d) $\sqrt{2g(H'-h_f)}$
5. The draught produced in terms of hot flue-gas column is given by
   (a) $H\left[\frac{m_a}{m_a+1}\times\frac{T_g}{T_a}-1\right]$
   (b) $353H\left[\frac{1}{T_a}-\frac{m_a+1}{m_a}\times\frac{1}{T_g}\right]$
   (c) $\sqrt{2gH'}$
   (d) $\sqrt{2g(H'-h_f)}$
6. The condition of maximum discharge through a chimney is given by
   (a) $\frac{T_g}{T_a}=2\left(\frac{H}{H+1}\right)$
   (b) $\frac{T_g}{T_a}=2\left(\frac{m_a+1}{m_a}\right)$
   (c) $\frac{T_g}{T_a}=2\left(\frac{m_a}{m_a+1}\right)$
   (d) none of the above
7. The chimney efficiency is given by
   (a) $H\left[\frac{m_a}{m_a+1}\times\frac{T_g}{T_a}-1\right]$
   (b) $gH\left[\frac{1}{T_a}-\frac{m_a+1}{m_a}\times\frac{1}{T_g}\right]$
   (c) $gH\left[\frac{m_a}{m_a+1}\times\frac{T_g}{T_a}-1\right]$
   (d) none of the above
8. The balanced draught is produced by
   (a) chimney draught + induced fan draught
   (b) forced fan draught and induced fan draught
   (c) forced fan draught + chimney draught
   (d) none of the above
9. The steam jet draught is used in
   (a) Cochran boiler
   (b) Lancashire boiler
   (c) Locomotive boiler
   (d) none of the above
10. The evaporation rate of a boiler is defined as
    (a) $\frac{\text{Mass of steam generated}}{\text{Time period in hours}}$
    (b) $\frac{\text{Mass of steam generated per hour}}{\text{Heating area of grate in m}^2}$
    (c) $\frac{\text{Mass of steam generated per hour}}{\text{Mass of fuel burned per hour}}$
    (d) all of the above

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11. The equivalent evaporation is defined as
    (a) steam generated at 100°C
    (b) dry and saturated steam generated at 100°C from feed water at 100°C
    (c) steam generated at 1 bar and at 100°C
    (d) none of the above
12. Which one of following is standard condition for equivalent evaporation?
    (a) Pressure 1 bar and generation at 100°C
    (b) From feed water at 100°C and 1 bar
    (c) From and at 100°C and 1 bar
    (d) None of the above
13. The enthalpy of evaporation at 100°C
    (a) 2527 kJ/kg
    (b) 2257 kJ/kg
    (c) 2276 kJ/kg
    (d) 2557 kJ/kg
14. The boiler efficiency is defined as
    (a) $\frac{\text{Heat utilised in steam generated/h}}{\text{Heat liberated by fuel burning}}$
    (b) $\frac{\text{Heat utilised in steam generated}}{\text{Heat liberated by fuel burning/h}}$
    (c) $\frac{\text{Heat utilised in steam generated/h}}{\text{Heat liberated by fuel burning/h}}$
    (d) none of above.

Answers

1. (d) 2. (a) 3. (c) 4. (d) 5. (a) 6. (b) 7. (d) 8. (b)
9. (c) 10. (d) 11. (b) 12. (c) 13. (b) 14. (c)

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20

Steam Engines

Introduction

The steam engine is a slow speed reciprocating machine, in which the energy of steam is converted into mechanical work. The steam is used as a working fluid. Steam is prepared outside the engine in the boiler, and thus steam engine is also called an *external combustion engine*.

Steam engines have played a very important role in India, being a major factor in the Industrial revolution and Indian Railways. Nowadays, steam engines are replaced by internal combustion engines, steam and gas turbines. In some plants, the steam engines are used to operate air compressors, pumps, and other auxiliaries. The exhaust steam from these engines is used for space heating and industrial process work.

Figure 20.1 shows the sectional view of steam-engine.

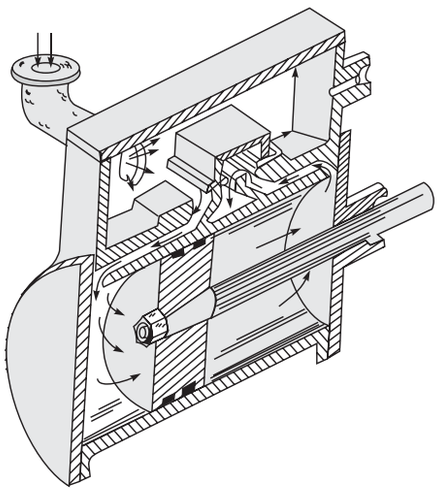

Fig. 20.1 *Sectional view of steam engine*

20.1 CLASSIFICATION OF STEAM ENGINES

Steam engines can be classified in the following manner:

1. According to position of cylinder axis
- (a) Vertical engine
- (b) Horizontal engine, and
- (c) Inclined steam engine

2. According to action of operation
- (a) Single acting engine
- (b) Double acting engine

3. According to expansion stages
- (a) Single cylinder engine
- (b) Compound steam engine

4. According to speed of operation
- (a) Low speed engine (less than 100 rpm)
- (b) Medium speed engine (100 to 250 rpm)
- (c) High speed engine (more than 250 rpm)

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5. According to method of steam exhausting

(a) *Condensing engine* Steam after expansion in the cylinder exhausted to condenser.

(b) *Non-condensing engine* Steam after expansion in the cylinder is exhausted into atmosphere.

20.2 CONSTRUCTION OF STEAM ENGINE

The constructional features of a simple steam engine as shown in Fig. 20.2 are given below.

1. Frame It is made of cast iron, and it supports the moving parts of the engine. It rests on the foundation.

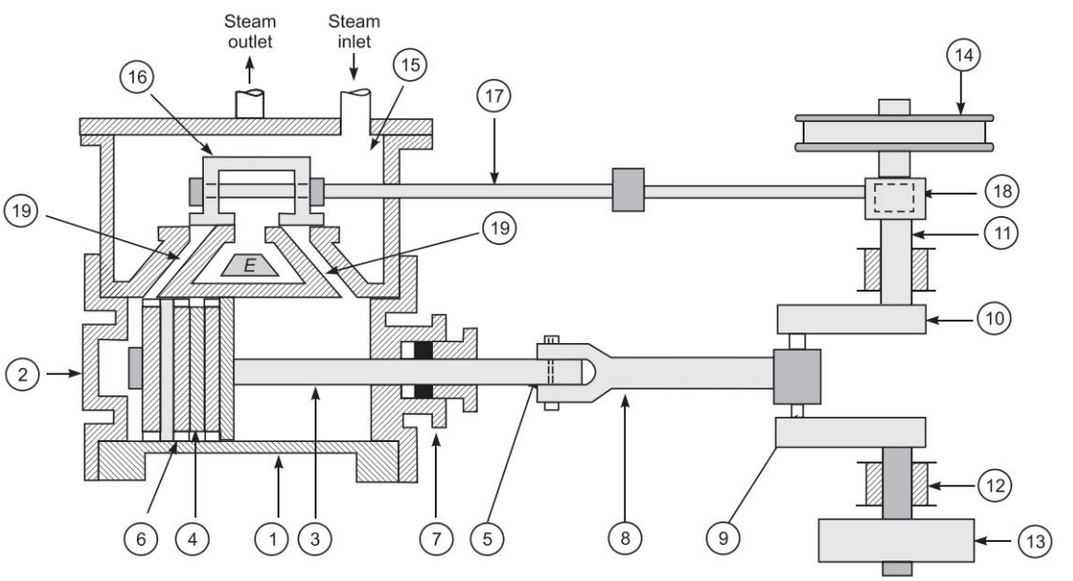
2. Cylinder It is the heart of the engine. It is attached with a steam chest. It is made of cast iron and it forms the chamber in which the piston reciprocates. One end of the cylinder is closed by a cylinder cover, known as the cover, end, while the other end is known as the crank end.

3. Piston It is a cylindrical part and is made of cast iron. It is fitted with piston rings to provide sealing between the cylinder and piston body. The piston reciprocates in the cylinder.

4. Piston Rod It is the link which connects the piston and cross head. The piston rod reciprocates inside the cylinder through the stuffing box and gland.

5. Cross-head and guide The cross-head provides bearings to a pin, which is joined to a connecting rod while it connects the piston rod rigidly. It reciprocates on parallel guides and provides linear motion to the piston rod.

6. Connecting Rod The connecting rod is made of carbon or alloy steel. It is a component which takes up the force of the piston through the piston rod and cross-head and transmits it to the crank. It converts reciprocating motion of the piston rod into rotary motion. The small end of the connecting rod is joined to the cross-head by pin (gudgeon pin) and the larger end is connected to crank by crank pin.



1. Cylinder 2. Cylinder cover 3. Piston rod 4. Piston 5. Cross-head 6. Piston ring 7. Stuffing box and gland 8. Connecting rod 9. Crank pin 10. Crank 11. Crank shaft 12. Main bearings 13. Fly wheel 14. Pulley 15. Steam chest 16. D-slide valve 17. D-slide valve rod 18. Eccentric 19. Steam ports E. Exhaust pipe

Fig. 20.2 *Simple steam engine*

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7. Crank shaft It is made of forged steel, it is supported on the main bearing and is provided with a crank pin supported with webs. It supports the fly wheel at one end and the output pulley on the other end. The rotary motion of the crank shaft is mechanical energy.

8. Eccentric It is mounted on the crankshaft and it connects the *D*-slide valve through the eccentric rod. The eccentric converts the rotary motion of the crank into reciprocating motion of the *D*-slide valve.

9. D-slide valve It is located in the steam chest. It controls the steam entry into the cylinder and its exhaust. The cross-section of the *D*-slide valve is like the letter D, and hence it is called *D-slide valve*.

10. Fly wheel It is mounted on the end of the crankshaft. It is made of cast iron. It stores energy in the form of inertia, when energy is in excess and it gives energy back, when it is in deficit. In other words, it minimises the speed fluctuations on the engine.

20.3 WORKING OF A DOUBLE-ACTING STEAM ENGINE

Steam engines are generally of double-acting type. Double acting means that both back and front faces of the piston are arranged as working faces.

The *D*-slide valve controls the steam entry into the cylinder on one end and its exhaust from the other end simultaneously.

Figure 20.3 illustrates one position of the *D*-slide valve which opens the steam port *A* for steam entry into the cylinder on the cover end, and at the same time it also opens the steam port *B* for escape of used steam from the cylinder to the exhaust pipe *E*.

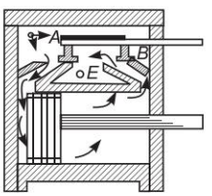

Fig. 20.3 *Steam port A opens for steam entry and steam port B for exhaust*

Figure 20.4 shows another position of the *D*-slide valve which covers both the steam ports for steam entry. It is the steam cut-off. The steam in the cylinder can expand and do work until the piston reaches the end of the stroke.

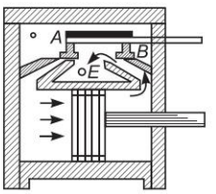

Fig. 20.4 *Port A closes for steam entry and port B opens for steam exhaust*

Figure 20.5 shows the position of the piston and *D*-slide valve at the completion of the stroke. The steam enters the port *B* while the used steam escapes from the port *A*.

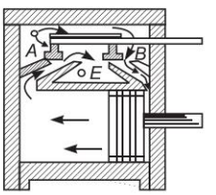

Fig. 20.5 *Port B opens for steam entry and port A opens for steam exhaust*

20.4 HYPOTHETICAL INDICATOR DIAGRAM OF STEAM ENGINE

The steam engine works on the modified Rankine cycle discussed in Chapter 12; Section 12.12. A hypothetical indicated diagram is drawn with the following assumptions:

1. Instantaneous opening and closing of ports,
2. Steam enters the engine at boiler pressure,
3. Steam leaves the engine at condenser pressure,
4. No throttling effect due to any restriction in the port,

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5. No compression of steam,
6. No heat transfer losses, and
7. Piston moves in the cylinder frictionlessly.

20.4.1 Hypothetical Indicator Diagram without Clearance

The hypothetical indicator diagram for a steam engine without clearance volume is shown in Fig. 20.6. The sequence of operations are as follows:

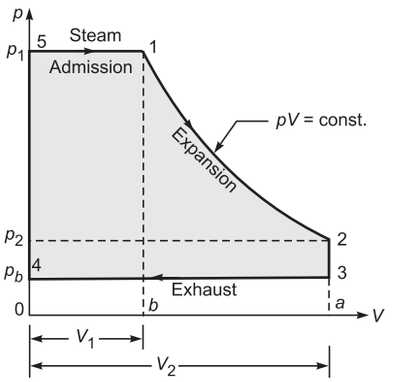
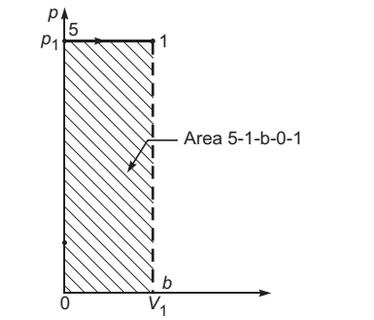


Fig. 20.6 *Hypothetical diagram of a steam engine without clearance volume*

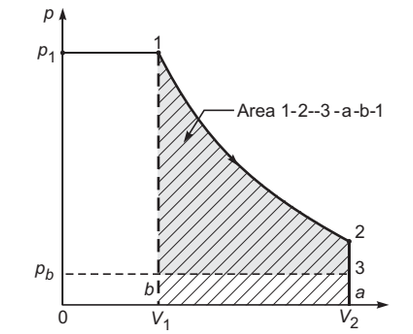
Steam Admission The steam at boiler pressure p_1 enters the cylinder at the state 5 and steam supply is cut off at the state 1 and it is known as the *cut-off point*. The area 5–1–b–0–5 represents the work done during steam entry into the cylinder as shown in Fig. 20.7(a).

Expansion The steam expands in the cylinder from the state 1 to the end of stroke of the piston at the state 2. The expansion is hyperbolic. Area under the curve 1–2 represents the work done during hyperbolic expansion as shown in Fig. 20.7(b).

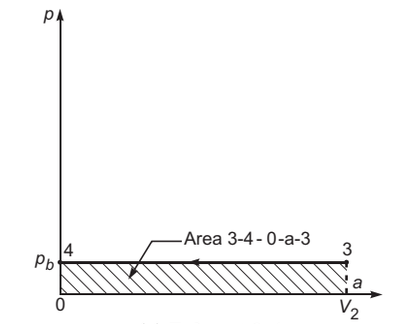
Steam Release The exhaust port opens at the point 2, and steam is released and pressure drops suddenly from p_2 to the condenser pressure p_b (called *back pressure*). The point 2 is known as *the point of steam release*. The line 2–3 represents the *steam release process*.



(a) Steam admission



(b) Expansion 1-2 and steam release 2-3



(c) Exhaust 3-4

Fig. 20.7 *The processes of steam engine on a p–V diagram*

Exhaust The line 3–4 represents the exhaust of steam into the condenser. The state 4 represents the piston position at the top dead centre (without clearance). The fresh steam enters the cylinder at the state 5 for the next cycle. The area 3–4–0–a–3 of Fig. 20.7(c) represents the work required for steam exhaust from the cylinder.

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Work Done Work done by expansion of steam per cycle in the steam engine;

W = Area of indicator diagram
= Area 5–1–b–0–5 + Area 1–2–3–a–b–1
– Area 3–4–0–a–3

$$= p_1V_1 + \int pdV - p_bV_2$$

or $$W = p_1V_1 + p_1V_1 \ln\left(\frac{V_2}{V_1}\right) - p_bV_2$$

or $$W = p_1V_1(1 + \ln r_e) - p_bV_2 \quad ...(20.1)$$

where $r_e = \dfrac{V_2}{V_1}$ known as *expansion ratio*.

Mean effective pressure of the cycle can be obtained as

$$p_m = \frac{\text{Work done}}{\text{Swept volume}}$$

$$= \frac{p_1V_1(1+\ln r_e) - p_bV_2}{V_2}$$

$$= p_1\frac{V_1}{V_2}\left[1+\ln r_e\right] - p_b$$

or $$p_m = p_1\rho\,[1 + \ln (r_e)] - p_b \quad ...(20.2)$$

where $$\rho = \frac{1}{r_e} = \frac{V_1}{V_2}, \text{ cut-off ratio.}$$

20.4.2 Hypothetical Indicator Diagram with Clearance Volume

Figure 20.8. shows a hypothetical indicator diagram for a steam engine cylinder with clearance. The

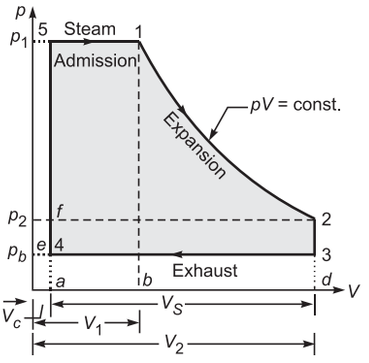

Fig. 20.8 *Hypothetical Indicator diagram with clearance and compression*

operation of all the processes are exactly same as that of the indicator diagram shown in Fig. 20.6, except the effect of the clearance volume, V_c.

The steam enters the cylinder at the point 4, the steam pressure in the cylinder suddenly rises to the boiler pressure p_1. This process is represented by the line 4–5.

Work done During the Cycle

W = Area 5–1–2–3–4–5 of indicator diagram
= Area a–5–1–b + Area b–1–2–3–d
– Area 3–4–a–d

$$= p_1(V_1 - V_c) + \int pdV - p_b(V_2 - V_c)$$

$$= p_1V_1 - p_1V_c + p_1V_1 \ln\left(\frac{V_2}{V_1}\right) - p_bV_s$$

$$= p_1V_1\,[1 + \ln (r_e)] - p_1V_c - p_bV_s \quad ...(20.3)$$

where $r_e = \dfrac{V_2}{V_1}$, expansion ratio.

Mean effective pressure, p_m

$$p_m = \frac{\text{Work done}}{\text{Stroke volume}} = \frac{W}{V_s}$$

$$p_m = \frac{p_1V_1}{V_s}\left[1 + \ln (r_e)\right] - \frac{p_1V_c}{V_s} - p_b\frac{V_s}{V_s} \quad ...(20.4)$$

Let $$c = \frac{V_c}{V_s}, \text{ clearance ratio.}$$

$$\rho = \frac{V_1 - V_c}{V_s}, \text{ cut-off ratio.}$$

$\therefore$ $$V_1 = \rho V_s + V_c$$

or $$\frac{V_1}{V_s} = \rho + c$$

Using, we get

$$p_m = p_1(\rho + c)\,[1 + \ln (r_e)] - p_1c - p_b \quad ...(20.5)$$

If neglecting the effect of clearance then

$$V_c = 0 \quad \text{and} \quad c = 0,$$

Then $$W = p_1V_1\,[1 + \ln (r_e)] - p_bV_2 \quad (\because\ V_2 = V_s)$$

and $$p_m = p_1\rho[1 + \ln (r_e)] - p_b$$

Thus we obtain the same relations given by Eqs. (20.2) and (20.3) of work done and mean effective pressure, respectively for indicator diagram without clearance.

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20.4.3 Hypothetical Indicator Diagram with Clearance and Compression

Figure 20.9 shows a hypothetical indicator diagram for a steam engine with compression of steam in clearance volume.

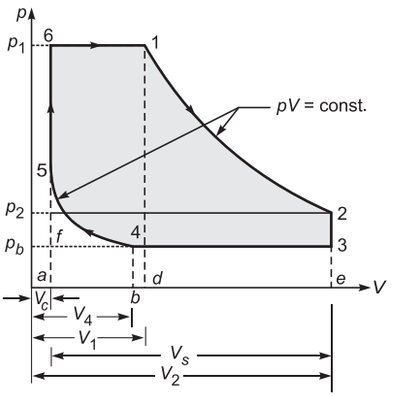

Fig. 20.9 *Hypothetical indicator diagram with clearance and compression*

Process 6–1 is steam admission, Process 1–2 hyperbolic expansion, Process 2–3 is steam release as considered in previous indicator diagrams.

Process 3–4 Exhaust of steam into condenser, the exhaust terminates at the state 4.

Process 4–5 Compression of remaining steam in the cylinder.

Process 5–6 Fresh steam enters from boiler at the state 5 for the next cycle. Its pressure rise suddenly to boiler pressure p_1.

Work Done

W = Area of indicator diagram 6–1–2–3–4–5–6.

= Area a–6–1–d + Area d–1–2–3–e – Area b–4–3–e– Area a–f–5–4–b

$$= p_1(V_1 - V_c) + p_1 V_1 \ln\left(\frac{V_2}{V_1}\right) - p_b(V_2 - V_4) - p_b V_4 \ln\left(\frac{V_4}{V_c}\right) \quad ...(20.6)$$

$$W = p_1 V_1 + p_1 V_1 \ln\left(\frac{V_2}{V_1}\right) - p_1 V_c - p_b(V_2 - V_4) - p_b V_4 \ln\left(\frac{V_4}{V_c}\right)$$

$$= p_1 V_1\left[1 + \ln\left(\frac{V_2}{V_1}\right)\right] - p_1 V_c - p_b(V_s + V_c - V_4) - p_b V_4 \ln\left(\frac{V_4}{V_c}\right) \quad ...(20.7)$$

Let $r_e = \dfrac{V_2}{V_1}$, expansion ratio

$\rho = \dfrac{V_1 - V_c}{V_s}$, cut-off ratio

$c = \dfrac{V_c}{V_s}$, clearance volume

$\alpha = \dfrac{V_4 - V_c}{V_s}$, fraction of stroke volume completed at the start of compression,

Using, we get

$V_1 = \rho V_s + V_c$ and $V_4 = \alpha V_s + V_c$

Therefore, work done,

$$W = p_1 V_1[1 + \ln(r_e)] - p_1 V_c - p_b[V_s + V_c - \alpha V_s - V_c] - p_b(\alpha V_s + V_c)\ln\left[\frac{\alpha V_s + V_c}{V_c}\right]$$

$$= p_1 V_1[1 + \ln(r_e)] - p_1 V_c - p_b(1 - \alpha)V_s - p_b(\alpha V_s + V_c)\ln\left(\frac{\alpha + c}{c}\right) \quad ...(20.8)$$

The mean effective pressure

$$p_m = \frac{\text{Work done}}{\text{Swept volume}} = \frac{W}{V_s}$$

$$= \frac{p_1 V_1}{V_s}[1 + \ln(r_e)] - \frac{p_1 V_c}{V_s} - p_b\frac{V_s}{V_s}(1 - \alpha) - p_b\left(\frac{\alpha V_s}{V_s} + \frac{V_c}{V_s}\right)\ln\left[\frac{\alpha + c}{c}\right]$$

$$= p_1(\rho + c)[1 + \ln(r_e)] - p_1 c - p_b(1 - \alpha) - p_b(\alpha + c)\ln\left(\frac{\alpha + c}{c}\right) \quad ...(20.9)$$

Case (*a*) If neglecting the effect of compression, i.e., $\alpha = 0$, we get

$W = p_1 V_1[1 + \ln(r_e)] - p_1 V_c - p_b V_s$

and $p_m = p_1(\rho + c)[1 + \ln(r_e)] - p_1 c - p_b$

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These are the relations of work done and mean effective pressure for the hypothetical indicator diagram Fig. 20.8, without compression.

Case (*b*) If neglecting the effect of compression and clearance, i.e., $\alpha = 0$ and $c = 0$, we get

$$W = p_1 V_1 [1 + \ln (r_e)] - p_b V_2 \quad [\because V_s = V_2]$$

and $$p_m = p_1 \rho [1 + \ln (r_e)] - p_b$$

These are relations for hypothetical diagram, Fig. 20.6 without clearance and compression.

20.5 ACTUAL INDICATOR DIAGRAM

Figures 20.6, 20.8 and 20.9 are hypothetical indicator diagrams for a steam engine. They are theoretical representations of the steam engine cycle where all practical losses are assumed negligible. However, an indicator diagram taken with the help of an instrument indicator as shown in Fig. 20.10, is called *actual indicator diagram.*

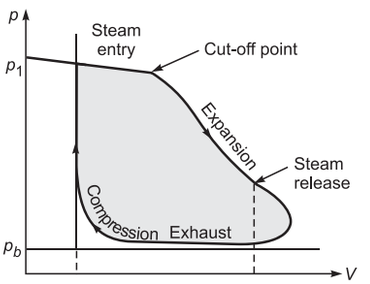

Fig. 20.10 *Actual indicator diagram*

20.5.1 Actual Indicator Diagram vs Hypothetical Indicator Diagram

The close examination of the two types of diagrams reveals the following facts:

1. The steam pressure drops considerably between the boiler and cylinder. This pressure drop is due to condensation of some quantity of steam into the pipeline and due to wire drawing (throttling) effect at the inlet port.
2. There is a gradual pressure drop during the steam supply into the cylinder up to the cut-off point. It is due to condensation of some steam in the cylinder due to heat transfer.
3. The inlet port does not close instantaneously, it closes gradually. Thus, the indicator diagram is rounded off at the cut-off point.
4. The expansion of steam in the cylinder is not true hyperbolic. It is due to continuous heat exchange between the steam and cylinder. During steam entry and early part of expansion, the steam is hotter and heat is transferred to cylinder walls, causing steam to condense. In the later part of the stroke, the steam is colder than cylinder walls, and thus heat is transferred to steam, causing reheating of steam.
5. The steam release occurs before the end of expansion stroke, and the exhaust port opens gradually, and thus the curve is rounded off at the toe of the diagram.
6. The exhaust pressure is slightly higher than the back pressure since the steam is being forced out of the cylinder.
7. Some steam remains in the cylinder, and is compressed before the fresh steam enters the cylinder.

20.5.2 Measure to Improve Workdone

The work lost per cycle can be reduced by

1. Rapid opening and closing of valves by using Corliss valves or drop valves
2. Steam jacketing, which makes the process nearly hyperbolic and minimises condensation
3. Reducing back pressure by liberal exhaust port opening.

20.5.3 Diagram Factor

The *diagram factor* is defined as the ratio of the area of actual indicator diagram to the area of the hypothetical indicator diagram. It is denoted by *K*.

$$K = \frac{\text{Area of the actual indicator diagram}}{\text{Area of hypothetical indicator diagram}} \quad ...(20.10)$$

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$$\text{or} \quad K = \frac{\text{Mean height of the actual indicator diagram}}{\text{Mean height of hypothetical indicator diagram}} \quad ...(20.11)$$

$$\text{or} \quad K = \frac{\text{actual mean effective pressure}}{\text{Hypothetical mean effective pressure}} \quad ...(20.12)$$

Therefore,

Actual work done/cycle

$= K \times$ hypothetical work done

and Actual MEP ($p_{m,act}$)

$= K \times$ hypothetical mean effective pressure

20.6 CYLINDER CONDENSATION

Steam engines are generally double acting. Steam from the boiler enters on both sides of the cylinder. Further, at the end of expansion, the steam temperature reduces due to decrease in steam pressure and the cylinder becomes cool. When the fresh steam enters the cylinder and it comes in contact of the colder surface of the cylinder, some steam condenses into the cylinder without doing any work. Some steam also condenses during expansion. This is called *cylinder condensation.* Due to condensation of steam in the early part of the working stroke, there is increase in steam consumption rate as high as 20%. The cylinder condensation may be reduced by

(i) Using superheated steam,
(ii) Steam jacketing of the cylinder, which keeps the cylinder surface at high temperature,
(iii) By compressing some portion of the exhausted steam,
(iv) By compounding of the steam engine, and
(v) By operating the engine at higher speed.

20.7 MASS OF STEAM IN CYLINDER

The total mass of steam in the cylinder is the sum of the mass of fresh steam admitted per stroke and the mass of steam left behind in the clearance volume from the previous cycle.

m_s = Mass of cushion steam + Mass of steam admitted per stroke

= Mass of cushion steam + $\frac{\text{Mass of steam used per hour}}{\text{No. of cycles per hour}}$ ...(20.13)

The mass of steam supplied per stroke is called *cylinder feed* and the steam left behind in the clearance space is known as *cushion steam* and it acts as a buffer for slowing down the piston at the end of the stroke. The mass of cushion steam can be obtained with the help of calibrated indicator diagram as shown in Fig. 20.11.

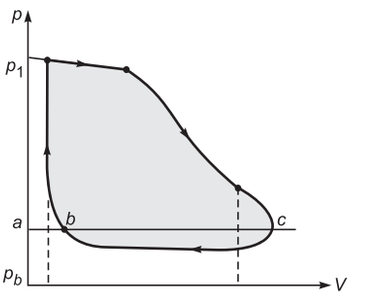

Fig. 20.11 *Calibrated indicator diagram*

On the compression curve, the steam may be assumed just dry and saturated, because, the compression tends to dry the steam. By choosing a point *b* on the compression curve, the volume and pressure are noted. From the steam table, the specific volume of dry steam at this pressure is used for calculating the mass of cushion steam.

Mass of cushion steam

$$= \frac{\text{Volume on the compression curve}}{\text{Sp. volume corresponding topre ssure}} \quad ...(20.14)$$

20.8 MISSING QUANTITY

As discussed in the previous section, some quantity of steam condenses inside the cylinder during the early part of the admission without doing any work. This quantity of condensed steam is considered as *missing quantity*.

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The magnitude of missing quantity can be obtained with the help of the indicator diagram of the steam engine shown in Fig. 20.12. An expansion curve is drawn assuming the steam to be dry and saturated at every point throughout the expansion stroke. It gives the theoretical volume occupied by steam at any point. It follows the saturation curve.

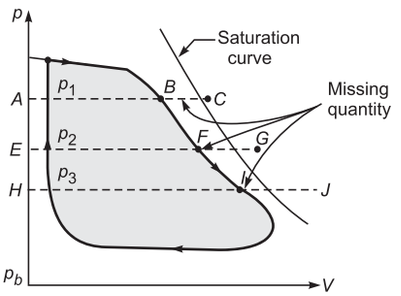

Fig. 20.12 *Saturation curve and missing quantity*

The horizontal difference between the actual expansion curve and saturation curve is the loss of volume due to wetness of steam and is considered as a missing quantity.

The line AC represents the volume of steam in the cylinder, if steam was dry and saturated at pressure p_1 and the line AB represents the actual volume of expanding steam in the cylinder at the same pressure p_1. Therefore, the missing quantity (volume of steam) at the point B is

$$BC = AC - AB$$
$$= v_g(1 - x) \qquad ...(20.15)$$

Sometimes, the missing quantity is also expressed as the difference between the actual mass of steam in the cylinder and indicated mass of dry and saturated steam.

$$\text{Missing mass of steam} = m_s(1 - x) \qquad ...(20.16)$$

The missing quantity is mainly due to condensation of steam but a small amount is due to leakage past the piston. Towards the end of the expansion stroke, the reheating of steam takes place, thus some missing quantity is recovered.

20.9 STEAM CONSUMPTION

The steam consumption of a steam engine may be defined as the amount of steam in kg consumed by the engine per hour. The steam consumption of an engine can be obtained from the theoretical indicator diagram as

Mass of steam admitted per cycle

m_s = Volume of steam swept up to cut-off × density of steam

$$= \frac{\pi}{4} d^2 L \times \rho \times \frac{1}{v} = (\rho V_s)\frac{1}{v} \text{ (kg)} \quad ...(20.17)$$

where

d = bore of cylinder, m

L = stroke length of piston, m

ρ = cut-off ratio,

v = specific volume of steam at admission pressure, m^3/kg

$= x v_g$ for wet steam

$= v_g$ for dry and saturated steam

$= v_g \frac{T_{sup}}{T_{sat}}$ for superheated steam

For a single-acting steam engine which makes N rotations per minute (rpm),

$$\dot{m}_s = \frac{\pi}{4} d^2 L \times \frac{1}{v} \times 60\,N \text{ (kg/h)} \qquad ...(20.18)$$

Equation (20.18) represents the steam consumption on one side of the piston. For a single-acting steam engine, it is the total steam consumption per hour. Generally, the steam engines are double acting. For double-acting steam engines, the total steam consumption is twice the above amount, provided

1. the cut-off ratio on both sides of the piston is same
2. the volume occupied by the piston rod is negligible

Thus, for a double-acting steam engine

$$\dot{m}_s = \frac{\pi}{4} d^2 L \times \frac{1}{v} \times (2 \times 60) N \text{ (kg/h)} \qquad ...(20.19)$$

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More often, the steam consumption of a steam engine is given in *specific steam consumption*. The specific steam consumption (*ssc*) or steam rate is defined as the amount of steam required in kg to develop 1 kWh (3600 kJ) of work. It can be obtained by dividing the amount of steam consumed per hour by the engine power (kW) units developed by the engine. It is measured in kg/kWh and may be expressed on the basis of either indicated power or brake power.

$$ssc = \frac{\text{Mass of steam consumption, kg/h}}{\text{Power produced by engine, kW}}$$

(kg/kWh) ...(20.20)

Figure 20.13 shows the specific steam consumption for throttle and cut-off governings.

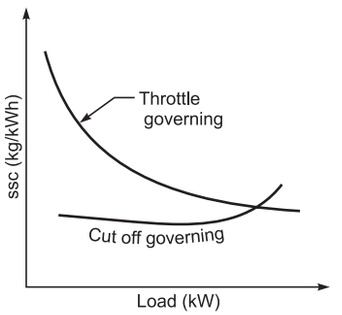

Fig. 20.13 *Load v/s specific steam consumption*

20.10 STEAM COMPRESSION IN THE CYLINDER

In order to minimise condensation of live steam during its admission into the cylinder, the exhaust port is closed earlier and some quantity of steam remaining in the cylinder is compressed. Steam pressure and temperature increase, causing the cylinder surface temperature to rise. Then fresh steam from the boiler enters the cylinder, thus reducing steam condensation. The work of compression is compensated by reduction in cylinder condensation.

20.11 GOVERNING OF STEAM ENGINES

Most steam engines are designed to work on rated speed at which their efficiency is maximum. The load applied on the engine corresponding to rated speed is fixed and is called full load.

During the operation of the engine, as load varies, the speed of the engine will also vary. If the mass flow rate of steam is kept constant, the efficiency of the engine decreases at loads other than the full load. Therefore, it is imperative to keep the engine speed constant by controlling the mass flow rate and other properties of steam by a device. Thus a device which controls the mass flow rate of steam in the engine according to applied load on the engine is called *governor*, and the process of controlling speed is called *governing*.

The function of a governor on a steam engine is to maintain the constant speed of the engine with minimum fluctuations, irrespective of the load on the engine. In other words, the engine should be able to adjust its power output according to the load with minimum fluctuations in speed.

There are two types of governing techniques used on steam engines:

(i) Throttle governing
(ii) Cut-off governing

20.11.1 Throttle Governing

In this method of governing, the cut-off point is kept constant, but steam pressure is regulated by a throttle valve. The steam from the boiler at a pressure p_1 is throttled to some lower pressure according to reduction in load on the engine as shown in Fig. 20.14. The lower pressure of steam during the admission process reduces the work developed by the engine.

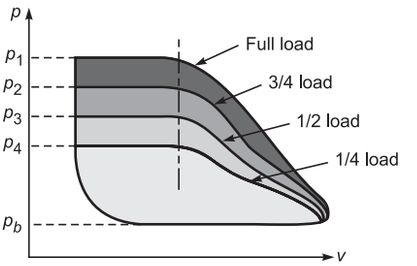

Fig. 20.14 *Throttle governing*

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During a throttle control governing, the indicated power developed by an engine is varied, and the steam consumption rate is also varied. Measurements reveal that the steam consumption rate is linearly proportional to the indicated power and thus is related as

$$\dot{m}_s = a\,IP + C \qquad ...(20.21)$$

where a is the slope of the straight line, C is the intersection point at no load.

Figure 20.15 shows a graph of the steam consumption and indicated power for a throttle governing. The linear line on the graph is called *Willan's line*.

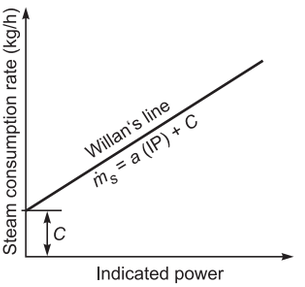

Fig. 20.15 *Load vs steam consumption for throttle governing*

20.11.2 Cut-off Governing

As indicated by its name, in this method of governing, the inlet steam pressure is kept constant, but the steam cut-off point is controlled according to load on the engine. As load falls on the engine, the steam cut-off point is shifted towards the left as shown in Fig. 20.16. The cut-off governing regulates the mass of steam entering the cylinder. The cut-off governing is made with the help of a slide valve operating under the control of a centrifugal governor.

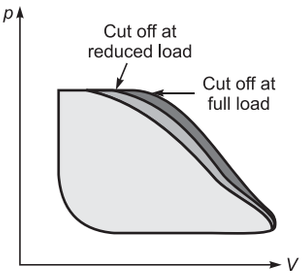

Fig. 20.16 *Cut-off Governing*

The variation of steam consumption rate against the indicated power developed in cut-off governing is shown in Fig. 20.17.

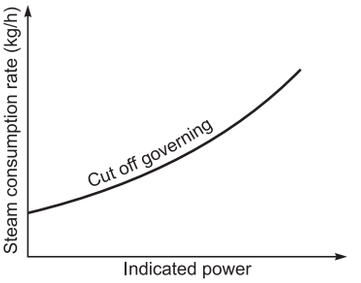

Fig. 20.17 *Load vs steam consumption for cut-off governing*

20.11.3 Comparision Between Throttle and Cut-off Governing

No.	*Throttle control governing*	*Cut-off governing*
1.	The throttling is an irreversible process, thus the available energy is lost during throttling.	During cut-off governing the pressure remains constant, thus the enthalpy of steam also remains constant.
2.	At part load, the steam enters cylinder at lower pressure, thus the steam consumption rate increases.	At part load, the mass of steam supplied is reduced by shifting the steam cut-off towards left, thus steam consumption rate is reduced.
3.	Due to throttle governing, the thermal efficiency of the engine is lowered.	Thermal efficiency of the engine does not change with cut-off governing.
4.	It is simple and easy to operate.	It is complicated and costly.

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20.12 POWER OUTPUT OF STEAM ENGINE

20.12.1 Piston Speed

It is the *linear distance travelled* by a piston per second and is expressed as

$$\mathcal{V} = \frac{2LN}{60} \text{ (m/s)} \qquad ...(20.22)$$

20.12.2 Theoretical Indicated Power

The theoretical power developed by the engine inside the cylinder as shown by the area of an indicated diagram is known as *indicated power*. It is designated as IP_{th} and expressed as the product of force and velocity of the piston.

IP_{th} = M.E.P × Cross sectional area of piston × Piston velocity

$$= p_m A \frac{2LN}{60} = p_m L A \frac{n}{60}$$

$$= p_m V_s \frac{n}{60} \qquad ...(20.23)$$

where $n = N$, for single-acting engine
$= 2N$, for double-acting engine
N = rotation per minute (rpm) of engine.

20.12.3 Actual Indicated Power

The actual indicated power developed by an engine,

$$IP = K\, p_m L A \frac{n}{60} \text{ (kW)} \qquad ...(20.24)$$

$$= p_{m,act}\ LA \frac{n}{60}$$

If mean effective pressure is used in kN/m^2, A in m^2, L in m, then the indicated power is expressed in kW.

20.12.4 Brake Power

It is the power available (output) at the crank shaft of the engine. However, it is less than the power developed in the cylinder due to various *frictional losses*.

The brake power (BP) is measured by dynamometers like rope brake, prony brake or hydraulic dynamometers. The *rope brake dynamometers* are extensively used in laboratories. A schematic of a rope brake dynamometer is shown in Fig. 20.18. A rope wrapped over the fly wheel (brake drum) applies frictional resistance to the rotation of the brake drum, mounted on the engine shaft.

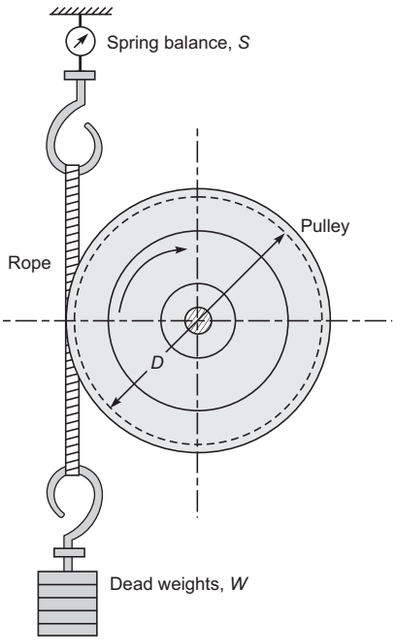

Fig. 20.18 *Rope-brake dynamometer*

Let W = Weight applied on rope, N (= mg)
S = Spring balance reading, N
N = Speed of crank shaft, *rpm*
R = Effective radius of brake drum, m

Then BP = Force × $\frac{\text{displacement}}{\text{time}}$

$$= \frac{(W - S)}{1000} \times 2\pi R \frac{N}{60} \text{ (kW)} \qquad ...(20.25)$$

20.12.5 Frictional Power

The frictional power is the difference between indicated power and brake power, i.e.,

$$FP = IP - BP \qquad ...(20.26)$$

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20.13 EFFICIENCIES OF A STEAM ENGINE

The efficiencies of a steam engine relate power developed and heat supplied by steam into the engine.

Heat supplied by steam to engine

$$\dot{Q}_s = \dot{m}_s \times (h_1 - h_2) \qquad ...(20.27)$$

where $\dot{m}_s$ = Mass flow rate of steam (kg/*s*)

h_1 = Specific enthalpy of steam entering the engine (kJ/kg)

h_2 = Specific enthalpy of exhaust steam (kJ/kg)

20.13.1 Indicated Thermal Efficiency

It is defined as the ratio of actual indicated power developed in the engine cylinder to heat supplied by steam.

$$\eta_{ith} = \frac{IP_{act}}{\dot{m}_s (h_1 - h_2)} \qquad ...(20.28)$$

20.13.2 Brake Thermal Efficiency

It is defined as the ratio of brake power to heat supplied by steam.

$$\eta_{bth} = \frac{BP}{\dot{m}_s (h_1 - h_2)} \qquad ...(20.29)$$

20.13.3 Mechanical Efficiency

It is defined as the ratio of brake power to indicated power developed by the steam engine.

$$\eta_{mech} = \frac{BP}{IP} = \frac{\eta_{bth}}{\eta_{ith}} \qquad ...(20.30)$$

Figure 20.19 shows the variation of mechanical and brake thermal efficiencies with increase of load.

20.13.4 Overall Efficiency

It is defined as the ratio of brake power to energy supplied by the fuel for steam generation in the boiler.

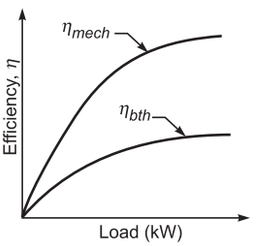

Fig. 20.19 *Variation of mechanical and brake thermal efficiencies with load*

$$\eta_{overall} = \frac{BP}{\dot{m}_f \times CV} \qquad ...(20.31)$$

$\dot{m}_f$ = Rate of fuel supply, kg/s

CV = Calorific value of fuel, kJ/kg

Example 20.1 *A double-acting steam engine has a cylinder bore of 250 mm and a stroke of 300 mm. The dry saturated steam is supplied from the boiler at a pressure of 10 bar. The condenser pressure is 0.3 bar. The engine has a speed of 160 rpm and a diagram factor of 0.8. If cut-off takes place at 0.35 of the stroke, neglect the clearance and calculate the indicated power of the engine.*

Solution

Given A double-acting steam engine without clearance

d = 250 mm L = 300 mm

p_1 = 10 bar = 1000 kN/m^2

p_b = 0.3 bar = 30 kN/m^2

K = 0.8 ρ = 0.35

N = 160 rpm n = 320 working strokes/min

To find Indicated power of the engine.

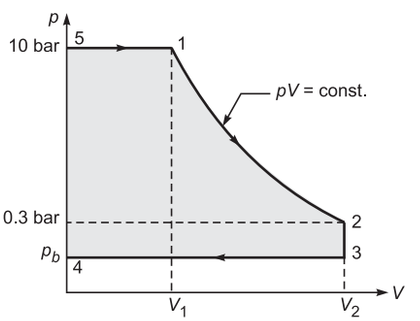

Fig. 20.20

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Analysis The stroke volume of the cylinder

$$V_s = V_2 = \frac{\pi}{4} d^2 \times L = \frac{\pi}{4} \times (0.25)^2 \times (0.3)$$
$$= 0.0147 \text{ m}^3$$

The volume of cylinder at cut-off

$$V_1 = \rho V_s = 0.35 \times 0.0147 = 0.0051 \text{ m}^3$$

$$\text{Expansion ratio, } r_e = \frac{V_2}{V_1} = \frac{0.0147}{0.0051} = 2.857$$

Theoretical mean effective pressure as

$$p_m = p_1 \rho \, [1 + \ln (r_e)] - p_b$$
$$= 10 \times 0.35 \, [1 + \ln (2.857)] - 0.3$$
$$= 6.874 \text{ bar}$$

Actual mean effective pressure

$$p_{m,\,act} = K \times p_m = 0.8 \times 6.874 = 5.499 \text{ bar}$$

Indicated power

$$IP = \frac{p_{m,\,act}\, LA}{60} \times \frac{n}{60}$$
$$= (5.499 \times 100) \times (0.0147) \times \left(\frac{320}{60}\right)$$
$$= \mathbf{43.19 \text{ kW}}$$

Example 20.2 *The steam enters a steam engine at 15 bar and exhausts at 1.5 bar. The steam supply is cut off at 40% of the stroke. The clearance volume is 5% of the swept volume. Calculate the mean effective pressure.*

Solution

Given A steam engine

$p_1 = 15$ bar $\qquad p_b = 1.5$ bar

$\rho = 0.4 \qquad V_c = 0.05\, V_s$

To find The mean effective pressure.

Assumption Diagram factor, $K = 1$

Analysis For indicator diagram shown in Fig. 20.21;

$$V_1 = V_c + \rho V_s = 0.05\, V_s + 0.4\, V_s = 0.45\, V_s$$
$$V_2 = V_c + V_s = 0.05\, V_s + V_s = 1.05\, V_s$$

Expansion ratio;

$$r_e = \frac{V_2}{V_1} = \frac{1.05 V_s}{0.45 V_s} = 2.333$$

Hypothetical work transfer

$$W = p_1 V_1 \left[1 + \ln (r_e)\right] - p_1 V_c - p_b V_s$$

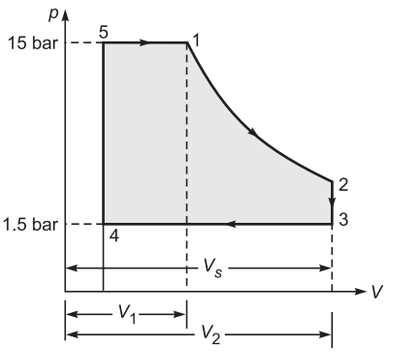

Fig. 20.21

The mean effective pressure is given by

$$p_m = \frac{\text{Work done}}{\text{Swept volume}} = \frac{W}{V_s}$$

$$\text{or} \quad p_m = \frac{15 \times 0.45 V_s \left[1 + \ln (2.33)\right] - 15 \times 0.05 V_s - 1.5 V_s}{V_s}$$

$$= 6.75\,(1.8473) - 0.75 - 1.5 = \mathbf{10.22 \text{ bar}}$$

Alternatively, the mean effective pressure for a steam engine cylinder with clearance can also be obtained by using Eq. (20.5),

$$p_m = p_1(\rho + c)\,[1 + \ln (r_e)] - p_1 c - p_b$$
$$= \mathbf{10.22 \text{ bar}}$$

Example 20.3 *The area of an indicator diagram is 25 cm^2. The swept volume is 0.15 m^3. Calculate the theoretical mean effective pressure. The indicator diagram is drawn to the following scales:*

1 cm = 1 bar along the pressure axis

1 cm = 0.02 m^3 along the volume axis

Solution

Given Indicator diagram parameters,

$A = 25 \text{ cm}^2 \qquad V_s = 0.15 \text{ m}^3$

Spring constant = 1 bar/cm

To find The mean effective pressure.

Analysis The length of the indicator diagram

$$= \frac{\text{Swept volume}}{\text{Scale on volume axis}}$$
$$= \frac{0.15 \text{ m}^3}{0.02 \text{ m}^3/\text{cm}} = 7.5 \text{ cm}$$

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The mean effective pressure is

$$p_m = \frac{\text{Area of indicator diagram}}{\text{Length of indicator diagram}} \times \text{Spring constant}$$

$$= \frac{25}{7.5} \times 1 = \mathbf{3.33\ bar}$$

Example 20.4 *A single-cylinder, double-acting steam engine gives an indicated power of 50 kW, when running at 210 rpm. The engine bore is 0.25 m and the stroke is 0.38 m. The steam is supplied at 10 bar, dry and saturated and the back pressure is 0.6 bar, while cut-off takes place at 20% of the stroke. Calculate the diagram factor.*

If steam consumption is 600 kg/h, calculate the indicated thermal efficiency.

Solution

Given The double-acting steam engines

$d = 0.25$ m $\quad L = 0.38$ m

$N = 210$ rpm $\quad n = 2N = 420$ cycles/min

$p_1 = 10$ bar $= 1000$ kPa

$p_b = 0.6$ bar $= 60$ kPa

$IP = 50$ kW

$\rho = 0.2$

$\dot{m}_s = 600$ kg/h

To find

(i) Diagram factor, and

(ii) Indicated thermal efficiency.

Assumptions

(i) Engine cylinder without clearance.

(ii) Neglecting effect of piston rod on underside of piston.

(iii) The steam condenses completely after expansion.

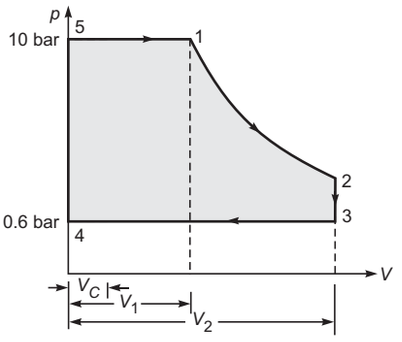

Fig. 20.22

Analysis

(i) *Diagram factor*

Expansion ratio in the cylinder

$$r_e = \frac{1}{\rho} = \frac{1}{0.2} = 5.0$$

The mean effective pressure in the cylinder

$$p_m = p_1 \rho\,[1 + \ln(r_e)] - p_b$$
$$= 1000 \times 0.2 \times [1 + \ln(5.0)] - 60$$
$$= 462 \text{ kPa}$$

Area of cylinder

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.25)^2 = 0.049 \text{ m}^2$$

The actual indicated power developed is given by

$$IP = p_{m,\,act}\, LA \frac{n}{60}$$

$$\therefore \quad 50 = p_{m,\,act} \times 0.38 \times 0.049 \times \frac{420}{60}$$

or $\quad p_{m,act} = 383$ kPa

The diagram factor,

$$K = \frac{p_{m,act}}{p_m} = \frac{383}{462} = \mathbf{0.828}$$

(ii) *Indicated thermal efficiency*

From steam table

At 10 bar, dry and saturated steam

$h_1 = h_g$ @ 10 bar $= 2777.08$ kJ/kg

At 0.6 bar $h_2 = h_f$ @ 0.6 bar $= 359.9$ kJ/kg

Heat supplied to steam engine,

$$\dot{Q}_s = \dot{m}_s \times (h_1 - h_2)$$
$$= (600 \text{ kg/h}) \times (2777.08 - 359.9)$$
$$= 1450308 \text{ kJ/h} = 402.86 \text{ kW}$$

Indicated thermal efficiency

$$\eta_{ith} = \frac{IP}{\dot{Q}_s} = \frac{50}{402.86}$$
$$= 0.1241 \quad \text{or} \quad \mathbf{12.41\%}$$

Example 20.5 *A rope-brake dynamometer is used to measure the brake power of a steam engine. The net weight applied at the rim of the brake drum of 1 m diameter is 130 kg. The engine runs at 300 rpm. Determine the brake power.*

Solution

Given A rope-brake dynamometer

$m = 130$ kg $\quad D = 1$ m

$N = 300$ rpm

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To find Brake power

Analysis The brake power is calculated as

$$BP = \frac{2\pi N T}{60}$$

where torque,

$$T = \text{Force} \times \text{Drum radius}$$
$$= mg \times \frac{D}{2} = 130 \times 9.81 \times \frac{1}{2} = 637.65 \text{ Nm}$$

Then $$BP = 2\pi \times \frac{300}{60} \times 637.65 = 20032 \text{ W}$$
$$= \mathbf{20.03\ kW}$$

Example 20.6 *A double-acting steam engine has steam admission at 12 bar with cut-off at 1/3 of stroke. The exhaust is at 0.2 bar. The cylinder bore is 0.3 m and the stroke is 0.4 m. The engine runs at 250 rpm. Determine the power developed considering the piston rod diameter is 6 cm on the crank side.*

Solution

Given A double-acting steam engine

$p_1 = 12$ bar $\quad p_2 = 0.2$ bar

$V_1 = \frac{1}{3}V_2$ or $\rho = \frac{1}{3}$

$\therefore \quad r_e = 3$

$d = 0.3$ m $\quad L = 0.4$ m

$N = 250$ rpm

$n = N = 250$ cycles for each side

$d_{cr} = 6$ cm

To find Power developed by the engine.

Assumptions

(i) Cylinder without clearance.

(ii) Diagram factor $K = 1$.

(iii) Same mean effective pressure on both sides of piston.

Analysis The mean effective pressure,

$$p_m = p\rho[1 + \ln(r_e)] - p_b$$
$$= 12 \times \frac{1}{3}[1 + \ln(3)] - 0.2 = 8.194 \text{ bar}$$

Area of cylinder on cover end,

$$A_1 = \frac{\pi}{4}d^2 = \frac{\pi}{4} \times (0.3)^2 = 70.68 \times 10^{-3} \text{ m}^2$$

Power from cover-end side

$$IP_1 = p_m LA \frac{N}{60}$$
$$= (8.194 \times 100) \times 0.4 \times 70.68 \times 10^{-3} \times \frac{250}{60}$$
$$= 96.\ 54 \text{ kW}$$

Effective area of cylinder on crank side

$$A_2 = \frac{\pi}{4}(d^2 - d_{cr}^2) = \frac{\pi}{4} \times (0.3^2 - 0.0\ 6^2)$$
$$= 0.0678 \text{ m}^2$$

Power developed by engine from crank side

$$IP_2 = p_m L A_2 \frac{N}{60}$$
$$(8.194 \times 100) \times 0.4 \times 0.0678 \times \frac{250}{60}$$
$$= 92.67 \text{ kW}$$

Total power developed by the engine

$$IP = IP_1 + IP_2 = 96.54 + 92.67 = \mathbf{189.21\ kW}$$

Example 20.7 *A double-acting steam engine with a bore of 30 cm and a stroke of 40 cm has steam admission at 12 bar and the exhaust is at 1 bar. Cut-off occurs when the total volume is equal to 0.25 of stroke volume. The engine runs at 250 rpm.*

If the clearance is 8% of strokes, calculate the power developed, neglecting the effect of piston rod.

Solution

Given A double-acting steam engine

$d = 30$ cm $= 0.3$ m $\quad L = 40$ cm $= 0.4$ m

$p_1 = 12$ bar $= 1200$ kPa $\quad p_b = 1$ bar $= 100$ kPa

$V_1 = 0.25\,V_s \quad N = 250$ rpm

$n = 2N = 500$ rpm

$V_c = 0.08\,V_s$ or $c = 0.08$

To find Power developed by the engine.

Assumptions

(i) Neglecting the compression of steam in the cylinder.

(ii) Diagram factor $K = 1$.

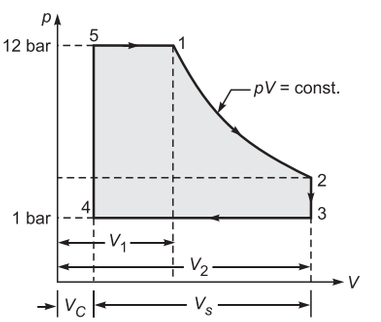

Fig. 20.23

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Analysis The volume of cylinder at the point of cut-off

$$V_1 = 0.25\,V_s$$

and $$V_2 = V_c + V_s = (0.08 + 1)\,V_s = 1.08\,V_s$$

Cut-off ratio $$\rho = \frac{V_1 - V_c}{V_s} = \frac{(0.25 - 0.08)\,V_s}{V_s} = 0.17$$

Expansion ratio $$r_e = \frac{V_2}{V_1} = \frac{1.08}{0.25} = 4.32$$

The mean effective pressure in the cylinder

$$p_m = p_1(\rho + c)\,[1 + \ln (r_e)] - p_1 c - p_b$$
$$= 12 \times (0.17 + 0.08)\,[1 + \ln (4.32)] - 12 \times 0.08 - 1$$
$$= 5.43 \text{ bar} = 543 \text{ kPa}$$

Power developed IP

$$= p_m\,LA\,\frac{n}{60} = 543 \times 0.4 \times \left(\frac{\pi}{4} \times (0.3)^2\right) \times \frac{500}{60}$$
$$= \mathbf{127.935\ kW}$$

Example 20.8 *Find the cylinder diameter and length of stroke for a double-acting steam engine developing 60 kW, when the cut-off takes place at 0.4 of stroke. The steam is supplied at 8 bar and it is exhausted at 1.1 bar. The clearance volume is 5% of the stroke volume. The engine runs at 120 rpm. Neglect the effect of compression. Take the diagram factor as 0.8 and stroke is 1.5 times of diameter.*

Solution

Given A double-acting steam engine with clearance.

$IP = 60$ kW $\qquad \rho = \frac{V_1 - V_c}{V_s} = 0.4$

$c = 0.05$ $\qquad p_1 = 8$ bar

$p_b = 1.1$ bar $\qquad K = 0.8$

$L = 1.5d$ $\qquad N = 120$ rpm

$n = 2N = 240$ cycle/min

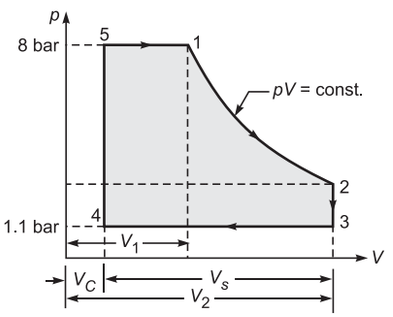

Fig. 20.24

To find

(i) Diameter of the cylinder.

(ii) Stroke length.

Analysis The volume after expansion

$$V_2 = V_c + V_s = cV_s + V_s$$
$$= 0.05\,V_s + V_s = 1.05\,V_s$$

and volume at cut-off

$$V_1 = \rho V_s + V_c = 0.4\,V_s + 0.05\,V_s = 0.45\,V_s$$

The expansion ratio, $r_e = \frac{V_2}{V_1} = \frac{1.05\,V_s}{0.45\,V_s} = 2.333$

The theoretical mean effective pressure

$$p_m = p_1(\rho + c)\,[1 + \ln (r_e)] - p_1 c - p_b$$
$$= 8 \times (0.4 + 0.05)\left[1 + \ln (2.333)\right] - 8 \times 0.05 - 1.1$$
$$= 5.15 \text{ bar}$$

Actual mean effective pressure

$$p_{m,\,act} = p_m \times K = 5.15 \times 0.8 = 4.12 \text{ bar}$$

Indicated power

$$IP = p_{m,\,act} \times LA\left(\frac{n}{60}\right)$$

$$60 = (4.12 \times 100) \times (1.5d) \times \left(\frac{\pi}{4} d^2\right) \times \left(\frac{240}{60}\right)$$

$$d^3 = \frac{60 \times 4 \times 60}{4.12 \times 100 \times 1.5 \times \pi \times 240} = 0.03090$$

$$d = 0.3138 \text{ m} = \mathbf{31.38\ cm}$$

Stroke $L = 1.5d = \mathbf{47.07\ cm}$

Example 20.9 *In a steam engine the clearance volume is 5% of the swept volume and the back pressure is 1.15 bar. If the compression is at 0.3 of the stroke, find the pressure at the end of compression stroke.*

Find also the mean effective pressure, if the steam supply pressure is 13.7 bar, and cut-off occurs at 4.0% of the stroke.

Solution

Given A steam engine with clearance and compression.

$V_c = 0.05\,V_s$ $\qquad c = 0.05$

$p_1 = 13.7$ bar $\qquad p_b = 1.15$ bar

$V_4 - V_c = 0.3\,V_s$ $\qquad \alpha = 0.3$ and $\rho = 0.4$

To find

(i) p_5, the pressure after compression.

(ii) p_m, the mean effective pressure of the cycle.

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Assumptions

(i) The compression and expansion are hyperbolic.

(ii) All processes are reversible.

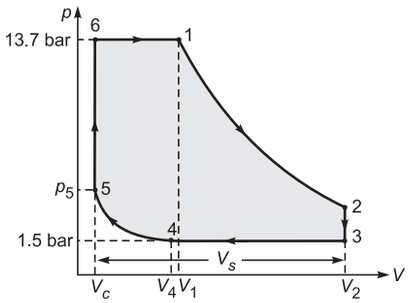

Fig. 20.25

Analysis

(i) *The pressure at end of compression* p_5

For the process 4–5, the pressure and volume can be related as

$$p_4 V_4 = p_5 V_5$$

where $V_4 = V_c + 0.3\,V_s = 0.05\,V_s + 0.3\,V_s$

$= 0.35\,V_s$

$V_5 = V_c = 0.05\,V_s$

Thus $p_5 = p_4 \dfrac{V_4}{V_5} = 1.15 \times \dfrac{0.35}{0.05} =$ **8.05 bar**

(ii) *The mean effective pressure*

$$p_m = p_1(\rho + c)[1 + \ln(r_e)] - p_1 c - p_b(1 - \alpha) - p_b(\alpha + c)\ln\left(\frac{\alpha + c}{c}\right)$$

where $r_e = \dfrac{V_2}{V_1} = \dfrac{V_c + V_s}{V_c + \rho V_s} = \dfrac{(0.05 + 1)V_s}{(0.0\,5 + 0.4)V_s}$

$= 2.333$

Thus $p_m = 13.7 \times (0.4 + 0.05)\,[1 + \ln(2.333)]$

$- 13.7 \times 0.05 - 1.15 \times (1 - 0.3) - 1.15 \times (0.3$

$+ 0.05) \times \ln\left(\dfrac{0.3 + 0.0\,5}{0.05}\right)$

$= 11.39 - 0.685 - 0.805 - 0.7832$

$=$ **9.115 bar**

Example 20.10 *The following data was obtained during a trial on a double-acting steam engine.*

Average mean effective pressure $= 105\ N/cm^2$

Speed $= 240$ *rpm*

Stroke $= 1.5 \times$ *Bore* $= 30$ *cm*

Net brake load $= 1.5$ *kN*

Effective brake diameter $= 160$ *cm*

Supply steam condition $=$ *Dry and Saturated at 9 bar*

Condensate collected $= 800$ *kg/h*

Condenser pressure $= 0.07$ *bar*

Exhaust steam quality $= 0.7$ *dry*

Determine

(a) Indicated power,

(b) Brake power,

(c) Mechanical efficiency,

(d) Indicated thermal efficiency,

(e) Brake thermal efficiency.

Solution

Given A trial on a double-acting steam engine

$p_m = 105\ N/cm^2 = 1050\ kN/m^2$

$N = 240$ rpm

$n = 2N$ (double acting engine) $= 480$ rpm

$d = 20$ cm $\qquad L = 1.5\,d = 30$ cm

$(W - s) = 1.5$ kN

$R = \dfrac{160\ \text{cm}}{2} = 80\ \text{cm} = 0.8\ \text{m}$

$p_1 = 9$ bar $\qquad p_2 = 0.07$ bar

$x_1 = 1.0 \qquad x_2 = 0.7$

$\dot{m}_s = 800$ kg/h

To find

(i) Indicated power, *IP*

(ii) Brake power, *BP*

(iii) Mechanical efficiency, η_{mech}

(iv) Indicated thermal efficiency, η_{ith}

(v) Brake thermal efficiency, η_{bth}.

Analysis

(i) *Indicated power*

$$IP = p_{m,\,act} \times LA \times \frac{n}{60}$$

$$= 1050 \times 0.3 \times \frac{\pi}{4} \times (0.2)^2 \times \frac{480}{60}$$

$=$ **79.168 kW**

(ii) *Brake power* $BP = (W - s)\,\dfrac{2\,\pi\,RN}{60}$

$= (1.5\ \text{kN}) \times 2\,\pi \times (0.8\ \text{m}) \times \dfrac{240}{60}$

$=$ **30.16 kW**

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(iii) *Mechanical efficiency*

$$\eta_{mech} = \frac{BP}{IP} = \frac{30.16}{79.168} = 0.3809 \text{ or } \mathbf{38.09\%}$$

(iv) *Indicated thermal efficiency*

From steam table

At 9 bar, $h_1 = h_g = 2773.94$ kJ/kg

At 0.07 bar, $h_{f_2} = 163.4$ kJ/kg, and

$h_{fg} = 2409.2$ kJ/kg

Enthalpy of exiting steam

$$h_2 = h_{f_2} + x_2 h_{fg_2}$$
$$= 163.4 + 0.7 \times 2409.2$$
$$= 1849.84 \text{ kJ/kg}$$

Heat supplied to steam engine,

$$\dot{Q}_s = \dot{m}_s \times (h_1 - h_2)$$
$$= (800 \text{ kg/h}) \times (2773.94 - 1849.84)$$
$$= 739280 \text{ kJ/h} = 205.35 \text{ kW}$$

$$\eta_{ith} = \frac{IP}{\dot{Q}_s} = \frac{79.168}{205.35} = 0.3855 \text{ or } \mathbf{38.55\%}$$

(v) *Brake thermal efficiency*

$$\eta_{bth} = \frac{BP}{\dot{Q}_s} = \frac{30.16}{205.38} = 0.1468 \text{ or } \mathbf{14.68\%}$$

Example 20.11 *During a trial on a vertical double-acting steam engine, the following observations were taken:*

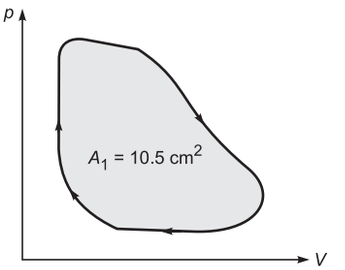
Indicator card area (cover-end side) $= 10.5$ *cm*^{2}
Indicator card area (crank-end side) $= 10.2$ *cm*^{2}
Length of indicator card $= 7.6$ *cm*
Spring constant $= 0.8$ *bar/m*
Speed $= 120$ *r.p.m*
Diameter of piston $= 21.5$ *cm*
Diameter of piston rod $= 3.7$ *cm*
Stroke $= 30.5$ *cm*

Calculate the indicator power. If the mechanical efficiency of the engine is 80% at the given load, find the brake torque required.

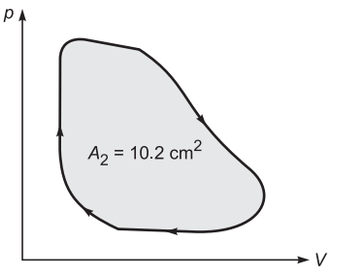
Solution

Given A double-acting vertical steam engine.

Indicator $A_1 = 10.5$ cm^2
$d = 21.5$ cm
$A_2 = 10.2$ cm^2
$d_{cr} = 3.7$ cm
$l = 7.6$ cm
$L = 30.5$ cm



Indicator diagram of cover end



Indicator diagram of crank end

Fig. 20.26

$k = 0.8$ bar/cm
$N = 120$ rpm $\quad \eta_{mech} = 0.8$

To find

(i) Indicator power, and
(ii) Brake torque.

Analysis

(i) The mean effective pressure from an indicator diagram is obtained as

$$p_m = \frac{\text{Area of indicator diagram}}{\text{Length of indicator card}} \times \text{spring constant}$$

On the cover side

$$p_{m_1} = \frac{A_1}{l} \times \text{spring constant } (k)$$
$$= \frac{10.5 \text{ cm}^2}{7.6 \text{ cm}} \times (0.8 \text{ bar/cm})$$
$$= 1.1052 \text{ bar} = 110.52 \text{ kPa}$$

$$A_{cover} = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.215)^2 = 0.0363 \text{ m}^2$$

and $$IP_1 = p_{m_1} L A_{cover} \frac{N}{60}$$
$$= 110.52 \times 0.305 \times 0.0363 \times \frac{120}{60}$$
$$= 2.447 \text{ kW}$$

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On the crank side

$$p_{m_2} = \frac{A_2}{l} \times k = \frac{10.2 \text{ cm}^2}{7.6 \text{ cm}} \times (0.8 \text{ bar/cm})$$

$$= 1.0737 \text{ bar} = 107.37 \text{ kPa}$$

$$A_{crank} = \frac{\pi}{4}(d^2 - d_{cr}^2) = \frac{\pi}{4} \times (0.215^2 - 0.037^2)$$

$$= 0.0352 \text{ m}^2$$

and $$IP_2 = p_{m_2} L A_{crank} \times \frac{N}{60}$$

$$= 107.37 \times 0.305 \times 0.0352 \times \frac{120}{60}$$

$$= 2.305 \text{ kW}$$

Total indicated power of engine

$$IP = IP_1 + IP_2 = 2.447 + 2.305$$

$$= \mathbf{4.752\ kW}$$

(ii) Brake power,

$$\eta_{mech} = \frac{BP}{IP}$$

or $$BP = \eta_{mech} \times IP = 0.8 \times 4.752$$

$$= \mathbf{3.80\ kW}$$

The brake power can also be expressed as

$$BP = \frac{2\pi N T}{60}$$

$$3.80 = 2\pi \times \frac{120}{60} \times T$$

Brake torque,

$$T = 0.3024 \text{ kN-m} = \mathbf{302.4\ Nm}$$

Example 20.12 *A throttle-governed steam engine uses 2820 kg of steam per hour, when developing an output power of 150 kW and 5000 kg/h of steam when developing 375 kW. Estimate the thermal efficiency of the engine when developing an output of 225 kW, assuming the steam supply is dry and saturated at 10 bar and the exhaust is at 0.25 bar.*

Solution

Given A throttle-governed steam engine

$\dot{m}_{s_1} = 2820$ kg/h

$IP_1 = 150$ kW

$\dot{m}_{s_2} = 5000$ kg/h

$IP_2 = 375$ kW

$IP_3 = 225$ kW

$p_1 = 10$ bar, dry and saturated

$p_b = 0.25$ bar

To find Thermal efficiency of the engine at given conditions.

Assumption Steam condenses completely, after expansion.

Analysis The graphical representation of mass consumption rate v/s power output of the engine is shown in Fig. 20.27.

Using the relation $\dot{m}_s = a\,IP + C$

$2820 = a \times 150 + C$

and $5000 = a \times 375 + C$

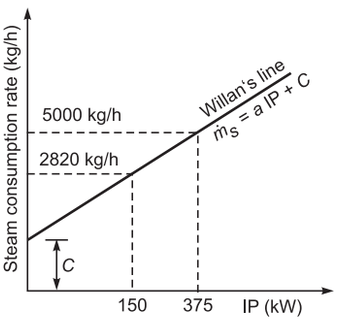

Fig. 20.27 *Fuel consumption vs power output*

Solving these equations, we get

$a = 9.6889$ kg/kWh

$C = 1366.67$ kg/h

The linear law

$$\dot{m}_s = 9.6889\,IP + 1366.67$$

At $IP_3 = 225$ kW

$$\dot{m}_s = 9.6889 \times 225 + 1366.67$$

$$= 3546.67 \text{ kg/h} = 0.985 \text{ kg/s}$$

Thermal efficiency $\eta_{th} = \dfrac{\text{Power output}}{\dot{m}_s \times (h_1 - h_2)}$

From steam table at 10 bar, dry and saturated steam

$h_1 = h_g$ @ 10 bar $= 2776.2$ kJ/kg

At 0.25 bar $h_2 = h_f$ @ 0.25 bar $= 272.0$ kJ/kg

Then $$\eta_{th} = \frac{225}{0.985 \times (2776.2 - 272)} = 0.0912 = \mathbf{9.12\%}.$$

Example 20.13 *The cylinder of a double-acting steam engine is 30 cm in diameter and has 44-cm stroke. The steam enters the cylinder at 6 bar and leaves at 1.2 bar. The steam is cut-off at 0.4 of stroke and compression starts from 0.8 of return stroke. The clearance*

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is 10% of the swept volume. The engine runs at 150 rpm. Assuming hyperbolic expansion and compression, calculate (a) Indicated power produced, and (b) the steam consumption per kWh. Assume diagram factor of 0.9.

Solution

Given A double-acting engine with clearance and compression.

$d = 30$ cm $\quad L = 44$ cm $\quad p_1 = 6$ bar
$\rho = 0.4 \quad V_4 - V_c = (1 - 0.8)\, V_s$
$c = 0.10$
$pV = C \quad p_b = 1.2$ bar
$K = 0.9$
$N = 150$ rpm $\quad n = 2N = 300$ rpm

To find

(i) Indicated power produced, and
(ii) Steam consumption kg/kWh.

Assumptions

(i) Dry and saturated steam eners the cylinder.
(ii) Negligible area of piston rod.

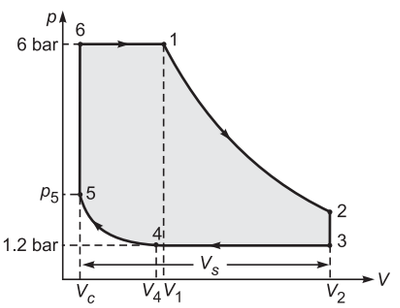

Fig. 20.28

Analysis The stroke volume

$$V_s = \frac{\pi}{4} d^2 \times L = \frac{\pi}{4} \times (0.3)^2 \times 0.44$$
$$= 0.0311 \text{ m}^3$$

Clearance volume,

$$V_c = 0.1 V_s$$

Cut-off volume

$$V_1 - V_c = 0.4\, V_s$$
$$V_1 = 0.4\, V_s + V_c$$
$$= (0.4 + 0.1)\, V_s = 0.5\, V_s$$

Volume after expansion

$$V_2 = V_s + V_c = 1.1\, V_s$$

Expansion ratio

$$r_e = \frac{V_2}{V_1} = \frac{1.1}{0.5} = 2.2$$

Volume at the start of compression

$$\alpha = \frac{V_4 - V_c}{V_s} = (1 - 0.8) = 0.2$$

$$\therefore \quad V_4 = 0.2\, V_s + V_c = 0.3\, V_s$$

Mean effective pressure,

$$p_m = p_1(\rho + c)\,[1 + \ln(r_e)] - p_1 c - p_b\left[(1-\alpha) + (\alpha + c)\ln\left(\frac{\alpha + c}{c}\right)\right]$$
$$= 6 \times (0.4 + 0.1)\left[1 + \ln(2.2)\right] - 6 \times 0.1 - 1.2\left[(1 - 0.2) + (0.2 + 0.1)\ln\left(\frac{0.2 + 0.1}{0.1}\right)\right]$$
$$= 6 \times 0.5 \times (1 + 0.788) - 0.6 - 1.2 \times [0.8 + 0.3 \times 1.0986]$$
$$= 5.36653 - 0.6 - 1.355 = 3.40 \text{ bar}$$

Actual mean effective pressure,

$$p_{m,act} = K\, p_m = 0.9 \times 3.40 = 3.06 \text{ bar.}$$

Indicated power developed,

$$IP = p_{m,act}\, LA \frac{n}{60} = p_{m,act}\, V_s \times \frac{n}{60}$$
$$= (3.06 \times 100) \times 0.0311 \times \frac{300}{60}$$
$$= \mathbf{47.6\ kW}$$

From steam tables at 6 bar, $v_g = 0.316$ m^3/kg

Theoretical mass per cycle,

$$\dot{m}_{th} = \frac{p_1 V_s}{v_g}$$
$$= \frac{0.4 \times 0.0311}{0.316} = 0.0393 \text{ kg/stroke}$$

Actual mass consumtion per cycle,

$$\dot{m}_1 = \frac{m_{th}}{K} = \frac{0.0393}{0.9} = 0.0437 \text{ kg/stroke}$$

Total steam consumption,

$$\dot{m}_s = \dot{m}_1 \times \text{No of effective strokes/hour}$$
$$= 0.0437 \times 300 \times 60 = 787.34 \text{ kg/h}$$

Specific steam consumption,

$$ssc = \frac{\dot{m}_s}{IP} = \frac{787.37}{47.6} = \mathbf{16.54\ kg/kWh}$$

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Example 20.14 *The following readings were taken during a trial of a single-cylinder, double-acting, non-condensing steam engine running at 240 rpm.*

Cylinder diameter,	$d = 300$ mm
Srtoke length,	$L = 450$ mm
Piston rod diameter,	$d_{cr} = 50$ mm
Cut-off	$= 30\%$ of stroke
Length of indicator diagram	$l = 55$ mm
Area of indicator diagram for cover end	$A_1 = 1620\ mm^2$
Area of indicator diagram for crank end	$A_2 = 1460\ mm^2$
Spring constant	$k = 0.12$ bar/mm
Effective circumference of the brake wheel,	$\pi D = 8.344$ m
Dead load on the brake	$W = 1900$ N
Reading of the spring balance	$S = 190$ N
Pressure of steam supplied	$p_1 = 10.5$ bar
Dryness fraction of steam supplied	$x = 0.92$

Find (a) indicated power, (b) brake power, (c) mechanical efficiency, (d) specific steam, consumption, and (e) brake thermal efficiecy.

Solution

Given Data as above

To find (i) *IP* (ii) *BP* (iii) η_{mech} (iv) specific steam consumption on *IP* and *BP* basis, and (v) brake thermal efficiency

Assumption Diagram factor, $K = 1$.

Analysis

(i) *Indicated power*

$$p_m = \frac{\text{Area of indicator diagram} \times \text{Spring constant}}{\text{Length of indicator diagram}}$$

On cover end;

$$P_{m_1} = \frac{A_1 \times k}{l} = \frac{1620 \times 0.12}{55} = 3.534 \text{ bar}$$

On crank end;

$$p_{m_2} = \frac{A_2 \times k}{l} = \frac{1460 \times 0.12}{55} = 3.815 \text{ bar}$$

Swept volume on the cover side

$$V_{s_1} = \frac{\pi}{4} d^2 \times L$$

$$= \frac{\pi}{4} \times (0.3)^2 \times 0.45 = 0.0318 \text{ m}^3$$

Swept volume on the crank side

$$V_{s_2} = \frac{\pi}{4} (d^2 - d_{cr}^2) \times L$$

$$= \frac{\pi}{4} (0.3^2 - 0.05^2) \times 0.45$$

$$= 0.0309 \text{ m}^3$$

The indicated power developed,

$$IP = p_{m_1} V_{s_1} \frac{N}{60} + p_{m_2} V_{s_2} \times \frac{N}{60}$$

$$= (3.534 \times 100) \times 0.0318 \times \frac{240}{60} + (3.185 \times 100) \times 0.0309 \times \frac{240}{60}$$

$$= 44.96 + 39.40 = \mathbf{84.36\ kW}$$

(ii) *Brake power developed by engine*

$$BP = (W - S) \frac{\pi D N}{60}$$

$$= (1900 - 190) \times 10^{-3} \times (8.344 \text{ m}) \times \frac{240}{60}$$

$$= \mathbf{57.0\ kW}$$

(iii) *Mechanical efficiency*

$$\eta_{mech} = \frac{BP}{IP} = \frac{57.0}{84.36} = 0.6766 \text{ or } \mathbf{67.66\%}$$

(iv) *Specific steam consumption*

Mass flow rate of steam

$$\dot{m}_s = \frac{\text{Cut-off volume}}{\text{Specific volume}} \times \text{No of effective stroke/h}$$

Total swept volume;

$$V_s = V_{s_1} + V_{s_2}$$

$$= 0.0318 + 0.0309 = 0.0627 \text{ m}^3$$

Cut-offv olume

$$V_1 = 0.3 \times V_s = 0.3 \times 0.0627$$

$$= 0.0188 \text{ m}^3$$

No. of effective stroke/cycle = 1 (on each side)

No. of cycle per hour = $240 \times 60 = 14400$

Specific volume of dry saturated steam at 10.5 bar;

$$v_g = 0.1854 \text{ m}^3/\text{kg}$$

Specific volume of supplied steam with $x = 0.92$ dry.

$$v = x v_{g\ @\ 10.5 \text{ bar}}$$

$$= 0.92 \times 0.1854 = 0.1705 \text{ m}^3/\text{kg}$$

Steam consumption rate,

$$\dot{m}_s = \frac{V_1}{v} = \frac{0.0188}{0.1705} \times 1 \times 14400 = 1587.17 \text{ kg/h}$$

Specific steam consumption on *IP* basis

$$= \frac{\dot{m}_s}{IP} = \frac{1587.17}{84.36} = \mathbf{18.81\ kg/kWh}$$

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Specific steam consumption on *BP* basis

$$= \frac{\dot{m}_s}{BP} = \frac{1587.17}{57.0}$$

$$= \mathbf{27.84\ kg/kWh}$$

(v) Brake thermal efficiency

$$\eta_{bth} = \frac{\text{Brake power}}{\text{Heat supply rate}}$$

Heat supplied/kg = $h_1 - h_2$

At 10.5 bar, $h_f = 772$ kJ/kg,

and $h_{fg} = 2005.9$ kJ/kg

$h_1 = 772 + 0.92 \times 2005.9$

$= 2617.42$ kJ/kg

$h_2 = 0$ (since engine is non condensing)

$q_{in} = 2617.42 - 0 = 2617.42$ kJ/kg

Total heat supplied

$$\dot{Q}_s = \dot{m}_s\, q_{in} \frac{1587.17}{3600} \times 2671.42 = 1177.77 \text{ kW}$$

$$\eta_{bth} = \frac{BP}{\dot{Q}_s} = \frac{57}{1177.77} = 0.0483 \text{ or } \mathbf{4.83\%}$$

Example 20.15 *A single-cylinder, double-acting steam engine is supplied with steam at 9 bar, dry and saturated and exhausted at 140 kPa. Cut-off takes place at 0.4 of stroke. Determine the necessary cylinder bore and piston stroke. The engine develops 22.5 kW. Assume a diagram factor of 0.8, stroke of 1.25 times the bore, speed of 240 rpm and hyperbolic expansion.*

If actual steam consumption is 1.5 times the indicated steam consumption, calculate the probable steam consumption and indicated thermal efficiency.

Solution

Given A double-acting steam engine

$p_1 = 9$ bar $\quad p_2 = 140$ kPa $= 1.4$ bar

$x_1 = 1.0 \quad \rho = 0.4$

$IP = 22.5$ kW $\quad K = 0.8$

$N = 240$ rpm

$n = 2N$ (double acting engine) $= 480$ rpm

$L = 1.25d \quad m_{act} = 1.5 m_{th}$

To find

(i) Bore of cylinder, d,

(ii) Piston stroke, L,

(iii) Actual steam consumtion, m_{act}.

(iv) Indicated thermal efficiency, η_{ith}.

Assumptions

(i) Negligible clearance in the cylinder.

(ii) Steam condenses completely at the end of expansion.

Analysis The expansion ratio,

$$r_e = \frac{1}{\rho} = \frac{1}{0.4} = 2.5$$

Theoretical mean effective pressure as

$$p_m = p_1 \rho [1 + \ln (r_e)] - p_b$$

$$= 9 \times 0.4 \times [1 + \ln (2.5)] - 1.4 = 5.5 \text{ bar}$$

Actual mean effective pressure,

$$p_{m,\,act} = K \times p_m = 0.8 \times 5.5 = 4.4 \text{ bar or } 440 \text{ kPa}$$

Indicated power,

$$IP = p_{m,\,act}\ LA \times \frac{n}{60}$$

$$22.5 = 440 \times (1.25\, d) \times \left(\frac{\pi}{4} d^2\right) \times \left(\frac{480}{60}\right)$$

or $\quad d^3 = 6.51 \times 10^{-3} \text{ m}^3$

(i) *Bore* $= 0.1867$ m or **187 mm**

(ii) *Stroke* $L = 1.25\, d =$ **234 mm**

Swept volume,

$$V_s = \frac{\pi}{4} d^2 L$$

$$= \frac{\pi}{4} \times (0.187)^2 \times (0.234) = 0.00642 \text{ m}^3$$

The steam consumption per cycle

From steam table, at 9 bar

$$v_g = 0.2148 \text{ m}^3/\text{kg}$$

Indicated mass per cycle,

$$\dot{m}_1 = \frac{\rho_1 V_s}{K v_g} = \frac{0.4 \times 0.00642}{0.8 \times 0.2148}$$

$= 0.0149$ kg/cycle

Steam consumption per hour,

$\dot{m}$ = $\dot{m}_1$ × No of effective strokes/hour

$= \dot{m}_1 \times (n \times 60)$

$= 0.0149 \times 480 \times 60 = 430.4$ kg/h

(iii) *Actual steam consumption*

$\dot{m}_s = 1.5 \times \dot{m} = 1.5 \times 430.4 = 645.58$ kg/h

(iv) *Indicated thermal efficiency*

From steam tables,

At 9 bar $\quad h_1 = h_g = 2773.94$ kJ/kg

At 1.4 bar $\quad h_2 = h_{f_2} = 458.4$ kJ/g

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Heat supplied to steam engine,

$$\dot{Q}_s = \dot{m}_s \times (h_1 - h_2)$$
$$= (645.58 \text{ kg/h}) \times (2773.94 - 458.4)$$
$$= 1494881.57 \text{ kJ/h} = 415.24 \text{ kW}$$

Then $\eta_{ith} = \dfrac{IP}{\dot{Q}_s} = \dfrac{22.5}{415.24} = 0.0541$ or **5.41%**

20.14 COMPOUND STEAM ENGINES

If steam expands completely in single expansion within the cylinder, the steam engine is called a simple steam engine.

Nowadays the modern boilers generate steam at very high pressure and the expansion of steam is carried out in more than one cylinder. Thus an engine is called a compound steam engine. A hypothetical *p–V* diagram for a triple expansion compound steam engine is shown in Fig. 20.29. The whole hyperbolic expansion is divided into three parts by horizontal lines. The high-pressure cylinder receives the steam from the boiler and it rejects exhaust steam directly to the intermediate pressure cylinder and the intermediate cylinder rejects the exhaust steam directly to the low-pressure cylinder. The three cylinders are designed in such a manner that the same amount of work is done in each cylinder.

As steam expands, its volume increases and hence a large space is required in low- pressure cylinders. The length of all cylinders is same, they all have the same length of stroke, but the volume must

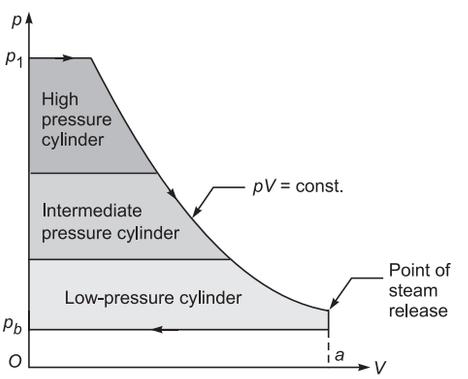

Fig. 20.29 *Hypothetical p–V diagram for compound steam engine*

be varied by varying cylinder diameters. Therefore, the diameter of the intermediate cylinder will be larger than the high-pressure cylinder, as steam pressure has been dropped and its volume has correspondingly increased. By the same reasoning, the low-pressure cylinder will have a still larger diameter.

20.14.1 Limitations of Simple Steam Engine

The compounding of steam engines is necessary because of the following limitations of a simple steam engine.

1. If high-pressure steam expands in a single stage of expansion to condenser pressure, it requires a larger expansion ratio and thereby bigger stroke length.
2. A steam engine with bigger stroke length will have larger engine size and more imbalance forces.
3. With use of high pressure steam, the pressure difference on two sides of the piston will increase, which will cause leakage past the piston.
4. To handle the high-pressure steam, the cylinder and other parts should have to be made stronger. It will further increase the weight of the engine and large variation in turning moment. Thus vibrations will occur on the engine.
5. With use of larger expansion ratio, the temperature variation in the cylinder would be large, which will result into larger steam condensation in the cylinder and thereby increase in steam consumption rate.

20.14.2 Advantages of Compound Steam Engine

Limitations of simple steam engines can be overcome by compounding a steam engine. The compound steam engines have following advantages.

(i) The pressure difference per cylinder is reduced, which reduces steam leakage and stroke length.

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(ii) With reduced pressure difference in a cylinder, the temperature variation is also reduced, which will result into less cylinder condensation.
(iii) Low pressure range in a cylinder also results in less variation in the turning moment.
(iv) Since high-pressure steam enters the HP cylinder, it should only be made stronger to withstand high pressures and conusequently, the successive cylinders will be lighter in weight. It reduces the weight-to-power ratio of the engine.
(v) The engine can be started at any position.
(vi) The thermal efficiency of the engine improves.
(vii) The cylinder condensation can also be reduced by reheating steam after expansion in each cylinder.
(viii) High speed of engine is possible due to perfect mechanical balancing.
(ix) Due to lighter reciprocating parts of an engine, the engine vibrations are reduced.
(x) In case of any breakdown, the engine can run at reduced load.

20.14.3 Disadvantages of Compound Steam Engine

Apart from several attractive advantages, a compound steam engine has the following disadvantages:

(i) It has high initial cost.
(ii) Its size is large and it requires large floor space.
(iii) It has complex construction, thus maintenance is difficult.
(iv) Radiation losses are large from a high-pressure cylinder.
(v) Its wear and tear are large and it has lubrication trouble.

Summary

- A steam engine is also called an *external combustion engine* that converts heat energy of steam into mechanical work.
- The steam engines are generally of double-acting type, both back and front faces of the piston are arranged as working faces.
- The *diagram factor K* is defined as the ratio of area of the actual indicator diagram to area of the hypothetical indicator diagram.
- When steam enters the cylinder and it comes in contact of the surface of the cylinder, which is colder than the steam, some steam condenses into the cylinder without doing any work. This quantity of steam is considered as a missing quantity.
- The function of a governor on a steam engine is to maintain the constant speed of the engine with minimum fluctuations, irrespective of the load on the engine. There are two types of governing techniques used on the steam engines:
  (i) Throttle governing
  (ii) Cut-off governing
- During a throttle-control governing, the mass of steam consumed by an engine is linearly proportional to indicated power and is thus related as
$$\dot{m}_s = a\,IP + C$$
- In cut-off governing, the inlet steam pressure is kept constant, but the steam cut-off point is controlled according to load on the engine.
- The mass of steam supplied per stroke is called *cylinder feed,* and the steam left behind in the clearance space is known as *cushion steam.*
- Specific steam consumption may be expressed on the basis of either indicated power or brake power as
$$ssc = \frac{\text{Mass of steam consumption, kg/h}}{\text{Power produced by engine, kW}} \quad \text{(kg/kWh)}$$
- Indicated power developed by an engine,
$$IP = K\,p_m\,L\,A\frac{n}{60} \quad \text{(kW)}$$
- Brake power is the power available (output) at the crank shaft of the engine,
$$BP = \frac{(W - S)}{1000} \times 2\pi R\,\frac{N}{60}\,\text{(kW)}$$

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Glossary

Cross-head Link between piston rod and connecting rod

D-slide valve It controls the steam entry into the cylinder and its exhaust

Eccentric It converts the rotary motion of the crank into reciprocating motion of the *D*-slide valve

Diagram factor Ratio of area of actual indicator diagram to area of hypothetical indicator diagram

Cylinder feed Mass of steam supplied per stroke

Cushion steam Steam left behind in the clearance space

Missing quantity Quantity of steam condenses inside the cylinder during admission without doing any work

Piston Speed Linear distance travelled by the piston per second

Review Questions

1. Describe the hypothetical and actual indicator diagram for a steam engine and explain what you understand by diagram factor.
2. Explain with neat diagrams the governing methods of a simple steam engine.
3. Explain 'Willan's line' for steam engine.
4. Draw the actual and hypothetical indicator diagram for a steam engine. Establish an expression for theoretical work done per stroke and the mean effective pressure. State any assumptions made.
5. Write the name of parts of a simple steam engine.
6. Draw an expression for indicated work of a double-acting steam engine without clearance volume.
7. What is cylinder condensation. How is it reduced?
8. Compare throttle control governing and cut-off governing.
9. What are limitations of a simple steam engine? State pointwise.
10. Why are steam engines compounded? State the advantages of compounding of steam engines.

Problems

1. The following observations were made during a trial on a single-cylinder, double-acting steam engine:

Bore = 250 mm; Stroke = 200 mm; Effective brake load = 800 N; Brake radius = 50 cm; Steam admission = 7 bar, 200°C; Mean effective pressure = 250 kPa; Speed = 200 rpm; Condensate collected = 200 kg/h; Temperature of condensate = 50°C.

Determine (i) brake power, (ii) mechanical efficiency, (iii) indicated specific steam consumption, and (iv) brake thermal efficiency.

[(i) 8.377 kW, (ii) 51.2%, (iii) 12.22 kg/kWh, (iv) 5.72%]

2. Find the mean effective pressure and work done per stroke of an engine working at an admission pressure of 6 bar and a back pressure of 1.0 bar if the cut-off ratio is 0.5 of the stroke. The cylinder diameter is 20 cm and its stroke is 30 cm. Neglect clearance and compression.

[4.08 bar; 384.3 Nm]

3. Determine the actual mean effective pressure for a reciprocating steam engine which receives steam at 10 bar and exhausts at 0.4 bar. The cut-off is at 0.4 of the stroke with clearance volume as 10% of the stroke. Assume diagram factor = 0.8.

[6.033 bar]

4. If in Problem (3), diameter of the cylinder is 21 cm and stroke length is 30 cm, determine the

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IP. of the engine is running at 150 rpm. Assume the engine to be double acting. *[31.34 kW]*

5. Calculate *IP*, *BP* and mechanical efficiency of a double-acting steam engine whose data as follows:

imep = 2.5 bar; diameter of the cylinder = 24 cm; stroke = 40 cm; speed = 120 rpm; net brake load = 1060 N; brake drum radius = 1 m.

[18.095 kW; 13.32 kW; 73.6%]

6. Find the required cylinder diameter and the length of the stroke for a steam engine to develop 60 kW of indicated power at 150 rpm. The boiler pressure is 10 bar, cut-off at 0.4 of the stroke, back pressure is 1 bar and the diagram factor is 0.7. Neglect clearance and compression and assume the stroke length to be 1.5 times the diameter of the cylinder. *[72.94 cm; 41.91 cm]*

7. A double-acting steam engine is supplied with steam at 9.8 bar and exhausts at a pressure of 1.05 bar. The expansion ratio for the cylinder is 2.5 and it develops 190 kW at a speed of 250 rpm. Determine the diameter and stroke length of the cylinder, if the stroke is 1.3 times its diameter. *[30.97 cm; 40.27 cm]*

8. The following are the readings taken during a trial on a double-acting steam engine.

Stroke = 30 cm; diameter of cylinder = 20 cm; speed = 119 rpm; imep = 8 N/cm^2; mass of fuel supplied = 0.08 kg/min; calorific value of fuel used = 30000 kJ/kg; net brake load = 300 N; steam used = 1.2 kg/min; back pressure = 10.33 N/cm^2; steam admission pressure = 40 N/cm^2, dry-saturated.

Calculate *IP, BP*, mechanical efficiency, indicated thermal efficiency and the overall efficiency of the plant. *[2.99 kW; 2.243 kW; 75.2%; 7.475%; 5.608%]*

Objective Questions

A. Choose the correct answer.

1. The ratio of clearance volume to the swept volume is known as
   (a) expansion ratio (b) cut-off ratio
   (c) compression ratio (d) clearance ratio
2. The ratio of the volume of cut-off to the swept volume is known as
   (a) expansion ratio (b) cut-off ratio
   (c) compression ratio (d) clearance ratio
3. Mean effective pressure is obtained if the work done is divided by
   (a) total volume of the cylinder
   (b) swept volume of the cylinder
   (c) clearance volume of the cylinder
   (d) none of the above
4. Diagram factor is defined as
   (a) $\dfrac{\text{Area of the theroetical indicator diagram}}{\text{Area of the actual indicator diagram}}$
   (b) $\dfrac{\text{area of the actual indicator diagram}}{\text{area of the theoretical indicator diagram}}$
   (c) product of the area of the actual indicator diagram and the theoretical indicator diagram
   (d) none of the above
5. The diagram factor is
   (a) less than unity (b) equal to unity
   (c) greater than unity (d) zero
6. The theoretical mean effective pressure is given by the expression
   (a) $\dfrac{p_1}{r}[1 - \ln(r)] - p_b$
   (b) $\dfrac{p_1}{r}[1 + \ln(r)] - p_b$
   (c) $\dfrac{p_1}{r}[1 + \ln(r)] + p_b$
   (d) $\dfrac{p_1}{r}[1 - \ln - (r)] + p_b$

   where p_1 = initial pressure of steam entering the cylinder
   P_b = back pressure of steam in the cylinder
   r = ratio of expansion of steam

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7. The actual mean effective pressure is equal to
   (a) the theoretical mean effective pressure plus diagram factor
   (b) the theroretical mean effective pressure minus diagram factor
   (c) the theroretical mean effective pressure multiplied by diagram factors
   (d) the theoretical mean effective pressure divided by diagram factor
8. For the same cylinder specifications, speed and initial pressure, and cut-off, the condensing engine develops-in comparison with a non-condensing engine.
   (a) less power (b) more power
   (c) the same power (d) none of the above
9. For the same cylinder specifications, speed, initial pressure, cut-off and back pressure, in comparison with single-acting engine, a double-acting engine develops
   (a) the same power
   (b) 50% more power
   (c) approximately 100% more power
   (d) none of the above
10. The indicator diagram is taken with the help of a/an
    (a) crankshaft (b) connecting rod
    (c) eccentric (d) indicator
11. The steam in a steam engine entrapped in the cylinder after the exhaust valve closes is known as
    (a) cushion steam
    (b) missing quantity
    (c) clearance volume steam
    (d) none of the above
12. The saturation curve is the curve showing the volume of the steam in the cylinder during expansion in that steam engine if the dryness fraction of the steam is
    (a) 0.85 (b) 0.90
    (c) 0.95 (d) unity
13. A steam engine is said to be a compound engine if
    (a) steam enters at the same initial pressure in more than one cylinders placed side by side
    (b) steam exhausts at the same exhaust pressure in more than one cylinder placed side by side
    (c) steam after expanding from one cylinder is admitted into another cylinder to expand further
    (d) none of the above
14. The large expansion ratio in a single cylinder of a steam engine results in
    (a) much condensation of steam
    (b) large stroke of piston
    (c) large temperature range
    (d) all of the above
15. By compounding a steam engine, the temperature range is
    (a) reduced (b) constant
    (c) increased (d) none of above
    (e) receiver type
16. In throttle governing, intake steam
    (a) pressure is varied
    (b) temperature is varied
    (c) volume is varied
    (d) none of the above
17. In cut-off governing, intake steam
    (a) pressure is varied
    (b) temperature is varied
    (c) mass is varied
    (d) none of the above

B. Fill in the blanks.

1. A steam engine works on modified _____ cycle.
2. When steam is admitted from both sides of a steam engine, it is called _____.
3. A double-acting engine of the same size and speed will produce _____ the power as a single-acting engine.
4. A double-acting engine requires _____ to connect the piston rod to the connecting rod.
5. A stuffing box and gland are used to reduce _____ of steam.
6. A fictitious constant pressure acting on the piston to produce the same amount of work as in the actual engine is called _____.
7. The expansion in the steam engine follows _____ law.
8. The popularly used steam valve is called _____.
9. The pressure at which the cylinder is connected to the exhaust is called _____.

6. The reduction in exhaust pressure will _____ the engine efficiency.
7. Throttle governing will cause an _____ in steam consumption.
8. If a well-designed governor is used, the engine speed will _____.
9. The cylinder temperature will _____ due to steam jacketing.
10. Cylinder condensation _____ due to steam jacketing.
11. A fly wheel _____ the cyclic torque variation.
12. The power developed _____ as speed is decreased, other factors remaining unchanged.
13. Steam consumption _____ due to clearance.
14. The MEP _____ if cylinder condensation increases.
15. The diagram factor _____ when cylinder condensation is reduced.

10. If the exhaust port is closed before the dead centre then the steam will be _____.
11. In order to have exhaust pressure below atmospheric pressure, a _____ is used.
12. Steam jacketing is used to reduce cylinder _____.
13. The ratio of actual mep to the theoretical MEP is called _____.

C. Answer using 'increases', 'decreases' or 'remains constant'.

1. If cut-off is decreased, the IP will _____.
2. If cut-off is decreased, the steam consumption will _____.
3. If the power developed is larger than the power required for the driven machine, the engine speed will _____.
4. If cut-off is increased, the efficiency will _____.
5. The reduction in exhaust pressure will _____ the power developed.

Answers A

1. (d) 2. (b) 3. (b) 4. (b) 5. (a) 6. (b) 7. (c) 8. (b)
9. (c) 10. (d) 11. (c) 12. (d) 13. (c) 14. (d) 15. (a) 16. (a)
17. (c)

Answers B

1. Rankine 2. double acting 3. twice
4. cross head and guide 5. leakage 6. mean effective pressure
7. hyperbolic 8. *D*-slide valve 9. release pressure
10. compressed 11. condenser 12. condensation
13. diagram factor

Answers C

Increases: 3, 5, 6, 7, 9, 13, 15
Decreases: 1, 2, 4, 10, 11, 12, 14
Remain constant: 8

21

Steam Nozzles

Introduction

A nozzle is a device of varying cross-sectional area, in which the pressure energy of fluid is converted into kinetic energy. The mass of steam passing through any section of the nozzle remains constant. When fluid passes through the nozzle, the pressure of fluid decreases with increase in velocity. Nozzles are used in stream turbines, gas turbines, Jet engines, flow measuring devices, fuel injection and carburetion systems of IC engines, spray paintings, etc.

21.1 TYPES OF STEAM NOZZLES

The steam nozzles are classified as (i) convergent nozzles, (ii) divergent nozzles, and (iii) convergent–divergent nozzles.

21.1.1 Convergent Nozzle

A convergent nozzle is shown in Fig. 21.1 Its cross-sectional area decreases continuously from its entrance to exit. It is used, when back pressure is equal to or greater then critical pressure.

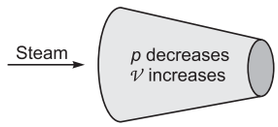

Fig. 21.1 *Convergent nozzle*

21.1.2 Divergent Nozzle

A divergent nozzle is shown in Fig. 21.2. Its cross-sectional area increases continuously from its entrance to exit. It is used when the back pressure is less than critical pressure.

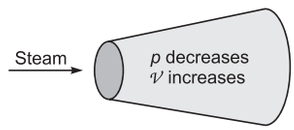

Fig. 21.2 *Divergent nozzle*

21.1.3 Convergent–divergent Nozzle

A convergent–divergent nozzle is shown in Fig. 21.3. Its cross sectional area first decreases from its entrance to throat and then increases from throat to exit. The *throat* is a portion of the nozzle

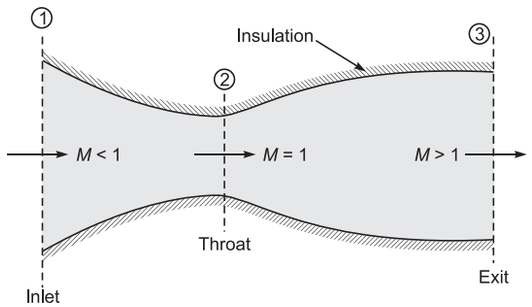

Fig. 21.3 *Converging–diverging nozzle*

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which has smallest cross-section. The convergent–divergent nozzle is used when back pressure is less than the critical pressure. It is widely used in steam and gas turbines.

21.2 STEAM FLOW THROUGH A NOZZLE

Steam flow through a nozzle is considered adiabatic, since during the expansion of steam in a nozzle, neither heat is supplied to the nozzle nor heat is rejected from the nozzle. However, the work is performed by increasing kinetic energy of steam.

21.2.1 Velocity of Steam

The mass-flow rate of steam passing through a nozzle is considered constant with respect to time. The mass-flow rate ($\dot{m}_s$) of any section of nozzle can be expressed as

$$\dot{m}_s = \frac{A_c \mathcal{V}}{v} \qquad \ldots(21.1)$$

where A_c = Cross-sectional area of nozzle, m^2
$\mathcal{V}$ = Velocity of steam, m/s
v = Specific volume of steam, m^3/kg

Accordingly, the velocity of steam at the entrance

$$\mathcal{V}_1 = \frac{\dot{m}_s v_1}{A_1} = \frac{4\dot{m}_s v_1}{\pi d_1^2} \qquad \ldots(21.2)$$

and velocity of steam at the exit

$$\mathcal{V}_2 = \frac{\dot{m}_s v_2}{A_2} = \frac{4\dot{m}_s v_2}{\pi d_2^2} \qquad \ldots(21.3)$$

Consider isentropic flow of steam through a nozzle as shown in Fig. 21.4. At the inlet of the nozzle, superheated steam at high pressure enters the nozzle with negligible velocity. As steam flows in the nozzle, the steam pressure decreases with increase in its velocity. At the same time, the enthalpy of steam also decreases. This enthalpy drop is utilised for converting potential energy into kinetic energy.

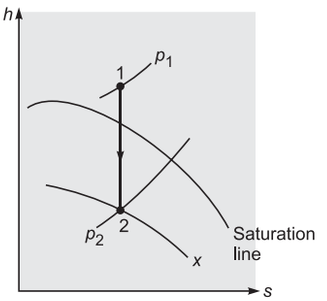

Fig. 21.4 *Isentropic flow through a nozzle*

The following point should be noted for flow through the nozzle.

$\dot{Q}$ = 0, there is no heat transfer.

$\dot{W}$ =0, no work interaction is involved during flow through nozzle.

Δpe = 0, fluid experiences usually very little or no change in its elevation between inlet and outlet.

Using steady-flow energy equation, we get,

$$h_2 - h_1 + \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} = 0 \qquad \ldots(21.4)$$

where $\mathcal{V}_1$ = Inlet velocity of steam, m/s
$\mathcal{V}_2$ = Exit velocity of steam, m/s
h_1 = Enthalpy of steam at nozzle entrance, J/kg
h_2 = Enthalpy of steam at nozzle exit, J/kg.

Using Eq. (21.4), the velocity of steam at the nozzle exit can be expressed as

$$\mathcal{V}_2 = \sqrt{\mathcal{V}_1^2 + 2(h_1 - h_2)} \qquad \ldots(21.5)$$

Normally, the **velocity of steam at nozzle entrance is very small as compared to velocity of steam at nozzle exit, hence entrance velocity $\mathcal{V}_1$ is neglected.** Thus, Eq. (21.5) is modified as

$$\mathcal{V}_2 = \sqrt{2(h_1 - h_2)} \qquad \ldots(21.6)$$

When the magnitude of enthalpies of steam is used in kJ/kg (as directly obtained from steam tables) then

$$\mathcal{V}_2 = \sqrt{2 \times 1000 \times (h_1 - h_2)}$$

$$= 44.72 \times \sqrt{(h_1 - h_2)} \text{ (m/s)} \qquad \ldots(21.7)$$

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21.2.2 Discharge Through an Isentropic Nozzle

Since the fluid flow through the nozzle is isentropic, therefore, the pressure and specific volume of steam are related as

$$pv^n = \text{C (constant)}$$

where n is the index of expansion.

For isentropic flow through the nozzle, the enthalpy drop $(h_1 - h_2)$ is equal to $-\int vdp$.

Therefore, the kinetic energy gain when velocity of approach is negligible;

$$\frac{\mathcal{V}_2^2}{2} = -\int vdp = \frac{n}{n-1}(p_1v_1 - p_2v_2)$$

$$= \frac{n}{n-1}p_1v_1\left(1 - \frac{p_2v_2}{p_1v_1}\right)$$

For isentropic flow

$$\frac{v_2}{v_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = \left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}} \quad \ldots(21.8)$$

then $$\frac{\mathcal{V}_2^2}{2} = \frac{n}{n-1}p_1v_1\left[1 - \frac{p_2}{p_1}\left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}}\right]$$

$$= \frac{n}{n-1}p_1v_1\left[1 - \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}}\right]$$

or $$\mathcal{V}_2 = \sqrt{\frac{2n}{n-1}p_1v_1\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]} \quad \ldots(21.9)$$

Using in continuity equation, the mass flow rate of steam through the nozzle

$$\dot{m}_s = \frac{A_2\mathcal{V}_2}{v_2}$$

$$= \frac{A_2}{v_2}\sqrt{\frac{2n}{n-1}p_1v_1\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]}$$

where A_2 = cross-sectional area, m^2, and v_2 = specific volume of steam, m^3/kg. Using v_2 from Eq. (21.8), we get

$$\dot{m}_s = \frac{A_2}{v_1\left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}}}\sqrt{\frac{2n}{n-1}p_1v_1\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]}$$

$$= \frac{A_2}{v_1}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}\sqrt{\frac{2n}{n-1}p_1v_1\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]}$$

$$= A_2\sqrt{\frac{2n}{n-1}\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} \times \frac{p_1v_1}{v_1^2}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]}$$

or $$\dot{m}_s = A_2\sqrt{\frac{2n}{n-1}\frac{p_1}{v_1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right]}$$

$\ldots(21.10)$

It is an expression for mass-flow rate of steam at the nozzle exit.

21.2.3 Condition for Maximum Discharge

Equation (21.10) gives the mass-flow rate of steam through a nozzle. It is evident that the mass-flow rate remains same throughout its cross-section.

The mass-flow rate of steam will be maximum at the throat, where the cross section A_2 is minimum. The value of pressure ratio $\frac{p_2}{p_1}$ at the throat is called the critical pressure ratio. For maximum discharge rate, differentiating Eq. (21.10) with respect to pressure ratio $\frac{p_2}{p_1}$ at throat and equating it to zero;

$$\frac{d\dot{m}}{d\left(p_2/p_1\right)} = 0$$

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$$0 = \frac{d}{d\left(p_2 / p_1\right)} \left\{ A_2 \sqrt{\frac{2n}{n-1} \frac{p_1}{v_1} \left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}} \right]} \right\}$$

The quantities A_2, p_1, v_1, n are constant with respect to $\frac{p_2}{p_1}$. Thus, we can write above equation as

$$0 = \frac{d}{d\left(p_2 / p_1\right)} \left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}} \right]$$

$$0 = \frac{2}{n} \left(\frac{p_2}{p_1}\right)^{\frac{2}{n}-1} - \frac{n+1}{n} \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}-1}$$

or $$2 \left(\frac{p_2}{p_1}\right)^{\frac{2-n}{n}} = (n+1) \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

or $$\left(\frac{p_2}{p_1}\right)^{\frac{2-n}{n}} \left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}} = \frac{n+1}{2}$$

or $$\left(\frac{p_2}{p_1}\right)^{\frac{1-n}{n}} = \frac{n+1}{2}$$

or $$\frac{p_2}{p_1} = \left(\frac{n+1}{2}\right)^{\frac{n}{1-n}} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}} \quad \ldots(21.11)$$

This pressure ratio is called critical *pressure ratio*, and the pressure p_2 at the throat is called *critical pressure*.

Substituting critical pressure in the expression of mass flow, Eq. (21.10), we get the maximum mass discharged through the nozzle

$$\dot{m}_s = A_2 \sqrt{\frac{2n}{n-1} \frac{p_1}{v_1}} \sqrt{\left[\left(\frac{2}{n+1}\right)^{\frac{n}{n-1} \times \frac{2}{n}} - \left(\frac{2}{n+1}\right)^{\frac{n}{n-1} \times \frac{n+1}{n}} \right]}$$

$$= A_2 \sqrt{\frac{2\,n}{n-1} \frac{p_1}{v_1} \left[\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}} - \left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}} \right]}$$

$$= A_2 \sqrt{\frac{2n}{n-1} \times \frac{p_1}{v_1} \times \left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}} \times \sqrt{\left[1 - \left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1} - \frac{2}{n-1}} \right]}$$

$$= A_2 \sqrt{\frac{2n}{n-1} \times \frac{p_1}{v_1} \times \left(\frac{2}{n+1}\right)^{\frac{2}{n-1}} \left[1 - \left(\frac{2}{n+1}\right)^{\frac{n-1}{n-1}} \right]}$$

$$= A_2 \sqrt{\frac{2n}{n-1} \times \frac{p_1}{v_1} \times \left(\frac{2}{n+1}\right)^{\frac{2}{n-1}} \left[1 - \left(\frac{2}{n+1}\right) \right]}$$

$$= A_2 \sqrt{\frac{2n}{n-1} \times \frac{p_1}{v_1} \times \left(\frac{2}{n+1}\right)^{\frac{2}{n-1}} \left[\frac{n+1-2}{n+1} \right]}$$

$$\dot{m}_{max} = A_2 \sqrt{\frac{2n}{n+1} \times \frac{p_1}{v_1} \times \left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}} \quad \ldots(21.12)$$

It is evident from Eq. (21.12), that the maximum mass flow rate of steam depends on initial state (p_1, v_1) of steam and throat area, A_2 and is independent of the final state of steam. Therefore, the diverging portion of the nozzle does not affect the mass-flow rate, but it accelerates the steam leaving the nozzle.

21.2.4 Critical Velocity

The velocity of steam at the throat corresponding to maximum mass flow rate can be obtained by using critical pressure ratio in Eq. (21.9).

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$$\mathcal{V}_{max} = \sqrt{\frac{2n}{n-1} p_1 v_1 \left[1 - \left(\frac{2}{n+1}\right)^{\frac{n}{n-1} \times \frac{n-1}{n}}\right]}$$

$$= \sqrt{\frac{2n}{n-1} p_1 v_1 \left[\frac{n+1-2}{n+1}\right]}$$

or $$\mathcal{V}_{max} = \sqrt{\frac{2n}{n+1} p_1 v_1} \qquad \ldots(21.13)$$

21.2.5 Critical Pressure Ratio

Generally, the pressure at the throat is designated as p^*. The *critical pressure ratio* is defined as the pressure ratio which maximizes the mass flow rate of steam to be discharged by the nozzle.

$$\frac{p^*}{p_1} = \frac{\text{Throat pressure}}{\text{Inlet pressure}} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$$

The values of critical pressures for different values of expansion index n are calculated below:

1. Saturated Steam When the steam is dry saturated at the nozzle inlet, the index of expansion is taken as 1.135;

$$\therefore \quad \frac{p^*}{p_1} = \left(\frac{2}{1.135+1}\right)^{\frac{1.135}{1.135-1}} = \mathbf{0.577} \qquad \ldots(21.14)$$

2. Superheated Steam When the steam is superheated at the nozzle inlet, $n = 1.30$.

$$\therefore \quad \frac{p^*}{p_1} = \left(\frac{2}{1.30+1}\right)^{\frac{1.30}{1.30-1}} = \mathbf{0.545} \qquad \ldots(21.15)$$

3. Wet Steam For wet stream at the nozzle inlet, the index of expansion is considered as $n = 1.113$.

$$\therefore \quad \frac{p^*}{p_1} = \left(\frac{2}{1.113+1}\right)^{\frac{1.113}{1.113-1}} = \mathbf{0.582} \qquad \ldots(21.16)$$

4. Air When air enters a nozzle inlet, the index of expansion is considered as $\gamma = 1.4$ and

$$\frac{p^*}{p_1} = \left(\frac{2}{1.4+1}\right)^{\frac{1.4}{1.4-1}} = \mathbf{0.528}$$

21.2.6 Significance of Critical Pressure Ratio

Consider a nozzle attached to a large reservoir as shown in Fig. 21.5. The reservoir contains steam at high and steady pressure p_1. Since the steam velocity at the inlet to nozzle is relatively small, thus it is considered negligible. The pressure at the exit plane of the nozzle is the back pressure p_b.

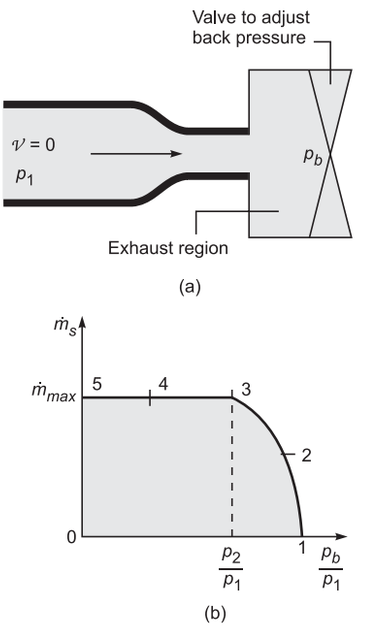

Fig. 21.5 *Effect of back pressure on mass-flow rate*

Let the pressure at the exit plane vary gradually. Its effects are the following:

1. When back pressure $p_b = p_1$, there is no mass flow $\dot{m}_s = 0$, through the nozzle. This corresponds to Case 1 of Fig. 21.5(b).
2. If the back pressure p_b is reduced to the state 2, it causes the pressure to decrease in flow direction which results in certain mass-flow rate of steam through the nozzle. Further, decrease in back pressure results in greater mass-flow rate.
3. If the back pressure p_b is further reduced, the mass-flow rate reaches its minimum possible value. This case is represented by state 3 on

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Fig. 21.5(b). At this state, the pressure is called *critical pressure* and the velocity of flow is called *sonic velocity*.

4. If the back pressure is further reduced to states 4 and 5, less than p_2, neither the mass flow rate nor flow velocity in the nozzle is affected and the nozzle is said to be *choked*. For a choked nozzle, the mass-flow rate reaches the maximum possible value for given conditions.

For a convergent–divergent nozzle, the sonic velocity reaches at the throat, where cross-section is minimum. The flow is subsonic in convergent portion, and supersonic in the divergent portion of the nozzle.

21.3 FLOW THROUGH ACTUAL NOZZLES

21.3.1 Effect of Irreversibilities on Nozzle Efficiency

In the preceeding section of this chapter, the flow through the nozzle is approximated to be isentropic. Actual expansion through nozzles is non-isentropic flow. It is due to presence of irreversibility at the surface of flow and within the fluid itself. The primary cause of irreversibility in nozzles is the presence of frictional effects, which are due to

(i) friction between fluid and wall surface of nozzle, and
(ii) friction within the fluid itself.

The friction between the wall surface and fluid molecules makes the expansion adiabatic but not isentropic. The energy lost in overcoming the friction is used to reheat the steam and the enthalpy and entropy of steam increases during the process.

The convergent portion of the nozzle is smaller than the divergent portion. Thus, the wall friction is small in the convergent portion as compared to the divergent portion. The fluid friction is also small in convergent portion than in the divergent portion, since the fluid velocity in the convergent portion is small. Thus, *most of the friction occurs in the divergent portion of the nozzle* and *h–s* diagram takes the shape as shown in Fig. 21.6. The *nozzle efficiency* for such a nozzle is defined as the ratio of actual enthalpy drop to the isentropic enthalpy drop

$$\eta_N = \frac{h_1 - h_3}{h_1 - h_{3s}} \qquad \ldots(21.17)$$

where h_1 is the enthalpy of steam at the nozzle inlet and h_3 is the enthalpy of fluid at the exit for the actual nozzle, while h_{3s} is the exit enthalpy for a nozzle under isentropic conditions. The nozzle efficiency varies from 90% to 99%. The larger nozzles have more efficiency than the smaller nozzles.

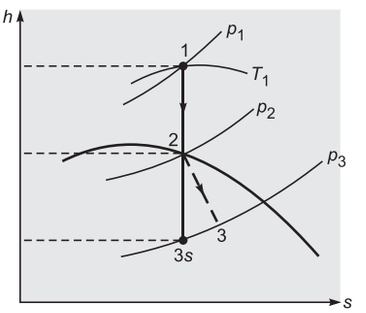

Fig. 21.6 *Effect of friction in divergent portion of nozzle*

The nozzle efficiency can also be defined in terms of actual kinetic energy and kinetic energy corresponds to isentropic flow at nozzle exit as

$$\eta_N = \frac{\text{Actual kinetic energy at nozzle exit}}{\text{Kinetic energy at nozzle exit for isentropic flow at same exit pressure}}$$

$$= \frac{\mathcal{V}_3^2}{\mathcal{V}_{3s}^2}$$

Another cause of irreversibility in the nozzles is flow separation, which induces strong turbulence near the nozzle wall. Flow separation occurs when the angle of divergence in a convergent–divergent nozzle is made too large. Consequently, the flow area increases faster than the fluid expands. Thus, the included (cone) angle of the divergent duct is usually kept less than 20°C.

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The frictional losses in the nozzle depends upon material of construction, size, shape and surface because the wall surface in the large nozzles occupies a smaller portion of total flow volume.

The *effect of friction in a nozzle* can be summarized as

(i) Reduction in enthalpy drop,
(ii) Reheating of fluid,
(iii) Reduction in exit velocity,
(iv) Increase in sepecific volume,
(v) Decrease in mass flow rate.

21.3.2 Actual Velocity

Actual flow velocity of steam through a nozzle is

$$\mathcal{V}_2 = 44.72 \times \sqrt{k(h_1 - h_2)} \text{ (m/s)} \quad \ldots(21.18)$$

where k is called friction coefficient of the nozzle.

21.3.3 Nozzle Velocity Coefficient

The *nozzle velocity coefficient* ($C_{\mathcal{V}}$) is also an important parameter and can be expressed as

$$C_{\mathcal{V}} = \frac{\text{Actual velocity at the nozzle exit}}{\text{Velocity at the nozzle exit with isentropic flow and same exit pressure}}$$

$$= \frac{\mathcal{V}_3}{\mathcal{V}_{3s}} = \sqrt{\eta_N} \quad \ldots(21.19)$$

It follows that the velocity coefficient is equal to the square root of the nozzle efficiency;

The mass-flow rate through the nozzle is the design consideration, because it is affected by irreversibilities. An important parameter, coefficient of discharge (C_D), relates the actual mass flow with mass flow under the isentropic conditions for the same nozzle.

$$C_D = \frac{\text{Actual mass-flow rate}}{\text{mass-flow rate with esentropic flow}}$$

$$= \frac{\dot{m}}{\dot{m}_s} \quad \ldots(21.20)$$

Example 21.1 *A nozzle is to be designed to expand steam at the rate of 0.10 kg/s from 500 kPa, 210°C to 100 kPa. Neglect inlet velocity of steam. For a nozzle efficiency of 0.9, determine the exit area of the nozzle.*

Solution

Given Expansion of steam through a nozzle

$\dot{m} = 0.1$ kg/s
$p_1 = 500$ kPa
$T_1 = 210°C = 483$ K
$p_2 = 100$ kPa
$\mathcal{V}_1 = 0$
$\eta_N = 0.9$

To find The exit area of the nozzle.

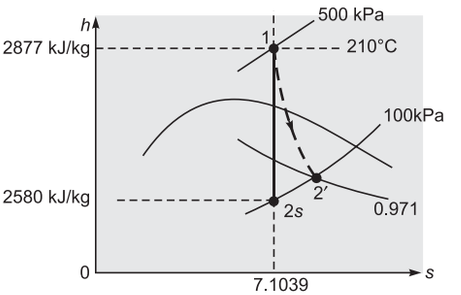

Fig. 21.7 *h-s diagram*

Assumption $\Delta pe = 0$

Analysis From Mollier diagram, the properties of steam

State 1: Superheated steam at 500 kPa and 210°C,

$h_1 = 2877$ kJ/kg $\quad s_1 = 7.1039$ kJ/kg·K

After isentropic expansion, $s_1 = s_2$

State 2: Wet steam; $p_2 = 100$ kPa

$h_{2s} = 2580$ kJ/kg $\quad s_{2s} = 7.1039$ kJ/kg·K

Isentropic enthalpy drop in nozzle

$\Delta h_{isentropic} = h_1 - h_{2s} = 2877 - 2580 = 297$ kJ/kg

Actual enthalpy drop;

$\Delta h_{act} = \eta_N \times \Delta h_{isentropic}$
$= 0.9 \times 297 = 267.3$ kJ/kg
$= 267.3 \times 10^3$ J/kg

Actual enthalpy at the state 2;

$h_2 = h_1 - \Delta h_{act}$
$= 2877 - 267.3 = 2609.7$ kJ/kg

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On the Mollier chart, locate the intersection point of the pressure of 100 kPa and specific enthalpy of 2609.7 kJ/kg. At this point, we get

$$x_2 = 0.971 \quad \text{and} \quad v_2 = 1.645 \text{ m}^3/\text{kg}$$

$$\text{Steam velocity } \mathcal{V}_2 = 44.72 \times \sqrt{\Delta h_{act}}$$

$$= 44.72 \times \sqrt{267.3} = 731.14 \text{ m/s}$$

$$A_2 = \frac{\dot{m} v_2}{\mathcal{V}_2} = \frac{0.10 \times 1.645}{731.14}$$

$$= \mathbf{0.225 \times 10^{-3} \text{ m}^2}$$

Example 21.2 *Calculate the critical pressure and throat area per unit mass-flow rate of steam, expanding through a convergent–divergent nozzle from 10 bar, dry saturated, down to atmospheric pressure of 1 bar. Assume that the inlet velcocity is negligible and that the expasion is isentropic.*

Solution

Given Isentropic expansion of dry saturated steam though convergent–divergent nozzle

$\mathcal{V}_1 = 0$ $\qquad$ $p_1 = 10$ bar

$\dot{m}_s = 1$ kg/s $\qquad$ $p_3 = 1$ bar, $s = C$

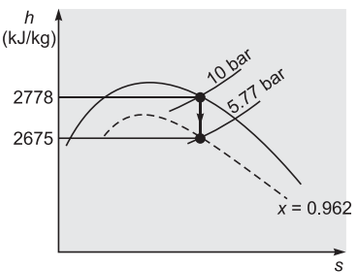

Fig. 21.8

To find

(i) Critical pressure, and

(ii) Throat area.

Assumptions

(i) The expansion is in thermal equilibrium.

(ii) Frictionless flow of steam through nozzle.

(iii) Isentropic index for expansion of dry saturated steam is 1.135.

Analysis

(i) *Critical pressure*

$$\frac{p^*}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$$

or $$p_2 = p^* = 10 \times \left(\frac{2}{1.135+1}\right)^{\frac{1.35}{1.35-1}}$$

$$= \mathbf{5.77 \text{ bar}}$$

(ii) *Throat area*

From Mollier chart, at 5.77 bar after isentropic expansion

$$h_2 = 2675 \text{ kJ/kg} \quad x_2 = 0.962$$

$$v_{g@\,5.77\,\text{bar}} = 0.328 \text{ m}^3/\text{kg (from steam table)}$$

Specific volume at throat;

$$v_2 = x_2 v_g = 0.962 \times 0.328$$

$$= 0.316 \text{ m}^3/\text{kg}$$

The exit velocity of steam

$$\mathcal{V}_2 = 44.72 \times \sqrt{(h_1 \quad h_2)}$$

$$= 44.72 \times \sqrt{2778 - 2675}$$

$$= 453.87 \text{ m/s}$$

Using continuity equation

$$A_2 = \frac{\dot{m}_s v_2}{\mathcal{V}_2} = \frac{1 \times 0.316}{453.87}$$

$$= 6.96 \times 10^{-4} \text{ m}^2 = \mathbf{696 \text{ mm}^2}$$

Example 21.3 *Steam enters a convergent–divergent nozzle at 2 MPa and 400°C with a negligible velocity and mass-flow rate of 2.5 kg/s and it exits at a pressure of 300 kPa. The flow is isentropic between the nozzle entrance and throat and overall nozzle efficiency is 93 per cent. Determine (a) throat, and (b) exit areas.*

Solution

Given Expansion of superheated steam through a convergent–divergent nozzle.

$$\dot{m}_s = 2.5 \text{ kg/s} \quad \eta_N = 0.93$$

Inlet conditions Superheated steam;

$$p_1 = 2 \text{ MPa} = 2000 \text{ kPa}$$

$$T_1 = 400°\text{C}, \quad \mathcal{V}_1 = 0$$

Exit conditions

$$p_2 = 300 \text{ kPa}$$

To find

(i) Throat area of nozzle, and

(ii) Exit area of nozzle.

Analysis

(i) *Throat area of nozzle*

The critical pressure ratio for superheated steam;

$$\frac{p_2}{p_1} = 0.546$$

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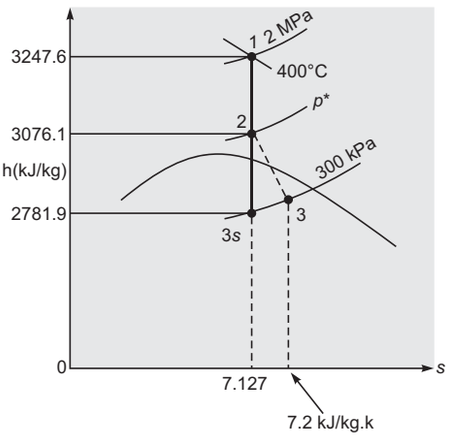

Fig. 21.9

or $p_2 = 0.546 \times 2000 = 1092$ kPa

The steam properties;

Inlet $p_1 = 2$ MPa

$T_1 = 400°C$

$h_1 = 3247.6$ kJ/kg

$s_1 = 7.127$ kJ/kg·K

Throat $p_2 = 1092$ kPa

$s_1 = s_2 = 7.127$ kJ/kg·K

$h_2 = 3076.1$ kJ/kg

$v_2 = 0.242$ m^3/kg

The velocity of steam at throat

$$\mathcal{V}_2 = 44.72 \times \sqrt{h_1 - h_2}$$
$$= 44.72 \times \sqrt{3247.6 - 3076.1}$$
$$= 585.66 \text{ m/s}$$

Throat area

$$A_2 = \frac{\dot{m}_s v_2}{\mathcal{V}_2} = \frac{2.5 \times 0.242}{585.66}$$
$$= 1.033 \times 10^{-3} \text{ m}^2 = \mathbf{10.33 \text{ cm}^2}$$

(ii) *Exit* Isentropic state 3*s*

$p_3 = 300$ kPa

$h_{3s} = 2781.9$ kJ/kg $\quad s_2 = s_{3s}$

The enthalpy of steam at the actual exit state 3 is determined by using nozzle efficiency

$$\eta_N = \frac{h_1 - h_3}{h_1 - h_{3s}}$$

or $h_3 = h_1 - \eta_N (h_1 - h_{3s})$

$= 3247.6 - 0.93 \times (3247.6 - 2781.9)$

$= 2814.5$ kJ/kg

Therefore, at actual exit state, the properties are

$p_3 = 300$ kPa $\quad h_3 = 2814.5$ kJ/kg

$v_3 = 0.6764$ m^3/kg

The velocity at nozzle exit

$$\mathcal{V}_3 = 44.72 \times \sqrt{h_1 - h_3}$$
$$= 44.72 \times \sqrt{3247.6 - 2814.5}$$
$$= 930.7 \text{ m/s}$$

And the exit area of nozzle

$$A_3 = \frac{\dot{m}_s v_3}{\mathcal{V}_3} = \frac{2.5 \times 0.6764}{930.7}$$
$$= 1.817 \times 10^{-3} \text{ m}^2 = \mathbf{18.17 \text{ cm}^2}$$

Example 21.4 *In a convergent–divergent nozzle, the steam enters at 15 bar and 300°C and leaves at a pressure of 2 bar. The inlet velocity to the nozzle is 150 m/s. Find the required throat and exit areas for mass-flow rate of 1 kg/s. Assume nozzle efficiency to be 90 per cent and $C_{ps} = 2.4$ kJ/kg·K*

Solution

Given Expansion of superheated steam through a convergent–divergent nozzle.

$\dot{m}_s = 1$ kg/s, $\quad \eta_N = 0.90$, $\quad C_{ps} = 2.4$ kJ/kg·K

Inlet conditions Superheated steam;

$p_1 = 15$ bar $= 1500$ kPa, $\quad T_1 = 300°C$

$\mathcal{V}_1 = 150$ m/s

Exit conditions

$p_3 = 2$ bar $= 200$ kPa

To find

(i) Throat area of nozzle,

(ii) Exit area of nozzle.

Schematic with given data

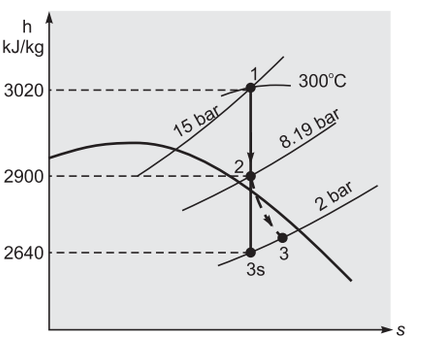

Fig. 21.10

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Analysis

(i) *Throat area of nozzle*

The critical pressure for superheated steam

$$\frac{p_2}{p_1} = 0.546$$

or $p_2 = 0.546 \times 15 \text{ bar} = 8.19 \text{ bar}$

The steam properties

Inlet $p_1 = 15$ bar, $T_1 = 300°C$,

$h_1 = 3020$ kJ/kg

Throat $p_2 = 8.19$ bar, $s_1 = s_2 = 7.127$ kJ/kg·K

$h_2 = 2900$ kJ/kg

$v_2 = 0.3 \text{ m}^3/\text{kg}$

Exit state Isentropic state 3*s*, $p_3 = 2$ bar

$h_{3s} = 2640$ kJ/kg $s_1 = s_{3s}$

Since steam enters the nozzle with a velocoty, therefore, the stagnation enthalpy at inlet

$$h_{01} = h_1 + \frac{\mathcal{V}_1^2}{2}$$

$$= (3020 \text{ kJ/kg}) + \frac{(150)^2}{2000} \text{ (kJ/kg)}$$

$$= 3031.25 \text{ kJ/kg}$$

The velocity of steam at throat

$$\mathcal{V}_2 = 44.72 \times \sqrt{h_{01} - h_2}$$

$$= 44.72 \times \sqrt{3031.25 \quad 2900}$$

$$= 512.34 \text{ m/s}$$

Throat area;

$$A_2 = \frac{\dot{m}_s v_2}{\mathcal{V}_2} = \frac{1 \times 0.3}{512.34}$$

$$= 5.855 \times 10^{-4} \text{ m}^2 = \mathbf{585.5 \text{ mm}^2}$$

The enthalpy of steam at the actual exit state 3 is determined by using nozzle efficiency

$$\eta_N = \frac{h_{01} - h_3}{h_{01} - h_{3s}}$$

or $h_{01} - h_3 = \eta_N (h_{01} - h_{3s})$

$$= 0.93 \times (3031.25 - 2640)$$

$$= 363.86 \text{ kJ/kg}$$

and $h_3 = 3031.25 - 363.86$

$$= 2667.38 \text{ kJ/kg}$$

The specific volume at the intersection point of pressure of 2 bar and specific enthalpy of 2667.38 kJ/kg is

$v_3 = 0.85 \text{ m}^3/\text{kg}$ (From Mollier chart)

The velocity at nozzle exit

$$\mathcal{V}_3 = 44.72 \times \sqrt{h_{01} - h_3}$$

$$= 44.72 \times \sqrt{3031.25 - 2667.38}$$

$$= 963.15 \text{ m/s}$$

And the exit area of nozzle

$$A_3 = \frac{\dot{m}_s v_3}{\mathcal{V}_3} = \frac{1 \times 0.85}{963.15}$$

$$= 8.82 \times 10^{-4} \text{ m}^2 = \mathbf{882 \text{ mm}^2}$$

Example 21.5 *Calculate the throat and exit diameters of a convergent–divergent nozzle, which will discharge 820 kg of steam per hour at a pressure of 8 bar superheated to 220°C into a chamber having a pressure of 1.5 bar. The friction loss in the divergent portion of the nozzle may be taken as 0.15 of the isentropic enthalpy drop.*

Solution

Given Expansion of superheated steam through a convergent–divergent nozzle.

$\dot{m}_s = 820 \text{ kg/h} = 0.2278 \text{ kg/s}$

Inlet conditions: Superheated steam;

$p_1 = 8$ bar $T_1 = 220°C$

Exit conditions: $p_3 = 1.5$ bar $\Delta h_f = 0.15 \, \Delta h_{2-3}$

To find

(i) Throat area of nozzle, and

(ii) Exit area of nozzle.

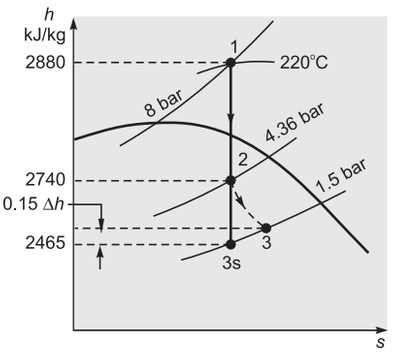

Fig. 21.11

Assumptions

(i) Inlet velocity of fluid is negligible.

(ii) No change in potential energy.

(iii) Flow of fluid in convergent portion is isentropic.

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Analysis The critical pressure ratio for expansion of superheated steam is

$$\frac{p^*}{p_1} = 0.546 \qquad \text{when } k = 1.3$$

The critical pressure $p^* = 0.546 \times 8$ bar $= 4.368$ bar

The critical pressure is greater than the exit pressure of steam, thus expansion will continue in divergent portion also.

The steam properties;

Inlet: $p_1 = 8$ bar $\quad T_1 = 220°C$;
$h_1 = 2880$ kJ/kg

Throat: $p_2 = 4.36$ bar, $\quad s_1 = s_2 = 7.127$ kJ/kg·K
$h_2 = 2740$ kJ/kg, $\quad v_2 = 0.435$ m^3/kg

Exit state: Isentropic state 3*s*
$p_3 = 1.5$ bar $\quad h_{3s} = 2465$ kJ/kg, $x = 0.94$

(i) *Throat area of nozzle*

The velocity of steam at throat

$$\mathcal{V}_2 = 44.72 \times \sqrt{h_1 - h_2}$$
$$= 44.72 \times \sqrt{2880 - 2740} = 529.13 \text{ m/s}$$

Throat area;

$$A_2 = \frac{\dot{m}_s v_2}{\mathcal{V}_2} = \frac{0.2278 \times 0.435}{529.13}$$
$$= 1.87 \times 10^{-4} \text{ m}^2 = \mathbf{187.2 \text{ mm}^2}$$

The enthalpy of steam at actual exit state 3 ($p_3 = 1.5$ bar)

$$h_3 = h_{3s} + \Delta h_f$$
$$= 2465 + 0.15 \times (2740 - 2465)$$
$$= 2506.25 \text{ kJ/kg}$$

The specific volume at intersection point of pressure of 1.5 bar and specific enthalpy of 2506.25 kJ/kg is

$v_3 = 1.16$ m^3/kg (From Mollier chart)

The velocity at nozzle exit

$$\mathcal{V}_3 = 44.72 \times \sqrt{h_1 - h_3}$$
$$= 44.72 \times \sqrt{2880 - 2506.25}$$
$$= 864.55 \text{ m/s}$$

(ii) *Exit area of nozzle*

$$A_3 = \frac{\dot{m} v_3}{\mathcal{V}_3} = \frac{0.2278 \times 1.16}{864.55}$$
$$= 3.0564 \times 10^{-4} \text{ m}^2 = \mathbf{305.64 \text{ mm}^2}$$

Example 21.6 *A convergent–divergent nozzle is required to discharge 2 kg of steam per second. The nozzle is supplied with the steam at 7 bar and 180°C and discharge takes place against the back pressure of 1 bar. The expansion up to throat is isentropic and the frictional resistance between throat and exit is equivalent to 63 kJ/kg of steam. The approach velocity to the nozzle is 75 m/s and throat pressure is 4 bar. Estimate*

(a) Suitable areas of throat and exit,

(b) Overall efficiency of the nozzle based on enthalpy drop between inlet pressure, temperature and exit pressure.

Solution

Given Expansion of superheated steam through a convergent–divergent nozzle.

$\dot{m}_s = 2$ kg/s

Inlet conditions: Superheated steam;
$p_1 = 7$ bar, $\quad T_1 = 180°C$, $\quad \mathcal{V}_1 = 75$ m/s

Throat condition: $p_2 = 4$ bar, $\quad s_1 = s_2$

Exit conditions: $p_3 = 1$ bar, $\quad \Delta h_f = 63$ kJ/kg

To find

(i) Throat area of nozzle,

(ii) Exit area of nozzle, and

(iii) Overall efficiency of the nozzle.

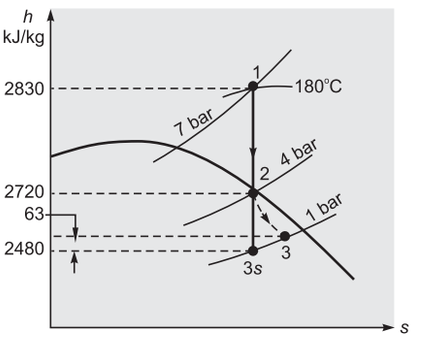

Fig. 21.12

Analysis

(i) *Throat area of nozzle*

The steam properties

Inlet: $p_1 = 7$ bar $\quad T_1 = 180°C$
$h_1 = 2830$ kJ/kg

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Throat: $p_2 = 4$ bar $\quad s_1 = s_2 = 7.127$ kJ/kg·K

$h_2 = 2720$ kJ/kg $\quad v_2 = 0.48$ m^3/kg

Exit state: Isentropic state 3*s*

$p_3 = 1$ bar $\quad h_{3s} = 2480$ kJ/kg

$s_1 = s_{3s}$

The stagnation enthalpy at inlet

$$h_{01} = h_1 + \frac{\mathcal{V}_1^2}{2}$$

$$= (2830 \text{ kJ/kg}) + \frac{(75)^2}{2000} \text{ (kJ/kg)}$$

$$= 2832.8 \text{ kJ/kg}$$

The velocity of steam at throat

$$\mathcal{V}_2 = 44.72 \times \sqrt{h_{01} - h_2}$$

$$= 44.72 \times \sqrt{2832.8 - 2720}$$

$$= 475 \text{ m/s}$$

(i) *Throat area*

$$A_2 = \frac{\dot{m}_s v_2}{\mathcal{V}_2} = \frac{2 \times 0.48}{475}$$

$$= 2.021 \times 10^{-3} \text{ m}^2$$

$$= \mathbf{2021 \text{ mm}^2}$$

The enthalpy of steam at the actual exit state 3 ($p_3 = 1$ bar)

$$h_3 = h_{3s} + \Delta h_f = 2480 + 63$$

$$= 2543 \text{ kJ/kg}$$

The specific volume at the intersection point of pressure of 1 bar and specific enthalpy of 2543 kJ/kg is

$v_3 = 1.6$ m^3/kg (From Mollier chart)

The velocity at nozzle exit

$$\mathcal{V}_3 = 44.72 \times \sqrt{h_{01} - h_3}$$

$$= 44.72 \times \sqrt{2832.8 - 2543}$$

$$= 761.31 \text{ m/s}$$

(ii) *Exit area of nozzle*

$$A_3 = \frac{\dot{m}_s v_3}{\mathcal{V}_3} = \frac{2 \times 1.6}{761.31}$$

$$= 4.203 \times 10^{-3} \text{ m}^2 = \mathbf{4203 \text{ mm}^2}$$

(iii) *Overall efficiency of the nozzle*

$$\eta_{overall} = \frac{h_1 - h_3}{h_1 - h_{3s}}$$

$$= \frac{2830 - 2543}{2830 - 2480} = \mathbf{0.82 \quad or \quad 82\%}$$

Example 21.7 *A turbine having a set of 16 nozzles receives steam at 20 bar and 400°C. The pressure of steam at the nozzle exit is 12.0 bar. If the discharge rate is 260 kg/min and the nozzle efficiency is 90%, calculate the cross-sectional area at the nozzle exit.*

If the steam has a velocity of 80 m/s at entry to the nozzle, find the percentage increase in discharge.

Solution

Given Expansion of steam through set of nozzles

Case 1 $\quad N = 16, \quad p_1 = 20$ bar, $\quad T_1 = 400$°C

$\eta_N = 0.9, \quad p_2 = 12$ bar, $\quad \dot{m}_s = 260$ kg/min

Case 2 $\quad \mathcal{V}_1 = 80$ m/s

To find

(i) Cross-sectional area at nozzle exit,

(ii) Percentage increase in discharge, if inlet velocity approaches 80 m/s.

Assumptions

(i) For the first case, the inlet velocity to the nozzle is negligible.

(ii) For superheated steam, the isentropic index is $k = 1.3$.

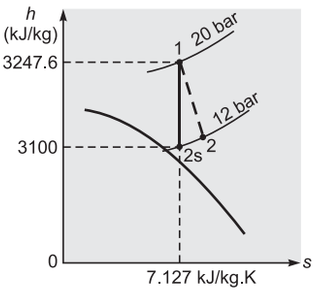

Fig. 21.13

Analysis The critical pressure ratio for expansion of superheated steam is

$$\frac{p^*}{p_1} = 0.546 \quad \text{when} \quad k = 1.3$$

The critical pressure $p^* = 0.546 \times 20$ bar $= 10.92$ bar

The critical pressure is less than the exit pressure of steam from nozzle. Thus the nozzle is convergent type only.

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From Mollier chart or steam tables

State 1: Nozzle entry $p_1 = 20$ $T_1 = 400°C$

$h_1 = 3247.6$ kJ/kg

State 2: Nozzle exit, $s_1 = s_{2s}$ $p_2 = 12$ bar

$h_{2s} = 3100$ kJ/kg

Case 1 Using nozzle efficiency

$$\eta_N = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

or $h_1 - h_2 = \eta_N (h_1 - h_{2s})$

$= 0.9 \times (3247.6 - 3100) = 132.84$ kJ/kg

and $h_2 = 3247.6 - 132.84 = 3114.76$ kJ/kg

From Mollier chart at $p_2 = 12$ bar, and $h_2 = 3114.76$ kJ/kg

Specific volume $v_2 = 0.275$ m^3/kg

The exit velocity of steam

$$\mathcal{V}_2 = 44.72 \times \sqrt{h_1 - h_2} = 44.72 \times \sqrt{132.84}$$

$= 515.42$ m/s

Mass-flow rate of steam per nozzle

$$\dot{m}_1 = \frac{\dot{m}_s}{N} = \left(\frac{260}{60} \text{ kg/s}\right) \times \frac{1}{16} = 0.271 \text{ kg/s}$$

Further, $\dot{m}_1 = \dfrac{A_2 \mathcal{V}_2}{v_2}$

or $$A_2 = \frac{m_1 v_2}{\mathcal{V}_2} = \frac{0.271 \times 0.275}{515.42}$$

$$= \mathbf{1.44 \times 10^{-4} \, m^2}$$

Case 2 If initial velocity of steam is taken as 80 m/s then initial stagnation enthalpy

$$h_{01} = h_1 + \frac{\mathcal{V}_1^2}{2000} = 3247.6 + \frac{(80)^2}{2000}$$

$= 3250.8$ kJ/kg

$$h_{01} - h_2' = \eta_N (h_{01} - h_{2s}) = 0.9 \times (3250.8 - 3100)$$

$= 135.72$ kJ/kg

The velocity of steam at nozzle exit

$$\mathcal{V}_2' = 44.72 \times \sqrt{h_{01} - h_2'} = 44.72 \times \sqrt{135.72}$$

$= 521$ m/s

Percentage increase in velocity

$$= \frac{\mathcal{V}_2 - \mathcal{V}_2'}{\mathcal{V}_2} \times 100 = \frac{521 - 515.42}{515.42} \times 100$$

$= \mathbf{1.08\%}$

Example 21.8 *The steam is supplied to a nozzle at a rate of 1 kg/s from an inlet condition of 10 bar, dry saturated and exit at 1 bar pressure. The efficiency of the nozzle for the convergent portion is 95 per cent and that of the divergent portion is 90 per cent. Determine*

(a) throat and exit diameters of nozzle,

(b) length of nozzle, if divergent cone angle of the nozzle is 14°,

(c) The power in kW corresponds to exit velocity of the steam.

Solution

Given Expansion of dry saturated steam through a convergent–divergent nozzle.

$\dot{m}_s = 1$ kg/s $p_1 = 10$ bar $x_1 = 1.0$

$p_2 = 1$ bar $\eta_{N,\,conv} = 0.95$ $\eta_{N,div} = 0.9$

To find

(i) Throat diameter of nozzle,

(ii) Exit diameter of nozzle,

(iii) Length of nozzle for cone angle $\alpha = 14°$,

(iv) Power at nozzle exit.

Assumptions

(i) Inlet velocity to nozzle is negligible.

(ii) Isentropic index of expansion of dry saturated steam is 1.135.

(iii) Steam expansion is in thermal equilibrium.

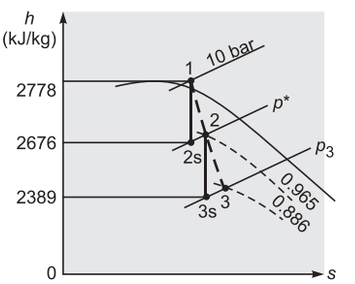

Fig. 21.14

Analysis

(i) The critical pressure ratio at throat

$$\frac{p^*}{p_1} = 0.577$$

or $p_2 = p^* = 0.577 \times 10 \text{ bar} = 5.77$ bar

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From Mollier chart

State 1: Nozzle entry

$p_1 = 10$ bar $\quad x = 1.0$

$h_1 = 2778$ kJ/kg

State 2: After isentropic expansion at throat

and $\quad p_2 = 5.77$ bar $\quad s_1 = s_{2s}$

$h_{2s} = 2676$ kJ/kg

Using nozzle efficiency in convergent portion

$$\eta_{N,\,conv} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

or $\quad h_2 = h_1 - \eta_{N,conv}(h_1 - h_{2s})$

$= 2778 - 0.95 \times (2778 - 2676)$

$= 2681.1$ kJ/kg

From Mollier chart at $p_2 = 5.77$ bar and $h_2 = 2681.1$ kJ/kg,

$x_2 = 0.965 \quad v_2 = 0.327$ m^3/kg

The velocity of steam at throat

$$\mathcal{V}_2 = 44.72 \times \sqrt{h_1 - h_2}$$

$$= 44.72 \times \sqrt{2778 - 2681.1}$$

$= 440.2$ m/s

The mass-flow rate of steam is given by

$$\dot{m}_s = \frac{A_2 \mathcal{V}_2}{v_2}$$

or $\quad A_2 = \dfrac{1 \times 0.327}{440.2}$

$= 7.428 \times 10^{-4}$ m^2 $= 7.428$ cm^2

Further, $\quad A_2 = \dfrac{\pi}{4} d_2^2$

or $\quad d_2 =$ **3.75 cm**

(ii) State 3*s*: After isentropic expansion in divergent section of the nozzle,

$h_{3s} = 2389.0$ kJ/kg $\quad$ at $s_2 = s_{3s}$

Using nozzle efficiency of divergent section

$$\eta_{N,\,div} = \frac{h_2 - h_3}{h_2 - h_{3s}}$$

or $\quad h_3 = h_2 - \eta_{N,\,div}(h_2 - h_{3s})$

$= 2681.1 - 0.9 \times (2681.1 - 2389)$

$= 2418.2$ kJ/kg

From Mollier chart at

$p_3 = 1$ bar $\quad h_3 = 2418.2$ kJ/kg

$x_3 = 0.886$

and $\quad v_3 = 1.694$ m^3/kg

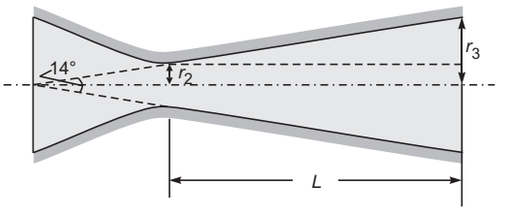

Fig. 21.15

Exit velocity of steam

$$\mathcal{V}_3 = 44.72 \times \sqrt{h_1 - h_3}$$

$$= 44.72 \times \sqrt{2778 - 2418.2}$$

$= 848.28$ m/s

and $\quad \dot{m}_s = \dfrac{A_3 \mathcal{V}_3}{v_3}$

or $\quad A_3 = \dfrac{1 \times 1.694}{848.28}$

$= 1.977 \times 10^{-3}$ m^2

$$A_3 = \frac{\pi}{4} d_3^2$$

It gives $\quad d_3 = 0.0504$ m $=$ **5.04 cm**

(iii) $\quad \tan(\alpha/2) = \dfrac{r_3 - r_2}{L} = \dfrac{d_3 - d_2}{2L}$

or $\quad L = \dfrac{5.04 - 3.75}{2 \tan 7°} =$ **8.0 cm**

(iv) Power at nozzle exit

= kinetic energy of steam

$$= \dot{m}_s \frac{\mathcal{V}_3^2}{2} = \frac{1 \times (848.28)^2}{2}$$

$= 359.79 \times 10^3$ J/s

$=$ **359.79 kW**

Example 21.9 *The steam is supplied to a nozzle at 15 bar, 350°C and exits at 1 bar. If the divergent portion of the nozzle is 80 mm long and the throat diameter is 6 mm, determine the cone angle of the divergent portion. Assume 12% of total enthalpy drop is lost in the friction in the divergent portion. Also calculate the velocity and temperature of steam at the throat.*

Solution

Given Expansion of steam through convergent–divergent nozzle.

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$p_1 = 15$ bar $\quad T_1 = 350°C \quad p_2 = 1$ bar
$\eta_N = 1 - 0.12 = 0.88 \quad L = 80$ mm $\quad d_2 = 6$ mm

To find

(i) Temperature of steam at throat,
(ii) Velocity of steam at throat, and
(iii) Cone angle of divergent portion of nozzle.

Assumptions

(i) Isentropic expansion in convergent portion of nozzle.
(ii) Inlet velocity of steam to nozzle is negligible.
(iii) Isentropic index for steam expansion as 1.3.

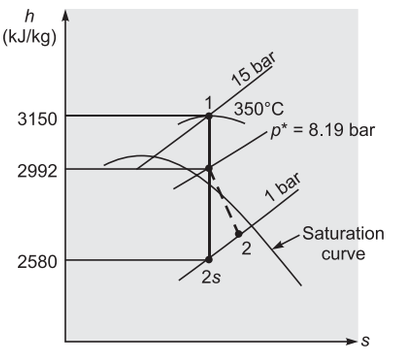

Fig. 21.16

Analysis The critical pressure ratio for nozzle

$$\frac{p^*}{p_1} = 0.546$$

Critical pressure of steam

$$p_2 = p^* = 0.546 \times 15 \text{ bar} = 8.19 \text{ bar}$$

which is greater than exit pressure of steam. Thus the steam expands in divergent portion of nozzle also.

From Mollier chart;

Nozzle entry: $p_1 = 15$ bar, $T_1 = 350°C$
$h_1 = h_{01} = 3150$ kJ/kg

Throat: (critical) condition: $p_2 = 8.19$ bar
$s_1 = s_2$
$h_2 = 2992$ kJ/kg
$v_2 = 0.24$ m^3/kg,
$T_2 = 270°C$

Nozzle exit: State 3*s*
$p_3 = 1$ bar, $s_1 = s_{3s}$
$h_{3s} = 2580$ kJ/kg

(i) Temperature of steam at throat
= **270°C**

(ii) Velocity of steam at throat

$$\mathcal{V}_2 = 44.72 \times \sqrt{h_1 - h_2}$$
$$= 44.72 \times \sqrt{3150 - 2992}$$
$$= \mathbf{562.14 \text{ m/s}}$$

The mass-flow rate of steam through nozzle

$$\dot{m}_s = \frac{A_2 \mathcal{V}_2}{v_2} = \frac{\left(\frac{\pi}{4}\right) d_2^2 \mathcal{V}_2}{v_2}$$
$$= \frac{\left(\frac{\pi}{4}\right) \times (6 \times 10^{-3})^2 \times 562.14}{0.24}$$
$$= 0.0662 \text{ kg/s}$$

Using nozzle efficiency for divergent portion

$$\eta_{N,\,div} = \frac{h_2 - h_3}{h_2 - h_{3s}}$$

or $\quad h_3 = h_2 - \eta_N (h_2 - h_{3s})$
$= 2992 - 0.88 \times (2992 - 2580)$
$= 2629.44$ kJ/kg

From Mollier chart at
$p_3 = 1$ bar $\quad h_3 = 2629.44$ kJ/kg
$v_3 = 1.75$ m^3/kg

The velocity at nozzle exit

$$\mathcal{V}_3 = 44.72 \times \sqrt{3150 - 2629.44}$$
$$= 1020.32 \text{ m/s}$$

The exit area of the nozzle

$$A_3 = \frac{\dot{m}_s v_3}{\mathcal{V}_3} = \frac{0.0662 \times 1.75}{1020.32}$$
$$= 1.1354 \times 10^{-4} \text{ m}^2$$
$$= 113.54 \text{ mm}^2$$

Further, $A_3 = \frac{\pi}{4} d_3^2 \Rightarrow d_3 = 12.02$ mm

(iii) *Cone angle of divergent portion* Refer Fig. 21.17.

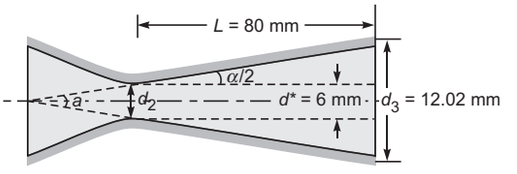

Fig. 21.17

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$$\tan\left(\frac{\alpha}{2}\right) = \frac{r_3 - r_2}{L} = \frac{d_3 - d_2}{2L}$$

$$= \frac{12.06 - 6}{2 \times 80} = 0.0383$$

or $$\frac{\alpha}{2} = 2.156°$$

Cone angle, $\alpha = \mathbf{4.31°}$

Example 21.10 *The nozzles of a turbine are supplied with superheated steam at 10 bar, 250°C. The steam leaves the nozzles at a pressure of 1.0 bar. The steam consumption for the turbine is 16 kg/kWh, when it develops 225 kW. If the throat diameter is 0.8 cm, determine the number of nozzles required and exit diameter of the nozzles, assuming that 10% of the total heat drop is lost in overcoming the friction in the divergent portion only. Neglect the velocity of approach.*

Solution

Given Expansion of steam through nozzles of a turbine

$p_1 = 10$ bar	$T_1 = 250°C$	$\mathcal{V}_1 = 0$
$p_3 = 1.0$ bar	$P = 225$ kW	$ssc = 16$ kg/kWh
$d_2 = 0.8$ cm	$\eta_N = 1 - 0.1 = 0.9$	

To find

(i) Number of nozzles used in turbine,

(ii) Exit diameter of the nozzle.

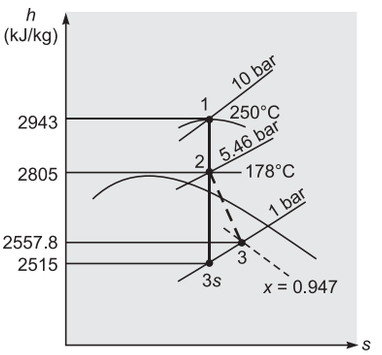

Fig. 21.18

Analysis The mass-flow rate through the nozzles is determined as

$$\dot{m}_s = \frac{sss \times P}{3600} = \frac{16 \times 225}{3600} = 1 \text{ kg/s}$$

Since steam is superheated at the entrance of the nozzles, thus isentropic index

$$k = 1.3$$

and critical pressure ratio, $\frac{p_2}{p_1} = 0.546$

Thus, throat pressure $p_2 = 0.546\, p_1 = 5.46$ bar

which is greater than exit pressure, thus the expansion remains continue in divergent protion.

Entry of steam: $p_1 = 10$ bar, $T_1 = 250°C$, $h_1 = 2943$ kJ/kg

Throat condition:

$p_2 = 5.46$ bar $\quad s_1 = s_2$

$h_2 = 2805$ kJ/kg $\quad v_2 = 0.366$ m^3/kg

Nozzle exit: $h_{3s} = 2515$ kJ/kg·K $\quad s_1 = s_3$

$x = 0.947 \quad v_{g_3} = 1.694$ m^3/kg

Velocity at throat; $\mathcal{V}_2 = 44.72 \times \sqrt{h_1 - h_2}$

$= 44.72 \times \sqrt{2943 - 2805}$

$= 525.35$ m/s

The mass flow rate per nozzle at throat,

$$\dot{m}_1 = \frac{A_2 \mathcal{V}_2}{v_2}$$

$$= \frac{\frac{\pi}{4}(0.8 \times 10^{-2})^2 \times 525.35}{0.366}$$

$= 0.0721$ kg/s

(i) Number of nozzles required

$$= \frac{\dot{m}_s}{\dot{m}_1} = \frac{1 \text{ kg/s}}{\text{g } 0.0721 \text{ k /s}} = 13.86 = \mathbf{14 \text{ nozzles}}$$

The enthalpy at the nozzle exit is determined by using nozzle efficiency

$$\eta_N = \frac{h_2 - h_3}{h_2 - h_{3s}}$$

or $h_3 = 2805 - 0.9 \times (2805 - 2515)$

$= 2544$ kJ/kg

The velocity at nozzle exit

$$\mathcal{V}_3 = 44.72 \times \sqrt{2943 - 2544}$$

$= 893.28$ m/s

The exit area of the nozzle

$$A_3 = \frac{\dot{m}_1 (x v_{g_3})}{\mathcal{V}_3} = \frac{0.0721 \times 0.947 \times 1.694}{893.28}$$

$= 1 \times 10^{-4}$ m^2

(ii) Exit diameter of the nozzle

$$A_3 = \frac{\pi}{4} d_3^2$$

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$$\text{or} \qquad d_3 = \sqrt{\frac{4\,A_3}{\pi}} = \sqrt{\frac{4\times 1.295\times 10^{-4}}{\pi}}$$

$$= 0.01284\,\text{m} = \mathbf{12.84\ mm}$$

Example 21.11 *A steam turbine develops 190 kW with a consumption of 18 kg/kWh. The pressure and temperature of steam enering the nozzle are 11.8 bar and 220°C, respectively. The steam leaves the nozzles at 1.18 bar. The diameter of nozzle at the throat is 8 mm. find the number of nozzles.*

If 8 per cent enthalpy drop is lost due to friction in diverging part of the nozzle, determine the diameter at the exit of the nozzle and exit velocity of leaving steam.

Solution

Given Expansion of steam through nozzles of a turbine

$p_1 = 11.8$ bar, $T_1 = 220°C$ $ssc = 18$ kg/kWh

$p_3 = 1.18$ bar $P = 190$ kW $d_2 = 8$ mm

$\eta_N = 1 - 0.08 = 0.92$

To find

(i) Number of nozzles used in turbine,

(ii) Exit diameter of the nozzle, and

(iii) Exit velocity of leaving steam.

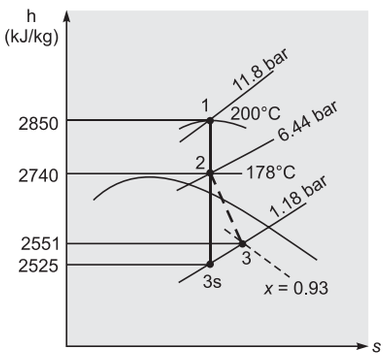

Fig. 21.19

Analysis The mass flow rate through the nozzles is determined as

$$\dot{m}_s = \frac{sss \times p}{3600} = \frac{18\times 190}{3600} = 0.95\ \text{kg/s}$$

Since steam is superheated at the entrance of the nozzles, thus isentropic index

$$k = 1.3$$

and critical pressure ratio,

$$\frac{p_2}{p_1} = 0.546$$

Throat pressure $p_2 = 0.546\,p_1$

$= 0.546 \times 11.8$ bar

$= 6.44$ bar

which is greater than exit pressure, hence the expansion continues in divergent portion.

Entry of steam: $p_1 = 11.8$ bar, $T_{01} = 220°C$

$h_1 = 2850$ kJ/kg

Throat condition: $p_2 = 6.44$ bar

$s_1 = s_2$

$h_2 = 2740$ kJ/kg

$v_2 = 0.368\ \text{m}^3/\text{kg}$

Nozzle exit: $h_{3s} = 2525$ kJ/kg·K

$s_1 = s_3, \quad x_3 = 0.93$

$v_{g_3} = 1.44\ \text{m}^3/\text{kg}$

Velocity at throat;

$$\mathcal{V}_2 = 44.72 \times \sqrt{2850 - 2740}$$

$$= 469.04\ \text{m/s}$$

The mass flow rate per nozzle at throat

$$\dot{m}_1 = \frac{A_2\mathcal{V}_2}{v_2} = \frac{\frac{\pi}{4}(8\times 10^{-3})^2 \times 469.04}{0.368}$$

$$= 0.064\ \text{kg/s}$$

(i) *Number of nozzles required*

$$= \frac{\dot{m}_s}{\dot{m}_1} = \frac{0.95\ \text{kg/s}}{0.064\ \text{kg/s}}$$

$$= 14.83 \approx \mathbf{15\ nozzles}$$

The enthalpy at the nozzle exit is determined by using nozzle efficiency

$$\eta_{N,\,div} = \frac{h_2 - h_3}{h_2 - h_{3s}}$$

or $h_3 = 2740 - 0.92 \times (2740 - 2525)$

$= 2542.2$ kJ/kg

(ii) *The velocity at nozzle exit*

$$\mathcal{V}_3 = 44.72 \times \sqrt{2850 - 2542.2}$$

$$= \mathbf{784.58\ m/s}$$

The exit area of the nozzle

$$A_3 = \frac{\dot{m}_1\,(x v_{g_3})}{\mathcal{V}_3}$$

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$$= \frac{0.064 \times (0.93 \times 1.44)}{784.58}$$

$$= 1.0924 \times 10^{-4}\,\text{m}^2$$

(iii) *Exit diameter of the nozzle*

$$A_3 = \frac{\pi}{4} d_3^2$$

or $$d_3 = \sqrt{\frac{4A_3}{\pi}} = \sqrt{\frac{4 \times 1.0924 \times 10^{-4}}{\pi}}$$

$$= 0.01179\,\text{m} = \mathbf{11.79\ mm}$$

Example 21.12 *Steam at a pressure of 15 bar and a dryness fraction of 0.97 is discharged through a convergent–divergent nozzle to a back pressure of 0.2 bar. The steam consumption is 9 kg/kWh. If the power developed is 220 kW, determine*

(a) Throat pressure,

(b) Number of nozzles required, if each nozzle has throat of rectangular cross-section of 4 mm × 8 mm.

(c) The cross-sectional area of the exit rectangle if 12% of the overall isentropic enthalpy drop reheats steam by friction in divergent portion

Solution

Given Expansion of steam through a convergent divergent nozzles

$p_1 = 15$ bar $\quad x_1 = 0.97 \quad ssc = 9$ kg/kWh

$p_3 = 0.2$ bar $\quad P = 220$ kW $\quad A_2 = 4\text{ mm} \times 8\text{ mm}$

$\eta_N = 1 - 0.12 = 0.88$

To find

(i) Throat pressure,

(ii) Number of nozzles used in turbine, and

(iii) Exit cross section of the nozzle.

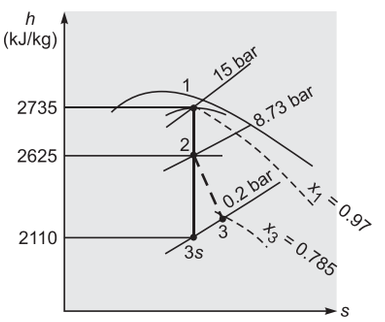

Fig. 21.20

Analysis The mass flow rate through the nozzles is determined as

$$\dot{m}_s = \frac{sss \times P}{3600}$$

$$= \frac{9 \times 220}{3600} = 0.55\ \text{kg/s}$$

(i) *Throat pressure*

Since steam is wet at the entrance of the nozzles, thus isentropic index

$$k = 1.113$$

and critical pressure ratio,

$$\frac{p_2}{p_1} = \left(\frac{2}{1.113+1}\right)^{\frac{1.113}{1.113-1}} = 0.582$$

Throat pressure

$$p_2 = 0.582\, p_1$$

$$= 0.582 \times 15\ \text{bar} = \mathbf{8.73\ bar}$$

which is greater than exit pressure, hence the expansion continues in divergent portion.

(ii) *Number of nozzles required*

Entry of steam: $p_1 = 15$ bar $\quad x_1 = 0.97$

$h_1 = 2735$ kJ/kg

Throat condition:

$p_2 = 8.73$ bar $\quad s_1 = s_2$

$h_2 = 2660$ kJ/kg

$v_2 = 0.24\ \text{m}^3/\text{kg}$

Nozzle exit: $h_{3s} = 2105$ kJ/kg·K $\; s_1 = s_3$,

$x_3 = 0.785$, $v_{g_3} = 6.1\ \text{m}^3/\text{kg}$.

Velocity at throat;

$$\mathcal{V}_2 = 44.72 \times \sqrt{2735 - 2660}$$

$$= 387.28\ \text{m/s}$$

The mass flow rate per nozzle at throat

$$\dot{m}_1 = \frac{A_2 \mathcal{V}_2}{v_2} = \frac{(8 \times 4) \times 10^{-6} \times 387.28}{0.24}$$

$$= 0.0516\ \text{kg/s}$$

Number of nozzle

$$= \frac{\dot{m}_s}{\dot{m}_1} = \frac{0.55\ \text{kg/s}}{0.0516\ \text{kg/s}}\ 10.65 \approx \mathbf{11\ nozzles}$$

The enthalpy at the nozzle exit is determined by using nozzle efficiency in divergent portion

$$\eta_{N,\,div} = \frac{h_1 - h_3}{h_1 - h_{3s}}$$

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or $h_3 = 2735 - 0.88 \times (2735 - 2110)$

$= 2185$ kJ/kg

(iii) *Exit cross-sectional area of the nozzle*

The velocity at nozzle exit

$$\mathcal{V}_3 = 44.72 \times \sqrt{2735 - 2185}$$

$$= \mathbf{1048.77\ m/s}$$

The exit area of the nozzle

$$A_3 = \frac{\dot{m}_1 (x v_{g_3})}{\mathcal{V}_3} = \frac{0.0516 \times (0.785 \times 6.1)}{1048.77}$$

$$= \mathbf{2.356 \times 10^{-4}\ m^2 = 235.6\ mm^2}$$

21.4 SUPERSATURATED EXPANSION OF STEAM

21.4.1 Supersaturation

When steam expands in the nozzle, its pressure and temperature drop simultaneously, but steam does not start condensing, and expands as superheated vapour even as it reaches the saturation line. This process is very quick, the residence time of steam velocity is very high, and there may not be sufficient time for necessary heat transfer and formation of liquid droplets. Consequently, the condensation of vapour is delayed for a little while. This phenomenon is known as *supersaturation* and steam, which exits in the wet region without containing any mositure is called *supersaturated steam*. Such expansion of steam is called a *supersaturated expansion*. The point at which condensation occurs may be within the nozzle or after the vapour leaves the nozzles.

21.4.2 Metastable State

The steam below the saturation curve up to the point where condensation begins neither in stable equilibrium nor in unstable equilibrium. Since fluid is a homogenous vapour below saturation temperature, the steam in this condition is said to be in a *metastable state*.

Consider superheated steam is expanding isentropically in a convergent–divergent nozzle as shown in Fig. 21.21. The expansion process is represented on *T–s* and *h–s* diagrams in Fig. 21.22(a) and (b), respectively. The point *a* is on saturation line, where in normal condition, the condensation of steam begins. However, if the point *a* is reached in the divergent section of the nozzle, the condensation does not occur at this point. The steam exists as a vapour between points *a* and *b*, but the temperature is lower than the saturation temperature for a given pressure. The point *b* is known as *metastable state*. The point *b* lies on the pressure line p_2 produced from the superheated region.

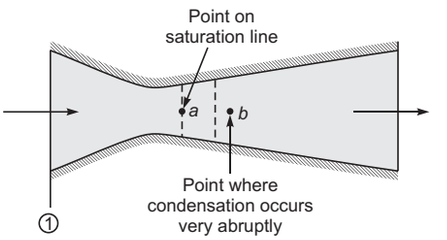

Fig. 21.21 *Supersaturation of steam in nozzle*

The temperature of supersaturated vapour at p_2 is T_b, which is less than the saturation temperature T_2 corresponding to p_2. This vapour is said to be *supercooled, and the degree of supercooling (or undercooling)* is given by $(T_2 - T_b)$.

The *degree of supersaturation* is defined as the ratio of actual pressure at the point *b* to the saturation pressure corresponding to the temperature T_b of steam at the point *b*.

$$\text{Degree of supersaturation} = \frac{p_b}{p_{b,sat}} \qquad ...(21.21)$$

The locus of points where the condensation will take place regardless of initial temperature and pressure of steam at the nozzle entrance is called the *Wilson line*. The Wilson line lies between 95 and 96 per-cent dryness fraction curves in the saturation region as shown on the *h–s* diagram Fig. 21.22(b) and it is approximated by the 96% dryness fraction line. Therefore, the high-velocity steam flowing through the nozzle is assumed to begin to condense when the 96% dryness fraction line in crossed.

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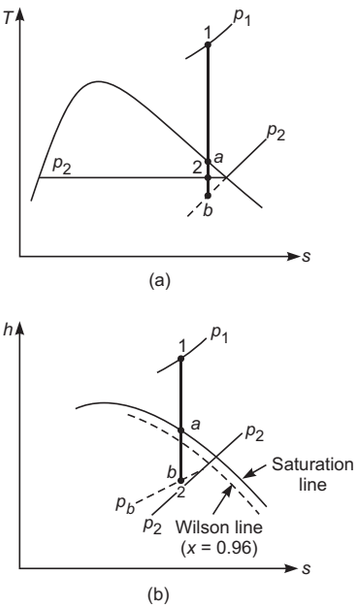

Fig. 21.22 *Supersaturated expansion of steam*

21.4.3 Effect of Supersaturation Flow

It is observed that the enthalpy drop in supersaturated flow ($h_1 - h_b$) is less than the enthalpy drop ($h_1 - h_2$) for expansion in equilibrium conditions. Thus, the exit velocity for supersaturated flow is less than that for equilibrium flow. However, the difference in enthalpy drop is small and square root of enthalpy drop is used for evaluating exit velocity. Thus, the effect on the exit velocity is also small.

Further, the specific volume at exit with supersaturated flow v_b is considerably less than the specific volume at exit with equilibrium flow, v_2. Now the mass-flow rate through an area A_2 as given earlier for equilibrium flow,

$$\dot{m}_s = \frac{A_2 \mathcal{V}_b}{v_b} \qquad \ldots(21.22)$$

Since $v_b < v_2$, thus the mass flow rate with supersaturated flow is greater than the mass flow rate with equilibrium flow.

The effect of supersaturation flow can be summarised below:

1. Actual heat drop is slightly reduced.
2. Exit velocity is slightly reduced.
3. The specific volume is reduced, thus the mass flow rate is increased by 2 to 5%.
4. Entropy is slightly increased, because abrupt condensation takes place at constant enthalpy line.

Since the steam during supersaturted expansion remains in superheated state, it can be treated as a gas with an isentropic index of 1.3 and problems of supersaturated flow can be solved without use of Mollier chart.

Example 21.13 *Determine the throat area, exit area and exit velocity for a steam nozzle to pass a mass flow of 0.2 kg/s, when inlet conditions are 10 bar and 250°C and the final pressure is 2 bar. Assume expansion is isentropic and inlet velocity is negligible. Use $pv^{1.3}$ = constant. Do not use h–s chart.*

Solution

Given Steam expansion through a nozzle

$\dot{m} = 0.2$ kg/s $\quad p_1 = 10$ bar

$T_1 = 250°C$ $\quad p_3 = 2$ bar

$s_1 = s_2$ $\quad pv^{1.3} = C, \mathcal{V}_1 = 0$

Specific volume of steam at 10 bar, 250°C, $v_1 = 0.233\ m^3/kg$ (from steam tables)

To find

(i) Throat area of nozzle,
(ii) Exit velocity of steam from nozzle, and
(iii) Exit area of nozzle.

Analysis

(i) For given, $n = 1.3$, the critical pressure ratio

$$\frac{p_2}{p_1} = 0.546$$

The pressure of steam at throat

$$p_2 = 0.546 \times 10 \text{ bar} = 5.46 \text{ bar}$$

The velocity of steam at the throat, Eq. (21.9);

$$\mathcal{V}_2 = \sqrt{\left(\frac{2n}{n-1}\right) p_1 v_1 \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]}$$

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$$= \sqrt{\left(\frac{2\times1.3}{0.3}\right)\times(10\times10^5\,\text{N/m}^2)} \sqrt{\times(0.233\,\text{m}^3/\text{kg})\times\left[1-\left(\frac{5.46}{10}\right)^{\frac{0.3}{1.3}}\right]}$$

$$= 513 \text{ m/s}$$

The specific volume at the throat

$$v_2 = v_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = 0.233\times\left(\frac{10}{5.46}\right)^{\frac{1}{1.3}}$$

$$= 0.371 \text{ m}^3/\text{kg}$$

Area of throat, using continuity equation

$$A_2 = \frac{\dot{m}v_2}{\mathcal{V}_2} = \frac{0.2\times0.371}{513}$$

$$= 1.446\times10^{-4}\text{m}^2$$

$$= \mathbf{1.446\ cm^2}$$

(ii) Exit velocity of steam,

$$\mathcal{V}_3 = \sqrt{\left(\frac{2n}{n-1}\right)p_1v_1\left[1-\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{n}}\right]}$$

$$= \sqrt{\left(\frac{2\times1.3}{0.3}\right)(10\times10^5\times0.233)\times\left[1-\left(\frac{2}{10}\right)^{\frac{0.3}{1.3}}\right]}$$

$$= \mathbf{791.5\ m/s}$$

(iii) The specific volume of steam at exit,

$$v_3 = v_1\left(\frac{p_1}{p_3}\right)^{\frac{1}{n}} = 0.233\times\left(\frac{10}{2}\right)^{\frac{1}{1.3}}$$

$$= 0.803 \text{ m}^3/\text{kg}$$

The exit area of nozzle,

$$A_3 = \frac{\dot{m}v_3}{\mathcal{V}_3} = \frac{0.2\times0.803}{791.5}$$

$$= 2.030\times10^{-4}\text{m}^2 = \mathbf{2.030\ cm^2}$$

Example 21.14 *5.2 kg/s of steam at 30 bar and 350°C is supplied to a group of six nozzles on a wheel diameter maintained at 4-bar pressure. Determine for metastable expansion,*

(a) Diameter of nozzles at exit without friction,
(b) Degree of under–cooling and supersaturation,
(c) Loss in available heat drop due to ireversibiltiy,
(d) Increase in entropy,
(e) Ratio of mass-flow rate with metastable expansion to that of expansion in thermal equilibrium.

Solution

Given Metastable expansion of steam through nozzles without friction

$\dot{m}_s$ = 5.2 kg/s N = 6 nozzles
p_1 = 30 bar T_1 = 350°C = 623 K
$p_2 = p_b$ = 4 bar

To find

(i) Diameter of nozzles at exit,
(ii) Degree of undercooling,
(iii) Degree of supersaturation,
(iv) Loss in available enthalpy drop,
(v) Entopy increase,
(vi) Ratio of mass-flow rate with metastable expansion to that of expanssion in thermal equilibrium.

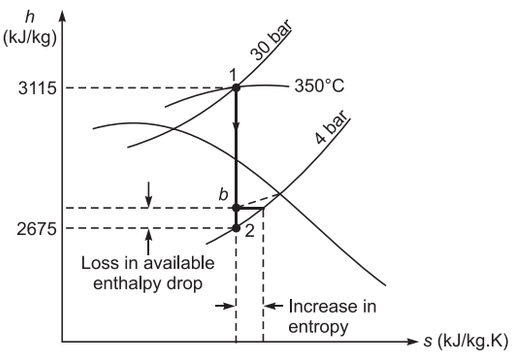

Fig. 21.23 *h–s diagram*

Assumptions

(i) Negligible inlet velocity of steam
(ii) Index of steam expansion, n = 1.3.

Properties of steam From Mollier chart,

Steam inlet: 30 bar, 350°C

h_1 = 3115 kJ/kg v_1 = 0.09 m^3/kg

State 2: After expansion in thermal equilibrium,

p = 4 bar
h_2 = 2675 kJ/kg v_2 = 0.46 m^2/kg

Analysis

(i) *Diameter of nozzles*

In metastable state, the steam behaves as gas,

$$h_1 - h_b = \left(\frac{n}{n-1}\right)p_1v_1\left[1-\left(\frac{p_b}{p_1}\right)^{\frac{n-1}{n}}\right]$$

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$$= \frac{1.3}{1.3-1} \times 30 \times 10^2 \times 0.09 \times \left[1 - \left(\frac{4}{30}\right)^{\frac{1.3-1}{1.3}}\right]$$

$$= 435 \text{ kJ/kg}$$

The velocity at the metastable state b,

$$\mathcal{V}_b = \sqrt{\left(\frac{2n}{n-1}\right) p_1 v_1 \left[1 - \left(\frac{p_b}{p_1}\right)^{\frac{n-1}{n}}\right]}$$

$$= \sqrt{\left(\frac{2 \times 1.3}{0.3}\right) \times (30 \times 10^5 \times 0.09)}$$

$$= \times \sqrt{\left[1 - \left(\frac{4}{30}\right)^{\frac{0.3}{1.3}}\right]}$$

$$= 932.8 \text{ m/s}$$

The specific volume of steam at the state b,

$$v_b = v_1 \left(\frac{p_1}{p_b}\right)^{\frac{1}{n}} = 0.09 \left(\frac{30}{4}\right)^{\frac{1}{1.3}}$$

$$= 0.424 \text{ m}^3/\text{kg}$$

Mass flow rate per nozzle

$$\dot{m} = \frac{\dot{m}_s}{N} = \frac{5.2}{6} = 0.867 \text{ kg/s}$$

The area at nozzle exit, (continuity equation)

$$A_b = \frac{\dot{m} v_2}{\mathcal{V}_b} = \frac{0.867 \times 0.424}{932.8}$$

$$= 3.939 \times 10^{-4} \text{ m}^2$$

and $$A_b = \frac{\pi}{4} d^2 \Rightarrow d = \sqrt{\frac{4 A_b}{\pi}}$$

$$= 0.0224 \text{ m} = \mathbf{22.4 \text{ mm}}$$

(ii) *Degree of undercooling*

Temperature T_b after expansion is determined as

$$\frac{T_b}{T_1} = \left(\frac{p_b}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{4}{30}\right)^{\frac{0.3}{1.3}} = 0.628$$

or $$T_b = 0.628 \times (623 \text{ K}) = 391.33 \text{ K}$$

$$= 118.33°\text{C}$$

From steam tables at 4 bar, $T_{sat} = 143.63°\text{C}$

Degree of subcooling (undercooling)

$$= T_{sat} - T_b = 143.63 - 118.33 = \mathbf{25.3°C}$$

(iii) *Degree of supersaturation*

Saturation pressure corresponding to 118.33°C, from steam tables; $p_{b,sat} \approx 1.88$ bar

Degree of supersaturation

$$\frac{p_b}{p_{b,sat}} = \frac{4 \text{ bar}}{\text{al.88 b r}} = \mathbf{2.12}$$

(iv) *Loss in available enthalpy drop*

Isentropic enthalpy drop,

$\Delta h_{isentropic} = h_1 - h_2 = 3115 - 2675$

$= 440$ kJ/kg

Loss in available enthalpy drop

$= \Delta h_{isentrop} - (h_1 - h_b) = 440 - 435$

$= \mathbf{5 \text{ kJ/kg}}$

(v) *Increase in entropy*

$$\Delta s = \frac{\text{Loss in enthalpy drop}}{(T_{sat} + 273)}$$

$$= \frac{5 \text{ kJ/kg}}{(143.63 + 273)} = \mathbf{0.012 \text{ kJ/kg} \cdot \text{K}}$$

(vi) *Ratio of mass-flow rate of two types of expansions*

Velocity at nozzle exit corresponds to isentropic enthalpy drop (expansion in thermal equilibrium)

$$\mathcal{V}_2 = \sqrt{2 \Delta h_{isentropic}} = \sqrt{2 \times 440 \times 10^3}$$

$$= 938 \text{ m/s}$$

Then the mass-flow rate for equilibrium conditions

$$\dot{m} = \frac{A_2 \mathcal{V}_2}{v_2} = \frac{A_b \mathcal{V}_b}{v_2} = \frac{3.939 \times 10^{-4} \times 938}{0.46}$$

$$= 0.803 \text{ kg/s}$$

Ratio of mass flow $= \frac{\dot{m}_s}{\dot{m}} = \frac{0.867}{0.803} = \mathbf{1.078}$

Example 21.15 *A convergent–divergent nozzle receives steam at 7 bar and 200°C and expands it isentopically to 3 bar. Neglect the inlet velocity, calculate the exit area required for a mass flow rate of 0.1 kg/s*

(a) when flow is in equilibrium throughout,

(b) when the flow is supersaturated with $pv^{1.3}$ = constant.

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Soltuion

Given Expansion of steam though a convergent–divergent nozzle.

$p_1 = 7$ bar $\quad T_1 = 200°C \quad \mathcal{V}_1 = 0$

$p_2 = 3$ bar $\quad \dot{m}_s = 0.1$ kg/s $\quad n = 1.3$

To find Exit area of nozzle, when

(i) Expansion of steam is in thermal equilibrium,

(ii) Expansion of steam is supersaturated.

Analysis

(i) *When expansion is in thermal equilibrium throughout*

From Mollier chart and steam table

$h_1 = 2846$ kJ/kg $\quad h_2 = 2682$ kJ/kg

$x_2 = 0.98 \quad v_{g_2} = 0.6057$ m^3/kg

The velocity of steam at nozzle exit

$$\mathcal{V}_2 = 44.72 \times \sqrt{2846 - 2682}$$
$$= 572.7 \text{ m/s}$$

Using continuity equation to obtain exit area of nozzle,

$$A_2 = \frac{\dot{m}_s (x v_{g_2})}{\mathcal{V}_2}$$
$$= \frac{0.1 \times (0.98 \times 0.6057)}{572.7}$$
$$= 1.036 \times 10^{-4} \text{m}^2$$
$$= \mathbf{103.6\ mm^2}$$

(ii) *When steam expansion is supersaturated*

Here, $h_1 = 2846$ kJ/kg

$v_1 = 0.3001$ m^3/kg (from steam tables)

After expansion of steam as a gas

$$v_b = v_1 \left(\frac{p_1}{p_b}\right)^{\frac{1}{n}} = 0.3001 \times \left(\frac{7}{3}\right)^{\frac{1}{1.3}}$$
$$= 0.576 \text{ m}^3/\text{kg}$$

The velocity of steam after supersaturation expansion

$$\mathcal{V}_b = \sqrt{\left(\frac{2n}{n-1}\right) p_1 v_1 \left[1 - \left(\frac{p_b}{p_1}\right)^{\frac{n-1}{n}}\right]}$$
$$= \sqrt{\left(\frac{2 \times 1.3}{0.3}\right) \times 7 \times 10^5 \times 0.3001}$$
$$= \times \sqrt{\left[1 - \left(\frac{3}{7}\right)^{\frac{0.3}{1.3}}\right]} = 568.63 \text{ m/s}$$

Using continuity equation, for exit area

$$A_b = \frac{\dot{m} v_b}{\mathcal{V}_b}$$
$$= \frac{\dot{m} v_b}{\mathcal{V}_b} = \frac{0.1 \times 0.576}{568.63}$$
$$= 1.0127 \times 10^{-4} \text{m}^2 = \mathbf{101.27\ mm^2}$$

Summary

- The nozzle is a duct of varying cross-sectional area, in which the pressure energy of a fluid is converted into kinetic energy.
- The properties of steam are functions of pressure and temperature. Thus ideal gas relations are not applicable to steam. An isentropic index γ is replaced by n, the ratio of specific heats, in certain relations for steam.

  $n = 1.135$ for dry saturated steam

  $n = 1.3$ for superheated steam
- The critical pressure ratio for dry saturated steam is 0.577 and that for superheated steam is 0.546.
- The velocity of approach to the nozzle is very small as compared to exit velocity of fluid at the nozzle exit. Therefore, the inlet velocity is considered negligible. The exit velocity of fluid from a nozzle can be expressed as

  $$\mathcal{V}_2 = 44.72 \times \sqrt{(h_1 - h_2)}$$
- The presence of friction on the nozzle wall surface and within the fluid makes the fluid flow non-isentropic but adiabatic. The nozzle efficiency is defined as

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$$\eta_N = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{\text{Acutal enthalpy drop}}{\text{Isentropic enthalpy drop}}$$

- The high-velocity steam does not start condensing, when it reaches the saturation line and exits as a supersaturated substance in the wet region. Such expansion is said to be supersaturated expansion. Supersaturation state is in non-equilibrium, and thus is called metastable state.
- The degree of supersaturation is defined as the ratio of actual pessure to the saturation pressure corresponding to temperature of steam at the location condensation.
- With supersaturation,
  1. Actual heat drop is slightly reduced
  2. Exit velocity is slightly reduced
  3. The specific volume is reduced, thus the mass flow rate is increased by 2 to 5%

Glossary

Critical pressure Pressure of steam at the throat

Critical velocity Velocity of steam at throat corresponds to maximum discharge condition

Supersaturation Existance of fluid in superheated state in wet region

Metastable State Supersaturation state of a fluid

Degree of supercooling Difference between saturation temperature and temperature of supersaturated vapour

Degree of supersaturation Ratio of actual pressure to the saturation pressure corresponding to temperature of steam condensate.

Review Questions

1. What is the function of a steam nozzle?
2. What are the types of nozzles? Explain with neat diagrams.
3. Derive an equation for discharge through the nozzle.
4. Derive an expression for condition of maximum discharge.
5. Define critical velocity, critical pressure ratio.
6. What is the significance of critical pressure ratio on discharge through the nozzle?
7. Discuss the effect of irreversibilities on nozzle efficiency with the help of a *T–s* diagram.
8. Discuss the supersaturated flow of steam.
9. Define metastable state and degree of supersaturation.
10. What are the effects of supersaturation flow?

Problems

1. Steam enters a convergent–divergent nozzle at 11 bar, dry and saturated at a rate of 0.75 kg/s, and expands isentropically to 2.7 bar. Neglect the inlet velocity and assuming the expansion follows a law $pv^{1.135}$ = constant, determine

(a) the area of the nozzle throat,

(b) the area of the nozzle exit.

[(a) 474 mm^2, (b) 646 mm^2]

2. Steam at a pressure of 10 bar and 0.95 dry expands in a convergent–divergent nozzle. The back pressure of the nozzle is maintaind at 0.85 bar. The throat area is 2.4 cm^2. Find the maximum mass-flow rate and the required exit area. Assume the index of expansion to be 1.35 throughout. *[0.3546 kg/s, 6.575 cm^2]*

3. Steam enters a convergent nozzle at a pressure of 5 bar and 180°C and a velocity of 200 m/s. The discharge pressure is maintained at 3 bar. Determine the required throat area for a mass-flow rate of 0.5 kg/s and a nozzle efficiency of 96%. *[6.284 cm^2]*

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4. Dry, saturated steam at a pressure of 10 bar is discharged through a nozzle at a back pressure of 1 bar. Determine the throat and exit diameters for a mass-flow rate of 1 kg/s and nozzle efficiency of 90%. *[6.99 cm^2; 18.17 cm^2]*

5. Steam at 12 bar and 220°C is supplied to a steam turbine at the rate of 6.5 kg per kW per hour when the turbine develops 220 kW. The back pressure is maintained at 1.2 bar. The nozzle throat diameter is 0.8 cm. The fricition losses in the convergent portion are negligible. Find the mass flow rate and number of the nozzle required and the diameter at the exit of the nozzle.

[0.07234 kg/s; 6 nozzles; 1.183 cm]

6. A convergent–divergent nozzle is supplied steam at a pressure of 10 bar, dry, saturated. The exit pressure is 1 bar. The friction losses are equivalent to 5% of heat drop in the convergent portion and 10% of heat drop in the divergent portion. For a throat diameter of 1.47 cm, determine the mass flow rate and the exit diameter of the nozzle. Also, find the semi cone angle of the divergent portion if its length is 2.85 cm.

[0.236 kg/s; 2.37 cm; 8.65°]

7. A nozzle is supplied with steam at 15 bar, 250°C with negligible velcosity and it leaves the nozzle at 1.2 bar pressure. There are 10% friction losses throughout the passage of the nozzle. Determine the maximum mass-flow rate and the exit diameter, if the throat diameter is 0.8 cm.

[0.1011 kg/s; 1.364 cm]

8. Eight nozzles are required to discharge steam to an impulse turbine from a pressure of 10 bar and 350°C. The back pressure of the turbine is maintained at 0.14 bar. The nozzle efficiency is 90%. If the throat diameter for each nozzle is 0.8 cm, find the total mass-flow rate and exit diameter of each nozzle. *[0.516 kg/s; 2.283 cm]*

9. Steam at 10 bar and 0.95 dry expands in a nozzle up to 0.12 bar pressure. The mass-flow rate is 0.5 kg/s. The index of expansion may be calculated using Zeuner's equation given by $n = 1.035 + 0.1x$ where, 'n' is the index and 'x' is the dryness fraction. *[2069 cm; 7.763 cm]*

10. 40.86 kg/min. of steam is supplied to a convergent–divergent nozzle at 11 bar and 220°C. If the expansion is under metastable conditions, determine the velocity and the area at the throat. Also, determine the degree of undercooling at the throat. *[486.8 m/s; 4.41 cm^2; 3°C]*

11. Steam is expanded in a nozzle from 8 bar, 200°C to a pressure of 1.4 bar. Determine the isentropic heat drop, actual heat drop and the degree of subcooling.

[316 kJ/kg; 299.6 kJ/kg; 66°C]

12. Steam at a pressure of 20 bar, 250°C expands in a convergent–divergent nozzle up to the exit pressure of 2 bar. Assuming a nozzle coefficient of 0.94 for supersaturated flow up to the throat and nozzle efficiency as 90%, find

(i) Velocity at throat,
(ii) Mass-flow rate, if the throat diameter is 1 cm,
(iii) Velocity and diameter of the nozzle at the exit.

[(i) 469.2 m/s, (ii) 0.205 kg/s, (iii) 1.55 cm]

13. Steam at a pressure of 15 bar and 200°C is supplied to a nozzle and it leaves at a pressure of 1 bar. Expansion is supersaturated up to the throat and in thermal equilibrium afterwards. For the mass-flow rate of 0.1 kg/s, find the exit velocity, exit area, degree of undercooling at the throat and its degree of supersaturation.

[956.6 m/s; 1.5 cm^2; 33°C; 2.375]

14. Steam at 15 bar and 200°C is supplied to a convergent–divergent nozzle against a back pressure of 4 bar. Expansion is supersatuated up to the throat and the nozzles are rectangular in shape, its width being 2.5 times the breadth. For a mass-flow rate of 0.3 kg/s, find

(i) Dimensions of the nozzle at the exit,
(ii) Degree of undercooling and supersaturation,
(iii) Increase in entropy.

[(i) 0.8415 cm, 2.1 c, (ii) 23°C, 2.375, (iii) 0.005 kJ/kg·K]

15. Steam is expanded in a set of nozzles from 10 bar and 200°C to 5 bar. What type of nozzle it is? Neglect the initial velocity, find the minimum area of nozzle required to allow a mass flow rate of 3 kg/s under the given conditions. Assume isentropic expansion of steam. *[21 cm^2]*

16. Dry, saturated steam at 9 bar expands through two nozzles to 1 bar. The throat diameter of the nozzle is 2.5 mm. Assuming nozzle efficiency of 90%, find the quantity of steam used per hour and power produced by steam at the nozzle exit.

[43.2 kg/h and 4 kW]

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Objective Questions

1. Steam nozzle converts
   (a) heat energy of steam into pressure energy
   (b) heat energy of steam into kinetic energy
   (c) pressure energy of steam into heat energy
   (d) pressure energy of steam into potential energy
2. A nozzle is designed for
   (a) maximum pressure at the outlet
   (b) maximum discharge
   (c) maximum pressure and maximum discharge
   (d) maximum kinetic energy at the outlet
3. Ideal expansion of steam through a nozzle is considered
   (a) isothermal
   (b) adiabatic
   (c) polytropic
   (d) none of the above
4. Presence of frictional effect during flow through the nozzle
   (a) reduces the exit velocity
   (b) increases the exit velocity
   (c) has no effect on exit velocity
   (d) none of the above
5. Frictional losses in the nozzle
   (a) reduces the heat drop
   (b) increases the heat drop
   (c) has no effect on heat drop
   (d) none of the above
6. Actual exit velocity of steam from a nozzle considering friction coefficient k and negligible inlet velocity, can be expressed as
   (a) $\mathcal{V}_2 = 91.53 \times \sqrt{k(h_1 - h_2)}$
   (b) $\mathcal{V}_2 = 100 \times \sqrt{k(h_1 - h_2)}$
   (c) $\mathcal{V}_2 = 44.72 \times \sqrt{k(h_1 - h_2)}$
   (d) $\mathcal{V}_2 = 9.81 \times \sqrt{k(h_1 - h_2)}$
7. Nozzle efficiency is defined as
   (a) ratio of isentropic heat drop to useful heat drop
   (b) ratio of useful heat drop to isentropic heat drop
   (c) product of useful heat drop and isentropic heat drop
   (d) none of the above
8. When a nozzle operates with maximum mass flow, it is said to be
   (a) under expanding flow
   (b) over expanding flow
   (c) choked flow
   (d) none of the above
9. For supersaturated flow in the nozzle, the discharge
   (a) increases
   (b) decreases
   (c) remains constant
   (d) none of the above
10. For the flow through the convergent and divergent nozzle, whole of friction loss is assumed
    (a) in the converging portion
    (b) in the divergent portion
    (c) between inlet and outlet
    (d) none of the above
11. The presence of friction in the nozzle
    (a) increases the final dryness fraction of steam
    (b) decreases the final dryness fraction of steam
    (c) it does not affect the dryness fraction of steam
    (d) none of the above
12. When the back pressure of a nozzle is below the critical pressure, the nozzle is said to be
    (a) under expanding flow
    (b) over expanding flow
    (c) choked flow
    (d) none of the above

Answers

1. (b) 2. (d) 3. (b) 4. (a) 5. (a) 6. (c) 7. (b) 8. (c)
9. (a) 10. (b) 11. (a) 12. (c)

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22

Steam Turbines

Introduction

A *steam turbine* is a prime mover in which rotating shaft work is developed by a steadily flowing fluid. The work is produced by changing the momentum of steam as it passes through a rotor of the turbine. The change in angular momentum of steam causes the torque on the rotor, thus the rotor spins. Therefore, a steam turbine is *a rotodynamic machine.*

The steam turbine essentially contains two parts: (i) nozzles, and (ii) moving curved blades. The high-velocity steam jet coming out of the nozzle impringes on the curved blades, and the steam glides tangentially on the curved blades fixed on the periphery of a moving wheel. The motive force is exterted on the blades due to change in direction of flowing steam which causes the blades to rotate.

Since the steam undergoes a continuous steady flow process and the speed of fluid is very high, thus a turbine handles a large mass of fluid and produces a large power. Steam turbines are used to drive electric generators in power plants to produce electricity. They are also used to propel large ships, ocean liners, submarines, and so on. The small steam turbines are used to drive the fans, compressors and pumps.

22.1 HISTORY OF STEAM TURBINES

The first steam turbine was made by Hero of Alexandria, 2000 years ago, i.e., in the first century AD. This turbine worked on the pure reaction principle and no power was produced by it. In fact, it was considered as an amazing toy as shown in Fig. 22.1. It consisted of a hollow ball mounted in between two pivots. The ball was equipped with two converging tubes (nozzles). The generated steam was supplied to the ball through one pivot. The steam was expanded through the converging tubes to atmospheric pressure, thus causing a reactive force on the ball and making it to rotate between the pivots.

Fig. 22.1 *Hero's turbine*

In the year 1888, Gustaf de Laval of Stockholm, one of the prominent pioneers of the steam turbine,

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built a reaction turbine on the principle of Hero's turbine, which attained a speed of 42000 rpm with a tip speed of 180 m/s. However, the efficiency was very low and hence commercial turbines could not be built on the pure reaction principle. In the year 1884, about four years prior to the development of the de Laval reaction turbine, Sir Charles Parson expressed that moderate surface velocities and rotor speeds were essential to turn it into a prime mover. This gave the principle of compounding of the turbine in series.

22.2 WORKING PRINCIPLE OF A STEAM TURBINE

Figure 22.2 shows a steam jet gliding on a vane tangentially at an angle α and leaving the vane at an angle β after gliding over the curved surface of the vane. Due to change in angular momentum of the steam jet, the blade moves with a linear velocity u. According to Newton's second law, the change in momentum is directly proportional to the force acting on the blades.

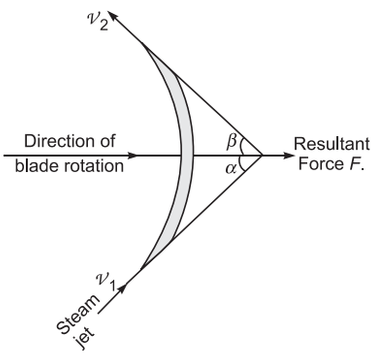

Fig. 22.2 *Forced exerted on moving blade*

Force applied on the vane
= Change in momentum of jet
= mass flow rate × change in velocity

$$F = \dot{m}_s[\mathcal{V}_1 \cos \alpha - (-\mathcal{V}_2 \cos \beta)]$$
$$= \dot{m}_s[\mathcal{V}_1 \cos \alpha + \mathcal{V}_2 \cos \beta] \quad ...(22.1)$$

where $\dot{m}_s$ = mass-flow rate of steam, kg/s
$\mathcal{V}_1$ = Velocity of steam jet entering, m/s
$\mathcal{V}_2$ = Velocity of steam leaving, m/s

The torque acting on the rotating wheel of radius R is given by

$$T = FR = \dot{m}_s R[\mathcal{V}_1 \cos \alpha + \mathcal{V}_2 \cos \beta] \quad ...(22.2)$$

The linear velocity of blades

$$u = \frac{2\pi NR}{60} = \frac{\pi DN}{60}$$

The rate of work done by the blades

Power, P = Force × linear blade velocity
$$= \dot{m}_s[\mathcal{V}_1 \cos \alpha + \mathcal{V}_2 \cos \beta] \times u$$
$$= \dot{m}_s[\mathcal{V}_1 \cos \alpha + \mathcal{V}_2 \cos \beta] \frac{2\pi NR}{60}$$

or $$P = \frac{2\pi NT}{60} (W) \quad ...(22.3)$$

where N = rotation per minute of rotating wheel.
$T = \dot{m}_s R(\mathcal{V}_1 \cos \alpha + \mathcal{V}_2 \cos \beta)$

22.3 CLASSIFICATION OF STEAM TURBINES

The steam turbines can be classified according to

1. Mode of Action
(i) *Impulse* turbines,
(ii) Reaction turbines, and
(iii) Impulse–reaction turbines.

2. Direction of Steam Flow Relative to Rotor
(i) Axial flow turbine,
(ii) Radial flow turbine, and
(iii) Mixed flow turbine.

3. Number of Stages
(i) Single stage turbine, and
(ii) Multi-stage turbine.

22.3.1 Principle of Impulse Turbine

The *impulse turbines* work on the principle of impulse. The steam coming out the boiler expands in the stationary nozzles, where the kinetic energy of steam increases at the cost of enthalpy drop. This high-velocity steam jet is directed to glide over the curved blades mounted on the periphery of the rotor (rotating wheel). When the steam jet glides tangentially over the curved surface of blades, the

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direction of steam flow is changed. It changes the momentum of the high-velocity steam jet and thus impulsive force is exerted in normal direction of steam jet, which ultimately provides motive force at the turbine shaft and it spins rapidly. Figure 22.3 shows a schematic of a simple impulse wheel. It consists of a rotor having a number of curved blades fixed round the circumference.

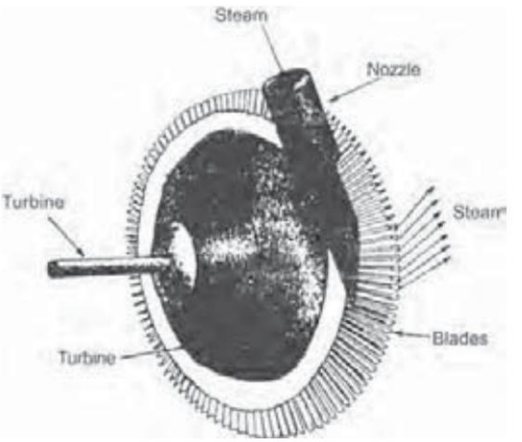

Fig. 22.3 *Characteristic of simple impulse turbine*

22.3.2 Principle of Reaction Turbine

The *reaction turbines* work on the principle of Newton's third law of motion; the reaction (backward) force is always developed opposite to a certain action. In these turbines the rotation is caused by the reaction force generated by the momentum change of fluid accelerating through the nozzles that are attached to the rotor (moving wheel), like a nozzle on rotating shaft as shown in Fig. 22.4 (a).

22.3.3 Principle of Impulse–Reaction Turbine

The impulse–Reaction turbine works on the principle of impulse as well as the principle of reaction.

- In this type, the drop in pressure takes place in fixed nozzles as well as in moving blades.

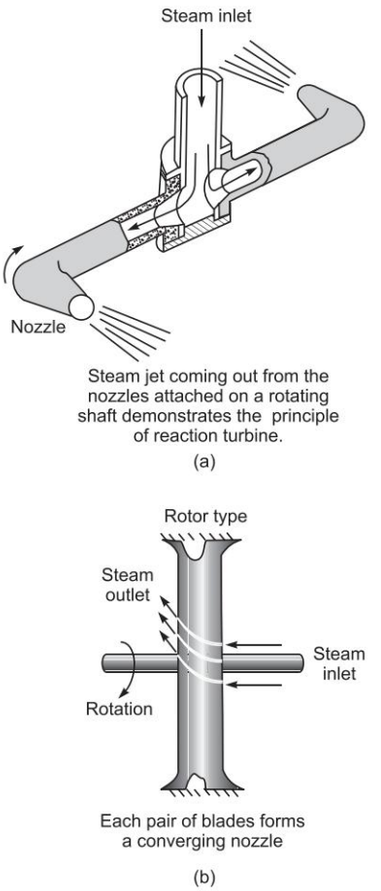

Fig. 22.4 *Working principle and passages of reaction turbine*

- The pressure drop suffered by steam while passing through the moving blades causes a further increase in kinetic energy within these blades, giving rise to a reaction and adds to the propelling force, which is applied through the rotor to the turbine shaft.
- The blade passage cross-sectional area is varied (converging type).

The shape of nozzles in the reaction turbine is not the same as is that of a simple impulse turbine. Instead, they are two-dimensional nozzles, like channels formed in the passage between the blades of each row. In the reaction turbine, the entire pressure

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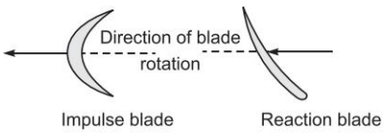

Fig. 22.5 *Shape of blades in impulse and reaction turbine*

drop is obtained gradually and continuously over a series of fixed and moving blades in succession. The fixed blades act as guide vanes to regulate the direction of steam flow as well as to expand the steam to a higher velocity. *Parson's turbine* is an example of an impulse–reaction turbine.

22.4 THE SIMPLE IMPULSE TURBINE

A simple impulse turbine consists of a set of nozzles, a rotor mounted on a shaft, one set of moving blades attached to the rotor and the casing.

A simple impulse turbine is schematically shown in Fig. 22.6. The uppermost portion of the diagram shows a longitudinal section of the upper half of the turbine, the middle portion of the diagram shows the shape of the nozzle and blading and the lower portion shows the variation of the absolute velocity and absolute pressure during the flow of steam through the nozzles and blades.

In the impulse turbine, the complete expansion of steam (from boiler pressure to condenser pressure) takes place in stationary nozzles, the enthalpy drops and kinetic energy of steam increases, which is converted in to shaft work. The steam passes on moving blades at constant pressure, but with gradual reduction in its velocity.

$$p_1 = p_2$$

In an impulse turbine, the absolute velocity of steam coming out the nozzles is about 1050 m/s. If this high-velocity steam jet is used on a single row of blades, the rotor speed may reach as 30,000 rpm, which is too high for practical considerations.

Further, the steam leaving the moving blade is also at high velocity, thus involves a loss of energy, which is commonly known as *carry-over loss* or *leaving loss.* Accordingly, the velocity of steam

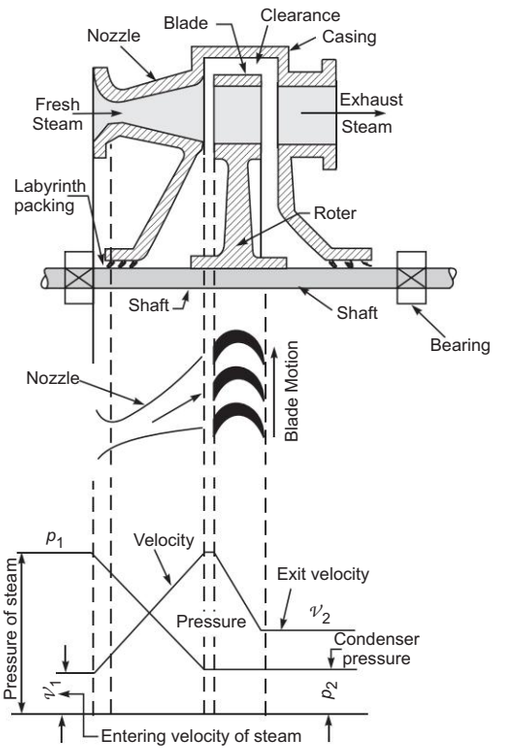

Fig. 22.6 *Schematic arrangement of simple impulse turbine*

jet cannot be effectively utilized in a single row blades. It is therefore, necessary to incorporate some improvement. It is achieved by using more than one set of nozzles, blades and rotors in series, so that either the steam jet pressure or the steam jet velocity is regulated by the turbine in stages.

22.4.1 Vector Diagrams of Velocities

The steam supplied to a single-wheel impulse turbine expands completely in the nozzle and leaves with a high absolute velocity. The steam is directed on the rotor at an angle α. Therefore, the absolute inlet velocity has two components tangential and axial as shown in Fig. 22.7. The tangential component (with direction of rotation) is called the *whirl velocity* and is mainly responsible for useful work, while axial (vertical) component is called *axial* or *flow velocity*, allows the flow of steam

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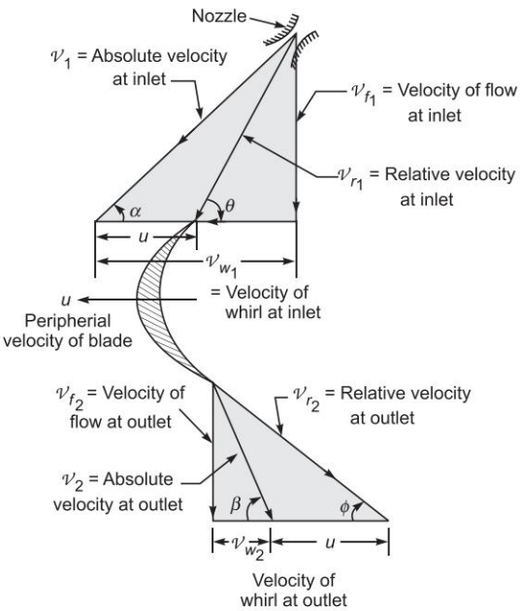

Fig. 22.7 *Velocity diagram for impulse turbine*

across the wheel. Since the blades are moving with a velocity u, then the velocity of steam relative to the blade at the inlet is called the *relative velocity,* which makes an angle θ with the blade.

To summarise, the various terms used in the vector diagram of velocity used in Fig. 22.7 and Fig. 22.8 are as given below:

(i) Inlet Vector Diagram

$\mathcal{V}_1$ = Absolute velocity of steam jet coming out the nozzle, m/s

α = Angle of steam jet in respect to plane of blade rotation, degree

u = Linear velocity of blade, m/s

$\mathcal{V}_{r_1}$ = Relative velocity of steam at inlet, m/s

θ = Blade inlet angle, degree

$\mathcal{V}_{w_1}$ = Whirl velocity, (component $\mathcal{V}_1$ in the direction of rotation), m/s

$\mathcal{V}_{f_1}$ = axial or flow velocity of steam, m/s

(ii) Exit Vector Diagram

$\mathcal{V}_2$ = Absolute velocity of steam at blade outlet, m/s

$\mathcal{V}_{r_2}$ = Relative velocity of steam at outlet, m/s

$\mathcal{V}_{f_2}$ = Axial velocity at discharge, m/s

$\mathcal{V}_{w_2}$ = Whirl velocity (component of $\mathcal{V}_2$ in the direction of rotation)

β = Steam jet outlet angle, degree

ϕ = Blade outlet angle, degree

The relative velocity at the inlet $\mathcal{V}_{r_1}$ is the vector difference of the absolute velocity $\mathcal{V}_1$ and the blade velocity u. The relative velocity $\mathcal{V}_{r_1}$ makes an angle θ with the axis of rotation. Since the steam does not expand in the moving blades, thus the relative velocity $\mathcal{V}_{r_2}$ at the exit in absence of any frictional effects, will be equal to relative velocity at the inlet. However, if there is friction then the two relative velocities are related as

$$\mathcal{V}_{r_2} = k\mathcal{V}_{r_1} \qquad ...(22.4)$$

where k is called the *blade velocity constant* and its value varies from 0.7 to 0.9, which takes into account the loss of velocity due to friction.

22.4.2 Combined Velocity Diagram

We have already drawn the velocity diagrams for velocities for moving blades at its inlet and exit on two different planes. Since the blade velocity u is common in both the vector diagrams of Fig. 22.7, thus both diagrams can be combined together on using common blade velocity u, as shown in Fig. 22.8 by superimposing one on the other.

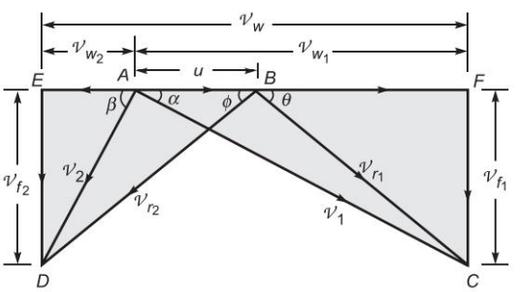

Fig. 22.8 *Combined inlet and outlet velocity diagram*

22.4.3 Procedure to Construct Combined Velocity Diagram

1. Draw a horizontal line. Mark a segment AB equal to blade velocity u to the scale.

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2. From the point A draw a line AC inclined at nozzle angle α and length equal to absolute velocity of steam $\mathcal{V}_1$ (to the scale).
3. Join points B and C. The line BC represents relative velocity $\mathcal{V}_{r_1}$ at the inlet.
4. From the point C, draw a vertical line CF. It represents axial (flow) velocity $\mathcal{V}_{f_1}$ at the inlet.
5. Measure the angle CBF as θ as blade inlet angle.
6. Calculate the relative velocity at the outlet $\mathcal{V}_{r_2} = k\mathcal{V}_{r_1}$.
7. From the point B, draw a line BD equal to BC inlined ϕ (blade ouitlet angle). The line BD represents relative velocity at the outlet.
8. Join points A and D. The line AD represents absolute velocity $\mathcal{V}_2$ of steam at the outlet.
9. Draw a vertical line DE from the point D. The line DE represents axial (flow) velocity $\mathcal{V}_{f_2}$ of steam at outlet.
10. Measure EA as whirl velocity $\mathcal{V}_{w_2}$ and AF as whirl velocity $\mathcal{V}_{w_1}$.

22.4.4 Analysis of Velocity Vector Diagrams (Refer Fig. 22.8)

(*i*) *Force on the Blade* By Newton's second law of motion, the tangential force is proportional to the rate of change of momentum, i.e.,

Tangential force

= Rate of mass flow × change in velocity in tangential direction

$$F = \dot{m}_s [\mathcal{V}_{w_1} - (-\mathcal{V}_{w_2})]$$

or $$F = \dot{m}_s [\mathcal{V}_{w_1} + \mathcal{V}_{w_2}] \quad ...(22.5)$$

where $\mathcal{V}_{w_2}$ is opposite in direction of $\mathcal{V}_{w_1}$.

$\dot{m}_s$ = Rate of mass flow of steam.

(*ii*) *Work done on the Blade*

Work = Force × displacement

Rate of work done,

$$\dot{W} = \text{Force} \times \text{blade velocity}$$

$$= \dot{m}_s (\mathcal{V}_{w_1} + \mathcal{V}_{w_2}) u = \dot{m}_s u \mathcal{V}_w \quad ...(22.6)$$

or $$\dot{W} = \dot{m}_s \frac{2\pi NR}{60} (\mathcal{V}_w) = \frac{2\pi NT}{60} (W) \quad ...(22.7)$$

where $T = \dot{m}_s R (\mathcal{V}_w)$ is turning moment that acts on the rotor.

$$\mathcal{V}_w = \mathcal{V}_{w_1} + \mathcal{V}_{w_2}$$

Horse power developed,

$$hp = \frac{\dot{W}}{746}$$

22.4.5 Axial Thrust

The vertical component of the absolute velocity is responsible to provide axial thrust on the bearings and other components and therefore, is undesirable. Axial thrust on the wheel is obtained due to difference between the velocities of flow (axial velocity) at entrance and outlet of blade.

Axial thrust

= Mass flow rate × change of axial velocity

$$= \dot{m}_s \times (\mathcal{V}_{f_1} - \mathcal{V}_{f_2}) \quad ...(22.8)$$

The axial thrust on the wheel must be balanced or thrust bearing may be used to take up the load.

22.4.6 Blade Efficiency

In a single-stage impulse turbine, the amount of energy supplied to the blades is equal to kinetic energy of entering steam, i.e., $\mathcal{V}_1^2/2$. Blade efficiency, also called *diagram efficiency*, can be calculated as

Blade efficiency,

$$\eta_b = \frac{\text{Work done on the blade}}{\text{Energy supplied to the blade}}$$

$$\eta_b = \frac{u \mathcal{V}_w}{\mathcal{V}_1^2/2}$$

or $$\eta_b = \frac{2u \mathcal{V}_w}{\mathcal{V}_1^2} \quad ...(22.9)$$

The blade efficiency may also be evaluated as

$$\eta_b = \frac{\text{Change in kinetic energy per kg}}{\text{Energy supplied per kg}}$$

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$$= \frac{\mathcal{V}_1^2 - \mathcal{V}_2^2}{\mathcal{V}_1^2} \qquad ...(22.10)$$

It will be maximum, when $\mathcal{V}_2^2$ is minimum, i.e., when $\beta = 90°$ or discharge is axial.

22.4.7 Stage Efficiency

A stage of an impulse turbine consists of a nozzle set and a moving wheel. The stage efficiency correlates the enthalpy drop in the nozzle and work done in the stage. The stage efficiency is sometimes referred as *gross efficiency.* It is expressed as

$$\eta_{stage} = \frac{\text{Work done on blade}}{\text{Energy supplied per stage}}$$

$$= \frac{u\mathcal{V}_w}{\Delta h} \qquad ...(22.11)$$

where $\Delta h = h_1 - h_2$ specific enthalpy drop of steam in the nozzle.

Further, the stage efficiency

= Blade efficiency × Nozzle efficiency

$$\eta_{stage} = \eta_b \times \eta_N \qquad ...(22.12)$$

Nozzle efficiency is given by

$$\eta_N = \frac{\mathcal{V}_1^2}{2(h_0 - h_1)} \qquad ...(22.13)$$

with h_0 = Enthalpy of steam at the entrance of nozzle, J/kg, and

h_1 = Enthalpy of steam at nozzle exit, J/kg

22.4.8 Net Efficiency or Overall Efficiency

The work delivered as a shaft output is always less than the work calculated from the velocity diagram due to mechanical friction. Thus, the *net efficiency* is defined as the ratio of work delivered at the shaft to total energy supplied in the stage.

$$\text{Net efficiency} = \frac{\text{Net work output at shaft}}{\text{total energy supplied to the stage}}$$

or $$\eta_{overall} = \eta_N \times \eta_{Stage} \times \eta_{mech.} \qquad ...(22.14)$$

Energy converted to heat by blade friction

= Loss of kinetic energy during flow over blades

$$= \dot{m}_s \times (\mathcal{V}_{r_1}^2 - \mathcal{V}_{r_2}^2) \qquad ...(22.15)$$

22.4.9 Blade-Velocity Coefficient

Due to presence of friction, when steam passes over the blades, the existing relative velocity is always less than inlet relative velocity.

Thus $\mathcal{V}_{r_2} = k\mathcal{V}_{r_1}$

where k is called the blade velocity coefficient.

22.5 OPTIMUM OPERATING CONDITIONS FROM BLADE-VELOCITY DIAGRAM

The work done per kg of steam flow in a single-stage impulse turbine is given by

$$w = u\mathcal{V}_w = u(\mathcal{V}_{w_1} + \mathcal{V}_{w_2})$$

From combined velocity triangle, Fig. 22.8;

$$\mathcal{V}_{w_1} = \mathcal{V}_1 \cos\alpha = \mathcal{V}_{r_1} \cos\theta + u$$

and $$\mathcal{V}_{w_2} = \mathcal{V}_2 \cos\beta = \mathcal{V}_{r_2} \cos\phi - u$$

Using above, we get work done per kg of steam in a single stage

$$w = u(\mathcal{V}_{r_1} \cos\theta + \mathcal{V}_{r_2} \cos\phi)$$

If there is no friction and for symmetrical blades,

$$\mathcal{V}_{r_1} = \mathcal{V}_{r_2} \quad \text{and} \quad \theta = \phi$$

Then the work done per kg of steam is

$$w = 2u\mathcal{V}_{r_1} \cos\theta = 2u(\mathcal{V}_1 \cos\alpha - u) \qquad ...(22.16)$$

Now blade or diagram efficiency is given by

$$\eta_b = \frac{2u(\mathcal{V}_1 \cos\alpha - u)}{\frac{\mathcal{V}_1^2}{2}}$$

$$= \frac{4u}{\mathcal{V}_1}\left(\cos\alpha - \frac{u}{\mathcal{V}_1}\right) \qquad ...(22.17)$$

Introducing the *blade-speed ratio;*

$$s = \frac{u}{\mathcal{V}_1}$$

$$\eta_b = 4s(\cos\alpha - s)^2$$

$$= 4s\cos\alpha - 4s^2 \qquad ...(22.18)$$

For a given steam velocity $\mathcal{V}_1$ and blade velocity u, it is clear from Eq. (22.16) that the work done per kg of steam would maximum, when cos α = 1 or α = 0. For zero value of α, the axial flow component would be zero. But it is essential to have

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an axial flow component to allow the steam to reach the blades and to clear the blades on leaving. As α increases, the work done on the blades reduces, but at the same time surface area of blades reduces, thus there are less frictional losses. With these conflicting requirements, the blade angle α is generally kept between 15° and 30°. For a given value of α, the optimum *blade-speed ratio, s* for maximum diagram efficiency can be obtained by differentiating Eq. (22.18) with respect to s and equating it to zero i.e.,

$$\frac{d\eta_b}{ds} = \frac{d}{ds}[4s\cos\alpha - 4s^2] = 0$$

or $$4\cos\alpha - 8s = 0$$

or $$s = \frac{u}{\mathcal{V}_1} = \frac{\cos\alpha}{2} \qquad ...(22.19)$$

It is the condition for maximum efficiency for impulse turbine. The maximum blade or diagram efficiency can be obtained by substituting $s = \frac{\cos\alpha}{2}$ in Eq. (22.18).

$$\eta_{b,\,max} = \frac{4\cos^2\alpha}{2} - \frac{4\cos^2\alpha}{4}$$

$$= \cos^2\alpha \qquad ...(22.20)$$

The variation of blade efficiency with blade-speed ratio, $s\left(=\frac{u}{\mathcal{V}_1}\right)$ is plotted in Fig. 22.9.

Further, the blade-speed ratio yields

$$\mathcal{V}_1 = \frac{2u}{\cos\alpha}$$

Substituting in Eq. (22.16), we get maximum work done on the blade

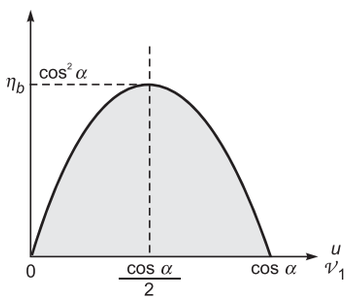

Fig. 22.9 *Diagram efficiency vs blade-speed ratio*

$$w_{max} = 2u\left(\frac{2u}{\cos\alpha}\times\cos\alpha - u\right)$$

$$= 2u^2 \qquad ...(22.21)$$

Example 22.1 *The steam expands isentropically in a simple impulse turbine from 12 bar, 250°C with an enthalpy of 2935 kJ/kg to an enthalpy of 2584 kJ/kg at 0.1 bar. The nozzle makes 20° with blade motion and the blades are symmetrical. Calculate the blade velocity that produces maximum efficiency for a turbine speed of 3600 rpm. Assume that the steam enters the nozzle with negligible velocity.*

Solution

Given A single-stage impulse turbine

$p_0 = 12$ bar $\quad T_0 = 250°C \quad p_b = 0.1$ bar

$h_0 = 2935$ kJ/kg $\quad h_1 = 2584$ kJ/kg $\quad \alpha = 20°$

$N = 3600$ rpm $\quad \phi = \theta \quad \mathcal{V}_0 = 0$

To find Blade velocity.

Analysis The change in specific enthalpy of steam

$$\Delta h = h_0 - h_1 = 2935 - 2584 = 351 \text{ kJ/kg}$$

The steam jet velocity

$$\mathcal{V}_1 = 44.72 \times \sqrt{\Delta h} \qquad (\because \mathcal{V}_0 = 0)$$

or $$\mathcal{V}_1 = 44.72 \times \sqrt{(351 \text{ kJ/kg})} = 837.83 \text{ m/s}$$

For maximum velocity, Eq. (22.19)

$$\frac{u}{\mathcal{V}_1} = \frac{\cos\alpha}{2}$$

or $$u = \frac{\mathcal{V}_1\cos\alpha}{2} = \frac{837.83\times\cos 20°}{2} = \mathbf{393.65\ m/s}$$

Example 22.2 *The steam leaves the nozzles of a single-row impulse turbine at 900 m/s. The nozzle angle is 20° and blade angles are 30° at inlet and outlet. Calculate the blade velocity and work done per kg of steam. Assume the flow over the blade is frictionless.*

Solution

Given A single-row impulse turbine

$\mathcal{V}_1 = 900$ m/s, $\quad \alpha = 20°$

$\theta = \phi = 30°$

To find

(i) Blade velocity, and

(ii) Work done per kg of steam.

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Analysis The velocity diagram is constructed as explained below:

(i) Draw a horizontal line and choose a point *A* and draw a line *AC* inlined at 20° to the horizontal line. Its length represents absolute steam velocity of 900 m/s.

(ii) Draw a vertical line *CD* through the point *C*. It represents inlet flow velocity, $\mathcal{V}_{f_1}$.

(iii) Draw a line *BC* through the point *C* making an angle of (90° – 30° =) 60° with vertical line *CD*. Line *BC* will also make blade inlet angle of $\theta = 30°$ with horizontal and will represents the relative velocity $\mathcal{V}_{r_1}$ at inlet.

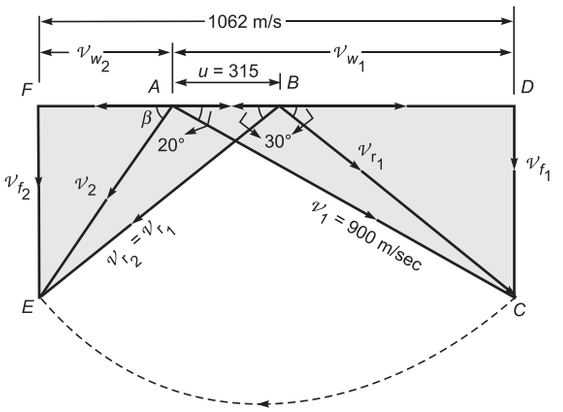

Fig. 22.10

(iv) Measure length of segment *AB* along horizontal line. Its magnitude represents *blade velocity u* = **315 m/s.**

(v) Draw the line *BE* through the point *B* at angle of $\phi = 30°$ and length is equal to $\mathcal{V}_{r_2} = \mathcal{V}_{r_1}$.

(vi) Join *A* and *E*. Line *AE* represents absolute velocity of steam at exit.

(vii) Draw a vertical line *EF* through point *E* to represent flow velocity $\mathcal{V}_{f_2}$ at outlet.

(viii) Measure the length of line $FD = \mathcal{V}_{w_2} + \mathcal{V}_{w_2}$ = 1062 m/s.

Work done per kg of steam

$$w = u\mathcal{V}_w = 315 \times 1062$$
$$= 334530 \text{ J/kg} = \textbf{334.53 kJ/kg}$$

Example 22.3 *In a single-stage, single-row impulse turbine, the steam is entering at a velocity of 1200 m/s with a nozzle angle of 20° and leaving the blade in the axial direction. The ratio of blade velocity to whirl velocity of steam is 0.6. Sketch the velocity diagram and calculate.*

(a) Blade velocity,

(b) Work done per kg of steam.

Solution

Given A single-stage, single-row impulse turbine

$\mathcal{V}_1 = 1200$ m/s $\qquad \alpha = 20°$

$\beta = 90°$ (Axial discharge) $\qquad \dfrac{u}{\mathcal{V}_w} = 0.6$

To find

(i) Blade velocity, and

(ii) Work done per kg of steam.

Assumption Adiabatic turbine and expansion of steam without friction.

Analysis The velocity diagram is constructed as explained below:

(i) Draw a horizontal line and choose a point *A* and draw a line *AC* inlined at 20° to the horizontal line. Its length represents absolute steam velocity of 1200 m/s.

(ii) Draw a vertical line *CD* through the point *C*. It represents inlet flow velocity, $\mathcal{V}_{f_1}$.

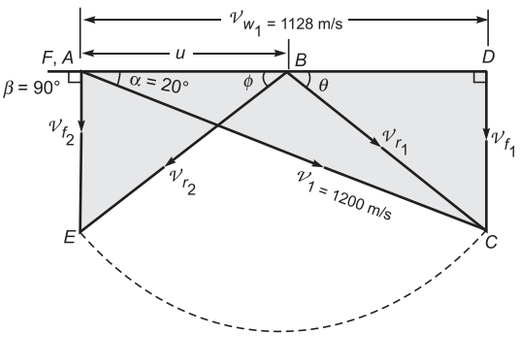

Fig. 22.11 *Velocity diagram for axial flow turbine*

(iii) For axial flow turbine $\mathcal{V}_{w_2} = 0$; hence $\mathcal{V}_w = \mathcal{V}_{w_1}$ = length of line *AD* = 1128 m/s.

Calculate $u = 0.6\mathcal{V}_w = 676.8$ m/s.

(iv) Mark a point *B* along the horizontal line *AD*. Length *AB* represents *blade velocity u* = **676.8 m/s.**

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Work done per kg of steam

$$w = u\mathcal{V}_w = 676.8 \times 1128$$
$$= 763430.4 \text{ J/kg} = \mathbf{763.43 \text{ kJ/kg}}$$

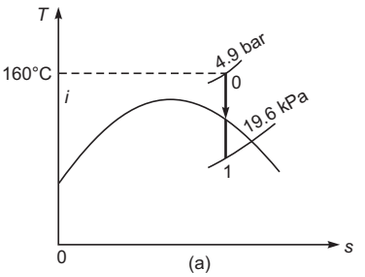
Example 22.4 *The steam at 4.9 bar and 160°C is supplied to a single-stage impulse turbine at a mass flow rate of 30 kg/min, from where it is exhausted to a condenser at a pressure of 19.6 kPa. The blade speed is 300 m/s. The nozzles are inclined as 25° to the plane of wheel and the outlet blade angle is 35°.*

Neglecting friction losses, determine (a) theoretical power developed by the turbine, (b) diagram efficiency, and (c) stage efficiency.

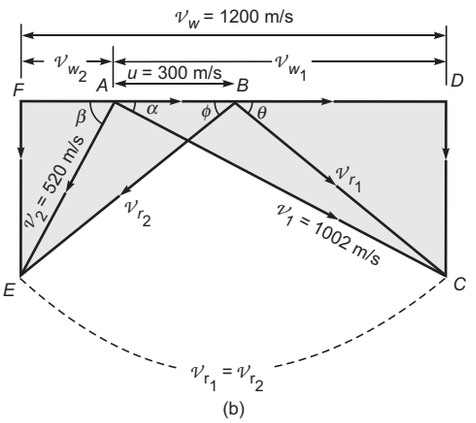
Solution

Given A single-stage impulse turbine

$p_0 = 4.9$ bar $\quad p_b = 19.6$ kPa
$T_0 = 160°C \quad \alpha = 25°$
$u = 300$ m/s $\quad \phi = 35°$
$\dot{m}_s = 30$ kg/min



(a)



(b)

Fig. 22.12

To find

(i) Power developed by turbine,
(ii) diagram efficiency, and
(iii) Stage efficiency.

Assumptions

(i) Total pressure drop in nozzle only,
(ii) Adiabatic turbine and isentropic expansion of steam.

Properties of steam from Mollier diagram

At 4.9 bar and 160°C

$h_0 = 2777$ kJ/kg $\quad s_0 = 6.82$ kJ/kg·K

At 19.6 kPa and $s_1 = s_0 = 6.82$

$h_1 = 2275$ kJ/kg

Analysis The change in specific enthalpy of steam

$$\Delta h = h_0 - h_1 \; 2777 - 2275 = 502 \text{ kJ/kg}$$

The steam jet velocity ($\mathcal{V}_0 = 0$)

$$\mathcal{V}_1 = 44.72 \times \sqrt{\Delta h}$$

or $$\mathcal{V}_1 = 44.72 \times \sqrt{(502 \text{ kJ/kg})} = 1002 \text{ m/s}$$

Velocity diagram

For vector diagram, using

$\mathcal{V}_1 = 1002$ m/s $\quad u = 300$ m/s
$\alpha = 25° \quad \beta = 35°$

(i) By measuring *EF*,

$$\mathcal{V}_w = 1200 \text{ m/s}$$

Power developed in the stage

$$P = \dot{m}_s \, u\mathcal{V}_w$$
$$= \left(\frac{30}{60} \text{ kg/s}\right) \times (300 \text{ m/s}) \times (1200 \text{ m/s})$$
$$= 180 \times 10^3 \text{ W} = \mathbf{180 \text{ kW}}$$

(ii) Diagram (blade) efficiency

$$\eta_b = \frac{2u\mathcal{V}_w}{\mathcal{V}_1^2}$$
$$= \frac{2 \times (300 \text{ m/s}) \times (1200 \text{ m/s})}{(1002 \text{ m/s})^2}$$
$$= 0.717 \text{ or } \mathbf{71.7\%}$$

(iii) Stage efficiency

$$\eta_{stage} = \frac{u\mathcal{V}_w}{\Delta h}$$

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$$= \frac{(300\,\text{m/s}) \times (1200\,\text{m/s})}{(502 \times 10^3\,\text{J/kg})}$$

$$= 0.717 \text{ or } \mathbf{71.7\%}$$

Example 22.5 *A single-stage impulse turbine is supplied with steam at 4 bar and 160°C and it is exhausted at a condenser pressure of 0.15 bar at the rate of 60 kg/min. The steam expands in a nozzle with an efficiency of 90%. The blade speed is 250 m/s and the nozzles are inclined at 20° to the plane of the wheel. The blade angle at the exit of the moving blade is 30°. Neglecting friction losses in the moving blades, determine,*

(a) Steam jet velocity,
(b) Power developed,
(c) Blade efficiency,
(d) Stage efficiency.

Solution

Given The single-stage impulse steam turbine with

$p_0 = 4$ bar, 160°C $p_b = 0.15$ bar
$\dot{m}_s = 60$ kg/min $\eta_N = 0.9$
$u = 250$ m/s $\alpha = 20°$
$\phi = 30°$

To find

(i) Steam jet velocity,
(ii) Power developed,
(iii) Blade efficiency, and
(iv) Stage efficiency.

Assumptions

(i) Isentropic expansion in the turbine.
(ii) No frictional losses, thus $\mathcal{V}_{r_1} = \mathcal{V}_{r_2}$.

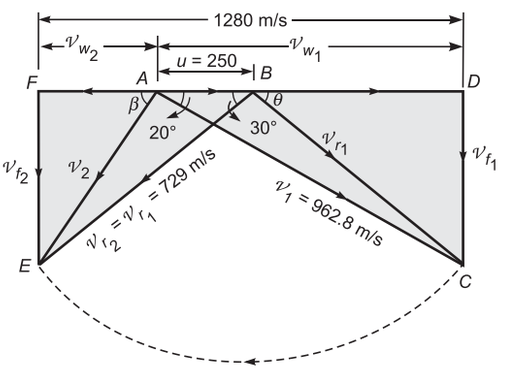

Fig. 22.13

Analysis

(i) From the Mollier diagram

h_0 @ 4 bar, 160°C = 2780 kJ/kg
h_1 @ 0.15 bar = 2265 kJ/kg,

when $s_0 = s_1$

$\Delta h = h_0 - h_1 = 2780 - 2265 = 515$ kJ/kg

Actual heat drop,

$$\Delta h_{act} = \eta_N \times \Delta h$$
$$= 0.9 \times 515 = 463.5 \text{ kJ/kg}$$

Steam jet velocity,

$$\mathcal{V}_1 = 44.72 \times \sqrt{\Delta h_{act}}$$
$$= 44.72 \times \sqrt{(463.5\,\text{kJ/kg})}$$
$$= \mathbf{962.8\ m/s}$$

The velocity diagram can be drawn as in Fig. 22.13.

By measurement we get,

$\mathcal{V}_{w_1} + \mathcal{V}_{w_2} = FD = 1280$ m/s

(ii) Power developed,

$$P = \dot{m}_s \mathcal{V}_w u$$
$$= \left(\frac{60}{60}\,\text{kg/s}\right) \times (1280\,\text{m/s}) \times (250\,\text{m/s})$$
$$= 320 \times 10^3\,\text{W} = \mathbf{320\ kW}$$

(iii) Blade efficiency,

$$\eta_b = \frac{2\mathcal{V}_w u}{\mathcal{V}_1^2}$$
$$= \frac{2 \times (1280\,\text{m/s}) \times (250\,\text{m/s})}{(962.8\,\text{m})^2}$$
$$= 0.6904 = \mathbf{69.04\%}$$

(iv) Stage efficiency,

$$\eta_s = \frac{\mathcal{V}_w u}{\Delta h}$$
$$= \frac{(1280\,\text{m/s}) \times (250\,\text{m/s})}{(515 \times 10^3\,\text{J/kg})}$$
$$= 0.621 = \mathbf{62.1\%}$$

Example 22.6 *The rotor of an impulse turbine is of 260 mm diameter and runs at 20,500 rpm. The nozzle angle is 20°, and issues a steam jet with a velocity of 910 m/s. The mass flow rate through the turbine nozzle's blading is 2.0 kg of steam per second. Draw the velocity diagram and calculate the (a) tangential force on blades,*

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(b) axial force on blades, (c) power developed by the turbine wheel, (d) efficiency of the blading, (e) inlet angle of blades for shockless inflow of steam.

Solution

Given A single-stage impulse turbine

$D = 260$ mm $\quad N = 20{,}500$ rpm $\quad \alpha = 20°$

$\mathcal{V}_1 = 910$ m/s $\quad \dot{m}_s = 2.0$ kg/s $\quad \theta = \phi$

To find

(i) Tangential force on the blades,
(ii) Axial force on the blades,
(iii) Power developed by the turbine wheel,
(iv) Efficiency of the blading, and
(v) Inlet angle of the blades for shockless inflow of steam.

Assumptions

(i) Adiabatic turbine and isentropic flow of steam in the turbine, and
(ii) No friction on the turbine blade, thus $\mathcal{V}_{r_1} = \mathcal{V}_{r_2}$.

Analysis The linear blade velocity

$$u = \frac{\pi DN}{60} = \pi \times (0.26\,\text{m}) \times \left(\frac{20500}{60}\right) \text{rps} = \mathbf{279\ m/s}$$

The measurements on velocity diagram is shown in Fig. 22.14, reveal that

Relative velocity at inlet, $\mathcal{V}_{r_1} = 654.5$ m/s
Axial velocity at inlet, $\mathcal{V}_{f_1} = 325.5$ m/s
Axial velocity at outlet, $\mathcal{V}_{f_2} = 325.5$ m/s
Whirl velocity at inlet, $\mathcal{V}_{w_1} = 854$ m/s
The total whirl velocity, $\mathcal{V}_w = 1114$ m/s
Inlet blade angle, $\theta = 29°$

No, friction loss, $\mathcal{V}_{r_2} = \mathcal{V}_{r_1} = 654.5$ m/s
For shockless flow, $\theta = \phi = 29°$

(i) Tangential force on the blade

$$F = \dot{m}_s (\mathcal{V}_{w_1} + \mathcal{V}_{w_2}) = \dot{m}_s \mathcal{V}_w$$
$$= (2\ \text{kg/s}) \times (1114\ \text{m/s})$$
$$= 2228\ \text{N} = \mathbf{2.228\ kN}$$

(ii) Axial force on the blade

$$= \dot{m}_s (\mathcal{V}_{f_1} - \mathcal{V}_{f_2})$$
$$= (2\ \text{kg/s}) \times (325.5 - 325.5) = 0$$

(iii) Power developed by the turbine

$$P = Fu$$
$$= (2.228\ \text{N}) \times (279\ \text{m/s})$$
$$= 621.6\ \text{kN m/s} = \mathbf{621.6\ kW}$$

(iv) Blade efficiency

$$\eta_b = \frac{2u\mathcal{V}_w}{\mathcal{V}_1^2}$$
$$= \frac{2 \times (279\,\text{m/s}) \times (1114\,\text{m/s})}{(910\,\text{m/s})^2}$$
$$= 0.75 = \mathbf{75\%}$$

(v) Inlet angle of blade for shockless inflow, $\theta = \mathbf{90°}$

22.6 EFFECT OF BLADE FRICTION ON VELOCITY DIAGRAM

In the earlier section, the effect of friction is neglected and relative velocity at outlet, $\mathcal{V}_{r_2}$ is assumed equal to the inlet relative velocity, $\mathcal{V}_{r_1}$. In actual practice, the blades offer frictional flow of steam over them, and thus the exit relative velocity

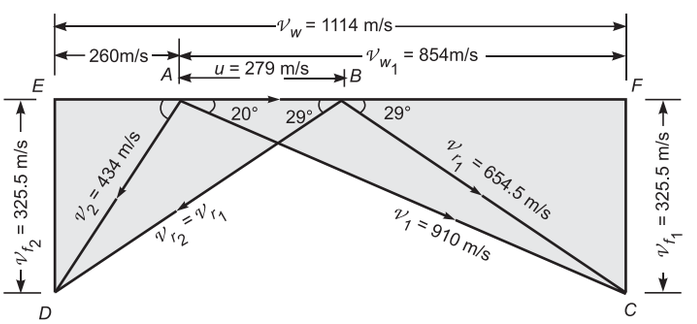

Fig. 22.14 *Velocity diagram*

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is always less than the inlet relative velocity. This frictional effect reduces the relative velocity in the order of 10 to 15% as steam passes over the blades. The two relative velocities are related as

$$\mathcal{V}_{r_2} = k\mathcal{V}_{r_1}$$

where k is called the *blade-velocity coefficient* or *friction coefficient*. The modified velocity diagram is shown in Fig. 22.15.

Mark the point G on BC such that

$$BG = k\,BC = k\mathcal{V}_{r_1}$$

Now with B as centre and BG as radius, draw an arc along BD to locate the position D. Then complete the outlet velocity triangle in the usual manner.

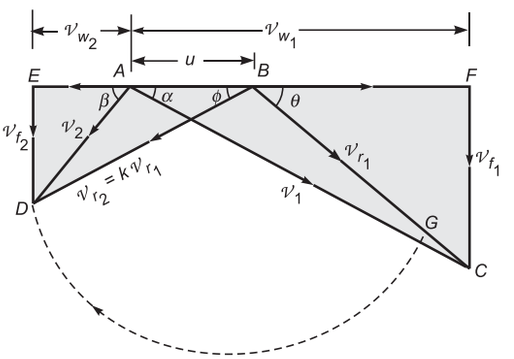

Fig. 22.15 *Actual velocity diagram*

22.7 CONDITION FOR AXIAL DISCHARGE

When the blades of an impulse turbine are designed in such a way that the steam leaving the blades at its tip at 90° to the direction of blade peripheral velocity u then the turbine is called *axial flow turbine*. For an axial flow turbine, $\beta = 0$; $\mathcal{V}_2 = \mathcal{V}_{f_2}$ and $\mathcal{V}_{w_2} = 0$.

A typical velocity diagram is shown in Fig. 22.16.

The power developed by axial flow turbine ($\mathcal{V}_{w_2} = 0$)

$$P = \dot{m}_s u\mathcal{V}_w = \dot{m}_s u\mathcal{V}_{w_1} \qquad ...(22.23)$$

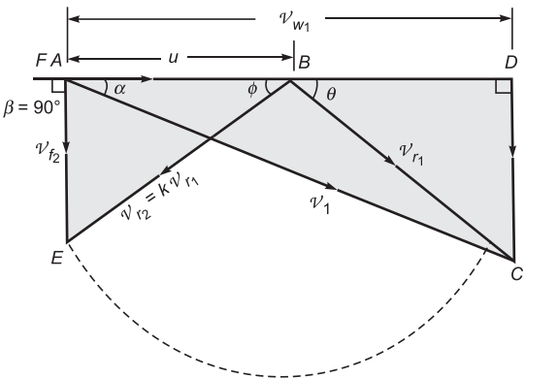

Fig. 22.16 *Velocity diagram for axial flow impulse turbine*

***Example* 22.7** *Steam leaves the nozzle of a single-stage impulse turbine at 840 m/s. The nozzle angle is 18° and the blade angles are 29° at the inlet and outlet. The friction coefficent is 0.9.*

Calculate

(a) Blade velocity,

(b) Steam mass flow rate in kg/h to develop 300 kW power.

Solution

Given The single-stage impulse steam turbine

$\mathcal{V}_1 = 840$ m/s $\qquad \alpha = 18°$

$\phi = \theta = 29°$ $\qquad k = \dfrac{\mathcal{V}_{r_1}}{\mathcal{V}_{r_2}} = 0.9$

$P = 300$ kW

To find

(i) Blade velocity, u (m/s)

(ii) The mass flow rate in (kg/h).

Analysis The velocity diagram can be drawn as shown in Fig. 22.17.

(i) Draw a horizontal line. At the point A, with $\alpha = 18°$ and $\mathcal{V}_1 = 840$ m/s (to the scale) draw a line AC.

(ii) At the point C, draw a vertical line CD. Through the point C with an angle $90° - \theta = 61°$, with vertical line; draw a line BC, whose measurement gives us $\mathcal{V}_{r_1} = 535$ m/s.

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(iii) With $\mathcal{V}_{r_2} = 0.9\ \mathcal{V}_{r_1} = 481.5$ and $\theta = 29°$, draw a line *BE*.

(iv) Meet $BE = \mathcal{V}_2$.

(v) Meet *EF*, its measurement gives $\mathcal{V}_{f_2}$.

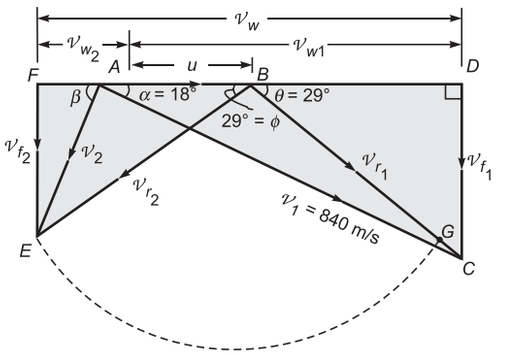

Fig. 22.17

The measurement shows that

$u = 330$ m/s

$\mathcal{V}_w = 890$ m/s

(b) Power developed

$$P = \frac{\dot{m}_s\, u\, \mathcal{V}_w}{1000} \text{ kW}$$

$$300 = \frac{\dot{m}_s \times 330 \times 890}{1000}$$

or $\dot{m}_s = 1.02$ kg/s

Alternatively, the velocities can be worked out trignometrically as follows:

From ΔACF,

$AD = \mathcal{V}_1 \cos\alpha$

or $\mathcal{V}_{w_1} = 840 \cos 18° = 798.9$ m/s

From $\Delta_s\, ACD$ and BCD

$CD = AC \sin\alpha = BC \sin\theta$

or $\mathcal{V}_1 \sin\alpha = \mathcal{V}_{r_1} \sin\theta$

$$\therefore \quad \mathcal{V}_{r_1} = \frac{840 \sin 18°}{\sin 29°} = 535.4 \text{ m/s}$$

$\mathcal{V}_{r_2} = k\mathcal{V}_{r_1} = 0.9 \times 535.4 = 481.9$ m/s

$u = \mathcal{V}_{w_1} - \mathcal{V}_{r_1} \cos 29°$

$= 798.9 - 535.4 \cos 29° = 330.6$ m/s

$\mathcal{V}_{w_2} = \mathcal{V}_{r_2} \cos\phi - u = 481.9 \times \cos 29° - 330.6$

$= 421.5 - 330.6 = 90.9$ m/s

$\mathcal{V}_w = \mathcal{V}_{w_1} + \mathcal{V}_{w_2} = 798.9 + 90.9 = 889.89$ m/s

Example 22.8 *The steam is supplied to a de-Laval turbine at a velocity of 1000 m/s at an angle of 20°. The blade velocity is 300 m/s and the blades are symmetrical. The mass-flow rate of the steam is 0.5 kg/s. Allowing a friction factor of 0.8, determine.*

(a) Blade efficiency

(b) Power developed

(c) Stage efficiency, if the nozzle efficiency is 95%

Solution

Given A de-Laval steam turbine

$\mathcal{V}_1 = 1000$ m/s $\quad \alpha = 20°$

$u = 300$ m/s $\quad \theta = \phi$

$\dot{m}_s = 0.5$ kg/s $\quad k = \dfrac{\mathcal{V}_{r_2}}{\mathcal{V}_{r_1}} = 0.8$

$\eta_N = 0.95$

To find

(i) Blade efficiency,

(ii) Power developed, and

(iii) Stage efficiency.

Assumptions

(i) Adiabatic expansion in the turbine.

(ii) Change in potential energy is negligible.

Analysis The velocity diagram for given data is drawn as shown below. The measurements reveal that

$\mathcal{V}_w = 1162$ m/s

(i) Blade efficiency,

$$\eta_b = \frac{2u\mathcal{V}_w}{\mathcal{V}_1^2}$$

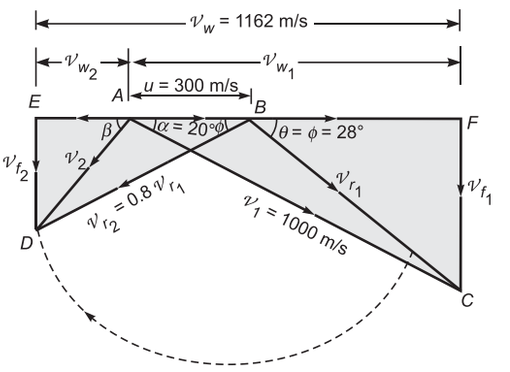

Fig. 22.18

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$$= \frac{2 \times 300 \times 1162}{(1000)^2} = 0.697 = \mathbf{69.7\%}$$

(ii) Power developed,

$$p = \dot{m}_s \frac{u \mathcal{V}_w}{1000}$$

$$= 0.5 \times \frac{300 \times 1162}{1000} = \mathbf{174.3\ kW}$$

(iii) Stage efficiency,

$$\eta_{stage} = \text{Nozzle efficiency} \times \text{Blade efficiency}$$

$$= 0.95 \times 0.697 = 0.662 = \mathbf{66.2\%}$$

Example 22.9 *In a de-Laval turbine, steam issues from the nozzles with a velocity of 850 m/s. The nozzle angle is 20°. Mean blade velocity is 350 m/s and the blades are equiangular. The mass-flow rate is 1000 kg/min. The friction factor is 0.8. Determine:*

(a) Blade angles,
(b) Axial thrust on the bearings,
(c) Power developed in kW,
(d) Blade efficiency,
(e) Stage efficiency, if nozzle efficiency is 93%.

Solution

Given The de-Laval turbine

$\mathcal{V}_1 = 850$ m/s $\quad \alpha = 20°$

$u = 350$ m/s

$\dot{m}_s = 1000$ kg/min $= 16.667$ kg/s

$\phi = \theta \quad \eta_N = 0.93$

$k = \dfrac{\mathcal{V}_{r_1}}{\mathcal{V}_{r_2}} = 0.8$

To find

(i) Blade angles,
(ii) Axial thrust on the bearings,
(iii) Power developed in kW,
(iv) Blade efficiency, and
(v) Stage efficiency.

Assumptions

(i) Adiabatic turbine.
(ii) Change in potential energy of the steam during expansion is negligible.

Analysis The velocity diagram for given data is drawn as shown in Fig. 22.19, with some convenient scale.

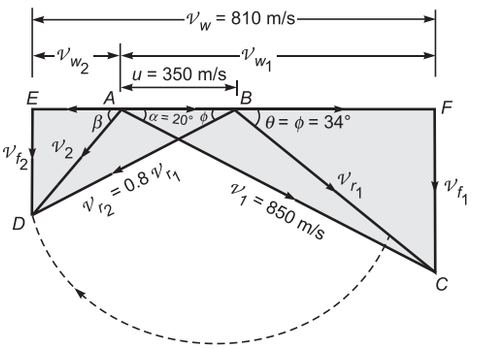

Fig. 22.19

(i) Draw a line AB equal to the blade velocity 350 m/s.

(ii) From the point A, draw a line AC equal to $\mathcal{V}_1 = 850$ m/s at an angle 20°.

(iii) Meet the points B and C, and measure angle CBF, $\theta = 34°$ and length BC ($\mathcal{V}_{r_1}$) = 535 m/s.

(iv) From the point B, draw a line BD equal to $0.8\ \mathcal{V}_{r_1}$ at an angle equal to $\phi = 34°$.

The measurements show that

inlet relative velocity, $\mathcal{V}_{r_1} = 535$ m/s

exit relative velocity, $\mathcal{V}_{r_2} = 0.8\ \mathcal{V}_{r_1}$

$= \mathbf{428\ m/s.}$

(i) Nozzle angle $\beta = \phi = \mathbf{34°}$

Axial velocities, $\mathcal{V}_{f_1} = 300$ m/s, and $\mathcal{V}_{f_2} = 240$ m/s

The total whirl velocity, $\mathcal{V}_w = \mathcal{V}_{w_1} + \mathcal{V}_{w_2}$

$= 790 + 20 = 810$ m/s

(ii) Axial thrust on bearings

Axial force $= \dot{m}_s (\mathcal{V}_{f_1} - \mathcal{V}_{f_2})$

$= (16.667\ \text{kg/s}) \times (300 - 240)$

$= \mathbf{1000\ N}$

(iii) Power developed,

$$P = \frac{\dot{m}_s u \mathcal{V}_w}{1000} = \frac{16.667 \times 350 \times 810}{1000}$$

$$= \mathbf{4725\ kW}$$

(iv) Diagram or blade efficiency,

$$\eta_b = \frac{2u\mathcal{V}_w}{\mathcal{V}_1^2}$$

$$= \frac{2 \times 350 \times 810}{(850)^2}$$

$= 0.7847$ or **78.47%**

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(v) Stage efficiency

$$\eta_{stage} = \eta_b \times \eta_N$$
$$= 0.7847 \times 0.93 = 0.7298 \text{ or } \mathbf{72.98\%}$$

Example 22.10 *The nozzles of a de-Laval turbine deliver 1.5 kg/s of steam at a speed of 800 m/s to a ring of moving blades having a speed of 200 m/s. The exit angle of the nozzle is 18°. If the blade velocity coefficient is 0.75 and the exit angle of the moving blades is 25°, calculate.*

(a) Inlet angle of moving and fixed blades,
(b) Diagram efficiency,
(c) Energy lost in blades per second,
(d) Power developed, and
(e) Axial thrust on the turbine rotor.

Solution

Given A de-Laval turbine

$\dot{m}_s = 1.5$ kg/s
$\alpha = 18°$ $\quad \phi = 25°$
$u = 200$ m/s $\quad \mathcal{V}_1 = 800$ m/s
$k = \dfrac{\mathcal{V}_{r_2}}{\mathcal{V}_{r_1}} = 0.75$

To find

(i) Inlet angle of moving and fixed blades,
(ii) Diagram efficiency,
(iii) Energy lost in blades per second,
(iv) Power developed, and
(v) Axial thrust on the turbine rotor.

Analysis The velocity diagram can be drawn as shown in Fig. 22.20. By measurement, we get,

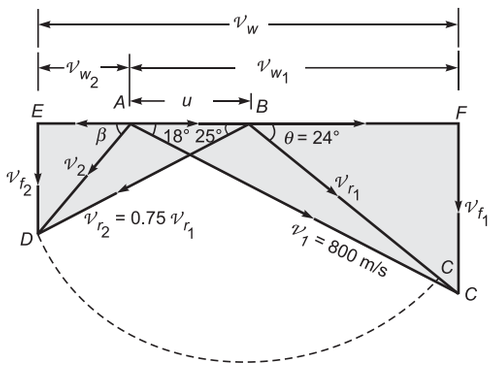

Fig. 22.20

(i) Inlet angle of moving blade, $\theta = 24°$.
Inlet angle of fixed blades, $\beta = \mathbf{42.5°}$
By measurement,

$\mathcal{V}_w = EF = 977$ m/s
$\mathcal{V}_{r_1} = AC = 612.5$ m/s
$\mathcal{V}_{r_2} = 0.75\,\mathcal{V}_{r_1} = 459.4$ m/s
$\mathcal{V}_{f_1} = FC = 247$ m/s
$\mathcal{V}_{f_2} = 194$ m/s

(ii) Diagram efficiency, $\eta_b = \dfrac{2u\mathcal{V}_w}{\mathcal{V}_1^2}$

or $\eta_b = \dfrac{2 \times 200 \times 977}{(800)^2}$
$= 0.6106$ or $\mathbf{61.06\%}$

(iii) Energy lost in blades

$$\Delta e = \frac{\mathcal{V}_{r_1}^2 - \mathcal{V}_{r_2}^2}{2}$$
$$= \frac{(612.5)^2 - (459.4)^2}{2}$$
$$= 82065 \text{ Nm/kg}$$
$$= \mathbf{82.065 \text{ kJ/kg}}$$

(iv) Power developed

$$P = \dot{m}_s \frac{\mathcal{V}_w\, u}{1000}$$
$$= 1.5 \times \frac{977 \times 200}{1000} = \mathbf{293.1 \text{ kW}}$$

(v) Axial thrust on rotor

$$F_a = \dot{m}_s (\mathcal{V}_{f_1} - \mathcal{V}_{f_2})$$
$$= 1.5 \times (247 - 194)$$
$$= \mathbf{79.5 \text{ N}}$$

Example 22.11 *A simple de Laval steam turbine, nozzle ring is suplied with 200 kg/h of dry steam at 10 bar. The exhaust pressure is 0.6 bar. The nozzle angle is 20° and nozzle efficiency is 85%. The blade outlet angle is 28° and blade speed is 200 m/s. The blade velocity coefficient is 0.78. The power loss due to fan action and disc friction is 1.5 kW. The losses reappear as heat. Calculate*

(a) Net output at the shaft
(b) Dryness fraction of steam in exhaust passage, when speed is small
(c) The dryness fraction of steam as it leaves the turbine rotor bladding

Solution

Given A simple steam turbine

$\dot{m}_s = 200$ kg/h $= 0.055$ kg/s

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$\alpha = 20°$ $\phi = 28°$

$u = 200$ m/s $p_0 = 10$ bar (dry steam)

$p_1 = 0.6$ bar $k = \dfrac{\mathcal{V}_{r_2}}{\mathcal{V}_{r_1}} = 0.78$

$\eta_N = 0.85$

Power loss in fan action and friction = 1.5 kW.

To find

(i) Net output at shaft,

(ii) Dryness fraction of exhaust steam, after heating by frictional heat, and

(iii) Dryness fraction of steam as leaves the turbine rotor.

Analysis The properties of steam at nozzle inlet and exit

At 10 bar (dry and saturated)

$h_1 = 2780$ kJ/kg (from Mollier diagram)

At 0.6 bar ($s_1 = s_2$)

$h_2 = 2300$ kJ/kg

$x_2 = 0.845$

Change in enthalpy,

$\Delta h = 2780 - 2300 = 480$ kJ/kg

(i) *The absolute velocity of steam at nozzle exit*

$$\mathcal{V}_1 = 44.72\sqrt{\eta_N(\Delta h)}$$
$$= 44.72 \times \sqrt{0.85 \times 480}$$
$$= 903.3 \text{ m/s}$$

The velocity diagram can be drawn as shown in Fig. 22.21. By measurement, inlet angle of moving blade, $\theta = 26°$.

$\mathcal{V}_w = EF = 1140$ m/s

$\mathcal{V}_{r_1} = BC = 731$ m/s

$\mathcal{V}_{r_2} = 0.78\,\mathcal{V}_{r_1} = 570$ m/s

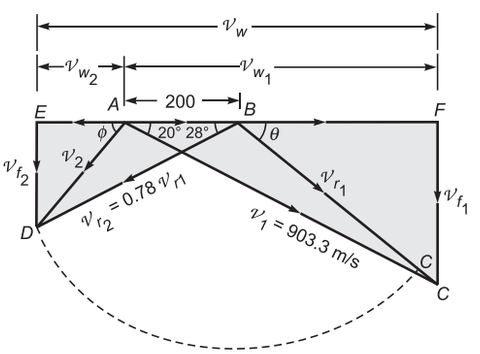

Fig. 22.21

$\mathcal{V}_{f_1} = FC = 320$ m/s

$\mathcal{V}_{f_2} = 272$ m/s

Theoretical power developed,

$P_{th} = \dot{m}_s\, u\, \mathcal{V}_w$

$= 0.350 \times 1140 = 12540$ Nm/s

$= 12.54$ kW

Net power output,

$P = 12.54 - 1.5 =$ **11.04 kW**

(ii) *Dryness fraction of exhaust steam, after heating by frictional heat*:

When the speed of the turbine is small, then all the frictional heat will be utilized to heat the steam. Therefore, the change in enthalpy will be

$$\Delta h_1 = 480 + \frac{1.5\,\text{kW}}{0.55\,\text{kg/s}} = 507.27 \text{ kJ/kg}$$

The enthalpy at the turbine exhaust

$h_{2'} = h_1 - \Delta h_1$

$= 2780 - 507.27 = 2272.73$ kJ/kg

From Mollier diagram at 0.6 bar and 2272.73 kJ/kg

$x_{2'} =$ **0.88**

(iii) *Dryness fraction of steam as it leaves the turbine rotor*

Dryness fraction of steam as it leaves the turbine rotor at $h = 2300$ kJ/kg and $s_1 = s_2$.

$x_2 =$ **0.845**

Example 22.12 *A simple impulse turbine has a mean blade-ring diameter of 70 cm and runs at 3000 rpm. The speed ratio is 0.46 and the discharge is axial. The nozzle angle is 21° and blade friction factor is 0.95. Determine*

(a) Blade angles,

(b) Theoretical specific power output.

Solution

Given A simple impulse turbine

$D = 70$ cm $N = 3000$ rpm

$\beta = 90°$

speed ratio, $s = \dfrac{u}{\mathcal{V}_1} = 0.46$

$\alpha = 21°$ $k = 0.95$

To find

(i) Blade angles, and

(ii) Theoretical specific power output.

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Analysis

(i) For axial discharge, $\beta = 90°$

The blade velocity,

$$u = \frac{\pi DN}{60} = \frac{\pi \times 70 \times 10^{-2} \times 3000}{60}$$
$$= 110 \text{ m/s}$$

The nozzle exit velocity,

$$\mathcal{V}_1 = \frac{u}{s} = \frac{110}{0.46} = 239 \text{ m/s}$$

The relation between two relative velocities.

$$\frac{\mathcal{V}_{r_2}}{\mathcal{V}_{r_1}} = 0.95$$

The velocity diagram can be constructed as follows:

(a) Draw a horizontal line *EF*. Mark $AB = u =$ 110 m/s.

(b) At point *A*, $\alpha = 21°$ and $\mathcal{V}_1 = 239$ m/s, draw a line *AC*.

(c) Meet *BC*, it gives θ **= 37.12°,** inlet blade angle.

(d) Using centre *B* and radius $BG = 0.95\ BC$, draw a chord to cut vertical line *AD* at point *D*.

(e) Meet *BD*, *DA* and *CF*.

The velocity diagram for given data can be drawn as shown in Fig. 22.22.

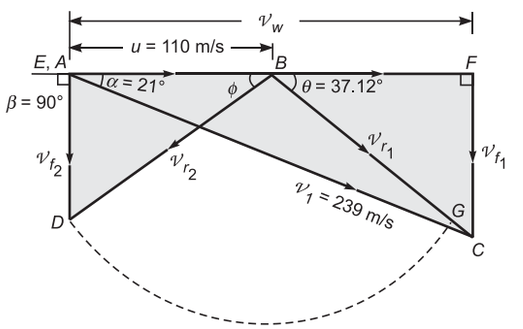

Fig. 22.22

The measurements on the velocity diagram give, whirl velocity $\mathcal{V}_w = 224$ m/s

The specific power output,

$$P = \frac{u\mathcal{V}_w}{1000} = \frac{110 \times 224}{1000}$$
$$= \mathbf{24.64\ kW/kg\ of\ steam}$$

Alternatively

$$\mathcal{V}_{w_1} = \mathcal{V}_1 \cos \alpha = 239 \times \cos 21$$
$$= 223.12 \text{ m/s}$$

$$\therefore \quad BF = \mathcal{V}_{w_1} - u = 223.12 - 110$$
$$= 113.12 \text{ m/s}$$

$$CF = \mathcal{V}_{f_1} = \mathcal{V}_1 \sin \alpha = 239 \sin 21$$
$$= 85.65 \text{ m/s}$$

$$\therefore \quad \mathcal{V}_{r_1} = \sqrt{(BF)^2 + (CF)^2}$$
$$= \sqrt{(113.12)^2 + (85.65)^2}$$
$$= 141.93 \text{ m/s}$$

$$\tan \theta = \frac{CF}{BF}$$

It gives the inlet angle of the moving blades

$$\theta = \tan^{-1}\left(\frac{85.65}{113.12}\right) = \mathbf{37.12°}$$

$$\mathcal{V}_{r_2} = k\mathcal{V}_{r_1} = 0.95 \times 141.93 = 133.95 \text{ m/s}$$

$$\cos \phi = \frac{u}{\mathcal{V}_{r_2}} = \frac{110}{133.95}$$

or, exit angle of the moving blades,

$$\phi = 34.8°$$
$$\mathcal{V}_{w_2} = 0$$

Example 22.13 *In an impulse turbine (with a single row wheel), the mean diameter of the blade is 1.05 m and speed is 3000 rpm. The nozzle angle is 18°. The ratio of blade speed to steam speed is 0.42 and ratio of relative velocity at outlet from the blades to that at inlet is 0.84. The outlet angle of the blade is to be made 3° less than the inlet angle. The steam-flow rate is 10 kg/s. Draw the velocity diagram for blades and derive the following:*

(a) Tangential thrust on the blades,

(b) Axial thrus on the blades,

(c) Power developed in the blades,

(d) Blading efficiency.

Solution

Given A simple impulse turbine

$D = 1.05$ m $\qquad N = 3000$ rpm

$\phi = \theta - 3°$

speed ratio, $s = \frac{u}{\mathcal{V}_1} = 0.42$

$\alpha = 18° \qquad k = 0.84$

$\dot{m}_s = 10$ kg/s

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To find

(i) Tangential thrust on the blades,
(ii) Axial thrus on the blades,
(iii) Power developed in the blades, and
(iv) Blading efficiency.

Analysis The blade velocity,

$$u = \frac{\pi DN}{60} = \frac{\pi \times 1.05 \times 3000}{60} = 164.93 \text{ m/s}$$

The nozzle exit velocity,

$$\mathcal{V}_1 = \frac{u}{s} = \frac{164.93}{0.42} = 392.7 \text{ m/s}$$

The relation between two relative velocities.

$$\frac{\mathcal{V}_{r_2}}{\mathcal{V}_{r_1}} = 0.84$$

The velocity diagram can be constructed as shown in Fig. 22.23.

(i) Draw a horizontal line EF. Mark $AB = u = 164.93$ m/s.
(ii) At the point A, $\alpha = 18°$ and $\mathcal{V}_1 = 392.7$ m/s, draw a line AC.
(iii) Meet BC, it gives $\theta = \mathbf{30°}$, inlet blade angle. Length of the line BC represents $\mathcal{V}_{r_1}$.
(iv) Draw the line BD to represent $\mathcal{V}_{r_2} = 0.84\,\mathcal{V}_{r_1}$ at an angle $\phi = \theta - 3° = 27°$.
(v) Meet AD to represent $\mathcal{V}_2$.
(vi) Meet ED to represent $\mathcal{V}_{f_2}$.
(vii) Meet CF to represent $\mathcal{V}_{f_1}$.

The measurements on the velocity diagram give

$\mathcal{V}_w = EF = 395$ m/s
$\mathcal{V}_{r_1} = BC = 242.5$ m/s
$\mathcal{V}_{r_2} = 0.84\,\mathcal{V}_{r_1} = 203.7$ m/s

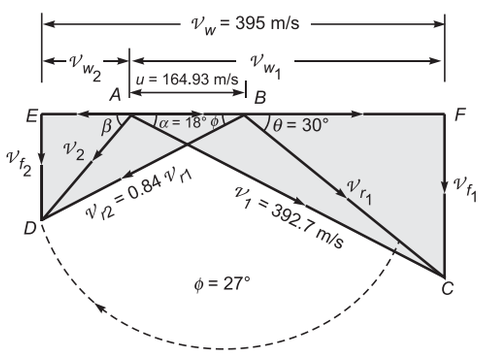

Fig. 22.23

$\mathcal{V}_{f_1} = FC = 122.5$ m/s
$\mathcal{V}_{f_2} = 98.5$ m/s.

(i) Tangential thrust on the blades,

$$F = \dot{m}_s (\mathcal{V}_{w_1} + \mathcal{V}_{w_2}) = 10 \times 395 = \mathbf{3950\ N}$$

(ii) Axial thrust on the blades,

$$F_a = \dot{m}_s (\mathcal{V}_{f_1} - \mathcal{V}_{f_2})$$
$$= \times (122.5 - 98.5) = \mathbf{240\ N}$$

(iii) Power developed in the blades,

$$P = \frac{\dot{m}_s u \mathcal{V}_w}{1000} = \frac{10 \times 164.93 \times 395}{1000}$$
$$= \mathbf{651.47\ kW}$$

(iv) Blading efficiency,

$$\eta_b = \frac{2u\mathcal{V}_w}{\mathcal{V}_1^2} = \frac{2 \times 164.93 \times 395}{(392.7)^2}$$
$$= 0.691 \text{ or } \mathbf{69.1\%}$$

Example 22.14 *The steam leaves the nozzle of a simple impulse turbine of outlet area 15.5 cm^2 at 920 m/s. The steam coming out of the nozzle is 0.91 dry at 150 kPa absolute. The blade inlet and outlet angles are 30°, and the blade velocity is 230 m/s. If the friction factor is 0.8, find (a) the nozzle angle, (b) the power developed, (c) the diagram efficiency, and (d) the axial thrust on the blading.*

Solution

Given A simple impulse turbine with

Nozzle outlet area	$A_c = 15.5$ cm^2
Inlet velocity of steam,	$\mathcal{V}_1 = 920$ m/s
Quality of steam,	$x = 0.91$
Steam pressure,	$p_1 = 150$ kPa
Blade angles,	$\phi = \theta = 30°$
Blade velocity,	$u = 230$ m/s
Friction factor,	$k = 0.8$

To find

(i) Nozzle angle α,
(ii) Power developed in the turbine,
(iii) Diagram efficiency,
(iv) Axial thrust on the blading.

Assumptions

(i) Adiabatic turbine.
(ii) Change in potential energy of the steam during expansion in negligible.

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Analysis The velocity diagram for the given data is drawn as shown in Fig. 22.24 with some convenient scale.

(i) Draw a horizontal line *EF*. Mark segment $AB = u = 230$ m/s.

(ii) From point *B* draw a line *BC* at angle 30°.

(iii) From point *A* with radius $AC = \mathcal{V}_1 = 920$ m/s, draw a chord to cut the line *BC* at point *C*.

(iv) With angle $\phi = 30°$ and $\mathcal{V}_{r_2} = 0.8\mathcal{V}_{r_2}$, draw a line *BD*.

(v) Join *AD*, *ED* and *CF* to complete the velocity triangle.

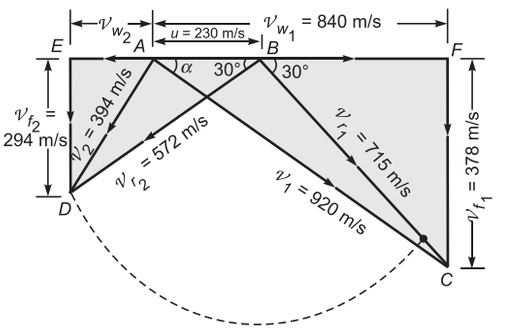

Fig. 22.24

The measurements show that inlet relative velocity,

$$\mathcal{V}_{r_1} = 715 \text{ m/s}$$

Exit relative velocity,

$$\mathcal{V}_{r_2} = 0.8, \mathcal{V}_{r_1} = 572 \text{ m/s}$$

(i) Nozzle angle $\alpha = \mathbf{23°}$

Axial velocities,

$$\mathcal{V}_{f_1} = 378 \text{ m/s}, \qquad \mathcal{V}_{f_2} = 294 \text{ m/s}$$

The total whirl velocity,

$$\mathcal{V}_w = \mathcal{V}_{w_1} + \mathcal{V}_{w_2} = 840 + 259 = 1099 \text{ m/s}$$

(ii) The specific volume of steam at 150 kPa

$$v_g = 1.15828 \text{ m}^3/\text{kg}$$

Mass-flow rate of steam

$$\dot{m}_s = \frac{A_c \mathcal{V}_1}{x v_g} = \frac{15.5 \times 10^{-4} \times 920}{0.9 \times 1.15828} = \mathbf{1.353 \text{ kg/s}}$$

Power developed,

$$P = \frac{\dot{m}_s u \mathcal{V}_w}{1000} = \frac{1.353 \times 230 \times 1099}{1000} = \mathbf{341.97 \text{ kW}}$$

(iii) Diagram or blade efficiency

$$\eta_b = \frac{2u\mathcal{V}_w}{\mathcal{V}_1^2} = \frac{2 \times 230 \times 1099}{(920)^2} = 0.597 \text{ or } \mathbf{59.7\%}$$

(iv) Axial thrust on the blading

$$F = \dot{m}_s (\mathcal{V}_{f_1} - \mathcal{V}_{f_2}) = 1.353 \times (378 - 294) = \mathbf{113.65 \text{ N}}$$

Example 22.15 *Steam issues from the nozzle of a simple impulse turbine with a velocity of 610 m/s. The nozzle angle is 20° and the diameter of the rotor is 62 cm and runs at 9500 rpm. The blade outlet angle is 30° and the friction factor is 0.8. Calculate the power developed per kg. of steam and the diagram efficiency.*

Solution

Given A single impulse steam turbine

$\mathcal{V}_1 = 610$ m/s, $\alpha = 20°$

$D = 62$ cm, $N = 9500$ rpm

$\phi = 30°$, $k = 0.8$

To find

(i) The power developed per kg of steam, and

(ii) Diagram efficiency.

Assumptions

(i) Adiabatic turbine.

(ii) No change in potential energy of steam.

(iii) Negligible steam velocity at the nozzle inlet.

Analysis The linear blade velocity

$$u = \frac{\pi DN}{60} = \frac{\pi \times (0.62 \text{ m}) \times (9500 \text{ rpm})}{60} = 308.4 \text{ m/s}$$

The velocity diagram is shown in Fig. 22.25. The measurements reveal that

The inlet relative velocity,

$$\mathcal{V}_{r_1} = \text{length of line } BC = 339 \text{ m/s}$$

Inlet blade angle, $\theta = 37°$

The relative velocity of steam at outlet,

$$\mathcal{V}_{r_2} = k\mathcal{V}_{r_1} = 0.8 \times 339 = 271.2 \text{ m/s}$$

The net whirl velocity,

$$\mathcal{V}_w = \mathcal{V}_{w_1} - \mathcal{V}_{w_2} = \text{length of line } EF = 510 \text{ m/s}$$

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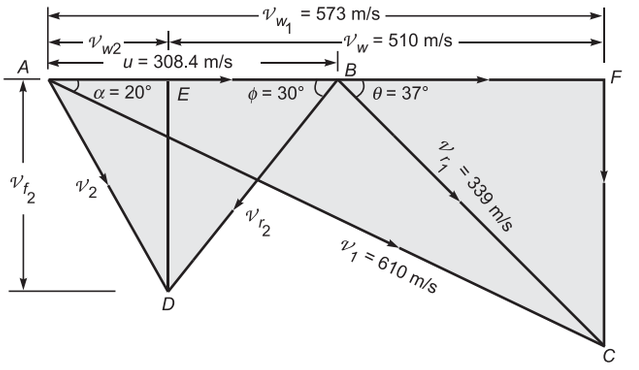

Fig. 22.25

(i) Power developed/kg of steam,

$$P = \frac{u\mathcal{V}_w}{1000} = \frac{308.4 \times 510}{1000}$$

$$= 157.28 \text{ kJ/kg}$$

(ii) Diagram or blade efficiency

$$\eta_b = \frac{2u\mathcal{V}_w}{\mathcal{V}_1^2} = \frac{2 \times 308.4 \times 510}{(610)^2}$$

$$= 0.845 = \textbf{84.5\%}$$

22.8 COMPOUNDING OF IMPULSE TURBINE

The compounding of an impulse turbine is done to reduce rotational speed of the turbine to the practical limits. In a simple impulse turbine, a rotor speed of 30,000 rpm is almost impractical to use in practice. The compounding of an impulse turbine is achieved by using more than one set of nozzles, blades, rotors in a series, keyed to a common shaft; so that either the steam pressure or steam jet velocity can be regulated by the turbine in stages. Thus, compounding of steam turbine improves the performance of the turbine.

There are three main types of compounding in impulse turbines.

(i) Velocity compounded steam turbine
(ii) Pressure compounded steam turbine
(iii) Pressure velocity compounded turbine

22.8.1 Velocity Compounded Impulse Turbine (Curtis Turbine)

This design was developed by an American engineer CG Curtis in year 1985. In velocity compounding, the steam is expanded to an exhaust pressure in a single set of nozzles. The kinetic energy of the steam is absorbed in two or three sets of moving blades. The pressure and velocity variation for a three sets of moving blades are shown in Fig. 22.26

It consists of a set of nozzles, three rows of moving blades and two rows of guide blades. The guide blades are arranged between moving blades in the reverse manner.

On passing through moving blades, the steam gives up its partial kinetic energy. During the passage through the moving blades, the steam pressure remains constant. Then the steam enters the stationary guide blades. The guide blades collect the steam coming out of the moving blades, adjust its direction of motion and direct it on the next row moving blades. Then steam enters the next row of guide blades and then moving blades in similar manner. During the passage through the guide blades, there is no change in pressure and velocity of steam.

The velocity of the steam jet is increased in the nozzles only and it drops gradually, after being utilized successively by all rows of moving blades till it is finally discharged to the condenser. The

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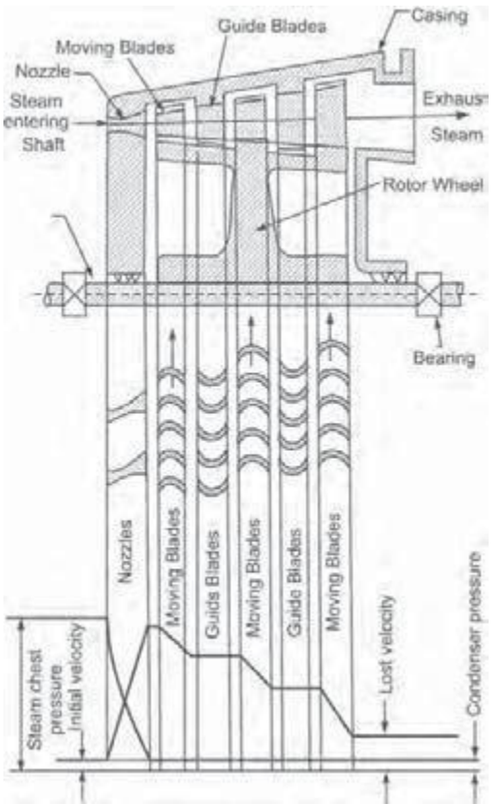

Fig. 22.26 *Three row velocity compounded impulse*

velocity, diagram for a two rows of moving blades is shown in Fig. 22.27.

Terminology for Vector Diagram of Velocity Compounded Steam Turbine The suffix *f* indicates quantity for first row and *s* indicates for second row.

u = Linear velocity of blade, m/s

$\mathcal{V}_{1f}$ = Absolute velocity of steam jet at inlet to blade, m/s

α_f = Inlet angle of steam jet in respect to plane of blade rotation, degree

$\mathcal{V}_{r_{1f}}$ = Relative velocity of steam at inlet, m/s

θ_f = Blade inlet angle, degree

$\mathcal{V}_{w_{1f}}$ = Whirl velocity at inlet, m/s

$\mathcal{V}_{f_{1f}}$ = Flow velocity of steam air inlet; m/s

$\mathcal{V}_{2_f}$, β_f, $\mathcal{V}_{r_{2f}}$, ϕ_f, $\mathcal{V}_{w_{2f}}$, $\mathcal{V}_{f_{2f}}$ are corresponding values at the outlet of the first row of moving blades and the inlet to the first row stationary guide blades.

$\mathcal{V}_{1s}$, α_s, $\mathcal{V}_{r_{1s}}$, θ_s, $\mathcal{V}_{w_{1s}}$, $\mathcal{V}_{f_{1s}}$ are corresponding values at the inlet to the second row of moving blades.

$\mathcal{V}_{2s}$, β_s, $\mathcal{V}_{r_{2f}}$, ϕ_f, $\mathcal{V}_{w_{2f}}$, $\mathcal{V}_{f_{2f}}$ are corresponding values at the outlet of the second row of moving blades and inlet to the second row stationary guide blades, and so on.

The work done per kg of steam on the first row of moving blades

$$w_f = u(\mathcal{V}_{w_{1f}} + \mathcal{V}_{w_{2f}})$$

The work done per kg of steam on the second row of moving blades

$$w_s = u(\mathcal{V}_{w_{1s}} + \mathcal{V}_{w_{2s}})$$

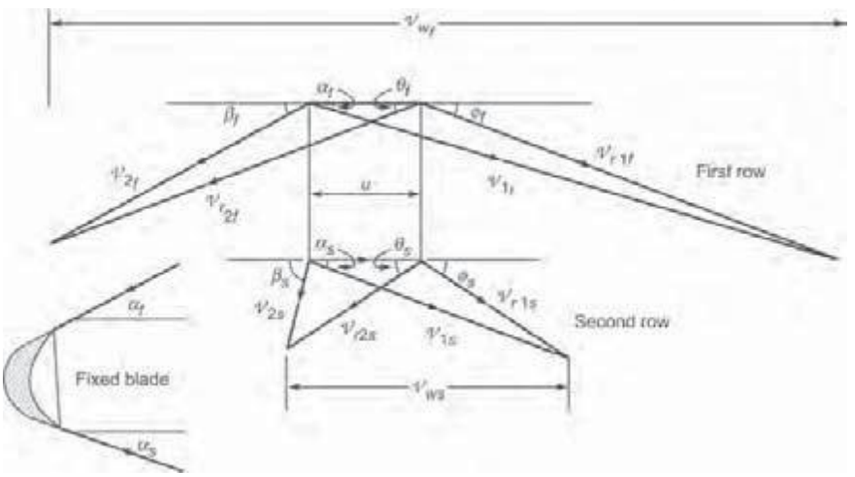

Fig. 22.27 *Velocity diagram for a two row velocity compounded impulse turbine*

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Total work done per kg of steam on the two-row wheel

$$w_{total} = w_f + w_s$$
$$= u(\mathcal{V}_{w_{1f}} + \mathcal{V}_{w_{2f}}) + u(\mathcal{V}_{w_{1s}} + \mathcal{V}_{w_{2s}})$$
$$= u(\mathcal{V}_{w_f} + \mathcal{V}_{w_s}) \quad ...(22.22)$$

Power output

$$P = \dot{m}_s u(\mathcal{V}_{w_f} + \mathcal{V}_{w_s}) \quad ...(22.23)$$

where $\mathcal{V}_{w_f} = \mathcal{V}_{w_{1f}} + \mathcal{V}_{w_{2f}}$, total whirl velocity of first row of moving blades, and

$\mathcal{V}_{w_s} = \mathcal{V}_{w_{1s}} + \mathcal{V}_{w_{2s}}$, total whirl velocity of second row of moving blades.

Blade of diagram efficiency

$$\eta_b = \frac{\text{Work done on the blade}}{\text{Energy supplied to the blade}}$$

$$\eta_b = \frac{u(\mathcal{V}_{w_f} + \mathcal{V}_{w_s})}{\frac{1}{2}\mathcal{V}_1^2} \quad ...(22.24)$$

Stage efficiency,

$$\eta_{stage} = \frac{\text{Work done on blade}}{\text{Energy supplied per stage}}$$
$$= \frac{u(\mathcal{V}_{w_f} + \mathcal{V}_{w_s})}{\Delta h} \quad ...(22.25)$$

22.8.2 Pressure Compounded Impulse Turbine (Rateau turbine)

Instead of expanding the steam completely in a single set of nozzles, the expansion of steam is splitted into a number of phases by arranging a number of moving and fixed blades in series, thus the rotor speed is obtained in practical range. The fixed blades act as nozzles. The steam expands equally in all rows of fixed blades. Thus, this arrangement can be viewed as a number of simple impulse machines in series on the same shaft, allowing the exhaust steam from one turbine to enter the nozzle of the succeeding turbine. Each of the simple impulse machines would be termed as a *"stage"* of the turbines, since each stage comprises its set of nozzles and moving blades. Fig. 22.28 shows schematic arrangement of three stage impulse turbine.

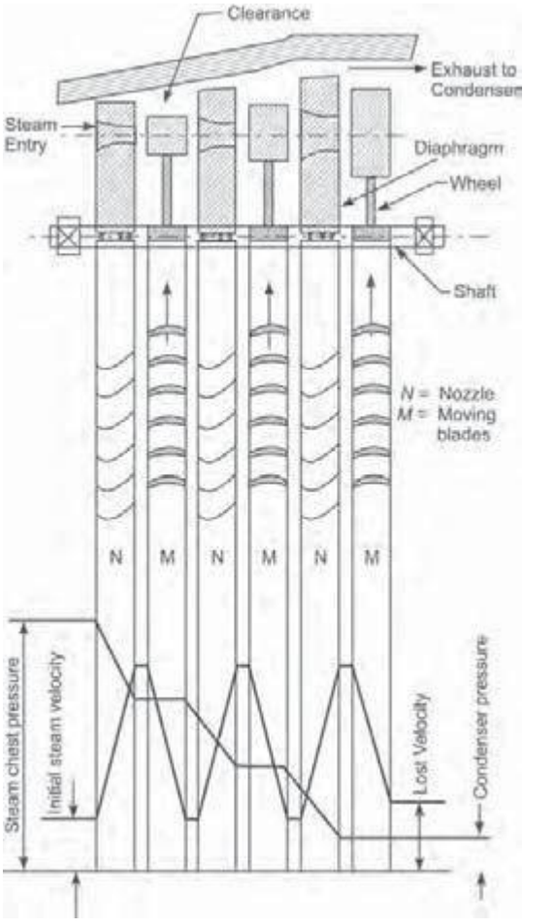

Fig. 22.28 *Three stage pressure compounded impulse turbine*

This arrangement is equivalent to splitting up the whole pressure drop into a series of small pressure drops, hence arrangement is known as *pressure compounding.*

The lower part of Fig. 22.28 shows the velocity and pressure distribution. Steam enters the first row of the nozzle, a small pressure drop takes place with increase in steam velocity. The steam passes over the first row of moving blades, The steam pressure remains constant, but steam velocity decreases. It constitutes one stage. The variation of pressure velocity gets repeated for a number of stages until condenser pressure is reached.

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In a pressure-compounding steam turbine, a partial enthalpy of steam is transformed into kinetic energy in each stage. Hence, steam velocity is much lower than the simple impulse and velocity-compounded steam turbines, and thus the operation is salient and more efficient. The example of pressure-compounded steam turbine is Rateau and Zeolly turbine.

22.8.3 Pressure–Velocity Compounded Impulse Turbine

This is a combination of pressure and velocity compoundings. The total pressure drop of steam is divided into a number of stages as done in pressure compounding. Each stage has a number of rows of fixed and moving blades working as an independent velocity compounded stage.

Each stage is separated from the adjacent stage by a row of stationary ring of nozzles for expansion of steam for the next stage. The set of moving and fixed blades is used for velocity compounding and a set of nozzle rings in between stages is utilized for pressure compounding. Such type of compounding offers a larger pressure drop in each stage with less number of stages. Therefore, the turbine is simple in construction and compact in size.

The diagrammatic arrangement shown in Fig. 22.29, explains the principle of working, construction and pressure-velocity diagrams.

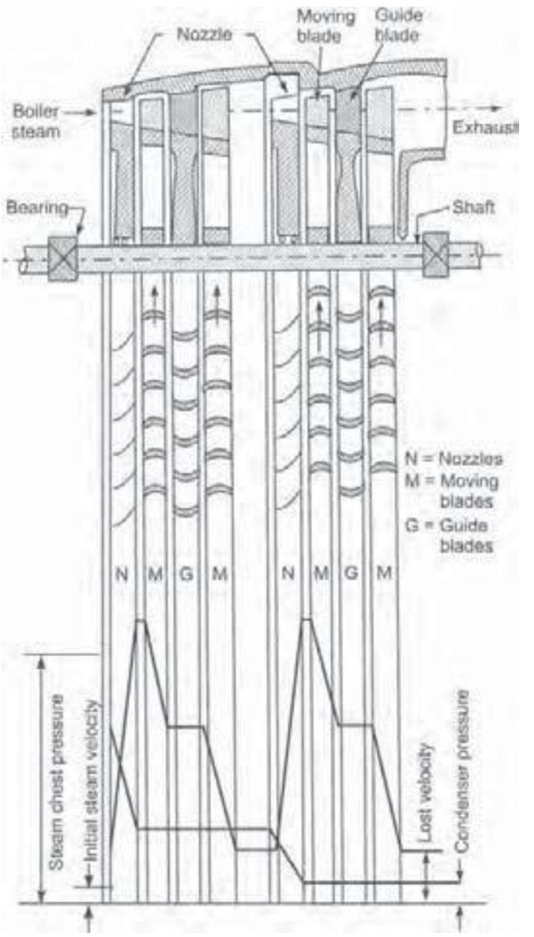

Fig. 22.29 *Pressure-Velocity compounded impulse turbine with two rotors*

Example 22.16 *In an impulse turbine, the steam issues from the nozzle with a speed of 600 m/s and blade speed is 120 m/s. The velocity is compounded by passing the steam through a ring of moving blades, through a ring of fixed blades and finally through a ring of moving blades.*

The nozzle angle is 18° and the blade exit angles and relative velocity coefficients are the following:

Blades	*exit angle*	*Velocity coefficient*
First row moving blades	*20°*	*0.8*
Fixed-row blades	*25°*	*0.85*
Second-row moving blades	*30°*	*0.9*

Find the diagram efficiency under these conditions and power output for steam flow rate of 5 kg/s.

What would be maximum possible diagram efficiency for given steam inlet velocity and nozzle angle?

Solution

Given A velocity compounded impulse turbine with

Nozzle angle $\alpha_f = 18°$

Inlet velocity of steam,

$V_{1f} = 600$ m/s

Blade velocity, $u = 120$ m/s

Mass flow rate, $\dot{m}_s = 5$ kg/s

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Moving Blade exit angles,

$$\phi_f = 20°, \quad \text{and} \quad \phi_s = 30°$$

Fixed blade exit angle, $\alpha_s = 25°$

Friction factors, $k_1 = 0.8$,

$$k_2 = 0.85, \quad \text{and} \quad k_3 = 0.9.$$

To find

(i) Diagram efficiency,
(ii) Power developed by the turbine,
(iii) Maximum possible diagram efficiency.

Assumptions

(i) Adiabatic turbine.
(ii) Change in potential energy of the steam during expansion in negligible.

Analysis The velocity diagram for the given data is drawn as shown in Fig. 22.30 with some convenient scale.

(i) Draw a horizontal line AB equal to blade velocity $u = 120$ m/s to a convenient scale.
(ii) From the point A draw a line AC to represent $\mathcal{V}_{1f} = 600$ m/s at angle $\alpha = 18°$ to horizontal line.
(iii) Join the point B and C by a line BC to represent $\mathcal{V}_{r_{1f}}$ relative velocity at inlet.
(iv) From the point B, draw line BD equal to $\mathcal{V}_{r_{2f}} = 0.8\mathcal{V}_{r_{1f}}$ at an angle $\phi_{1f} = 20°$.
(v) Join points A and D to represent absolute velocity of steam at exit of first stage.

The measurements show the following

Inlet relative velocity,

$$\mathcal{V}_{r_{1f}} = 480 \text{ m/s}$$

Exit relative velocity,

$$\mathcal{V}_{r_{2f}} = 0.8, \mathcal{V}_{r_{1f}} = 368 \text{ m/s}$$

First row moving blade inlet angle

$$\phi_f = 20°,$$

Absolute velocity of steam at exit of first stage

$$\mathcal{V}_{2f} = 280 \text{ m/s}$$

The total whirl velocity of first stage

$$\mathcal{V}_{w_f} = \mathcal{V}_{w_{1f}} + \mathcal{V}_{w_{2f}} = 790 \text{ m/s}$$

The *second-row velocity diagram* is also constructed in a similar manner.

Blade velocity, $u = 120$ m/s

Steam velocity, $\mathcal{V}_{1s} = 0.85\,\mathcal{V}_{2f} = 238$ m/s at an angle $\alpha_s = 25°$

Exit relative velocity,

$$\mathcal{V}_{r_{2s}} = 0.9\mathcal{V}_{r_{1s}}$$

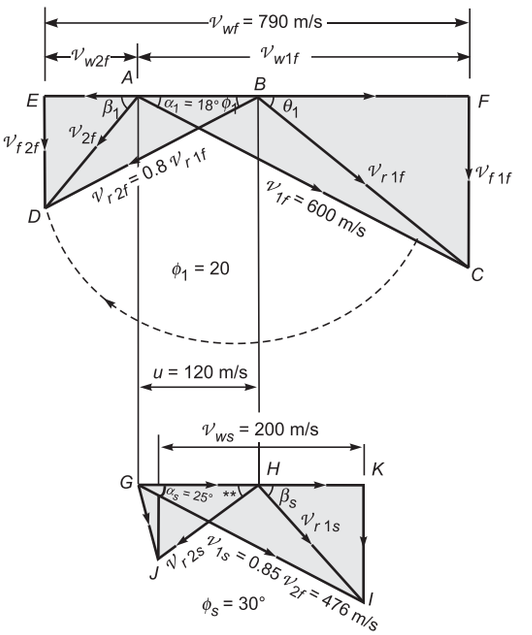

Fig. 22.30

Second-row moving blade angle

$$\phi_s = 30°,$$

The total whirl velocity of first stage; on measurement

$$\mathcal{V}_{w_s} = \mathcal{V}_{w_{1s}} + \mathcal{V}_{w_{2s}} = 200 \text{ m/s}$$

(i) Blade or diagram efficiency,

$$\eta_b = \frac{2u(\mathcal{V}_{w_f} + \mathcal{V}_{w_s})}{\mathcal{V}_{1f}^2}$$

$$= \frac{2 \times 120 \times (790 + 200)}{(600)^2}$$

$$= 0.66 \quad \text{or} \quad \mathbf{66\%}$$

(ii) Power developed by the turbine,

$$P = \frac{\dot{m}_s\, u(\mathcal{V}_{w_f} + \mathcal{V}_{w_s})}{1000}$$

$$= \frac{5 \times 120 \times (790 + 200)}{1000} = \mathbf{594 \text{ kW}}$$

(iii) Maximum possible diagram efficiency:

$$\eta_{b,max} = \cos^2\alpha_f = \cos^2(18°)$$

$$= 0.9045 \quad \text{or} \quad \mathbf{90.45\%}$$

Example 22.17 *The following particulars relate to a two-row velocity compounded impulse wheel:*

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Steam velocity at nozzle outlet = 650 m/s
Mean blade velocity = 125 m/s
The nozzle outlet angle = 16°
Outlet angle of first row of moving blades = 18°
Outlet angle of fixed guide blades = 22°
Outlet angle of second moving blades = 36°
Steam flow rate = 2.5 kg/s

The ratio of relative velocity at the outlet to that at the inlet is 0.84 for all blades.

Determine the following:

(a) Axial thrust on blades,
(b) The power developed, and
(c) The efficiency of the wheel.

Solution

Given A two-row velocity compounded impulse turbine with

Nozzle angle $\alpha_f = 16°$
Inlet velocity of steam, $\mathcal{V}_{1f} = 650$ m/s
Blade velocity, $u = 125$ m/s
Mass flow rate, $\dot{m}_s = 2.5$ kg/s
Moving-blade exit angles,
$\phi_f = 18°$, and $\phi_s = 36°$
Fixed blade exit angle, $\alpha_s = 22°$,
Friction factors, $k_1 = k_2 = k_3 = 0.84$.

To find

(i) Axial thrust on the blades,
(ii) Power developed, and
(iii) Efficiency of wheel.

Assumptions

(i) Adiabatic turbine.
(ii) Change in potential energy of the steam during expansion is negligible.

Analysis The velocity diagram for the given data is drawn as shown in Fig. 22.31 with some convenient scale.

(i) Draw a horizontal line AB equal to the blade velocity $u = 125$ m/s to a convenient scale.
(ii) From the point A draw a line AC to represent $\mathcal{V}_{1f} = 650$ m/s at angle $\alpha = 16°$ to horizontal line.
(iii) Join points B and C by a line BC to represent $\mathcal{V}_{r_{1f}}$ relative velocity at inlet.
(iv) From the point B, draw line BD equal to $\mathcal{V}_{r_{2f}} = 0.8\mathcal{V}_{r_{1f}}$ at an angle $\phi_f = 18°$.
(v) Join points A and D to represent absolute velocity of steam at exit of first stage.

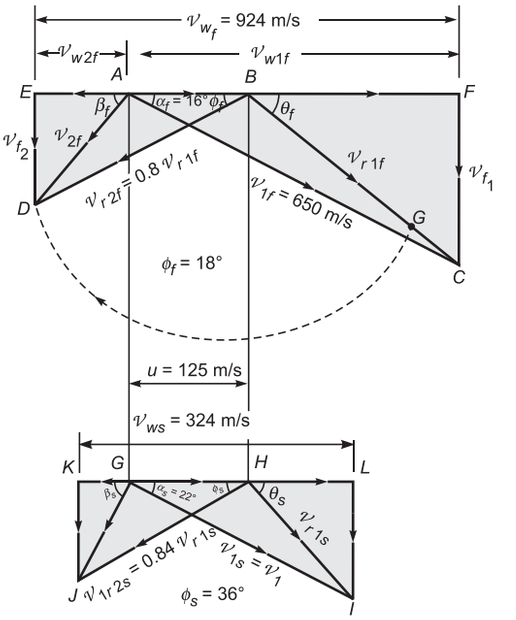

Fig. 22.31

The measurements show the following:

Given that

$$\mathcal{V}_{r_{2f}} = 0.84\,\mathcal{V}_{r_{1f}} \quad \text{and} \quad \theta_f = 20°$$

Flow velocities of steam for first row wheel

$$\mathcal{V}_{f_{1f}} = 180 \text{ m/s}, \quad \text{and} \quad \mathcal{V}_{f_{2f}} = 138 \text{ m/s}.$$

The total whirl velocity of first stage

$$\mathcal{V}_{w_f} = \mathcal{V}_{w_{1f}} + \mathcal{V}_{w_{2f}} = 924 \text{ m/s}$$

The *second-row velocity diagram* is also constructed in a similar manner.

Blade velocity, $u = 125$ m/s
Steam velocity, $\mathcal{V}_{1s} = 0.84\,\mathcal{V}_{2f}$ an angle
$\alpha_s = 22°$
Exit relative velocity,

$$\mathcal{V}_{r_{2s}} = 0.84\,\mathcal{V}_{r_{1s}} \quad \text{and} \quad \phi_s = 30°,$$

Flow velocities of steam for second-row wheel

$$\mathcal{V}_{f_{1s}} = 122 \text{ m/s}, \quad \text{and} \quad \mathcal{V}_{f_{2s}} = 107 \text{ m/s}.$$

The total whirl velocity of first stage

$$\mathcal{V}_{w_s} = \mathcal{V}_{w_{1s}} + \mathcal{V}_{w_{2s}} = 324 \text{ m/s}$$

(i) Axial thrust on the blades,

$$F_a = \dot{m}_s\,[(\mathcal{V}_{f_{1f}} - \mathcal{V}_{f_{2f}}) + (\mathcal{V}_{f_{1s}} - \mathcal{V}_{f_{2s}})]$$
$$= 2.5 \times [(180 - 138) + (122 - 107)]$$
$$= \mathbf{142.5\ N}$$

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(ii) Power developed by the wheel,

$$P = \frac{\dot{m}_s\, u(\mathcal{V}_{w_f} + \mathcal{V}_{w_s})}{1000}$$

$$= \frac{2.5 \times 125 \times (924 + 324)}{1000}$$

$$= \mathbf{390\ kW}$$

(iii) Blade or diagram efficiency,

$$\eta_b = \frac{2u(\mathcal{V}_{w_f} + \mathcal{V}_{w_s})}{\mathcal{V}_{1f}^2}$$

$$= \frac{2 \times 125 \times (924 + 324)}{(650)^2}$$

$$= 0.7384 \quad \text{or} \quad \mathbf{73.84\%}$$

Example 22.18 *In a two stage velocity compounded impulse turbine, the steam leaves the nozzle at a velocity of 675 m/s, when blade speed is 150 m/s. The nozzle angle is 20°, while, the discharge angles are 25° for first-row blades, 25° for fixed blades and 30° for the second-row blades. There is 10% loss in velocity during passage of steam through each row of blades. For the steam flow rate of 5 kg/s, calculate the power output of the turbine and diagram efficiency.*

Solution

Given A two-stage, velocity-compounded steam turbine

$\mathcal{V}_{1f} = 675$ m/s $\quad u = 150$ m/s $\quad \alpha_f = 20°$

$\alpha_s = 25°$ $\quad \theta_f = 25°$ $\quad \theta_s = 30°$

$\dot{m}_s = 5$ kg/s $\quad k = 0.9$

To find

(i) Power developed, and

(ii) Blade efficiency.

Analysis The velocity diagram for first stage of turbine is drawn as shown in Fig. 22.32(a).

The measurements reveal that

Inlet velocities

$u = 150$ m/s Line segment AB

$\mathcal{V}_{1_f} = 675$ m/s Line AC at angle $\alpha = 20°$

$\mathcal{V}_{r_{1f}} = 536.5$ m/s Line BC through B to meet C

$\mathcal{V}_{f_{1f}} = 231$ m/s Length of line CF

$\mathcal{V}_{w_{1f}} = 634.5$ m/s Line AF through A to meet F

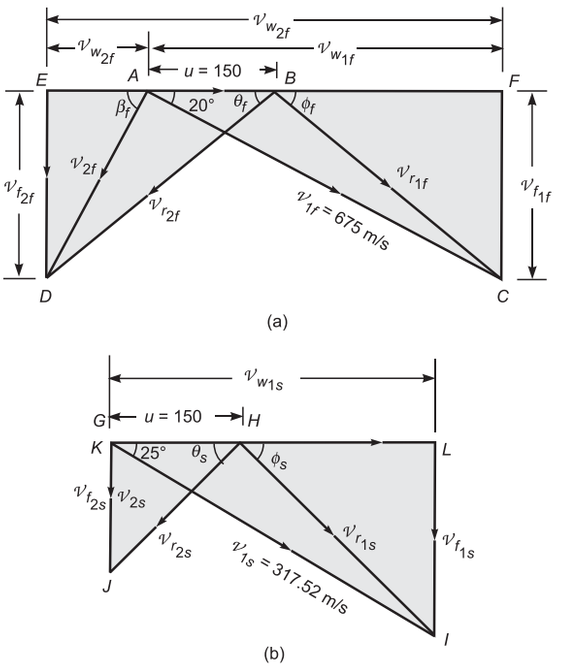

Fig. 22.32

Exit velocities

$\mathcal{V}_{r_{2f}} = k\mathcal{V}_{r_{1f}} = 0.9 \times 536.5$ m/s $= 482.85$ m/s

= Line BD through point B at angle, $\theta_f = 25°$

$\mathcal{V}_{2_f} = 352.8$ m/s Line AD through the point A to meet D.

$\mathcal{V}_{w_{2f}} = 287.5$ m/s Length of the line AE.

For the *second* stage;

$u = 150$ m/s Line GH,

$\mathcal{V}_{1s} = k\mathcal{V}_{2_f} = 0.9 \times 352.8$ m/s $= 317.52$ m/s

= Line GI at angle $\alpha = 25°$

By measurement;

$\mathcal{V}_{r_{1s}} = 192$ m/s Line HI through H to meet I.

$\mathcal{V}_{w_{1s}} = 287.5$ m/s Line GL through G to meet L.

Exit velocities

$\mathcal{V}_{r_{2s}} = k\mathcal{V}_{r_{1s}} = 0.9 \times 192$ m/s $= 172.8$ m/s,

= Line HJ through point H at angle, $\theta_s = 30°$

$\mathcal{V}_{w_{2s}} = 0$,

Total whirl velocities;

$\mathcal{V}_{w_f} = \mathcal{V}_{w_{1f}} + \mathcal{V}_{w_{2f}}$

$= 634.5 + 287.5 = 922$ m/s

$\mathcal{V}_{w_s} = \mathcal{V}_{w_{1s}} + \mathcal{V}_{w_{2s}}$

$= 287.5 + 0 = 287.5$ m/s

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Power output

$$P = \dot{m}_s\, u\, (\mathcal{V}_{w_f} + \mathcal{V}_{w_s})$$
$$= 5 \times 150 \times (922 + 287.5)$$
$$= 907125 \text{ W or } \mathbf{907.125\ kW}$$

Blade or diagram efficiency

$$\eta_b = \frac{u(\mathcal{V}_{w_f} + \mathcal{V}_{w_s})}{\frac{1}{2}\mathcal{V}_1^2}$$
$$= \frac{(150\text{ m/s}) \times (922\text{ m/s} + 287.5\text{ m/s})}{\frac{1}{2}(675\text{ m/s})^2}$$
$$= 0.796 \quad \text{or} \quad \mathbf{79.6\%}$$

Example 22.19 *The nozzles of a two-row velocity compounded impulse turbine, have outlet angle of 22° and utilise an isentropic enthalpy drop of 220 kJ/kg of steam. All moving and guide blades are symmetrical and mean blade velocity is 150 m/s. Assume an isentropic efficiency of for the nozzle as 90%. Calculate the specific power output produced by each kg of steam. The velocity at the inlet to nozzle can be neglected.*

Solution

Given A two-stage velocity compounded steam turbine

$u = 150$ m/s
$\alpha_f = 22°$
$\Delta h = 220$ kJ/kg

For symmetrical blades

$\theta_f = \phi_f$ $\quad \beta_f = \beta_s$
$\theta_s = \phi_s$ $\quad \eta_N = 0.9$ $\quad \mathcal{V}_0 = 0$

To find Specific power output.

Assumption No friction between blades and steam, $k = 0$.

Analysis The exit velocity from nozzles

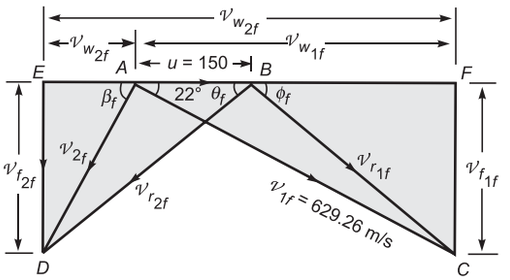
$$\mathcal{V}_1 = 44.72 \times \sqrt{\eta_N\, \Delta h}$$
$$= 44.72 \times \sqrt{0.9 \times 220} = 629.26 \text{ m/s}$$

The velocity diagram for first stage of turbine is drawn as shown in Fig. 22.33(a).

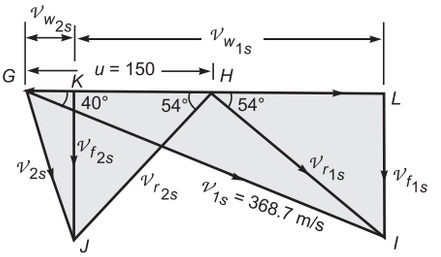
The measurements reveal that

Inlet velocities

$u = 150$ m/s Line AB
$\mathcal{V}_{1_f} = 629.26$ m/s Line AC at angle $\alpha = 22°$
$\beta_f = \phi_f = 28.5°$ (For symmetrical blades)



(a) Velocity diagram for first row



(b) Velocity diagram for second row

Fig. 22.33

$\mathcal{V}_{r_{1f}} = 493.5$ m/s Line BC through B to meet C, angle $\phi_f = 28.5°$
$\mathcal{V}_{w_{1f}} = 583.5$ m/s Line AF through A to meet F

Exit velocities

$\mathcal{V}_{r_{2f}} = k\mathcal{V}_{r_{1f}} = 1 \times 493.5$ m/s $= 493.5$ m/s
= Line BD through point B at angle, $\theta_f = 28.5°$
$\mathcal{V}_{w_{2f}} = 283.5$ m/s Line EA through A to meet E
$\mathcal{V}_{2_f} = 368.7$ m/s Line AD at angle $\beta_f = 40°$

From velocity diagram of second stage of turbine

Inlet velocities

$u = 150$ m/s Line GH
$\mathcal{V}_{1s} = \mathcal{V}_{2_f} = 368.7$ m/s Line GI at angle $\alpha_s = 40°$
$\mathcal{V}_{w_{1s}} = 282.5$ m/s Line GL through G to meet L
$\mathcal{V}_{r_{1s}} = 224.7$ m/s Line HI through H to meet I, angle $\phi_s = 54°$

Exit velocities

$\mathcal{V}_{r_{2s}} = \mathcal{V}_{r_{1s}} = 224.7$ m/s, Line HJ at an angle $\theta_s = 54°$
$\mathcal{V}_{w_{2s}} = -18$ m/s, Length of line GK.

Specific power output

$$= u\,(\mathcal{V}_{w_{1f}} + \mathcal{V}_{w_{2f}} + \mathcal{V}_{w_{1s}} + \mathcal{V}_{w_{2s}})$$
$$= 150 \times (583.5 + 283.5 + 282.5 - 18)$$
$$= 169725 \text{ W} \quad \text{or} \quad \mathbf{169.725\ kW}$$

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Example 22.20 *A compounded impulse turbine has two rows of moving blades separated by a fixed row of blades. The steam leaves the nozzle at an angle of 20° with the direction of motion of blades. The blades exit angles are I^st^ moving 30°, fixed 22° and 2^nd^ moving 30°.*

If the adiabatic heat drop of nozzle is 186.2 kJ/kg with nozzle efficiency of 90%, calculate the blade velocity necessary, if the final velocity of steam is to be axial. Assume a loss of 15% in the relative velocity of all blade passages. Also obtain the blade efficiency and stage efficiency.

Solution

Given A velocity compounded impulse turbine with

Nozzle angle $\alpha_f = 20°$

Moving blade exit angles,

$\phi_f = 30°$, and $\phi_s = 30°$

Fixed blade exit angle, $\alpha_s = 22°$, and $\beta_s = 90°$

Friction factors, $k_1 = k_2 = k_3 = 0.85$.

$\Delta h = 186.2$ kJ/kg, $\eta_N = 0.9$

To find

(i) Blade velocity,

(ii) Diagram efficiency, and

(iii) Stage efficiency.

Analysis The velocity of steam coming out of the nozzle

$$\mathcal{V}_{1f} = 44.72\sqrt{\Delta h \eta_N} = \sqrt{186.2 \times 0.9}$$

$$= 578.93 \text{ m/s}$$

The velocity diagram for axial discharge turbine is drawn in Fig. 22.34.

Second-row velocity triangle

1. Draw a horizontal line *GK*. Mark the segment *GH* as unknown blade velocity *u*.
2. From the point *G* draw a vertical line *GJ* to represent $\mathcal{V}_{2s}$ for axial flow turbine.
3. Draw a line *HJ* at an angle of 30° as second-row moving blade angle. The length *HJ* represents outlet relative velocity $\mathcal{V}_{r_{2s}}$ of second row blaes.
4. The relative velocity at inlet to second-row moving blades $\mathcal{V}_{r_{1s}} = \dfrac{\mathcal{V}_{r_{2s}}}{k_3} = \dfrac{\text{Length } HJ}{0.85}$.
5. Now draw a line *GI* at an angle of 22° fixed row blade angle.

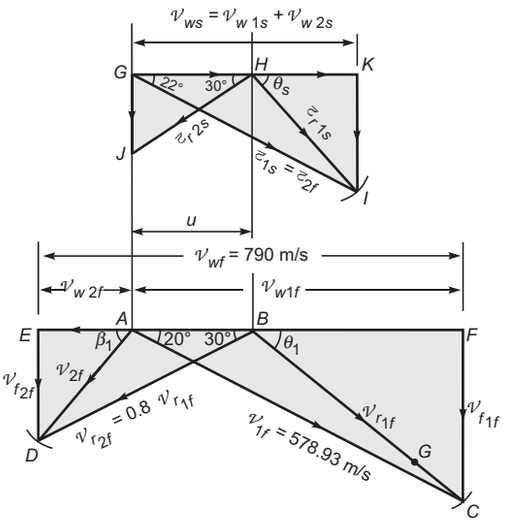

Fig. 22.34

6. From point *H* with radius equal to $\mathcal{V}_{r_{1s}}$ draw an arc to cut the line *GI* at point *I*. The length *GI* represent absolute velocity of steam at inlet to second row.
7. Calculate the absolute velocity of steam at exit to first stage

$$\mathcal{V}_{2f} = \frac{\mathcal{V}_{1s}}{k_2} = \frac{\text{Length } GI}{0.85}$$

First-row velocity triangle

1. Draw a horizontal line *AB* equal to the blade velocity *u*.
2. From the point *A* draw an arc with the radius equal to $\mathcal{V}_{2f}$.
3. From the point *B* draw a line *BD* at angle $\alpha = 30°$.
4. Join points *A* and *D* to represent $\mathcal{V}_{2f}$.
5. Join points *B* and *D* by a line *BD* to represent $\mathcal{V}_{r_{2f}}$ relative velocity at outlet.
6. Calculate the relative velocity of steam at inlet to first stage

$$\mathcal{V}_{r_{1f}} = \frac{\mathcal{V}_{r_{2f}}}{k_1} = \frac{\text{Length } BD}{0.85}$$

7. From the point *B*, draw an arc radius equal to $\mathcal{V}_{r_{1f}}$.
8. From the point *A*, draw line *AC* at an angle $\alpha_{1f} = 20°$ to cut the arc at point *C*.
9. Joint points *B* and *C* to complete the velocity triangle of first stage.

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10. Measure the length of line AC and calculate the scale of triangle as

$$\text{Scale} = \frac{\text{Length } AC}{578.93}$$

11. Measure the length AB and calculate the blade velocity u by using scale of triangle. The measurements show that

(i) Blade velocity u = **117 m/s** (Length AB)

The total whirl velocity of first stage

$$\mathcal{V}_{w_f} = \mathcal{V}_{w_{1f}} + \mathcal{V}_{w_{2f}} = 762 \text{ m/s}$$

The total whirl velocity of first stage

$$\mathcal{V}_{w_s} = \mathcal{V}_{w_{1s}} + \mathcal{V}_{w_{2s}} = 234 \text{ m/s}$$

(ii) Blade or diagram efficiency

$$\eta_b = \frac{2u(\mathcal{V}_{wf} + \mathcal{V}_{ws})}{\mathcal{V}_{1f}^2}$$

$$= \frac{2 \times 117 \times (762 + 234)}{(578.93)^2}$$

$$= 0.6954 \quad \text{or} \quad \mathbf{69.54\%}$$

(iii) Stage efficiency:

$$\eta_{stage} = \frac{u(\mathcal{V}_{wf} + \mathcal{V}_{ws})}{\Delta h}$$

$$= \frac{117 \times (762 + 234)}{186.2 \times 10^3}$$

$$= 0.6258 \quad \text{or} \quad \mathbf{62.58\%}$$

22.9 REACTION TURBINE (IMPULSE REACTION TURBINE)

In a reaction turbine, the moving blades have converging steam passage. Therefore, when steam passes over the moving blades, it expands with a drop in steam pressure and increase in kinetic energy. Thus in a reaction turbine, the steam jet leaves the moving blades with higher velocity than it enters the blades. The higher velocity steam jet coming out of the moving blades, reacts on the blades and causes them to rotate in opposite direction. The *Parson turbine* which is named after its inventor, Sir Charles A Parson, in 1884 is a good example of a reaction turbine. It developed 7.5 kW running at 17000 rpm.

In modern steam turbines, both impulse and reaction principles are used simultaneously. In this turbine, the steam does not expand completely in the stationary nozzle, but it expands in the fixed as well as moving blades, both acting as nozzles. The motive force is partly impulsive and partly reaction force due to continuous expansion of

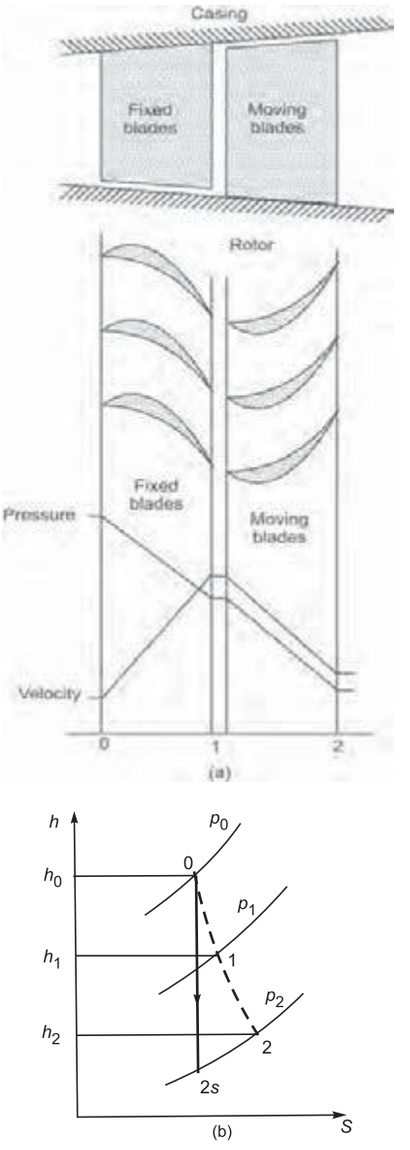

Fig. 22.35 *Fixed and moving blades of a reaction turbine blade pair showing (a) pressure and velocity variation, and (b) the process on an h–s chart*

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steam in fixed and moving blades. The work output from the turbine is due to both impulsive and reactive forces. Therefore, these turbines are also called the *impulse-reaction turbines.* However, the work produced by reactive force is higher than the impulsive force, and hence these turbines are also simply referred as *reaction turbines.*

The pressure falls continuously as the steam flows over the fixed and moving blades of each stage. The steam velocity increases in each set of the fixed blades while it decreases in the moving blades.

There are a number of rows of moving blades attached to the rotor and an equal number of fixed blades attached to the casing. Thus these are referred as *rotor blades* and *stator blades*, respectively. The fixed blades are set in the reversed direction of moving blades as shown in Fig. 22.35(a). Due to the fixed blades at the entrance, the steam is admitted for the whole circumference and hence there is a full admission. Fig. 22.35(b) shows the effect of friction, when steam glides over the fixed and moving blades. The actual enthalpy drop is less than the isentropic enthalpy drop.

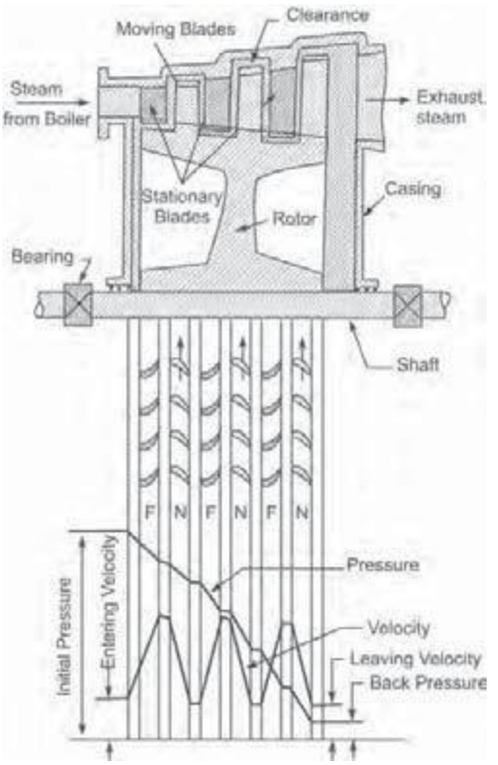

Fig. 22.36 *Three stage reaction turbine*

The reaction turbines are also compounded to reduce the rotor speed. The variation of pressure and velocity through a three-stage reaction turbine is shown in Fig. 22.36.

In an impulse turbine, the steam pressure remains constant while steam flows through the moving blades and no thrust is exerted by the steam in the direction of the rotor axis, while in the reaction turbine, the axial thrust is considerable due to pressure difference of either sides of moving blades. Dummy pistons and thrust bearings are used to balance this axial thrust.

22.9.1 Velocity Diagram for Reaction Turbine

As discussed above, in reaction turbines, the steam expands continuously while passing over the rings of fixed and moving blades. It is accomplished by using a tapered rotor with progressively increasing blade height. The effect of expansion of steam on the moving blade is to increase the relative velocity at the exit. Therefore, the relative velocity $\mathcal{V}_{r_2}$ is always greater than the relative velocity at the inlet $\mathcal{V}_{r_1}$. Thus the inlet and outlet velocity diagrams are shown with common blade velocity u in Fig. 22.37.

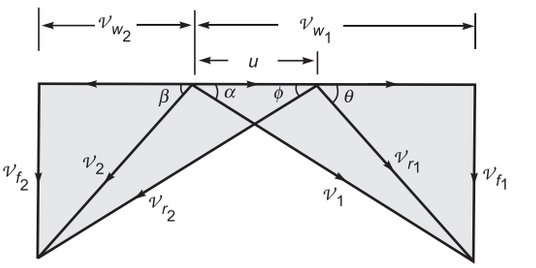

Fig. 22.37 *Velocity triangle diagram for reaction turbine*

22.9.2 Degree of Reaction

The degree of reaction of a reaction turbine is defined as the ratio of the enthalpy drop in moving

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blades to the total enthalpy drop in the stage. It is designated as Λ and may be defined as

$$\Lambda = \frac{\text{Enthalpy drop in the movig blades}}{\text{Total ehthalpy drop in the stage}} \quad \ldots(22.26)$$

Figure 22.35(b) shows the actual enthalpy drop when steam glides over the fixed and moving blades.

h_0 = Enthalpy of the steam at inlet of fixed blades,

h_1 = Enthalpy of the steam at entry of moving blades, and

h_2 = Enthalpy of the steam at exit from the moving blades.

The total enthalpy drop in a stage

= Enthalpy drop in fixed blades + Enthalpy drop in moving blades

$= h_1 - h_0 + h_2 - h_1$

$= \Delta h_f + \Delta h_m$

The total enthalpy drop for a stage ($\Delta h_m + \Delta h_f$) is equal to work done by the steam in the stage and it equals to,

$$\Delta h_m + \Delta h_f = u(\mathcal{V}_{w_1} + \mathcal{V}_{w_2}) \quad \ldots(22.27)$$

Hence,

Degree of reaction,

$$\Lambda = \frac{\mathcal{V}_{r_2}^2 - \mathcal{V}_{r_1}^2}{2u(\mathcal{V}_{w_1} + \mathcal{V}_{w_2})} \quad \ldots(22.28)$$

$$\mathcal{V}_{r_2} = \mathcal{V}_{f_2} \operatorname{cosec} \phi,$$

and $$\mathcal{V}_{r_1} = \mathcal{V}_{f_1} \operatorname{cosec} \theta$$

$$(\mathcal{V}_{w_1} + \mathcal{V}_{w_2}) = \mathcal{V}_{f_1} \cot\theta + \mathcal{V}_{f_2} \cot\phi$$

The velocity of flow is generally constant while the steam passes over the blade ring,

i.e., $\mathcal{V}_{f_1} = \mathcal{V}_{f_2} = \mathcal{V}_f$ (say)

$\therefore$ degree of reaction,

$$\Lambda = \frac{(\mathcal{V}_f \operatorname{cosec}\phi)^2 - (\mathcal{V}_f \operatorname{cosec}\theta)^2}{2u\mathcal{V}_f(\cot\theta + \cot\phi)}$$

or $$\Lambda = \frac{\mathcal{V}_f(\operatorname{cosec}^2\phi - \operatorname{cosec}^2\theta)}{2u(\cot\phi + \cot\theta)}$$

or $$\Lambda = \frac{\mathcal{V}_f}{2u}\left[\frac{(\cot^2\phi + 1) - (\cot^2\theta + 1)}{\cot\phi + \cot\theta}\right]$$

$$= \frac{\mathcal{V}_f}{2u}\left[\frac{\cot^2\phi - \cot^2\theta}{\cot\phi + \cot\theta}\right]$$

It gives $$\Lambda = \frac{\mathcal{V}_f}{2u}[\cot\phi - \cot\theta] \quad \ldots(22.29)$$

In a reaction turbine, the terms 'degree of reaction' is a measure of proportion of work done by the reaction effect. For the simple impulse turbine, $\Lambda = 0$, no heat drop in moving blades; for the Parson's type of blading, which has the same section for both the fixed and moving blades, $\Lambda = 0.5\,(\Delta h_f = \Delta h_m)$, then

$$\frac{1}{2} = \frac{\mathcal{V}_f}{2u}[\cot\phi - \cot\theta]$$

It gives $$u = \mathcal{V}_f(\cot\phi - \cot\theta) \quad \ldots(22.30)$$

The blade velocity u can also be written as

$$u = \mathcal{V}_f(\cot\alpha - \cot\beta) \quad \ldots(22.31)$$

Comparing Eqs. (22.30) and (22.31), we get

$$\theta = \beta \quad \text{and} \quad \phi = \alpha$$

It indicates that a reaction turbine with 50% degree of reaction will have moving and fixed blades of same shape.

For Parson's reaction turbine, the inlet and outlet velocity diagram are symmetrical with common blade velocity u as shown in Fig. 22.38 and

$$\mathcal{V}_1 = \mathcal{V}_{r_2} \quad \text{and} \quad \mathcal{V}_2 = \mathcal{V}_{r_1}$$

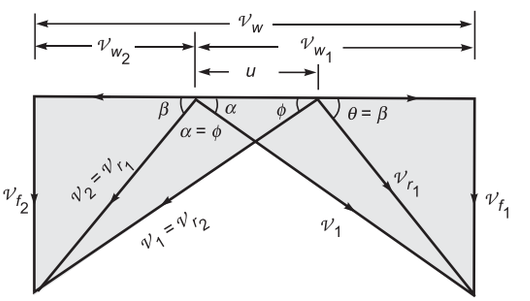

Fig. 22.38 *Velocity diagram for Parson's reaction turbine*

22.9.3 Blade or Diagram Efficiency of a Reaction Turbine

The blade or diagram efficiency of a turbine is given as

$$\eta_b = \frac{\text{Work done in stage}}{\text{Energy supplied in the stage}}$$

$$= \frac{u(\mathcal{V}_{w_1} + \mathcal{V}_{w_2})}{\Delta h_f + \Delta h_m} = \frac{w}{\Delta h} \quad \ldots(22.32)$$

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Refer velocity triangle, Fig. 22.38;

The work done per kg of steam

$$w = u(\mathcal{V}_{w_1} + \mathcal{V}_{w_2})$$
$$= u(\mathcal{V}_1 \cos\alpha + \mathcal{V}_{r_2}\cos\phi - u) \quad \ldots(22.33)$$

We have $\alpha = \phi, \mathcal{V}_{r_2} = \mathcal{V}_1$, then

$$w = u(\mathcal{V}_1 \cos\alpha + \mathcal{V}_1 \cos\alpha - u)$$
$$= u(2\mathcal{V}_1 \cos\alpha - u)$$
$$= \mathcal{V}_1^2\left[\frac{2u\mathcal{V}_1}{\mathcal{V}_1^2}\cos\alpha - \frac{u^2}{\mathcal{V}_1^2}\right]$$

Using $s = \dfrac{u}{\mathcal{V}_1}$, *blade-speed ratio*, we get

$$w = \mathcal{V}_1^2(2s\cos\alpha - s^2) \quad \ldots(22.34)$$

Energy input to the blades in a stage

Δh = Kinetic energy supplied to the fixed blades + Kinetic energy supplied to the moving blades

Kinetic energy supplied to fixed blades

$$= \frac{\mathcal{V}_1^2}{2}$$

Kinetic energy supplied to the moving blades

$$= \frac{\mathcal{V}_{r_2}^2 - \mathcal{V}_{r_1}^2}{2}$$

$$\therefore \quad \Delta h = \frac{\mathcal{V}_1^2}{2} + \frac{\mathcal{V}_{r_2}^2 - \mathcal{V}_{r_1}^2}{2} \quad \ldots(22.35)$$

For the symmetrical triangles,

$$\mathcal{V}_{r_2} = \mathcal{V}_1$$
$$\therefore \quad \Delta h = \frac{\mathcal{V}_1^2}{2} + \frac{\mathcal{V}_1^2 - \mathcal{V}_{r_1}^2}{2}$$
$$= \mathcal{V}_1^2 - \frac{\mathcal{V}_{r_1}^2}{2} \quad \ldots(22.36)$$

Refer Fig. 22.38 for ΔABC

$$\mathcal{V}_{r_1}^2 = u^2 + \mathcal{V}_1^2 - 2u\mathcal{V}_1\cos\alpha$$

Then
$$\Delta h = \mathcal{V}_1^2 - \frac{u^2 + \mathcal{V}_1^2 - 2u\mathcal{V}_1\cos\alpha}{2}$$
$$= \frac{\mathcal{V}_1^2 - u^2 + 2u\mathcal{V}_1\cos\alpha}{2}$$
$$= \frac{\mathcal{V}_1^2}{2}\left[1 - \left(\frac{u}{\mathcal{V}_1}\right)^2 + 2\frac{u}{\mathcal{V}_1}\cos\alpha\right]$$

Using $s = \dfrac{u}{\mathcal{V}_1}$, we get

$$\Delta h = \frac{\mathcal{V}_1^2}{2}[1 - s^2 + 2s\cos\alpha] \quad \ldots(22.37)$$

Substituting Eqs (22.34) and (22.37) in Eq. (22.32), then the blade or diagram efficiency is

$$\eta_b = \frac{\mathcal{V}_1^2(2s\cos\alpha - s^2)}{\frac{\mathcal{V}_1^2}{2}[1 - s^2 + 2s\cos\alpha]} = \frac{2(2s\cos\alpha - s^2)}{1 - s^2 + 2s\cos\alpha}$$

$$= \frac{2(1 + 2s\cos\alpha - s^2) - 2}{1 + 2s\cos\alpha - s^2}$$

$$\eta_b = 2 - \frac{2}{1 + 2s\cos\alpha - s^2} \quad \ldots(22.38)$$

22.9.4 Condition for Maximum Efficiency

From Eq. (22.38), it is evident that the blade efficiency would be maximum, when the term $(1 + 2s\cos\alpha - s^2)$ becomes maximum.

Using the condition of maxima, differentiating the above term with respect to blade speed ratio, s and equating to zero.

$$\frac{d}{ds}(1 + 2s\cos\alpha - s^2) = 0$$
$$2\cos\alpha - 2s = 0$$

or
$$s = \cos\alpha \quad \ldots(22.39)$$

Substituting in Eq. (22.38), we get

$$\eta_{b,\,max} = 2 - \frac{2}{1 + 2\cos^2\alpha - \cos^2\alpha}$$
$$= 2 - \frac{2}{1 + \cos^2\alpha}$$
$$= \frac{2 + 2\cos^2\alpha - 2}{1 + \cos^2\alpha}$$
$$= \frac{2\cos^2\alpha}{1 + \cos^2\alpha} \quad \ldots(22.40)$$

The variation of diagram efficiency with blade speed ratio, s for the simple impulse turbine and a reaction stage shown in Fig. 22.39. The efficiency curve for reaction turbine is flat for maximum value of blade speed ratio.

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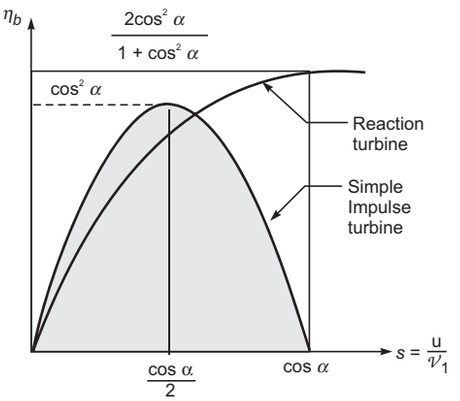

Fig. 22.39 *Diagram efficiency against blade speed ratio for impulse and reaction turbines*

22.9.5 Blade Height

Figure 22.40 shows the blade arrangement in a reaction turbine. The blade height of a particular stage can be determined by continuity equation, i.e.,

Mass-flow rate of steam,

$$\dot{m}_s = \frac{\text{Area flow} \times \text{flow velocity}}{\text{Specific volume of steam}}$$

Area of flow,

A_c = Mean circumference × Blade height

or $A_c = 2\pi r h$ or $[\pi(D + h)]\,h\,(m^2)$…(22.41)

Velocity of flow $= \mathcal{V}_{f_1}$ (m/s)

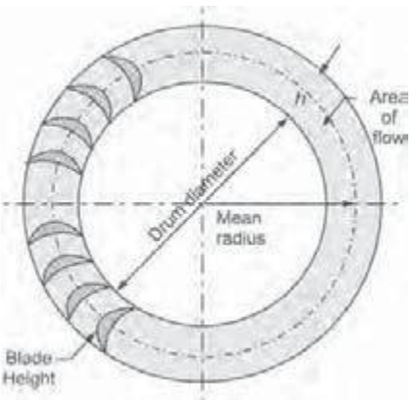

Fig. 22.40 *Arrangement of drum and blade on rotor*

Sp. Volume of steam $= v_g$ @ steam pressure

Then $$\dot{m}_s = \frac{2\pi r h \mathcal{V}_{f_1}}{v_g} \quad \text{…(22.42)}$$

Example 22.21 *Show that for a Parson's reaction turbine, the degree of reaction is 50%.*

Solution The degree of reaction of a reaction turbine is defined as

$$\Lambda = \frac{\text{Enthalpy drop in the moving blades}}{\text{Total enthalpy drop in the stage}}$$

$$= \frac{\Delta h_m}{\Delta h_m + \Delta h_f}$$

In terms of velocities, the enthalpy drop in moving blades

$$\Delta h_m = \frac{\mathcal{V}_{r_2}^2 - \mathcal{V}_{r_1}^2}{2} \quad \text{…(i)}$$

Enthalpy drop in fixed blades, with assumption that the velocity of steam entering the fixed blades is equal to the absolute velocity of steam leaving the previous moving blades.

$$\Delta h_f = \frac{\mathcal{V}_1^2 - \mathcal{V}_2^2}{2} \quad \text{…(ii)}$$

The velocity diagram for Parson's reaction turbine is shown in Fig. 22.38.

$\theta = \beta$ and $\phi = \alpha$

Thus, the inlet and outlet velocity diagram are symmetrical with common blade velocity u, thus using in Eq. (ii)

$\mathcal{V}_1 = \mathcal{V}_{r_2}$ and $\mathcal{V}_2 = \mathcal{V}_{r_1}$

$$\Delta h_f = \frac{\mathcal{V}_{r_2}^2 - \mathcal{V}_{r_1}^2}{2}$$

Total enthalpy drop

$$\Delta h_m + \Delta h_f = \frac{\mathcal{V}_{r_2}^2 - \mathcal{V}_{r_1}^2}{2} + \frac{\mathcal{V}_{r_2}^2 - \mathcal{V}_{r_1}^2}{2}$$

$$= 2\Delta h_m \quad \text{…(iii)}$$

Degree of reaction of Parson reaction turbine

$$\Lambda = \frac{\Delta h_m}{2\Delta h_m} = 0.5 \text{ or } \mathbf{50\% \text{ Proved}}$$

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Example 22.22 *A Parson reaction turbine running at 400 rpm with 50% reaction develops 75 kW per kg of the steam. The exit angle of the blade is 20° and the steam velocity is 1.4 times the blade velocity. Determine*

(a) Blade velocity,

(b) Blade inlet angle.

Solution

Given A Parson reaction turbine

$\alpha = \phi = 20°$ $\quad \dot{m}_s = 1$ kg/s $\quad V_1 = 1.4\,u$

$N = 400$ rpm $\quad P = 75$ kW $\quad \Lambda = 0.5$

To find

(i) Blade velocity, and

(ii) Blade inlet angle.

Analysis

(i) Blade velocity, u as shown in Fig. 22.41

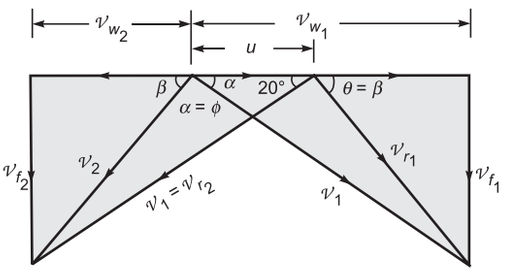

Fig. 22.41

Inlet whirl velocity,

$$V_{w_1} = V_1 \cos\alpha = V_1 \cos 20° = 0.9397\,V_1$$
$$= 0.9397 \times 1.4\,u = 1.31558\,u$$

Exit whirl velocity,

$$V_{w_2} = V_{w_1} - u \qquad (\text{since } \alpha = \phi, V_{r_2} = V_1)$$
$$= 1.31558\,u - u = 0.31558\,u$$

Power developed

$$P = \frac{\dot{m}_s\,u\,(V_{w_1} + V_{w_2})}{1000}\ \text{kW}$$

or $$75 = \frac{1 \times u[1.31558\,u + 0.31558\,u]}{1000}$$

or $$u^2 = 45.979 \times 10^3$$

It gives $u =$ **214.42 m/s**

(ii) Inlet blade angle,

$$V_1 = 1.4\,u = 1.4 \times 214.42 = 300.2\ \text{m/s}$$

$$V_{f_1} = V_{f_2} = V_1 \sin\alpha,\ V_{w_1} = V_1 \cos\alpha$$

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u} = \frac{V_1 \sin\alpha}{V_1 \cos\alpha - u}$$

$$= \frac{300.2 \sin 20°}{300.2 \cos 20° - 214.42} = 1.51715$$

or $\theta =$ **56.61°**

Example 22.23 *In a Parson reaction turbine, the angles of receiving tips are 35° and of discharging tips, 20°. The blade speed is 100 m/s. Calculate the tangential force, power developed, diagram efficiency, and axial thrust of the turbine, if its steam consumption is 1 kg/min.*

Solution

Given A Parson reaction turbine

$\alpha = \phi = 20°$ $\quad \dot{m}_s = 1$ kg/min $\quad u = 100$ m/s

$\theta = \phi = 35°$ $\quad \Lambda = 0.5$

To find

(i) Tangential force,

(ii) Power developed,

(iii) Diagram efficiency, and

(iv) Axial thrust.

Analysis Velocity diagram is constructed as shown in Fig. 22.42.

(i) Draw a horizontal line AB to some convenient scale to represent to blade velocity $u = 100$ m/s.

(ii) Through the point A, draw a line AC at an angle of 20°.

(iii) Through the point B, draw a line BC at an angle of 35° which intersects at the point C.

(iv) Measure length of $AC = V_1 =$ **220** m/s and $BC = V_{r_1} =$ **130** m/s.

(v) Through the point C, draw a vertical line CF to represent the flow velocity at inlet.

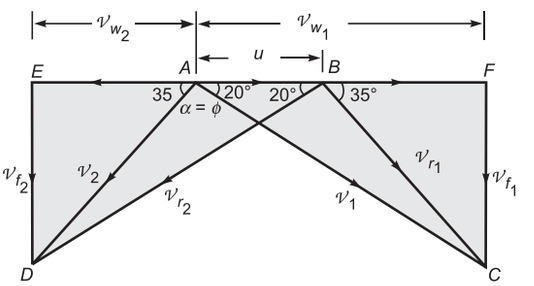

Fig. 22.42

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(vi) Measure length of $AF = \mathcal{V}_{w_1} = \mathbf{207}$ m/s.

(vii) Since $\mathcal{V}_{r_2} = \mathcal{V}_1$, and $\alpha = \theta$; draw a line BD at angle of 20°.

(viii) Join point A with a line AD, which will represent absolute velocity of steam at outlet i.e., $\mathcal{V}_2 = \mathbf{130}$ m/s.

(ix) Draw a vertical line DE through point D to represent flow velocity $\mathcal{V}_{f_2}$ at outlet.

(x) Measure length of segment $EA = \mathcal{V}_{w_2} = \mathbf{107}$ m/s.

(a) Tangential force

$$= \dot{m}_s (\mathcal{V}_{w_1} + \mathcal{V}_{w_2})$$
$$= (1/60) \times (207 + 107)$$
$$= \mathbf{5.233\ N}$$

(b) Power developed;

$$P = \frac{\dot{m}_s\, u (\mathcal{V}_{w_1} + \mathcal{V}_{w_2})}{1000} \text{ kW}$$
$$= \frac{(1/60) \times 100 \times [207 + 107]}{1000}$$
$$= \mathbf{0.5244\ kW}$$

(c) Diagram efficiency,

$$\eta_b = \frac{\text{Work done in a stage}}{\text{Energy supplied in the stage}}$$

The work done per kg of steam in a stage

$$w = u(\mathcal{V}_{w_1} + \mathcal{V}_{w_2})$$
$$= 100 \times (207 + 107) = 31{,}400 \text{ J/kg}$$

Energy supplied per kg of steam in a stage Eq. (22.36)

$$\Delta h = \mathcal{V}_1^2 - \frac{\mathcal{V}_{r_1}^2}{2}$$
$$= (220 \text{ m/s})^2 - \frac{(130 \text{ m/s})^2}{2}$$
$$= 39950 \text{ J/kg}$$

Thus $\eta_b = \dfrac{31400 \text{ J/kg}}{39950 \text{ J/kg}} = 0.786$ or **78.6%**

(d) Axial thrust; $F_a = \dot{m}_s (\mathcal{V}_{f_2} - \mathcal{V}_{f_1}) = \mathbf{0}$ ($\because \mathcal{V}_{f_2} = \mathcal{V}_{f_1}$)

Example 22.24 *A stage of steam turbine with Parson blading delivers dry, saturated steam at 2.7 bar from the fixed blades at 90 m/s. The mean blade height is 40 mm, and the moving blade's exit angle is 20°. The axial velocity of steam is three quarter of the blade velocity at the mean radius. The steam is supplied to the stage at the rate of 9000 kg/h. The effect of the blade tip thickness on the annulus area can be neglected. Calculate*

(a) the rotational speed of the wheel,

(b) the diagram power,

(c) the diagram efficiency,

(d) the enthalpy drop of the steam in this stage.

Solution

Given A reaction turbine.

$p_1 = 2.7$ bar, (dry and saturated steam)

$h = 40$ mm 0.04 m,

$\mathcal{V}_1 = 90$ m/s $\quad \alpha = 20°$ $\quad \mathcal{V}_f = \dfrac{3}{4} u$

$\dot{m}_s = 9000$ kg/h $= 2.5$ kg/s

To find

(i) Rotational speed of the wheel,

(ii) Diagram power,

(iii) Diagram efficiency, and

(iv) Enthalpy drop of the steam in this stage.

Assumptions

(i) Degree of reaction in the turbine is 50%.

(ii) Flow of steam in the turbine is frictionless and adiabatic.

Analysis The velocity diagram is shown in Fig. 22.43

(i) The flow velocity is given as

$$\mathcal{V}_f = \frac{3}{4} u = \mathcal{V}_1 \sin\alpha = 90 \sin 20°$$
$$= 30.78 \text{ m/s}$$

Then $u = \dfrac{4}{3} \mathcal{V}_f = 41.04$ m/s

The mass-flow rate of the steam is given by

$$\dot{m}_s = \frac{\mathcal{V}_f A}{v}$$

where A = annular area,

$\mathcal{V}_f$ = flow velocity, and

$v = v_g$ @ 2.7 bar $= 0.6686$ m^3/kg

Therefore, $2.5 \text{ kg/s} = \dfrac{30.78\, A}{0.6686}$

It gives, $A = 0.054$ m^3

Further, $A = 2\pi r h = 2\pi \times r \times 0.04$

It gives $r = 0.215$ m

Then the wheel rotational speed

$$N = \frac{\text{Blade speed}}{2\pi r} = \frac{u}{2\pi r}$$

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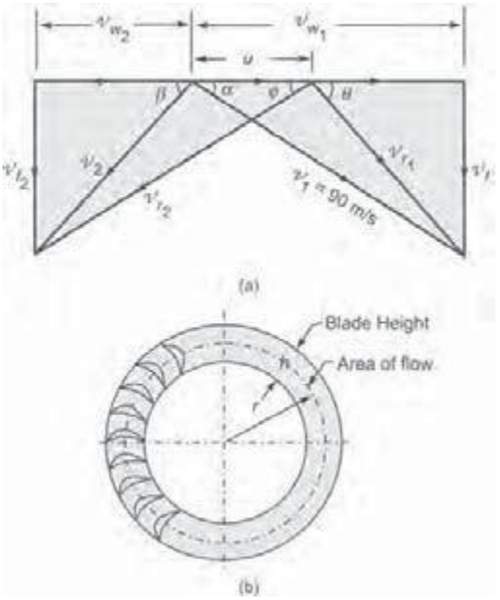

Fig. 22.43

$$= \frac{41.04}{2\pi \times 0.216} = 30.24 \text{ rev/s}$$

$$= \mathbf{1814 \text{ rpm}}$$

(ii) The diagram power

$$P = \frac{\dot{m}_s \, u \, \mathcal{V}_w}{1000}$$

We have

$$\mathcal{V}_{w_1} = \mathcal{V}_1 \cos\alpha = 90 \cos 20° = 84.57 \text{ m/s}$$

$$\mathcal{V}_{w_2} = \mathcal{V}_{r_2} \cos\phi - u = \mathcal{V}_1 \cos\phi - u \quad (\because \alpha = \phi \text{ and } \mathcal{V}_{r_1} = \mathcal{V}_1)$$

$$= 90 \cos 20° - 41.04 = 43.53 \text{ m/s}$$

$$\therefore \quad \mathcal{V}_w = \mathcal{V}_{w_1} + \mathcal{V}_{w_2} = 84.57 + 43.53$$

$$= 128.10 \text{ m/s}$$

and $$P = \frac{2.5 \times 41.04 \times 128.1}{1000} = \mathbf{13.14\,kW}$$

(iii) *Diagram efficiency or stage efficiency*

The relative efficiency at the inlet

$$\mathcal{V}_{r_1}^2 = \mathcal{V}_1^2 + u^2 - 2\mathcal{V}_1 u \cos\alpha$$

$$= 90^2 + 41.04^2 - 2 \times 90 \times 41.04 \cos 20°$$

$$= 2842.58$$

or $\mathcal{V}_{r_1} = 53.31$ m/s

Note: The magnitude of $\mathcal{V}_{f_1}$, $\mathcal{V}_{w_1}$, $\mathcal{V}_{w_2}$, $\mathcal{V}_{r_1}$, can also be obtained by measurements on vector diagram.

The energy input to moving blade per stage per kg of steam

$$= \mathcal{V}_1^2 - \frac{\mathcal{V}_{r_1}^2}{2}$$

$$= 90^2 - \frac{53.31^2}{2} = 6679.02 \text{ J/kg}$$

Total energy supplied to moving blade

$$= 2.5 \times 6679.02 = 16697.55 \text{ J/s}$$

Then $$\eta_{stage} = \frac{\text{Work done}}{\text{Energy supplied}}$$

$$= \frac{13.14 \times 10^3 \text{ J/s}}{16697.55 \text{ J/s}} = 0.787 = \mathbf{78.7\%}$$

(iv) Enthalpy drop in the moving blades can be obtained from stage efficiency as

$$\eta_{stage} = \frac{\text{Workdone per kg of steam}}{\text{Enthalpy dorp per stage}}$$

or Enthalpy drop $= \dfrac{13.14/2.5}{0.787} = \mathbf{8.348\ kJ/kg}$

Example 22.25 *At a stage in a reaction turbine, the mean blade ring diameter is 1 m. The turbine runs at 3000 rpm. The blades are designed for a degree of reaction of 50% with exit angles of 30° and inlet angles of 50°. The turbine is supplied with a steam at 10000 kg/h and the stage efficiency is 85%. Determine*

(a) power output of the stage,
(b) specific enthalpy drop in kJ/kg,
(c) percentage increase in relative velocity of steam over moving blades,
(d) the specific steam consumption.

Solution

Given A 50% reaction turbine with

$\Lambda = 0.5$ $D = 1$ m $N = 3000$ rpm

$\alpha = \phi = 30°$ $\theta = \beta = 50°$

$\dot{m}_s = 10000$ kg/h $= 2.778$ kg/s $\eta_{stage} = 0.85$

To find

(i) The power output of the stage,
(ii) Specific enthalpy drop (Δh) in kJ/kg,
(iii) Percentage increase in relative velocity of steam over moving blades, and
(iv) Specific steam consumption.

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Analysis The blade velocity is given by

$$u = \frac{\pi D N}{60} = \frac{\pi \times (1\text{m}) \times (3000\,\text{rpm})}{60}$$
$$= 157.08 \text{ m/s}$$

The velocity diagram for given turbine is constructed and shown in Fig. 22.44.

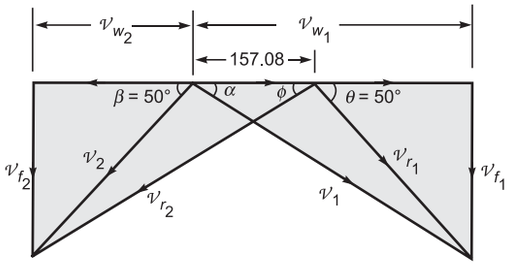

Fig. 22.44

From the measurements

$\mathcal{V}_1 = 351.8$ m/s, $\mathcal{V}_{w_1} = 304.7$ m/s

$\mathcal{V}_{f_1} = 229.6$ m/s $\mathcal{V}_{w_2} = 157.62$ m/s

The total whirl velocity

$$\mathcal{V}_w = \mathcal{V}_{w_1} + \mathcal{V}_{w_2} = 304.7 + 157.62$$
$$= 462.32 \text{ m/s}$$

(i) The power developed by the turbine

$$P = \frac{\dot{m}_s u \mathcal{V}_w}{1000}$$
$$= 2.778 \times \frac{157.08 \times 462.32}{1000}$$
$$= \mathbf{201.72\ kW}$$

(ii) The stage efficiency is given by

$$\eta_{stage} = \frac{\text{Power developed}}{\text{Total enthalpy drop per stage}}$$
$$= \frac{P}{\dot{m}_s \Delta h}$$

Thus the specific enthalpy drop can be obtained as

$$\Delta h = \frac{P}{\dot{m}_s \eta_{stage}} = \frac{201.72}{2.778 \times 0.85}$$
$$= \mathbf{85.43\ kJ/kg}$$

(iii) The exit relative velocity can be expressed as

$$\mathcal{V}_{r_2} \cos\phi = \mathcal{V}_{w_2} + u$$

or $$\mathcal{V}_{r_2} = \frac{157.62 + 157.08}{\cos 30} = 363.38 \text{ m/s}$$

The percentage increase in relative velocity of steam over moving blades

$$= \frac{\mathcal{V}_{r_2} - \mathcal{V}_{r_1}}{\mathcal{V}_{r_1}} \times 100$$
$$= \frac{363.38 - 229.6}{229.6°} \times 100 = \mathbf{58.27\%}$$

(iv) The specific steam consumption

$$ssc = \frac{\dot{m}_s}{P} \times 3600$$

Since $1\,\text{kWh} = 3600\,\text{kJ} = \frac{ssc\, u \mathcal{V}_w}{1000} = \frac{ssc \times P}{\dot{m}_s}$

Thus $$ssc = \frac{2.778 \times 3600}{201.72} = \mathbf{49.58\ kg/kWh}$$

Example 22.26 *A reaction turbine runs at 3000 rpm and the steam consumption is 20000 kg/h. The pressure of steam at a certain pair is 2 bar, its dryness fraction is 0.93 and the power developed by the pair is 50 kW. The discharge blade angle is 20° for both the fixed and moving blades and the axial velocity of flow is 0.72 times the blade velocity. Find the drum diameter and the blade height. Take the tip leakage steam as 8%. Neglect the blade thickness.*

Solution

Given A reaction turbine with

$N = 3000$ rpm $\quad \dot{m}_s = 20000$ kg/h

$p = 2$ bar $\quad x = 0.93$

$P = 50$ kW $\quad \alpha = \phi = 20°$

$\mathcal{V}_{f_1} = \mathcal{V}_{f_2} = 0.72\,u$

Leakage of steam = 8% = 0.08

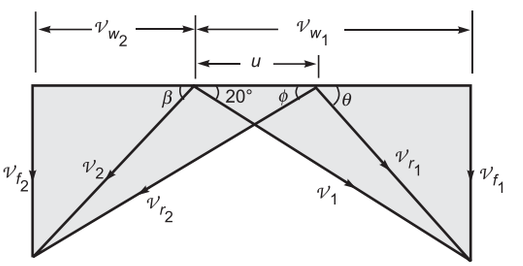

Fig. 22.45

To find

(i) Drum diameter, and

(ii) Blade height.

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Assumptions

(i) 50% Parson reaction turbine, and
(ii) No heat loss form the casing of the turbine.

Analysis

The actual mass flow rate of steam,

$$\dot{m}_{act} = \text{theoretical mass flow} \times (1 - \text{tip leakage})$$
$$= \frac{20000}{3600} \times (1 - 0.08) = 5.11 \text{ kg/s}$$

The specific volume of steam

$$v_s = x v_g \text{ @ } 2 \text{ bar} = 0.93 \times 0.8857$$
$$= 0.8237 \text{ m}^3/\text{kg}$$

The blade speed, $u = \dfrac{\pi (D + h) N}{60}$

$$u = \frac{\pi (D + h)}{60} \times 3000 = 157.08 (D + h) \quad \ldots(\text{i})$$

The axial flow velocity

$$\mathcal{V}_{f_1} = 0.72 u = 113.09 (D + h) \quad \ldots(\text{ii})$$

From velocity triangle, ΔACF

$$\frac{\mathcal{V}_{f_1}}{\mathcal{V}_{w_1}} = \tan 20°$$

or $$\mathcal{V}_{w_1} = \frac{113.09 (D + h)}{\tan 20°} = 310.73 (d + h) \quad \ldots(\text{iii})$$

and $$\mathcal{V}_{w_2} = \mathcal{V}_{w_1} - u = 310.73 (D + h) - 157.08 (d + h)$$
$$= 153.65 (d + h)$$

The total whirl velocity

$$\mathcal{V}_w = \mathcal{V}_{w_1} + \mathcal{V}_{w_2}$$
$$= 370.73 (D + h) + 153.65 (D + h)$$
$$= 464.381 (D + h)$$

Power developed by the turbine

$$P = \frac{\dot{m}_{act} u \mathcal{V}_w}{1000}$$

or $$50 = \frac{5.11 \times 157.08 (D + h) \times 464.38 (D + h)}{1000}$$

or $$(D + h)^2 = 0.1341 \quad \ldots(\text{iv})$$

or $$D + h = 0.3662 \quad \ldots(\text{v})$$

Further, the theoretical mass-flow rate

$$\dot{m}_s = \frac{\pi (D + h) h \mathcal{V}_{f_1}}{v_s}$$

$$\frac{20000}{3600} = \frac{\pi \times (D + h) h \times 113.09 (D + h)}{0.8237}$$

Using the value of $(d + h)^2$ from Eq. (iv)

$$\frac{20000}{3600} = \frac{\pi \times 0.1341 h \times 113.09}{0.8237}$$

$$h = 0.096 \text{ m}$$

Thus, $$D = 0.3662 - 0.096 = \mathbf{0.270\ m}$$

22.10 COMPARISON BETWEEN IMPULSE AND REACTION TURBINES

The differences between the impulse and reaction turbines are summarized in Table 22.1 given below.

Table 22.1 *Comparison between impulse and reaction turbines*

S.No.	*Impulse Turbines*	*Reaction Turbines*
1.	Pressure drop takes place in nozzles only and pressure remains constant in moving blades. This means that the relative velocity remains same in moving blades or reduces slightly on account of friction.	Pressure drop takes place in both fixed blades (nozzles) and moving blades. Thus, pressures value is different on the two sides of the moving blades. Continuous expansion of steam means relative velocity in moving blades increases and volume of flow changes.
2.	Blade shape is profile type and its manufacturing is simple.	Blade shape is aerofoil type and its manufacturing is difficult.
3.	Blade passage is of constant cross-sectional area since there is no expansion.	Blade passage is of variable cross-sectional area (converging type) due to expansion.
4.	Diaphragm contains the nozzles and the rotor construction is disc or wheel type.	Fixed blades attached to casing serve as nozzles and the rotor construction is drum type.

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5.	Because of large pressure drop in nozzles, the number of stages are less.	Because of small pressure drop in each stage, the number of stages are larger for the same pressure drop. Reaction turbines are multi-stage turbines only.
6.	Because of large pressure drop, the blade speed and steam speed are larger. Also, the diagram efficiency decreases rapidly with the change in designed blade speed/ steam speed ratio.	Because of small pressure drop, the blade speed and steam speed are small. The diagram efficiency vs blade speed/steam speed ratio curve is flat for large values, so greater working range is available.
7.	Overall friction losses are more compared to leakage losses.	Leakage losses are more compared to friction losses.
8.	Occupies less space per unit power.	Occupies more space per unit power.
9.	Suitable for small power.	Suitable for medium and large power.

22.11 LOSSES IN STEAM TURBINES

The total energy contents of steam supplied to a turbine is not completely converted into mechanical energy. There are certain energy losses, which occur inside a turbine. These are

1. Admission losses
2. Leakage losses
3. Friction losses
4. Exhaust losses
5. Radiation and convection losses
6. Losses due to moisture
7. Carry over losses

22.11.1 Admission Losses

In actual practice, the flow through the nozzle is not isentropic, but accompanied with losses, causing a decrease in kinetic energy of steam coming out of the nozzle. The decrease in kinetic energy is caused due to these causes.

1. Heat loss from steam before entering the nozzle.
2. The friction in the nozzle reduces available enthalpy drop, which is less than isentropic enthalpy drop. Thus, the actual velocity leaving the nozzle is less than that obtained with isentropic expansion.
3. Viscous friction between steam particles.
4. Flow deflection in the nozzle.
5. Development of boundary layer in nozzle.
6. Turbulence in the nozzle.

22.11.2 Leakage Losses

Steam leaves the boiler and reaches the condenser after passing through the main valve, regulating valves, nozzles, clearance spaces between nozzles and moving blades, diaphragm and rotating shaft, etc. Further, there is large pressure difference inside and outside, from one location to another location across these devices. Therefore, the steam leakages take place through (i) main valve and regulating valves, (ii) seals and glands, (iii) spaces between nozzles and moving blades, (iv) space between diaphragm and shafts of turbine, and (v) space between moving blade rings and turbine casing. The steam leakages through these devices carry energy, which is a wastage of steam.

22.11.3 Frictional Losses

Frictional resistance is offered during flow of steam through nozzles, on moving and stationary blades. In most of the turbines, the wheel is rotating in a space full of steam. The viscous friction at the wheel surface causes admission loss as steam passes from nozzle to wheel. The effect of partial admission creates eddies in the blade channels and blade windage losses.

The surface of the curved moving blades and stationary blades offers resistance, which increases with increase in relative velocity between them and roughness of blade surface.

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The energy loss also takes place when the steam jet turns along the curvature of the blade surface. The turning losses depend on the angle of turning.

22.11.4 Exhaust Losses

The energy content of steam is not fully utilized in the turbine. The exhaust steam coming out of the turbine and entering the condenser carries some of kinetic energy and useful enthalpy, which is direct energy loss.

22.11.5 Radiation and Convection Losses

The steam turbine is operated at a relatively high temperature. Therefore, some of the heat energy of steam is radiated and convected from the body of the turbine to its surroundings. These are the direct losses and are minimized by proper insulation.

22.11.6 Loss Due to Moisture

The steam passing through the last stage of the turbine has very high velocity and it contains large moisture. The liquid particles have lesser velocity than that of vapour particles. Hence the liquid particles obstruct the flow of vapour particles in the last stage of turbine and therefore, a part of kinetic energy of steam is lost. If the dryness fraction of steam falls below 0.88, the erosion and corrosion of blades can also take place.

22.11.7 Carry-over Losses

When steam passes from one stage to the next stage through the diaphram, some energy loss takes place, which is referred as carry-over loss. Therefore, the kinetic energy of steam available at succeeding row of moving blades for utilization is less than that at the exit of preceding row. It is due to formation of eddies in the annular space between the nozzle (Or fixed blades) and moving blades.

12.12 GOVERNING OF STEAM TURBINE

The purpose of governing of steam turbine is to maintain its speed as constant, irrespective of its load. The turbine speed is controlled by varying the steam flow rate by means of valves interposed between the boiler and turbine. The steam turbine may be governed by the following possible methods:

1. Throttle governing
2. Nozzle control governing
3. By-pass governing
4. Combination of any of the above two methods
5. Emergency governing

12.12.1 Throttle Governing

In throttle governing, the pressure of steam is reduced by passing the steam through a restricted passage like partially opened steam stop valve in order to maintain the speed of the turbine constant at part load. The throttle governing is most widely used on small turbines, because of its low initial cost and simple mechanism.

For the turbines of small power plants, the valves are light and a centrifugal governor shown in Fig. 22.46 may be used to actuate the valve directly. The centrifugal governor consists of flying balls attached on the arm connected to the sleeve. The sleeve moves up and down axially on a rotating shaft geared to the turbine shaft.

As the load on the turbine decreases, the turbine shaft speed increases. With increase in speed, the flying ball of the governor flies apart and raises the sleeve which operates a lever through fulcrum and actuates the main valve to close partially and reduce the mass flow rate of steam.

For large steam turbines, there is considerable friction at the valves and an oil-operated servo system can be used to boost the sensitivity of the lever connected to the governor sleeve. The small deflection of the lever of the governor is amplified with the help of the relay system shown in Fig. 22.47. It consists of a throttle valve (*A*), relay piston (*B*), differential lever (*C*), frictionless pilot valves (*D*), servomotor and piston spindle (*E*) connected to the throttle valve. A differential lever is attached at one end of the governor sleeve and the other end to

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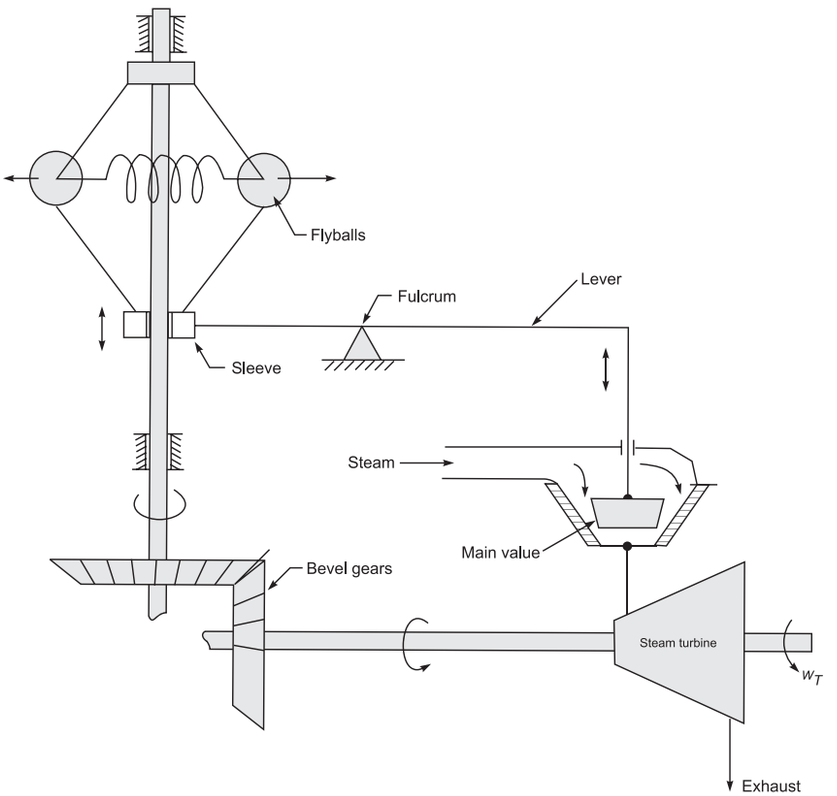

Fig. 22.46 *Simple centrifugal governor for throttle governing*

the throttle valve spindle and at some intermediate point, to the pilot valve spindle. The pilot valves are two small piston valves, which cover two ports in an oil chamber without any lap. Pipes *G* are open to oil drain tank. Any deflection of the sleeve causes the displacement of pilot valves in the oil chamber through a differential lever. Accordingly, the oil under the pressure enters either the upper half or lower half of the chamber.

If the load on the turbine decreases, the excessive power of the turbine will accelerate the rotor and causes the governor sleeve to lift. Thus, the lever deflection will raise the pilot valves spindle (D_1) and the upper port is opened to oil entry and the lower port for oil exit. The high-pressure oil enters the cylinder and pushes the relay piston (*B*) downward. Thus, the piston spindle (*E*) descends and closes the throttle valve partially. The downward movement of piston spindle (*E*) also lowers the pilot valves (*D*), both ports are covered and the relay piston position is locked. The steam pressure and its flow rate reduce, and thus the turbine power output reduces till its speed falls fairly near to normal value.

If the load on the turbine increases, the lever deflection opens the lower half passage for oil entry and an opposite operation of the piston opens the throttle valve for more steam flow and thus more power output of turbine till the speed again matches with normal value.

The measurement shows that the steam consumption rate, $\dot{m}_s$ (kg/h) is linearly proportional to the turbine load during throttle-control governing. Figure 22.48(a) shows a graph of the steam consumption and load on turbine. The linear line on the

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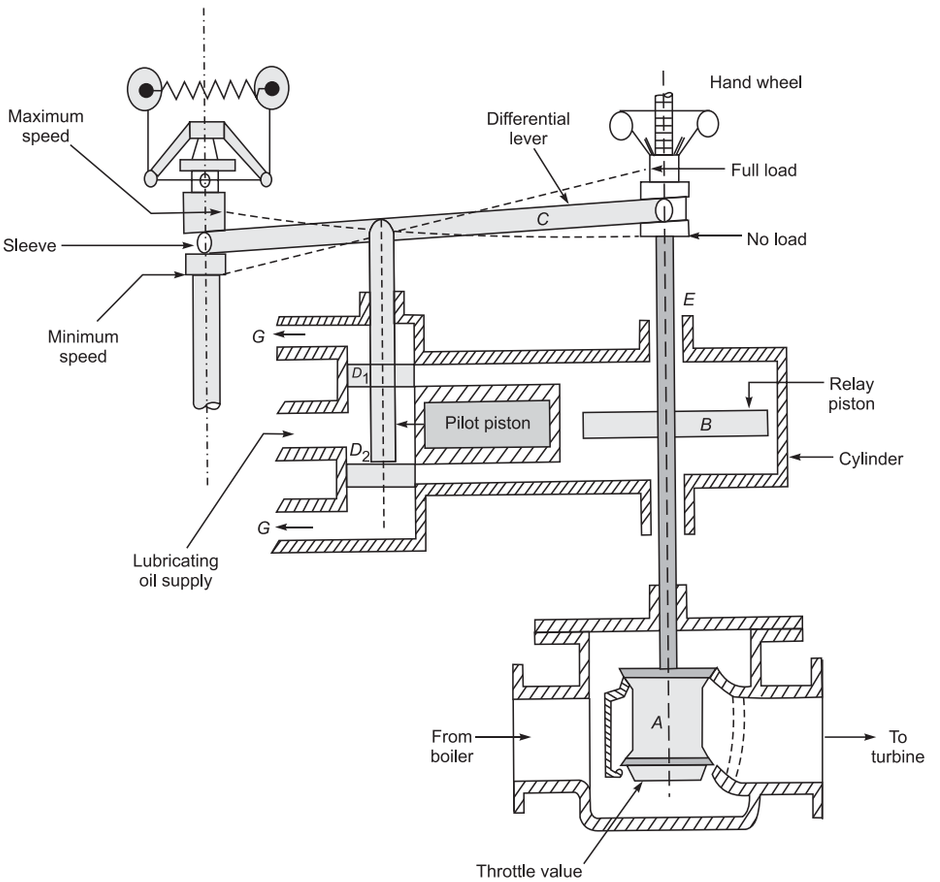

Fig. 22.47 *Servomotor-controlled throttle governing*

graph is called *Willan's line* given by

$$\dot{m}_s = a\,L + C \qquad ...(22.43)$$

where a is the steam rate, kg/kWh

L is load on turbine, kW

C is no load steam consumption

In throttle governing, the pressure of steam is reduced at turbine entry. Thus, the energy availability is reduced. Further, if throttle governing is used at low loads, the turbine efficiency is drastically reduced.

12.12.2 Nozzle Control Governing

In nozzle control governing, the nozzles are grouped in sets of two, three and more groups and each set of nozzles is controlled by a separate valve. When load on the turbine decreases, the required number of nozzles can be shut off. The nozzle control usually affects the first stage of the turbine only. Other stages remain unaffected. In nozzle control governing, the mass flow rate of steam is regulated rather than regulating steam pressure. It has the advantage of using steam at full boiler pressure and reduction in steam consumption rate at part loads.

Figure 22.49 shows a schematic arrangement of nozzle control governing. It consists of three sets of nozzle as N_1, N_2 and N_3 having 5, 4 and 12 nozzles, respectively. The opening and closing of these sets of nozzles is controlled by valves V_1, V_2 and V_3.

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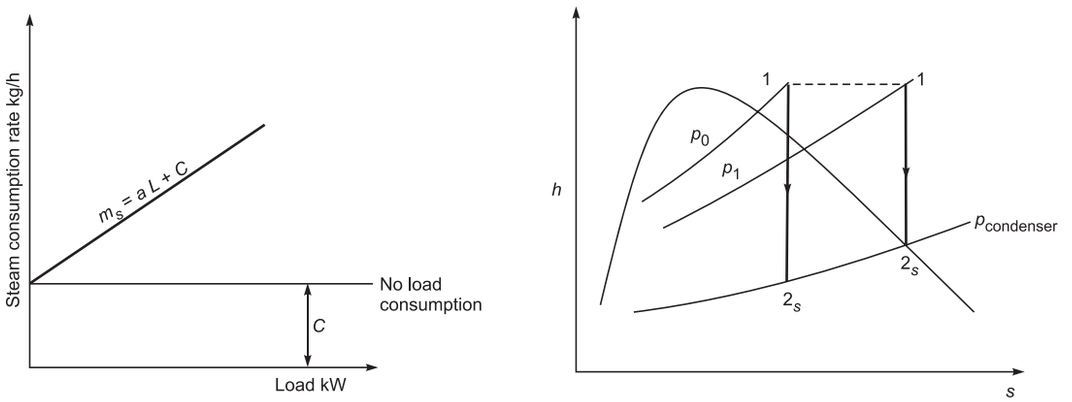

Fig. 22.48

12.12.3 By-Pass Governing

When a steam turbine is overloaded, additional fresh steam is admitted through a by-pass valve to later stages of the turbine. Within the economical loads, the turbine is governed by a speed governor through nozzle control or throttle governing. But for all loads greater than economical load, a by-pass valve is opened, allowing some fresh steam from the steam chest to enter the later stage of the turbine.

12.12.4 Emergency Governing

Every steam turbine is also provided with an emergency governor, which comes in operation, when

1. speed of shaft exceeds beyond 110%,
2. balancing of turbine is disturbed,
3. lubrication system fails, and
4. the vacuum in the condenser is too less or coolant supply to the condenser is inadequate.

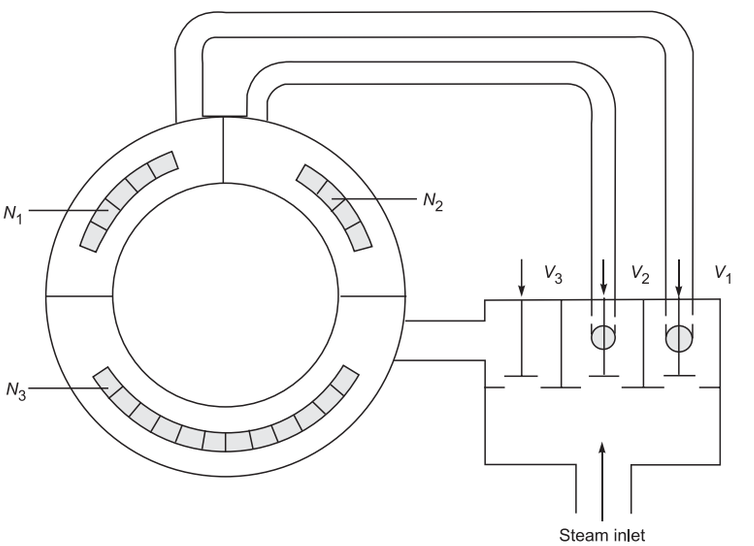

Fig. 22.49 *Nozzle controlled governing*

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22.13 SPECIAL FORMS OF TURBINES

There are dual demand of power and process heating simultaneously in several industries, such as papermaking, textile, sugar, chemical dying, refining, pulp, brewing, etc. For power generation, high pressure, superheated steam is required in a turbine, while the saturated steam at moderate pressure fulfils the requirement of process heating.

The steam power plants convert approximately 40% of heat supplied into useful work and reject remaining 60% of heat as waste heat to cooling water in the condenser. For process heating, the low-pressure saturated steam gives away its latent heat at constant temperature. In this situation, it is not economical to install two boilers to generate steam at different pressures for power generation and process heating. It is better to design one plant to offer both services by suitable modification. Such a plant is known as a combined heat and power or *co-generation plant*.

There are two types of steam turbines most widely used in combined power and process plants: back-pressure and the extraction-condensing types, shown in Fig. 22.50.

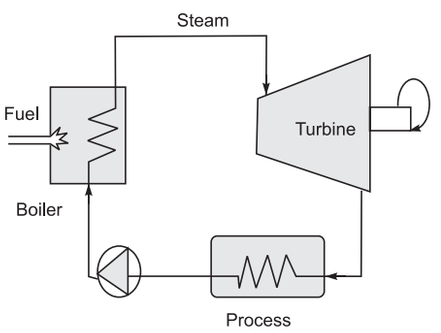
22.13.1 Back-Pressure Turbine

In a *back pressure turbine,* steam enters at boiler pressure and after expansion to some intermediate pressure, it is exhausted into a pipe, which leads to a process plant. This system may be used when power generated by expanding steam from boiler pressure to heating pressure is equal to or greater than the power requirement of the plant.

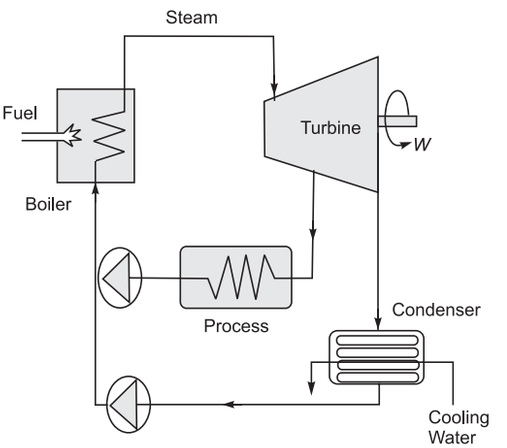
22.13.2 Extraction Turbine

In a pass-out or extraction turbine, the steam enters at boiler pressure, and after its expansion to some intermediate pressure between inlet and exhaust, some steam is extracted for process heating and the remaining steam is allowed to expand in the turbine to condenser pressure.

The choice between back-pressure turbine and extraction-condensing turbine depends mainly on the quantities of power and heat, quality of heat, and economic factors. The extraction points of steam from the turbine could be more than one, depending on the temperature levels of heat required by the processes.



(i) Back-pressure turbine



(ii) Extraction condensing turbine

Fig. 22.50

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Summary

- A steam turbine is a *rotodynamic machine*, in which a mechanical rotating shaft work is developed by steady flow of steam through it. The *motive force* in a steam turbine is obtained by changing the momentum of a high-velocity jet of steam, impinged on the curved blades fixed on the rotor.
- An *impulse turbine* works on the principle of impulse, Newton's second law of motion. The impulse force is generated by the change in momentum of steam gliding on the blades, causing the rotor to spin.
- A *reaction turbine* works on the principle of Newton's third law of motion, in which the rotation is caused by the reaction force generated by the momentum change of the flowing steam.
- Compounding of steam turbine is done to reduce the rotational speed of the turbine to the practical limits. There are three types of compounded impulse turbines.

  In the *velocity-compounded impulse turbine*, the steam expands to exhaust pressure in a single set of nozzles and the kinetic energy of the steam is absorbed by two or three sets of moving blades.

  In the *pressure-compounded impulse turbine,* the steam expands gradually into a number of stages. Each stage comprises of its own set of nozzles and moving blades.

  The *pressure-velocity compounded impulse turbine,* is a combination of pressure and velocity compounding.
- The rate of work done on the blade in a stage of impulse turbine is given by

$$\dot{W} = \dot{m}_s\, u\,(\mathcal{V}_{w_1} + \mathcal{V}_{w_2}) = \dot{m}_s\, u\,\mathcal{V}_w \text{ (W)}$$

- The blade efficiency

$$\eta_b = \frac{\text{Work done on the blade}}{\text{Energy supplied to the blade}} = \frac{2u\mathcal{V}_w}{\mathcal{V}_1^2} = \frac{\mathcal{V}_1^2 - \mathcal{V}_2^2}{\mathcal{V}_1^2}$$

- It becomes maximum, when blade speed ratio, $s = \frac{u}{\mathcal{V}_1}$ becomes equal to $\frac{\cos\alpha}{2}$.

$$\eta_{stage} = \frac{\text{Work done on the blade}}{\text{Energy supplied per stage}} = \frac{u\,\mathcal{V}_w}{\Delta h}$$

- Axial thrust on the blade

$$F_a = \dot{m}_s\,(\mathcal{V}_{f_1} - \mathcal{V}_{f_2})$$

- The actual reaction turbines do not operate on pure reaction, but they utilize the principle of impulse and reaction simultaneously. Therefore, these are also called *impulse-reaction turbines.* The Parson's reaction turbine operates on 50% of degree of reaction.
- The degree of reaction is defined as

$$\Lambda = \frac{\text{Enthalpy drop in the moving blades}}{\text{Total enthalpy drop in the stage}} = \frac{\Delta h_m}{\Delta h}$$

- The blade efficiency of a reaction turbine is given as

$$\eta_b = \frac{u(\mathcal{V}_{w_1} + \mathcal{V}_{w_2})}{\Delta h} = 2 - \frac{2}{1 + 2s\cos\alpha - s^2}$$

- It becomes maximum, when $s = \cos\alpha$

Glossary

Turbine A rotadynamic machine

Impulse turbine Steam expands in nozzles only and blades spin due to change in momentum of steam

Reaction turbine Steam expands in fixed and moving blades, thus reactive force acts on blades to cause them to rotate

Whirl velocity Tangential component of velocity, responsible for useful work

Flow velocity Axial component of velocity, allows the steam flow across the wheel

Relative velocity Velocity of steam relative to blade speed

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Stage A set of fixed blades (nozzles) and moving blades

Stage efficiency Ratio of work done on blade to energy supplied per stage

Blade-efficiency Ratio of work done on blade to energy supplied to blade

Blade-speed ratio Ratio of blade velocity to absolute steam velocity

Blade-velocity coefficient Ratio of outlet relative velocity to inlet relative velocity

Axial flow turbine Turbine, in which steam leaves blades at 90° to blade velocity u

Compounding Using more than one set of nozzles and moving blades in series keyed on same shaft

Degree of reaction Ratio of enthalpy drop in moving blades to total enthalpy drop in a stage

Review Questions

1. Distinguish between impulse and reaction turbines.
2. Explain the principle of working of an impulse turbine.
3. Write the details of a simple impulse turbine.
4. Draw the velocity-triangle diagram for an impulse turbine blades and derive the expressions for work done and axial thrust.
5. Define blade efficiency and stage efficiency.
6. Derive an expression for maximum blade efficiency for an impulse turbine in terms of blade speed ratio.
7. Why are steam turbines compounded? Explain.
8. Describe the various methods of compounding in an impulse turbine.
9. What is the pressure–velocity compounding? Write its advantages.
10. Prove that the net efficiency of a simple turbine is given by

$$\eta_{net} = \eta_{stage} \times \eta_N \times \eta_{mech.}$$

11. Explain the working of an impulse reaction turbine.
12. Draw the velocity triangle for a 50% reaction turbine and derive the expressions for work done and blade efficiency.
13. Define degree of reaction.
14. Prove that for a 50% reaction turbines, $\alpha = \phi$ and $\theta = \beta$.
15. Prove that the shape of fixed blades and moving blades is identical in a 50% reaction turbine.
16. Prove that $\mathcal{V}_1 = \mathcal{V}_{r_2}$ and $\mathcal{V}_2 = \mathcal{V}_{r_1}$ in a 50% reaction turbine.
17. Derive an expression for optimum stage efficiency of a reaction turbine.
18. Explain how the flow rate can be obtained from given blade height, mean diameter and steam condition at that stage.

Problems

1. Steam issues from the nozzles of a single impulse turbine at 850 m/s. The blades are moving at 350 m/s. The blade tip angles at inlet and exit are each 36°. The steam enters the blades without shock and the flow over the blades is frictionless. Determine (a) the angle at which the nozzles are inclined to the direction of motion of the blades, and (b) the diagram efficiency.

[(a) 22°, (b) 86%]

2. Steam leaves the nozzle of a single impulse wheel turbine at 900s. The nozzle angle is 20° and the blade angles are 30° at inlet and outlet. What is the blade velocity and the work done per kg of steam? Assume the flow over the blades as frictionless. *[270 m/s, 307.8 kJ/kg]*

3. The mean diameter of the blades of an impulse turbine with a single-row wheel is 1 m and the speed of rotation is 3,000 rpm. The nozzle angle

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is 18°, the ratio of blade speed to steam speed is 0.42, the ratio of the relative velocity at the outlet from the blades to that at inlet is 0.84. The outlet angle of the blade is to be 3° less than the inlet angle. The steam flow is 7 kg/s. Determine (a) tangential force on the blades, (b) power developed on the blades, (c) blading efficiency, and (d) axial thrust on the blades.

[(a) 2205 N (b) 346.18 kW (c) 84.8% (d) 8.5 N]

4. In a single-stage impulse turbine, the steam jet leaves the nozzles at 20° to the plane of the wheel at a speed of 670 m/s and it enters the moving blades at an angle of 35° to the drum axis. The moving blades are symmetrical in shape. Determine the blade velocity and diagram efficiency.

[300 m/s; 89.4%]

5. Steam with a velocity of 800 m/s enters an impulse turbine ring and drives the rotor at 3000 rpm. The jet angle is 20° and the mean drum diameter is 1.4 m. Assuming that inlet and exit angles of the moving blades are equal and a blade velocity coefficient of 0.85, find

(a) the blade angles
(b) diagram efficiency
(c) power developed per kg per second of steam flow
(d) stage efficiency, if the nozzle efficiency is 95%

[(a) 28° (b) 56% (c) 179.37 kW (d) 53.2%]

6. The velocity of jet of steam entering a de-Laval turbine is 500 m/s. The nozzles are inclined at 20° to the direction of blades. The blade speed is 200 m/s and the exit angle of the moving blades is 25°. For mass flow rate of 5 kg/s, determine

(a) exit velocity of steam
(b) diagram efficiency
(c) power developed.

[(a) 162 m/s (b) 90.4% (c) 225.95 kW]

7. Show that in an impulse turbine having a fixed nozzle angle α, equiangular blades with blade velocity coefficient 'k', the maximum blade efficiency is given by,

$$\eta_{max} = \left(\frac{1+k}{2}\right)\cos^2\alpha$$

8. The blade angles for a de-Laval turbine are designed for the maximum efficiency for which the blade velocity is fixed at 200 m/s and the blade inlet and discharge tip angles are 30°. Determine the required steam jet velocity and the nozzle angle, neglecting friction over the ring of moving blades. Hence, determine the diagram efficiency of the turbine.

[$\mathcal{V}_1$ = 407.5 m/s; α = 16°; η = 92.4%]

9. Steam at a velocity of 400 m/s relative to the moving blades enters an impulse turbine at an angle of 30°. The blade velocity is 250 m/s. The work developed in the blades is estimated to be 165.54 kW/kg. Assuming the blades to be symmetrical in shape, determine the blade efficiency and blade-velocity coefficient.

[83.5%; 0.9]

10. From the pair of blades of an impulse turbine, the steam leaves the moving blades at an absolute velocity of 130 m/s. The nozzles have a discharge angle of 20° and the steam enters the blades without shock. Determine the following for the maximum efficiency conditions.

(a) The blade angles
(b) The steam jet and blade velocity
(c) The stage efficiency, if the nozzle efficiency is 90%.

Assume equiangular moving blades.

[(a) 36°, (b) $\mathcal{V}_1$ = 435 m/s; u = 204 m/s; (c) 79.2%]

11. A single-stage impulse turbine nozzle issues a steam jet at a velocity of 450 m/s at an angle of 18° to the plane of wheels. The blade-speed ratio is 0.42. The blade velocity coefficient is 0.9 and the exit angle of the moving blades is 27°. Draw a velocity diagram for the stage and determine

(a) axial thrust on the bearings,
(b) power developed.

Assume the mass-flow rate of steam to be 10 kg/s. *[(a) 221.7 N, (b) 872.6 kW]*

12. In an impulse turbine, the steam enters a ring of moving blades with a relative velocity of 209 m/s. The nozzles are inclined at 20° to the plane of the wheel. The moving blades are symmetrical in shape and the blade velocity is 157 m/s. The friction losses in the blades are equivalent to

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14%. For an axial thrust of 171.6 N, determine the following:

(a) Mass flow rate of steam,
(b) Steam jet velocity,
(c) Power developed.

[(a) 98.1 kg/s, (b) 350 m/s, (c) 489.3 kW]

13. In a reaction turbine, the diameter of the rotor is 2 m and its speed is 840 rpm. The steam consumption amounts to 870 kg/min. The height of the blade at a particular stage is 15 cm. The exit angle of the nozzle and the moving blades is 25°. The pressure at this stage is 0.3 bar and steam is 0.98 dry. Estimate the power developed and the heat drop in kJ/s. *[199.4 kW; 17.63 kJ/s]*

14. In a stage of an impulse turbine provided with a single-row wheel, the mean diameter of the blade ring is 80 cm and the speed of rotation is 3,000 rpm. The steam issues from the nozzle with a velocity of 275 m/s and the nozzle angle is 20°. The inlet and outlet angles of the blades are equal, and due to friction in the blade channels the relative velocity of the steam at outlet from the blade is 0.86 times the relative velocity of steam entering the blades. What is the power developed in the blading when the axial thrust on the blades is 12.2 kg?

15. Steam issues from the nozzles of a de-Laval turbine with a velocity of 920 m/s. The nozzle angle is 20°, the mean diameter of the blades is 25 cm and the speed of rotation is 20,000 rpm. The steam flow through the turbine is 0.18 kg/s. If the ratio of relative velocity at outlet from the blades to that at inlet is 0.82, calculate

(a) Tangential force on blades
(b) Work done on blades per second
(c) Power of the wheel
(d) Efficiency of blading
(e) Axial force on blades,
(f) Inlet angle of blades for shock-less in flow of steam.

Assume that the outlet angle of blades is equal to the inlet angle.

16. In Problem 15, in a Parson's reaction turbine the exit angles of the moving blades is 20° and the absolute discharge velocity of steam from the moving blades is 50 m/s in the direction at 115° to the motion of the blades. Calculate the power developed if the mass-flow rate is 5 kg/s.

[89.2 kW]

17. One expansion in a reaction turbine has 8 pairs of blades of an outlet angle of 20°. The mean diameter of the expansion is 50 cm and the rotor speed is 3000 rpm. The ratio of blade speed to steam speed is 0.8. The efficiency for the stage is 80%. Determine the power developed and the adiabatic heat drop during the expansion for a steam-flow rate of 5 kg/s.

[320.17 kW; 76.8 kJ/kg]

18. A reaction turbine runs at 600 rpm and consumes 18000 kg/h of steam. The exit angle of the fixed and moving blades are 20°. The axial velocity of flow is 0.75 times the blade velocity. Determine the drum diameter and blade height of a particular stage where steam pressure is 2 bar and it is 0.95 dry. Assume the power developed to be 15 kW and the tip leakage of steam as 7%.

[0.9728 m; h = 5.05 cm]

19. Deduce an expression for work done per stage of a reaction turbine and determine the condition for maximum efficiency.

If the blade speed of a Parson's reaction turbine is 288 m/s and exit angles are 20°, determine the work done per kg and the maximum diagram efficiency of the turbine.

[$u = V_1 \cos \alpha$; 82678 Nm]

20. In a stage of 50% Parson's reaction turbine, the steam consumption is 1800 kg/h and it runs at 300 rpm. The discharge blade tip angles are 20° for both fixed and moving blades. The axial velocity of flow is 0.7 times the blade velocity. Determine the drum diameter and blade height of a particular turbine pair where the pressure is 2.0 bar of steam, 0.95 dry and the power developed amounts to 3.75 kW.

[91.89 cm, 11.41 cm]

21. At a particular stage of 50% reaction turbine, the pressure is 1.4 bar and steam is 0.9 dry. The inlet and outlet angles are 35° and 20°, respectively. The blade velocity is 67 m/s. Determine the blade height, if the ratio of drum diameter to blade height is 8.0 for a mass-flow rate of 4.5 kg/s. Also, find the power developed.

[5.9 cm; 63.93 kW]

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22. In a stage of impulse reaction turbine, operating with 50% degree of reaction, the blades are identical in shape. The outlet angle of moving blades is 19° and the absolute discharge velocity of steam is 100 m/s in the direction at 100° to the motion of blades. If the rate of flow of steam through the turbine is 15000 kg/h, calculate the power developed by the turbine in kW.

23. In a stage of implulse reaction turbine provided with a single-row wheel, the mean diameter of blades is 1 m, it runs at 3000 rpm. The steam issues from the nozzle at a velocity of 350 m/s and the nozzle angle is 20°. The rotor blades are equiangular. The blade friction factor is 0.86. Determine the power developed, if axial thrust on blade-end bearing of the rotor is 118 N.

24. In a two-stage, velocity-compounded steam turbine, the mean blade speed is 150 m/s, while steam velocity as it is issued from the nozzle is 675 m/s. The nozzle angle is 20°, the exit angle of first-row moving blades, fixed blades and second-row moving blades are 25°, 25°, and 30°, respectively. The blade friction coefficient is 0.9. If the steam-flow rate is 4.5 kg/s, determine
(i) power output, and (ii) diagram efficiency

Objective Questions

1. The steam turbines is a
   (a) rotary machine
   (b) reciprocating machine
   (c) rotodynamic machine
   (d) none of the above
2. Thermodynamic efficiency of a steam turbine is
   (a) less than a steam engine
   (b) less than a Diesel engine
   (c) less than a petrol engine
   (d) none of the above
3. From inlet to exit of steam nozzle, the pressure
   (a) increases (b) decreases
   (c) remains constant (d) none of the above
4. From inlet to exit of moving blades in case of impulse turbine, the pressure
   (a) increases (b) decreases
   (c) remains constant (d) none of the above
5. From inlet to exit of moving blades in case of a reaction turbine, the pressure
   (a) increases (b) decreases
   (c) remains constant (d) none of the above
6. In an impulse turbine, steam expands
   (a) in the nozzle only
   (b) in the moving blades only
   (c) in the fixed and moving blades
   (d) none of the above
7. In a reaction turbine, steam expands
   (a) in the nozzle only
   (b) in the moving blades only
   (c) in the fixed and moving blades
   (d) none of the above
8. A simple impulse turbine consists of
   (a) one set of nozzles and one set of moving blades
   (b) two sets of nozzle and one set of moving blades
   (c) one set each of fixed and moving blades
   (d) none of the above
9. Tangential components of velocity is called
   (a) relative velocity (b) flow velocity
   (c) whirl velocity (d) absolute velocity
10. Axial components of velocity is called
    (a) relative velocity (b) flow velocity
    (c) whirl velocity (d) absolute velocity
11. The stage efficiency is also called
    (a) blade efficiency
    (b) diagram efficiency
    (c) gross efficiency
    (d) none of the above
12. The blade efficiency is also called
    (a) stage efficiency
    (b) diagram efficiency

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(c) gross efficiency
(d) none of the above

13. The stage efficiency of a steam turbine is also given by
(a) $\dfrac{\text{Nozzle efficiency}}{\text{Blade efficiency}}$
(b) $\dfrac{\text{Blade efficiency}}{\text{Nozzle efficiency}}$
(c) nozzle efficiency × blade efficiency
(d) none of the above

14. The overall efficiency of a steam turbine is given by
(a) $\dfrac{\eta_{Nozzle}}{\eta_{Blade}} \times \eta_{stage}$
(b) $\dfrac{\eta_{Blade}}{\eta_{Nozzle}} \times \eta_{stage}$
(c) $\eta_{Nozzle} \times \eta_{Stage} \times \eta_{Mech}$
(d) none of the above

15. The blade-speed ratio is given by
(a) $\dfrac{\text{Absolute velocity}}{\text{Blade velocity}}$
(b) $\dfrac{\text{Blade velocity}}{\text{Absolute velocity}}$
(c) $\dfrac{\text{Blade velocity}}{\text{Absolute velocity}}$
(d) none of the above

16. In a velocity-compounded impulse steam turbine, steam expands in
(a) one set of nozzles only
(b) more than one set of nozzles
(c) fixed and moving blades
(d) none of the above

17. In a pressure-compounded impulse steam turbine, steam expands in
(a) nozzles and fixed blades only
(b) moving blades only
(c) fixed and moving blades both
(d) none of the above

18. In a pressure-compounded impulse steam turbine, pressure drop over each ring of moving blades
(a) remains constant (b) is increasing
(c) is decreasing (d) none of the above

19. In a pressure-compounded impulse steam turbine, as compared to velocity compounding, the number of stages is
(a) less (b) more
(c) same (d) none of the above

20. The degree of reaction is defined as
(a) $\dfrac{\text{Enthalpy drop in moving blades}}{\text{Enthalpy drop in fixed blades}}$
(b) $\dfrac{\text{Enthalpy drop in moving blades}}{\text{Total enthalpy drop}}$
(c) $\dfrac{\text{Enthalpy drop in fixed blades}}{\text{Total enthalpy drop}}$
(d) none of the above

21. For a Parson reaction turbine, the degree of reaction is
(a) 80% (b) 50%
(c) 75% (d) none of the above

22. For a Parson reaction turbine, the condition for maximum efficiency is given by
(a) $\dfrac{u}{v_1} = 2\cos\alpha$
(b) $\dfrac{u}{v_1} = \cos\alpha$
(c) $\dfrac{u}{v_1} = \cos^2\alpha$
(d) none of the above

23. In throttle governing of steam turbines
(a) mass-flow rate of steam is regulated
(b) pressure of steam is regulated
(c) additional steam is supplied
(d) none of the above

24. In nozzle control governing of steam turbines
(a) mass-flow rate of steam is regulated
(b) pressure of steam is regulated
(c) additional steam is supplied
(d) none of the above

25. In by-pass governing of steam turbines
(a) mass-flow rate of steam is regulated
(b) pressure of steam is regulated
(c) additional steam is supplied
(d) none of the above

Answers

1. (c)	2. (d)	3. (b)	4. (c)	5. (b)	6. (a)	7. (c)	8. (a)	9. (c)	10. (b)
11. (c)	12. (b)	13. (c)	14. (c)	15. (b)	16. (a)	17. (a)	18. (a)	19. (b)	20. (b)
21. (b)	22. (b)	23. (b)	24. (a)	25. (c)					

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23

Steam Condensers

Introduction

In thermal power plants, condense are used to condense the exhaust steam from a steam turbine to obtain maximum efficiency and also to convert the turbine exhaust steam into pure water so that it may be reused in the steam generator as feed water.

In this chapter, the classification, construction and working of jet and surface condensers are explained in the first part. Then the condenser and vacuum efficiency with the methods of their augmentation are discussed. The cooling tower and cooling pond are discussed at the end of the chapter.

23.1 CONDENSER

A condenser is a device in which vapour condenses to liquid phase at saturation temperature and constant pressure. During condensation, the working substance changes its phase from vapour to liquid and rejects latent heat. A condenser maintains a very low pressure, due to sudden decrease in specific volume of the working substance. In a steam condenser, the cooling of steam is accomplished by circulating water as a cooling agent in the condenser. The exhaust pressure in the condenser is maintained nearly 7 to 8 kPa which corresponds to condensate temperature of nearly 40°C.

23.2 FUNCTIONS OF A CONDENSER

In a steam power plant, the condenser plays a very important role. The main functions of a steam condenser are listed below:

1. The condenser lowers the back pressure at the turbine exhaust. Thus, steam expands through a higher pressure ratio across the turbine. It results into
   (i) increased work done per cycle,
   (ii) improved thermal efficiency of the cycle, and
   (iii) reduced steam consumption.
2. The condenser enables the recovery and recirculation of pure feed water into the plant. Thus,
   (i) the cost of water softening plant is reduced, and
   (ii) it also saves the cost of fresh water to be supplied to the boiler.
3. The condenser enables the removal of air and non-condensable gases from steam. Thus the heat-transfer rate is improved and tube corrosion is reduced.

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23.3 ELEMENTS OF A CONDENSING PLANT

A condenser unit consists of a condenser, air-extraction pump, cooling-water circulating pump, feed-water extraction pump and a cooling tower as shown in Fig. 23.1. The brief function of each element is given below:

(*i*) *Condenser* It condenses the exhaust steam from the turbine with the help of circulating water.

(*ii*) *Condensation Extraction Pump* It extracts the condensate from the bottom of the condenser and supplies it to the hot well. The feed water from the hot well is further pumped to the boiler.

(*iii*) *Air-extraction Pump* It removes the air and non-condensable gases from the condenser.

(*iv*) *Circulating Pump* It takes in the water from the sump of cooling tower and circulates it through the condenser tubes.

(*v*) *Cooling Tower* The hot water coming out of the condenser is sprayed from a certain height. A small quantity of water evaporates in air and the remaining water is cooled in the cooling tower. This cooled water is again recirculated into the condenser.

23.4 TYPES OF CONDENSERS

There are mainly two types of condensers:

(i) Jet condenser
(ii) Surface condenser

23.5 JET CONDENSER

In jet condensers, the exhaust steam and cooling water come in direct contact and mix up together. Thus, the final temperature of condensate and cooling water leaving the condenser is same. The cooling water is sprayed on the exhaust steam to cause rapid condensation.

A jet condenser is very simple in design and cheaper than a surface condenser. It can be used when cooling water is cheaply and easily available. However, the condensate cannot be reused in the boiler, because it contains impurities like dust, oil, metal particles, etc.

The jet condensers are also classified as

(a) Low-level jet condenser
  (i) Counter-flow type
  (ii) Parallel-flow type
(b) High-level jet condenser
(c) Ejector jet condenser

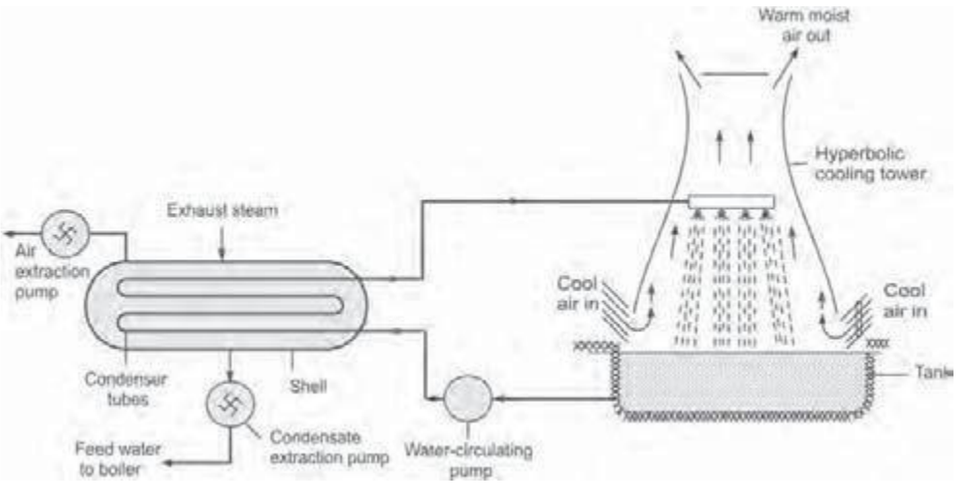

Fig. 23.1 *Elements of a condensing plant*

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23.5.1 Low-level Counter-flow Jet Condenser

The schematic diagram of a low-level counter-flow jet condenser is shown in Fig. 23.2. Exhaust steam is supplied from the bottom side of the condenser and it flows upwards while the cooling water is supplied from the top of the condenser.

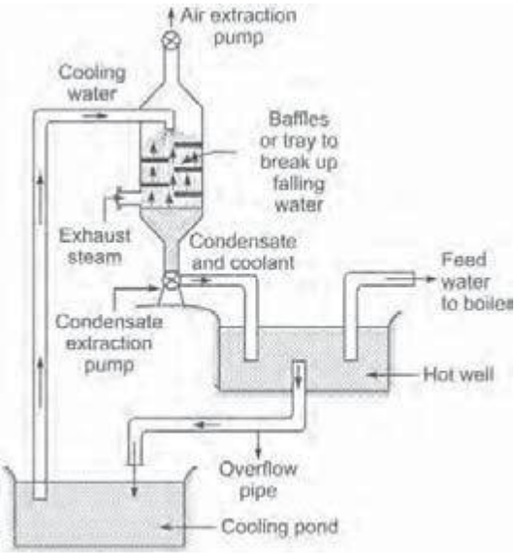

Fig. 23.2 *Low-level counter-flow jet condenser*

The water flows downward through a series of baffles or trays. As steam comes in contact with falling water, it gets condensed. The air-extraction pump, located at the top of the condenser sucks the air and any uncondensed vapour. The air pump maintains enough vacuum in the condenser shell, causing the cooling water to be lifted up to a height of approximately 5.5 m. A pump for water supply is only needed if it is to be lifted more than 5.5 m in height. The condensate extraction pump at the bottom of the shell extracts the liquid condensate and cooling water and discharges it to a hot well, from where it may be fed to the boiler, if cooling water suits it. The excess amount of condensate from the hot well flows into the cooling pond by an overflow pipe.

23.5.2 Low-level Parallel-flow Jet Condenser

In this type of condenser, the exhaust steam and cooling water both flow in the same direction. The steam usually enters at the top of the condenser and the cooling water just below it from the side as shown in Fig. 23.3. Other arrangements are similar to a counter-flow jet condenser. The mixture of condensate, coolant and air is extracted with the help of a wet air pump. This limits the vacuum created in the condenser up to 600 mm of Hg (approx 6 kPa).

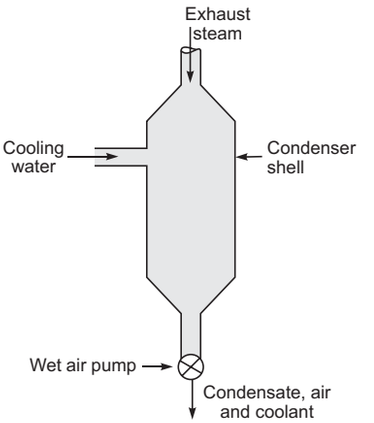

Fig. 23.3 *Low-level parallel-flow jet condenser*

23.5.3 High-level Jet Condenser

The schematic of a high-level jet condenser is shown in Fig. 23.4. This condenser is also called a *barometric condenser*. The condenser shell is installed at a height greater than that of atmospheric pressure in water column, i.e., 10.33 m. A long tail pipe, more than 10.33 m in height, is attached between the bottom of the condenser and the hot well. The pressure at the bottom of the pipe is equal to the atmospheric pressure, while at its top in the condenser shell, the vacuum is maintained. This allows the condensate and coolant to fall from the condenser under gravity without any extraction pump.

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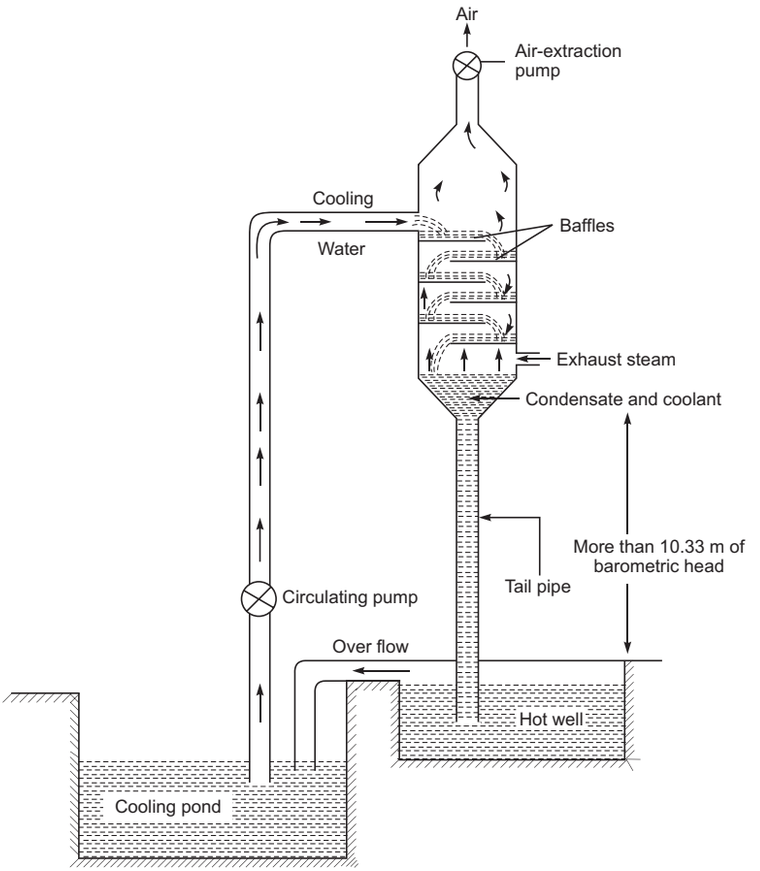

Fig. 23.4 *High-level jet condenser*

The cooling water is supplied by a water-circulation pump. The water enters from the top of the condenser and the exhaust steam enters at the bottom of the condenser shell. The water stream is broken into fine spray of droplets by baffles. The air released from steam and water flows towards the top, where it is extracted by an air-extraction pump.

23.5.4 Advantages and Disadvantages of a High-Level Jet Condenser

Advantages

(i) It requires less floor space.

(ii) It requires only a circulating pump of low capacity. The condensate falls due to the action of gravity.

Disadvantages

(i) Due to the high level of the condenser, long pipelines are installed. Hence, initial cost is high.

(ii) It requires more head room with costly support structure.

(iii) It is not readily accessible for maintenance.

23.5.5 Ejector Condenser

In this condenser, the momentum of flowing water is used to remove the mixture of condensate and coolant from the condenser without the use of any extraction pump. The schematic of an ejector condenser is shown in Fig. 23.5. The exhaust steam enters the condenser shell at a side through

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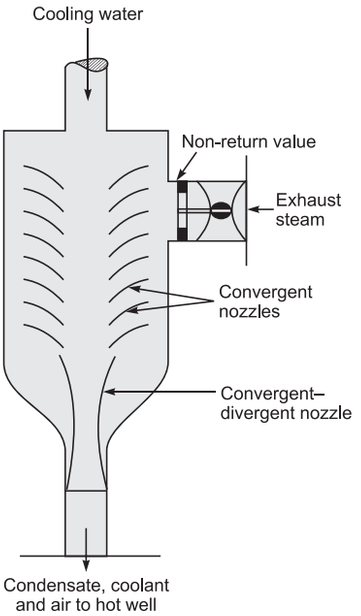

Fig. 23.5 *Ejector condenser*

a non-return valve, the cooling water enters from the top of the condenser under a water head of 5 to 6 m (approx. 1.5 bar) and passes over a series of converging nozzles and attains a high velocity. At the same time, vacuum is created in the side gap of the nozzles, drawing in the exhaust steam through the truncated divergent cones (nozzles).

The mixing of steam and water causes the condensation and hence the vacuum further increases in the side of the nozzles. In the central passage, the water and condensate get momentum, which forces the mixture of condensate and water and air out of the shell.

23.6 SURFACE CONDENSER

In the *surface condenser*, the exhaust steam and cooling water do not come in physical contact, rather they are separated by a heat-transfer wall. The exhaust steam passes over the outer surface of the tubes and the cooling water flows through the tubes. Since the cooling water does not mix with the condensing steam, the condensate remains pure and it can be used as a feed water in the boiler.The surface condensers are classified in two groups:

(i) Shell-and-tube type, and
(ii) Evaporative type.

23.6.1 Shell-and-tube type Surface Condenser

In this condenser, a large number of tubes are packed in a shell, with their axes parallel to the shell. The cooling water flows inside the tubes, while steam enters the shell side and condenses over the tubes. The baffles are commonly placed in the shell to force the steam to flow along path across the shell and to increase the heat-transfer rate and to maintain the uniform spacing between the tubes.

Shell-and-tube type surface condenser can be further classified according to the number of shell-and-tube passes involved. A surface condenser in which water makes one U turn in the water box, is a *one-shell pass and two-tube pass* surface condenser as shown in Fig. 23.6. Similarly, the surface condenser that involves two passes in the shell and four passes of the tube is called a *two-shell pass and four-tube pass condenser.* The surface condensers require two pumps, i.e., wet air pump to remove air and condensate and a water pump to circulate the cooling water under the pressure through the tubes. These are mainly used in large power plants and chemical industries.

The surface condensers are classified according to direction of steam flow:

(i) down flow, (ii) central flow, and (iii) inverted flow.

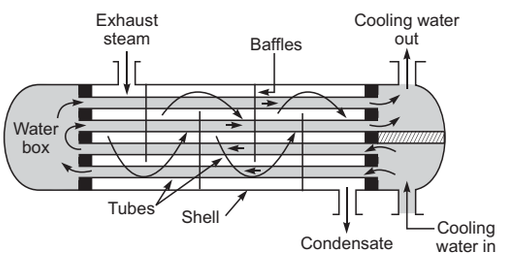

Fig. 23.6 *Shell and tube type condenser*

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(*i*) *Down-flow Surface Condenser* The sectional view of a down-flow surface condenser is shown in Fig. 23.7. The exhaust steam enters the top of the condenser shell and flows downward over the water tube. The water tubes are double passed. The cold water flows in the lower side first and then in the upper side in reverse direction. It enables the maximum heat transfer rate for a condenser. The extraction pump connected at the bottom of the condenser draws the condensate out of the condenser.

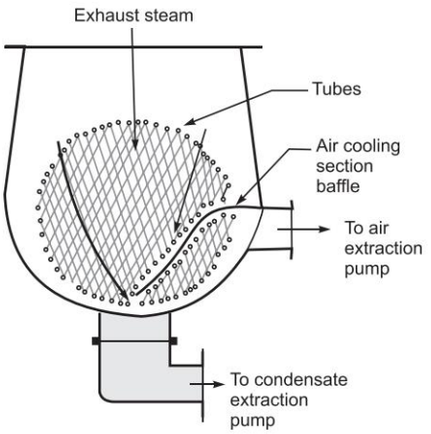

Fig. 23.7 *Down-flow surface condenser*

The air pump extracts the air and non-condensable gases from the condenser. The separated air under the baffle is cooled to maximum extent. Due to cooling of air, its specific volume is reduced. Therefore, the power consumption for operation of pump is reduced.

(*ii*) *Central-flow Surface Condesner* The sectional view of a central-flow surface condenser is shown in Fig. 23.8. The suction pipe of an air extraction pump is located at the centre of the condenser tubes. The steam flows radially inward. The condensate is collected at the bottom of the shell from where it is taken out by the condensate extraction pump. The steam gets access to the entire periphery of tubes, and thus a large surface area for heat transfer is available as compared to a down flow condenser.

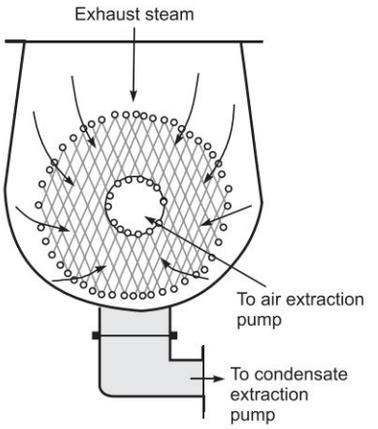

Fig. 23.8 *Central-flow surface condenser*

(*iii*) *Inverted flow Surface Condenser* The steam enters the bottom of the shell and air extraction pump connected at the top. Steam flows upward first and subsequently, returns to the bottom of the condenser. The condensate extraction pump is connected at the bottom of the shell to extract the condensate.

23.6.2 Evaporative Condenser

In this type of condenser, the evaporation of some cooling water provides cooling effect, thereby steam condenses. An evaporative condenser is shown in Fig. 23.9.

The steam to be condensed is passed through the finned (grilled) tubes. The cooling water is

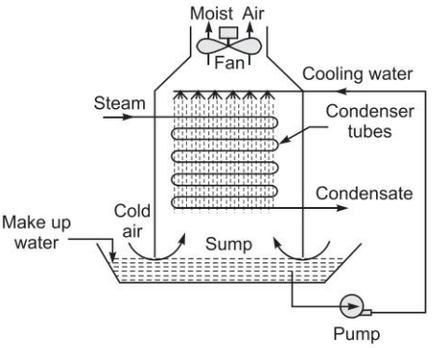

Fig. 23.9 *Evaporative steam condenser*

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sprayed over the outer surface of the tubes. Cool dry air is also blown over the tube surface, which causes the evaporation of a thin film of water over the tubes. This evaporation cools the water and extracts latent heat from the steam in the pipe, thus steam condenses. The water collected in the sump is mixed with make-up water and again pumped for spray. The condensate is extracted with the help of a wet pump. The air passing over the tubes carries moisture and it is drawn by an induced draft fan located at the top.

The evaporative condensers are most suitable for small plants, where supply of cold water is limited. These are most popular with refrigerant condensing units and for chemical equipments.

23.6.3 Advantages and Disadvantages of Surface Condensers

Advantages

1. The surface condenser lowers the back pressure (7 to 8 kPa) of steam at the turbine exit, and thus allows the expansion of steam through a higher pressure ratio.
2. It has high vacuum efficiency, and is thus suitable for large power plants.
3. It gives a pure condensate which can be recirculated as feed water to the boiler.
4. Since the condensate is reused, it saves the cost of fresh water to be circulated and the cost of its chemical treatment.
5. It requires low power input for air-extraction pump.
6. Since the cooling water is in indirect contact of steam, low quality cooling water can be used in the condenser.
7. The cooling of condensate can be controlled by regulating the flow of cooling water.

Disadvantages

1. Indirect cooling takes place in the condenser, and thus large cooling water is required.
2. Construction is complicated, requires higher installation cost.
3. It requires costly maintenance and skilled workers.
4. It requires large floor area.

23.6.4 Comparison between Jet and Surface Condensers

	Jet condenser (mixing type)	*Surface condenser (non-mixing type)*
1.	Exhaust steam and cooling water mix together, thus the steam condenses due to physical contact.	Exhaust steam condenses on outer surface of tubes through which water flows. Hence, the two fluids do not mix.
2.	Condensate contains impurities, and cannot be reused.	Condensate is pure and can be used repeatedly as feed water to the boiler.
3.	Heat is exchanged by direct contact of cooling water, and thus for condensation of steam, less quantity of water is required.	Indirect heat exchange, thus a large quantity of cooling water is required.
4.	Construction of a jet condenser is simple, thus the condenser is less costly.	Construction is slightly complicated, thus initial higher installation cost.
5.	Maintenance is simple and cheap.	Costly maintenance, requires skilled worker.
6.	It requires small floor space.	It requires large floor space.
7.	More power is required for air pump.	Less power is needed for air pump.
8.	Less power is needed for water pumping.	More power is needed for water pumping
9.	It has low vacuum efficiency, thus is less suitable for large plants.	It has high vacuum efficiency, thus is more suitable for large plants.

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23.7 ESTIMATION OF COOLING WATER REQUIRED

The steam condenser is used to condense several thousand kilograms of steam per hour in a power plant. The steam releases its latent heat of condensation and sometimes a portion of sensible heat also. Therefore, a huge amount of cooling water is required for steam condensation. Approximately, 30 to 50 kg of cooling water is required per kg of steam to be condensed.

Figure 23.10 shows a schematic of a steam condenser with steam and cooling water flowing through it.

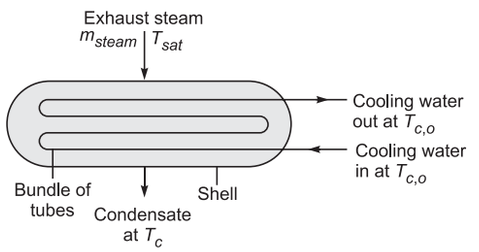

Fig. 23.10 *Steam condenser with steam and cooling water flow*

Usually, the condensate loses some of its sensible heat, and thus the temperature T_c of the condensate leaving the condenser is less than the saturation temperature of the condensate. The temperature difference $(T_{sat} - T_c)$ is known as *undercooling* or subcooling of the condensate.

The rate of heat loss by exhaust steam

$\dot{Q}_{steam}$ = sum of latent heat and subcooling heat of the condensate

$= \dot{m}_{steam}\ [x h_{fg} + C_{pw}(T_{sat} - T_c)]$

$= \dot{m}_{steam}\ [C_{pw} T_{sat} + x h_{fg} - C_{pw} T_c]$

Using $C_{pw} T_{sat} = h_f$ (sensible heat of exhaust steam), and

$C_{pw} T_c = h_{fc}$

= sensible heat of condensate leaving the condenser in a hot well

Then $\dot{Q}_{steam} = \dot{m}_{steam}\ [h_f + x h_{fg} - h_{fc}]$ ...(23.1)

The rate of heat gain by cooling water

$\dot{Q}_w$ = mass flow rate × specific heat × temp. rise of water

$= \dot{m}_w \times C_{pw}(T_{c,o} - T_{c,i})$ ...(23.2)

The energy balance on a condenser leads to

$\dot{Q}_w = \dot{Q}_{steam}$

or $\dot{m}_w \times C_{pw}(T_{c,o} - T_{c,i})$

$= \dot{m}_{steam}\ [h_f + x h_{fg} - h_{fc}]$

or $$\dot{m}_w = \frac{\dot{m}_{steam}\,(h_f + x h_{fg} - h_{fc})}{C_{pw}\,(T_{sat} - T_c)} \quad ...(23.3)$$

where C_{pw} is the specific heat of water = 4.187 kJ/ kg·K and $\dot{m}_w$ and $\dot{m}_{steam}$ are the mass-flow rate of cooling water and exhaust steam respectively into the condenser.

23.8 CONDENSER EFFICIENCY

The condenser is a heat exchanger and its efficiency can be defined as the ratio of actual temperature rise to the maximum possible temperature rise of cooling water.

$$\eta_{condenser} = \frac{\text{Actual temperature rise of cooling water}}{\text{Maximum possible temperature rise of cooling water}}$$

or $$\eta_{condenser} = \frac{(\Delta T)_c}{(\Delta T)_{max}} \quad ...(23.4)$$

For a condenser, the actual temperature rise of water is

$(\Delta T)_c$ = Exit temperature of water – Inlet temperature of water

$= T_{c,o} - T_{c,i}$ ...(23.5)

and the maximum possible temperature rise of cooling water is

ΔT_{max} = Sat. temp. corresponding to absolute pressure in condenser – Water inlet temperature

$= T_{sat} - T_{c,i}$

Then $$\eta_{condenser} = \frac{T_{c,o} - T_{c,i}}{T_{sat} - T_{c,i}} \quad ...(23.6)$$

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Example 23.1 *A surface condenser is designed to handle 12000 kg of steam per hour. The steam enters at 8 kPa, 0.9 dry. The condensate leaves the condenser at the corresponding saturation temperature. Calculate the rate of cooling water, if cooling water temperature rise is limited to 12°C.*

Solution

Given Data of a surface condenser

$\dot{m}_{steam} = 12000$ kg/h $\quad p_{sat} = 8$ kPa

$x = 0.9 \quad \Delta T_c = 12°C$

To find The mass-flow rate of cooling water in the condenser.

Assumptions

(i) No air present in the condenser.

(ii) No heat loss from the condenser body.

(iii) The specific heat of cooling water as 4.187 kJ/kg·K.

Analysis The properties of steam at 8 kPa (from steam tables A-13):

$T_{sat} = 41.5°C,$

$h_{fg} = 2403.1$ kJ/kg

The energy balance for the condenser

Heat given by exhaust steam

= Heat gain by cooling water ...(i)

Heat given by exhaust steam

$= \dot{m}_{steam} \times$ Latent heat of condensation

$= \dot{m}_{steam} [x h_{fg}]$ ...(ii)

Heat gain by cooling water

$= \dot{m}_w C_{pw} \Delta T_c$ ...(iii)

Using Eqs. (ii) and (iii) in Eq. (i), with numerical values,

$$12000 \times 0.9 \times 2403.1 = \dot{m}_w \times 4.187 \times 12$$

Mass of cooling water;

$$\dot{m}_w = \frac{12000 \times 0.9 \times 2403.1}{4.187 \times 12}$$

$$= \mathbf{5,16,548.84\ kg/h}$$

Example 23.2 *The following data refers to a test of the surface condenser of a steam turbine.*

Absolute pressure of the steam entering the condenser = *5.628 kPa*

Temperature of condensate leaving the condenser = *32°C*

Inlet temperature of cooling water = *15°C*

Outlet temperature of cooling water = *30°C*

Mass of cooling water per kg of steam = *32 kg*

Assuming that all the heat lost by the exhaust steam is taken up by the circulating water, determine the dryness fraction of the steam as it enters the condenser.

Solution

Given Data from a trial on a surface condenser

$p_{sat} = 5.628$ kPa $\quad T_c = 32°C$

$T_{c,i} = 15°C \quad T_{c,o} = 30°C$

$m_w = 32$ kg/kg of steam

$m_{steam} = 1$ kg

To find Dryness fraction of the steam entering the condenser.

Assumptions

(i) No air present in the condenser.

(ii) No heat loss from the condenser body.

(iii) The specific heat of cooling water as 4.187 kJ/kg·K.

Analysis The properties of steam from steam tables:

At 5.628 kPa $\quad h_f = 146.66$ kJ/kg,

$h_{fg} = 2418.62$ kJ/kg

At 32°C $\quad h_{fc} = 134.2$ kJ/kg

The energy balance on the condenser

Heat given by exhaust steam

= Heat gain by cooling water ...(i)

Heat given by exhaust steam

$= m_{steam} [h_f + x h_{fg} - h_{fc}]$ ...(ii)

Heat gain by cooling water $= m_w C_{pw} (T_{c,o} - T_{c,i})$

Using Eq. (ii) and (iii) in Eq. (i), with numerical values,

$1 \times [146.66 + 2418.62\, x - 134.2]$

$= 32 \times 4.187 \times (30 - 15)$

or $\quad 2418.62\, x = 2009.76 - 12.46 = 1997.3$

Dryness fraction of steam; $x =$ **0.826**

Example 23.3 *The steam is supplied to a steam turbine at 3.0 MPa and 300°C. The expansion of steam is carried out isentropically to a condenser vacuum of 713 mm of Hg. The barometer reads 758 mm of Hg. The condenser temperature is 20°C and rise in temperature of cooling water is 12°C. Determine*

(a) quality of steam entering the condenser, and

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(b) quantity of cooling water circulated per kg of steam.

Assume no air is present in the condenser.

Solution

Given Data from steam turbine and condenser

$p_1 = 30.0$ MPa $\quad T_1 = 300°C$

$p_g = 713$ mm of Hg $\quad p_{atm} = 758$ mm of Hg

$T_c = 20°C$ $\quad \Delta T_w = 12°C$

$m_{steam} = 1$ kg $\quad$ No air in condenser

To find

(i) Dryness fraction of the steam entering the condenser.

(ii) Quantity of cooling water circulated per kg of steam.

Assumptions

(i) The specific heat of cooling water as 4.187 kJ/kg·K.

(ii) No heat loss from the condenser body.

Analysis The absolute pressure in the condenser,

$$p_{abs} = p_{atm} - p_g = 758 - 713 = 45 \text{ mm of Hg}$$

$$= 45 \text{ mm of Hg} \times \frac{101.325}{760 \text{ mm of Hg}} \approx 6.0 \text{ kPa}$$

The quality of steam entering the condenser can be found by using Mollier diagram for isentropic expansion.

Choose coordinates 30.0 MPa and 300°C on the Mollier diagram and draw a vertical line till it intersects the 6 kPa line. Read the dryness fraction at the intersection point. It is found to be

$$x = \mathbf{0.77}$$

The properties of steam from steam tables:

At 6.0 kPa $\quad h_f = 151.5$ kJ/kg,

$\quad h_{fg} = 2416.0$ kJ/kg

At 20°C $\quad h_{fc} = 83.9$ kJ/kg

Heat given by exhaust steam

$$= m_{steam}\,[h_f + x\,h_{fg} - h_{fc}] \qquad ...(i)$$

Heat gain by cooling water $= m_w C_{pw} \Delta T_w$ $\qquad ...(ii)$

Equating Eq. (i) and (ii) and using numerical values

$$1 \times [151.5 + 0.77 \times 2416 - 83.9] = m_w \times 4.187 \times 12$$

or $\quad 1927.92 = 50.244\, m_w$

Mass of cooling water; $\quad m_w = \dfrac{1927.92}{50.244}$

$$= \mathbf{38.37 \text{ kg/kg of steam}}$$

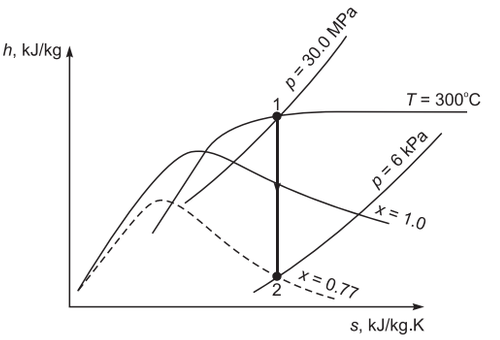

Fig. 23.11 *Isentropic expansion in turbine*

Example 23.4 *The following data were recorded from a test of a surface condenser:*

Inlet temperature of circulating water = 21°C

Exit temperature of circulating water = 35°C

Vacuum in the condenser = 704.7 mm of Hg

Barometer reading = 760 mm of Hg

Calculate the efficiency of the condenser.

Solution

Given Data from a test of a surface condenser

$T_{c,\,i} = 21°C$ $\quad T_{c,\,o} = 35°C$

$p_g = 704.7$ mm of Hg $\quad p_{atm} = 760$ mm of Hg

To find Condenser efficiency.

Assumptions

(i) Density of mercury, $\rho = 13600$ kg/m^3.

(ii) Acceleration due to gravity, $g = 9.81$ m/s^2.

Analysis The absolute pressure in the condenser

$$p_{abs} = p_{atm} - p_g = 760 - 704.7 = 55.3 \text{ mm of Hg}$$

This absolute pressure can be expressed in kPa as

$$p_{abs} = 55.3 \text{ mm of Hg} \times \frac{101.325 \text{ kPa}}{760 \text{ mm of Hg}}$$

$$= 7.38 \text{ kPa}$$

From steam tables; the saturation temperature at 7.38 kPa

$$T_{sat} = 40°C$$

Temperature rise of cooling water,

$$(\Delta T)_c = T_{c,\,o} - T_{c,\,i} = 35 - 21 = 14°C$$

Maximum possible temperature rise of water,

$$(\Delta T)_{max} = T_{sat} - T_{c,\,i} = 40 - 21 = 19°C$$

Thus $\quad \eta_{condenser} = \dfrac{(\Delta T)_c}{(\Delta T)_{max}} = \dfrac{14}{19} = 0.736$ or $\mathbf{73.6\%}$

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23.9 ANALYSIS OF CONDENSER OPERATION

23.9.1 Vacuum Creation

When the steam condenses in a closed vessel, the vapour phase of working substance (water) changes to liquid phase, and thus its *specific volume reduces to more than one thousand times*. Due to change in specific volume, the absolute pressure in the condenser falls below the atmospheric pressure and a high vacuum is created.

This lower pressure in the condenser permits maximum expansion of steam in the turbine, and more work is developed. The minimum pressure that can be attained depends on the temperature of condensate and air present in the condenser.

The term *vacuum* means *the pressure below the atmospheric pressure.* The vacuum in the condenser is generally measured in mm of mercury (Hg). The vacuum is the negative gauge pressure and is equal to the differences of barometric pressure and absolute pressure in the condenser as shown in Fig. 23.12.

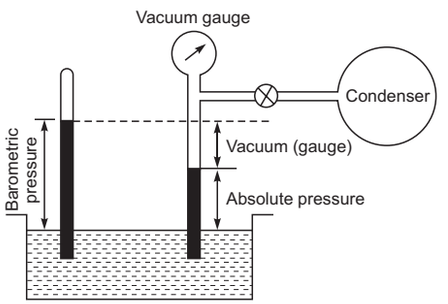

Fig. 23.12 *The vacuum is the difference between barometric and absolute pressure*

The absolute pressure in the condenser

= Atmospheric pressure – Vacuum gauge pressure ...(23.7)

= Barometer reading – Vacuum gauge reading

The vacuum is usually referred to a standard atmospheric pressure as 101.325 kPa or 760 mm of Hg, i.e.,

$$1 \text{ kPa} = \frac{760}{101.325} \text{ mm of Hg}$$

At any barometric reading other than standard reading of 760 mm of Hg, the vacuum gauge reading is corrected to standard barometric reading as

Corrected vacuum in mm of Hg

= Standard barometer reading

– (Actual barometer reading

– Vacuum gauge reading) ...(23.8)

The abosolute pressure in the condenser can be determined as

p_{abs} = 760 mm – corrected vacuum in mm of Hg ...(23.9)

23.9.2 Dalton's law of Partial Pressures

For a mixture of non-reactive gases, the total (mixture) is given by

$$p = \frac{m_A R_A T}{V} + \frac{m_B R_B T}{V}$$
$$= p_A + p_B + \ldots + \Sigma p_i \quad ...(23.10)$$

where p_i is the pressure of i_{th} gas component at the mixture temperature T and volume V.

According to Dalton's law, the total pressure of a non-reactive mixture is equal to the sum of partial pressure of its components, if each component would occupy the same volume alone at the mixture temperature.

The condenser has a mixture of wet steam and air. The total pressure p in the condenser is the sum of partial pressure of steam, p_{sat} and air, p_a. The saturation pressure of steam p_{sat} can be obtained from the steam table at condensate (saturation) temperature. According to Dalton's law,

$$p = p_{sat} + p_a$$

or $$p_a = p - p_{sat} \quad ...(23.11)$$

With the help of partial pressure of air, the mass of air can be determined as

$$m_a = \frac{p_a V}{R_a T_{sat}} \quad ...(23.12)$$

where V is the volume of the condenser, T_{sat} is the condensate temperature and R_a is the characteristic gas constant of air, which is equal to 0.287 kJ/kg·K.

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23.9.3 Vacuum Efficiency

In the steam condenser, a mixture of steam and air is present. Therefore, the absolute pressure in the condenser is the sum of partial pressures of steam and air. The presence of air in the condenser disturbs the vacuum, and thus the actual vacuum is less than that could be attained in the condenser, if only steam is present. If no air is present in the condenser then absolute pressure in the condenser is the saturation pressure of steam and thus the maximum vacuum would be attained.

The vacuum efficiency is defined as ratio of the actual vacuum to the maximum possible vacuum.

$$\eta_{vacuum} = \frac{\text{Actual vacuum in the condenser}}{\text{Maximum possible vacuum (gauge)}}$$

$$= \frac{p_g}{p_{g,max}} \qquad ...(23.13)$$

where p_g is the actual vacuum

= Barometric pressure – Absolute pressure in condenser

$= p_{atm} - p$

Maximum possible vacuum

$p_{g,\,max}$ = Barometric pressure – Sat. pressure of steam corresponding to condensate temperature

$= p_{atm} - p_{sat}$

23.9.4 Sources of Air in the Condenser

All types of condensers require air extraction pump to remove the air present in the condenser. The main sources of air present in the condenser are the following:

(i) The ambient air leaks to the condenser chamber at the joints and glands which are internally under pressure lower than that of ambient. It can be reduced by taking utmost care while designing and making vacuum joints.

(ii) Another source of air is the dissolved air with feed water. The dissolved air in feed water enters into the boiler and it travels with steam into the condenser. Its quantity depends upon the quality of feed water.

(iii) In case of a jet condenser, some air comes in with the injected water in which it is dissolved.

23.9.5 Effects of Air Leakage into the Condenser

The presence of air in the condenser is a far serious concern. It affects the performance of the condenser to a great extent, thereby, the performance of steam power plant. The presence of air into the condenser puts the following effects:

(i) The presence of air lowers the vacuum in the condenser. Thus the back pressure of the plant increases, and consequently, *the work output of the turbine reduces.*

(ii) The presence of air also lowers the partial pressure of steam and hence lower saturation temperature. The steam with lower saturation temperature has higher latent heat. A *large quantity of cooling water is required* to get the desired result in the condenser.

(iii) Air forms the film adjacent to the tube surface in the condenser. Air has very poor thermal conductivity. Hence, the *rate of heat transfer from vapour to cooling medium is reduced.*

(iv) The presence of air in the condenser corrodes to the metal surfaces. Therefore, the *life of condenser is reduced*.

Example 23.5 *A vacuum of 712 mm was obtained with the barometer reading of 753 mm of Hg. Correct the vacuum to a standard barometer reading of 760 mm of Hg.*

Solution

Given Vacuum reading = 712 mm of Hg
Actual barometer reading = 753 mm of Hg
Standard barometer reading = 760 mm of Hg

To find Corrected vacuum in the condenser.

Analysis The corrected vacuum in the condenser is given by

= Standard barometer reading
– (Actual barometer reading – Actual vacuum reading)
= 760 mm – (753 mm – 712 mm) = **719 mm of Hg**

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Example 23.6 *Steam enters a condenser at 35°C. The barometer reading is 760 mm of mercury. If the vacuum of 690 mm is recorded, calculate the vacuum efficiency.*

Solution

Given Vacuum in a steam condenser

T_{sat} = 35°C

Barometer reading = 760 mm of Hg

Vacuum reading, p_g = 690 mm of Hg

To find Vacuum efficiency

Analysis The saturation pressure of steam at 35°C; (from steam table A)

$$p_{sat} = 5.628 \text{ kPa}$$

$$= (5.628 \text{ kPa}) \times \frac{760 \text{ mm of Hg}}{101.325 \text{ kPa}}$$

$$= 42.21 \text{ mm of Hg}$$

Max. possible vacuum

= Barometer reading – Sat. pressure of condensing steam

$$p_{g,\,max} = 760 - 42.21 = 717.79 \text{ mm of Hg}$$

The vacuum efficiency

$$\eta_{vacuum} = \frac{\text{Actual vacuum in the condenser}}{\text{Maximum possible vacuum (gauge)}}$$

$$= \frac{690 \text{ mm of Hg}}{717.79 \text{ mm of Hg}} = 0.961 \text{ or } \mathbf{96.1\%}$$

Example 23.7 *A barometric jet condenser deals with 3000 kg of steam per hour. The steam is 0.92 dry. The absolute pressure in the condenser is 100 cm of mercury. Cooling water enters at 15°C and mixed condensate and cooling water leaves the condenser at 40°C. Calculate the quantity of water required per minute and the minimum height of tail pipe required above the level of condensate in the hot well.*

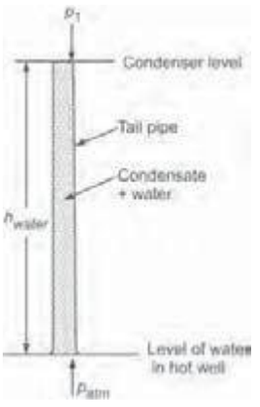

Fig. 23.13 *Schematic tail pipe*

Solution

Given Data from a barometric jet condenser

$\dot{m}_{steam}$ = 3000 kg/h

p = 100 mm of Hg

x = 0.92

$T_{c,\,i}$ = 15°C $\qquad$ $T_{c,\,o}$ = 40°C

To find

(i) Quantity of cooling water circulated per minute.

(ii) Minimum height of tail pipe above condensate level in hot well.

Assumptions

(i) Standard atmospheric pressure of 760 mm of Hg.

(ii) No heat loss from the condenser body.

(iii) The specific heat of cooling water as 4.187 kJ/kg·K.

(iv) The density of water as 1000 kg/m^3.

Analysis

(i) *Quantity of cooling water circulated per min*

The absolute pressure in the condenser in kPa;

$$p_1 = 100 \text{ mm of Hg} \times \frac{101.325}{760 \text{ mm of Hg}}$$

$$= 13.33 \text{ kPa}$$

The properties of steam from steam tables:

At 13.33 kPa h_f = 215.5 kJ/kg,

h_{fg} = 2373.65 kJ/kg

At 40°C h_{fc} = 165.54 kJ/kg

Heat given by exhaust steam,

$$Q_w = m_{steam}\,[h_f + x h_{fg} - h_{fc}] \qquad ...(i)$$

Heat gain by cooling water;

$$Q_w = m_w C_{pw} (T_{c,\,o} - T_{c,\,i}) \qquad ...(ii)$$

Equating Eq. (i) and (ii) and using numerical values

$$\left(\frac{3000}{60}\right) \times [215.5 + 0.92 \times 2373.65 - 167.54]$$

$$= m_w \times 4.187 \times (40 - 15)$$

or $111585.9 = 104.675\, m_w$

Mass of cooling water

$$m_w = \frac{111585.9}{104.675} = \mathbf{1066 \text{ kg/min}}$$

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(ii) *Height of tail pipe above condensate level in hot well*

The atmospheric pressure acts at the condensate level in the hot well. It can be expressed as

p_{atm} = abs. pressure in condenser + gauge pressure offered by water column in tail pipe

or $p_{atm} = p_1 + p_g = p_1 + \rho_{water}\, g h_{water}$

or $101.325 \times 10^3 = 13.33 \times 10^3 + 1000 \times 9.81 \times h_{water}$

Height of tail pipe;

$$h_{water} = \frac{101.325 \times 10^3 - 13.33 \times 10^3}{1000 \times 9.81}$$

$$= \mathbf{8.97\ m}$$

Example 23.8 *3000 kg of wet steam with a dryness fraction of 0.95 is condensed per hour in a barometric condenser. The minimum height of the tail race above the hot well is 8.5 m. The barometric pressure is 760 mm of Hg. The cooling water enters the condenser at 25°C and the mixture of condensate and cooling water exit temperature is 50°C. Calculate*

(a) Vacuum in the condenser in mm of Hg,

(b) Absolute pressure in the condenser in kPa,

(c) Mass of cooling water required without undercooling.

Solution

Given A barometric jet condenser

$\dot{m}_{steam}$ = 3000 kg/h

x = 0.95

H_{water} = 8.5 m

Barometer reading = 760 mm of Hg

$T_{c,\,i}$ = 25°C

$T_{c,\,o}$ = 50°C

To find

(i) Vacuum in the condenser in mm of Hg.

(ii) Absolute pressure in the condenser in kPa.

(iii) Mass of cooling water required without undercooling.

Assumptions

(i) No heat loss from the condenser body.

(ii) The specific heat of cooling water as 4.187 kJ/kg·K.

(iii) The density of water as 1000 kg/m^3.

(iv) The density of mercury as 13590 kg/m^3.

Analysis

(i) *The vacuum in the condenser*

The mass of water in the tail pipe acts downward, thus creating vacuum in the condenser. The gauge (vacuum) pressure in fluid column is expressed as

$$p_g = \rho g h$$

Therefore, equating the gauge (vacuum) pressure in water and mercury columns;

$$\rho_{water}\, g h_{water} = \rho_{Hg}\, g h_{Hg}$$

Using the numerical values;

$1000 \times 9.81 \times (8.5\ \text{m}) = 13590 \times 9.81 h_{Hg}$

or $$h_{Hg} = \frac{1000 \times 9.81 \times 8.5}{13590 \times 9.81}$$

= 0.625 m or **625 mm of Hg**

(ii) *Absolute pressure in the condenser*

The absolute pressure in the condenser

$p_{abs} = p_{atm} - p_{vacuum}$

= 760 mm – 625 mm

= 135 mm of Hg

This pressure can be obtained in kPa as

$$p_{abs} = 135\ \text{mm of Hg} \times \frac{101.325\ \text{kPa}}{760\ \text{mm of Hg}}$$

$\approx$ **18 kPa**

(iii) *Mass of cooling water required without undercooling*

The properties of steam from steam tables:

At 50°C h_f = 209.31 kJ/kg,

h_{fg} = 2382.75 kJ/kg

The energy balance on the condenser

Heat given by exhaust steam without undercooling,

$$\dot{Q}_{steam} = \dot{m}_{steam}\, [h_f + x\, h_{fg}]$$

Heat gain by cooling water,

$$\dot{Q}_w = \dot{m}_w\, C_{pw}\, (T_{c,\,o} - T_{c,\,i})$$

Making energy balance using numerical values

$3000 \times [209.31 + 0.95 \times 2382.75]$

$= \dot{m}_w \times 4.187 \times (50 - 25)$

or $7418767.5 = 104.67\, \dot{m}_w$

Mass of water needed;

$$\dot{m}_w = \frac{7418767.5}{104.67} = \mathbf{70877.68\ kg/h}$$

Example 23.9 *The temperature in a surface condenser is 37.31°C and the vacuum is 698 mm of Hg and the barometer reads 755.2 mm of Hg. Correct the vacuum reading to a standard barometer of 760 mm and hence, determine (i) the partial pressure of steam and air, and*

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(*ii*) *the mass of air associated with one kg of steam. Take R = 0.287 kJ/kg·K for air.*

Solution

Given A surface condenser

$T_{sat} = 37.31°C$

Vacuum, $p_g = 698$ mm of Hg

$R_a = 0.287$ kJ/kg·K

Barometer pressure, $p_{atm} = 755.2$ mm of Hg

Standard barometer reading = 760 mm of Hg

To find

(i) Correct vacuum reading to standard atmospheric pressure,

(ii) Partial pressure of steam and air, and

(iii) Mass of air associated with 1 kg of steam.

Analysis

(i) *Corrected vacuum reading*

= Standard barometer reading – (Actual barometer reading – Vacuum reading)

= 760 – (755.2 – 698)

= **702.8 mm of Hg**

(ii) *Partial pressure of steam and air*

The saturation pressure of steam at 37.31°C (from steam tables)

$p_{sat} = 6.439$ kPa

$v_g = 22.585$ m^3/kg of steam

The partial pressure of steam is p_{sat} = **6.439 kPa**

The absolute pressure of steam in the condenser

$$p = p_{atm} - p_g = 755.2 - 698$$

$$= 57.2 \text{ mm of Hg}$$

$$= 57.2 \text{ mm of Hg} \times \frac{101.325 \text{ kPa}}{717.79 \text{ mm of Hg}}$$

$$= 7.63 \text{ kPa}$$

Using Dalton's law of partial pressure, the pressure of air in the condenser

$$p_a = p - p_{sat} = 7.63 \text{ kPa} - 6.439 \text{ kPa}$$

$$= \mathbf{1.187 \text{ kPa}}$$

(iii) *Mass of air associated with 1 kg of steam*

Specific volume of air = Specific volume of steam

$v_a = 22.585$ m^3/kg of steam

$$m_a = \frac{p_a v_a}{R_a T_{sat}}$$

$$= \frac{(1.187 \text{ kPa}) \times (22.585 \text{ m}^3/\text{kg})}{(0.287 \text{ kJ/kg·K}) \times (37.31 + 273)(\text{K})}$$

$$= \mathbf{0.301 \text{ kg/kg of steam}}$$

Example 23.10 *A condenser of 0.75 m^3 capacity contains saturated steam and air at a temperature of 45°C and a pressure of 0.13 bar abs. Air also leaks further into the condenser, increases the pressure to 0.28 bar abs. and temperature falls to 38°C. Calculate the mass of air which has leaked in. Take R for air as 0.287 kJ/kg·K.*

Solution

Given Steam and air mixture in a condenser

$V = 0.75$ m^3 $\quad T_{sat_1} = 45°C$

$p_1 = 0.13$ bar = 13 kPa

After air leaks into the condenser

$p_2 = 0.28$ bar $\quad T_{sat_2} = 38°C$

$R = 0.287$ kJ/kg·K

To find Mass of air leaks the into the condenser.

Analysis The saturation pressure of steam at 45°C (from steam tables A-12)

$p_{sat_1} = 9.59$ kPa

The initial partial pressure of air in the condenser

$$p_{a_1} = p_1 - p_{sat_1} = 13 \text{ kPa} - 9.59 \text{ kPa}$$

$$= 3.41 \text{ kPa}$$

Mass of air present intially in the condenser

Volume of air = Volume of steam

$V_a = 7.5$ m^3

Mass of air; $m_{a_1} = \dfrac{p_{a_1} V_a}{R_a T_{sat_1}}$

$$= \frac{(3.41 \text{ kPa}) \times (7.5 \text{ m}^3)}{(0.287 \text{ kJ/kg·K}) \times (45 + 273)(\text{K})}$$

$$= 0.28 \text{ kg}$$

After air leaks into the condenser, the saturation pressure of steam at 38°C,

$p_{sat_2} = 6.62$ kPa

The final partial pressure of air in the condenser,

$$p_{a_2} = p_2 - p_{sat_2} = 28 \text{ kPa} - 6.62 \text{ kPa}$$

$$= 21.38 \text{ kPa}$$

Mass of air present finally in the condenser,

$$m_{a_2} = \frac{p_{a_2} V_a}{R_a T_{sat_2}} = \frac{(21.38 \text{ kPa}) \times (7.5 \text{ m}^3)}{(0.287 \text{ kJ/kg·K}) \times (38 + 273)(\text{K})}$$

$$= 1.796 \text{ kg}$$

The air leaking into the condenser,

$$= m_{a_2} - m_{a_1} = 1.796 \text{ kg} - 0.28 \text{ kg} = \mathbf{1.516 \text{ kg}}$$

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Example 23.11 *A vacuum of 710 mm of Hg was recorded in a condenser when the barometer reads 755 mm of Hg. The temperature of the condensate was 25°C. Calculate the pressure of steam and air in the condenser and mass of air per kg of steam. Also, calculate the vacuum efficiency.*

Solution

Given A steam condenser

Vacuum, $p_g = 710$ mm of Hg

$p_{atm} = 755$ mm of Hg

$T_{sat} = 25°C$

To find

(i) Partial pressure of steam and air,

(ii) Mass of air associated with 1 kg of steam,

(iii) Vacuum efficiency.

Assumption R for air as 0.287 kJ/kg·K.

Analysis

(i) *The absolute pressure in the condenser*

$$p = p_{atm} - p_g = 755 - 710 = 45 \text{ mm of Hg}$$

$$= 45 \text{ mm of Hg} \times \frac{101.325 \text{ kPa}}{760 \text{ mm of Hg}}$$

$$\approx 6 \text{ kPa}$$

The saturation pressure of steam at 25°C (from steam tables)

$p_{sat} = 3.17$ kPa

$v_g = 43.36 \text{ m}^3/\text{kg}$

The partial pressure of steam in the condenser:

$p_{sat} = \mathbf{3.17 \text{ kPa}}$

The partial pressure of air in the condenser

$$p_a = p - p_{sat} = 6 \text{ kPa} - 3.17 \text{ kPa}$$

$$= \mathbf{2.83 \text{ kPa}}$$

(ii) *Mass of air present in the condenser*

Sp. Volume of air

= Specific volume of steam

$= 43.36 \text{ m}^3/\text{kg}$

$$\therefore \quad m_a = \frac{p_a v_a}{R_a T_{sat}} = \frac{(2.83 \text{ kPa}) \times (43.36 \text{ m}^3/\text{kg})}{(0.287 \text{ kJ/kg·K}) \times (25 + 273)(\text{K})}$$

$$= \mathbf{0.0143 \text{ kg/kg of steam}}$$

(iii) *Vacuum efficiency*

Actual vacuum,

$p_g = 710$ mm of Hg

Ideal vacuum

$p_{g,\,max}$ = Standard barometer reading – Steam pressure

$$= 760 - 3.17 \text{ kPa} \times \frac{760 \text{ mm of Hg}}{101.325 \text{ kPa}}$$

$$= 736.24 \text{ mm of Hg}$$

$$\therefore \quad \eta_{vacuum} = \frac{p_g}{p_{g,max}} = \frac{710 \text{ mm of Hg}}{736.24 \text{ mm of Hg}}$$

$$= 0.964 \quad \text{or} \quad \mathbf{96.4\%}$$

Example 23.12 *The absolute pressure in the condenser is 11.56 kPa when the barometer reads 1 bar. The condenser temperature is 40°C. Calculate the partial pressure of air, vacuum efficiency and mass of air present in the condenser per kg of steam.*

Solution

Given Absolute pressure in the condenser,

$p = 11.56$ kPa

Barometric pressure, $p_{atm} = 1$ bar $= 100$ kPa

The condenser temperature, $T_{sat} = 40°C$

To find

(i) Partial pressure of air,

(ii) Vacuum efficiency, and

(iii) Mass of air present in the condenser per kg of steam.

Assumption The gas constant for air as 0.287 kg/kg·K.

Analysis

(i) *Partial pressure of air in the condenser*

The saturation pressure of steam at condensate temperature of 40°C

$p_{sat} = 7.384$ kPa

The partial pressure of air,

$$p_a = p - p_{sat} = 11.56 \text{ kPa} - 7.384 \text{ kPa}$$

$$= \mathbf{4.176 \text{ kPa}}$$

(ii) *Vacuum efficiency*

The vacuum pressure in the condenser,

$p_g = p_{atm} - p = 100 \text{ kPa} - 11.56 \text{ bar} = 88.44$ kPa

Maximum possible vacuum in the condenser

$$p_{g,\,max} = p - p_{sat} = 100 - 7.384$$

$$= 92.616 \text{ kPa}$$

$$\eta_{vacuum} = \frac{p_g}{p_{g,max}} = \frac{88.44 \text{ kPa}}{92.616 \text{ kPa}}$$

$$= 0.9549 \quad \text{or} \quad \mathbf{95.49\%}$$

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(iii) *Mass of air present in the condenser per kg of steam*

Volume of air/kg of steam = Specific volume of steam at 40°C

or v_a = 19.5219 m^3/kg of steam (from steam tables)

The mass of air

$$m_a = \frac{p_a\, v_a}{R_a T_{sat}} = \frac{(4.176\ \text{kPa}) \times (19.5219\ \text{m}^3/\text{kg})}{(0.287\ \text{kJ/kg}\cdot\text{K}) \times (40+273)(\text{K})}$$

= **0.9705 kg/kg of steam**

Example 23.13 *During a trial on a condenser, the following readings were recorded:*

Barometer reading = 766 mm of Hg
Actual vacuum recorded by gauge = 716 mm of Hg
Temperature of exhaust steam = 35°C
Temperature of hot well = 29°C
Inlet temperature of cooling water = 15°C
Outlet temperature of cooling water = 24°C

Calculate

(a) Corrected vacuum to standard barometer reading of 760 mm of mercury,
(b) Vacuum efficiency,
(c) Undercooling of condensate, and
(d) Condenser efficiency.

Solution

Given Data from the trial on a steam condenser:

Actual barometer reading = 766 mm of Hg
Standard barometer reading = 760 mm of Hg
Actual vacuum in the condenser, p_g = 716 mm of Hg
Exhaust steam temperature, T_{sat} = 35°C
Cooling water $T_{c,i}$ = 15°C
$T_{c,o}$ = 24°C
Hot well temperature, T_c = 29°C

To find

(i) Corrected vacuum,
(ii) Vacuum efficiency,
(iii) Undercooling of condensate, and
(iv) Condenser efficiency.

Assumption No air present in the condenser.

Analysis

(i) The corrected vacuum
= Standard barometer reading – (Actual barometer reading – Actual vacuum reading)
= 760 mm – (766 mm – 716 mm)
= **710 mm of Hg**

(ii) The saturation pressure of steam corresponds to temperature, T_{sat} = 35°C

p_{sat} = 5.628 kPa

$$= 5.628 \times \left(\frac{760\ \text{mm of Hg}}{101.325\ \text{kPa}}\right)$$

= 42.21 mm of Hg

Maximum possible vacuum;

$p_{g,max}$ = Actual barometer reading – saturation pressure
= 766 mm – 42.21 mm
= 723.79 mm of Hg

The vacuum efficiency;

$$\eta_{vacuum} = \frac{\text{Actual vacuum}}{\text{Max. possible vacuum}} = \frac{p_g}{p_{g,max}}$$

$$= \frac{716\ \text{mm}}{723.79\ \text{mm}} = 0.9892 = \mathbf{98.92\%}$$

(iii) Undercooling of condensate
= Condensate temperature – hot well temperature
= Exhaust steam temp. – temperature of condensate coming out the condenser
= 35°C – 29°C = **6°C**

(iv) Condenser efficiency;

$$\eta_{condenser} = \frac{\text{Actual temperature rise of cooling water}}{\text{Maximum possible temperature rise of cooling water}}$$

$$= \frac{T_{c,o} - T_{c,i}}{T_{sat} - T_{c,i}} = \frac{24-15}{35-15}$$

= 0.45 or **45%**

Example 23.14 *During trial on a steam condenser, the following observations were recorded.*

Condenser vacuum = 680 mm of Hg
Barometer reading = 764 mm of Hg
Mean condenser temperature = 36.2°C
Hot well temperature = 30°C
Condensate formed per hour = 1780 kg
Inlet temperature of cooling water = 20°C
Outlet temperature of cooling water = 32°C
Quantity of cooling water circulated = 1250 kg/min

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Determine

(a) Condenser vacuum corrected to standard barometer,

(b) Vacuum efficiency,

(c) Condenser efficiency, and

(d) Condition of steam entering the condenser.

Assume R *for air* $= 0.287$ *kJ/kg·K,*

Specific heat of water $= 4.186$ *kJ/kg·K*

Solution

Given Data from a trial on a surface condenser:

Barometer reading, $p_{atm} = 764$ mm of Hg

Condenser vacuum, $p_g = 680$ mm of Hg

Condensate mean temp. $T_{sat} = 36.2°C$

Hot well temperature, $T_c = 30°C$

Cooling water $T_{c,\,i} = 20°C$

$T_{c,\,o} = 32°C$

Mass of steam condensed, $\dot{m}_{steam} = 1780$ kg/h

Mass of water circulated, $\dot{m}_w = 1250$ kg/min

$R = 0.287$ kJ/kg·K

$C_{pw} = 4.186$ kJ/kg·K

To find

(i) Condenser vacuum corrected to standard barometer reading,

(ii) Vacuum efficiency, η_{vacuum}

(iii) Condenser efficiency, $\eta_{condenser}$

(iv) Quality of exhaust steam, x.

Analysis

(i) *Condenser vacuum corrected to standard barometer reading*

= Standard barometer reading
– (Actual barometer reading
– Vacuum reading)

$= 760 - (764 - 680)$

$=$ **676 mm of Hg**

(ii) *The vacuum efficiency*

The saturation pressure of steam corresponding to condensate temperature of 36.2°C

$$p_{sat} = 6 \text{ kPa} = 6 \times \frac{760 \text{ mm}}{101.325 \text{ kPa}}$$

$= 45$ mm of Hg

The maximum possible vacuum in the condenser

$p_{g,\,max}$ = Actual barometer reading
– sat. pressure of condensate

$= 764 - 45 = 719$ mm of Hg

Then

$$\eta_{vacuum} = \frac{\text{Actual vacuum in the condenser}}{\text{Maximum possible vacuum}}$$

$$= \frac{676 \text{ mm}}{719 \text{ mm}} = 0.9402 \quad \text{or} \quad \mathbf{94.02\%}$$

(iii) *Condenser efficiency*

Actual temperature rise of water,

$$(\Delta T)_c = T_{c,\,o} - T_{c,\,i} = 32 - 20 = 12°C$$

The absolute pressure in the condenser

p = barometer reading – Vacuum gauge reading

$= 764 - 680 = 84$ mm of Hg

$$= 84 \times \frac{101.325 \text{ kPa}}{760 \text{ mm}} = 11.12 \text{ kPa}$$

Saturation temperature of steam corresponding to $p_{sat} = 11.12$ kPa (from steam tables)

$$T_{sat} = 48°C$$

Maximum possible temperature rise of cooling water

$$(\Delta T)_{max} = T_{sat} - T_{c,\,i} = 48 - 20 = 28°C$$

Then $\eta_{condenser} = \dfrac{(\Delta T)_c}{(\Delta T)_{max}} = \dfrac{12}{28}$

$= 0.4286$ or **42.86%**

(iv) *The quality of exhaust steam, x*

From steam tables,

At 11.12 kPa, $h_f = 200.1$ kJ/kg and

$h_{fg} = 2387.2$ kJ/kg

At 30°C $h_{fc} = 125.75$ kJ/kg

Heat energy given by steam

= Heat gain by circulating water

Heat energy given by steam,

$$\dot{Q}_{steam} = \dot{m}_{steam}\,[h_f + x\,h_{fg} - h_{fc}\,)]$$

Heat gain by cooling water,

$$\dot{Q}_w = \dot{m}_w\,C_{pw}\,(\Delta T)_c$$

Using numerical values and equating two quantities of heat energy;

$$\frac{1760}{60} \times [200.1 + x \times 2387.2 - 125.75]$$

$$= 1250 \times 4.186 \times (32 - 20)$$

or $2387.2\,x = 2140.57 - 74.35 = 2066.22$

or $x = \dfrac{2066.22}{2387.2} =$ **0.865**

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Example 23.15 *The following observation were made during a test on a surface condenser:*

Barometer reading = *760 mm of Hg*
Condenser vacuum = *705 mm of Hg*
Mean temperature of the condensate = *35°C*
Condensate collected = *2000 kg/h*
Quantity of cooling water circulated = *60,000 kg/h*
Rise in temperature of cooling water = *16°C*
Hot well temperature = *28°C*
Inlet temperature of cooling water = *20°C*

Calculate

(a) Vacuum efficiency,
(b) Condenser efficiency,
(c) Quality of steam entering the condenser,
(d) Mass of air present per m^3 of the condenser volume.

Solution

Given Data from the test on a surface condenser:

Barometer reading p_{atm} = 760 mm of Hg
Condenser pressure, p_g = 705 mm of Hg
Condensate mean temperature T_{sat} = 35°C
Mass of steam condensed, $\dot{m}_{steam}$ = 2000 kg/h
Mass of water circulated, $\dot{m}_w$ = 60,000 kg/h
$(\Delta T)_c$ = 16°
Hot well temperature, T_c = 28°C
$T_{c,\,i}$ = 20°C

To find

(i) Vacuum efficiency, η_{vacuum}
(ii) Condenser efficiency, $\eta_{condenser}$
(iii) Quality of exhaust steam, x
(iv) Mass of air present in condenser, m_a per m^3 condenser volume.

Assumptions

(i) The specific heat of water as 4.187 kJ/kg·K.
(ii) The specific gas constant for air as 0.287 kJ/kg·K.

Analysis The absolute pressure in the condenser

p = barometer reading – Vacuum gauge reading
= 760 – 705 = 55 mm of Hg
$= 55 \times \dfrac{101.325 \text{ kPa}}{760 \text{ mm}} = 7.332$ kPa

Saturation pressure of steam, corresponds to condensate temperature of 35°C

p_{sat} = 5.628 kPa
$= 5.628 \text{ kPa} \times \dfrac{760 \text{ mm}}{101.325 \text{ k a}} = 42.21$ mm of Hg

The maximum possible vacuum in the condenser,

$p_{g,\,max}$ = barometer reading – saturation pressure of condensate
= 760 – 42.21 = 717.79 mm of Hg

(i) *The vacuum efficiency*

$$\eta_{vacuum} = \frac{\text{Actual vacuum in the condenser}}{\text{Maximum possible vacuum}}$$

$$= \frac{705 \text{ mm}}{717.79 \text{ mm}} = 0.982 \quad \text{or} \quad \mathbf{98.2\%}$$

(ii) *Condenser efficiency*

Actual temperature rise of water, $(\Delta T)_c$ = 16°C
Saturation temperature of steam corresponds to p_{sat} = 7.332 bar (from steam tables)

T_{sat} = 40°C

Maximum possible temperature rise of cooling water

$(\Delta T)_{max} = T_{sat} - T_{c,\,i} = 40 - 20 = 20$°C

Then $\eta_{condenser} = \dfrac{(\Delta T)_c}{(\Delta T)_{max}} = \dfrac{16}{20} = 0.8$
$= \mathbf{80\%}$

(iii) *The quality of exhaust steam, x*

Heat given by steam
= Heat gain by circulating water

Heat given by steam

$\dot{Q}_{steam} = \dot{m}_{steam}\,[\,h_f + x\,h_{fg} - h_{fc}\,]$

Heat gain by cooling water,

$\dot{Q}_w = \dot{m}_w\,C_{pw}\,(\Delta T)_c$

From steam tables:

At 35°C, h_f = 146.66 kJ/kg and
h_{fg} = 2418.62 kJ/kg
At 28°C h_{fc} = 117.4 kJ/kg

Using the numerical values

$2000 \times [146.66 + 2418.62\,x - 117.4]$
$= 60{,}000 \times 4.187 \times 16$

or $2418.62\,x = 2009.76 - 29.26 = 1980.5$

or $x = \dfrac{1980.5}{2418.62} = \mathbf{0.8188}$

(iv) *Mass of air present in the condenser per m^3 of condenser volume*

The partial pressure of air

$p_a = p - p_{sat}$

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$$= 7.332 \text{ kPa} - 5.628 \text{ kPa}$$
$$= 1.705 \text{ kPa}$$
$$m_a = \frac{p_a V_a}{RT_{sat}}$$
$$= \frac{(1.705 \text{ kPa}) \times (1 \text{ m}^3)}{(0.287 \text{ kJ/kg} \cdot \text{K}) \times (35 + 273)(\text{K})}$$
$$= \mathbf{0.0193\,kg}$$

23.10 AIR EXTRACTION

The presence of air increases the back pressure in the condenser. Therefore, the air should be removed continuously out of the condenser. For removal of air from the condenser, there are two types of systems used:

(i) Dry-air extraction system, and
(ii) Wet-air extraction system.

(*i*) *Dry-air Extraction System* In this system, the dry pump removes only moist air from the condenser. A separate pump is used for extraction of condensate from the bottom of the condenser.

The air pump is usually connected at the top of the condenser. The opening of air extraction pump is separated from opening for incoming steam by a screen or baffle. It avoids the steam being directly removed by the air extraction pump and permits the cooling of air prior to entry in the air-extraction pump.

(*ii*) *Wet air Extraction System* In this system, a wet air extraction pump extracts -air with condensate from the condenser. The air is extracted at the temperature of condensate without cooling. The pump is connected at the bottom of condenser.

The pump may be reciprocating or rotary type. The following two pumps are commonly used in a steam power plant.

1. Edward's air pump 2. Steam jet air ejector

23.10.1 Air Pump

The most commonly used reciprocating type of air extraction pump is Edward's air pump as shown in Fig. 23.14. It is a wet-air extraction pump. It is

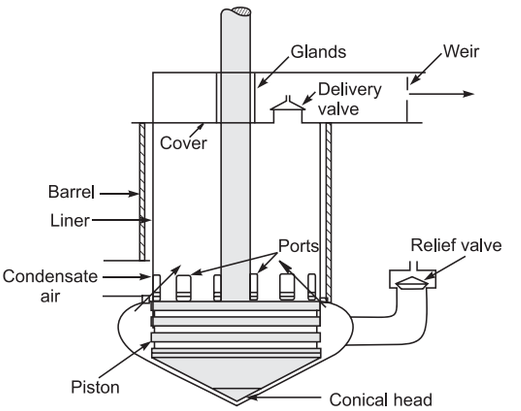

Fig. 23.14 *Edward's air pump*

capable to maintain a relatively low vacuum, thus it is only suitable for small power plants.

It consists of solid bucket-type plunger, which is flat at the top and conical at the bottom surface. The plunger reciprocates in the pump barrel. The delivery valves are mounted in the cover of the pump barrel. These valves are also called *head valves*. The plunger has several ports along its periphery. These ports communicate with the condenser.

When the plunger is at the top position, the condensate and air from the condenser is collected in the conical portion of the barrel. When the piston moves down, the partial vacuum is created above the plunger. The air and condensate rush into the barrel above the plunger through ports in it. At this position the head valves are in a closed position and sealed by water. When the piston moves up, the condensate with air is compressed slightly above atmospheric pressure and delivery valves open. The condensate-air mixture is discharged above the barrel cover. The condensate flows over the weirs to the hot well.

A relief valve is provided at the base of the cylinder. This valve opens to the atmosphere to release the pressure in case the pressure inside increases above atmospheric pressure.

23.10.2 Steam-Jet Air Ejector

The working principle of a steam-jet air ejector is shown in Fig. 23.15. It consists of a convergent–

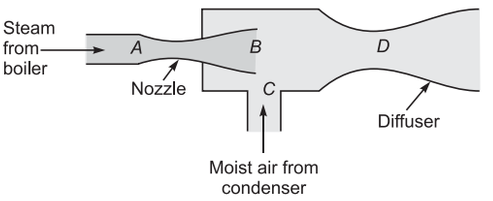

Fig. 23.15 *Steam jet air ejector*

divergent nozzle and a diffuser. The high-pressure steam from the boiler enters the nozzle *A*, where its kinetic energy increases and pressure reduces. The pipe *C* is connected to the condenser from where moist air is sucked by low-pressure steam at the location *B*. The momentum of steam jet carries the mixture of steam and moist air in to the diffuser *D*, where its velocity decreases and pressure increases before discharge.

If sufficient number of ejectors are installed, a very low pressure can be maintained in the condenser. The steam ejectors are simple in construction, cheap, highly efficient and without any moving parts. These are suitable for large power plants, where a high condenser vacuum is required.

Example 23.16 *During a trial on a condenser, the following readings were recorded:*

Condenser vacuum = *716 mm of Hg*
Barometer reading = *760 mm of Hg*
Mean condenser Temperature = *30°C*
Condensate collected = *22000 kg/h*

The air entering the condenser is given by the equation

$$\dot{m}_a = \left(\frac{\text{Steam condensed in kg/h}}{2000} + 2\right) \text{kg/h}$$

Calculate the discharge capacity of wet air pump, which removes air and condensate in m^3*/min. Take volumetric efficiency of the pump as 80%.*

Solution

Given Data from the test on a surface condenser:

Barometer reading, p_{atm} = 760 mm of Hg
Condenser vacuum pressure, p_g = 700 mm of Hg
Hot well temperature, T_c = 30°C
Mass of steam condensed, $\dot{m}_{steam}$ = 22000 kg/h
Volumetric efficiency, η_{vol} = 0.8

To find Capacity of air pump in m^3/min.

Assumption The specific gas constant for air as 0.287 kJ/kg·K.

Analysis The mass-flow rate of air in the condenser

$$\dot{m}_a = \left(\frac{\text{Steam condensed in kg/h}}{2000} + 2\right) \text{kg/h}$$

$$= \frac{(22000 \text{ kg/h})}{2000} + 2 = 13 \text{ kg/h}$$

From steam tables:

At condensate temperature of 30°C

p_{sat} = 4.25 kPa and v_f = 0.001004 m^3/kg

The absolute pressure in the condenser

$$p_{abs} = p = p_{atm} - p_g = 760 - 700 = 60 \text{ mm of Hg}$$

$$= 60 \text{ mm of Hg} \times \frac{101.325 \text{ kPa}}{717.79 \text{ mm of Hg}} = 8 \text{ kPa}$$

Using Dalton's law of partial pressure of air in the condenser

$$p_a = p - p_{sat} = 8 \text{ kPa} - 4.25 \text{ kPa} = 3.75 \text{ kPa}$$

From ideal-gas relation, the volume flow rate of air

$$\dot{V}_a = \frac{\dot{m}_a R_a T_a}{p_a}$$

$$= \frac{(13 \text{ kg/h}) \times (0.287 \text{ kJ/kg} \cdot \text{K}) \times (30 + 273)(\text{K})}{(3.75 \text{ kPa})}$$

$$= 301.46 \text{ m}^3\text{/h}$$

The volume rate of steam condensed

$$\dot{V}_{steam} = \dot{m}_{steam} \times v_f$$

$$= 22000 \times 0.001004 = 22.088 \text{ m}^3\text{/h}$$

Total volume to be handled by wet pump per hour

$$\dot{V} = \dot{V}_a + \dot{V}_{steam} = 301.46 + 22.088$$

$$= 323.55 \text{ m}^3\text{/h} = 5.393 \text{ m}^3\text{/min.}$$

The discharge capacity of wet pump

$$= \frac{\text{Volume flow rate}}{\text{Volumetric efficiency}}$$

$$= \frac{5.392 \text{ m}^3 / \text{min}}{0.8} = \mathbf{6.74 \text{ m}^3\text{/min}}$$

Example 23.17 *The air leakage into a surface condenser operating with a steam turbine is estimated as 84 kg/h. The vacuum near the inlet of the air pump is 700 mm of Hg, when the barometer reads 760 mm of Hg. The temperature at the inlet of vacuum pump is 20°C. Calculate*

(a) minimum capacity of air pump in m^3*/h,*

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(b) The diameter of reciprocating air pump to remove the air, if it runs at 200 rpm with L/D ratio of 1.5 and volumetric efficiency of 100%,

(c) Mass of vapour extracted per minute.

Solution

Given A surface condenser:

$T_{sat} = 20°C$ $\quad \dot{m}_a = 84$ kg/h

$\eta_{vol} = 1.0$ $\quad L/D = 1.5$

$N = 200$ rpm $\quad p_{atm} = 760$ mm of Hg

$p_g = 700$ mm of Hg

To find

(i) Capacity of air pump,

(ii) Cylinder diameter, and

(iii) Mass of vapour extracted.

Assumptions

(i) For air $R = 0.287$ kJ/kg·K.

(ii) Air pump handles dry air only.

Analysis The mass-flow rate of air into the condenser per minute

$$\dot{m}_a = \frac{84}{60} = 1.4 \text{ kg/min}$$

At condensate temperature of 20°C,

$$p_{sat} = 2.34 \text{ kPa} \quad \text{and} \quad v_f = 0.001002 \text{ m}^3/\text{kg}$$

The absolute pressure in the condenser,

$$p_{abs} = p = p_{atm} - p_g = 760 - 700$$
$$= 60 \text{ mm of Hg}$$
$$= 60 \text{ mm of Hg} \times \frac{101.325 \text{ kPa}}{717.79 \text{ mm of Hg}}$$
$$= 8 \text{ kPa}$$

Using Dalton's law, the partial pressure of air in the condenser

$$p_a = p - p_{sat} = 8 \text{ kPa} - 2.34 \text{ kPa}$$
$$= 5.66 \text{ kPa}$$

(i) *Capcaity of air pump*

From ideal gas relation, the volume flow rate of air

$$\dot{V}_a = \frac{\dot{m}_a R_a T_a}{p_a}$$
$$= \frac{(1.4 \text{ kg/min}) \times (0.287 \text{ kJ/kg·K}) \times (20 + 273)(\text{K})}{(5.66 \text{ kPa})}$$
$$= 20.8 \text{ m}^3/\text{h}$$

The discharge capacity of wet pump

$$= \frac{\text{Volume flow rate}}{\text{Volumetric efficiency}}$$
$$= \frac{20.8 \text{ m}^3/\text{min}}{1.0}$$
$$= \mathbf{20.8 \text{ m}^3/\text{min}}$$

(ii) *Diameter of pump*

The pump extraction rate can be expressed as

$$\dot{V}_a = \left(\frac{\pi}{4}\right) D^2 L \, N \, \eta_{vol}$$

or $$20.8 = \left(\frac{\pi}{4}\right) D^2 \times (1.5D) \times 200 \times 1.0$$

or $$D^3 = 0.088278 \text{ m}^3 = 88278 \text{ cm}^3$$

Diameter $D = \mathbf{44.52 \text{ cm}}$

(iii) *Mass of vapour extracted*

Assuming some steam also flows with air through the air-extraction pump, the mass flow rate

$$\dot{m}_{steam} = \frac{\dot{V}_a}{v_f} = \frac{20.8}{0.001002}$$
$$= \mathbf{20778.2 \text{ kg/min}}$$

Example 23.18 *A surface condenser deals with 13625 kg of steam per hour at a pressure of 0.09 bar. The steam enters 0.85 dry and the temperature of the condensate and air extraction pipes is 36°C. The air leakage amount to be 7.26 kg/h.*

Determine

(a) the surface area required, if the average heat transmission rate is 3.97 kJ/cm²

(b) the cylinder diameter of dry air pump, if it is to be a single-acting reciprocating type, runs at 60 rpm with a stroke-to-bore ratio of 1.25 and a volumetric ratio of 0.85

Solution

Given A surface condenser:

$\dot{m}_{steam} = 13625$ kg/h $\quad p = 0.09$ bar $= 9$ kPa

$x = 0.85$ $\quad T_{sat} = 36°C$

$\dot{m}_a = 7.26$ kg/h $\quad \eta_{vol} = 0.85$

$L/D = 1.25$ $\quad q = 3.97$ kJ/cm²

$N = 60$ rpm

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To find

(i) Surface area of condenser, and

(ii) Cylinder diameter of dry air pump.

Assumptions

(i) For air $R = 0.287$ kJ/kg·K.

(ii) Air pump handles air only.

Analysis From steam tables,

At 0.09 bar = 9 kPa

$h_f = 183.3$ kJ/kg and

$h_{fg} = 2397.7$ kJ/kg

At 36°C, $p_{sat} = 5.94$ kPa

$h_{fc} = 150.86$ kJ/kg

The heat transfer rate from steam per second

$$\dot{Q} = \dot{m}_{steam}\ [h_f + x\,h_{fg} - h_c]$$

$$= \frac{13625}{3600} \times [183.3 + 0.85 \times 2397.7 - 150.86]$$

$$= 7836.2 \text{ kJ/s}$$

The surface area required,

$$A = \frac{\dot{Q}}{q} = \frac{7836.2 \text{ kJ/s}}{3.97 \text{ kJ/cm}^2.\text{s}} = \mathbf{1973.85\ cm^2}$$

The mass-flow rate of air into the condenser per minute

$$\dot{m}_a = \frac{7.26}{60} = 0.121 \text{ kg/min}$$

The partial pressure of air into the condenser,

$$p_a = p - p_{sat} = 9 - 5.94 = 3.06 \text{ kPa}$$

The volume-flow rate into the condenser,

$$\dot{V}_a = \frac{\dot{m}_a R_a\ T_{sat}}{p_a}$$

$$= \frac{(0.121 \text{ kg/s}) \times (0.287 \text{ kJ/kg} \cdot \text{K}) \times (36 + 273)(\text{K})}{(3.06 \text{ kPa})}$$

$$= 3.506 \text{ m}^3/\text{min}$$

The pump extracts the air with this discharge rate, i.e.,

$$\dot{V}_a = \left(\frac{\pi}{4}\right) D^2 L\ N\ \eta_{vol}$$

or $$3.506 = \left(\frac{\pi}{4}\right) D^2 \times (1.25D) \times 60 \times 0.85$$

or $$D^3 = 0.0700 \text{ m}^3 = 0.07 \times 10^6 \text{ cm}^3$$

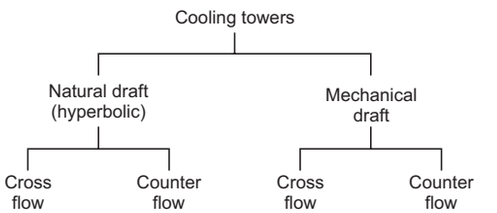
or $$D = \mathbf{41.2\ cm}$$

23.11 COOLING TOWERS

Power plants, air-conditioning systems and some industries produce a large quantity of waste heat in the form of hot water. In the present scenario, in most of the places, the water supply is limited and thermal pollution is also a serious concern. Therefore, the waste heat from water must be rejected for its re-use.

The cooling tower is a direct-contact type of heat exchanger. It is a semi-enclosed, evaporative cooler. The water is sprayed through a certain height and an air current passes over it. Some water evaporates and the heat of evaporation is extracted from falling water and surrounding air, thus both water and air cool.

The cooling towers are classified as shown:



23.11.1 Natural-draft Cooling Tower

A natural-draft cooling tower is tall and hyperbolic in construction. It looks like a large chimney and works as an ordinary chimney. The chimney effect is created by the hyperbolic profile of the tower. As air comes in contact of heated falling water, air is heated, becomes lighter and rises up in the tower. The cold and heavier outside air fills the vacant space creating an air flow. These towers do not require any external power to induce the air, but their construction cost is much more than mechanical-draft cooling towers. These are constructed from RCC and have base diameters ranging 75 to 100 m and heights between 100 m to 150 m.

In the *cross-flow* arrangement, the water is sprayed downward near the base of the tower on a fill material. Air is induced radially inward and flows in across the water.

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In the *counter-flow* arrangement as shown in Fig. 23.16, air enters through a peripheral section at the bottom of the tower and flows upward through a descending water spray made from a suitable height.

The heat exchanger section in cross-and counter-flow natural-draft cooling towers occupies a small portion of the tower, while the remaining structure is used to promote the chimney effect. These towers are only suitable in desert regions, where moisture content in air is very low.

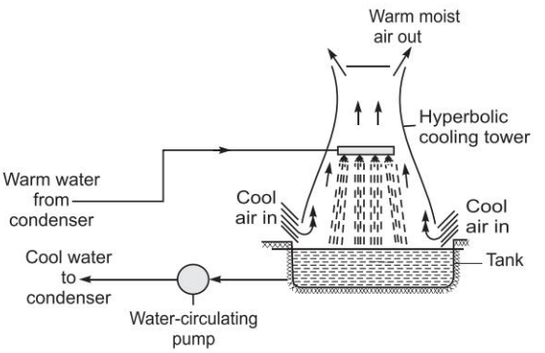

Fig. 23.16 *Natural-draft cross-flow cooling tower*

23.11.2 Mechanical or Induced-draft Cooling Towers

These are induced-draft cooling towers. These are compact in size with low height. The air movement through the tower is created by induced fans, located at the top of the tower.

An *induced draft counter flow* cooling tower is shown in Fig. 23.17. The warm water coming from the condenser is pumped to the top of the tower and is sprayed into air stream. The falling water passes through a series of baffles, thus the water breaks into fine droplets to promote evaporation. The atmospheric air is drawn in by an induced fan flowing upward, counter to the direction of the falling water droplets. As the two streams interact, a small fraction of water stream evaporates into moist air and cools the remaining falling water. The cooled water is collected at the bottom of the tower and is pumped back to the condenser. The moist air

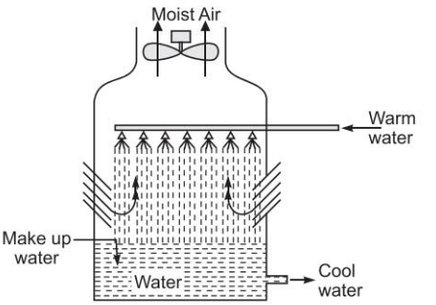

Fig. 23.17 *Induced draft cooling tower*

is discharged from the top of the tower. Since some of the warm water is evaporated into the air stream, an equivalent amount of make-up water is added to the cycle.

23.11.3 Comparison Between Natural-draft and Mechanical-draft Cooling Towers

1. The natural-draft cooling towers have a higher construction cost than mechanical draft towers.
2. Natural-draft cooling towers require unreasonable high height. They use only a small portion for heat exchange between hot water and air, while mechanical draft cooling towers utilize the whole height and space for heat exchange.
3. Natural-draft cooling towers cannot control the temperature of outlet water precisely. However, the mechanical cooling towers enable better control on heat transfer process, which is favourable for varying load on power plant and changing ambient conditions.

23.12 COOLING POND

The cooling ponds are large open tanks. The hot water is sprayed into air and is cooled by air as it falls into the pond as shown in Fig. 23.18. The cold water from the pond is recirculated to the condenser with the help of a pump. In the cooling pond, the cooling effect is mainly due to evaporation of

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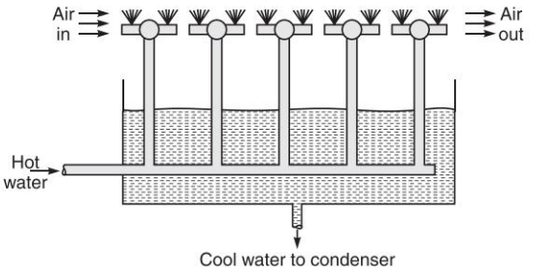

Fig. 23.18 *Cooling tower*

sprayed water and water from the upper surface of the pond.

The cooling ponds are suitable for small capacity steam plants or where land is cheaply available. The water loss due to air drift (windage) is high and it is unprotected against dust and dirt.

Summary

- *A condenser* is a device, used for vapour condensation to liquid state at saturation temperature and constant pressure. The condensers are of basically two types: jet and surface condensers.
- In a jet condenser, the exhaust steam and cooling water mix together. The cooling water is sprayed on the steam, which causes rapid condensation. These are further classified as
  (i) low level,
  (ii) high level, and
  (iii) ejector-type jet condensers.
- In the *surface condensers,* the steam and cooling water do not physically interact. The exhaust steam condenses over the outer surface of the tubes, through which cooling water flows. The condensate coming out of the condenser is pure, thus it is used as feed water in the boiler.
- The evaporative condensers are suitable for small power plants, refrigerating plant and for chemical equipments.
- The cooling towers are used to cool the circulating water in the large surface condenser. It uses the principle of water evaporation.

$$\eta_{condenser} = \frac{\text{Actual temperature rise of cooling water}}{\text{Maximum possible temperature rise of water}}$$

- The vacuum is the pressure measured below atmospheric pressure. The vacuum is maintained in the condenser in order to provide large expansion of steam through the turbine. The vacuum efficiency is defined as

$$\eta_{vacuum} = \frac{\text{Actual vacuum in the condenser}}{\text{Maximum obtained vacuum}} = \frac{p_g}{p_{g,\,max}}$$

- Due to presence of air in the condenser, the vacuum is disturbed and actual vacuum is less than the vacuum that could be obtained in absence of air in the condenser.

  Actual vacuum = Barometric pressure – Absolute measured pressure in the condenser

  Maximum vacuum = Barometric pressure – Saturation pressure of steam at condenser temperature.

- The dry and wet air pumps are used to remove the air from the condenser. The removal air helps to maintain the desired vacuum in the condenser.
- The dry air pump only extracts the moist air from the condenser and the condensate is extracted by the condensate extraction pump, while the wet air pump is used to remove both condensate and air from the condenser. Edward's wet air pump is suitable for small power plants, while steam-jet air ejectors help to maintain very low back pressure in the condenser, and are thus useful for large power plants.

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Glossary

Condenser A device in which vapour condenses to liquid at saturation temperature and constant pressure

Jet condensers A device in which the exhaust steam and cooling water come in direct contact and mix up together

Barometric Condenser A high-level jet condenser having a tail pipe longer than 10.33 m.

Surface condenser Exhaust steam and cooling water interact indirectly for heat transfer

Condenser efficiency Ratio of actual temperature rise to the maximum possible temperature rise of cooling water

Vacuum The pressure below the atmospheric pressure

Vacuum efficiency Ratio of the actual vacuum to the maximum possible vacuum

Review Questions

1. Write the function and applications of the condenser.
2. Why is the steam condenser used in each power plant unit?
3. Classify the steam condenser.
4. Describe the details and working of a low-level jet condenser.
5. Explain the working principle of a jet condenser.
6. Explain the working of a barometric jet condenser.
7. Explain the working of an ejector-type condenser.
8. Why does a barometric jet condenser not require a water-extraction pump? Explain.
9. Why does an ejector-type jet condenser not require a water-extraction pump? Explain.
10. Explain the principle of working of a surface condenser.
11. Explain the working of a shell-and-tube type surface condenser.
12. Explain the working of an evaporative condenser.
13. Explain the working of a cooling pond with a diagram.
14. Explain the working of a natural-draft cross-flow cooling tower with a diagram.
15. Define condenser efficiency.
16. Define vacuum and how it is corrected to standard barometric reading?
17. Define vacuum efficiency?
18. Write the source of air into a condenser.
19. Differentiate between jet and surface condensers.
20. State Dalton's law of partial pressures.
21. Explain the determination of quantity of circulating water in a condenser.
22. What do you mean by the terms 'undercooling of condensate' and 'hot well'?

Problems

1. Exhaust steam having a dryness fraction of 0.85 enters a surface condenser at a pressure of 9.81 kPa and is condensed to water at 38°C. The circulating water enters at 15°C and leaves at 30°C. Calculate the mass of cooling water required per kg of exhaust steam. *[32.88 kg]*

2. The vacuum at the air-extraction pump in a condenser is 706 mm of mercury and the temperature is 37.31°C. The air leakage into the condenser is 5 kg per 10,000 kg of steam. Determine (a) the volume of air to be dealt with the dry air pump per kg of steam entering the condenser, and (b) the mass of water vapour associated with this air. The barometric reading is 760 mm of Hg. *[(a) 0.0628 m^3, (b) 0.00267 kg]*

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3. 80% dry steam enters a surface condenser where the vacuum is 92.85 kPa, the barometer reads 101.18 kPa and is condensed to 37.31°C. The temperature of the hot well is 32.2°C. The circulating water enters at 15.5°C and leaves at 30°C. Determine

(a) the mass of air extracted per kg of steam,

(b) the mass of circulating water required per kg of steam, and

(c) the vacuum efficiency.

[(a) 0.4 kg, (b) 32.14 kg/kg of steam, (c) 98%]

4. The following observations were made on a condensing plant in which the temperature of condensation was measured directly by thermometers; the recorded vacuum was 710 mm of Hg and the barometer reads 765 mm of Hg. The mean temperature of condensation and hot well was 34.25°C and 28°C, respectively. The condensate leaves the condenser at the rate of 2000 kg/h. The cooling water circulates in the condenser at the rate of 64,000 kg/h and its inlet and outlet temperature was recorded 14.5°C and 30°C, respectively. Calculate (a) the state of steam entering the condenser, and (b) the mass of air present per m^3 of condenser volume.

[(a) 0.85, (b) 0.497 kg]

5. A vacuum of 712 mm was obtained with a barometer reading of 753 mm of Hg. Correct the vacuum to a standard barometric reading of 76 cm of Hg. *[71.9 cm of Hg]*

6. Following data were recorded during testing of a condenser:

Vacuum = 700 mm;
Barometer reading = 754 mm of Hg;
Condensate temperature = 18°C.

Find the partial pressure of air and steam in the condenser and the mass of air/kg of steam.

[p_a = 0.05135 bar; p_s = 0.071976 bar; 3.775 kg/kg of steam]

7. The vacuum reading of a condenser is 70.5 cm of Hg when the barometer shows 76 cm of Hg and the condensate temperature is 31°C. Find the vacuum efficiency. *[97.06%]*

8. The following observations were made on a steam condensing plant:

Barometer reading = 760 mm of Hg; Recorded vacuum = 700 mm of Hg; Mean temperature of condensation = 34°C; Hot-well temperature = 27°C. Mass of condensate = 2120 kg/h; Mass of cooling water = 66000 kg/h; Rise in temperature of cooling water = 16°C.

Find

(a) the state of steam entering the condenser,

(b) mass of air present m^3 of condenser, and

(c) vacuum efficiency.

[(a) x = 0.8492 (b) m_a = 0.0304 kg/m^3 (c) h_{vaccum} = 97.2%]

9. The following observations were recorded during a test on a steam condenser:

Recorded condenser vacuum = 71 cm of Hg; Barometer reading = 76.5 cm of Hg, Mean condenser temperature = 34°C, Condensate collected = 1800 kg/h; Mass of cooling water = 5700 kg/h; Cooling water temperature rise = 17.5°C.

Calculate

(a) Corrected vacuum to standard barometer of 76 cm of Hg,

(b) vacuum efficiency,

(c) condenser efficiency, and

(d) state of steam entering the condenser.

Assume inlet temperature of cooling water = 8.5°C. *[(a) 70.5 cm of Hg (b) 97.91% (c) 55.76% (d) 0.9667]*

10. The vacuum gauge reading of a condenser is 71 cm of Hg and the mean temperature of the condenser is 35°C. The air leakage into the condenser is 1 kg per 2000 kg of steam. Determine the volume of air to be handled by the dry air pump per kg of steam and the mass of vapour associated with air. Barometer reading = 76 cm.

[0.0424 m^3/kg of steam, 0.00168 kg]

11. The following observations were made on a steam condensing plant:

Barometer reading = 76 cm of Hg; Recorded condenser vacuum = 70 cm of Hg; Mean temperature of condensate = 35°C; Condensate collected 16.75 kg/min; Cooling water = 39600 kg/h; Rise in temperature of cooling water = 14°C; Inlet temperature of cooling water = 25°C.

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Calculate

(a) Mass of air present per m^3 of condenser volume,

(b) Dryness fraction of steam at entry to condenser,

(c) The vacuum efficiency, and

(d) The effectiveness (efficiency) of the condenser.

[(a) 0.02692 kg/m^3 (b) 0.87 (c) 97.51% (d) 84.64%]

12. Exhaust steam having a dryness fraction of 0.8 enters a surface steam condenser where the vacuum is 696.5 mm of mercury (barometer reading = 759 mm of Hg) and is condensed to 37.31°C. The temperature of the hot well is 32.2°C. The circulating water enters at 15.5°C and leaves at 30°C. Determine (a) the mass of air extracted per kg of steam, (b) the mass of circulating water required per kg of steam, and (c) the vacuum efficiency.

13. A surface condenser is designed to handle 10,000 kg of steam per hour. The steam enters at 0.08 bar and 0.9 dry and the condensate leaves at corresponding saturation temperature. the pressure is constant throughout the condenser. Calculate the cooling water flow rate per hour, if the cooling water temperature rise is 10°C.

Objective Questions

1. The condensation of steam in a condenser takes place at
   (a) constant pressure
   (b) constant temperature
   (c) constant pressure and constant temperature
   (d) none of the above
2. During condensation process, the temperature of the condensing fluid
   (a) remains constant
   (b) decreases
   (c) increases
   (d) none of the above
3. The function of a condenser in a steam power plant is
   (a) to reduce back pressure
   (b) to condense the exhaust steam
   (c) to reduce specific volume of fluid
   (d) all of the above
4. In a high-level jet condenser, the condenser shell is installed at a height of
   (a) more than 5.5 m
   (b) more than 10.33 m
   (c) less than 10.33 m
   (d) none of the above
5. The surface condensers are preferred in steam power plant, because
   (a) they require less coolant
   (b) condensate can be reused
   (c) they are more efficient
   (d) none of the above
6. In evaporative condensers, the condensing of steam is achieved
   (a) by rejecting heat to surrounding air
   (b) by rejecting heat to coolant
   (c) by evaporation of some coolant
   (d) none of the above
7. Dalton's law of partial pressure, applicable to condensers, states that
   (a) $p_a = p_{abs} + p_{sat}$
   (b) $p_{sat} = p_{abs} + p_a$
   (c) $p_{abs} = p_a + p_{sat}$
   (d) none of the above
8. The absolute pressure in a condenser is given by
   (a) $p_{abs} = p_{atm} + p_{vacuum}$
   (b) $p_{abs} = p_{atm} - p_{vacuum}$
   (c) $p_{abs} = p_{vacuum}$
   (d) none of the above
9. The vacuum efficiency of a condenser is defined as
   (a) $\dfrac{\text{Actual absolute pressure in condeser}}{\text{Atmospheric pressure}}$

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(b) $\frac{\text{Atmospheric pressure}}{\text{Absolute pressure in condenser}}$

(c) $\frac{\text{Actual vacuum in the condeser}}{\text{Maximum possible vacuum}}$

(d) none of the above

10. The condenser efficiency is defined as

(a) $\frac{\text{Temp. rise of coolant}}{\text{Temp. drop of condensate}}$

(b) $\frac{\text{Temp. drop of condensate}}{\text{Temp. rise of coolant}}$

(c) $\frac{\text{Actual temp. rise of coolant}}{\text{Maximum possible temp. rise of coolant}}$

(d) none of the above

11. Air leakage into the condenser reduces
    (a) turbine output
    (b) cooling capacity
    (c) life of condenser
    (d) all of the above
12. The vacuum maintained in a condenser depends on
    (a) pressure of cooling water
    (b) temperature of cooling water
    (c) back pressure maintained in the condenser
    (d) all of the above

Answers

1. (c) 2. (a) 3. (d) 4. (b) 5. (b) 6. (c) 7. (c) 8. (b)
9. (c) 10. (c) 11. (d) 12. (b)

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24

Internal Combustion Engines

Introduction

An *internal combustion engine* is a machine that converts chemical energy in a fuel into mechanical energy. Fuel is burnt in a combustion chamber, releases its chemical energy in the form of heat, which is converted into mechanical energy with the help of a reciprocating piston and crank mechanism.

Two principal types of reciprocating internal combustion engines are in general use: the Otto-cycle engine and the Diesel engine. The Otto-cycle engine, named after its inventor, the German technician *Nikolaus August Otto*, is the familiar gasoline engine used in automobiles and airplanes. The Diesel engine, named after the French-born German engineer *Rudolf Christian Karl Diesel*, operates on a different principle and usually uses oil as a fuel. It is employed in electric-generation and marine-power plants, in trucks and buses, and in other automobiles. Both Otto-cycle and Diesel-cycle engines are manufactured in two-stroke and four-stroke cycle models.

24.1 CLASSIFICATION OF IC ENGINES

The internal combustion engines are usually of reciprocating type. The reciprocating internal combustion engines are classified on the basis of the thermodynamic cycle and mechanical method of operation, type of fuel used, type of ignition, type of cooling system and cylinder arrangement, etc. The detailed classification is given below:

1. *According to piston strokes in the working cycle*
   (a) Four-stroke engine
   (b) Two-stroke engine
2. *According to fuel used in the cycle*
   (a) Petrol engine (b) Diesel engine
   (c) Gas engine (d) Multi–fuel engine
3. *According to method of ignition*
   (a) Spark ignition
   (b) Compression ignition
4. *According to fuel-feeding system*
   (a) Carburetted engine
   (b) Engine with fuel injection
5. *According to charge feeding system*
   (a) Naturally aspirated engine
   (b) Supercharged engine
6. *According to cooling system*
   (a) Air-cooled engine
   (b) Water-cooled engine
7. *According to number of cylinders*
   (a) Single cylinder engine
   (b) Multi-cylinder engine

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8. *According to speed of the engine*
   (a) Low-speed engine
   (b) Medium-speed engine
   (c) High-speed engine
9. *According to position of engine*
   (a) Horizontal engine
   (b) Vertical engine
   (c) V-engine

The petrol engines use low compression ratio. The fuel and air mixture as a charge is ignited by a high intensity spark. Therefore, they are also called *spark ignition (SI) engines.* The Diesel engines use high compression ratio, and the compressed charge is autoignited. Therefore, they are also called *compression ignition (CI) engines.*

24.2 COMPONENTS OF ENGINES

The essential parts of Otto-cycle and Diesel-cycle engines are the same. Actually an internal combustion engine consists of a large number of parts and each part has its own function. A few of them are shown in Fig. 24.1 and listed below:

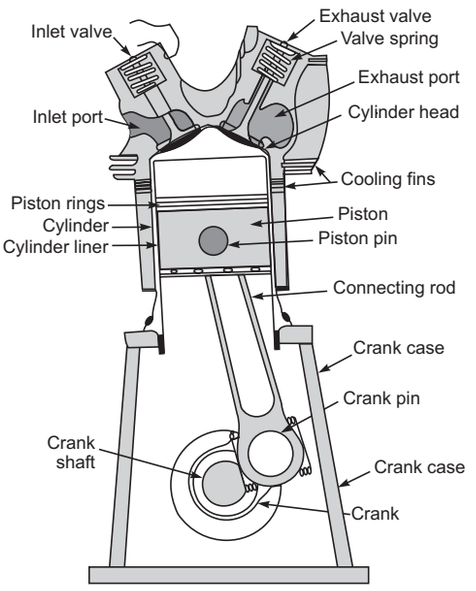

Fig. 24.1 *Details of an internal combustion engine*

1. Cylinder It is the heart of the engine. The piston reciprocates in the cylinder. It has to withstand high pressure and temperature, and thus it is made strong. Generally, it is made from cast iron. It is provided with a cylinder liner on the inner side and a cooling arrangement on its outer side. For two-stroke engines, it houses exhaust and transfer port.

2. Cylinder Head The top cover of the cylinder, towards *TDC*, is called *cylinder head*. It houses the spark plug in petrol engines and fuel injector in Diesel engines. For four stroke cycle engines, the cylinder head has the housing of inlet and exhaust valves.

3. Piston It is the reciprocating member of the engine. It reciprocates in the cylinder. It is usually made of cast iron or aluminium alloys. Its top surface is called *piston crown* and bottom surface is called *piston skirt*. Its top surface is made flat for four-stroke engines and deflected for two-stroke engines.

4. Piston Rings Two or three piston rings are provided on the piston. The piston rings seal the space between the cylinder liner and piston in order to prevent leakage (blow by losses) of high-pressure gases, from cylinder to crank case.

5. Crank It is a rotating member. It makes circular motion in the crank case (its housing). Its one end is connected with a shaft called *crank-shaft* and the other end is connected with a connecting rod.

6. Crank Case It is the housing of the crank and body of the engine to which cylinder and other engine parts are fastened. It also acts as a ground for lubricating oil.

7. Connecting Rod It is a link between the piston and crank. It is connected its one end with a crank and on the other end with a piston. It transmits power developed on the piston to a crank shaft through crank. It is usually made of medium carbon steel.

8. Crank Shaft It is the shaft, a rotating member, which connects the crank. The power developed by

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the engine is transmitted outside through this shaft. It is made of medium carbon or alloy steels.

9. Cooling Fins or Cooling Water Jackets During combustion, the engine releases a large amount of heat. Thus the engine parts may be subjected to a temperature at which engine parts may not sustain their properties such as hardness, etc. In order to keep the engine parts within safe temperature limits, the cylinder and the cylinder head are provided with a cooling arrangement. The cooling fins are provided on light duty engines, while a cooling water jacket is provided on medium and heavy duty engines.

10. Cam Shaft It is provided on four-stroke engines. It carries two cams, for controlling the opening and closing of inlet and exhaust valves.

11. Inlet Valve This valve controls the admission of charge into the engine during a suction stroke.

12. Exhaust Valve The removal of exhausted gases after doing work on the piston is controlled by the valve.

13. Inlet Manifold It is the passage which carries the charge from carburettor to the engine.

14. Exhaust mainfold It is the passage which carries the exhaust gases from the exhaust valve to the atmosphere.

15. **(*a*)** ***Spark Plug*** It is provided on petrol engines. It produces a high-intensity spark which initiates the combustion process of the charge.

(*b*) ***Fuel Injector*** It is provided on Diesel engines. The Diesel fuel is injected in the cylinder at the end of the compression through a fuel injector under very high pressure.

16. **(*a*)** ***Carburettor*** It is provided with a petrol engine for preparation of a homogeneous mixture of air and fuel (petrol). This mixture, as a charge, is supplied to engine cylinder through suction valve or port.

(*b*) ***Fuel Pump*** It is provided with a Diesel engine. The diesel is taken from the fuel tank, its pressure is raised in the fuel pump and then it is delivered to fuel injector.

17. Fly wheel It is mounted on the crank shaft and is made of cast iron. It stores energy in the form of inertia, when energy is in excess and it gives back energy when it is in deficit. In other words, it minimizes the speed fluctuations on the engine.

24.3 OTTO CYCLE ENGINES: PETROL ENGINES

The ordinary Otto-cycle engine is a four-stroke engine; that is, its piston makes four strokes, two toward the cylinder head (TDC) and two away from the head (BDC). By suitable design, it is possible to operate an Otto-cycle as a two-stroke cycle engine with one power stroke in every revolution of the engine. Thus, the power of a two-stroke cycle engine is theoretically double that of a four-stroke cycle engine of comparable size. These engines are also called *spark ignition engines.*

24.3.1 Two-stroke Petrol Engine

All essential operations are carried out in one revolution of the crank shaft or two strokes of the piston. Therefore, the engine is called a two-stroke or two-stroke cycle engine.

(*a*) *Construction Details*

A two-stroke petrol engine is shown in Fig. 24.2. It consists of a cylinder, cylinder head, piston, piston rings, connecting rod, crank, crank case, crank shaft, etc. The charge (air–fuel mixture) is prepared outside the cylinder in the carburettor. In the simplest type of two-stroke engine, the ports are provided for charge inlet and exhaust outlet, which are uncovered and closed by the moving piston. The suction port *S* with a reed-type valve is used for

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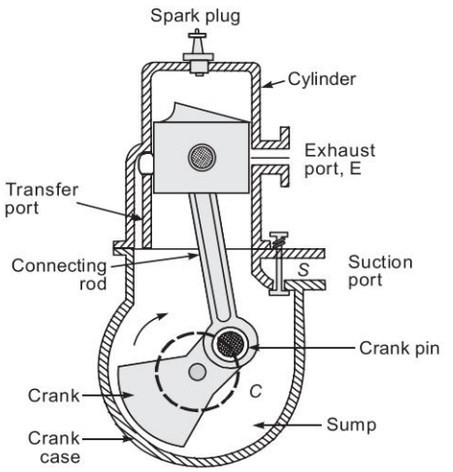


Fig. 24.2 *Two stroke petrol engine*

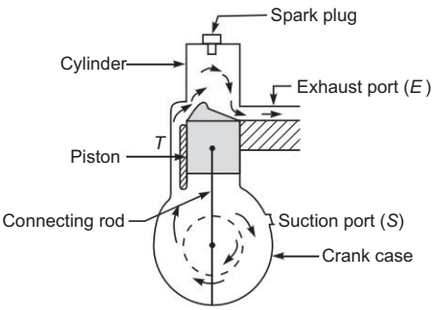
induction of charge into the crank case, the transfer port *T* is used for transfer of charge from the crank case to the cylinder and the exhaust port *E* serves the purpose of discharging the burnt gases from the cylinder. The spark plug is located in the cylinder head.

(b) Operation

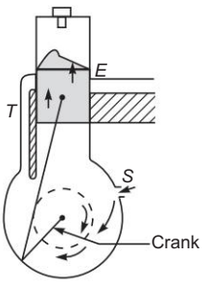
In a two-stroke engine, the inlet, transfer and exhaust ports are covered and uncovered by a moving piston. The following operations take place in a two-stroke or in one cycle of the engine.

(*i*) *Charge Transfer and Scavenging* When the piston is nearer to the crank case (bottom dead centre), the transfer port and exhaust port are uncovered by the piston as shown in Fig. 24.3(a). A mixture of air and fuel as a charge, slightly compressed in the crank case, enters through the transfer port *T* and drives out the burnt gases of the previous cycle through the exhaust port *E*.

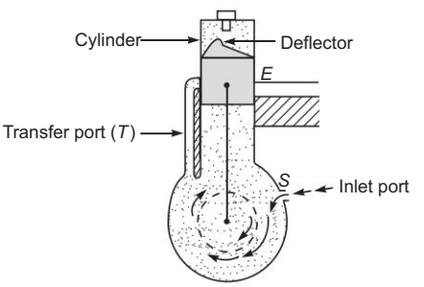
In a two-stroke engine, the piston top is made deflected. Therefore, the incoming charge is



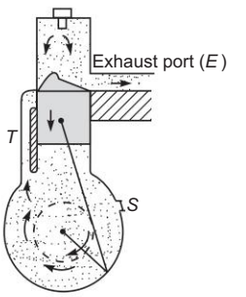
(a) Charge transfer and scavenging



(b) Start of compression



(c) Compression and Suction



(d) Power and exhaust

Fig. 24.3 *Operations of a two-stroke petrol engine*

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directed upward, and aids in sweeping of the burnt gases out of the cylinder. This operation is known as *scavenging* (a gas-exchange process).

As the piston moves upward, the fresh charge passes into the cylinder for $1/6^{th}$ of the revolution and the exhaust port remains open a little longer than the transfer port.

(*ii*) *Compression and Suction* As the piston moves upward, both the transfer port and exhaust port are covered by the piston and the charge trapped in the cylinder is compressed by the piston's upward movement as shown in Fig. 24.3(b). At the same time, a partial vacuum is created into the crank case, the suction port *S* opens by moving the crank and the fresh charge enters the crank case [Fig. 24.3 (c)].

(*iii*) *Combustion* When the piston reaches at its end of stroke nearer to the cylinder head or at the top dead centre, a high-intensity spark from the spark plug ignites the charge and initiates the combustion in the cylinder. The burning of the charge generates the pressure in the cylinder.

(*iv*) *Power and Exhaust* The burning gases apply pressure on the top of the piston, and the piston is forced downward as a result of pressure generated.

As the piston descends through about 80% of the expansion stroke, the exhaust port *E* is uncovered by the piston, and the combustion gases leave the cylinder by pressure difference and at the same time, the underside of the piston causes compression of charge taken into crank case as shown in Fig. 24.3(d).

(*v*) *Charging* The slightly compressed charge in the crank case passes through the transfer port and enters the cylinder as soon as it is uncovered by the descending piston and when it approaches the bottom dead centre, the cycle is completed.

The *p-V* diagram and port-timing diagram for a two-stroke petrol engine are shown in Figs. 24.4 and 24.5.

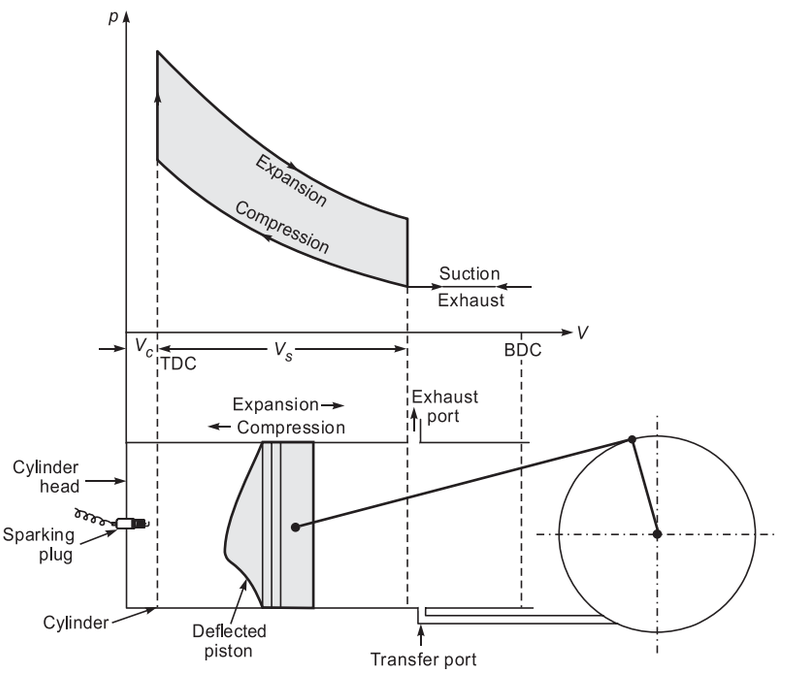

Fig. 24.4 *p–V diagram and schematic of a two-stroke petrol engine*

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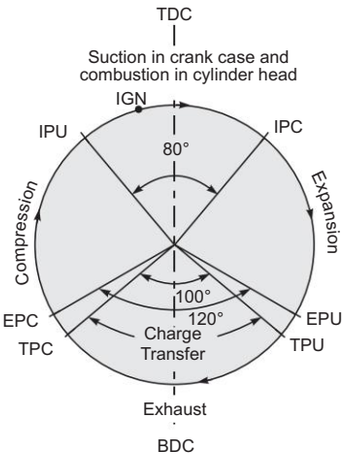
(*c*) *Port-Timing Diagram*



IPU = Inlet port uncovered 40° before TDC
IPC = Inlet port closed 40° after TDC
EPU = Exhaust port uncovered 60° before BDC
EPC = Exhaust port closed 60° after BDC
TPU = Transfer port uncovered 50° before BDC
TPC = Transfer port closed 50° after BDC
IGN = Spark ignition 15-20º before TDC

Fig. 24.5 *Port-timing diagram for a two-stroke petrol engine*

(*d*) *Applications*

Two-stroke gasoline engines are used where simplicity and low cost are main considerations. These engines have a little higher specific fuel consumption.

1. 50 cc–70 cc engines are used in mopeds, lawn moovers and non-gear vehicles.
2. 100–150 cc engines are commonly used in scooters and motor cycles.
3. 250 cc two-stroke engines are used in high-powered (racing) motor cycles.
4. These engines can also be used in small electric generator sets, pumping sets and motor boats.

24.3.2 Four-Stroke Cycle Petrol Engine

All operations are carried out in four strokes of the piston, i.e., two revolutions of the crank shaft. Therefore, the engine is called a four stroke engine.

(*a*) *Constructional Details*

Similar to a two-stroke engine, it also consists of a cylinder, cylinder head attached with spark plug, piston attached with piston ring, connecting rod, crank, crank shaft, etc., as shown in Fig. 24.6. In a four-stroke engine, valves are used instead of ports. There are suction and exhaust valves. These valves are operated by cams attached on a separate shaft, called a *cam shaft*. It is rotated at half the speed of a crank shaft.

(*b*) *Operation*

The travel of the piston from one dead centre to another is called *piston stroke* and a four-

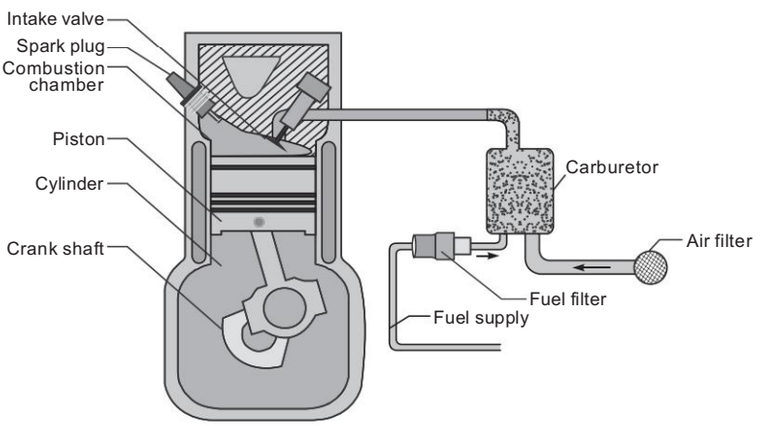


Fig. 24.6 *Four-stroke petrol engine*

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stroke cycle consists of four strokes as suction, compression, expansion and exhaust strokes.

1. Suction Stroke The suction valve opens, exhaust valve remains closed as shown in Fig. 24.7(a). The piston moves from the top dead centre to the bottom dead centre, the *charge* (mixture of fuel and air prepared in the carburettor) is drawn into the cylinder.

2. Compression Stroke When the piston moves from the bottom dead centre to top dead centre, and the suction valve is closed, exhaust valve remains closed as shown in Fig. 24.7(b). The trapped charge in the cylinder is compressed by the upward moving piston. As the piston approaches the top dead centre, the compression stroke completes.

3. Expansion Stroke At the end of the compression stroke, the compressed charge is ignited by a high-intensity spark created by a spark plug, combustion starts and the high-pressure burning gases force the piston downward as shown in Fig. 24.7(c). The gas pressure performs work, therefore, it is also called *working stroke* or *power stroke*. When the piston approaches the bottom dead centre in its downward stroke then this stroke is completed. In this stroke, both valves remain closed.

4. Exhaust Stroke When the piston moves from the bottom dead centre to the top dead centre, only the exhaust valve opens and burnt gases are expelled to surroundings by upward movement of the piston as shown in Fig. 24.7(d). This stroke is completed when the piston approaches the top dead centre. Thus, one cycle of a four stroke petrol engine is completed. The next cycle begins with piston movement from the top dead centre to the bottom dead centre.

Figure 24.8 shows the *p–V* diagram with a schematic of a four-stroke petrol engine.

(*c*) *Valve Timing*

Theoretically, in a four-stroke cycle engine, the inlet and exhaust valves open and close at dead centres as shown in Fig. 24.9(a).

A typical valve-timing diagram for a four-stroke petrol engine is shown in Fig. 24.9(b). The angular positions in terms of crank angle with respect to TDC and BDC position of piston are quoted on the diagram.

When the inlet valve and exhaust valve remain open simultaneously, it is called a *valve operlap*.

(*d*) *Applications*

These engines are mostly used on automobiles, motor cycles, cars, buses, trucks, aeroplanes, small pumping sets, mobile electric generators, etc.

Nowadays, the four-stroke petrol engines have been replaced by four-stroke Diesel engines for most applications.

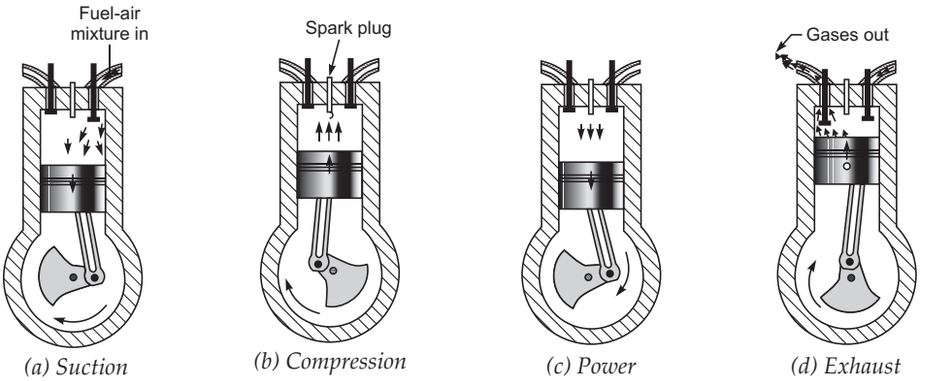

Fig. 24.7 *Operations of a four-stroke petrol engine*

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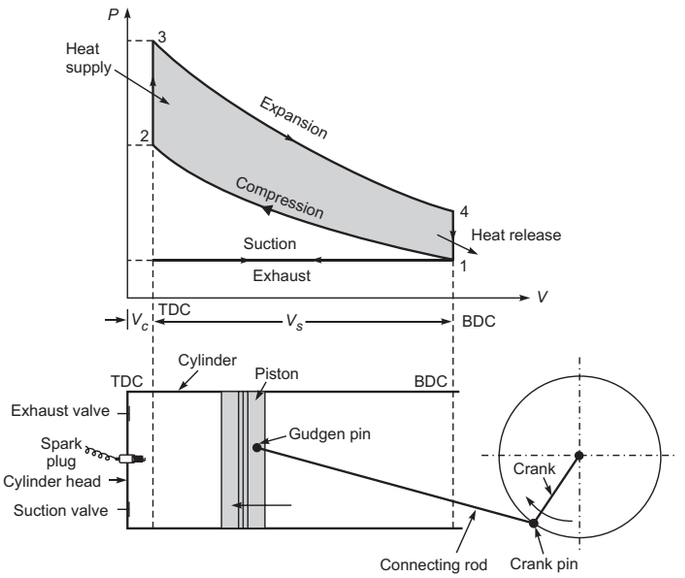
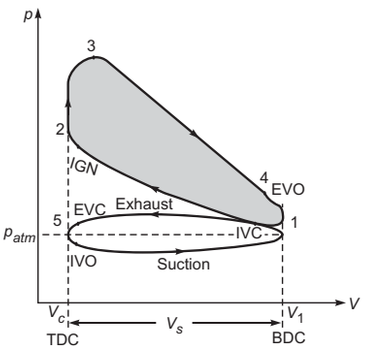
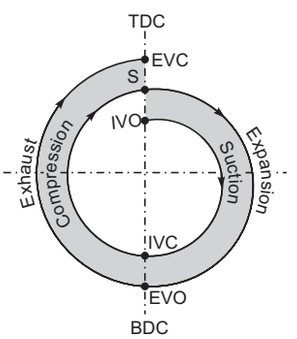


(a) Theoretical p–V diagram for a four-stroke petrol engine



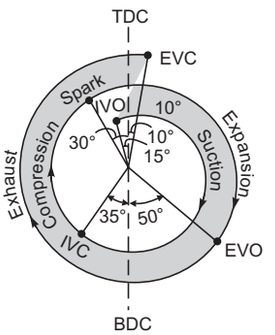
(b) Actual p–V diagram for four stroke petrol engine

Fig. 24.8



(a) Theoretical valve timing diagram of a four stroke cycle

IVO = Inlet valve opens when piston at TDC
IVC = Inlet valve closes, when piston reaches BDC
S = Spark produces, when piston reaches TDC
EVO = Exhaust valve opens when piston at BDC
EVC = Exhaust valve closes, when piston at TDC



(b) The typical valve timing diagram for a four stroke petrol engine

IVO : Inlet valve opens about 15° before TDC
IVC : Inlet valve closes 20° – 40° after BDC to take advantage of rapidly moving gas
S : Spark occurs 20°–40° before TDC
EVO : Exhaust valve opens about 50° before BDC
EVC : Exhaust valve close about 0° to 10° after TDC

Fig. 24.9

24.4 DIESEL ENGINES

All engines using diesel as a fuel operate on the Diesel cycle. They work similar to a petrol engine except they take in only air as charge during suction, and fuel is injected at the end of the compression stroke. The Diesel engines have a fuel injector instead of a spark plug in the cylinder head

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as shown in Fig. 24.10. The diesel engines use a high compression ratio in the range of 14 to 21. The temperature of intake air reaches quite a high value at the end of compression. Therefore, the injected fuel is self ignited. The Diesel engines use a hetrogeneous air–fuel mixture, ratio ranging from 20 to 60.

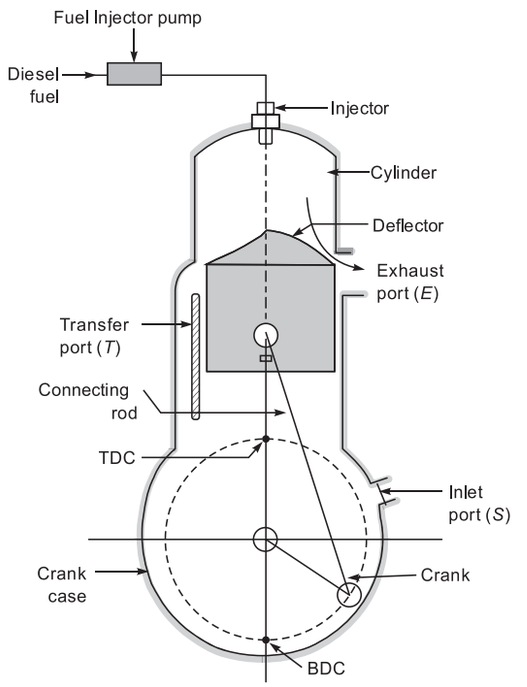

Fig. 24.10 *Schematic of two-stroke Diesel engine*

24.4.1 Two-stroke Diesel Engine

The operation of a two-stroke Diesel engine is similar to a petrol engine, except it takes air as charge and fuel is injected at the end of the compression stroke. It uses a high compression ratio. Therefore, the injected fuel is self-ignited.

Operation

Both inlet and exhaust take place through the cylinder ports which are covered and uncovered by the piston.

(*i*) *Charge Transfer and Scavenging* When the piston is nearer to the crank case (bottom dead centre), the transfer port and exhaust port are uncovered by the piston and the slightly compressed air enters into the cylinder through the transfer port and helps to scavenge the remaining burnt gases from the cylinder as shown in Fig. 24.11(a). The charge transfer and scavenging continue till the piston completes its downward stroke and further, it moves upward and covers the transfer port.

(*ii*) *Compression and Suction* After covering the transfer port, the exhaust port is also covered by the upward moving piston. As both ports are covered by the piston in Fig. 24.11(b), the air trapped in the cylinder is compressed during the forward stroke of the piston. As the piston moves towards the cylinder head, a partial vacuum is created in the crank case, the inlet port opens and fresh air enters the crank case, Fig. 24.11(c).

(*iii*) *Combustion and Power* Near the end of the compression stroke, the fuel is injected at a very high pressure with the help of the fuel pump and injector. The injected fuel is self ignited in the presence of hot air and combustion starts. The piston is forced downward by very high pressure of burnt gases and power is transmitted to the crank shaft.

(*iv*) *Exhaust* Near the end of the power stroke, the exhaust port is uncovered first by the piston and the products of combustion start leaving the cylinder as a result of pressure difference as shown in Fig. 24.11(d).

(*v*) *Charging* The slightly compressed air in the crank case passes through the transfer port and enters the cylinder as soon as it is uncovered by the descending piston and when it approaches the bottom dead centres, the cycle is completed.

Theoretical p–V Diagram

The theoretical *p–V* digaram for a two-stroke Diesel engine is shown in Fig. 24.12. The valve timing diagram for a two-stroke Diesel engine is very similar to that of a two-stroke petrol engine as shown in Fig. 24.9 except, only air is inducted in the crank case and fuel is injected at the end of

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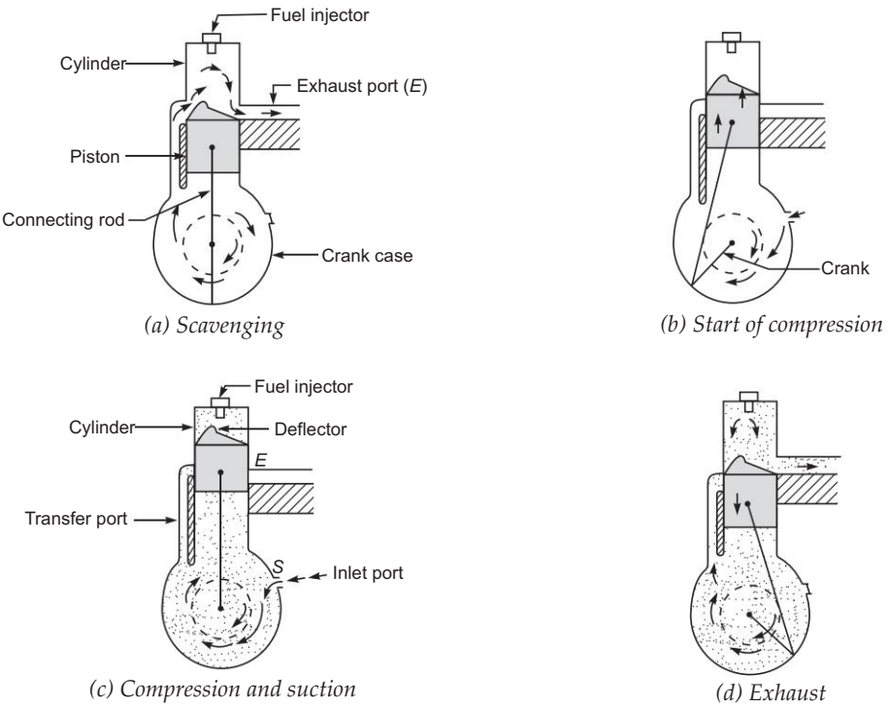

Fig. 24.11 *Operations of a two-stroke Diesel engine*

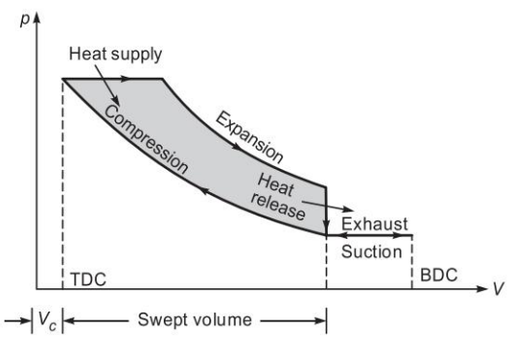

Fig. 24.12 *p–V diagram for two-stroke Diesel engine*

the compression stroke, instead of spark from the spark plug.

24.4.2 Four-stroke Diesel Engine

A four-stroke Diesel engine contains a fuel injector, fuel pump, cylinder, cylinder head, inlet and exhaust valves, piston attached with piston rings, connecting rod, crank shaft, cams, camshaft, etc. shown in Fig. 24.13. One cycle of a four-stroke Diesel engine is completed in four strokes of the piston or two revolutions of the crank shaft.

(a) Working of Engine

The four-stroke Diesel engine operates in a similar manner as a four-stroke petrol engine. A schematic of a four-stroke Diesel engine is shown in Fig. 24.13. The details of operations are discussed below.

1. Suction Stroke The inlet (suction) valve opens, the exhaust valve remains closed, only air is drawn into the cylinder as the piston moves from the top dead centre to the bottom dead centre. This stroke ends as the piston approaches the bottom dead centre. Fig. 24.14(a)

2. Compression Stroke As the piston moves from the bottom dead centre to the top dead centre, the inlet valve closes, exhaust valve remains closed as shown in Fig. 24.14(b). The air trapped into the

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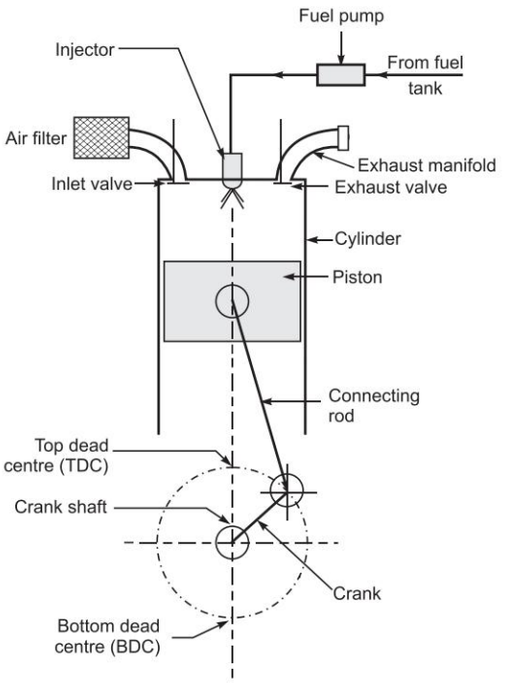


Fig. 24.13 *Schematic of a four-stroke Diesel engine*

cylinder is compressed in the cylinder till the piston approaches the top dead centre. The air temperature reaches about 800°C by compression. At the end of the compression stroke, the fuel is injected at very high pressure into the compressed hot air. The temperature of hot compressed air is sufficient to ignite the injected fuel. Thus, ignition takes place inside the cylinder.

3. Expansion Stroke During this stroke, both valves remain closed as shown in Fig. 24.13(c). The piston at the top dead centre is pushed by expansion of burning gases. Actual work is obtained during this stroke due to the force obtained by high pressure burning gases. Therefore, this stroke is called *power stroke or working stroke*.

4. Exhaust Stroke During this stroke, the piston moves from the bottom dead centre to the top dead centre, exhaust valve opens and the inlet valve remains closed. Burnt gases of the previous

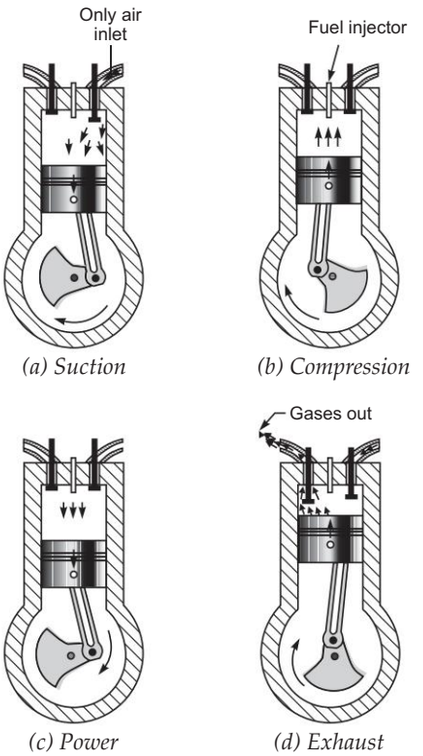


Fig. 24.14 *Operations of four-stroke diesel engine*

stroke are expelled out from the cylinder by upward movement of the piston.

The theoretical p–V diagram is shown in Fig. 24.15 for a four-stroke Diesel engine operation.

(*b*) *Valve-Timing Diagram*

Theoretically, the inlet and exhaust valves open at dead centres as shown in Fig. 24.9(a). A typical valve-timing diagram for a four-stroke Diesel engine is shown in Fig. 24.16.

(*c*) *Applications*

The four-stroke Diesel engine is one of the most popular prime movers. It is manufactured from 50 mm to 1000 mm cylinder bore with speeds ranging from 100 rpm to 4500 rpm. It has wide applications. Some of these are

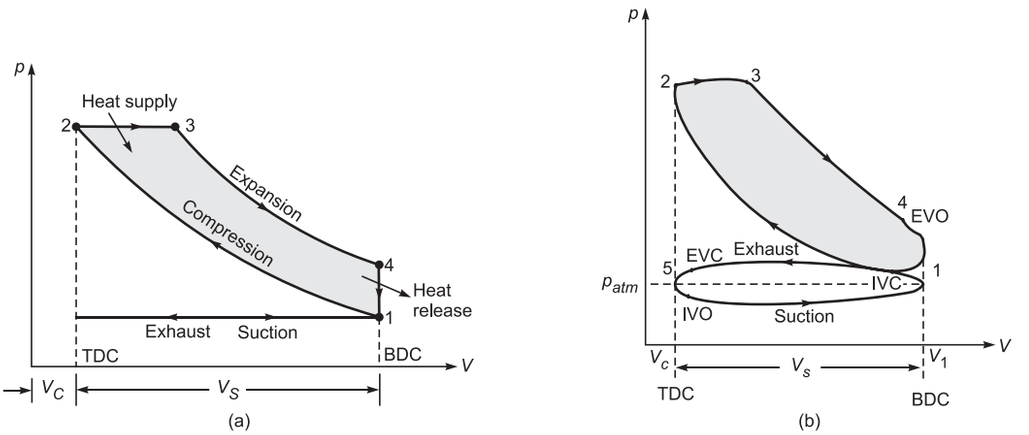
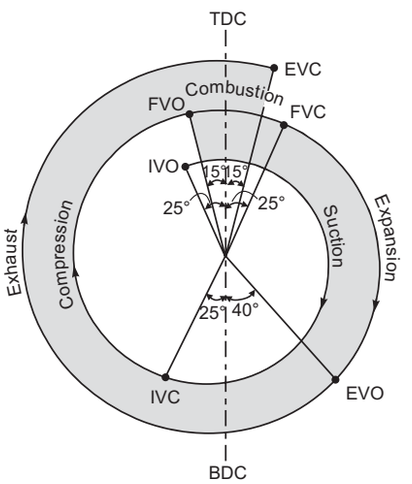


Fig. 24.15 *(a) Theoretical p-V diagram (b) Actual p-V diagram engine*



IVO : Inlet valve opens 10° to 30° before TDC
IVC : Inlet valve closes 20°–50° after BDC
EVO : Exhaust valve opens approximately 40° before BDC
EVC : Exhaust valve closes aproximately 15° after TDC
FVO : Fuel injection starts 5° to 15° before TDC
FVC : Fuel injection stops 15° to 25° after TDC

Fig. 24.16 *Typical valve-timing diagram for a four-stroke Diesel engine*

1. Small pumping sets for agriculture,
2. Construction machinery,
3. Air compressor and drilling jigs,
4. Tractors, jeeps, cars, taxies, buses, trucks,
5. Diesel-electric locomotives,
6. Small power plants, mobile electric generating plants,
7. Boats and ships,
8. Power saws, Bulldozers, tanks, etc.

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24.5 COMPARISON BETWEEN PETROL AND DIESEL ENGINES

According to	*Petrol Engine*	*Diesel Engine*
1. Basic cycle	It operates on constant-volume cycle.	It operates on constant-pressure cycle.
2. Fuel used	It uses gasolene or petrol as fuel.	It uses diesel and oils as a fuel.
3. Fuel induction	The air–fuel mixture is prepared in the carburettor and inducted into the engine cylinder during the suction stroke.	The Diesel engine takes in only air during the suction stroke, and it is compressed. At the end of the compression stroke the fuel is injected under the high pressure by a fuel injector.
4. Ignition of charge	The charge (air–fuel mixture) is ignited by a high-intensity spark produced at the spark plug.	Fuel is injected in very hot air, therefore, it is self-ignited.
5. Compression ratio	It uses less compression ratio, usually range of 4 to 10.	It uses high compression ratio, range of 14 to 21.
6. Pressure rise	Lower and controlled rate of pressure rise; therefore, operation is salient and smooth.	High rate of pressure variation, so engine operation is rough, and noisier.
7. Efficiency	For the same compression ratio, the efficiency of petrol engine is better.	It has lower efficiency for same compression ratio.
8. Pollution	Comparatively lower pollution for same power output.	Higher pollution for same power output.
9. Weight	It has comparatively less number of parts, thus is less in weight.	It uses large number of sturdier parts, thus engine is heavy.
10. Cost	Engines are cheaper.	Costlier engine due to complicated parts.
11. Maintenance	It requires less and cheaper maintenance.	It requires costlier and large maintainance.
12. Starting	Very easy to start due to lower compression ratio.	Very difficult to start due to higher compression ratio.

24.6 COMPARISON BETWEEN TWO-STROKE AND FOUR-STROKE ENGINES

According to	*Two-Stroke Engine*	*Four-Stroke Engine*
1. Working stroke	There is one working stroke in each revolution. Hence engine has more even torque and reduced vibration.	There is one working stroke in two revolutions. Hence engine has uneven torque and large vibration.
2. Engine design	It uses ports and hence engine design is simple.	It uses valves, therefore, mechanism involved is complex.
3. Mechanical efficiency	The working cycle completes in one revolution and hence it has high mechanical efficiency.	Working cycle completes in two revolution, hence, it has more friction, thus less mechanical efficiency.
4. Scavenging	The burnt gases are not completely driven out. It results in dilution of fresh charge.	It has separate stroke for explusion of burnt gases, thus ideally no dilution of fresh charge.

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5. Thermal efficiency	Poor thermal efficiency due to poor scavenging and escaping of charge with exhaust gases.	Very good thermal efficiency.
6. Cost	Less cost due to less parts in engine.	More cost due to large number of parts.
7. Maintenance	Cheaper and simple.	Costlier and slightly complex.
8. Weight	Lighter engine body.	Heavier engine body.

24.7 ADVANTAGES AND DISADVANTAGES OF TWO-STROKE CYCLE ENGINES

Advantages The two-stroke cycle engines have the following advantages over four-stroke cycle engine:

(i) The two-stroke engines are simple in design, manufacturing and operation. Hence they are used in small power engines such as scooters, mopeds, motor cycles and autorickshas, etc.

(ii) The two-stroke cycle engine develops net work as positive because it works above atmospheric pressure.

(iii) The turning moment of the two-stroke cycle engine is more uniform, thus a smaller flywheel is required.

(iv) There are less frictional losses due to absence of valves, thus higher mechanical efficiency is obtained in two-stroke cycle engines.

(v) All the operations in two-stroke cycle engines complete in two strokes, i.e., one revolution of the crank shaft, thus theoretically, a two-stroke cycle engine develops twice the power of the four-stroke cycle engine for the same size and same speed of the engine.

(vi) The two-stroke cycle engine is much lighter and more compact for same power output than a four-stroke cycle engine.

(vii) Due to less parts in a two-stroke cycle engine, the maintenance required is very less. Further, the maintenance cost is very low.

(viii) Actually, the gas dilution in a two-stroke cycle engine is comparatively less than the four-stroke cycle engine.

(ix) Engine cost is also less than the four-stroke cycle engine.

Disadvantages The two-stroke cycle engines have the following disadvantages over four-stroke cycle engines:

(i) The effective compression in a two-stroke cycle engine is lower for the same stroke.

(ii) Due to less effective cooling arrangement in two-stroke cycle engines, some amount of lubrication oil burns during the combustion of charge, thus more lubrication oil is consumed.

(iii) A two-stroke cycle engine is more noisy because of sudden release of exhaust gases in the port.

(iv) In a two-stroke cycle engine, the scavenging (gas exhange process) is poor. The fresh charge pushes the burnt gases out. In this process, some fresh charge always flows with the exhaust gases.

(v) More wear and tear take place in a two-stroke cycle engine.

(vi) Thermal efficiency of a two-stroke cycle engine is less due charge loss during scavenging.

24.8 AIR–FUEL MIXTURE

The *air–fuel ratio* is defined as the *ratio of mass of air to the mass of fuel*. The engines operating on

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different loads and speeds require different air–fuel ratios. The fuel and air are mixed to form the three types of mixtures.

(i) Chemically correct mixture (stoichiometric mixture)
(ii) Rich mixture
(iii) Lean mixture

Chemically correct or *stoichiometric air–fuel mixture* is one, which has just sufficient air for complete combustion of fuel. The mass of air required for complete combustion of 1 kg of particular fuel (A/F ratio) is computed from a chemical equation. This computed value for most of the hydrocarbons is usually approximated to be 15.

A mixture which contains more air than the stoichiometric requirement is called a *lean mixture* (example A/F ratio of 17 : 1, 19 : 1, etc.)

A mixture which contains less air than the stoichiometric required air is called a *rich mixture*, example A/F ratio of 12 : 1, 10 : 1, etc.

A petrol engine can operate on A/F ratios in the range 10 : 1 to 20 : 1, but it does not perform satisfactorily at extreme ratios. The air–fuel ratio used in the engine has a considerable influence on the engine performance. The mixture corresponding to maximum power is called the *best power mixture* and its A/F ratio is approximately 13 : 1. The mixture corresponding to minimum fuel consumption per kWh of power is called the *best economy mixture* and its A/F ratio is approximately 16 : 1. Figure 24.17 shows a typical A/F mixture requirement by a carburetted engine at constant speed and full throttle opening.

A petrol engine requires a rich mixture for starting and idling (no load condition). When an engine operates at part loads of up to 75% of the designed load, the mixture required is slightly weak. But on acceleration and maximum load conditions, the engines requires a rich mixture (such as A/F ratio of 13).

In a CI engine, irrespective of load condition at a given speed, the engine sucks almost a constant quantity of air. According to load condition, the quantity of fuel is injected into the cylinder, thus A/F ratio varies from cycle to cycle. The overall A/F ratio that is used in a CI engine ranges from 80 : 1 at no load and 20 : 1 at full load. Actually, in a CI engine, the mixture is heterogeneous with different A/F ratios in different areas of the combustion chamber. There may be some pockets where there is only air or fuel, having rich or lean mixture. But there are always certain areas where the A/F ratio is within the combustible limit and combustion starts from such areas and spreads over the combustion chamber.

For a CI engine, the amount of fuel required varies directly with load on the engine, thus the curve of *bsfc* remains almost horizontal with brake power as shown in Fig. 24.18. Figure 24.19 shows the gross fuel consumption against brake power. This line in the graph is called *Willan's line* and it indicates that the fuel consumption of a CI engine increases as load increases.

Fuel System The fuel system provides the correct air–fuel mixture in the cylinder for efficient

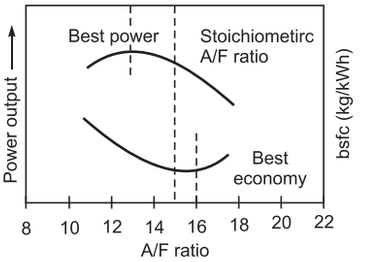

Fig. 24.17 *Variation of power output and bsfc with A/F ratio for an S.I. engine*

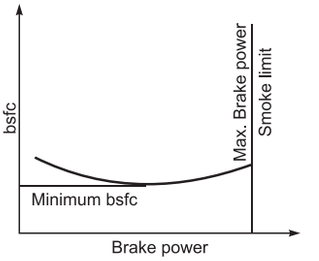

Fig. 24.18 *The bsfc against BP for CI engine*

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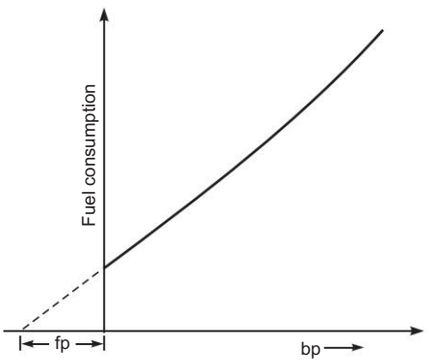

Fig. 24.19 *Fuel consumption against brake power for a CI engine*

burning. There are basically two methods available; one is called *carburation* and is particularly used for petrol engines, and the other is *fuel injection* used for Diesel engines. There are various designs of each system, but only the basic principle will be discussed in the chapter.

24.9 CARBURATION

The process of preparation of combustible mixture by mixing the proper amount of fuel with air before admission to the engine cylinder is called *carburation*. A device which atomises the fuel and mixes it with air and prepares the charge for Otto cycle engine is called carburetter.

When the piston descends in the cylinder, it creates vacuum, the charge is drawn through the carburetter into the cylinder through intake valve and induction manifold.

24.9.1 Factors Affecting Carburation

The factors affecting the carburation process are

(i) the engine speed

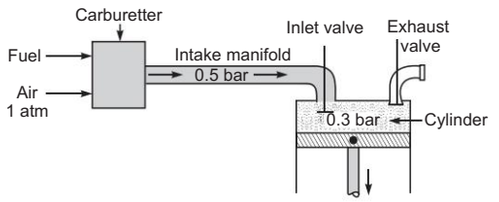

Fig. 24.20 *Schematic of carburation system*

(ii) the vapourisation characteristics of fuel
(iii) the temperature of the incoming air, and
(iv) the carburetter design.

24.9.2 Simple Carburetter

The carburetters are used on most SI engines for preparation of combustible air–fuel mixture as a charge. Figure 24.21 shows a simple carburetter, which provides an air–fuel mixture for normal range at a single speed. It consists of a float chamber, fuel discharge nozzle and metering orifice, a venturi, a throttle valve and a chock valve.

How it Works The basic carburetter as shown in Fig. 24.21 is built around a hollow tube called a *throat*. The downward motion of the piston creates a partial vacuum inside the cylinder that draws air into the carburetter's throat and past a nozzle that sprays fuel. The mixture of air and fuel produced inside the carburetter is delivered to the cylinder for combustion.

Throttle Valve A throttle valve at the base of the carburetter controls the amount of charge sucked through the engine by the partial vacuum in the cylinder. The driver opens the throttle valve by pressing down the accelerator. When throttle valve opens wider, more air flows through the carburetter, delivering larger amounts of fuel to the engine. The driver can regulate or close the opening of the throttle valve by decreasing pressure on the accelerator.

Venturi The *venturi* is a gradually decreasing cross-sectional hollow tube. The venturi is the narrowing of the carburettor's throat. Air rushing through the narrow part speeds up. At the same time, air pressure against the sides of the passage-way decreases, creating a partial vacuum inside the throat. This partial vacuum draws fuel through the nozzle and into the air.

Float Chamber The fuel that enters the carburetter is stored in a reservoir called a *float chamber* or *float bowl*. A float and needle valve system maintains a constant level of gasolene in the float chamber.

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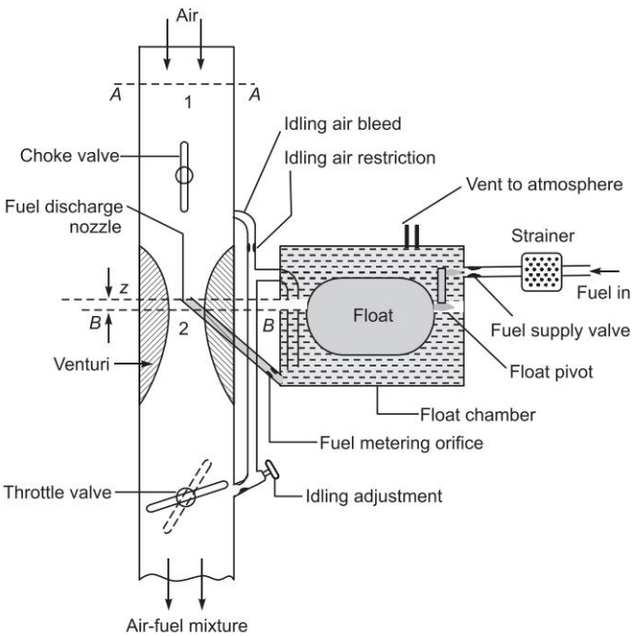

Fig. 24.21 *Simple Carburetter*

The float floats on the gasolene surface and closes the needle valve. If the level of fuel in the float chamber falls below the designed level, the float goes down, thereby opening the fuel supply valve, and fuel enters the float chamber. As fuel reaches the designed level, the needle closes the fuel supply valve. The fuel level in the float chamber is maintained slightly below the tip of the discharge jet of the nozzle in order to avoid overflow through the jet, when the carburetter is not in operation.

Idle and Transfer Ports In addition to the main nozzle in the venturi portion of the carburetter, two other nozzles, or ports, deliver fuel to the engine. The idle port is located below the venturi and allows the engine to get fuel when air flow through the carburetter is minimum, such as when the engine is idling at a low speed. Fuel from the idle port is drawn into the cylinder by the engine vacuum. The idle port supplies enough fuel to keep the engine running at slow speeds. Fuel from the main nozzle is necessary to run the engine at normal operating speeds.

Choke and Cold Starts The choke is a device that can partially block air from entering into the carburetter. If the throttle valve is open and the choke valve is closed, the vacuum from the engine is strong enough inside the carburetter to draw more fuel from all the nozzles. This added fuel produces a rich air–fuel mixture to help a cold engine get started. Once the engine warms–up, the choke is shut off.

Air–fuel Ratio A carburetter can be adjusted to mix larger or smaller amounts of air with the fuel. An idling engine at normal operating temperature requires an air-to-fuel ratio of about 15 : 1 (by weight) to completely burn the fuel. Raising or lowering the air flow makes the mixture either lean (containing less fuel) or rich (containing more fuel). A lean mixture produces a cleaner, hotter combustion for normal speeds, but not enough fuel for starting the engine efficiently or allowing it to produce more power. A rich mixture burns easily in the engine but produces more pollutants as by-products.

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The carburetter is adjusted to provide a rich mixture for cold engine starts because the rich mixture burns easier and longer. As the engine warms up, the carburetter alters the air–fuel ratio for a leaner mixture.

24.9.3 Limitations of a Simple Carburetter

A simple carburetter discussed above suffers from the following drawbacks.

1. It can only provide required A/F ratio at one throttle position at constant speed. As speed or throttle position changes, the mixture is either richer or leaner.
2. At a wider open throttle, the air flow rate increases at the venturi throat, the air density and the pressure of air decrease, while the fuel density remains constant. Thus, a simple carburetter provides a progressively rich mixture with increasing throttle opening or speed of engine.
3. It fails to supply a richer mixture at the time of engine start.
4. It is not able to supply a rich mixture during idling.
5. It fails to supply a rich mixture during acceleration and overload.

24.10 FUEL-INJECTION SYSTEM

The fuel-injection system is the most important component in the working of a CI engines. The engine performance, i.e., power output, fuel economy, etc., depend on the effectiveness of a fuel injection system. A typical fuel injection system functions in the following ways.

1. It measures the correct quantity of fuel to be injected per cycle.
2. It injects fuel in the cylinder at the correct time.
3. It controls the rate of fuel injection.
4. It atomizes the fuel into very fine droplet-like spray.
5. It gives a proper spray pattern to the fuel droplets in order to mix it into air in a short period.
6. It supplies equal quantity of fuel to all cylinders in case of a multicylinder engine.

24.10.1 Classification of Fuel-injection Systems

In CI engines, two methods of fuel injection are used:

(a) Air injection system, and
(b) Solid injection system.

(*a*) *Air-injection System* This system is also called *indirect injection* (IDI) system. In this method, the fuel is injected into the cylinder by means of compressed air. The air is compressed in a compressor to a pressure higher than that at the end of the compression stroke. Then air and fuel are injected through the fuel nozzle into the engine cylinder. The advantages and disadvantages of air injection systems are given below.

Advantages

1. It provides better atomisation and distribution of fuel.
2. Inferior-quality fuel can also burn efficiently.
3. The charge burns completely and the engine gives better thermal efficiency.
4. The chance of choking of the fuel valve is negligible due to compressed air supply.

Disadvantages

The air-injection system is little used nowadays due to the following reasons:

1. It requires multistage compression for high pressure of air. The large number of parts, the intercoolers, etc., make the system heavy, complicated and expensive.
2. Separate mechanical linkage is required for operation of fuel valve and compressor.
3. Due to large number of moving parts, the mechanical efficiency of the engine reduces.

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4. More space is required for the engine.
5. Charge burns in the combustion chamber very near to the injection nozzle, which may lead to overheating and burning of the valve and its seat.

(*b*) *Solid-injection System* It is also called *airless injection system or direct injection* (DI) system. In this system, the liquid fuel is directly injected into the combustion chamber without use of compressed air. It is also called *mechanical injection*.

Advantages

1. It is compact and simple in construction.
2. It does not require compressed air.
3. It has better control on the quantity of fuel to be injected.

Disadvantages

1. It requires very high accuracy in the fuel injector and fuel pump.
2. With this system, inferior quality of fuel cannot be injected.
3. The prepared charge is more heterogenous.

24.10.2 Main Components of Solid-injection Systems

The fuel-injection system consists of mainly the following components.

(i) Fuel tank
(ii) Fuel feed pump to supply fuel from the fuel tank to the injector
(iii) Fuel filter to separate dust and dirt in the fuel
(iv) Fuel-injection pump to meter and pressurise the fuel for injection
(v) Governor to ensure the correct quantity of fuel according to load and speed
(vi) Fuel piping and injectors to take the fuel from the pump and distribute in the combustion chamber by atomizing it into fine droplets

A typical arrangement of various components for a solid fuel-injection system used in Diesel engines is shown in Fig. 24.22.

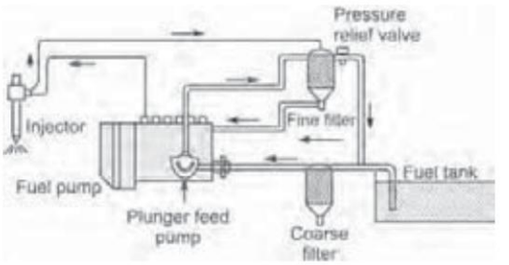

Fig. 24.22 *Typical fuel feed system for a CI engine*

The fuel-injection systems for Diesel engines employ a high-pressure fuel pump, which increases the pressure of the fuel to about 120 to 200 bar and this fuel is injected through the nozzle(s) into the hot air present into the combustion chamber at the end of the compression stroke.

24.10.3 Simple Fuel-Injection Pump

The basic principle of a fuel-injection system can be understood with the help of Fig. 24.23. It consists of a spring loaded, plunger-type pump. The plunger is activated through a push rod from the cam shaft.

When the follower on the push rod is at the minimum lift position of the cam, the spring forces the plunger for its lowest position. Thus a suction is created in the barrel and the fuel from the main tank flows into the barrel through the fuel filter. When the cam rotates and reaches its maximum lift, the

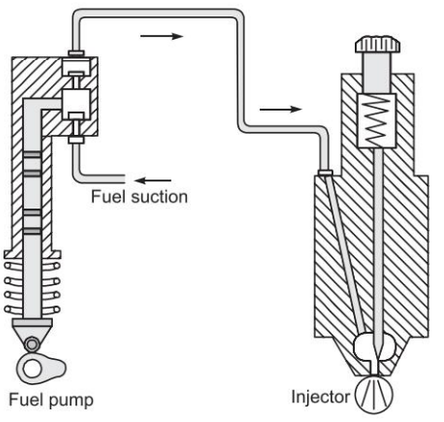

Fig. 24.23 *Schematic diagram of a fuel feed pump*

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plunger is lifted upwards, the inlet valve closes and the fuel is forced through the delivery valve.

When the fuel-operating pressure is reached, the fuel from the injector is injected into the cylinder. The spring pressure above the valve rod in the injector is used to set the fuel injection pressure.

24.10.4 Jerk-type Fuel-Injection Pump

Nowadays, the '*jerk pump*' fuel-injection system is universally used over the whole range of CI engines. The jerk pump as shown in Fig. 24.24, is a precision equipment and consists of plunger in a barrel, a very close fit. A cam gives vertical movement to the plunger, while a rack controls its angular movement in the barrel. The plunger is provided with a helical groove and the barrel has a supply port, spill port and a spring-loaded delivery valve.

This system functions in the following way:

1. It uses a variable stroke of the plunger.
2. It measures the correct quantity of fuel at the beginning of the plunger stroke and spilling back the excess fuel.
3. The axial distance travelled by the plunger in each stroke is the same. The angular rotation of the plunger by rack decides the length of effective stroke for fuel injection.
4. Using the plunger stroke, the rack brings the end of the fuel delivery by suddenly spilling off the fuel from the cylinder.

The upper part of the helical groove in the plunger controls the uncovering of the spill port. The timing of the opening of spill port by the helix is thus decided by the angular movement of the plunger. The plunger is rotated by the rack, which is moved in or out by the governor. By changing the angular position of the helical groove in the plunger, the length of the stroke, during which fuel is delivered, can be varied and thereby the quantity of fuel to be delivered to the cylinder is also varied accordingly.

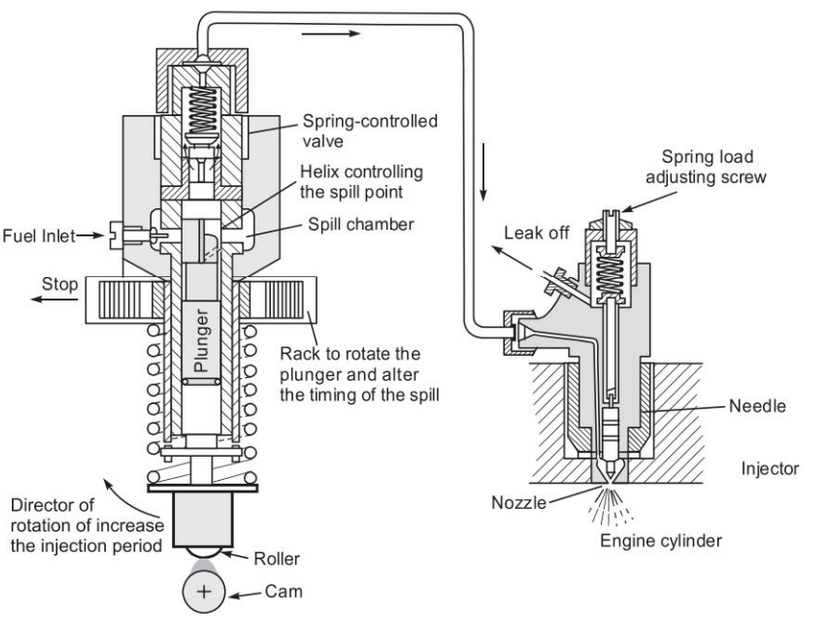

Fig. 24.24 *Fuel injector pump and fuel injector for a CI engine*

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The fuel at high pressure passes to the spring loaded injector, where the needle is set to lift at a predetermined pressure in the delivery line.

Operation of Fuel Pump The operation of a fuel pump as plunger undergoes a stroke is illustrated with the help of Fig. 24.25.

Fig. 24.25 (a) and (b)

1. The plunger is at its down stroke.
2. The fuel inlet port *F* is uncovered and the fuel enters the barrel above the plunger and into the recesses in the plunger, Fig. 24.25(a).
3. As the plunger rises up, it closes the port *F*
4. Fuel flows out through the spill port *S*, because recesses in the plunger align with the port *S* as shown in Fig. 24.25(b).
5. A spring force acts on the check valve and it does not open.
6. The effective stroke ends and no fuel is delivered to the injector.

Fig. 24.25 (c) and (d), For full load on the engine

1. During the upward stroke of the plunger, the rack rotates the plunger in such a way that the fuel inlet port and spill port, both are closed.

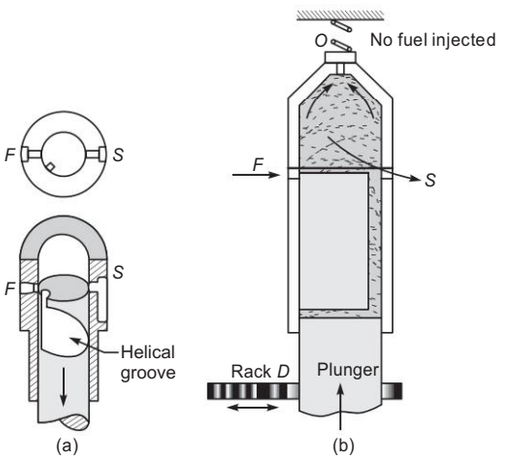

Fig. 24.25 *(a) and (b) fuel pump operation for no load*

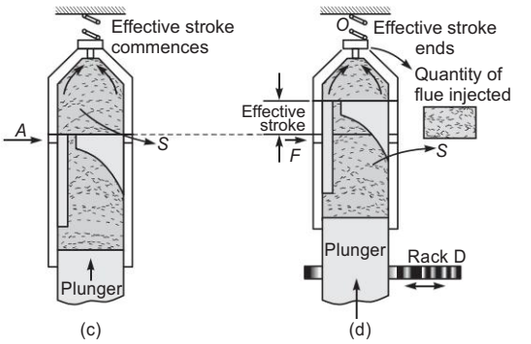

Fig. 24.25 *(c) and (d) for load on the engine*

2. The trapped fuel in the barrel is now forced to pass the check valve through the orifice *O* to the injector as shown in Fig. 24.25(c).
3. The injection of fuel starts.
4. The injection continues during the upward stroke of the plunger till the helical recess on the plunger uncovers the spill port *S*.
5. The check valve closes the orifice, the fuel injection stops. The effective stroke of the plunger ends as shown in Fig. 24.25(d).
6. The remaining fuel escapes through the spill port.

Fuel Pump Operation for Partial Load

1. During upward stroke, the plunger is rotated by the rack to the position shown in Fig. 24.25(e).
2. The trapped fuel is forced to pass the check valve through the orifice *O*.

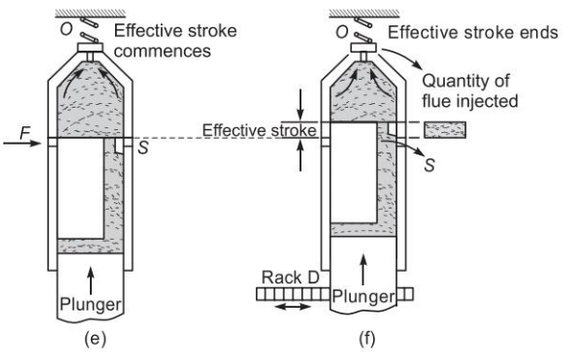

Fig.24.25 *(e) and (f) fuel pump operation for partial load*

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3. The fuel injection starts.
4. But for partial load on the engine, the spill port *S* is uncovered sooner as shown in Fig. 24.25(f).
5. The effective stroke is shortened.
6. The quantity of fuel injected is small.

It is to remember that the axial distance travelled by the plunger in each stroke is the same. The angular rotation of the plunger by the rack decides the length of the effective stoke.

24.10.5 Fuel Injector

The fuel pump in its effective stroke supplies the fuel to the fuel injector. The fuel injector delivers the fuel under pressure into the combustion chamber. The fuel injector serves to fulfil the following tasks:

1. It atomizes the fuel into fine droplets.
2. It distributes the fuel uniformly by proper spray pattern.
3. It prevents the injection of fuel on the cylinder walls and piston top.
4. It controls the start and stop of fuel instantaneorusly.

A cross-sectional view of a typical fuel injector is shown in Fig. 24.26. The injector assembly consists of

(i) a needle valve
(ii) compression spring
(iii) a nozzle
(iv) adjusting screw with lock nut
(v) an injector body

The compression spring exerts the force on the nozzle valve through the spindle to close it. When the fuel is supplied under high pressure by a fuel pump, as it overcomes the spring force, the nozzle valve lifts and the fuel is sprayed into the combustion chamber in finely atomized particles. As the fuel pressure falls, the nozzle valve is pushed on its seat by a spring force. The amount of fuel injected is regulated by the duration of open period of the nozzle valve. The pressure of fuel injection can be adjusted by spring tension by means of an adjusting screw.

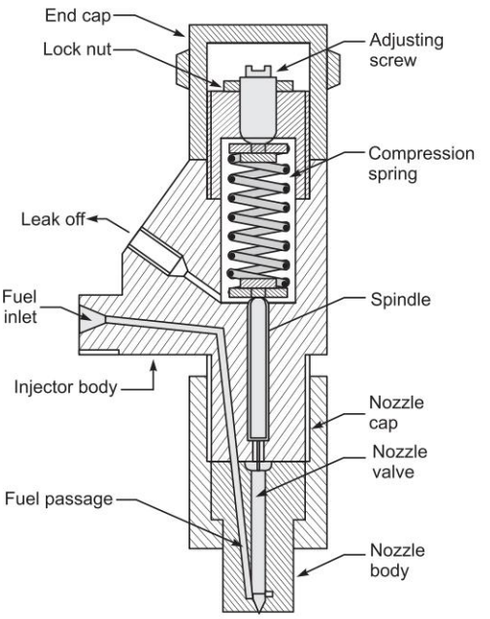

Fig. 24.26 *Fuel injector (Bosch)*

24.11 COMBUSTION

24.11.1 Combustion in SI Engines

The homogeneous mixture of fuel and air prepared in carburetter as charge is inducted in the spark ignition engine. This charge is compressed in the cylinder during compression stroke. At the end of the compression stroke, the combustion is initiated by a high intensity spark produced by an electric discharge. The typical stages of combustion are shown in Fig. 24.27.

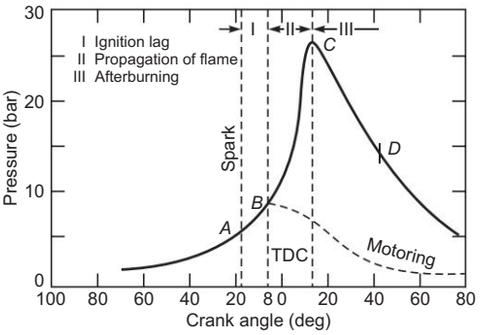

Fig. 24.27 *Stages of combustion in S.I. engine*

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First Stage A–B It is referred as the preparation phase (*ignition lag*) in which fuel elements become ready to react chemically with the oxygen present in compressed air. The chemical process depends on temperature and pressure and nature of charge. The growth and development of the propagating nucleus of flame takes place in this phase.

Second Stage B–C The second stage occurs when the piston is at the top dead centre and the flame is propagated at a faster rate throughout the combustion chamber and reaches to the farthest end of the cylinder. The rate of heat release in this stage depends on turbulence intensity and reaction rate. The rate of pressure rise is proportional to the rate of heat release. The initial gas force is exerted on the piston for its power stroke.

Third Stage C–D This stage starts from the instant at which maximum pressure is reached in the cylinder. This stage occurs during earlier part of the expansion stroke and the flame velocity decreases, and the rate of combustion becomes low. There is no pressure rise during this stage.

24.11.2 Detonation in SI Engine

The engine knock occurs when some of the unburnt gases ahead of the flame in SI engine are ignited spontaneously. The unburnt gas ahead of the flame is compressed as flame propagates through the mixture, and the temperature and pressure of the unburnt mixture rise. If the temperature of the unburnt mixture at some instant exceeds the self-ignition temperature of fuel remaining in the mixture, the auto-ignition of unburnt mixture occurs. This phenomenon is called *detonation*, or knocking.

Detonation in the combustion chamber generates a shock wave which traverses from the end gas region and an expansion wave which traverses into the end gas region. The two waves collide at the boundary of the combustion chamber and interact to produce high-amplitude severe pressure pulses.

The phenomenon of detonation may be illustrated with the help of Fig. 24.28. Figure 24.28(a)

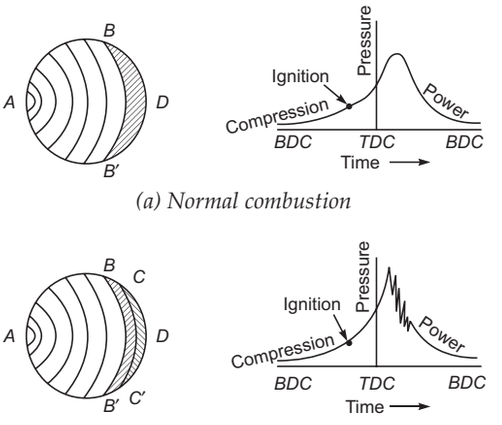

Fig. 24.28 *Normal combustion and detonation in SI engine*

shows normal combustion when the flame travels from the spark plug to the farthest end of the combustion chamber and pressure wave travels in one direction only.

Figure 24.28(b) shows the combustion with detonation. The advancing flame front compresses the unburned charge *BB'D* in the end zone of the combustion chamber, thus raising its pressure and temperature. The temperature of this charge is also increased due to heat transfer from the burning charge. If the temperature of the end charge reaches the self-ignition temperature of the fuel, and remains for some duration, the auto-ignition of charge takes place leading to knocking combustion.

During auto-ignition, another pressure wave starts traveling in the opposite direction to the main pressure wave. When the two pressure waves collide, a severe pressure pulse is generated. The gas in the combustion chamber is subjected to vibration along the pressure pulse until the pressure pulse is subsidized to an equilibrium state. The gas vibration can force the combustion chamber to vibrate. An objectionable audible sound can be heard from the engine.

Detonation during combustion can cause total engine failure. The gas vibration can scrub the chamber walls causing increased heat loss.

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24.11.3 Combustion in CI Engines

In compression ignition engines, only air is sucked and compressed to a high pressure and high temperature. The fuel is injected into this highly compressed air in the combustion chamber. The jet of fuel disintegrates into tiny droplets. These droplets evaporate in the presence of compresed air and a mixture of air and fuel vapour formed at some locations. As soon as the temperature of this mixture attains self-ignition temperature, the auto-ignition takes place.

The combustion in CI engines can be considered to take place in four stages as shown in Fig. 24.29:

1. Ignition delay period,
2. Period of uncontrolled combustion,
3. Period of controlled combustion, and
4. Period of after burning.

1. Ignition Delay Period This phase is also called *preparatory phase*. In this phase, the injected fuel droplets air first heated and evaporated by absorbing heat from the surrounding compressed air. It reduces the temperature of the thin layer of surrounding air and some time elapses before the temperature of the mixture again reaches the self ignition temperature.

The time period required to start actual burning of mixture after injection of the first fuel droplets into the combustion chamber is called *ignition delay period*. The ignition delay period is the combination of physical and chemical delays.

The *physical delay* is the time lapse between the beginning of fuel injection and start of chemical reaction. During this period, fuel jet is atomized, evaporated, mixed with compressed air and raises its self-ignition temperature.

The *chemical delay* is the time lapse for development of inflammation after the beginning of reaction. The chemical delay depends on temperature of compressed air. It is analogous to ignition lag in an SI engine.

The ignition delay period influences the engine performance, combustion rate, exhaust quality and engine knock.

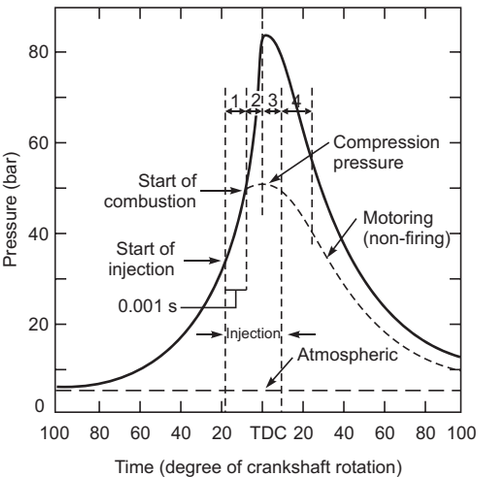

Fig. 24.29 *Stages of combustion in CI engines*

2. Period of Uncontrolled Combustion During the ignition delay period, most of the fuel admitted would have been evaporated and formed a combustible mixture with compressed air. By that time the pre-flame reactions would have been also completed. The auto-ignition of the combustible mixture starts from different places of the combustion chamber simultaneously. It leads to uncontrolled combustion and rapid pressure rise.

The pressure rise during uncontrolled combustion depends on the length of the delay period. Longer the delay period, rapid the pressure rise, since more fuel would have been accumulated in the combustion chamber.

3. Period of Controlled Combustion The uncontrolled combustion is followed by controlled combustion. The temperature and pressure in the second stage are already quite high. Hence, the ignition delay reduces and fuel injected in the combustion chamber burns faster as it gets oxygen. During this stage, the pressure rise can be controlled by controlling the fuel-injection rate.

4. Period of after Burning Combustion does not come to an end with the end of the fuel-injection process. The burning of unburnt and partially burnt fuel particles left in the combustion chamber

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continue as they come in contact of oxygen. The duration of this process is called the period of after burning.

24.12 GOVERNING OF IC ENGINES

The internal combustion engines are governed to operate them at constant speed at all load conditions.

24.12.1 Quantity Governing

The SI engines are *quantity governed* by the opening and closing of the throttle valve, which regulates the mass-flow rate of charge into the cylinder. The position of the throttle valve is regulated by a centrifugal-type speed governor.

The working principle of a *centrifugal governor* is illustrated in Fig. 24.30. Two arms are hinged at the top of the spindle and two revolving balls are fitted on the other ends of these arms. The arms are connected to sleeve through two links. The speed of the crank shaft is transmitted to the spindle through a pair of bevel gears. The rotation of the spindle of the governor causes the weights to move centrally outward due to a centrifugal force. It makes the sleeve move in an upward direction. This movement of the sleeve is transmitted to the throttle valve through a connected lever. Thus the throttle valve is partially closed. It minimizes the charge supply to the engine, and thus speed reduces. So the engine speed can be adjusted to normal speed.

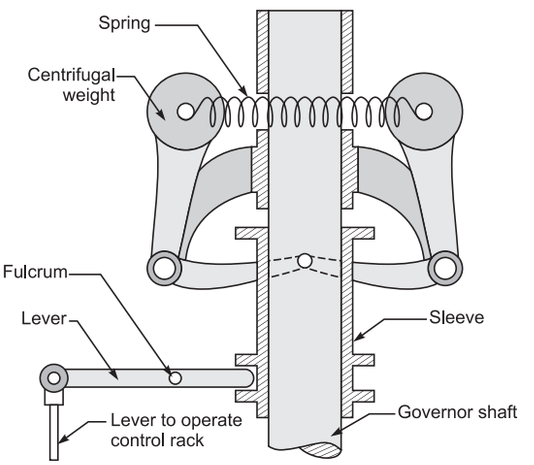

Fig. 24.30 *Principle of mechanical governor*

24.12.2 Quality Governing

The CI engines are governed by controlling the quantity of fuel to be injected per cycle. Since the amount of air inducted per cycle into the cylinder remains almost constant. By varying the quantity of fuel, the quality of charge is varied according to load condition. Therefore, the CI engines are called *quality governed engines.*

The governor in CI engines controls the movement of the rack which rotates the plunger in the barrel. The rotation of the plunger governs the helix opening with a spill port. As illustrated in Fig. 24.30, the balls of the governor rotate at the speed of the engine shaft. When the load on the engine decreases, the speed of the shaft suddenly increases. Therefore, the rotating balls fly out, causing the sleeve to raise through connected levers. The upward movement of the sleeve will draw the rack of fuel injection pump. The rack drives the pinion and the plunger rotates axially, bringing the helix in communication of spill port. Thus, the effective stroke of the plunger ends and reduces the amount of fuel delivered to the engine.

When the load on the engine increases, the speed of the governor decreases, and the sleeve position comes down on the spindle. The lever controls the rack for longer effective stroke of the plunger. Thus more fuel is injected into the cylinder to meet the load condition.

24.12.3 Hit-and-Miss Governning

It is used for governing the speed of gas engines. When the load on the gas engine suddenly decreases, the speed of the engine increases to a very high value. The governor sleeve rises, the lever attached to the sleeve lifts the pecker block so that the gas valve does not open. Hence only air is supplied to the engine and no power is developed inside cylinder. The engine speed comes back to normal and vice-versa.

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24.13 IGNITION SYSTEM

The SI engines use lower compression ratio and self-ignition temperature of gasoline is higher. For initiation of combustion of charge, an ignition system is required. The ignition system provides the high intensity spark needed to ignite the fuel. The ignition system produces, distributes, and regulates electric sparking that ignites fuel vapour in the combustion chambers.

Types of Ignitions Systems Two broad categories of ignition systems are defined as

1. Battery ignition system
2. Magneto ignition system

The whole ignition system can be divided into *primary* and *secondary* circuits. The primary circuit consists of a battery or *magneto* as a source of primary current, ignition switch, ballast resistance, primary windings, contact breaker points and capacitor. One end of the capacitor is connected to the contact breaker and the other is grounded (to the engine its elf).

The secondary circuit consists of a secondary winding (of large number turns of fine wires), distributor and spark plugs.

24.13.1 Components of an Ignition System

An ignition system consists of the following components:

1. Source of electric current—battery or magneto
2. Ignition switch
3. Ballast resistance
4. Ignition coil
5. Contact breaker
6. Capacitor
7. distributor
8. Spark plug

1. Battery The battery is the source of electrical energy. It produces electrical energy from chemical energy stored in it. A 16–12 V battery is used to supply the voltage to the primary circuit of an ignition system. It is charged by an engine-driven dynamo.

2. Ignition Switch It is an electrical switch which is used to allow and stop the flow of electrical energy from source to primary circuit by turning it on or off.

3. Induction Coil The induction coil is an iron core, wrapped with primary and secondary windings of differently sized wires. The primary winding has 100 to 200 turns of relatively thick wire, whereas the secondary winding has approximately 20,000 turns of fine wire.

4. Ballast Resistance The ballast resistance is an electrical resistance which is provided in series of primary winding to regulate the current in a primary circuit.

5. Capacitor A capacitor is a device that temporarily stores electric charge. In the ignition system, a capacitor helps to produce a sharply defined cut-off current when the breaker points open. The capacitor also absorbs the surge of high-voltage electricity as it moves from the coil to the points. In doing so, the capacitor minimizes arcing across the breaker points when they open, thus increasing their service life.

6. Distributor The distributor serves two primary functions. It routes high-voltage pulses to individual cylinders in the correct sequence and with precise timing. It also houses a mechanical switching system involving breaker points—two contact points that open to interrupt the flow of electric current.

A wire conductor carries the pulses of current from the coil to the distributor, which routes them through other wires to individual spark plugs. The *spark plugs* deliver sparks that ignite the fuel.

24.13.2 Battery Ignition System

The battery ignition system is shown in Fig. 24.31. The battery is used as a source of electrical energy.

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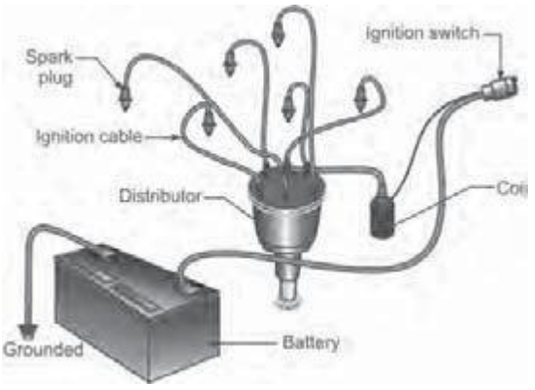
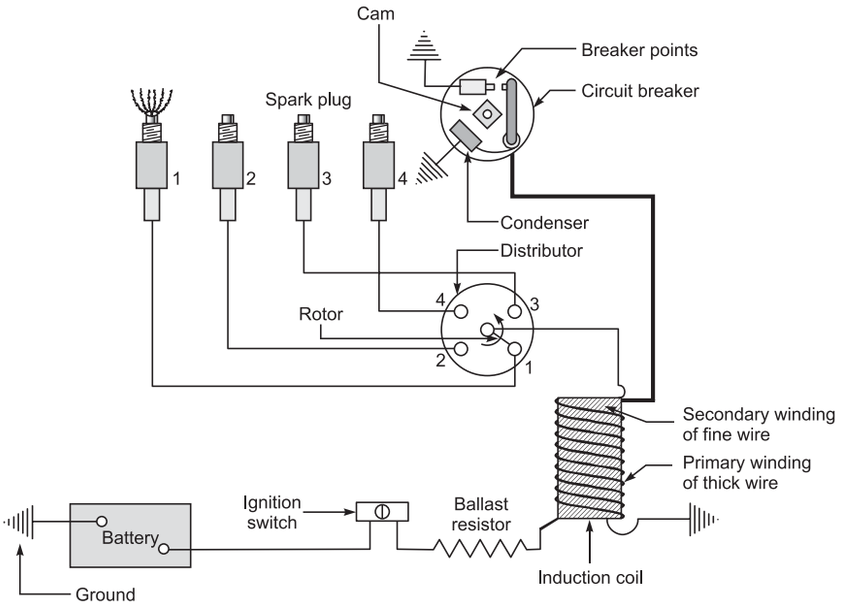


(a) Pictorial view of Battery Ignitions system



(b) Schematic of battery ignitions system

Fig. 24.31

When the ignition switch is turned on and the rotating cam makes the contact on breaker points, a 12-V current flows from the battery in *primary winding* through the ignition switch, and the primary circuit is completed through the ground. The magnetic field is set up around the secondary winding.

When the rotating cam opens the breaker points, the flow of low-voltage current stops and the magnetic field collapses, inducing a high-voltage surge of about 20,000 volts in the *secondary winding.* This high voltage current passes to the distributor which connects the spark plug in correct sequence, depending upon the firing order of the engine.

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Advantages of Battery Ignitions System

1. It gives constant voltage irrespective of speed of the engine.
2. It gives better spark at low speed and starting of engine.
3. It is reliable and requires very less maintenance, except battery and contact points.
4. The battery is charged by dynamo run by engine.

Disadvantages of Battery Ignition System

1. The system is bulky and occupies more space.
2. At higher speed, the sparking voltage decreases.
3. If battery discharges, the engine cannot be started as induction coil fails to operate.

24.13.3 Magneto Ignition System

A magneto is an electric generator that provides the current for ignition in systems that do not have batteries. It is mounted to the engine. When engine is started, it supplies kinetic energy to the magneto. The magneto then converts this energy to electrical energy. The schematic of magneto ignition system is shown in Fig. 24.32.

The difference between battery and magneto ignition systems is in the source of electrical energy, all other components being the same. A magneto system is popular on motorcycles, racing cars, and a variety of small engines.

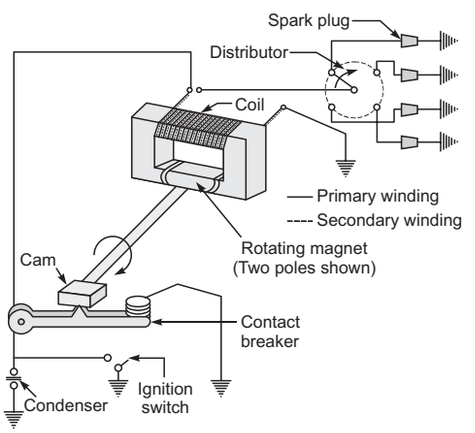

Fig. 24.32 *Magneto ignition system*

The advantages and disadvantages of magneto ignition systems are listed below.

Advantages

1. The system is more reliable due to absence of battery and connecting cables.
2. This system generates secondary voltage according to engine speed, thus it is more suitable for medium and high speed engines.
3. The system is very light and compact, and requires very less space.
4. The spark intensity improves as engine speed increases.
5. The system is much cheaper.
6. It require very less and cheaper maintenance.

Disadvantages

1. The system is not useful at low speed and starting of the engine, because at low speed, it produces poor spark.
2. The powerful spark at high speed may cause burning of electrodes of the spark plug.

24.13.4 Comparison between Battery Ignition System and Magneto Ignition System

The battery ignition system uses a battery which converts the stored chemical energy into electrical energy. A magneto is an electrical generator which generates low voltage. The graphical comparison of current produced in two systems with engine speed is shown in Fig. 24.33.

It is evident from Fig. 24.33 that for a magneto ignition system, the starting current is poor. But as speed increases, the current also increases. For a battery ignition system, the starting current is excellent and it gives good spark at low speeds. Therefore, for low speed engines, the battery ignition system is more suitable.

The diffrences between battery ignition systems and magneto ignition systems are given in the table as follows.

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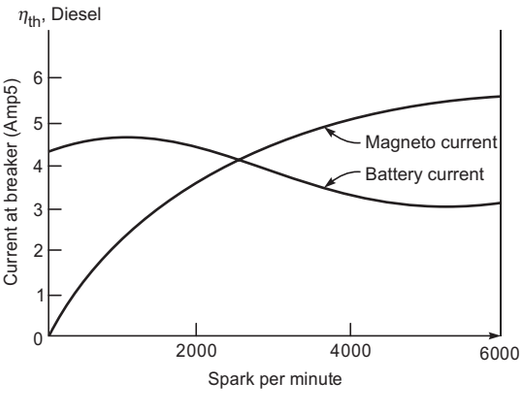

Fig. 24.33 *Speed vs Breaker current for Battery and magneto ignition systems*

24.13.6 Spark Plug

The spark plug is a device which conducts a high-voltage current from ignition system and produces spark to ignite the compressed charge into the combustion chamber. The spark plug as shown in Fig. 24.34, mainly consists of

- Central electrode,
- Ceramic (insulated) body, and
- Outer or ground electrode.

The central electrode is surrounded by a ceramic body. It has a threaded base, which is screwed at the top of the engine cylinder. Two electrodes on the base of the spark plug project into the combustion chamber. The upper end of the central electrode is connected to a high tension cable from the distributor. The other electrode is located at a small distance from the central electrode and is welded to a steel shell of the plug and is grounded with the engine body. The gap between the two electrodes is known as *spark gap*. The high-tension current passes through the central electrode and jumps over the spark gap to grounded electrode and produces

Sl. No.	*Aspect*	*Battery Ignition system*	*Magneto Ignition system*
1.	Source of energy	Conversion of chemical energy into electrical energy.	Conversion of kinetic energy into electrical energy.
2.	Primary current	Obtained from battery	Obtained from magneto.
3.	Starting	Easy start, because battery gives good spark.	Poor spark at start and low speed of engine.
4.	Low speed	Good spark, thus no problem.	Poor spark at low speed, thus engine runs eratically.
5.	High Speed	Current for spark decreases as engine speed increases.	Current for spark increases with speed of engine, thus excellent spark at high speed.
6.	Space required	System is bulky and requires more space.	System is light weight and compact, thus less space is required.
7.	Maintenance	System requires more maintenance and it is difficult to start engine when battery is in discharge condition.	Very less and cheap maintenance, due to absence of battery.
8.	Applications	In cars, light commercial vehicles.	On two wheelers and racing cars, aircrafts, etc.

24.13.5 Electronic Ignition

Electronic ignition systems use semiconductors and other solid-state electronic components to switch current flow on and off in the coil, eliminating the need for breaker points. Automobile manufacturers began installing electronic ignition systems in the 1970s and 1980s in an effort to produce cleaner, more efficient engines.

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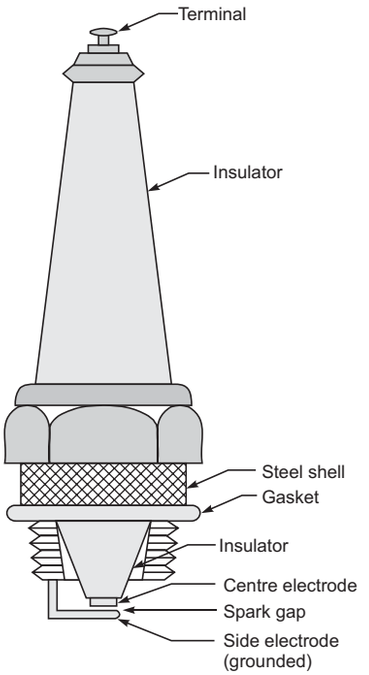

Fig. 24.34 *Spark-plug*

the high intensity spark to ignite the fuel vapour in the combustion chamber. The intensity of spark is greatly affected by the spark gap.

1. If the gap is too large, there may be possibility of no spark due to insufficient high voltage.
2. If the gap is too small, there may be possibility of very less spark, which is insufficient to ignite the charge.

Requirements of a Good Spark Plug

1. It must withstand high temperature attained in the combustion chamber.
2. It must produce a good spark under all working conditions.
3. It must provide suitable insulation between two electrodes to prevent short circuiting.
4. It must maintain proper gap between the two electrodes.
5. It must offer very high resistance to current leakage.
6. It must offer maximum resistance to erosion, burning away the spark points.
7. It must be heat resistant not to cause pre-ignition of charge within cylinder, when it is hot.

24.14 FIRING ORDER

The firing order is a sequence in which the firing takes place in various cylinders of a multicylinder engine. The firing order of a multicylinder engine is arranged to obtain an uniform torque and power by burning of charge, reduced vibrations and balancing of the complete engine. The firing order depends on the number of cylinders in the engine and it may differ from engine to engine. The firing order for some mullicylinder engines are given below:

Number of cylinders	*Firing order*
2-cylinder engine	1, 2
3-cylinder engine	1, 3, 2
4-cylinder engine	1, 3, 4, 2 or 1, 2, 4, 3
6-cylinder engine	1, 5, 3, 6, 2, 4 or 1, 4, 2, 6, 3, 5
7-cylinder engine	1, 6, 2, 5, 8, 3, 7, 4 or 1, 8, 7, 3, 6, 5, 4, 2

24.15 ENGINE-COOLING SYSTEM

24.15.1 Necessity of Cooling

The temperature of gases inside the engine cylinder, due to combustion, reaches a quite high value (more than 2500°C) during a cycle. If an engine is allowed to run without external cooling, the cylinder walls, cylinder liner, and piston will tend to attain the average temperature of the hot gases to which they are exposed, which may be of the order of 1000 to 1500°C. At such high temperatures, obviously, the metal will lose its strength and piston will expand considerably and sieze the liner. Of course, theoretically, the thermal the efficiency of the engine will be better without cooling, but in actual practice, this engine will not run longer. If the cylinder wall temperatrue exceeds 265°C, the lubricating oil

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starts losing its viscosity, starts evaporating, thus tending towards lubrication failure. Also, the high temperature may cause excessive stresses in some parts, making them useless for further operations. Therefore, the internal combustion engines are provided with cooling arrangement, which keeps the engine temperature well within the safe working temperature limits. A typical heat balance for a reciprocating internal combustion engine is shown in Fig. 24.35.

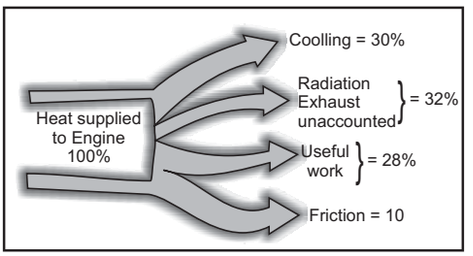

Fig. 24.35 *A typical heat distribution for an internal combustion engine*

24.15.2 Effects of Engine Overheating

The following are the harmful effects of engine overheating:

1. High temperature reduces the strength of the piston and cylinder liner.
2. Overheating may lead to burning of lubricants, thus there may be possibility of lubrication failure and metal to metal contact, thus more heat generation due to friction.
3. The overheating may cause uneven expansion of the piston and cylinder that may lead to piston seizure.
4. The overheated cylinder or piston may lead to pre-ignition of charge in SI engines.
5. As temperature of the cylinder increases, the volumetric efficiency decreases, and hence the power output of the engine is reduced.

24.15.3 Effects of Engine Undercooling

The cooling system in an internal combustion engine should provide adequate cooling but not excessive cooling. However, the excessive cooling is not as harmful as overheating. But undercooling is undesirable due to the following reasons.

1. At low temperature, starting of the engine becomes difficult.
2. At low temperature, there is poor vaporisation of fuel, the combustion is not proper and the engine runs eratically.
3. At low temperature, the viscosity of lubricating oil increases, it offers more frictional resistance, and thus the output of the engine decreases.
4. Undercooling of the engine may change the valve clearance and settings.
5. Overcooling may reduce engine life due to corrosion and carbon deposits.

In general, undercooling affects the economy and life of the engine.

There are two basic types of cooling systems used in reciprocating engines to absorb and dissipate the heat from hot cylinders.

1. Air-cooling system, and
2. Liquid-cooling system

24.15.4 Air-Cooling System

In an air-cooling system, the outer surface of the cylinder and cylinder head is cooled by air flowing over them. To increase the heat transfer rate from the surface, the metallic fins are cast on the cylinder and cylinder head. These fins increase the heat transfer area, and thereby heat transfer rate.

Air cooling system is a very simple, reliable and maintenance-free cooling system, with no operating cost. It is very suitable for small engines of automobiles.

Applications

1. It is used in small engines, i.e., motor cycles, scooters, mopeds, aeroplanes, and combat tanks, where speed of the vehicle gives a good velocity to the air to cool the engine.
2. It is also used in small stationary engines used for agriculture and industries.

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Advantages

1. The design of the engine becomes simpler with an air-cooling system.
2. There is no cooling pipe radiator, fan pump and liquid cooling jacket, and hence the engine has less weight.
3. In an air-cooled engine, the cylinder wall temperature is relatively higher. Thus there is more output from the engine.
4. No danger of coolant leakage, coolant freezing, etc.
5. Installation, assembly, dismantling of the engine are quick and simple.
6. The weight per kW of an air-cooled engine is less than that of a water-cooled engine.
7. The engine is almost maintenance free.

Disadvantages

1. Non-uniform cooling of engine.
2. The compression ratio of the engine is limited due to high wall temperature.
3. The volumetric efficiency of the engine is less than a water-cooled engine.
4. It produces aerodynamic noise.
5. It can be used for only small-sized engines due to its small capacity of heat dissipation.

24.15.5 Liquid-Cooling System

In a liquid-cooling system, water or other solutions flow through the water jacket around the cylinder and cylinder head to absorb the heat. The hot liquid coming out of the water jacket is cooled in the radiator, where circulated air absorbs the heat from the radiator. The cold liquid coming out of the radiator is again pumped to the water jacket for absorbing heat.

The water-cooling system is explained with the help of a block diagram shown in Fig. 24.36.

The water-cooling system mainly consists of a radiator, fan water pump and thermostat.

1. Pump The pump maintains the water circulation through the water jacket around the engine. The bottom side of the radiator is connected to the suction side of the pump. The pump is mounted on the engine and driven by a crank shaft with a fan belt.

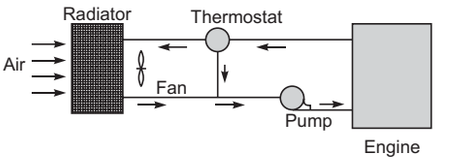

Fig. 24.36 *Schematic of the forced circulation water-cooling system*

2. Fan The fan is mounted in front of the radiator and is driven by a belt-pulley arrangement. The fan draws air through the spaces between the radiator tube and fins, thus bringing down the temperature of the water flowing in the tubes.

3. Water Jacket The water passages between the double walls of the cylinder and cylinder head are called the water jacket. The water jacket is usually cast as an integral part of the cylinder block and head. The jacket covers the entire length of the stroke in order to avoid unequal thermal expansion of cylinder and to prevent the breakdown of lubricating oil film by excessive temperature.

4. Radiator The radiator is basically a compact heat exchanger. It is provided with a large surface area for effective heat transfer. It consists of an upper header and lower header. Between these headers, there is a core of the radiator, which consists of a large number of elliptical or circular brass tubes, pressed into a large number of brass fins. As hot water flows from top to down in the radiator core, it transfers its heat to the radiator fins from where the heat is picked up by circulating air.

5. Thermostat The lower cylinder temperature results into poor performance and rough operation of the engine. In cold starting, if water is circulated through the radiator, as the engine starts, the circulation of cold water in the water jacket brings down the cylinder temperature continuously and the engine will take a long time to reach the safe operating temperature.

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The thermostat is an instrument which automatically maintains the preset minimum temperature and permits a quick warm up of the engine after starting. The thermostat is located in the upper hose connection and its opening and closing is controlled by the water temperature in the cooling system. During the warm-up period, the thermostat is closed and the water pump circulates the water through the water jacket only as shown in Fig. 24.36. When the preset operating temperature (70°C) is reached, the thermostatic valve opens and allows the water to circulate through the radiator. The preset operating temperature range varies from 60°C to 76°C.

6. Pressure Cap In order to avail the advantages of higher boiling temperature of water, the pressurised water is used in the radiator. The pressure is built up within the system due to continuous pumping. A pressure cap is fitted with two valves, a safety valve loaded by compression spring, and a vacuum valve. When the coolant is cold, both valves are closed. But as the engine warms up, the coolant temperature rises and reaches the desired preset pressure. If pressure in the system exceeds the preset value, the safety valve opens and releases some of the gases and liquid to maintain the desired pressure.

24.15.6 Advantages and Disadvantages of Liquid Cooling System

Advantages

1. Efficient cooling as compared to air-cooling system.
2. Fuel consumption of liquid-cooled engines is less than that of air-cooled engines.
3. Liquid-cooled engines require less frontal area.
4. For water-cooled engines, the cooling system can be located conveniently anywhere on the automobile. Some vehicles have it at the rear, while in air cooled engines, it is not possible.
5. Size does not pose a serious concern in the water cooled engine, while in a high-output engine, it is difficult to circulate the correct quantity of air in an air-cooled engine.

Limitations

1. It requires pure water supply for proper functioning.
2. The pump absorbs considerable power and it reduces the output of the engine.
3. In case of failure of cooling system, the engine may get a serious demage.
4. Cost of the system is considerably high.
5. The system requires continuous maintenance of its parts.

24.16 ENGINE LUBRICATION

The lubrication is the supply of oil between two surfaces having relative movement. The objects of lubrication are

1. To minimize the friction between the parts having relative motion.
2. To reduce the wear and tear of moving parts.
3. To cool the surfaces by carrying away the heat generated due to friction.
4. To seal the space between piston rings and cylinder liner.
5. To absorb the shocks between bearings and other parts and consequently, reduce noise.
6. To act as cleaning agent and remove dirt, grit and any deposits that might be present between the moving parts.

24.16.1 Properties of Lubricants

A good lubricating oil should have the following characteristics:

1. Suitable Viscosity The viscosity of the oil should not be changed with temperature rise.

2. Oilness It ensures the adherence to the bearings and spread over the surface. This property makes the oil smooth and very important in boundary lubrication.

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3. Strength The oil must have high strength to avoid metal to metal contact and seizure under heavy loads.

4. Chemical Stability The oil should not react with surfaces and any deposit in the cylinder.

5. Pour Point It should be low to allow the flow of lubricant at low temperature to the oil pump.

6. Flash Point and Fire Point The lubricating oil should not burn inside the cylinder, otherwise it will leave heavy deposit and poisonous exhaust. Therefore, the flash point and fire point of the lubricating oil must be high.

7. Neutralisation The oil should not have a tendency to form deposits by reacting with air, water, fuel or the products of combustion.

8. Cleaning The oil should act as cleaning agent inside the engine and should carry any deposits with it. It should also have non-foaming characteristics, low cost, and be non-toxic.

24.16.2 Parts of Engine Requiring Lubrication

The main parts of the engine which need lubrication are

(i) Main crank shaft bearings
(ii) Big end or crank pin bearings
(iii) Gudgen pin bearings
(iv) Piston rings and cylinder walls
(v) Timing gears
(vi) Cam shaft and cam shaft bearings
(vii) Valve mechanism
(viii) Valve guide, valve tappets, rocker arms
(ix) Governor
(x) Water pump bearing

24.17 LUBRICATION SYSTEMS

The various lubrication systems used for lubricating the above parts of an internal combustion engine are classified as

(i) Mist lubrication system,
(ii) Wet sump lubrication system, and
(iii) Dry sump lubrication system

24.17.1 Mist Lubrication System

It is a very simple system of lubrication. In this system, the small quantity of lubricating oil (usually 2 to 3%) is mixed with the fuel (preferably gasoline). The oil and fuel mixture is introduced through the carburetter. The gasoline is vaporised and oil in the form of mist enters the cylinder via the crank case. The droplets of oil strike the crank case, lubricate the main and connecting rod bearings and the rest of the oil lubricates the piston, piston rings and cylinder.

The system is preferred in two-stroke engines where crank case lubrication is not required. In a two-stroke engine, the charge is partially compressed in a crank case, so it is not possible to have the oil in crank case.

This system is simple, low cost and maintenance-free because it does not require any oil pump, filter, etc. However, it has certain serious disadvantages. Therefore, it is not popular among the lubrication system. Its *disadvantages* are the following:

1. During combustion in the engine, some lubricating oil also burnt and it causes heavy exhaust and forms deposits on the piston crown, exhaust port and exhaust system.
2. Since the lubricating oil comes in contact of acidic vapours produced during the combustion, it gets contaminated and may result in the corrosion of the bearings surface.
3. When the vehicle is moving downhill, the throttle is almost closed, and the engine suffers lack of lubrication as supply of fuel is less. It is a very serious drawback of this system.
4. There is no control over the supply of lubricating oil to the engine. In normal operating conditions, the two-stroke engines

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are always over-oiled. Thus consumption of oil is also more.

5. This system requires thorough mixing of oil and fuel prior to admission into the engine. It requires either separate mixing or use of some additives.

24.17.2 Wet-Sump Lubrication System

In the wet-sump lubrication system, the bottom of the crank case contains an oil pan or sump that serves as oil supply, oil storage tank and oil cooler. The oil dripping from the cylinders, bearings and other parts, falls under gravity back into the sump, from where it is picked up by pump and recirculated through the engine lubrication system. There are three types of wet-sump lubrication system.

(i) Splash system,
(ii) splash and pressure system, and
(iii) full-pressure system.

(*i*) *Splash Lubrication System* It is used on small, stationary four-stroke engines. In this system, the cap of the big end bearing on the connecting rod is provided with a scoop which strikes and dips into the oil-filled troughs at every revolution of the crank shaft and oil is splashed all over the interior of crank case into the piston and over the exposed portion of the cylinder as shown in Fig. 24.37. A hole is drilled through the connecting rod cap through which the oil passes to the bearing surface. Oil pockets are provided to catch the splashed oil over all the main bearings and also the cam shaft bearing. From these pockets, oil passes to the bearings through drilled hole. The surplus oil dripping from the cylinder flows back to the oil sump in the crank case.

(*ii*) *Splash and Pressure System* This system is a combination of splash and pressure system as shown in Fig. 24.38. In this system, the lubricating oil is supplied under the pressure to main and cam shaft bearings. The oil is also directed in the form of spray from nozzle or splashed by a scoop or dipper on the big end to lubricate bearings at the

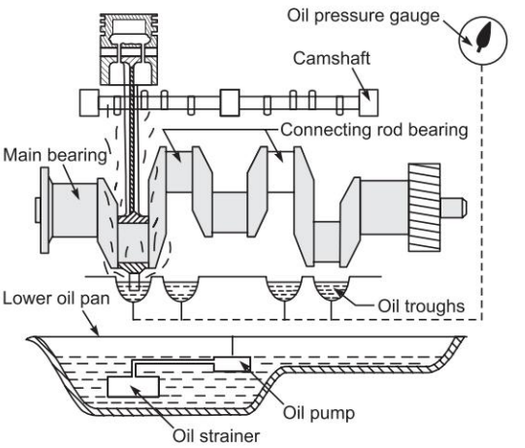

Fig. 24.37 *Splash lubrication system*

big end of the connecting rod, crank pin, gudgen pin, piston rings and cylinder.

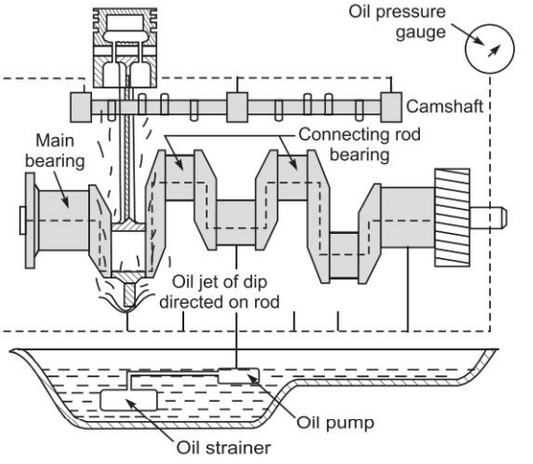

Fig. 24.38 *Splash and pressure lubrication system*

(*iii*) *Pressurized Lubrication System* In this system, the lubricating oil is supplied by a pump under pressure to all parts requiring lubrication as shown in Fig. 24.39. The oil under the pressure is supplied to main bearings of the crank shaft and camshaft. Holes drilled through the main crank shaft bearing journals, communicate oil to big end bearing and small end bearings through a hole drilled in the connecting rod. A pressure gauge is

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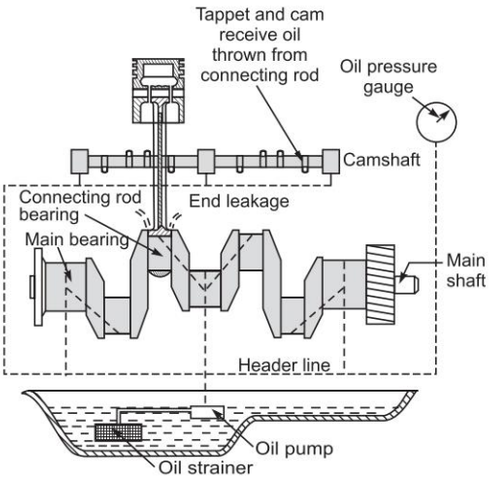

Fig. 24.39 *Pressurized lubrication system*

provided to confirm the circulation of oil to various parts.

This system provides sufficient lubrication to all parts and is favoured by most of the engine manufacturers. Thus, it is used in most heavy-duty and high-speed engines.

24.17.3 Dry-Sump Lubrication System

In this system, the oil supply is carried from an external tank. The oil from the sump is pumped by means of a scavenging pump through filters to the external storage tank. The oil from the storage tank is pumped to the engine cylinder through an oil cooler. The oil pressure may vary from 3 to 8 bar.

The dry-sump lubrication system is generally used for heavy-duty engines.

24.18 PERFORMANCE OF INTERNAL COMBUSTION ENGINES

An engine is selected for a particular application on the basis of its power output and rated speed. Other factors include capital cost and operational cost. Therefore, certain measurements and calculations are required to judge the performance of an engine. These are

1. Indicated power
2. Brake power
3. Frictional power
4. Fuel consumption
5. Air consumption
6. Brake thermal efficiency
7. Indicated thermal efficiency
8. Mecahnical Efficiency
9. Volumetric efficiency
10. Air–fuel ratio

The distribustion of energy produced by the combustion of fuel is shown in Fig. 24.40.

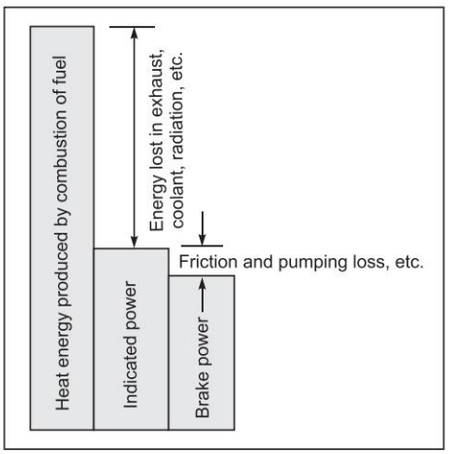

Fig. 24.40 *Heat energy distribustion in an IC engine*

24.18.1 Indicated Power (*IP*)

It is defined as the *rate of work done* on the piston by burning of charge inside the cylinder. It is evaluated from an indicated diagram obtained from the engine. It is the *gross power* produced by the engine. It is calculated as

IP = Indicated mean effective pressure × Swept volume rate

$$= \frac{p_{mi}\, L\, A\, n\, k}{60} \text{ (kW)} \qquad ...(24.1)$$

where k = Number of cylinders

p_{mi} = Indicated mean effective pressure, (kPa or kN/m^2)

L = Stroke length, (m)

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A = $(\pi/4)\,d^2$, cross-sectional area of cylinder of bore, d, (m)

n = Number of working strokes per minute, when engine has a speed of N rotations per minute

= N for two-stroke engines

= $\frac{N}{2}$ for four-stroke engines

24.18.2 Brake Power (*BP*)

It is the *net power* available at the engine shaft for external use. It is measured by the brake or dynamometer (refer rope brake dynamometer, Fig. 20.18), which can be loaded to measure the brake power of the engine. It is calculated as

Brake power = brake load (F) × velocity of brake drum ($2\pi R\,N/60$)

$$\text{or} \quad BP = \frac{2\pi N(FR)}{60{,}000} = \frac{2\pi NT}{60{,}000}\ (\text{kW}) \quad ...(24.2)$$

where, F = Braking force, (N)

R = Effective radius of the brake drum (m)

= 1/2 (Dia. of brake drum + Dia. of the rope)

T = FR, the torque is the product of force F and effective radius R of the brake drum

N = Speed of the engine in rpm

Brake power can also be obtained in terms of brake mean effective pressure (p_{mb}) as

BP = Brake mean effective pressure × Swept volume rate

$$= \frac{p_{mb}\,L\,A\,n\,k}{60}\ (\text{kW}) \quad ...(24.3)$$

where, k = Number of cylinders

p_{mb} = Brake mean effective pressure, (kPa or kN/m^2)

L = Stroke length, (m)

A = $(\pi/4)\,d^2$, cross-sectional area of cylinder of bore, d, (m)

n = Number of working strokes per minute, when engine has a speed of N rotation per minute

= N for two-stroke engine

= $\frac{N}{2}$ for a four-stroke engine

24.18.3 Friction Power (*FP*)

It is the part of the indicated power which is used to *overcome the frictional effects* within the engine. The friction power also includes power required to operate the fuel pump, lubrication pump, valves, etc. Therefore, it is given as the difference between the indicated power and brake power.

$$FP = IP - BP \quad ...(24.4)$$

24.18.4 Fuel Consumption

A calibrated burette can be used for fuel-consumption measurement as shown in Fig. 24.41. A quantity of fuel is taken from the fuel tank through a valve and then the valve is closed. The fuel flows into the running engine only from graduated burette. The time, Δt is recorded for a known volume of fuel-consumption. The fuel consumption rate is then given as

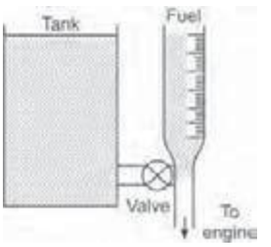

Fig. 24.41 *Fuel consumption measurement*

$$\dot{m}_f = \frac{V_{fuel} \times \rho_{fuel} \times 3600}{\Delta t}\ (\text{kg/h}) \quad ...(24.5)$$

where, V_{fuel} = Volume of fuel in m^3 used in time Δt,

ρ_{fuel} = Density of fuel, and

Δt = Time in seconds.

24.18.5 Specific Fuel Consumption (*sfc*)

It is defined as the ratio of the mass of fuel consumed per hour per unit power output (BP).

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It is also designated as *Bsfc* (*brake specific fuel consumption*). It is a parameter which decides the economical power production from an engine.

$$Bsfc \text{ or } sfc = \frac{\dot{m}_f \text{ (kg/h)}}{BP \text{ (kW)}} \text{ (kg/kWh)} \quad ...(24.6)$$

The specific fuel consumption in kg/kWh based on the indicated power (*IP*) is called the *Isfc* (*indicated specific fuel consumption*) and is expressed as

$$Isfc = \frac{\dot{m}_f \text{ (kg/h)}}{IP \text{ (kW)}} \text{ (kg/kWh)} \quad ...(24.7)$$

24.18.6 Air Consumption

Air-consumption rate of an engine can be effectively calculated by means of an orifice meter installed in an air box.

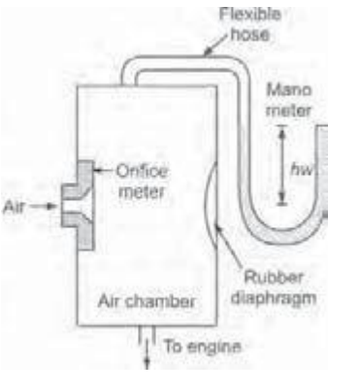

Fig. 24.42 *Air-consumption measurement*

An orifice is fitted to an air-tight air box. Inlet manifold is connected to the air box through a flexible pipe. A U-tube manometer is installed on the air box to measure the pressure depression in the water column (h_w), when the engine sucks the air. A rubber diaphragm is also installed to minimize the pressure pulsation.

The volume flow rate of air passing through the orifice, in m^3/s, can be calculated by using the relation

$$\dot{V}_a = A_{orifice} \times \text{velocity of air}$$

which can be expressed as

$$\dot{V}_a = \frac{\pi}{4} d_o^2 C_d \sqrt{2 g h_a} \quad ...(24.8)$$

where, $\dot{V}_a$ = Volume flow rate of air, m^3/s
d_o = Diameter of orifice, m
C_d = Coefficient of discharge of orifice
g = 9.81 m/s^2, acceleration due to gravity
h_a = Head of air = $\frac{\rho_w h_w}{\rho_{air}}$ (m),
h_w = Water column in m and
ρ_w = density of water,
ρ_{air} = density of air.

The mass-flow rate of air through the orifice can be calculated as

$$\dot{m}_a = \rho_{air} \times \dot{V}_a \quad ...(24.9)$$

24.18.7 Air–Fuel Ratio (A/F)

It is the ratio between the mass of the air and mass of the fuel supplied to the engine. It is expressed as

$$\text{A/F} = \frac{\dot{m}_a \text{ (mass flow rate of air)}}{\dot{m}_f \text{ (mass flow rate of fuel)}} \quad ...(24.10)$$

Theoretically, the correct (stoichiometric) air–fuel ratio is 15. But the combustion of air–fuel mixture can take place in A/F ratio ranges from 12 to 19 for petrol engines and 20 to 60 in Diesel engines.

24.19 EFFICIENCIES OF IC ENGINES

24.19.1 Brake Thermal Efficiency (η_{bth})

The power output of an engine is obtained from the combustion of charge. Thus the *overall efficiency* of an engine is given by brake thermal efficiency, i.e.,

$$\eta_{bth} = \frac{\text{Brake power}}{\text{Energy supply rate}} = \frac{BP}{\dot{m}_f \times CV} \quad ...(24.11)$$

where, $\dot{m}_f$ = mass flow rate of the fuel (kg/s)
CV = Calorific value of fuel, (kJ/kg)

24.19.2 Indicated Thermal Efficiency (η_{ith})

The indicated thermal efficiency is defined as the ratio of the indicated power to the heat supply rate, i.e.,

$$\eta_{ith} = \frac{IP}{\dot{m}_f \times CV} \quad ...(24.12)$$

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24.19.3 Mechanical Efficiency (η_{mech})

It is the ratio of the brake power and indicated power.

$$\eta_{mech} = \frac{BP}{IP} \qquad ...(24.13)$$

It can also be expressed as

$$\eta_{mech} = \frac{\text{Brake thermal efficiency}}{\text{Indicated thermal efficiency}}$$

$$= \frac{\eta_{bth}}{\eta_{ith}} \qquad ...(24.14)$$

and $$\eta_{mech} = \frac{\text{Brake mean effective pressure}}{\text{Indicated mean effective pressure}}$$

$$= \frac{p_{mb}}{p_{mi}} \qquad ...(24.15)$$

24.19.4 Relative Efficiency

It is the ratio of actual thermal efficiency to air standard efficiency of the engine. It is sometimes referred as *efficiency ratio*. It is expressed as

$$\eta_{Relative} = \frac{\text{Brake thermal efficiency}}{\text{Air standard efficiency}} \qquad ...(24.16)$$

Relative efficiency for most of the engines varies from 75 to 95% with air standard efficiency.

24.19.5 Volumetric Efficiency (η_{vol})

It is defined as the ratio of the mass of the actual charge inducted into the cylinder to the mass of the charge corresponding to the swept volume, or

$$\eta_{vol} = \frac{\text{Actual mass flow rate of the charge}}{\text{Density} \times \text{Swept volume per second}}$$

$$= \frac{\dot{m}_a \text{ (kg/s)}}{\rho_a \left(\frac{\pi}{4} d^2 L\right) \frac{n}{60}} \qquad ...(24.17)$$

where, ρ_a = Density of inlet charge,
d = bore, L = stroke
n = Number of effective suction strokes per cycle per minute
$n = N$ for a two-stroke engine
$= \frac{N}{2}$ for a four-stroke engine

The volumetric efficiency can also be defined as the ratio of the volume of the charge inducted in the cylinder, measured at NTP to the swept volume of the cylinder.

$$\eta_{vol} = \frac{V_{act}}{V_s} \qquad ...(24.18)$$

Example 24.1 *A rope-brake dynamometer was used to measure the brake power of a single cylinder, four-stroke cycle petrol engine. It was found that the torque due to brake load was 175 Nm and the engine makes 500 rpm. Determine the brake power developed by the engine.*

Solution

Given A single-cylinder, four-stroke petrol engine with a rope-brake dynamometer

$k = 1$
$T = 175$ Nm $\qquad N = 500$

To find Brake power

Analysis The brake power is given by

$$BP = \frac{2\pi N T}{60{,}000} = \frac{2\pi \times 500 \times 175}{60{,}000} = \mathbf{9.16\ kW}$$

Example 24.2 *A four-cylinder, four-stroke petrol engine develops indicated power of 14.7 kW at 1000 rpm. The mean effective pressure is 5.5 bar. Calculate the bore and stroke of the engine, if the stroke is 1.5 times the bore.*

Solution

Given A four-cylinder, four-stroke cycle petrol engine

$k = 4$ $\qquad n = \frac{N}{2}$
$IP = 14.7$ kW $\qquad N = 1000$ rpm
$p_m = 5.5$ bar $= 550$ kPa
$L = 1.5\,d$

To find (i) Bore (ii) Stroke

Analysis The power of an engine is given by

$$IP = p_m \frac{L A n k}{60}$$

$$14.7 = 550 \times \frac{(1.5\,d)}{60} \times \left(\frac{\pi}{4} d^2\right) \times \frac{1000}{2} \times 4$$

$$d^3 = 6.806 \times 10^{-4} \text{ m}^3$$

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Bore, $d = 0.08796$ m or **87.96 mm**
Stroke $L = 1.5d =$ **131.94 mm.**

Example 24.3 *A four-cylinder, two-stroke cycle petrol engine develops 30 kW at 2500 rpm. The mean effective pressure on each piston is 8 bar and mechanical efficiency is 80%. Calculate the diameter and stroke of each cylinder, if the stroke to bore ratio is 1.5. Also calculate the fuel consumption of the engine, if the brake thermal efficiency is 28%. The calorific value of the fuel is 43900 kJ/kg.*

Solution

Given A four-cylinder, two-stroke cycle petrol engine

$k = 4$ $BP = 30$ kW
$N = 2500$ rpm $n = N$
$\eta_{mech} = 0.8$ $L = 1.5d$
$\eta_{bth} = 0.28$ $CV = 43900$ kJ/kg
$p_m = 8$ bar $= 800$ kPa

To find

(i) Bore of cylinder,
(ii) Stroke of pistion, and
(iii) Fuel consumption rate (*Bsfc*).

Analysis The mechanical efficiency is given as

$$\eta_{mech} = \frac{BP}{IP}$$

$$\therefore \quad IP = \frac{30 \text{ kW}}{0.8} = 37.5 \text{ kW}$$

(i) The indicated power is expressed as

$$IP = \frac{p_m L A nk}{60}$$

or $$37.5 = \frac{800 \times (1.5d) \times \left(\frac{\pi}{4} d^2\right) \times 2500 \times 4}{60}$$

or $d^3 = 0.000238 \text{ m}^3$

Bore $d = 0.062$ m or **62 mm**

(ii) Stroke $L = 1.5d =$ **93 mm**

(iii) The brake thermal efficiency is given by

$$\eta_{bth} = \frac{BP}{\dot{m}_f CV}$$

or $$\dot{m}_f = \frac{30 \text{ kW}}{0.28 \times (43900 \text{ kJ/kg})}$$

$= 0.00244$ kg/s or 8.78 kg/h

The brake specific fuel consumption

$$Bsfc = \frac{\dot{m}_f \text{(kg/h)}}{BP \text{(kW)}} = \frac{8.78}{30}$$

$=$ **0.293 kg/kWh**

Example 24.4 *The following results were obtained from a test on a single-cylinder, four-stroke Diesel engine. Diameter of the cylinder is 30 cm, stroke of the piston is 45 cm, indicated mean effective pressure is 540 kPa and engine speed is 240 rpm. Calculate the indicated power of the engine.*

Solution

Given A single-cylinder, four-stroke Diesel engine

$d = 30$ cm $= 0.3$ m $L = 45$ cm $= 0.45$ m
$p_{mi} = 540$ kPa $N = 240$ rpm
$k = 1$
$n = \frac{N}{2} = 120$ working stroke per minute

To find Indicated power

Analysis The cross-sectional area of the cylinder

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.3)^2 = 0.07068 \text{ m}^2$$

Indicated power (*IP*)

$$IP = \frac{p_{mi} L A n k}{60}$$

$$= \frac{(540 \text{ kPa}) \times (0.45 \text{ m}) \times (0.07068 \text{ m}^2) \times 120 \times 1}{60}$$

$=$ **34.353 kW**

Example 24.5 *In a test of a single-cylinder, four-stroke Diesel engine, the following data were recorded.*

Indicated mean effective pressure $= 755$ kPa
cylinder diameter $= 10$ cm
piston stroke $= 15$ cm
engine speed $= 480$ rpm
brake wheel diameter $= 62.5$ cm
net load on the brake wheel $= 170$ N

Calculate

(a) indicated power,
(b) brake power, and
(c) the mechanical efficiency of the engine.

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Solution

Given A single-cylinder, four-stroke Diesel engine

$p_{mi} = 755$ kPa $\quad N = 480$ rpm

$n = \dfrac{N}{2} = 240$ working stroke/min.

$k = 1$ $\quad d = 10$ cm $= 0.1$ m

$L = 15$ cm $= 0.15$ m $\quad D_{brake} = 62.5$ cm

or $R_{brake} = 31.25$ cm $= 0.3125$ m

$F = 170$ N

To find

(i) Indicated power,
(ii) brake power, and
(iii) mechanical efficiency

Analysis The cross-sectional area of the cylinder

$$A = \frac{\pi}{4} \times (0.1\text{ m}) = 7.854 \times 10^{-3}\text{ m}^2$$

(i) Indicated power (IP)

$$IP = \frac{p_{mi}\ L\ A\ n\ k}{60}$$

$$= \frac{755 \times 0.15 \times 7.854 \times 10^{-3} \times 240 \times 1}{60}$$

$$= \mathbf{3.557\ kW}$$

(ii) Brake power,

$$BP = \frac{2\pi\ N\ T}{60{,}000} = \frac{2\pi\ N\ (F\ R_{brake})}{60{,}000}$$

$$= \frac{2\pi \times (480\text{ rpm}) \times (170\text{ N} \times 0.3125\text{ m})}{60{,}000}$$

$$= \mathbf{2.67\ kW}$$

(iii) Mechanical efficiency (η_{mech})

$$\eta_{mech} = \frac{BP}{IP} = \frac{2.67}{3.557} = 0.750 = \mathbf{75\%.}$$

Example 24.6 *The following results refer to a test on a petrol engine:*

Indicated power = 30 kW
brake power = 26 kW
Engine speed = 1000 rpm
Bsfc = 0.35 kg/kWh
CV of fuel used = 43900 kJ/kg

Calculate

(a) Indicated thermal efficiency,
(b) Brake thermal efficiency, and
(c) Mechanical efficiency.

Solution

Given A petrol engine

$IP = 30$ kW $\quad BP = 26$ kW

$N = 1000$ rpm $\quad Bsfc = 0.35$ kg/kWh

$CV = 43900$ kJ/kg

To find

(i) Indicated thermal efficiency, η_{ith}
(ii) Brake thermal efficiency, η_{bth}
(iii) Mechanical efficiency, η_{mech}

Analysis Fuel consumption rate,

$$\dot{m}_f = Bsfc \times BP$$

$$= 0.35 \times 26 = 9.1\text{ kg/h} = 2.53 \times 10^{-3}\text{ kg/s}$$

(i) Indicated thermal efficiency (η_{ith}),

$$\eta_{ith} = \frac{IP}{\dot{m}_f \times CV}$$

$$= \frac{30}{2.53 \times 10^{-3} \times 43900}$$

$$= 0.27 = \mathbf{27\%.}$$

(ii) Brake thermal efficiency (η_{bth}),

$$\eta_{bth} = \frac{BP}{\dot{m}_f\ CV}$$

$$= \frac{26}{2.53 \times 10^{-3} \times 43900}$$

$$= 0.234 = \mathbf{23.4\%.}$$

(iii) Mechanical efficiency (η_{mech}),

$$\eta_{mech} = \frac{BP}{IP} = \frac{26}{30} = 0.867 = \mathbf{86.7\%.}$$

Example 24.7 *The mechanical efficiency of a single-cylinder, four-stroke engine is 80%. The friction power is estimated to be 26 kW. Calculate the indicated power and brake power developed by the engine.*

Solution

Given A single-cylinder, four-stroke engine

$\eta_{mech} = 0.80$ $\quad FP = 26$ kW

To find

(i) Indicated power, and
(ii) Brake power.

Analysis The mechanical efficiency of an engine is given by

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$$\eta_{mech} = \frac{BP}{IP} \quad \text{or} \quad BP = 0.8\,IP$$

The friction power of the engine is given as

$$FP = IP - BP$$

$$26 \text{ kW} = IP - 0.8\,IP = 0.2\,IP$$

or $$IP = \frac{26}{0.2} = \mathbf{130\ kW}$$

Then $$BP = IP - FP = 130 - 26 = \mathbf{104\ kW.}$$

Example 24.8 *A Diesel engine has a brake thermal efficiency of 30%, if the calorific value of the fuel is 42000 kJ/kg. Calculate the brake specific fuel consumption.*

Solution

Given A Diesel engine with

$\eta_{bth} = 0.3$

$CV = 42000$ kJ/kg

To find Brake specific fuel consumption.

Analysis The brake thermal efficiency of an engine is expressed as

$$\eta_{bth} = \frac{BP}{\dot{m}_f \times CV}$$

or $$\frac{BP}{\dot{m}_f} = \eta_{bth}\,CV$$

$$= 0.3 \times 42000 = 12600 \text{ kJ/kg}$$

or $$\frac{\dot{m}_f}{BP} = 7.936 \times 10^{-5} \text{ kg/kJ}$$

and $$Bsfc = \frac{\dot{m}_f}{BP} \times 3600 = \mathbf{0.287\ kg/kWh}$$

Example 24.9 *A two-stroke, Diesel engine develops a brake power of 420 kW. The engine consumes 195 kg/h of fuel and air–fuel ratio is 22:1. Calorific value of the fuel is 42000 kJ/kg. If 76 kW of power is required to overcome the frictional losses, calculate*

(a) Mechanical efficiency,

(b) Air consumption,

(c) Brake thermal efficiency.

Solution

Given A two-stroke Diesel engine with

$BP = 420$ kW $\qquad FP = 76$ kW

A/F $= 22 : 1$ $\qquad CV = 42000$ kJ/kg

$\dot{m}_f = 195$ kg/h

To find

(i) Mechanical efficiency, η_{mech}

(ii) Air consumption, and

(iii) Brake thermal efficiency, η_{bth}

Analysis

(i) Mechanical efficiency (η_{mech})

Indicated power,

$$IP = BP + FP = 420 + 76 = 496 \text{ kW}$$

$$\eta_{mech} = \frac{BP}{IP} = \frac{420}{496} = 0.8467$$

$$= \mathbf{84.67\%.}$$

(ii) Fuel consumption rate,

$$\dot{m}_a = \dot{m}_f \times \frac{A}{F} = (195 \text{ kg/h}) \times \frac{22}{1}$$

$$= 4290 \text{ kg/h} \quad \text{or} \quad \mathbf{71.5\ kg/min}$$

(iii) The brake thermal efficiency of an engine is expressed as

$$\eta_{bth} = \frac{BP}{\dot{m}_f \times CV}$$

$$= \frac{420 \text{ kW}}{\left(\frac{195}{3600} \text{ kg/s}\right) \times (42000 \text{ kJ/kg})}$$

$$= 0.1846 \quad \text{or} \quad \mathbf{18.46\%}$$

Example 24.10 *An engine is used in a process which requires 100 kW of brake power with a mechanical efficiency of 78%. The engine uses 1 kg of fuel per minute. If a simple modification in design reduces the engine friction by 8 kW then what will be the percentage saving in fuel consumption? Assume indicated thermal efficiency remains same.*

Solution

Given An engine with

$BP = 100$ kW $\qquad \eta_{mech} = 0.78$

$\dot{m}_{f_1} = 1$ kg/min. $\qquad \eta_{ith} =$ constant

Change in $FP = 8$ kW

To find Percentage saving in fuel consumtion.

Analysis

(i) Mechanical efficiency (η_{mech}),

$$\eta_{mech} = \frac{BP}{IP}$$

or $$IP_1 = \frac{BP}{\eta_{mech}} = \frac{100}{0.78} = 128.21 \text{ kW}$$

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Friction power

$$FP_1 = IP - BP = 128.21 - 100$$
$$= 28.21 \text{ kW}$$

After modified design of engine,

$$FP_2 = 28.21 - 8 = 20.21 \text{ kW}$$
$$IP_2 = BP + FP_2 = 100 + 20.21 = 120.21$$

For same indicated thermal efficiency

$$\eta_{ith_1} = \eta_{ith_2}$$

or $$\frac{IP_1}{\dot{m}_{f_1} \times CV} = \frac{IP_2}{\dot{m}_{f_2} \times CV}$$

or $$\frac{128.21}{1 \times CV} = \frac{120.21}{\dot{m}_{f_2} \times CV}$$

or $$\dot{m}_{f_2} = 0.9376 \text{ kg/min}$$

Percentage saving in fuel consumption

$$= \frac{\dot{m}_{f_1} - \dot{m}_{f_2}}{\dot{m}_{f_1}} \times 100 = \frac{1 - 0.9376}{1} \times 100$$
$$= \mathbf{6.24\%}$$

Example 24.11 *Calculate the brake mean effective pressure of a four-cylinder, four -stroke Diesel engine having a 100-mm bore and 120-mm stroke which develops a power of 42 kW at 1200 rpm.*

Solution

Given A four-cylinder, four-stroke Diesel engine with

$k = 4$ $\quad$ $d = 100 \text{ mm} = 0.10 \text{ m}$

$N = 1200 \text{ rpm}$ $\quad$ $L = 120 \text{ mm} = 0.12 \text{ m}$

$BP = 42 \text{ kW}$ $\quad$ $n = \frac{N}{2}$

To find The brake mean effective pressure.

Analysis The brake mean effective pressure relates brake power as

$$BP = \frac{p_{mb} \, L \, A \, n \, k}{60}$$

where, p_{mb} is brake mean effective pressure

$$\therefore \quad 42 \text{ kW} = \frac{p_{mb} \times 0.12 \times \left(\frac{\pi}{4}\right) \times (0.1)^2 \times 1200 \times 4}{60 \times 2}$$

or $$p_{mb} = 1114.08 \text{ kPa} = \mathbf{11.14 \text{ bar}}$$

Example 24.12 *A single cylinder, 4-stroke Diesel engine running at 1800 rpm has an 85 mm bore and a 110-mm stroke. It takes 0.56 kg of air per minute and develops a brake power of 6 kW, while the air–fuel ratio is 20 : 1. CV of fuel is 42550 kJ/kg and the ambient air density is 1.18 kg/m^3. Calculate*

(a) The volumetric efficiency,

(b) Brake specific fuel consumption.

Solution

Given A single-cylinder, four-stroke Diesel engine with

$k = 1$ $\quad$ $d = 85 \text{ mm}$

$L = 110 \text{ mm}$ $\quad$ $n = \frac{N}{2} = \frac{1800}{2} = 900$

$\dot{m}_a = 0.56 \text{ kg/min}$ $\quad$ $\text{A/F} = 20 : 1$ $\quad$ $P = 6 \text{ kW}$

$CV = 42550 \text{ kJ/kg}$ $\quad$ $\rho_a = 1.18 \text{ kg/m}^3$

To find

(i) The volumetric efficiency, and

(ii) Brake specific fuel consumption.

Analysis

(i) *Volumetric efficiency*

Swept volume rate,

$$\dot{V}_s = \frac{\pi}{4} d^2 \, L \, n \, k = \frac{\pi}{4} \times (0.085)^2 \times 0.11 \times 900 \times 1$$
$$= 0.5617 \text{ m}^3/\text{min}$$

Mass of air,

$$\dot{m}_a = \dot{V}_s \, \rho_a = 0.5617 \times 1.18$$
$$= 0.663 \text{ kg/min}$$

Volumetric efficiency,

$$\eta_{vol} = \frac{\text{Actual mass of air}}{\text{Mass correponds to swept volume}}$$
$$= \frac{0.56}{0.663} \times 100 = \mathbf{84.5\%}$$

(ii) *Brake specific fuel consumption:*

$$\dot{m}_f = \frac{\text{Actual mass of air}}{\text{A/F ratio}} = \frac{0.56}{20}$$
$$= 0.028 \text{ kg/min or } 1.68 \text{ kg/h}$$

$$Bsfc = \frac{\dot{m}_f \text{ (kg/h)}}{BP \text{(kW)}} = \frac{0.028}{6} = \mathbf{0.28 \text{ kg/kWh}}$$

Example 24.13 *Calculate the brake mean effective pressure of a four-cylinder, two-stroke engine of 100 mm bore, 125 mm stroke, when it develops a torque of 490 Nm.*

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Solution

Given A four-cylinder, two-stroke engine with

$k = 4$ $d = 100$ mm

$n = N$ $L = 125$ mm

$T = 490$ Nm

To find The brake mean effective pressure of the engine

Analysis The brake power is given by

$$BP = \frac{2\pi N T}{60,000} = \frac{p_{mb} L A n k}{60}$$

Here $n = N$ for two-stroke engine

$$p_{mb} = \frac{2\pi T}{1000 L A k}$$

$$= \frac{2\pi \times 490}{1000 \times 0.125 \times \left(\frac{\pi}{4}\right) \times (0.1)^2 \times 4}$$

$$= 784 \text{ kPa}$$

Thus, the break mean effective pressure is **7.84 bar**.

Example 24.14 *A single-cylinder, C1 engine with a brake thermal efficiency of 30% uses diesel oil having a calorific value of 42000 kJ/kg. If its mechanical efficiency is 80%, calculate (a) Bsfc, (b) Isfc, and (c) η_{ith}.*

Solution

Given A single-cylinder *CI* engine with

$\eta_{bth} = 0.3$ $CV = 42000$ kJ/kg

$\eta_{mech} = 0.8$

To find

(i) *Bsfc* (ii) *Isfc* (iii) η_{ith}

Analysis

(i) The brake thermal efficiency relates brake power as

$$\eta_{bth} = \frac{BP}{\dot{m}_f CV}$$

$$\text{or } \frac{\dot{m}_f}{BP} = \frac{1}{\eta_{bth} CV} = \frac{1}{0.3 \times 42000} = \frac{1}{12600} \text{ kg/kJ}$$

$$\text{and } Bsfc = \frac{\dot{m}_f}{BP} \times 3600 = \frac{1}{12600} \times 3600$$

$$= \mathbf{0.286 \text{ kg/kWh}}$$

(ii) $\eta_{mech} = \dfrac{Isfc}{Bsfc}$

Thus $Isfc = 0.8 \times 0.286 = \mathbf{0.229 \text{ kg/kWh}}$

(iii) The mechanical efficiency is expressed as

$$\eta_{mech} = \frac{BP}{IP} = \frac{\eta_{bth}}{\eta_{ith}}$$

$$\text{or } \eta_{ith} = \frac{\eta_{bth}}{\eta_{mech}} = \frac{0.3}{0.8} = 0.375 = \mathbf{37.5\%}$$

Example 24.15 *A four-cylinder, four-stroke petrol engine has a 10-cm bore, 15-cm stroke and uses a compression ratio of 6. The engine develops 25 kW indicated power at 2000 rpm. Find the mean indicated pressure and air standard efficiency.*

Also calculate the fuel consumption per hour, if the indicated thermal efficiency is 30%. Take the calorific value of fuel as 42 MJ/kg.

Solution

Given A four-cylinder, four-stroke petrol engine

$k = 4$ $d = 10$ cm $= 0.1$ m

$L = 15$ cm $= 0.15$ m $r = 6$

$IP = 25$ kW $N = 2000$ rpm

$n = \dfrac{N}{2}$ $\eta_{ith} = 0.3$

$CV = 42$ MJ/kg

To find

(i) Air standard efficiency,

(ii) Indicated thermal efficiency, and

(iii) Fuel consumption per hour.

Assumptions

(i) The petrol engine works on air standard Otto cycle.

(ii) The ratio of specific heats $\gamma = 1.4$ for air.

Analysis

(i) *Air standard efficiency of Otto cycle:*

$$\eta_{Otto} = 1 - \frac{1}{r^{\gamma - 1}}$$

$$= 1 - \frac{1}{(6)^{1.4-4}} = 0.511 \text{ or } \mathbf{51.1\%}$$

(ii) *Indicated mean effective pressure*

$$IP = \frac{p_{mi} L A n k}{60}$$

$$25 \text{ kW} = \frac{p_{mi} \times (0.15\text{m}) \times (\pi/4) \times (0.1\text{m})^2 \times 2000 \times 4}{60 \times 2}$$

or $p_{mi} = 318.3$ kPa $= \mathbf{3.183 \text{ bar}}$

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(iii) *Fuel consumption per hour*

We have $\eta_{ith} = \dfrac{IP}{\dot{m}_f \times CV}$

or $\dot{m}_f = \dfrac{IP}{\eta_{ith}\, CV} = \dfrac{25 \text{ kW}}{0.3 \times 42 \times 10^3 \text{ kJ/kg}}$

$= 1.984 \times 10^{-3}$ kg/s = **7.14 kg/h**

Example 24.16 *A four-cylinder, four-stroke petrol engine has a bore of 57 mm and a stroke of 90 mm. Its rated speed is 2800 rpm, torque is 55.2 Nm. The fuel consumption is 6.74 lit/h. The density of the petrol is 735 kg/m^3 and petrol has a calorific value of 44200 kJ/kg. Calculate BP, bmep, brake thermal efficiency and brake-specific fuel consumption.*

Solution

Given A four-cylinder, four-stroke petrol engine with

$d = 57$ mm $= 0.057$ m $\quad L = 90$ mm $= 0.09$ m

$T = 55.2$ Nm $\quad N = 2800$ rpm

$k = 4$ $\quad n = \dfrac{N}{2}$

$\dot{V}_f = 6.74$ lit/h $\quad \rho_f = 735$ kg/m^3

$CV = 44200$ kJ/kg

To find

(i) *BP,*

(ii) Brake *mep,*

(iii) η_{bth}, and

(iv) Brake-specific fuel consumption.

Analysis

(i) The brake power of the engine can be obtained as

$$BP = \frac{2\pi N T}{60{,}000} = \frac{2\pi \times 2800 \times 55.2}{60{,}000}$$

$$= \mathbf{16.18 \text{ kW}}$$

(ii) The *Bmep* (brake mean effective pressure) is given by

$$BP = p_{mb} \frac{L A n k}{60}$$

$$16.18 = p_{mb} \times 0.09 \times \frac{\pi}{4} \times \frac{(0.057)^2 \times 2800}{60 \times 2} \times 4$$

or $p_{mb} = 755.1$ kPa = **7.55 bar**

(iii) The mass flow rate of the petrol

$$\dot{m}_f = \rho_f \dot{V}_f$$

$= (735 \text{ kg/m}^3) \times (6.74 \times 10^{-3} \text{ m}^3\text{/h})$

$= 4.954$ kg/h $= 1.376 \times 10^{-3}$ kg/s

The brake thermal efficiency

$$\eta_{bth} = \frac{BP}{\dot{m}_f \times CV}$$

$$= \frac{16.18}{1.376 \times 10^{-3} \times 44200} = 0.266$$

$= \mathbf{26.6\%}$

(iv) The brake-specific fuel consumption

$$Bsfc = \frac{\dot{m}_f \text{ (kg/h)}}{BP \text{ (kW)}} = \frac{4.954}{16.18}$$

$= \mathbf{0.306 \text{ kg/kWh}}$

Example 24.17 *The following data and results refer to a test on a four-cylinder, four-stroke car engine:*

cylinder bore = 7.5 cm

piston stroke = 9 cm

engine-to-rear-axle ratio = 39 : 8

wheel diameter with tyre fully inflated = 65 cm

The petrol consumption is 0.250 kg for a distance of 4 km, when the car was travelling at a speed of 60 kmph. CV of fuel is 44200 kJ/kg.

If the mean effective pressure is 5.625 bar, calculate the indicated power and indicated thermal efficiency.

Solution

Given A four-cylinder, four-stroke car engine with

$d = 7.5$ cm $= 0.075$ m

$L = 9$ cm $= 0.09$ m

$T = 55.2$ Nm

$k = 4$

$D_{wheel} = 65$ cm $= 0.65$ m

Distance $= 4$ km

Engine rear axle ratio $= 39 : 8$

speed $= 60$ km/h $= 16.667$ m/s

$m_f = 0.250$ kg

$CV = 44200$ kJ/kg

$Bmep = 5.625$ bar $= 562.5$ kPa

$n = \dfrac{N}{2}$

To find

(i) Indicated power, and

(ii) Indicated thermal efficiency.

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Analysis

$$\text{Velocity of car} = \frac{60 \times 1000}{60} = 1000 \text{ m/min}$$
$$= \pi D_{wheel} N_{tyre}$$

Revolution of tyre,

$$N_{tyre} = \frac{1000}{\pi \times 0.65}$$
$$= 490 \text{ rotation per min.}$$

Using rear-to-axle ratio, the speed of car engine

$$N = \frac{490 \times 39}{8} = 2388 \text{ rpm}$$

Cross-sectional area of the cylinder

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.075)^2$$
$$= 0.00441 \text{ m}^2$$

(i) Indicated power

$$IP = p_{mi} \frac{L\, A\, n\, k}{60}$$
$$= 562.5 \times \frac{0.09 \times 0.00441 \times 2388 \times \frac{1}{2} \times 4}{60}$$
$$= \mathbf{17.776 \text{ kW}}$$

(ii) Indicated thermal efficiency (η_{ith}),

Time for 4-km distance

$$= \frac{\text{Distance}}{\text{Car speed}} = \frac{4 \text{ km}}{\frac{60}{3600} \text{ km/s}}$$
$$= 240 \text{ seconds}$$

$$\text{Fuel rate } \dot{m}_f = \frac{\text{Fuel mass}}{\text{time}} = \frac{0.250 \text{ kg}}{240 \text{ s}}$$
$$= 0.00104 \text{ kg/s}$$

$$\eta_{ith} = \frac{IP}{\dot{m}_f CV} = \frac{17.776}{0.00104 \times 44200}$$
$$= 0.386 \quad \text{or} \quad \mathbf{38.6\%}$$

Example 24.18 *The following data and results refer to a test on a single-cylinder, two-stroke cycle engine:*

indicated mean effective pressure = 550 kPa
cylinder diameter = 21 cm
piston stroke = 28 cm
engine speed = 360 rpm
brake torque = 628 Nm
fuel consumption = 8.16 kg/h
calorific value of fuel = 42700 kJ/kg

Calculate

(a) mechanical efficiency,
(b) the indicated thermal efficiency,
(c) the brake thermal efficiency, and
(d) brake-specific fuel consumption in kg/kWh.

Solution

Given A single-cylinder, two-stroke cycle engine

$p_{mi} = 550$ kPa
$T = 628$ Nm
$N = 360$ rpm
$n = N = 360$ working stroke/min
$d = 21$ cm $= 0.21$ m
$L = 28$ cm $= 0.28$ m
$\dot{m}_f = 8.16$ kg/h
$CV = 42700$ kJ/kg
$k = 1$

To find

(i) Mechanical efficiency,
(ii) Indicated thermal efficiency,
(iii) Brake thermal efficiency, and
(iv) Brake-specific fuel consumption.

Analysis Cross-sectional area of the cylinder

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.21)^2 = 0.0346 \text{ m}^2$$

(i) Indicated power

$$IP = p_{mi} \frac{L\, A\, n\, k}{60}$$
$$= 550 \times \frac{0.28 \times 0.0346 \times 360 \times 1}{60}$$
$$= 32.0 \text{ kW}$$

Brake power

$$BP = \frac{2\pi\, N\, T}{60{,}000} = \frac{2\pi \times 360 \times 628}{60{,}000}$$
$$= 23.675 \text{ kW}$$

Mechanical efficiency, η_{mech}:

$$\eta_{mech} = \frac{BP}{IP} = \frac{(23.675 \text{ kW})}{(32.0 \text{ kW})}$$
$$= 0.7397 \approx \mathbf{74\%}$$

(ii) Indicated thermal efficiency (η_{ith});

$$\eta_{ith} = \frac{IP}{\dot{m}_f CV}$$
$$= \frac{(32.0 \text{ kW})}{\left(\frac{8.16}{3600} \text{ kg/s}\right) \times (42700 \text{ kJ/kg})}$$
$$= 0.330 = \mathbf{33\%}$$

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(iii) Brake thermal efficiency (η_{bth});

$$\eta_{bth} = \frac{BP}{\dot{m}_f \, CV}$$

$$= \frac{(23.675 \text{ kW})}{\left(\frac{8.16}{3600} \text{ kg/s}\right) \times (42700 \text{ kJ/kg})}$$

$$= 0.244 = \mathbf{24.4\%}$$

(iv) Brake-specific fuel consumption ($Bsfc$);

$$Bsfc = \frac{\dot{m}_f \text{ (kg/h)}}{BP \text{ (kW)}} = \frac{(8.16 \text{ kg/h})}{(23.675 \text{ kW})}$$

$$= \mathbf{0.3446 \text{ kg/kWh}}$$

Example 24.19 *The following observations were recorded during a test on a single cylinder, four-stroke oil engine:*

Bore	*300 mm*
Stroke	*450 mm*
Speed	*300 rpm*
Imep	*6 bar*
brake load	*1.5 kN*
Brake drum diameter	*1.8 m*
Brake rope diameter	*2 cm*

Calculate

(a) Indicated power,

(b) brake power, and

(c) mechanical efficiency.

Solution

Given A single-cylinder oil engine with following data:

$d = 300 \text{ mm} = 0.3 \text{ m}$

$L = 450 \text{ mm} = 0.45 \text{ m}$

$N = 300 \text{ rpm}$

$p_{mi} = 6 \text{ bar} = 600 \text{ kPa}$

$W_{brake} = 1.5 \text{ kN}$

$D_{brake} = 1.8 \text{ m}$, $d_{rope} = 2 \text{ cm} = 0.02 \text{ m}$

$k = 1$

$n = \frac{N}{2}$

To find

(i) IP (ii) BP (iii) η_{mech}

Analysis

(i) The indicated power,

$$IP = p_{mi} \frac{L \, A \, n \, k}{60}$$

$$= 600 \times 0.45 \times \frac{\pi}{4} \times \frac{(0.3)^2}{60} \times \frac{300}{2} \times 1$$

$$= \mathbf{47.71 \text{ kW}}$$

(ii) The brake power,

$$BP = \frac{2\pi \, N \, T}{60,000}$$

where, T = brake load × effective brake radius

$$R_{brake} = \frac{D_{brake} + d_{rope}}{2} = \frac{1.8 + 0.0\,2}{2}$$

$$= 0.91 \text{ m}$$

$$T = 1.5 \times 0.91 = 1.365 \text{ kNm}$$

$$= 1365 \text{ Nm}$$

Thus $$BP = \frac{2\pi \times 300 \times 1365}{60,000} = \mathbf{42.88 \text{ kW}}$$

(iii) Mechanical efficiency,

$$\eta_{mech} = \frac{BP}{IP} = \frac{42.88}{47.71} = 0.8988$$

$$= \mathbf{89.88\%}$$

Example 24.20 *A single-cylinder, four-stroke Diesel engine works on the following data:*

Cylinder bore	*= 15 cm*
Stroke	*= 25 cm*
Speed	*= 250 rpm*
Area of indicator diagram	*= 6 cm^2*
Length of the indicator diagram	*= 9 cm*
Spring constant	*= 7.5 bar/cm*
Brake specific fuel consumption	*= 0.24 kg/kWh*
CV of fuel	*= 42000 kJ/kg*
Diameter of brake wheel	*= 70 cm*
Rope diameter	*= 3.5 cm*
Brake load	*= 40 kg (392.4 N)*

Calculate

(a) brake power,

(b) indicated mean effective pressure,

(c) indicated power,

(d) mechanical efficiency, and

(e) indicated thermal efficiency.

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Solution

Given A single-cylinder, four-stroke Diesel engine:

$n = \frac{N}{2}$ $k = 1$

$d = 15$ cm $= 0.15$ m $L = 25$ cm $= 0.25$ m

$N = 250$ rpm $A_i = 6$ cm^2

$l = 9$ cm Spring constant $= 7.5$ bar/cm

$Bsfc = 0.24$ kg/kWh $CV = 42000$ kJ/kg

$D_{brake} = 70$ cm $d_{rope} = 3.5$ cm

$W_{brake} = 392.4$ N

To find

(i) BP (ii) $Imep$ (iii) IP

(iv) η_{mech} (v) η_{ith}

Analysis

Effective brake radius

$$R_{brake} = \frac{D_{brake} + d_{rope}}{2} = \frac{70 + 3.5}{2}$$

$= 36.75$ cm $= 0.3675$ m

$T =$ Load × Effective brake radius

$= W_{brake}\, R_{brake}$

$T = 392.4 \times 0.3675 = 144.2$ Nm

(i) The brake power

$$BP = \frac{2\pi N T}{60,000}$$

$$\therefore \quad BP = \frac{2\pi \times 250 \times 144.2}{60000} = \mathbf{3.775\ kW}$$

(ii) Indicated mean effective pressure (*Imep*)

$$p_{mi} = \frac{\text{Area of indicator diagram}}{\text{Length of the indicator diagram}} \times \text{Spring constant}$$

$$= \frac{6\ \text{cm}^2}{9\ \text{cm}} \times 7.5\ \text{bar/cm}$$

$= \mathbf{5.0\ bar}$ or **500kPa**

(iii) Indicated power (*IP*)

$$IP = p_{mi} \frac{L\, A\, n\, k}{60}$$

$$= 500 \times 0.25 \times \frac{\pi}{4} \times \frac{(0.15)^2}{60} \times \frac{250}{2} \times 1$$

$= \mathbf{4.6\ kW}$

(iv) Mechanical efficiency (η_{mech})

$$\eta_{mech} = \frac{BP}{IP} = \frac{3.775}{4.6} = 0.820 = \mathbf{82.0\%}$$

(v) Indicated thermal efficiency (η_{ith});

Given $Bsfc = \frac{\dot{m}_f}{BP}$

or $0.24 = \frac{\dot{m}_f}{3.775}$

or $\dot{m}_f = 0.906$ kg/h $= 2.516 \times 10^{-4}$ kg/s

Thus $\eta_{ith} = \frac{IP}{\dot{m}_f \times CV}$

$$= \frac{4.6}{2.516 \times 10^{-4} \times 42000}$$

$= 0.435$ or **43.5%**

Example 24.21 *A full-load test was conducted on a two-stroke engine and the following results were obtained:*

Speed $= 500$ *rpm*

Brake load $= 500$ *N*

imep $= 3$ *bar*

Oil consumtion $= 5$ *kg/h*

Jacket water temperature rise $= 35°C$

Jacket water flow rate $= 7$ *kg/min.*

A/F ratio by mass $= 30$

Exhaust gas temperature $= 350°C$

Room temperature $= 25°C$

Atmospheric pressure $= 1$ *bar*

Cylinder diameter $= 22$ *cm*

Stroke $= 28$ *cm*

Brake diameter $= 1.6$ *m*

CV of fuel $= 42000$ *kJ/kg*

Proportion of H_2 *by mass in fuel* $= 15\%$

Specific heat of exhaust gas $= 1.0$ *kJ/kg·K*

Specific heat of dry steam $= 2.0$ *kJ/kg·K*

Calculate

(a) indicated thermal efficiency

(b) Specific fuel consumtion

(c) Volumetric efficiency based on atmospheric conditions

Draw up a heat balance sheet for test.

Solution

Given A single-cylinder, two-stroke Diesel engine:

$n = N = 500$ rpm A/F $= 30$

$d = 22$ cm $= 0.22$ m $L = 28$ cm $= 0.28$ m

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p_{mi} = 3 bar = 300 kPa $\dot{m}_f$ = 5 kg/h
CV = 42000 kJ/kg W_{brake} = 500 N
D_{brake} = 1.6 m $\dot{m}_w$ = 7 kg/min
T_g = 350°C $(\Delta T)_w$ = 35°C
T_a = 25°C = 298 K p_a = 1 bar = 100 kPa
C_{pg} = 1.0 kJ/kg·K C_{ps} = 2.0 kJ/kg·K

To find

(i) Indicated thermal efficiency,
(ii) Specific fuel consumption,
(iii) Volumetric efficiency based on atmospheric conditions, and
(iv) Heat balance sheet.

Assumptions

(i) Single cyclinder engine, i.e. $k = 1$
(ii) Specific gas constant for air, $R = 0.287$ kJ/kg·K
(iii) Specific heat of water $C_{pw} = 4.187$ kJ/kg·K

Analysis The swept volume rate

$$\dot{V}_s = \frac{\pi}{4} d^2 L n k = \frac{\pi}{4} \times (0.22)^2 \times 0.28 \times \frac{500}{60} \times 1$$
$$= 0.0887 \text{ m}^3/\text{min}$$

(i) *Indicated power*

$$IP = p_{mi} \dot{V}_s = 300 \times 0.0887 = 26.6 \text{ kW}$$

Heat supplied by fuel

$$\dot{Q}_{in} = \dot{m}_f CV = 5 \times 42000$$
$$= 210000 \text{ kJ/h} = 58.33 \text{ kW}$$

Indicated thermal efficiency

$$\eta_{ith} = \frac{IP}{\dot{Q}_{in}} = \frac{26.6 \text{ kW}}{(58.33 \text{ kJ/s})}$$
$$= 0.456 \quad \text{or} \quad \mathbf{45.6\%}$$

(ii) *Specific fuel consumption*

The brake drum radius

$$R_{brake} = \frac{D_{brake}}{2} = \frac{1.6}{2} = 0.8 \text{ m}$$

Brake torque T = Load × Effective brake radius
$= W_{brake} R_{brake}$
$= 500 \times 0.8 = 400$ N-m

Brake power

$$BP = \frac{2\pi N T}{60{,}000} = \frac{2\pi \times 500 \times 400}{60000}$$
$$= 20.94 \text{ kW}$$

The brake-specific fuel consumtion

$$Bsfc = \frac{\dot{m}_f}{BP} = \frac{(5 \text{ kg/h})}{(20.94 \text{ kW})}$$
$$= \mathbf{0.238 \text{ kg/kWh}}$$

(iii) *Volumetric efficiency*

Actual mass flow rate into engine

$$\dot{m}_a = (\text{A/F})\ \dot{m}_f = 30 \times 5 = 150 \text{ kg/h}$$
$$= 0.4167 \text{ kg/s}$$

Actual volume flow rate of air;

$$\dot{V}_a = \frac{\dot{m}_a R T_a}{p_a} = \frac{0.4167 \times 0.287 \times 298}{100}$$
$$= 0.03563 \text{ m}^3/\text{s}$$

The volumetric efficiency

$$\eta_{vol} = \frac{\dot{V}_a}{\dot{V}_s} = \frac{0.03563}{0.0887}$$
$$= 0.4017 \quad \text{or} \quad \mathbf{40.17\%}$$

Heat Balnace sheet: On minute basis;

Heat supplied per minute by fuel

$$\dot{Q}_{in} = \dot{m}_f CV = \left(\frac{5}{60} \text{ kg/min}\right) \times 42000$$
$$= 3500 \text{ kJ/min}$$

Heat equivalent to BP

$$\dot{Q}_1 = BP \times 60 = 20.96 \times 60$$
$$= 1257.6 \text{ kJ/min}$$

Heat lost to cooling water

$$\dot{Q}_2 = \dot{m}_w C_{pw} (\Delta T) = 7 \times 4.187 \times 35$$
$$= 1025.81 \text{ kJ/min}$$

Heat lost to exhaust gases

Mass of exhaust gases formed/kg of fuel

$$m_{ex} = \dot{m}_f \times \frac{A}{F} = \left(\frac{5}{60} \text{ kg/min}\right) \times 30$$
$$= 2.5 \text{ kg/min}$$

Mass of H_2O formed during combustion

m_{ex} = 9H_2 × Mass of fuel used per minute
$= 9 \times 0.15 \times \frac{5}{60} = 0.1125$ kg/min

Total mass of exhaust gases (dry) per minute

m_g = Mass of wet exhaust – Mass of H_2O formed
= 2.5 – 0.1125 = 2.3875 kg/min

(a) Heat lost to dry exhaust gases

$$\dot{Q}_3 = \dot{m}_g C_{pg} (\Delta T)_g$$
$$= 2.3875 \times 1.0 \times (350 - 25)$$
$$= 775.93 \text{ kJ/min}$$

(b) H_2O formed exit in superheated state at 350°C, thus heat carried away by steam

$$\dot{Q}_4 = \dot{m}_{H_2O} (C_{pw} (T_{sat} - T_a) + h_{fg} + C_{ps} (T_g - T_a))$$
$$= 0.1125 \ [(4.187 \times (100 - 25) + 2257 + 2.0 \times (350 - 25)]$$
$$= 362.36 \text{ kJ/min}$$

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Heat Balance Sheet

Particulars	*Quntity*	*Percentage*
Credit (input)		
Heat supplied by fuel	*3500 kJ/min*	*100%*
Debit (output)		
Heat equivalent to BP	1256.4 kJ/min	35.9%
Heat carried by coolant	1025.81 kJ/min	29.3%
Heat carried away by dry flue gases	773.93 kJ/min	22.17%
Heat carried away by steam	362.3 kJ/min	10.35%
Unaccounted heat lost	79.8 kJ/min	2.28%

***Example* 24.22** *The brake power of a 6-cylinder, 4-stroke Diesel engine, absorbed by a hydraulic dynamometer is given by*

$$BP = \frac{WN}{20{,}000}\ (kW)$$

where W is the brake load in newtons and N is the rotational speed in rpm. The air consumption is measured by an air box with a sharp edge orifice. The following readings were recorded.

Bore of cylinder = 9.5 cm
Stroke of piston = 12 cm
Speed of engine = 2400 rpm
Brake load = 550 N
Ambient pressure = 75 cm of Hg
Fuel density = 0.83 kg/litre
Time for 100 cc fuel consumtion = 19.3 seconds
Room temperature = 25°C
C : H ratio in fuel mass = 83 : 17
Orific diameter = 3 cm
C_d *(orific)* = 0.6
Manometer head across the orifice = 14.5 cm of Hg

Calculate

(a) Brake mean effective pressure
(b) Brake specific fuel consumtion
(c) Percentage excess air
(d) Volumetric efficiency

Assume R = 0.287 kJ/kg·K for air.

Solution

Given A six-cylinder, four-stroke Diesel engine:

$n = \frac{N}{2}$, $R = 0.287$ kJ/kg·K
$k = 6$
$d = 9.5$ cm = 0.095 m
$L = 12$ cm = 0.12 m
$N = 2400$ rpm
$W = 550$ N
$p_a = 75$ cm of Hg
$\rho_f = 0.83$ kg/lit.
$t = 19.3$ s/100 cc consumtion
$T_a = 25°C = 298$ K
$d_o = 3$ cm = 0.03 m
$C_d = 0.6$ cm
$h_o = 14.5$ cm of Hg
C : H ratio = 83 : 17 by mass in fuel

To find

(i) *Brake mean effective pressure,*
(ii) Brake-specific fuel consumption, (*Bsfc*),
(iii) *Percentage excess air*, and
(iv) Volumetric efficiency, η_{vol}.

Analysis

(i) *The brake mean effective pressure*

The brake power is given by

$$BP = \frac{WN}{20{,}000}\ (\text{kW})$$

Using numerical values,

$$BP = \frac{550 \times 2400}{20{,}000} = 66 \text{ kW}$$

The brake power is also given by

$$BP = \frac{p_{mb} L A n k}{60}$$

$$\text{or}\quad p_{mb} = \frac{66 \times 60}{0.12 \times \frac{\pi}{4} \times (0.095)^2 \times \frac{2400}{2} \times 6}$$

$$= 646.61 \text{ kPa} \quad \text{or} \quad \mathbf{6.47 \text{ bar}}$$

(ii) *Brake specific fuel consumtion*

The mass-flow rate of fuel

$$\dot{m}_f = \frac{100 \text{ cc}}{(1000 \text{ cc/lit})} \times (0.83 \text{ kg/lit}) \times \left(\frac{3600 \text{ s/h}}{19.3 \text{ s}}\right)$$

$$= 15.48 \text{ kg/h}$$

$$Bsfc = \frac{\dot{m}_f \text{ (kg/h)}}{BP \text{ (kW)}} = \frac{15.48}{66} = \mathbf{0.2345 \text{ kg/kWh}}$$

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(iii) *Prercentage excess air*

Pressure of ambient air;

$$p_a = \frac{750 \text{ mm of Hg}}{760 \text{ mm of Hg}} \times 101.325 \text{ kPa}$$

$$= 99.9917 \cong 100 \text{ kPa}$$

Density of ambient air,

$$\rho_a = \frac{p_a}{RT_a} = \frac{100}{0.287 \times 298} = 1.17 \text{ kg/m}^3$$

Since fuel contains 0.83 kg of carbon and 0.17 kg of hydrogen in 1 kg of fuel, thus the theoretical mass of air required per kg of fuel for complete combustion

$$m_{th} = \frac{100}{23}\left[\frac{8}{3}C + 8H\right]$$

$$= \frac{100}{23} \times \left[\frac{8}{3} \times 0.83 + 8 \times 0.17\right]$$

$$= 15.53 \text{ kg/kg of fuel}$$

Theoretical air supply rate to burn the fuel completely

$$\dot{m}_{th} = m_{th} \times \dot{m}_f$$

$$= 15.53 \times 15.48 = 240.4 \text{ kg/h}$$

The head of air in orifice meter

$$h_a = \frac{h_{Hg}\rho_{Hg}}{\rho_a} = \frac{(0.145 \text{ m}) \times (13600 \text{ kg/m}^3)}{1.17 \text{ kg/m}^3}$$

$$= 1685.4 \text{ m}$$

Actual volume of air supplied through orificre meter

$$\dot{V}_a = \frac{\pi}{4} d_o^2 C_d \sqrt{2g h_a}$$

$$= \frac{\pi}{4} \times (0.03)^2 \times 0.6 \times \sqrt{2 \times 9.81 \times 1685.4}$$

$$= 0.077 \text{ m}^3/\text{s}$$

The actual mass-flow rate of air

$$\dot{m}_a = \rho_a \dot{V}_a = 1.17 \times 0.077$$

$$= 0.0902 \text{ kg/s} \quad \text{or} \quad 324.85 \text{ kg/h}$$

Percentage excess air supplied

$$= \frac{324.85 - 240.4}{240.4} \times 100 = \mathbf{35.12\%}$$

(iv) *Volumetric efficiency* (η_{mech})

Swept volume per minute

$$\dot{V}_s = \frac{\pi}{4} \times d^2 L n k$$

$$= \frac{\pi}{4} \times (0.095)^2 \times 0.12 \times \frac{2400}{2} \times 6$$

$$= 6.124 \text{ m}^3/\text{min}$$

Actual volume suction rate

$$\dot{V}_f = 0.077 \text{ m}^3/\text{s} \times (60 \text{ s/min}) = 4.62 \text{ m}^3/\text{min}$$

$$\eta_{vol} = \frac{\dot{V}_a}{\dot{V}_s} = \frac{4.62}{6.124} = 0.754 \quad \text{or} \quad \mathbf{75.4\%}$$

Example 24.23 *A 6-cylinder, four-stroke Diesel engine has a bore of 33.75 cm and a stroke of 37.5 cm. It is tested at half-load conditions and gave the following observations:*

Brake power	$= 142$ *kW*
Engine speed	$= 350$ *rpm*
Indicated mean effective pressure	$= 3.72$ *bar*
Fuel consumption rate	$= 44$ *kg/h*
Calorific value of fuel	$= 44800$ *kJ/kg*
Air consumption rate	$= 38.6$ *kg/min*
Water flow rate in the jacket	$= 60.2$ *kg/min*
Rise in temperature of water	$= 32°C$
Piston cooling oil flow rate	$= 34.96$ *kg/min*
C_p *of piston oil*	$= 2.1$ *kJ/kg·K*
Rise in cooling oil temperature	$= 20\,°C$
Exhaust gas temperature	$= 210\,°C$
Ambient air temperature	$= 25°C$
C_p *of exhaust gases*	$= 1.05$ *kJ/kg·K*
C_p *of cooling water*	$= 4.187$ *kJ/kg·K*

Fuel contains 14% H_2 by mass.

Prepare the heat balance sheet on minute and percentage basis . Calculate the specific fuel consumption at full load, assuming frictional power and indicated thermal efficiency do not change with load variation.

Assume partial pressure of water vapour in exhaust gases to be 0.06 bar.

Solution

Given Six-cylinder, four-stroke Diesel engine:

$k = 6$	$N = 350$ rpm, $n = \frac{N}{2}$
$d = 33.75$ cm $= 0.3375$ m	$L = 37.5$ cm $= 0.375$ m
$p_{mi} = 3.72$ bar $= 372$ kPa	$BP = 142$ kW
Loading = Half	$\dot{m}_f = 44$ kg/h
$CV = 44800$ kJ/kg	$\dot{m}_a = 38.6$ kg/min
$\dot{m}_w = 60.2$ kg/min	$\dot{m}_{oil} = 34.96$ kg/min
$(\Delta T)_w = 32°C$	$(\Delta T)_{oil} = 20°C$
$T_g = 210°C$	$T_a = 25°C = 298$ K

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p_v = 0.06 bar = 6 kPa C_{pw} = 4.187 kJ/kg·K

C_{pg} = 1.05 kJ/kg·K $C_{p,oil}$ = 2.1 kJ/kg·K

mass of $H_2 = 0.14\, m_f$

To find

(i) Heat balance sheet, and

(ii) Specific fuel consumtion at full load.

Analysis

(i) *Indicated power*

$$IP = p_{mi} L A \frac{n}{60} k$$

$$= 372 \times 0.375 \times \frac{\pi}{4} \times (0.3375)^2 \times \frac{350}{2 \times 60} \times 6$$

$$= 218.4 \text{ kW}$$

Heat supplied by fuel

$$Q_{in} = \dot{m}_f CV = \frac{44}{60} \times 44800$$

$$= 32853.3 \text{ kJ/min}$$

The heat equivalent to *BP*

$$\dot{Q}_1 = 142 \times 60 = 8520 \text{ kJ/min}$$

Heat carried by cooling water

$$\dot{Q}_2 = \dot{m}_w C_{pw} (\Delta T)_w$$

$$= 60.2 \times 4.187 \times 32$$

$$= 8065.83 \text{ kJ/min}$$

Heat carried by cooling oil

$$\dot{Q}_3 = \dot{m}_{oil} C_{p,oil} (\Delta T)_{oil}$$

$$= 34.96 \times 2.1 \times 20$$

$$= 1468.32 \text{ kJ/min}$$

Mass of exhaust gases formed per minute

$$\dot{m}_{ex} = \dot{m}_a + \dot{m}_f$$

$$= 38.6 + \frac{44}{60} = 39.33 \text{ kg/min}$$

The amount of water vapour formed;

$$\dot{m}_{H_2O} = 9\, H_2 = 9 \times 0.14 \times \frac{44}{60}$$

$$= 0.924 \text{ kg/min}$$

Mass of dry exhaust gases;

$$\dot{m}_g = \dot{m}_{ex} - \dot{m}_{H_2O} = 39.33 - 0.924$$

$$= 38.409 \text{ kg/min}$$

Heat carried by dry exhaust gases

$$\dot{Q}_4 = \dot{m}_g C_{pg} (T_g - T_a)$$

$$= 38.409 \times 1.05 \times (210 - 25)$$

$$= 7460.95 \text{ kJ/min}$$

Heat carried by superheated water vapour at 0.06 bar in exhaust gases

$$\dot{Q}_5 = \dot{m}_{H_2O} h_{sup}$$

where $h_{sup} = h_{g@0.06 \text{ bar}} + C_{ps}(T_{sup} - T_a)$

$$= 2567.5 + 2.1 \times (210 - 25)$$

$$= 2985.5 \text{ kJ/min}$$

Then $\dot{Q}_5 = 0.924 \times 2985.5 = 2758.6$ kJ/min

Heat Balance Sheet

Particulars	*Quantity*	*Percentage*
Credit (input) *Heat supplied by fuel,* $\dot{Q}_{in}$	*32853.3 kJ/min*	*100%*
Debit (output)		
Heat equivalent to BP, $\dot{Q}_1$	8520 kJ/min	25.93%
Heat carried by cooling water, $\dot{Q}_2$	8065.83 kJ/min	24.55%
Heat carried by cooling oil, $\dot{Q}_3$	1468.32 kJ/min	4.47%
Heat carried away by dry flue gases, $\dot{Q}_4$	7460.95 kJ/min	22.70%
Heat carried away by steam formed, $\dot{Q}_5$	2758.6 kJ/min	8.40%
Unaccounted heat lost = $\dot{Q}_{in} - \dot{Q}_1 - \dot{Q}_2 - \dot{Q}_3 - \dot{Q}_4 - \dot{Q}_5$	4579.6 kJ/min	13.94%

(ii) *Specific fuel consumtion at full load:*

Brake power at full load conditions

$BP_{full} = 2\, BP_{half} = 2 \times 142 = 284$ kW

Frictional power at half-load condition

$FP = IP_{half} - BP_{half} = 218.4 - 142 = 76.4$ kW

Since the *FP* remains constant, thus *IP* at full-load condtions

$IP_{full} = BP_{full} + FP = 284 + 76.4 = 360.4$ kW

Indicated thermal efficiency at half-load conditions

$$\eta_{ith} = \frac{IP_{half}}{\dot{Q}_{in}} = \frac{(218.4 \times 60 \text{ kJ/min})}{(32853.6 \text{ kJ/min})}$$

$$= 0.3988$$

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The heat input rate at full load

$$\dot{Q}_{in,\,full} = \frac{IP_{full}}{\eta_{ith}} = \frac{360.4\text{ kW}}{0.3988} = 903.71\text{ kW}$$

Fuel consumption rate at full load

$$\dot{m}_{f,\,full} = \frac{\dot{Q}_{in,full}}{CV} = \frac{903.71\text{ kW}}{44800\text{ kJ/kg}}$$
$$= 0.02017\text{ kg/s or } 72.62\text{ kg/h}$$

The brake specific fuel consumtion

$$bsfc = \frac{\dot{m}_{f,\,full}}{BP_{full}} = \frac{(72.62\text{ kg/h})}{(284\text{ kW})}$$
$$= \mathbf{0.255\text{ kg/kWh}}$$

Example 24.24 *A 4-cylinder, four-stroke engine has an 85-mm bore and 130-mm stroke. It develops 30 kW of power at 1500 rpm and it runs at 20% rich mixture. If the volume of air in the cylinder measured at 17°C and 762 mm of mercury is 70% of the swept volume, theoretical air–fuel ratio is 14.8, calorific value of fuel is 45000 kJ/kg and mechanical efficiency of the engine is 90%, calculate (a) indicated thermal efficiency, and (b) brake mean effective pressure.*

Assume R = 0.287 kJ/kg·K for air.

Solution

Given Four-cylinder, four-stroke engine

$k = 4$
$N = 1500$ rpm, $n = \frac{N}{2}$
$d = 85\text{ mm} = 0.085\text{ m}$
$L = 130\text{ mm} = 0.13\text{ m}$
$\dot{m}_{f,act} = 1.2\ \dot{m}_{f,th}$
$BP = 30\text{ kW}$
$CV = 45000\text{ kJ/kg}$
$T_a = 17°\text{C} = 290\text{ K}$
$p_a = 762\text{ mm of Hg}$
$V_{act} = 0.7\,V_s$
$A/F = 14.8$
$\eta_{mech} = 0.90$
$R = 0.287\text{ kJ/kg·K}$

To find

(i) Indicated thermal efficiency,
(ii) Brake mean effective pressure.

Analysis The atmospheric pressure corresponds to 762 mm of Hg

$$p_a = \frac{762}{760} \times 101.325\text{ kPa} = 101.59\text{ kPa}$$

Swept volume of four cylinders of engine

$$V_s = 4 \times \frac{\pi}{4} d^2 L = 4 \times \frac{\pi}{4} \times (0.085)^2 \times 0.13$$
$$= 2.95 \times 10^{-3}\text{ m}^3$$

Actual volume of air drawn

$$V_a = 0.7\,V_s = 0.7 \times 2.95 \times 10^{-3}$$
$$= 2.065 \times 10^{-3}\text{ m}^3$$

The mass of air inducted in cylinders

$$m_a = \frac{p_a V_a}{RT_a} = \frac{101.59 \times 2.065 \times 10^{-3}}{0.287 \times 290}$$
$$= 2.52 \times 10^{-3}\text{ kg}$$

Theoretical mass of air inducted per minute

$$\dot{m}_a = m_a \frac{N}{2} = 2.52 \times 10^{-3} \times \frac{1500}{2}$$
$$= 1.89\text{ kg/min}$$

Theoretical fuel used per minute

$$\dot{m}_{f,\,th} = \frac{\dot{m}_a}{\text{A/F}} = \frac{1.89}{14.8} = 0.1277\text{ kg/min}$$

When using 20% rich mixture, then

$$\dot{m}_{f,\,act} = 1.2\,\dot{m}_{f,\,th}$$
$$= 1.2 \times 0.1277 = 0.1532\text{ kg/min}$$

or $= 2.554 \times 10^{-3}\text{ kg/min}$

Indicated power

$$IP = \frac{BP}{\eta_{mech}} = \frac{30}{0.9} = 33.33\text{ kW}$$

(i) *Indicated thermal efficiency*

Heat supplied by fuel

$$\dot{Q}_{in} = \dot{m}_{f,\,act}\,CV = 2.554 \times 10^{-3} \times 45000$$
$$= 114.93\text{ kW}$$

Indicated thermal efficiency of the engine

$$\eta_{ith} = \frac{IP}{\dot{Q}_{in}} = \frac{33.33\ kW}{114.93\ kW} = 0.29 \text{ or } \mathbf{29\%}$$

(ii) *Brake mean effective pressure*

Brake power is given by

$$BP = p_{mb} L A \frac{n}{60} k = p_{mb} V_s \frac{n}{60}$$

or $$p_{mb} = \frac{BP}{V_s \frac{n}{60}} = \frac{30}{2.95 \times 10^{-3} \times \frac{1500}{2 \times 60}}$$
$$= \mathbf{813.56\text{ kPa}}$$

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24.20 SUPERCHARGING

The charging of an engine refers to supply of charge into the engine cylinder during suction stroke. In a naturally aspirated engine, the charge is inducted into the cylinder below atmospheric pressure. Thus the mass of charge is less than that corresponding to atmospheric pressure. Therefore, the volumetric efficiency of the engine is less than 100%.

Supercharging is a process which supplies the charge above atmospheric pressure. Super means better. So supercharge means increased mass of charge that is inducted into the cylinder during the suction stroke. The mass of charge can be increased by increasing the density of charge. The charge is compressed prior to admission into the cylinder, thereby enabling the greater mass of charge with same total piston displacement. The device used for compressing the charge is called supercharger.

The supercharging is used in racing cars and in heavy-duty diesel engines.

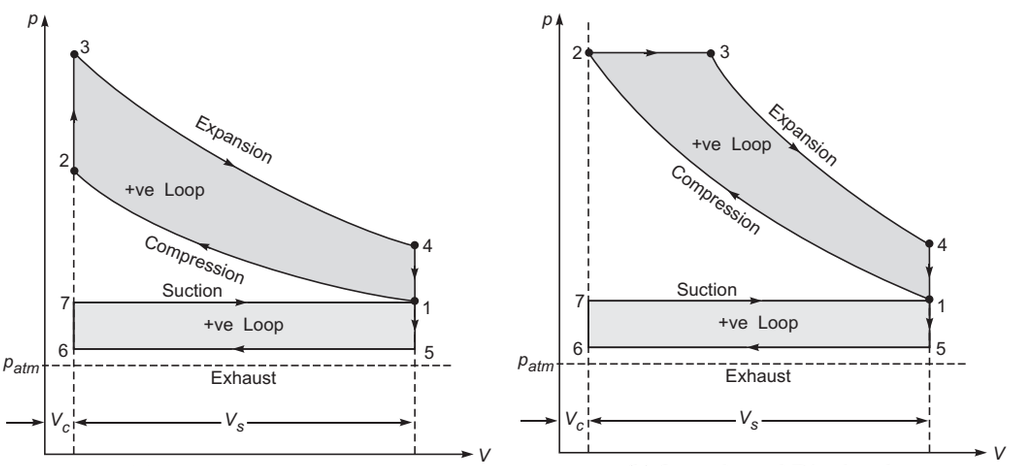
Objectives of Supercharging

1. It is used to overcome the effects of high altitude, such as high aircrafts and engines run on mountains.
2. It reduces the weight of engine per kW power developed.
3. It is used to reduce the size of engine to fit in a limited space such as in marine engines.
4. It is used to increase the power output from an existing engine.

The supercharged Otto and Diesel cycles are shown in Fig. 24.43.

Difference between Naturally Aspirated Engine and Supercharged Engine

Sr. No.	*Aspect*	*Naturally aspirated or conventional engine*	*Supercharged engine*
1.	Volumetric efficiency	It is always less than 100%	In most of the cases, it is more than 100%
2.	Mechanical efficiency	Less	Sightly better than naturally aspirated engines
3.	Combustion	Depends on turbulence of charge	Always good due to close spacing of fuel molecules
4.	Output	Less	More power output
5.	Peak pressure	Less	Higher
6.	Peak temperature	Less	Much higher



(a) Supercharge otto cycle *(b) Supercharged Diesel cycle*

Fig. 24.43 *p–V diagrams for supercharged engines*

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Summary

- An internal combustion engine is a heat engine that converts chemical energy of fuel into mechanical energy.
- The two-stroke engine completes its working cycle in two strokes of the piston. The three ports; inlet, transfer, and exhaust ports are used for suction, transfer and discharge of charge, respectively. A deflector-shapped piston is used to direct the charge inside the cylinder.
- The four-stroke engine completes its working cycle in four strokes of the piston as suction, compression, expansion and exhaust stroke. They are widely used on motor cycles, cars, buses, trucks and aeroplanes. Due to good thermal efficiency of four-stroke engines, the specific fuel consumption is less.
- Petrol engines use low compression ratio in the range of 4 to 10, while Diesel engines use high compression ratio usually in the range of 14 to 21. The petrol engines induct carburetted homogeneous air–fuel mixture as charge into the cylinder while Diesel engines induct only air during suction, and diesel is injected at the end of the compression stroke. The fuel burns in the presence of hot air. Therefore, the Diesel engines are also called compression ignition (CI) engines.
- The petrol engines are quantity-governing engines, while Diesel engines are quality-governing engines. The hit-and-miss governing is used for high speed gas engines.
- The ignition system is used to produce the high-intensity spark for initiation of combustion in the petrol engine. The battery ignition system is used on heavy-duty engines, while magneto ignition system is used on two wheelers, racing cars and aircrafts.
- The internal combustion engines are subjected to very high temperature during combustion of charge. Due to overheating of engine, there may be uneven expansion in some parts, burning of lubricant, valve seats, etc. Therefore, the engine should be provided with adequate cooling arrangement.
- The air-cooling system is simple, cheaper and maintenance-free and is widely used on two wheelers and light-duty engines.
- The heavy-duty engines release a large amount of heat during combustion, and therefore, they are water cooled. The water-cooling arrangement consists of a pump, a fan, a water jacket around the engine and a radiator.
- The lubrication minimizes the friction between moving parts. The mist lubrication system is used in two-stroke engines, while all four-stroke engines use wet or dry sump lubrication system.
- The supercharging refers to a process which supplies the charge to the cylinder above atmospheric pressure. Thus, the volumetric efficiency of the engine improves and the engine produces more power.
- Scavenging is a gas exchange process into the cylinder of the engine where the admitted charge pushes the combustion products out of the cylinder. Scavenging takes place in two-stroke engines at the end of the expansion stroke. Poor scavenging may lead to dilution of charge, thus less power is obtained from the engine.

Glossary

A/F Mass ratio of air to fuel

BDC Bottom dead centre

Bore Internal diameter of cylinder

Brake power Power available from the engine for external use

CI engine Compression ignition (Diesel) engine

Carburation Preparation of combustible mixture for petrol engine

Clearance volume Volume left in the centre, when piston is at TDC

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Compression ratio Ratio of maximum volume to minimum volume in the cylinder

Friction power Difference between indicated power and brake power

Fuel injector Device to inject the pressurised fuel into cylinder

Fuel pump Device to pressurise the fuel for injection

IC engines Internal combustion reciprocating engines

Indicated power Power developed on the piston by combustion gases inside the cylinder

Magneto An electric generator, which converts kinetic energy into electrical energy

Mean effective pressure Ratio of net work done to swept volume in the cycle

Mechanical efficiency Ratio of brake power to the indicated power

SI engine Spark ignition (petrol) engine

Scavenging Gas exchange process in two-stroke engines

Specific fuel consumption Mass of fuel (kg/h) consumed per unit power output (BP)

Stoichiometric air Amount of air just sufficient for complete combustion

Stroke Linear distance between TDC and BDC

Supercharging Charging of engine cylinder above atmospheric pressure

Swept volume Piston displacement volume in cylinder

TDC Top dead centre

Thermal efficiency Ratio of brake power to the heat-supply rate

Thermostat An instrument which controls the temperature

Review Questions

1. What is an internal combustion engine?
2. Write the classification of internal combustion engines.
3. List the parts of an IC engine.
4. Explain the construction, working and applications of a two-stroke petrol engine.
5. Explain construction, and working of a two-stroke Diesel engine.
6. Explain the working of a four-stroke petrol engine.
7. Discuss the construction of a four-stroke petrol engine.
8. Explain the working of a four-stroke Diesel engine.
9. Why are four-stroke engines preferred over two-stroke engines?
10. Compare petrol and Diesel engines.
11. Compare two-stroke and four-stroke engines.
12. Define charge. What do you mean by
    (a) Stoichiometric mixture
    (b) Rich mixture
    (c) Lean mixture
13. Draw the curves for best power and best economy on A/F ratio vs Power output plot.
14. Define brake-specific fuel consumption.
15. What is carburation? Draw a schematic for fuel-injection system in a Diesel engine.
16. Draw a simple carburetter and write the function of its parts.
17. Draw a neat diagram of a jerk-type fuel-injection pump and explain its working.
18. How are the following engines governed?
    (i) SI engines (ii) CI engines
19. Explain the abnormal combustion in SI engines.
20. What are the harmful effects of overheating of an engine? Explain.
21. Why is the cooling arrangement provided with each internal combustion engine?
22. What are the undesirable effects if an engine is undercooled?
23. Discuss the air-cooling system with its merits and demerits.
24. How is the liquid-cooling arrangement better than the air-cooling arrangement on an internal combustion engine?
25. Discuss water-cooling system for an internal combustion engine.
26. What are the advantages of increasing pressure of the coolant in a liquid-cooling system?

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27. What are the functions of lubrication?
28. What are the desirable properties of lubricating oil?
29. Explain the mist lubrication system.
30. Explain splash lubrication system.
31. Discuss the pressure-feed lubrication system.
32. What is an ignition system? What are the requirements of an ignition system for the SI engines?
33. Why is the battery ignition system preferred on most of the automobiles?
34. Explain the battery ignition system with a neat diagram.
35. Explain magneto ignition system.
36. What is supercharging? What are the effects of supercharging on engine performance?
37. Explain the working of a fuel pump.
38. Explain the working of a fuel injector.
39. What is scavenging? How does scavenging take place in a two-stroke petrol engine? Explain.
40. Explain (a) quality governing, and (b) quantity governing of IC engine.

Problems

1. The engine of a car has four cylinders of 70 mm bore, and 75 mm stroke. The compression ratio is 8. Determine the cubic capacity of the engine and the clearance volume of each cylinder.
[1154 cc, 41.21 cc]

2. A four-cylinder, four-stroke petrol engine has a bore of 80 mm and a stroke of 80 mm. The compression ratio is 8. Calculate the cubic capacity of the engine and clearance volume of each cylinder. What type of engine is this?
[(a) 1608.4 cc, (b) 57.4 cc, (c) Square engine]

3. A four-cylinder, four-stroke Diesel engine is to develop 30 kW at 1000 rpm. The stroke is 1.4 times the bore and the indicated mean effective pressure is 6.0 bar. Determine the bore and stroke of the engine. *[176 mm, 246 mm]*

4. A 42.5-kW engine has a mechanical efficiency of 85%. Find the indicated power and friction power. If the friction power is constant with the load, what will be the mechanical efficiency at 60% of the load? *[50 kW, 7.5 kW, 77.3%]*

5. Calculate the brake mean effective pressure of a four-cylinder, four-stroke Diesel engine having a 150-mm bore and 200-mm stroke which develops a brake power of 73.6 kW at 1200 rpm.
[5.206 bar]

6. An engine is using 5.2 kg of air per minute, while operating at 1200 rpm. The engine requires 0.2256 kg of fuel per hour to produce an indicated power of 1 kW. The air–fuel ratio 15 : 1. The indicated thermal efficiency is 38% and the mechanical efficiency is 80%. Calculate (a) brake power, and (b) heating value of the fuel.
[(a) 73.7 kW, (b) 41992.9 kJ/kg]

7. An engine develops an indicated power of 125 kW and delivers a brake power of 100 kW. Calculate (a) frictional power, and (b) mechanical efficiency of the engine. *[(a) 25 kW, (b) 80%]*

8. A single-cylinder, four-stroke cycle oil engine is fitted with a rope brake dynamometer. The diameter of the brake wheel is 600 mm and the rope diameter is 26 mm. The brake load is 170 N. If the engine runs at 450 rpm, calculate the brake power of the engine. *[2.5 kW]*

9. A single-cylinder, four-stroke Diesel engine having a displacement volume of 790 cc is tested at 300 rpm. When a braking torque of 49 Nm is applied, analysis of the indicator diagram gives a mean effective pressure of 980 kPa. Calculate the brake power and mechanical efficiency of the engine. *[(a) 1.54 kW, (b) 79.4%]*

10. A four-stroke petrol engine delivers a brake power of 36.8 kW with a mechanical efficiency of 80%. The air–fuel ratio is 15 : 1 and the fuel consumption is 0.4068 kg/kWh. The calorific value of the fuel is 42000 kJ/kg. Calculate (a) indicated power, (b) friction power, (c) brake thermal efficiency, (d) indicated thermal efficiency, and (e) total fuel consumption.
[(a) 46 kW, (b) 9.2 kW, (c) 21%, (d) 26.25%, (e) 15.12 kg/h]

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11. During a trial of four-stroke Diesel engine, the following observations were recorded:

Area of indicator diagram = 475 mm^2
Length of the indicator diagram = 62 mm
Spring constant = 1.1 bar/mm
Bore = 100 mm
Stroke = 150 mm
Speed = 375 rpm

Determine (a) indicated mean effective pressure, and (b) indicated power.

[(a) 8.43 bar, (b) 3.1 kW]

12. A four-stroke, gasolene engine develops a brake power of 410 kW. The engine consumes 120 kg of fuel in one hour and air consumption is 40 kg/min. The mechanical efficiency is 87% and the fuel heating value is 43000 kJ/kg. Determine the following:

(a) Air fuel ratio
(b) Indicated and brake thermal efficiencies

[(a) 20, (b) 32.875, 28.6%]

13. A twin cylinder, two-stroke internal combustion engine is operating with a speed of 4000 rpm. The fuel consumption is 10 litres per hour. The indicated mean effective pressure is 7.5 bar. Specific gravity of the fuel is 0.78. *CV* of fuel = 42 MJ/kg, *A/F* = 16, η_{vol} = 75%, η_{mech} = 80%, average piston speed = 600 m/min. Determine the dimensions of the cylinder. Also, calculate the brake thermal efficiency.

[d = 69.1 mm, L = 75 mm, η_{ith} = 30.91%]

14. A single-cylinder, four-stroke CI engine has a bore and stroke of 75 mm and 100 mm respectively. Find the *bmep*, if the torque is 25 N-m.

[35.55 bar]

Objective Questions

1. Which one of the following parts does not exist in an IC engine?
   (a) Crank shaft (b) Cam shaft
   (c) Piston rod (d) Connecting rod
2. Stoichiometric air–fuel ratio of petrol is roughly
   (a) 50 : 1 (b) 25 : 1
   (c) 15 : 1 (d) 1 : 1
3. A two-stroke engine has
   (a) two ports (*b*) three ports
   (c) two valves (d) three valves
4. In a two-stroke engine, one power stroke is obtained in
   (a) one revolution of the crank shaft
   (b) two revolutions of the crank shaft
   (c) four revolutions of the crank shaft
   (d) none of the above
5. In a four-stroke engine, one power stroke is obtained in
   (a) one revolution of the crank shaft
   (b) two revolutions of the crank shaft
   (c) four revolutions of the crank shaft
   (d) none of the above
6. In a two-stroke engine, the gas exchange process is called
   (a) charging (b) scavenging
   (c) combustion (d) none of the above
7. In a four–stroke Diesel engine, during suction stroke
   (a) fuel–air mixture is inducted
   (b) only fuel is inducted
   (c) only air is inducted
   (d) none of the above
8. The function of venturi in the carburetter is
   (a) to decrease the air velocity
   (b) to increase the velocity
   (c) to decrease the fuel flow
   (d) to increase the manifold vacuum
9. Which one of the following is true for a Diesel engine?
   (a) It has high compression ratio.
   (b) It does not have a spark plug.
   (c) It has large noise and vibrations.
   (d) All of the above
10. The air–fuel ratio of a petrol engine is controlled by
    (a) fuel injector (b) fuel pump
    (c) carburetter (d) none of the above

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11. In an IC engine, the exhaust pressure in the cylinder is
    (a) equal to the atmospheric pressure
    (b) below the atmospheric pressure
    (c) above the atmospheric pressure
    (d) none of the above
12. In a two-stroke IC engine, the piston top has a deflector for
    (a) better combustion of fuel
    (b) better scavenging of exhaust gases
    (c) better mixing of air and fuel
    (d) better charging of the cylinder
13. The pumping power is required in the following stroke(s):
    (a) Suction stroke
    (b) Exhaust stroke
    (c) Compression stroke
    (d) Suction and exhaust stroke
14. In a Diesel engine, fuel consumption against brake power is
    (a) parabolic (b) linear
    (c) hyperbolic (d) non-predictable
15. Which one of following is not a governing method used in IC engines?
    (a) Quality governing
    (b) Quantity governing
    (c) Injection governing
    (d) Hit-and-miss governing
16. Which one of following is a governing method used on Diesel engines?
    (a) Quality governing
    (b) Quantity governing
    (c) Injection governing
    (d) Hit-and-miss governing
17. Which one of following is a governing method used on petrol engines?
    (a) Quality governing
    (b) Quantity governing
    (c) Injection governing
    (d) Hit and miss governing
18. Increase in compression ratio in an Otto cycle engine may cause
    (a) misfiring
    (b) detonation
    (c) knocking
    (d) longer ignition delay
19. In a supercharged engine, induction air
    (a) is supplied at higher density
    (b) mixed with fuel
    (c) performes better scavenging
    (d) none of the above
20. In a Diesel engine, ignition occurs due to high
    (a) density of charge
    (b) temperature of air–fuel mixture
    (c) temperature of compressed air
    (d) intensity spark on spark plug
21. Thermal efficiency of an IC engine indicates percentage of
    (a) *BP* converted into *IP*
    (b) heat converted into work
    (c) *IP* converted into *BP*
    (d) heat lost into exhaust
22. Mechanical efficiency of an IC engine indicates percentage of
    (a) *BP* converted into *IP*
    (b) heat converted into work
    (c) *IP* converted into *BP*
    (d) heat lost into exhaust
23. By decreasing clearance volume, the volumetric efficiency of an IC engine
    (a) remains constant (b) increases
    (c) decreases (d) cannot say
24. By use of lubrication, which efficiency of an IC engine improves
    (a) volumetric efficiency
    (b) mechanical efficiency
    (c) charging efficiency
    (d) indicated thermal efficiency
25. By use of cooling, which efficiency of an IC engine decreases
    (a) volumetric efficiency
    (b) mechanical efficiency
    (c) charging efficiency
    (d) thermal efficiency

Answers

1. (c)	2. (c)	3. (b)	4. (a)	5. (b)	6. (b)	7. (c)	8. (b)	9. (d)	10. (c)
11. (c)	12. (b)	13. (d)	14. (b)	15. (c)	16. (a)	17. (b)	18. (b)	19. (a)	20. (c)
21 (b)	22. (c)	23. (b)	24. (b)	25. (d)					

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25

Reciprocating Air Compressor

Introduction

An *air compressor* is a machine which takes in atmospheric air, compresses it with the help of some mechanical energy and delivers it at higher pressure. It is also called *air pump*. An air compressor increases the pressure of air by decreasing its specific volume using mechanical means. Thus compressed air carries an immense potential of energy. The controlled expansion of compressed air provides motive force in air motors, pneumatic hammers, air drills, sand-blasting machines and paint sprayers, etc.

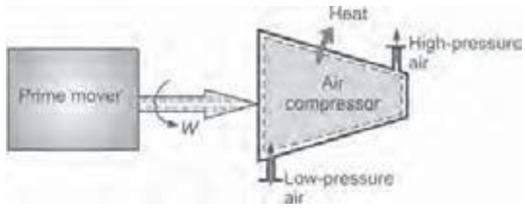

Fig. 25.1 *Schematic of an air compressor*

The schematic of an air compressor is shown in Fig. 25.1. The compressor recieves energy input from a prime mover (an engine or electric motor). Some part of this energy input is used to overcome the frictional effects, some part is lost in the form of heat and the remaining part is used to compress air to a high pressure.

25.1 USES OF COMPRESSED AIR

Compressed air has wide applications in industries as well as in commercial equipment. It is used in

1. Air refrigeration and cooling of large buildings,
2. Driving pneumatic tools in shops like drills, rivetters, screw drivers, etc.
3. Driving air motors in mines, where electric motors and IC engines cannot be used because of fire risks due to the presence of inflammable gases, etc.
4. Cleaning purposes,
5. Blast furnaces,
6. Spray painting and spraying fuel in Diesel engines,
7. Hard excavation work, tunneling, boring, mining, etc.
8. Starting of heavy-duty diesel engines,
9. Operating air brakes in buses, trucks and trains etc.
10. Inflating automobile and aircraft tyres,
11. Supercharging internal combustion engines,
12. Conveying solid and powder materials in pipelines,

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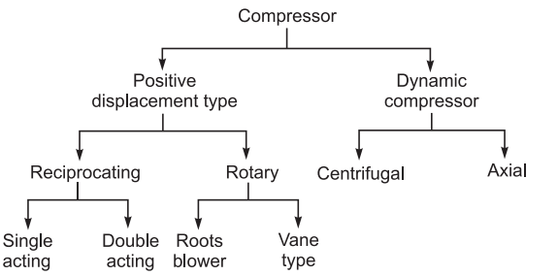
13. Process industries,
14. Operating lifts, hoists, crains and to operate pumps etc.
15. Pump sets for oil and gas transmission line,
16. Automobile suspension system.

25.2 CLASSIFICATION

The compressors are mainly classified as

(i) Reciprocating compressors, and
(ii) Rotary compressors.

The air compressors can broadly be classified as



A *reciprocating compressor* is used to produce high-pressure gas. It uses the displacement of piston in the cylinder for compression. It handles a low mass of gas and a high pressure ratio.

The *rotary compressors* are used for low and medium pressures. They usually consist of a bladed wheel or impeller that spins inside a circular housing. They handle a large mass of gas.

These compressors may be single stage or multistage to increase the pressure ratio.

25.3 RECIPROCATING COMPRESSOR TERMINOLOGY

In connection to reciprocating compressors, the following terms are defined:

1. Single-acting compressor is a compressor in which suction, compression and delivery of a gas take place only on one side of the piston during a cycle of one revolution of the crank shaft.

2. Double-acting compressor is a compressor in which suction, compression and delivery of gas take place on both sides of the piston and two cycles take place during one revolution of the crank shaft.

3. Single-stage compressor is a compressor in which the compression of gas to final delivery pressure is carried out in one cylinder only.

4. Multistage compressor is a compressor in which the compression of gas to the final pressure is carried out in more than one cylinder in series.

5. Pressure ratio is defined as the ratio of absolute discharge pressure to absolute suction pressure.

6. Free air is the air that exists under atmospheric condition.

7. Compressor displacement volume is the volume created when the piston travels a stroke. It is given as

$$V = \frac{\pi}{4} d^2 L \qquad ...(25.1)$$

where d is the bore of the cylinder and L the is stroke of the piston.

8. Induction-volume rate or volume-flow rate into the compressor is expressed in m^3/s and is given as

$\dot{V}$ = Volume inducted per cycle × No. of inductions per revolution × Number of revolutions per second

For the *single-acting reciprocating compressor*, only one cycle (thus, one induction) takes place for each revolution of the crank. Thus, for a compressor without clearance

$$\dot{V} = \frac{\pi}{4} d^2 L \frac{N}{60} \qquad ...(25.2)$$

For the *double-acting reciprocating compressor*, the induction takes place on both sides of the piston for each revolution. Thus,

$$\dot{V} = \frac{\pi}{4} d^2 L \left(\frac{2N}{60}\right) \qquad ...(25.3)$$

9. Capacity of a compressor is the actual quantity of air delivered per unit time at atmospheric conditions.

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***10. Free Air delivery* (*FAD*)** It is the discharge volume of the compressor corresponding to ambient conditions.

11. Piston speed is the linear speed of the piston measured in m/min. It is expressed as

$$\mathcal{V}_{piston} = 2LN \qquad ...(25.4)$$

25.4 COMPRESSED AIR SYSTEMS

Compressed air systems consist: intake air filters, inter-stage coolers, after coolers, air dryers, moisture drain traps, receivers, piping network, control valves and lubricators.

1. Intake Air Filters They prevent dust from entering the compressor. Dust causes sticking of valves, scoured cylinders, excessive wear, etc.

2. Inter-stage Coolers These are placed between consecutive stages of multistage compressor. They reduce the temperature of compressed air, before it enters the next stage of compression.

3. After Coolers They remove heat of compression and moisture in the air by reducing the temperature in a water-cooled heat exchanger, after compression is completed.

4. Air-dryers The remaining traces of moisture, after an after-cooler are removed by using air dryers, for using compressed air in instruments and pneumatic equipment. The moisture is removed by using adsorbents like silica gel or activated carbon, or refrigerant dryers, or heat of compression dryers.

5. Moisture Drain Traps Moisture drain traps are used for removal of moisture in the compressed air. These traps are manual drain cocks, timer based/ automatic drain valves, etc.

6. Air Receivers are cylindrical tanks into which the compressed air is discharged after final stage of compression from the air compressor. Receiver acts as storage tank and it helps to reduce pulsations and pressure variations from the compressed in the discharge line.

25.5 RECIPROCATING AIR COMPRESSOR

A machine which takes in air or gas during suction stroke at low pressure and then compresses it to high pressure in a piston–cylinder arrangement is known as a reciprocating compressor. External work must be supplied to the compressor to achieve required compression. This work is used to run the compressor. A part of the work supplied to the compressor is lost to overcome the frictional resistance between rubbing surfaces of the piston and cylinder. The cylinder of air compressor is cooled to minimise the work input.

The air compressed by a reciprocating compressor cannot directly be used for an application. The reciprocating motion of the piston gives rise to pulsating flow through the discharge valve of the compressor. Thus, the compressed air is discharged from the air compressor to an air receiver.

25.5.1 Construction

Figure 25.2 shows the sectional view of a single-stage air compressor. It consists of a piston, cylinder with cooling arrangement, connecting rod, crank, inlet and delivery valves. The piston fitted with piston rings, reciprocates in the cylinder. The prime mover (an engine or electric motor) drives the crank shaft, the crank rotates and converts rotary motion into reciprocating motion of piston

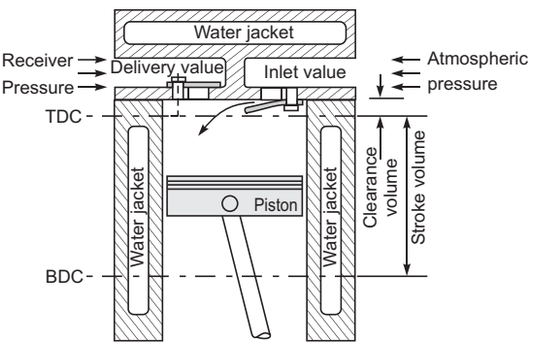

Fig. 25.2 *Sectional view of single-stage reciprocating air compressor*

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with the help of a connecting rod. The cylinder head consists of spring-loaded inlet and delivery valves, which are operated by a small pressure difference across them. The light spring pressure gives a rapid closing action. The piston rings seal the gap between the piston and cylinder wall. The cylinder is surrounded by a water jacket or metallic fins for proper cooling of air during compression.

The double-acting air compressor is shown in Fig. 25.3. Its construction is very similar to that of a single-acting air compressor, except for two inlet and two delivery valves on two ends of the cylinder in order to allow air entry and delivery on two sides of the piston. When the piston compresses the air on its one side, it creates suction on the other side. Thus, the suction and compression of air take place on two sides of the piston simultaneously.

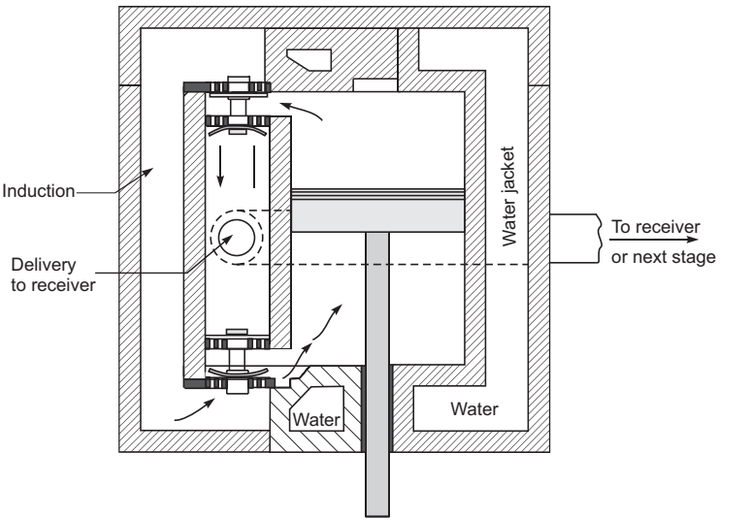

Fig. 25.3 *Double-acting reciprocating air compressor*

25.5.2 Working of a Single-Acting Air Compressor

As the piston moves in a downward stroke (from TDC to BDC), any residual compressed air left in the cylinder from the previous cycle expands first. On further movement of the piston, the pressure in the cylinder falls below the atmospheric pressure. The atmospheric air pushes the inlet valve to open and fresh air enters the cylinder as shown in Fig. 25.4. The line *c*-1 represents the induction stroke. During this stroke, the compressed air in the storage tank acts on the delivery valve, thus it remains closed. As the piston begins its return stroke from BDC to TDC, the pressure in the cylinder increases, and closes the inlet valve. The air in the cylinder is compressed by piston as shown by the curve 1-2.

During the compression stroke, as air pressure reaches a value, which is slightly more than the pressure of compressed air acting outside the delivery valve, the delivery valve opens and the compressed air is discharged from the cylinder to storage tank. At the end of the compression stroke, the piston once again moves downward, the pressure in the cylinder falls below the atmospheric pressure, the delivery valve closes and inlet valve opens for next cycle. The suction, compression and delivery of air take place with two strokes of the piston which is one revolution of the crank.

Figure 25.4 shows the p–V diagram for a reciprocating compressor without clearance. The processes are summarized below:

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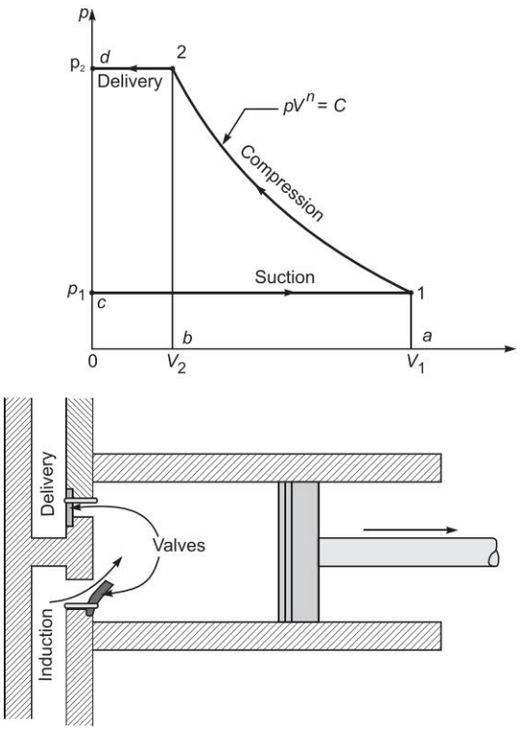

Fig. 25.4 *p–V diagram for a reciprocating compressor without clearance*

Process c–1 Suction stroke—inlet valve opens and air enters the compressor at constant pressure p_1

Process 1–2 Polytropic compression of air from pressure p_1 to pressure p_2

Process 2–d Discharge of compressed air through delivery valve at const. pressure p_2

Process d–c No air in the cylinder and return of piston for suction stroke

25.5.3 Indicated Work for a Single-acting Compressor without Clearance

The theoretical $p–V$ diagram for single-stage, single-acting reciprocating air compressor without clearance is shown in Fig. 25.4. The net work done in the cycle is equal to the area behind the curve on $p–V$ diagram and it is the work done on air.

Indicated work done on the air per cycle

= Area behind the curve, i.e., area c –1–2–d–c

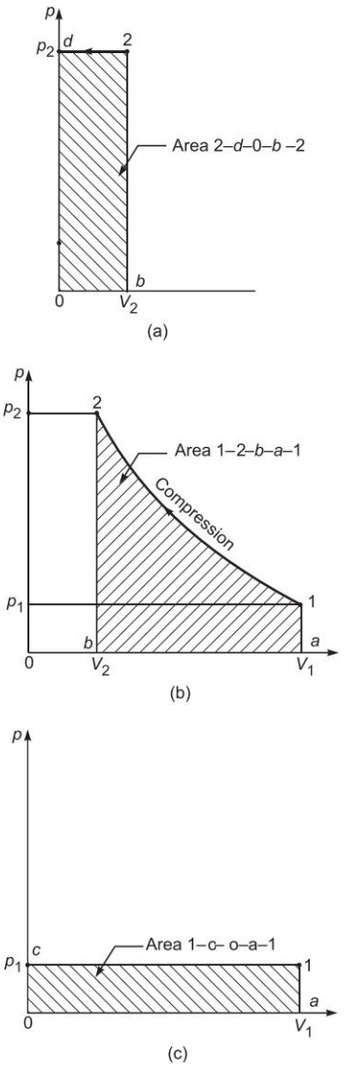

Fig. 25.5 *Three process areas on p–V diagram*

= Area 2–d –0 –b–2 + Area 1–2–b –a –1
– Area 1–c –0 –a –1

These three areas are shown in Fig. 25.5 as (a), (b) and (c), respectively.

Area 2–d –0 –b–2 = p_2V_2 (Flow work during discharge at constant pressure p_2)

Area 1–2–b–a –1 = $-\int pdV$ (Piston dispalcement work from p_1 to p_2, –

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sign is taken for compression)

Area $1–c–0–a–1 = p_1V_1$ (Flow work during suction at constant pressure p_1)

During compression process 1–2; the pressure and volume are related as

$$pV^n = \text{C (constant)}$$

Thus we get $-\int pdV = \dfrac{p_2V_2 - p_1V_1}{n-1}$

Therefore, the total indicated work input to compressor is

$$W_{in} = p_2V_2 + \frac{p_2V_2 - p_1V_1}{n-1} - p_1V_1 \quad ...(25.5)$$

$$= (p_2V_2 - p_1V_1)\left[\frac{1}{n-1} - 1\right]$$

$$W_{in} = \frac{n}{n-1}(p_2V_2 - p_1V_1) \text{ (kJ/cycle)} \quad ...(25.6)$$

Using charecteristic gas equation as

$$pV = m_aRT$$

Equation (25.6) can be modified as

$$W_{in} = \frac{n}{n-1} m_aR(T_2 - T_1) \text{ (kJ/cycle)} \quad ...(25.7)$$

Other expression for indicated work can be derived by arranging Eq. (25.7) as

$$W_{in} = \frac{n}{n-1} m_aRT_1\left[\frac{T_2}{T_1} - 1\right]$$

It is convenient to express the temperature T_2 in terms of delivery and intake pressure ratio.

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

Then $$W_{in} = \frac{n}{n-1} m_aRT_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] \text{(kJ/cycle)} \quad ...(25.8)$$

or $$W_{in} = \frac{n}{n-1} p_1V_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] \text{(kJ/cycle)} \quad ...(25.9)$$

where V_1 is the volume inducted per cycle.

25.5.4 Mean Effective Pressure (p_m)

It is a hypothetical average pressure, which if acted on the piston during the entire compression stroke will require the same power input as required during the actual cycle.

Net work input in a cycle,

$$W_{in} = p_m \times \text{(Swept volume)}$$
$$= p_m \times V_s$$

Thus $$p_m = \frac{W_{net}}{V_s} = \frac{\text{Work output}}{\text{Swept volume}} \quad ...(25.10)$$

From a given indicator diagram the indicated mean effective pressure can be obtained as

$$p_m = \frac{\text{Area of indicator diagram (mm}^2\text{)}}{\text{Length of the indicator diagram (mm)}} \times \text{Spring constant (kPa/mm)} \quad ...(25.11)$$

25.5.5 Power and Mechanical Efficiency

***1. Indicated Power* (IP)** The work done on air per unit time is called *indicated power input to the compressor.* The power required by an air compressor, running at N rpm is given as

Indicated power IP

= Work input per cycle × No. of cycles per unit time

or $$IP = \frac{W_{in}\, N\, k}{60} \text{ (kW)} \quad ...(25.12)$$

From an indicated diagram, It is calculated as

IP = Indicated mean effective pressure × Swept volume rate

$$= \frac{p_{mi}\, L\, A\, N\, k}{60} \text{ (kW)} \quad ...(25.13)$$

where for Eq. (25.12) and Eq. (25.13);

W_{in} = Indicated work input per cycle

p_{mi} = Indicated mean effective pressure, (kPa or kN/m^2)

L = Stroke length, (m)

$A = (\pi/4)d^2$, cross-sectional area of cylinder of bore, d, (m)

N = number of rotation per minute

k = number of suction per revolution of crank shaft

= 1 for single-acting reciprocating compressor

= 2 for double-acting reciprocating compressor

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2. Brake Power (BP) The actual power (brake power or shaft power) input to the compressor is more than the indicated power because some work is required to overcome the irreversibilities and mechanical frictional effects.

Brake power;

$$BP = \text{Indicated power} + \text{Frictional power} \qquad ...(25.14)$$

3. Mechanical Efficiency η_{mech} The mechanical efficiency of the compressor is given by

$$\eta_{mech} = \frac{\text{Indicated power}}{\text{Brake power}} \qquad ...(25.15)$$

The brake power is derived from a driving motor or engine. The input of a driving motor can be expressed as

$$\text{Motor power} = \frac{\text{Shaft power (or brake power)}}{\text{Mechanical efficiency of motor and drive}} \qquad ...(25.16)$$

Example 25.1 *A single-stage reciprocating air compressor takes in 1.4 kg of air per minute at 1 bar and 17°C and delivers it at 6 bar. Assuming compression process follows the law $pV^{1.35}$ = constant, calculate indicated power input to compressor.*

Solution

Given A single-stage reciprocating air compressor

$\dot{m}_a = 1.4$ kg/min $\quad p_1 = 1$ bar

$T_1 = 17°C = 290$ K $\quad p_2 = 6$ bar

$n = 1.35$ bar

Law $pV^{1.35} = C$

To find Indicator power input to compressor.

Assumptions

(i) Negligible clearance volume in the compressor.
(ii) No throttling effects on valve opening and closing.
(iii) Air as an ideal gas with $R = 0.287$ kJ/kg·K.

Analysis The delivery temperature of air

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = (290\text{ K}) \times \left(\frac{6}{1}\right)^{\frac{1.35-1}{1.35}}$$

$$= 461.46\text{ K}$$

The rate of work input to compressor, Eq. (25.7)

$$W_{in} = \frac{n}{n-1}\dot{m}_a R(T_2 - T_1)$$

$$= \frac{1.35}{1.35-1} \times (1.4\text{ kg/min}) \times (0.287\text{ kJ/kg·K}) \times (461.46 - 290)\text{ (K)}$$

$$= 265.72\text{ kJ/min}$$

Indicated power input;

$$IP = \frac{W_{in}}{60} = \frac{(265.72\text{ kJ/min})}{(60\text{ s/min})} = \mathbf{4.43\ kW}$$

Note: The indicated work input to compressor can also be calculated by using Eq. (25.8).

Example 25.2 *A single-acting, single-cylinder reciprocating air compressor has a cylinder diameter of 200 mm and a stroke of 300 mm. Air enters the cylinder at 1 bar; 27°C. It is then compressed polytropically to 8 bar according to the law $pV^{1.3}$= constant. If the speed of the compressor is 250 rpm, calculate the mass of air compressed per minute, and the power required in kW for driving the compressor.*

Solution

Given A single-acting, single-cylinder reciprocating air compressor

$d = 200$ mm $= 0.2$ m $\quad L = 300$ mm $= 0.3$ m

$p_1 = 1$ bar $= 100$ kPa $\quad p_2 = 8$ bar

$N = 250$ rpm $\quad T_1 = 27°C = 300$ K

$n = 1.3$

To find

(i) The mass of air compressed in kg/min,
(ii) Power input to compressor in kW.

Assumptions

(i) Negligible clearance volume in the cylinder.
(ii) Air as an ideal gas with $R = 0.287$ kJ/kg·K.

Analysis The swept volume of the cylinder per cycle

$$V_s = V_1 = \left(\frac{\pi}{4}\right)d^2 L$$

$$= \left(\frac{\pi}{4}\right) \times (0.2\text{ m})^2 \times (0.3\text{ m})$$

$$= 9.424 \times 10^{-3}\text{ m}^3$$

The mass of air, using perfect gas equation

$$m_a = \frac{p_1 V_1}{R T_1} = \frac{(100\text{ kPa}) \times (9.424 \times 10^{-3}\text{ m}^3)}{(0.287\text{ kJ/kg·K}) \times (300\text{ K})}$$

$$= 0.0109\text{ kg/cycle}$$

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The mass flow rate of air;

$\dot{m}_a$ = mass of air × number of suction/min = $m_a N$

= 0.0109 × 250 = **2.74 kg/min**

Temperature of air after compression

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$= (300\,\text{K}) \times \left(\frac{8}{1}\right)^{\frac{1.3-1}{1.3}} = 484.75\ \text{K}$$

The work input to compressor, Eq. (25.7)

$$W_{in} = \frac{n}{n-1} m_a R (T_2 - T_1)$$

$$= \frac{1.3}{1.3-1} \times (2.74\ \text{kg/min}) \times (0.287\ \text{kJ/kg}\cdot\text{K}) \times (484.75 - 300)\ (\text{K})$$

= 629.56 kJ/min or **10.49 kW**

Example 25.3 *A single-acting, single-cylinder reciprocating air compressor is compressing 20 kg/min. of air from 110 kPa, 30°C to 600 kPa and delivers it to a receiver. Law of compression is $pV^{1.25}$ = constant. Mechanical efficiency is 80%. Find the power input to compressor, neglecting losses due to clearance, leakages and cooling.*

Solution

Given A single-stage reciprocating air compressor

$\dot{m}_a$ = 20 kg/min p_1 = 110 kPa

T_1 = 30°C = 303 K p_2 = 600 kPa

Law $pV^{1.25}$ = C

η_{mech} = 0.8

To find Power input to compressor.

Assumptions

(i) Negligible clearance volume in the compressor.

(ii) No throttling effects on valve opening and closing.

(iii) Air as an ideal gas with R = 0.287 kJ/kg·K.

Analysis The delivery temperature of air

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = (303\ \text{K}) \times \left(\frac{600}{110}\right)^{\frac{1.25-1}{1.25}}$$

= 425.4 K

The indicated power input to compressor,

$$IP = \frac{n}{n-1} \dot{m}_a R (T_2 - T_1)$$

$$= \frac{1.25}{1.25-1} \times \left(\frac{20}{60}\ \text{kg/s}\right) \times (0.287\ \text{kJ/kg}\cdot\text{K}) \times (425.4 - 303)\ (\text{K})$$

= 58.55 kW

The motor (brake) power

$$BP = \frac{IP}{\eta_{mech}} = \frac{58.55\ \text{kW}}{0.8} = \mathbf{73.18\ kW}$$

Example 25.4 *A single-cylinder, double-acting, reciprocating air compressor receives air at 1 bar; 17°C, compresses it to 6 bar according to the law $pV^{1.25}$ = constant. The cylinder diameter is 300 mm. The average piston speed is 150 m/min at 100 rpm. Calculate the power required in kW for driving the compressor. Neglect clearance.*

Solution

Given A double-acting, single-cylinder reciprocating air compressor

d = 300 mm = 0.3 m p_1 = 1 bar = 100 kPa

p_2 = 6 bar N = 100 rpm

T_1 = 17°C = 290 K n = 1.25

k = 2 $\mathcal{V}_{piston}$ = 150 m/min

To find Power input to compressor in kW.

Analysis The piston speed is given as

$\mathcal{V}_{piston} = 2LN$

Stroke of piston;

$$L = \frac{\mathcal{V}_{piston}}{2N} = \frac{150\ \text{m/min}}{2 \times (100\ \text{rotation/min})}$$

= 0.75 m

The swept volume of the cylinder per cycle

$$V_s = V_1 = \left(\frac{\pi}{4}\right) d^2 L = \frac{\pi}{4} \times (0.3\ \text{m})^2 \times (0.7\ 5\text{m})$$

= 0.053 m^3

The indicated work input to compressor by Eq. (25.9)

$$W_{in} = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1.25}{1.25-1} \times 100 \times 0.053 \times \left[\left(\frac{6}{1}\right)^{\frac{1.25-1}{1.25}} - 1\right]$$

= 11.42 kJ/cycle

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For a double-acting reciprocating compressor, the indicated power

$$IP = \frac{W_{in}\, N\, k}{60} \text{ (kW)}$$

$$= \frac{11.42 \times 100 \times (2 \text{ for double acting})}{60}$$

$$= \mathbf{38.1\ kW}$$

Example 25.5 *A single-stage, single-acting, reciprocating air compressor takes in 1 m^3 air per minute at 1 bar and 17°C and delivers it at 7 bar. The compressor runs at 300 rpm and follows the law $pV^{1.35}$= constant. Calculate the cylinder bore and stroke required, assuming stroke-to-bore ratio of 1.5. Calculate the power of the motor required to drive the compressor, if the mechanical efficiency of the compressor is 85% and that of motor transmissions is 90 %. Neglect clearance volume and take R = 0.287 kJ/kg·K for air.*

Solution

Given A single-stage, single-acting, reciprocating air compressor

$\dot{V}$ = 1 m^3/min p_1 = 1 bar = 100 kPa

T_1 = 17°C = 290 K p_2 = 7 bar

N = 300 rpm n = 1.35

$\eta_{transmission}$ = 0.9 η_{mech} = 0.85

L/d = 1.5 R = 0.287 kJ/kg·K

To find

(i) Cylinder bore, and strokes,

(ii) Motor power.

Analysis Volume sucked in per cycle

$$V_s = \frac{\dot{V}}{N\, k} = \frac{1 \text{ m}^3/\text{min}}{(300 \text{ rpm}) \times 1} = \frac{1}{300} \text{m}^3$$

The cylinder (swept) volume also given as,

$$V_s = \frac{\pi}{4} d^2 L = \frac{\pi}{4} d^2 (1.5d) = 1.5 \times \left(\frac{\pi}{4}\right) d^3$$

Equating two equations

$$1.5 \times \left(\frac{\pi}{4}\right) d^3 = \frac{1}{300}$$

We get

cylinder bore,

d = 0.1414 m = **141.4 mm**

and stroke;

L = 1.5 d = **212.10 mm**

The mass flow rate of air per minute,

$$\dot{m}_a = \frac{p_1 \dot{V}}{R T_1} = \frac{(100 \text{ kPa}) \times (1 \text{ m}^3)}{(0.287 \text{ kJ/kg} \cdot \text{K}) \times (290 \text{ K})}$$

= 1.2 kg/min

The temperature of air after compression

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = (290\,\text{K}) \times \left(\frac{7}{1}\right)^{\frac{1.35-1}{1.35}}$$

= 480.28 K

The rate of work input to compressor, Eq. (25.7)

$$\dot{W} = \frac{n}{n-1} \dot{m}_a R (T_2 - T_1)$$

$$= \frac{1.35}{1.35 - 1} \times 1.2 \times 0.287 \times 480.28 - 290$$

= 252.77 kJ/min

Indicated power required;

$$IP = \frac{(252.77 \text{ kJ/min})}{(60 \text{ s/min})} = \mathbf{4.21\ kW}$$

The brake power input to compressor;

$$\text{Brake power} = \frac{IP}{\eta_{mech}} = \frac{(4.21 \text{ kW})}{0.85} = 4.956 \text{ kW}$$

The motor power required;

$$\text{Motor power} = \frac{\text{Brake power}}{\eta_{transmission}} = \frac{(4.956 \text{ kW})}{0.9}$$

$$= \mathbf{5.5\ kW}$$

25.6 MINIMIZING COMPRESSION WORK

The work done on the gas for compression can be minimized when the compression process is executed in an internally reversible manner, i.e., by minimizing the irreversibilities. The other way of reducing the compression work is to keep the specific volume of gas as small as possible during compression process. It is achieved by keeping the gas temperature as low as possible during the compression. Since specific volume of gas is proportional to temperature, therefore, the cooling arrangement is provided on the compressor to cool the gas during the compression.

For better understanding of the effect of cooling during compression process, we consider three types of compression processes executed

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between same pressure levels (p_1 and p_2); an isentropic compression 1–2″ (involves no cooling), a polytropic compression 1–2 (involves partial cooling) and an isothermal compression 1–2′ (involves perfect cooling) as shown in Fig. 25.6.

(a) The indicated compression work per cycle for a polytropic compression process 1–2 is given by Eq. (25.9)

$$W_{Poly} = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

(b) Isentropic compression process 1–2″

An equation for indicated work input can be obtained as Eq. (25.9) by replacing n by γ. That is,

$$W_{isentropic} = \frac{\gamma}{\gamma-1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

...(25.17)

(c) Isothermal compression process 1–2′: With perfect cooling ($T_2 = T_1$);

Indicated work input for isothermal compression is given by area c–1–2′–d–c.

Area c–1–2′–d–c = Area a–1–2′–b + Area b–2′–d–0 – Area a–1–c–0

$$W_{iso} = -\int_{V_1}^{V_2} pdV + p_2V_2 - p_1V_1$$

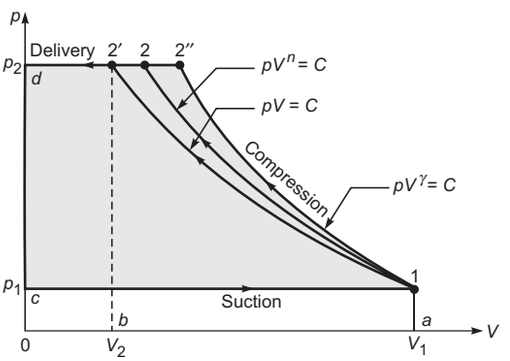

Fig. 25.6 *Three types of compression processes on a p–V diagram*

For an isothermal process; using

$$p = \frac{C}{V} \text{ (Since } pV = C\text{)}$$

We get $-\int_{V_1}^{V_2} pdV = p_1V_1 \ln\left(\frac{V_1}{V_2}\right)$

For isothermal process;

$$p_2V_2 = p_1V_1 \text{ and } \frac{V_1}{V_2} = \frac{p_2}{p_1}$$

$$\therefore \quad W_{iso} = p_1V_1 \ln\left(\frac{p_2}{p_1}\right) \qquad ...(25.18)$$

where V_1 is the volume of the air inducted per cycle.

The three processes are plotted on a p–V diagram in Fig. 25.6 for same inlet state and exit pressure. The area of the indicator diagram is the measure of compression work. The only type of compression can influence the magnitude of area of indicator diagram and length of the line 2–d.

It is interesting to observe from this diagram that among the three processes considered, the area with isentropic compression is *maximum.* Thus it requires maximum work input and with isothermal compression, the area of indicator diagram is *minimum.* Thus, the *compressor with isothermal compression will require minimum* work input.

25.6.1 Adiabatic Efficiency

This term is seldom used in practice for reciprocating compressors. The adiabatic efficiency of an air compressor is defined as the ratio of isentropic work input to actual work input.

$$\eta_{adiabatic} = \frac{\text{Isentropic work input}}{\text{Actual work input}} \qquad ...(25.19)$$

25.6.2 Compressor Efficiency

It compares the indicated work input to isothermal work input to the compressor and it is defined as ratio of isothermal work input to indicated work input

$$\eta_{comp} = \frac{\text{Isothermal work input}}{\text{Indicated work input}} \qquad ...(25.20)$$

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25.6.3 Isothermal Efficiency

It compares the actual work done on the gas with isothermal compression work, and is defined as the ratio of isothermal work input to actual work input during compression, i.e.,

$$\eta_{iso} = \frac{\text{Isothermal work input}}{\text{Actual work input}} \quad ...(25.21)$$

25.6.4 Methods for Improving Isothermal Efficiency

As illustrated with the help of Fig. 25.6, the compression of gas in isothermal manner requires minimum work input. With isothermal compression, the temperature remains constant throughout the compression process.

$$T_2 = T_1$$

Isothermal compression is only possible when all the heat generated during compression is dissipated to cooling medium around the cylinder wall. It is possible, when the compressor runs very slowly.

In actual practice, the compression process should approach isothermal compression even with high-speed compressors. The various methods are adopted to reduce the temperature of gas during the compression and keep it more closely to isothermal compression.

1. Water Spray It was an old method in which water was injected into the cylinder during compression of air to keep the temperature of air constant. But this method has certain disvantages, and thus became obsolete.

2. Water Jacketing It is commonly and successfully used practice for all types of reciprocating compressors. The water is circulated around the cylinder through the water jacket which helps to cool the air during compression.

3. External Fins For a small-capacity compressor, the effective cooling can be achieved by attaching fins of conducting material around the cylinder. The fins increase surface area of the cylinder for heat transfer.

4. Inter-cooler If the very high pressure ratio is required then air is compressed in stages. The intercooler is used between two stages of compression for cooling of compressed air after one stage, before entering the next stage. The water jackets are also used around the cylinder of compressor of each stage.

5. By Suitable Cylinder Proportions If the compressor has large surface to volume ratio, the greater surface area will be available for heat transfer and cooling will be more effective.

It is possible by choosing a cylinder with large bore and short piston stroke. The large cylinder head dissipates heat in a much more effective way, which contains hottest compressed air all the time.

Example 25.6 *A single-acting, single-stage reciprocating air compressor of 250–mm bore and 350–mm stroke runs at 200 rpm The suction and delivery pressures are 1 bar and 6 bar, respectively. Calculate the theoretical power required to run the compressor under each of the following conditions of compression:*

(a) isothermal,
(b) polytropic n = 1.3, and
(c) isentropic, $\gamma = 1.4$.

Neglect the effect of clearance and also calculate isothermal efficiency in each of the above cases.

Solution

Given A single-acting, single-stage reciprocating air compressor:

$p_1 = 1$ bar $= 100$ kPa $p_2 = 6$ bar
$d = 250$ mm $= 0.25$ m $L = 350$ mm
$N = 200$ rpm

and three types of compression with $n = 1$, 1.3 and 1.4

To find

(i) Theoretical power required to run the compressor for
  (a) Isothermal compression,
  (b) Polytropic compression,
  (c) Isentropic compression.
(ii) Isothermal efficiency in each case.

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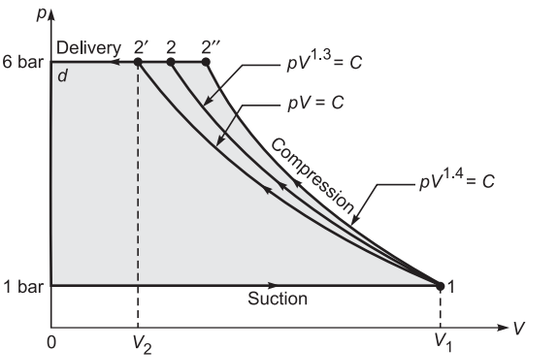

Fig. 25.7

Analysis The cylinder volume (without clearance)

$$V_1 = V_s = \left(\frac{\pi}{4}\right) d^2 L = \left(\frac{\pi}{4}\right) \times (0.25\ \text{m})^2 \times (0.35\ \text{m})$$

$$= 0.0171\ \text{m}^3$$

The volume flow rate of air

$\dot{V}_1$ = Cylinder volume × No of suctions per second

$$= 0.0171 \times \frac{200}{60} = 0.0572\ \text{m}^3/\text{s}$$

(i) *Theoretical power required*

(a) For isothermal compression,

$$\text{Power } \dot{W}_{iso} = p_1 \dot{V}_1 \ln\left(\frac{p_2}{p_1}\right)$$

$$= 100 \times 0.0572 \times \ln\left(\frac{6}{1}\right)$$

$$= 10.261\ \text{kJ/s or } \mathbf{10.261\ kW}$$

(b) Polytropic compression,

$$\dot{W}_{Poly} = \frac{n}{n-1} p_1 \dot{V}_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \left(\frac{1.3}{1.3-1}\right) \times 100 \times 0.0572 \times \left[\left(\frac{6}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$= \mathbf{12.7\ kW}$$

(c) Isentropic compression,

$$\dot{W}_{isentropic} = \frac{\gamma}{\gamma-1} p_1 \dot{V}_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$= \left(\frac{1.4}{1.4-1}\right) \times 100 \times 0.0572 \times \left[\left(\frac{6}{1}\right)^{\frac{1.4-1}{1.4}} - 1\right]$$

$$= \mathbf{13.38\ kW}$$

(ii) *Isothermal efficiency*

$$\eta_{iso} = \frac{\text{Isothermal work}}{\text{Actual work}} \times 100$$

(a) For isothermal compression,

$$\eta_{iso} = \mathbf{100\%}$$

(b) For polytropic compression,

$$\eta_{iso} = \frac{10.261}{12.7} \times 100 = \mathbf{80.8\%}$$

(c) For isentropic compression,

$$\eta_{iso} = \frac{10.261}{13.38} \times 100 = \mathbf{76.67\%}$$

25.7 CLEARANCE VOLUME IN A COMPRESSOR

The *clearance volume* is the space left in the cylinder when the piston reaches its topmost position, i.e., TDC. It is provided

(i) to avoid the piston striking the cylinder head, and

(ii) to accommodate the valve's actuation inside the cylinder, because suction and delivery valves are located in the clearance volume.

A compressor should have the smallest possible clearance volume, because the compressed air left in the clearance volume, first re-expands in the cylinder during suction, thus reducing suction capacity.

The ratio of clearance volume to swept volume is defined as the *clearance ratio* or *percentage clearance*. The value of clearance ratio may vary from 2 to 10%.

25.7.1 Effects of Clearance Volume

1. The volume of air taken in per stroke is less than the swept volume, thus the volumetric efficiency decreases.
2. More power input is required to drive the compressor for same pressure ratio, due to increase in volume to be handled.
3. The maximum compression pressure is controlled by the clearance volume.

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25.7.2 Indicated Compression Work with Clearance

The clearance volume is generally kept very small. The work done on the air in the clearance space during compression stroke is approximately equal to the work done by the air when it re-expands during suction stroke. Therefore, the work of compression is not affected by clearance space in the compressor. But the mass of air inducted is reduced, and thus the volumetric efficiency of the compressor will be less.

Figure 25.8 shows an indicator diagram for a reciprocating air compressor with clearance. After delivery of compressed air, the air remaining in the clearance volume at pressure p_2 expands, when the piston proceeds for the next suction stroke. As soon as the pressure p_1 reaches at the state 4, the induction of fresh charge starts and continues to the end of the stroke at state 1.

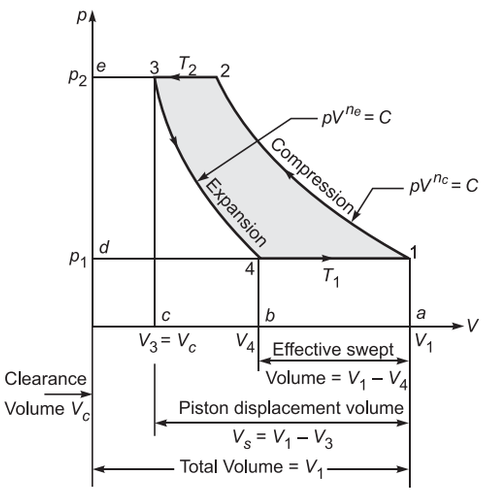

Fig. 25.8 *Indicator diagram for reciprocating air compressor with clearance*

The indicated work done is given by the area 1–2–3–4–1 on a p–V diagram.

Indicated work

= Area 1–2–3–4–1

= Area 1–2–*e*–*d* – Area 3–*e*–*d*–4

Similar to derivation of Eq. (25.9), the compression work equivalent to area 1–2–*e*–*d,* can be obtained with index of compression n_c.

Area 1–2–*e*–*d*;

$$W_{Comp} = \frac{n_c}{n_c - 1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n_c - 1}{n_c}} - 1 \right] \quad ...(25.22a)$$

The work done by gas during expansion (index n_e) can be also obtained as above

Area 3–*e*–*d*–4;

$$W_{Expan} = \frac{n_e}{n_e - 1} p_4 V_4 \left[\left(\frac{p_3}{p_4} \right)^{\frac{n_e - 1}{n_e}} - 1 \right]$$

$$= \frac{n_e}{n_e - 1} p_1 V_4 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n_e - 1}{n_e}} - 1 \right] \quad ...(25.22b)$$

Since $p_4 = p_1$ and $p_3 = p_2$

Net work of compression;

$$W_{in} = \frac{n_c}{n_c - 1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n_c - 1}{n_c}} - 1 \right]$$

$$- \frac{n_e}{n_e - 1} p_1 V_4 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n_e - 1}{n_e}} - 1 \right] \quad ...(25.23)$$

If indices of compression and expansion are same, i.e., $n_c = n_e = n$, then

$$W_{in} = \frac{n}{n-1} p_1 (V_1 - V_4) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \quad ...(25.24)$$

***Example* 25.7** *An ideal single-stage, single-acting reciprocating air compressor has a displacement volume of 14 litre and a clearance volume of 0.7 litre. It receives the air at a pressure of 1 bar and delivers it at a pressure of 7 bar. The compression is polytropic with an index of 1.3 and re-expansion is isentropic with an index of 1.4. Calculate the net indicated work of a cycle.*

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Solution

Given A single-stage, single-acting reciprocating air compressor.

$p_1 = 1$ bar $= 100$ kPa $p_2 = 7$ bar
$V_s = 14$ litre $V_c = 0.7$ litre
$n_c = 1.3$ $n_e = 1.4$

To find Indicated work input per cycle.

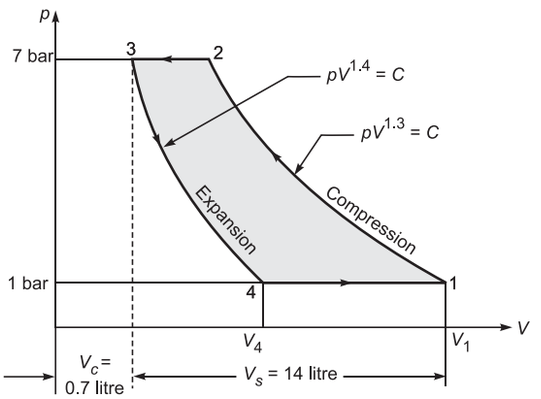

Fig. 25.9

Analysis The total volume of cylinder;

$$V_1 = V_s + V_c$$
$$= 14 + 0.7 = 14.7 \text{ litre} \quad \text{or} \quad 0.0147 \text{ m}^3$$

The volume V_4 after re-expansion of compressed air in clearance space

$$V_4 = V_3 \left(\frac{p_2}{p_1}\right)^{\frac{1}{n_e}} = 0.7 \times \left(\frac{7}{1}\right)^{\frac{1}{1.4}}$$
$$= 2.81 \text{ litre} \quad \text{or} = 0.00281 \text{ m}^3$$

Indicated work input per cycle, Eq. (25.23)

$$W_{in} = \frac{n_c}{n_c - 1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n_c - 1}{n_c}} - 1\right]$$
$$- \frac{n_e}{1 - n_e} p_1 V_4 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n_e - 1}{n_e}} - 1\right]$$
$$= \frac{1.3}{1.3 - 1} \times 100 \times 0.0147 \times \left[\left(\frac{7}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$
$$- \frac{1.4}{1.4 - 1} \times 100 \times 0.00281 \times \left[\left(\frac{7}{1}\right)^{\frac{1.4-1}{1.4}} - 1\right]$$
$$= 3.61 - 0.731 = \mathbf{2.88 \text{ kJ/cycle}}$$

Example 25.8 *A single-stage, double-acting reciprocating air compressor takes in 14 m³ of air per minute measured at 1.013 bar and 15°C. The delivery pressure is 7 bar and the compressor speed is 300 rpm. The compressor has a clearance volume of 5% of swept volume with a compression and re-expansion index of n = 1.3. Calculate the swept volume of the cylinder, the delivery temperature and the indicated power.*

Solution

Given A single-stage, double-acting reciprocating air compressor, with

$\dot{V}_1 = 14$ m³/min $T_1 = 15°C = 288$ K
$N = 300$ rpm $n = 1.3$
$p_1 = 1.03$ bar $= 101.3$ kPa $p_2 = 7$ bar
$V_c = 0.05 V_s$

To find

(i) The swept volume of cylinder,
(ii) The delivery temperature, and
(iii) Indicated power.

Assumptions

(i) No throttling effect on valve opening and closing.
(ii) Effect of piston rod on underside of cylinder is negligible.
(iii) Air as an ideal gas with specific gas constant $R = 0.287$ kJ/kg·K.

Analysis The p–V diagram for given data of compressor is shown in Fig. 25.10.

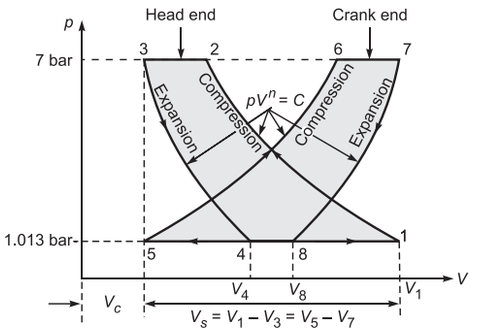

Fig. 25.10 *p–V diagram for double-acting reciprocating air compressor*

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(i) *Swept volume of cylinder*

The total volume of cylinder,

$$V_1 = V_s + V_c = V_s + 0.05\ V_s = 1.05\ V_s$$

The volume inducted per cycle

$$V_1 - V_4 = \frac{\text{Volume induction per minute}}{\text{No. of suction per revolution} \times \text{No. of revolution per minute}}$$

$$= \frac{14\ \text{m}^3/\text{min}}{2\ \text{suction per rev.} \times 300\ \text{rev./min}}$$

$$= 0.0233\ \text{m}^3/\text{cycle}$$

The volume V_4 after re-expansion of compressed air.

$$V_4 = V_3\left(\frac{p_3}{p_4}\right)^{\frac{1}{n}} = V_c\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$= 0.05\ V_s \times \left(\frac{7}{1.013}\right)^{\frac{1}{1.3}}$$

$$= 0.221\ V_s$$

Then $V_1 - V_4$

$$= 1.05\ V_s - 0.221\ V_s = 0.829\ V_s$$

or $0.829\ V_s = 0.0233\ \text{m}^3/\text{cycle}$

$\therefore$ swept volume,

$$V_s = \frac{0.0233}{0.829} = \mathbf{0.0281\ m^3/cycle}$$

The swept volume of the cylinder is 0.0281 m^3.

(ii) *The delivery temperature of air*

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$= 288 \times \left(\frac{7}{1.013}\right)^{\frac{1.3-1}{1.3}}$$

$$= 450\ \text{K} = \mathbf{177°C}$$

(iii) *Indicated power*

$$IP = \frac{n}{n-1}\,p_1 \dot{V}\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1.3}{1.3-1} \times (101.3\ \text{kPa}) \times \left(\frac{14}{60}\ \text{kg/s}\right)\left[\left(\frac{7}{1.013}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$= \mathbf{57.58\ kW}$$

25.8 ACTUAL INDICATOR DIAGRAM

The actual indicator diagram on a p–V plane for a single-stage reciprocating air compressor is shown in Fig. 25.11. It is similar to theoretical one (Fig. 25.8) except for induction and delivery processes. The variation during these processes is due to valve action effects.

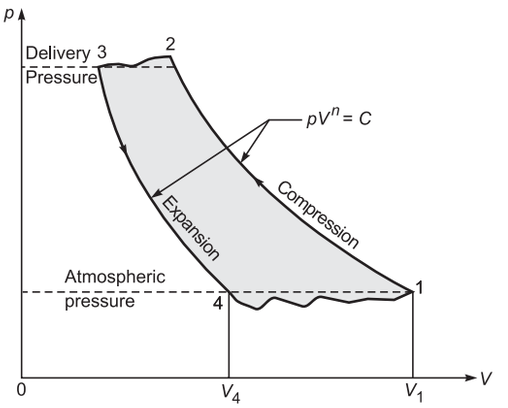

Fig. 25.11 *Actual indicator diagram for reciprocating compressor*

There must be some pressure difference across the valves to operate them. During the suction process 4–1, the pressure drops in the cylinder until the inlet valve is forced by atmospheric air to open. During the suction stroke, the piston creates *vacuum* in the cylinder. Thus pressure reduces and atmospheric air enters the cylinder.

Similarly, during delivery process 2–3, some more pressure is required to open the delivery valve and to displace the compressed air through narrow valve passage. Thus, *gas throttling* takes place during delivery, reducing the pressure gradually to the state 3.

The waviness of lines during these processes is due to valve bounce and wire drawing effect through the valves.

25.9 VOLUMETRIC EFFICIENCY

Actual volume sucked into the cylinder during the suction stroke is always less than the swept volume.

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It is due to

(i) resistance offered by inlet valve to incoming air,
(ii) temperature of incoming air, and
(iii) back pressure of residual gas left in the clearance volume.

The *volumetric efficiency,* η_{vol} of the air compressor is defined as the ratio of actual volume of air sucked into the compressor, measured at atmospheric pressure and temperature to the piston displacement volume.

In terms of mass ratio, the volumetric efficiency is defined as the ratio of actual mass of air sucked per stroke to the mass of air corresponding to piston displacement volume at atmospheric conditions.

$$\eta_{vol} = \frac{\text{Actual mass sucked}}{\text{Mass corresponding to swept volume at Atmospheric pressure and temperature}} \quad ...(25.25)$$

$$= \frac{\text{Effective swept volume}}{\text{Piston displacement volume}} \quad ...(25.26)$$

Figure 25.12 shows an indicator diagram for a reciprocating air compressor showing effective swept volume, and piston displacement volume. The volumetric efficiency can be expressed in terms of effective volume and piston displacement volume as;

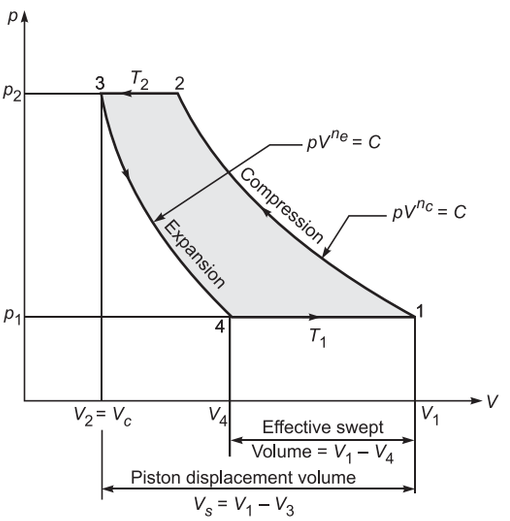

Fig. 25.12 *Indicator diagram for a reciprocating compressor showing effective swept volume, and piston displacement volume*

$$\eta_{vol} = \frac{V_1 - V_4}{V_1 - V_c} = \frac{V_s + V_c - V_4}{V_s + V_c - V_c} = \frac{V_s + V_c - V_4}{V_s}$$

$$= 1 + \frac{V_c}{V_s} - \frac{V_4}{V_s} \times \frac{V_c}{V_c} \quad ...(25.27)$$

Introducing $c = \frac{V_c}{V_s}$ as clearance ratio, and using $V_c = V_3$, then

$$\eta_{vol} = 1 + c - c\left(\frac{V_4}{V_3}\right) \quad ...(25.28)$$

For expansion of gas in clearance volume

$$\frac{V_4}{V_3} = \left(\frac{p_3}{p_4}\right)^{\frac{1}{n_e}} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{n_e}}$$

Then $$\eta_{vol} = 1 + c - c\left(\frac{p_2}{p_1}\right)^{\frac{1}{n_e}} \quad ...(25.29)$$

If index of expansion and index of compression are same, then

$$\left(\frac{p_2}{p_1}\right)^{\frac{1}{n_e}} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{n_c}} = \frac{V_1}{V_2}$$

and $$\eta_{vol} = 1 + c - c\left(\frac{V_1}{V_2}\right) \quad ...(25.30)$$

The volumetric efficiency decreases with pressure ratio $\left(\frac{p_2}{p_1}\right)$ in the compressor, its variation is shown in Fig. 25.13. The factors which lower volumetric efficiency are the following:

1. Too Large Clearance Volume Re-expansion of residual compressed air in the clearance space will reduce effective suction stroke $(V_1 - V_4)$ and therefore, the mass of fresh air entering into the cylinder reduces and volumetric efficiency decreases.

2. Obstruction at Intet Valve Obstruction due to narrow valve passage causes throttling of air in the cylinder. Throttling reduces the pressure in the

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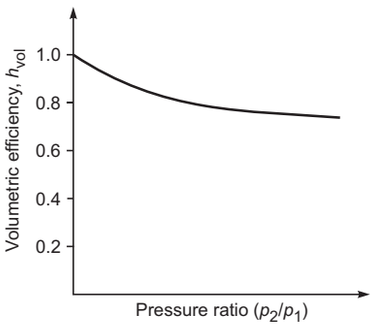

Fig. 25.13 *Effect of pressure ratio on volumetric efficiency of a reciprocating compressor*

cylinder during intake stroke and discharge pressure during delivery stroke, and thus the pressure ratio in the cylinder decreases. It leads to reduced FAD and volumetric efficiency.

3. High speed of Compressor With high speed of the compressor, the pressure drop across the inlet valve and delivery valve increases. Further, the temperature of compressed air increases due to less available time for cooling. Both these factors reduce volumetric efficiency of the compressor.

4. Heated Cylinder Walls The heated cylinder walls increase the temperature of the intake air. Thus, the specific volume of air increases which will reduce FAD and volumetric efficiency of the compressor.

5. Leakage Past the Piston Leakages across the piston will reduce the vacuum during suction and mass of compressed air above the piston. Both these effects will increase compression work input and decrease in volumetric efficiency.

25.10 FREE AIR DELIVERY (FAD)

The volume of compressed air corresponding to atmospheric conditions is known as *free air delivery* (FAD). FAD is the volume of compressed air measured in m^3/min, reduced to atmospheric pressure and temperature.

The free air delivered volume is less than the compressor displacement volume due to the following reasons:

1. *Obstruction at inlet valve* It offers the resistance to air flow through the narrow passage of valve.
2. *Re-expansion of high pressure air in clearance volume* It reduces effective suction stroke.
3. *Presence of hot cylinder walls of compressor* Air gets heated as it enters the cylinder. Thus, it expands and reducing the mass of air sucked into the cylinder.

In the actual indicator diagram as shown in Fig. 25.11, the air is sucked at a pressure and temperature which are lower than that of free (atmospheric) air. Using the property relation for an ideal gas as

$$\frac{p_f V_f}{T_f} = \frac{p_1 (V_1 - V_4)}{T_1}$$

Then free air delivery (FAD)

$$V_f = \frac{p_1 T_f}{p_f T_1} (V_1 - V_4) \qquad ...(25.31)$$

where the suffix *f* denotes free (ambient) conditions while the suffix 1 indicates actual suction conditions. Then volumetric efficiency with respect to free air delivery;

$$\eta_{vol,\ overall} = \frac{V_f}{V_1 - V_c} = \frac{p_1 T_f}{p_f T_1} \left(\frac{V_1 - V_4}{V_1 - V_c} \right)$$

$$= \frac{p_1 T_f}{p_f T_1} \left[1 + c - c \left(\frac{p_2}{p_1} \right)^{\frac{1}{n_e}} \right] \qquad ...(25.32)$$

Example 25.9 *A single-stage, single-acting reciprocating air compressor receives air at 1.013 bar, 27°C and delivers it at 9.5 bar. The compressor has a bore = 250 mm, and stroke = 300 mm and it runs at 200 rpm. The mass-flow rate of air is 200 kg/h. Calculate the volumetric efficiency of the compressor.*

Solution

Given A single-stage, single-acting reciprocating air compressor

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p_1 = 1.03 bar = 101.3 kPa T_1 = 27°C = 300 K
N = 200 rpm p_2 = 9.5 bar
d = 250 mm = 0.25 m L = 300 mm = 0.3 m
$\dot{m}_{act}$ = 200 kg/h

To find The volumetric efficiency of the compressor.

Assumptions

(i) Neglecting clearance volume.
(ii) Specific gas constant of air, R = 0.287 kJ/kg·K.

Analysis The volume swept per cycle

$$V_1 = \left(\frac{\pi}{4}\right) d^2 L$$

$$= \left(\frac{\pi}{4}\right) \times (0.25 \text{ m})^2 \times (0.3 \text{ m})$$

$$= 0.0147 \text{ m}^3$$

The mass of air inducted per cycle

$$m_a = \frac{p_1 V_1}{R T_1} = \frac{(101.3 \text{ kPa}) \times (0.0147 \text{ m}^3)}{(0.287 \text{ kJ/kg·K}) \times (300 \text{ K})}$$

$$= 0.0173 \text{ kg/cycle}$$

The mass-flow rate per hour

$\dot{m}_a$ = mass per cycle × No. of suctions/revolution × No. of revolutions/h
= (0.0173 kg/cycle) × (1 suction/rev) × (200 × 60 rev/h)
= 207.6 kg/h

The mass of air actually sucked, $\dot{m}_{act}$ = 200 kg/h

Thus $$\eta_{vol} = \frac{\text{Actual mass sucked}}{\text{Mass corresponds to swept volume at atmospheric pressure and temperature}}$$

$$= \frac{200 \text{ kg}}{207.6 \text{ kg}} = 0.963 \quad \text{or} \quad \mathbf{96.3\%}$$

Example 25.10 *A single-stage, double-acting reciprocating air compressor has a FAD of 14 m^3/min measured at 1.013 bar and 27°C. The pressure and temperature of the cylinder during induction are 0.95 bar and 45°C. The delivery pressure is 7 bar and the index of compression and expansion is 1.3. Calculate the indicated power required and volumetric efficiency. The clearance volume is 5% of the swept volume.*

Solution

Given A single-stage, double-acting reciprocating air compressor

p_f = 1.03 bar = 101.3 kPa
T_f = 27°C = 300 K
p_2 = 7 bar
FAD, V_f = 14 m^3/min
p_1 = 0.95 bar = 95 kPa
T_1 = 45°C = 318 K
$n_c = n_e = n$ = 1.3
$V_c = 0.05 V_s$

To find

(i) Indicated power, and
(ii) Volumetric efficiency.

Assumptions

(i) The compression and expansion are reversible.
(ii) Air as an ideal gas with R = 0.287 kJ/kg·K.

Analysis The p–V diagram for given data is shown in Fig. 25.14.

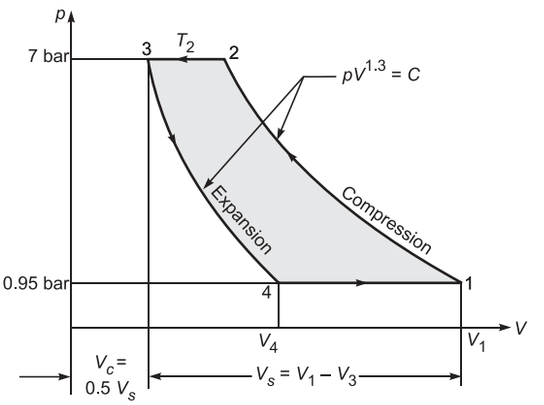

Fig. 25.14

The mass-flow rate corresponding to FAD at atmospheric conditions, p_f and T_f;

$$\dot{m}_a = \frac{p_f V_f}{R T_f}$$

$$= \frac{101.3 \times 14}{0.287 \times 300} = 16.47 \text{ kg/min}$$

The temperature T_2 after compression,

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$= (318 \text{ K}) \times \left(\frac{7}{0.95}\right)^{\frac{1.3-1}{1.3}} = 504.18 \text{ K}$$

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(i) The indicated power,

$$IP = \frac{n}{n-1}\dot{m}_a R(T_2 - T_1)$$

$$= \frac{1.3}{1.3-1} \times \left(\frac{16.47}{60}\text{kg/s}\right) \times (0.287\ \text{kJ/kg}\cdot\text{K}) \times (504.18\ \text{K} - 318\ \text{K})$$

$$= \mathbf{63.56\ kW}$$

(ii) The volumetric efficiency can be obtained by using Eq. (25.32)

$$\eta_{vol,\ overall} = \frac{p_1 T_f}{p_f T_1}\left[1 + c - c\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}\right]$$

$$= \frac{0.95 \times 300}{1.013 \times 318} \times \left[1 + 0.05 - 0.05 \times \left(\frac{7}{0.95}\right)^{\frac{1}{1.3}}\right]$$

$$= 0.723 \text{ or } \mathbf{72.3\%}$$

Example 25.11 *A single-stage, single-acting reciprocating air compressor has a bore of 20 cm and a stroke of 30 cm. The compressor runs at 600 rpm. The clearance volume is 4% of the swept volume and index of expansion and compression is 1.3. The suction conditions are at 0.97 bar and 27°C and delivery pressure is 5.6 bar. The atmospheric conditions are at 1.01 bar and 17°C. Determine,*

(a) The free air delivered in m^3/min

(b) The volumetric efficiency referred to the free air conditions

(c) The indicated power

Solution

Given Single-stage, single-acting reciprocating air compressor

$d = 20\ \text{cm} = 0.2\ \text{m}$ $L = 30\ \text{cm} = 0.3\ \text{m}$

$N = 600\ \text{rpm}$ $n = 1.3$

$p_1 = 0.97\ \text{bar} = 97\ \text{kPa}$ $T_1 = 27°\text{C} = 300\ \text{K}$

$p_f = 1.01\ \text{bar} = 101\ \text{kPa}$ $T_f = 17°\text{C} = 290\ \text{K}$

$V_c = 0.04\ V_s$ $p_2 = 5.6\ \text{bar}$

To find

(i) Free air delivery, m^3/min,

(ii) Volumetric efficiency at ambient conditions; $\eta_{vol,\ overall}$, and

(iii) Indicated power.

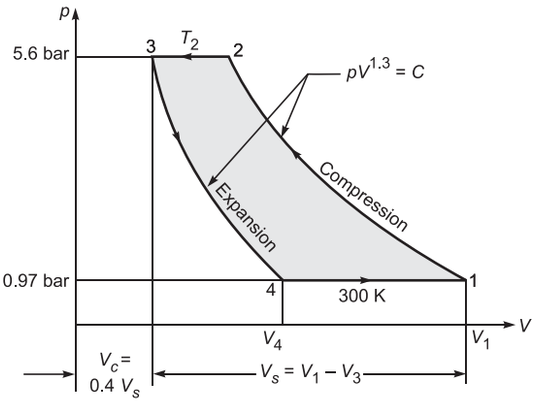

Fig. 25.15

Analysis

(i) *Free air delivery* (FAD)

The swept volume;

$$V_s = V_1 - V_3 = \left(\frac{\pi}{4}\right)d^2 L$$

$$= \left(\frac{\pi}{4}\right) \times (0.2\ \text{m})^2 \times (0.3\ \text{m})$$

$$= 0.009425\ \text{m}^3$$

Clearance volume

$$V_c = 0.04\ V_s = 0.04 \times 0.009425$$

$$= 3.77 \times 10^{-4}\ \text{m}^3$$

Total volume

$$V_1 = V_s + V_c$$

$$= 0.009425 + 3.77 \times 10^{-4}$$

$$= 0.0098\ \text{m}^3$$

The volume V_4, after re-expansion of compressed air in clearance space,

$$\frac{V_4}{V_3} = \left(\frac{p_3}{p_4}\right)^{\frac{1}{n}} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$V_4 = 3.77 \times 10^{-4} \times \left(\frac{5.6}{0.97}\right)^{\frac{1}{1.3}}$$

$$= 0.00145\ \text{m}^3$$

Effective swept volume,

$$V_1 - V_4 = 0.0098 - 0.00145 = 0.00835\ \text{m}^3$$

The free air delivery per cycle,

$$V_f = \frac{p_1 T_f}{p_f T_1}(V_1 - V_4)$$

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$$= \frac{0.97 \times 290}{1.01 \times 300} \times 0.00835$$

$$= 0.00775 \text{ m}^3/\text{cycle}$$

Free air delivered per minute

$$= V_f \times \text{No. of cycle per minute}$$

$$= 0.00775 \times 600 = \mathbf{4.65\ m^3/min}$$

(ii) The *volumetric efficiency referred to free air conditions*

$$\eta_{vol,\ overall} = \frac{V_f}{V_s} = \frac{0.00775}{0.009425}$$

$$= 0.822 \quad \text{or} \quad \mathbf{82.2\%}$$

(iii) *Indicated power*

Indicated work input using Eq. (25.24)

$$W_{in} = \frac{n}{n-1} p_1 (V_1 - V_4) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.3}{1.3-1} \times 97 \times 0.00835 \times \left[\left(\frac{5.6}{0.97} \right)^{\frac{1.3-1}{1.3}} - 1 \right]$$

$$= 1.75 \text{ kJ/cycle}$$

Since N cycles take place within a minute, the indicated power

$$IP = W_{in} \left(\frac{N}{60} \right)$$

$$= (1.75 \text{ kJ/cycle}) \times \left(\frac{600}{60} \text{ cycle/s} \right)$$

$$= \mathbf{17.5\ kW}$$

***Example* 25.12** *A single-stage, double-acting air compressor delivers air at 7 bar. The pressure and temperature at the end of the suction stroke are 1 bar and 27°C. It delivers 2 m³ of free air per minute when the compressor is running at 300 rpm. The clearance volume is 5% of the stroke volume. The pressure and temperature of the ambient air are 1.03 bar and 20°C. The index of compression is 1.3, and index of expansion is 1.35. Calculate*

(a) Volumetric efficiency of the compressor,

(b) Indicated power of the compressor,

(c) Brake Power, if mechanical efficiency is 80%.

Solution

Given A single-stage, double-acting reciprocating air compressor

$p_1 = 1$ bar $= 100$ kPa — $p_2 = 7$ bar

$N = 300$ rpm — $T_1 = 27°C = 300$ K

$\dot{V}_f = 2$ m³/min — $p_f = 1.03$ bar $= 103$ kPa

$V_c = 0.05\ V_s$ — $T_f = 20°C = 293$ K

$\eta_{mech} = 0.8$ — $n_c = 1.3$, and

$n_e = 1.35$

To find

(i) Volumetric efficiency,

(ii) Indicated power, and

(iii) Brake Power.

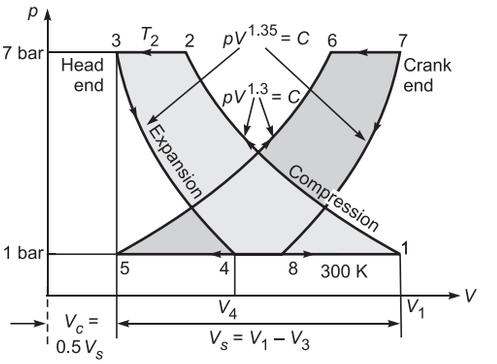

Fig. 25.16 *p-V diagram for double-acting compressor*

Analysis

(i) *Volumetric efficiency*

Clearance volume; $V_c = V_3 = 0.05\ V_s$

The volume V_4 after re-expansion of compressed air in clearance space

$$V_4 = V_3 \left(\frac{p_2}{p_1} \right)^{\frac{1}{n_e}}$$

$$= 0.05\ V_s \times \left(\frac{7}{1} \right)^{\frac{1}{1.35}} = 0.2113 V_s$$

The total volume of cylinder;

$$V_1 = V_s + V_c = 1.05\ V_s$$

Effective swept volume

$$V_1 - V_4 = 1.05\ V_s - 0.2113\ V_s = 0.8386\ V_s$$

The volumetric efficiency

$$\eta_{vol} = \frac{V_1 - V_4}{V_s} = \frac{0.8386 V_s}{V_s}$$

$$= 0.8386 \quad \text{or} \quad \mathbf{83.86\%}$$

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Free air delivered per cycle is given as

$$V_f = \frac{p_1 T_f}{p_f T_1} (V_1 - V_4)$$

Thus volume of air inducted per min at suction condition

$$\dot{V}_1 - \dot{V}_4 = \dot{V}_f \frac{p_f T_1}{p_1 T_f}$$

$$= (2 \text{ m}^3/\text{min}) \times \frac{(1.03 \text{ bar}) \times (300 \text{ K})}{(1 \text{ bar}) \times (293 \text{ K})}$$

$$= 2.109 \text{ m}^3/\text{min}$$

It is the volume sucked per minute, which can also be expressed as

$$\dot{V}_1 - \dot{V}_4 = \eta_{vol} V_s \times N \times \text{No. of suction per revolution}$$

$$2.109 \text{ m}^3/\text{min} = 0.8386 \, V_s \times (300 \text{ rpm}) \times 2$$

or $V_s = 0.00419 \text{ m}^3$

Now $V_1 = 1.05 \times 0.00419 = 0.0044 \text{ m}^3$

and $V_4 = 0.2113 \times 0.00419$

$= 8.856 \times 10^{-4} \text{ m}^3$

(ii) *Indicated power*

Indicated work input per cycle, Eq. (25.23)

$$W_{in} = \frac{n_c}{n_c - 1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n_c - 1}{n_c}} - 1 \right] - \frac{n_e}{1 - n_e} p_1 V_4 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n_e - 1}{n_e}} - 1 \right]$$

$$= \frac{1.3}{1.3 - 1} \times 100 \times 0.0044 \times \left[\left(\frac{7}{1} \right)^{\frac{1.3 - 1}{1.3}} - 1 \right] - \frac{1.35}{1.35 - 1} \times 100 \times 8.856 \times 10^{-4} \times \left[\left(\frac{7}{1} \right)^{\frac{1.35 - 1}{1.35}} - 1 \right]$$

$= 1.080 - 0.224 = 0.856$ kJ/cycle

The indicated power input to compressor

$$IP = W_{in} \left(\frac{N}{60} \right) \times 2 \text{ (for double acting)}$$

$$= (0.856 \text{ kJ/cycle}) \times \left(\frac{300}{60} \text{ cycle/s} \right) \times 2$$

$$= \mathbf{8.558 \text{ kW}}$$

(iii) The brake (shaft) power

$$\text{Brake power} = \frac{\text{Indicated power}}{\eta_{mech}} = \frac{8.558}{0.8}$$

$$= \mathbf{10.7 \text{ kW}}$$

Example 25.13 *A single-stage, double-acting reciprocating air compressor works between 1 bar and 10 bar. The compression follows the law $pV^{1.35}$ = constant. The piston speed is 200 m/min and the compressor speed is 120 rpm. The compressor consumes an indicated power of 62.5 kW with volumetric efficiency of 90%. Calculate*

(a) diameter and stroke of the cylinder

(b) Clearance volume as percentage of stroke volume

Solution

Given A single-stage, single-acting reciprocating air compressor

$p_1 = 1 \text{ bar} = 100 \text{ kPa}$ $\quad p_2 = 10 \text{ bar}$

$N = 120 \text{ rpm}$ $\quad \mathcal{V}_{piston} = 200 \text{ m/min}$

$\eta_{vol} = 0.9$ $\quad IP = 62.5 \text{ kW}$

and $pV^{1.35} = C$

To find

(i) Cylinder bore and piston stroke,

(ii) Clearance ratio.

Analysis

(i) *Cylinder bore* and *piston stroke*

The indicated work input per cycle for double acting compressor can be obtained as

$$W_{in} = IP \left(\frac{60}{2N} \text{ s/cycle} \right)$$

$$= (62.5 \text{ kJ/s}) \times \left(\frac{60}{2 \times 120} \text{ s/cycle} \right)$$

$$= 15.625 \text{ kJ/cycle}$$

Further,

$$W_{in} = \frac{n}{n - 1} p_1 (V_1 - V_4) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n - 1}{n}} - 1 \right]$$

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$$15.625 = \frac{1.35}{1.35-1} \times (100 \text{ kPa}) \times (V_1 - V_4) \times \left[\left(\frac{10}{1} \right)^{\frac{1.35-1}{1.35}} - 1 \right]$$

or $(V_1 - V_4) = 0.0496 \text{ m}^3$

The volumetric efficiency is given as

$$\eta_{vol} = \frac{V_1 - V_4}{V_s}$$

Stroke volume

$$\text{or} \qquad V_s = \frac{0.0496 \text{ m}^3}{0.9} = 0.0551 \text{ m}^3$$

The piston speed is given by

$$\mathcal{V}_{piston} = 2\,LN$$

Stroke length;

$$L = \frac{200}{2 \times 120} = 0.833 \text{ m or } \mathbf{833 \text{ mm}}$$

Further, the stroke volume can be expressed as

$$V_s = \left(\frac{\pi}{4} \right) d^2 L$$

$$\text{Bore} \qquad d = \sqrt{\frac{0.0551}{(\pi/4) \times 0.833}}$$

$$= 0.290 \text{ m} \quad \text{or} \quad \mathbf{290 \text{ mm}}$$

(ii) *Clearance ratio*

The volumetric efficiency is given by

$$\eta_{vol} = 1 + c - c \left(\frac{p_2}{p_1} \right)^{\frac{1}{n}}$$

$$\text{or} \qquad 0.9 = 1 + c - c \times \left(\frac{10}{1} \right)^{\frac{1}{1.35}}$$

Clearance ratio;

$$c = 0.222 \quad \text{or} \quad \mathbf{2.22\%}$$

Example 25.14 *A single-stage, single-acting reciprocating air compressor delivers 0.6 kg/min of air at 6 bar. The temperature and pressure at the suction stroke are 30°C and 1 bar, respectively. The bore and stroke are 100 mm and 150 mm respectively. The clearance volume is 3% of the swept volume and index of expansion and compression is 1.3. Determine,*

(a) the volumetric efficiency of compressor,

(b) the power required, if mechanical efficiency is 85%,

(c) speed of the compressor.

Solution

Given A single-stage, single-acting reciprocating air compressor

$p_1 = 1$ bar $\qquad p_2 = 6$ bar

$\dot{m}_a = 0.6$ kg/min $\qquad T_1 = 30°\text{C} = 303$ K

$L = 150$ cm $\qquad d = 100$ mm

$c = 0.03 \qquad n = 1.3$

$\eta_{mech} = 0.85$

To find

(i) Volumetric efficiency,

(ii) Power required by compressor, and

(iii) Speed of the compressor.

Assumptions

(i) Suction takes place at free air conditions.

(ii) The specific gas constant for air $R = 0.287$ kJ/kg·K.

Analysis

(i) The *volumetric efficiency* is given by

$$\eta_{vol} = 1 + c - c \left(\frac{p_2}{p_1} \right)^{\frac{1}{n}}$$

$$\text{or} \qquad = 1 + 0.03 - 0.03 \times \left(\frac{6}{1} \right)^{\frac{1}{1.3}}$$

$$= 0.910 \quad \text{or} \quad \mathbf{91.0\%}$$

(ii) *Power* required by compressor

Indicated power input

$$IP = \frac{n}{n-1} \dot{m}_a R\, T_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.3}{1.3-1} \times \left(\frac{0.6 \text{ kg}}{60 \text{ s}} \right) \times 0.287 \times 303 \times \left[\left(\frac{6}{1} \right)^{\frac{1.3-1}{1.3}} - 1 \right]$$

$$= 1.929 \text{ kW}$$

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Brake power required;

$$BP = \frac{IP}{\eta_{mech}} = \frac{1.929}{0.85} = \mathbf{2.27\ kW}$$

(iii) *Speed of the compressor*

The volume-flow rate of air at suction conditions

$$\dot{V}_1 - \dot{V}_4 = \frac{\dot{m}_a R T_1}{p_1}$$

$$= \frac{0.6 \times 0.287 \times 303}{100}$$

$$= 0.5217\ \text{m}^3/\text{min}$$

Piston-displacement volume rate

$$\dot{V}_s = \frac{\dot{V}_1 - \dot{V}_4}{\eta_{vol}} = \frac{0.5217\ \text{m}^3/\text{min}}{0.91}$$

$$= 0.5733\ \text{m}^3/\text{min}$$

Further, $\dot{V}_s = \left(\frac{\pi}{4}\right) d^2 L N$

$\therefore$ 0.5733

$$= \left(\frac{\pi}{4}\right) \times (0.1\ \text{m})^2 \times (0.1\ 5\text{m}) \times N$$

Speed of compressor;

$$N = \mathbf{487\ rpm}$$

Example 25.15 *A single-stage, double-acting reciprocating air compressor is driven by an electric motor consuming 40 kW, compresses 5.5 m^3/min air according to the law $pV^{1.3}$ = constant. It receives atmospheric air at 20°C and 745 mm of Hg barometer and delivers at a gauge pressure of 700 kPa. Calculate isothermal, volumetric, mechanical and overall efficiencies for the following data from an indiactor diagram for head and tail end.*

Length of the indicator diagram = 6.75 cm
Area of the head end = 7.6 cm^2
Area of the tail end = 7.8 cm^2
Spring scale = 200 kPa/cm

The diameter of the piston and piston rod are 25 and 2.5 cm, respectively. The stroke length is 30 cm. The compress or runs at 300 rpm.

Solution

Given A single-stage, double-acting reciprocating air compressor.

p_1 = 745 mm of Hg — p_{g_2} = 700 kPa
N = 300 rpm — $\dot{V}_f$ = 5.5 m^3/min
BP = 40 kW — L = 30 cm
d = 25 cm — d_{rod} = 2.5 cm, n = 1.3
$T_1 = T_f$ = 20°C = 293 K — n = 1.3 cm

For indicator diagram,

a_1 = 7.6 cm^2 — a_2 = 7.8 cm^2
l = 6.75 cm — k = 200 kPa/cm

To find

(i) Overall efficiency,
(ii) Isothermal efficiency,
(iii) Mechanical efficiency, and
(iv) Volumetric efficiency.

Assumptions

(i) Suction takes place at free air conditions.
(ii) Negligible clearance volume on head and tail side of the cylinder.
(iii) Specific gas constant for air R = 0.287 kJ/kg·K.

Analysis The suction pressure corresponds to 745 mm of Hg;

$$p_1 = \frac{745}{760} \times (101.3\ \text{kPa}) = 99.3\ \text{kPa}$$

Absolute delivery pressure,

$$p_2 = p_1 + p_{g_2} = 99.3 + 700 = 799.3\ \text{kPa}$$

Mass of air inducted per second

$$\dot{m}_a = \frac{p_1 \dot{V}_f}{R T_f} = \frac{(9\ 93\ \text{kPa}) \times (5.5\ \text{m}^3/\text{min})}{(0.287\ \text{kJ/kg}\cdot\text{K}) \times (293\ \text{K})}$$

$$= 6.49\ \text{kg/min or } 0.1082\ \text{kg/s}$$

Temperature after polytropic compression

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$= (293\ \text{K}) \times \left(\frac{793.3}{93.3}\right)^{\frac{1.3-1}{1.3}}$$

$$= 474.12\ \text{K}$$

Actual power input to air in the compressor;

$$\dot{W}_{in} = \frac{n}{n-1} \dot{m}_a R (T_2 - T_1)$$

$$= \frac{1.3}{1.3-1} \times 0.1082 \times 0.287 \times (474.12 - 293)$$

$$= 24.37\ \text{kW}$$

(i) *Overall efficiency*

$$\eta_{Overall} = \frac{\dot{W}_{in}}{BP} = \frac{24.37\ \text{kW}}{40\ \text{kW}}$$

$$= 0.6093 \quad \text{or} \quad \mathbf{60.93\%}$$

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Isothermal power input to the compressor;

$$\dot{W}_{iso} = p_1 \dot{V}_f \ln\left(\frac{p_2}{p_1}\right)$$

$$= (99.3 \text{ kPa}) \times \left(\frac{5.5 \text{ m}^3}{60 \text{ s}}\right) \times \ln\left(\frac{793.3}{93.3}\right)$$

$$= 19.48 \text{ kW}$$

(ii) *Isothermal efficiency*

$$\eta_{iso} = \frac{\dot{W}_{iso}}{BP} = \frac{19.48}{40} = 0.487 \quad \text{or} \quad \mathbf{48.7\%}$$

For head end of the cylinder

Cross-sectional area,

$$A_1 = \left(\frac{\pi}{4}\right) d^2 = \left(\frac{\pi}{4}\right) \times (0.25 \text{ m})^2$$

$$= 0.049 \text{ m}^2$$

Indicated mean effective pressure;

$$p_{m_1} = \frac{a_1}{l} k = \frac{7.6 \text{ cm}^2}{6.75 \text{ cm}} \times (200 \text{ kPa/cm})$$

$$= 225.18 \text{ kPa}$$

For tail end of the cylinder

Cross-sectional area,

$$A_2 = \left(\frac{\pi}{4}\right)\left[d^2 - d_p^2\right]$$

$$= \left(\frac{\pi}{4}\right) \times \left[(0.25 \text{ m})^2 - (0.025 \text{ m})^2\right]$$

$$= 0.0486 \text{ m}^2$$

Indicated mean effective pressure;

$$p_{m_2} = \frac{a_2}{l} k = \frac{7.8 \text{ cm}^2}{6.75 \text{ cm}} \times (200 \text{ kPa/cm})$$

$$= 231.11 \text{ kPa}$$

Total indicated power input to head and tail sides of the compressor

$$IP = \frac{p_{m_1} L A_1 N}{60} + \frac{p_{m_2} L A_2 N}{60}$$

$$= \frac{225.18 \times 0.3 \times 0.049 \times 300}{60} + \frac{231.11 \times 0.3 \times 0.0486 \times 300}{60}$$

or $IP = 4.965 + 5.054 = 33.4 \text{ kW}$

(iii) *Mechanical efficiency*

$$\eta_{mech} = \frac{IP}{BP} = \frac{33.4 \text{ kW}}{40 \text{ kW}}$$

$$= 0.835 \quad \text{or} \quad \mathbf{83.5\%}$$

Total displacement volume per minute from head and tail ends of the cylinder;

$$\dot{V}_{total} = (A_1 + A_2) L N$$

$$= (0.049 + 0.0486) \times 0.3 \times 300$$

$$= 8.78 \text{ m}^3/\text{min}$$

(iv) *Volumetric efficiency*

$$\eta_{vol} = \frac{\dot{V}_f}{\dot{V}_{total}} = \frac{5.5 \text{ m}^3/\text{min}}{8.78 \text{ m}^3/\text{min}}$$

$$= 0.626 \quad \text{or} \quad \mathbf{62.6\%}$$

Example 25.16 *During the overhauling of an old compressor, a distance piece of 9 mm thickness is inserted accidentally between the cylinder head and cylinder. Before overhaul, the clearance volume was 3 per cent of the swept volume. The compressor receives atmospheric air at 1 bar and it is designed to deliver air at a gauge pressure of 7 bar with a stroke of 75 cm. If the compression and re-expansion follow the law $pV^{1.3}$ = constant, determine the percentage change in*

(a) volume of free air delivered,

(b) power necessary to drive the compressor.

Solution

Given A reciprocating air compressor before and after overhaul

$p_1 = 1$ bar $\qquad p_{g_2} = 7$ bar

$t = 9$ mm $\qquad V_c = 0.03 V_s$

$L = 75$ cm $\qquad n = 1.3$

To find

(i) Percentage change in volume of free air delivered,

(ii) Percentage change in power necessary to drive the compressor.

Analysis The absolute pressure of delivered air

$$p_2 = p_1 + p_{g_2} = 1 \text{ bar} + 7 \text{ bar} = 8 \text{ bar}$$

The clearance space

Before overhaul,

$$L_{c_1} = 0.03L = 0.03 \times 75 = 2.25 \text{ cm}$$

After overhaul,

$$L_{c_2} = L_{c_1} + t = 2.25 \text{ cm} + 0.9 \text{ cm}$$

$$= 3.15 \text{ cm}$$

The clearance volume V_4 after re-expansion of compressed air in clearance space

$$V_4 = V_3 \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} = V_3 \times \left(\frac{8}{1}\right)^{\frac{1}{1.3}} = 4.95 V_c$$

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Using cylinder cross-section area A, before overhaul;

$$V_4 = 4.95 \times 2.25\ A = 11.14\ A\ \text{cm}^3$$

After overhaul,

$$V_{4a} = 4.95 \times 3.15\ A = 15.59\ A\ \text{cm}^3$$

Total volume of compressor

Before overhaul;

$$V_1 = V_s + AL_{c_1} = 75A + 2.25A$$
$$= 77.25\ A\ \text{cm}^3$$

After overhaul;

$$V_{1a} = V_s + AL_{c_2} = 75A + 3.15A = 78.15\ A\ \text{cm}^3$$

The effective suction volume

Before overhaul;

$$V_1 - V_4 = 77.25\ A\ \text{cm}^3 - 11.14\ A\ \text{cm}^3$$
$$= 66.11\ A\ \text{cm}^3$$

After overhaul;

$$V_{1a} - V_{4a} = 78.15\ A\ \text{cm}^3 - 15.59\ A\ \text{cm}^3$$
$$= 62.56\ A\ \text{cm}^3$$

(i) Percentage change in FAD

$$= \frac{66.11A - 62.56A}{66.11A} \times 100 = \mathbf{5.37\%}$$

(ii) Indicator work input

$$W_1 = \frac{n}{n-1} p_1(V_1 - V_4)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

In calculation of indicated work input, all terms except $(V_1 - V_4)$ will remain constant before and after overhaul. Therefore, percentage change in indicated work input will be equal to percentage change in FAD, i.e. % change in work input = **5.37%**

Example 25.17 *A 4-cylinder, single-stage, double acting air compressor delivers air at 7 bar. The pressure and temperature at the end of the suction stroke are 1 bar and 27°C. It delivers 30 m^3 of free air per minute when the compressor is running at 300 rpm. The pressure and temperature of the ambient air are 1 bar and 17°C. The clearance volume is 4% of the stroke volume. The stroke-to-bore ratio is 1.2. The index of compression and expansion is 1.32. Calculate*

(a) Volumetric efficiency of the compressor,
(b) Indicated power of the compressor,
(c) Size of motor, if mechanical efficiency is 85%,
(d) Cylinder dimensions.

Solution

Given A 4-cylinder, single-stage, double-acting reciprocating air compressor

No. of cylinders = 4 $\quad k = 2$
$p_1 = 1$ bar = 100 kPa $\quad T_1 = 27°C = 300$ K
$p_2 = 7$ bar $\quad \dot{V}_f = 30$ m^3/min
$N = 300$ rpm $\quad p_f = 1$ bar
$c = 0.04$ $\quad T_f = 17°C = 290$ K
$\eta_{mech} = 0.85$ $\quad n = 1.32$
$L/d = 1.2$

To find

(i) Volumetric efficiency,
(ii) Indicated power,
(iii) Size of motor (brake power), and
(iv) Bore of cylinder and piston stroke.

Analysis

(i) *Volumetric efficiency*

Clearance ratio;

$$c = 0.04$$

The volumetric efficiency;

$$\eta_{vol} = 1 + c - c\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$
$$= 1 + 0.04 - 0.04 \times \left(\frac{7}{1}\right)^{\frac{1}{1.32}}$$
$$= 0.8653 \quad \text{or} \quad \mathbf{86.53\%}$$

(ii) *Indicated power*

The effective swept volume;

$$\dot{V}_1 - \dot{V}_4 = \frac{p_f \dot{V}_f}{T_f} \times \frac{T_1}{p_1}$$
$$= \frac{(1\ \text{bar}) \times (30\ \text{m}^3/\text{min})}{(290\ \text{K})} \times \left(\frac{300\ \text{K}}{1\ \text{bar}}\right)$$
$$= 31.03\ \text{m}^3/\text{min} \quad \text{or} \quad 0.517\ \text{m}^3/\text{s}$$

Indicated power;

$$IP = \frac{n}{n-1} p_1(\dot{V}_1 - \dot{V}_4)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$
$$= \frac{1.32}{1.32 - 1} \times (100\ \text{kPa})$$
$$\times (0.517\ \text{m}^3/\text{s}) \times \left[\left(\frac{7}{1}\right)^{\frac{1.32-1}{1.32}} - 1\right]$$
$$= \mathbf{128.60\ kW}$$

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(iii) The motor (brake) power

$$BP = \frac{IP}{\eta_{mech}} = \frac{128.6}{0.85} = 151.3 \text{ kW}$$

(iv) *Cylinder dimensions*

The piston displacement volume of one cylinder can be obtained by using volumetric efficiency as

$$V_s = \frac{\dot{V}_1 - \dot{V}_4}{\text{No. of cylinders} \times \eta_{vol}}$$

$$= \frac{31.03}{4 \times 0.8653} = 8.965 \text{ m}^3/\text{min}$$

The piston displacement volume rate with $L = 1.2\,d$; for double-acting cylinder can be expressed as

$$V_s = \left(\frac{\pi}{4}\right) d^2 L N k$$

$$= \left(\frac{\pi}{4}\right) d^2 \times (1.2\,d) \times 300 \times 2$$

Bore $$d = \sqrt[3]{\frac{8.965}{(\pi/4) \times 1.2 \times 300 \times 2}}$$

$= 0.251$ m or **251 mm**

Stroke; $L = 1.2\,d =$ **301.4 mm**

Example 25.18 *A twin-cylinder single-stage, single-acting reciprocating air compressor running at 300 rpm has pressure ratio of 8. The clearance is 3 per cent of the swept volume. It compresses 30 m^3/min free air at 101.3 kPa and 20°C according to $pV^{1.3}$ = constant. The temperature rise during suction stroke is 25°C . The loss of pressure through intake and discharge valve is 8 and 150 kPa, respectively. Determine*

(a) Indicated power required by the compressor,

(b) Brake power, assuming compression efficiency of 85% and mechanical efficiency of 95% ,

(c) Cylinder dimesions for same bore and stroke size.

Solution

Given A single-stage, single-acting reciprocating air compressor

No. of cylinders = 2 $c = 0.03$

$N = 300$ rpm $n = 1.3$

$\dot{V}_f = 30$ m^3/min $p_f = 101.3$ kPa

$T_f = 20°C = 293$ K $p_f - p_1 = 8$ kPa

$\frac{p_2}{p_1} = 8$ $L = d$

$T_1 = 20°C + 25°C = 45°C$ or 318 K

$p_2 - p_d = 150$ kPa

$\eta_{comp} = 0.85$

$\eta_{mech} = 0.95$

To find

(i) Indicated power necessary to drive the compressor,

(ii) Brake power, and

(iii) Cylinder dimesions for same bore and stroke size.

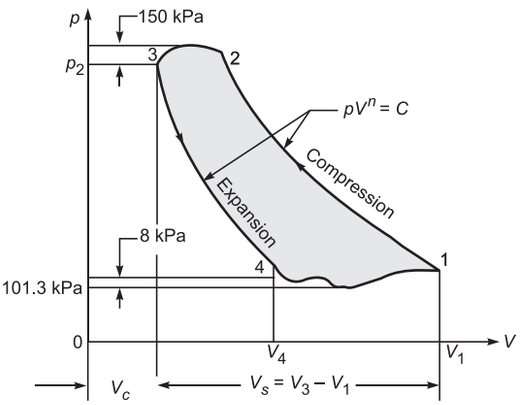

Fig. 25.17

Analysis The suction pressure

$$p_1 = p_f - 8 \text{ kPa} = 101.3 - 8 = 93.3 \text{ kPa}$$

Using the property relation for an ideal gas as

$$\frac{p_f \dot{V}_f}{T_f} = \frac{p_1 (\dot{V}_1 - \dot{V}_4)}{T_1}$$

It gives

$$\dot{V}_1 - \dot{V}_4 = \frac{p_f \dot{V}_f}{T_f} \times \frac{T_1}{p_1} = \frac{101.3 \times 30}{293} \times \frac{318}{93.3}$$

$$= 35.35 \text{ m}^3/\text{min}$$

(i) *The indicated power input to the compressor*

$$IP = \frac{n}{n-1} p_1 (\dot{V}_1 - \dot{V}_4) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.3}{1.3-1} \times 93.3 \times \left(\frac{35.5}{60} \right) \times \left[(8)^{\frac{1.3-1}{1.3}} - 1 \right]$$

$=$ **146.7 kW**

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(ii) *The brake power input to the compressor*
Brake power;

$$BP = \frac{\text{Indicated power}}{\eta_{comp} \times \eta_{mech}} = \frac{146.7}{0.85 \times 0.95}$$

$$= \mathbf{181.67\ kW}$$

(iii) *Cylinder dimesions for same bore and stroke size:*
The volumetric efficiency is given as

$$\eta_{vol} = 1 + c - c\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$= 1 + 0.03 - 0.03 \times (8)^{1/1.3}$$

$$= 0.8814$$

Further, the volumetric efficiency is also given as

$$\eta_{vol} = \frac{(\dot{V}_1 - \dot{V}_4)}{\dot{V}_s}$$

$\therefore$ total displacement volume for two cylinders;

$$\dot{V}_s = \frac{(\dot{V}_1 - \dot{V}_4)}{\eta_{vol}} = \frac{35.35}{0.8814}$$

$$= 40.1\ \text{m}^3/\text{min}$$

For a two-cylinder, single-acting reciprocating air compressor, the displacement volume per minute is also expressed as

$$\dot{V}_s = 2 \times \left(\frac{\pi}{4}\right) d^2 L N$$

For $d = L$;

$$40.1 = 2 \times \left(\frac{\pi}{4}\right) d^3 \times 300$$

It gives bore and stroke sizes as

$$d = L = 0.44\ \text{m} \quad \text{or} \quad \mathbf{440\ mm}$$

25.11 LIMITATIONS OF SINGLE-STAGE COMPRESSION

Usually, the pressure ratio for a single-stage reciprocating air compressor is limited to 7. Increase in pressure ratio in a single-stage reciprocating air compressor causes the following undesirable effects:

1. Greater expansion of clearance air in the cylinder and as a consequence, it decreases effective suction volume ($V_1 - V_4$) and therefore, there is a decrease in fresh air induction.
2. With high delivery pressure, the delivery temperature increases. It increases specific volume of air in the cylinder, thus more compression work is required.
3. Further, for high pressure ratio, the cylinder size would have to be large, strong and heavy working parts of the compressor will be needed. It will increase balancing problem and high torque fluctuation will require a heavier flywheel installation.

All the above problems can be reduced to minimum level with multistage compression.

25.12 MULTISTAGE COMPRESSION

As discussed in preceding sections, the compressor requires minimum work input with isothermal compression. But the delivery temperature T_2 increases with pressure ratio and the volumetric efficiency decreases as pressure ratio increases.

All the above problems can be reduced to minimum level by compressing the air in more than one cylinders with intercooling between stages, for the same pressure ratio. The compression of air in two or more cylinders in series is called *multistage compression.* Air cooling between stages provides the means to achieving an appreciable reduction in the compression work and maintaining the air temperature within safe operating limits.

25.12.1 Advantages of Multistage Compression

1. The gas can be compressed to a sufficiently high pressure.
2. Cooling of air is more efficient with intercoolers and cylinder wall surface.
3. By cooling the air between the stages of compression, the compression can be brought to isothermal and power input to the compressor can be reduced considerably.

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4. By multistaging, the pressure ratio of each stage is lowered. Thus, the air leakage past the piston in the cylinder is also reduced.
5. The low pressure ratio in a cylinder improves volumetric efficiency.
6. Due to phasing of operation in stages, in a multistage compressor, the negative and positive forces are balanced to a large extent. Thus, more uniform torque and better mechanical balance can be achieved.
7. Due to low pressure ratio in stages, the compressor speed could be higher for same isothermal efficiency.
8. Low working temperature in each stage helps to sustain better lubrication.
9. The low-pressure cylinder is made lighter and high-pressure cylinders are made smaller for reduced pressure ratio in each stage.

25.12.2 Work Done in Multistage Compressor with Intercooler

Figure 25.18 shows a schematic for two-stage compression. The air at p_1 and T_1 is first drawn into the first stage or low pressure (LP) cylinder. It is partially compressed to some intermediate pressure, p_2 and temperature T_2 and is then discharged to an intercooler which ideally cools the air to its initial temperature T_1. The cooled air then enters the second stage or high pressure (HP) cylinder and is compressed to a delivery pressure p_3 and temperature T_3. The corresponding indicator diagram is shown on a p–V plane in Fig. 25.19.

The cycle 1–2–3–4–1 represents first-stage compression cycle. The air is discharged from LP

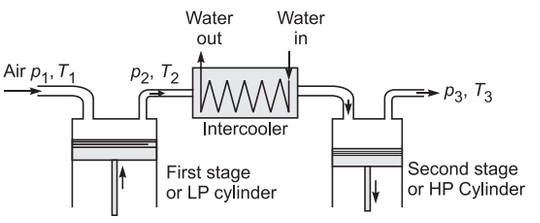

Fig. 25.18 *Two stage compression with intercooler*

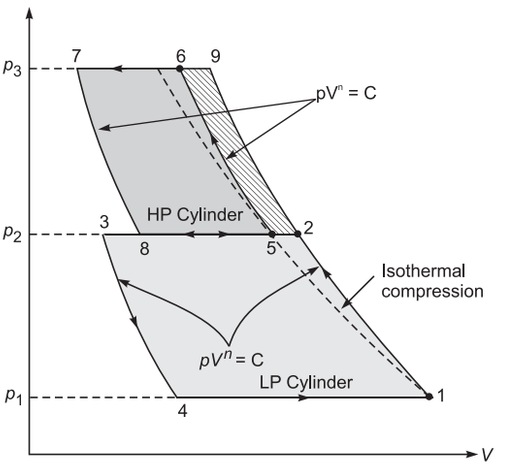

Fig. 25.19 *Indicator diagram for two-stage compressor*

cylinder at intermediate pressure p_2 and temperature T_2, related with inlet pressure p_1 and temperature T_1 as

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

The air is then cooled in an intercooler, if intercooling is complete (perfect), the air will enter the HP cylinder at the same temperature at which it enters the LP cylinder. The second-stage compression cycle in an HP cylinder is shown by cycle 5–6–7–8–5. The line 1–2–9 represents the single-stage compression from initial pressure p_1 to delivery pressure p_3. The shadded area 2–9–6–5–2 represents the saving in compression work obtained by intercooling.

The total indicator work

$$W_{in} = W_{LP} + W_{HP}$$

$$= \frac{n}{n-1} p_1 (V_1 - V_4)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$+ \frac{n}{n-1} p_1 (V_5 - V_8)\left[\left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 1\right]$$

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In terms of mass of air inducted per cycle;

$$W_{in} = \frac{n}{n-1} m_a R T_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] + \frac{n}{n-1} m_a R T_1 \left[\left(\frac{p_3}{p_2} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$W_{in} = \frac{n}{n-1} \times m_a R T_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2} \right)^{\frac{n-1}{n}} - 2 \right] \text{(kJ/cycle)} \quad ...(25.33)$$

where, $m_a = \frac{p_f V_f}{R T_f} = \frac{p_1 (V_1 - V_4)}{R T_1} = \frac{p_2 (V_5 - V_8)}{R T_1}$

Since same mass of air is handled by both cylinders, the suffix f represents free air conditions.

For given mass-flow rate $\dot{m}_a$ (kg/s), the indicated power

$$IP = \frac{n}{n-1} \dot{m}_a R T_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2} \right)^{\frac{n-1}{n}} - 2 \right] \text{(kW)} \quad ...(25.34)$$

25.12.3 Heat Rejected per Stage of Compression

In a two-stage air compressor, air rejects heat (i) during compression process, and (ii) after compression in intercooler.

$$Q_{stage} = Q_{comp} + Q_{cooling}$$

Heat rejected during polytropic compression process

$$Q_{comp} = m_a C_n (T_2 - T_1) \text{ (kJ)}$$

where $C_n = \frac{C_p - n C_v}{n - 1}$, the polytropic specific heat, measured in kJ/kg · K.

For perfect intercooling, the temperature of air after cooling should be reduced to initial temperature T_1. Therefore, the heat rejected in intercooler

$$Q_{cooling} = m_a C_p (T_2 - T_1) \text{ (kJ)} \quad ...(25.35)$$

Using $C_p = \gamma C_v$, we get

$$C_n = \frac{\gamma C_v - n C_v}{n-1} = \frac{(\gamma - n)}{n-1} C_v$$

Then total heat rejected;

$$Q_{stage} = m_a \frac{(\gamma - n)}{n-1} C_v (T_2 - T_1) + m_a C_p (T_2 - T_1)$$

$$= m_a \left[\frac{(\gamma - n)}{n-1} C_v + C_p \right] (T_2 - T_1) \text{ (kJ)} \quad ...(25.36)$$

For heat rejected per kg of air

$$q_{stage} = \left[\frac{(\gamma - n)}{n-1} C_v + C_p \right] (T_2 - T_1) \text{ (kJ/kg)} \quad ...(25.37)$$

25.12.4 Actual Indicator Diagram for a Two-Stage Compressor

The actual indicator diagram on a p–V plane for a two-stage reciprocating air compressor is shown in Fig. 25.20. The variation during suction and delivery processes is due to valve action effects.

The indicator diagrams for low-pressure and high-pressure cylinders overlap due to pressure drop taking place in intercooler and clearance effect.

During the suction process, pressure drops in the cylinder until the inlet valve is forced to open by air. Similarly, during delivery process , some more pressure is required to open the delivery valve and to displace the compressed air through a narrow valve passage. Thus, *gas throttling* takes place during delivery, which reduces the pressure gradually.

25.12.5 Condition for Minimum Compression Work: Optimum Intermediate Pressure

The intermediate pressure p_2 influences the work to be done on the gas and its distribution between

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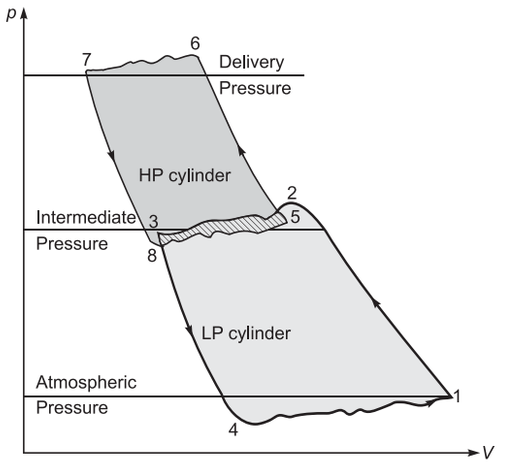

Fig. 25.20 *Actual indicator diagram for two-stage reciprocating air compressor*

stages. The intermediate pressure, which makes work input minimum, is always important.

The total power input to a two-stage reciprocating air compressor with complete intercooling is given by Eq. (25.34);

$$IP = \frac{n}{n-1}\,\dot{m}_a R T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 2\right]$$

If p_1, T_1 and p_3 are fixed then the optimum value of intermediate pressure p_2 for minimum work input can be obtained by applying condition of minima, i.e.,

$$\frac{d(IP)}{dp_2} = 0$$

$$\frac{n}{n-1}\dot{m}_a R T_1 \frac{d}{dp_2}\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 2\right] = 0$$

$$\left(\frac{1}{p_1}\right)^{\frac{n-1}{n}} \frac{d}{dp_2}(p_2)^{\frac{n-1}{n}} + (p_3)^{\frac{n-1}{n}} \frac{d}{dp_2}(p_2)^{\frac{1-n}{n}} = 0$$

$$\left(\frac{1}{p_1}\right)^{\frac{n-1}{n}} \left(\frac{n-1}{n}\right)(p_2)^{\frac{n-1}{n}-1} + (p_3)^{\frac{n-1}{n}} \left(\frac{1-n}{n}\right)(p_2)^{\frac{1-n}{n}-1} = 0$$

or $$\left(\frac{1}{p_1}\right)^{\frac{n-1}{n}} (p_2)^{-\frac{1}{n}} - (p_3)^{\frac{n-1}{n}} (p_2)^{\frac{1-2n}{n}} = 0$$

or $$(p_1)^{\frac{1-n}{n}} (p_2)^{-\frac{1}{n}} = (p_3)^{\frac{n-1}{n}} (p_2)^{\frac{1-2n}{n}}$$

or $$(p_2)^{-\frac{1}{n}+\frac{2n-1}{n}} = (p_1)^{\frac{n-1}{n}} (p_3)^{\frac{n-1}{n}}$$

or $$(p_2)^{\frac{2(n-1)}{n}} = (p_1 p_3)^{\frac{n-1}{n}}$$

Therefore,

$$p_2^2 = p_1 p_3 \qquad ...(25.38)$$

or $$\frac{p_2}{p_1} = \frac{p_3}{p_2} \qquad ...(25.39)$$

It is proved that for minimum compression work, the conditions required are the following

1. The pressure ratio of each stage should be the same.
2. The pressure ratio of any stage is the square root of overall pressure ratio, for a two stage compressor.
3. Air after compression in each stage should be cooled to intial temperature of air intake.
4. The work input to each stage is same.

Consider multistage compression with z stages. Then

$$\frac{p_2}{p_1} = \frac{p_3}{p_2} = \frac{p_4}{p_3} = \ldots = \frac{p_{(z+1)}}{p_z} = X \text{ (say)} \qquad ...(25.40)$$

Then $p_2 = Xp_1;\ p_3 = Xp_2 = X^2 p_1;$

$p_4 = Xp_3 = X^2 p_2 = X^3 p_1$

and $p_{(z+1)} = Xp_z = \ldots = X^z p_1$

or $$X^z = \frac{p_{(z+1)}}{p_1}$$

or $$X = \sqrt[z]{\frac{p_{(z+1)}}{p_1}}$$

$$= \sqrt[z]{\text{(Pressure ratio through compressor)}} \qquad ...(25.41)$$

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25.12.6 Minimum Compression Work Input for Two-Stage Compression

Inserting Eq. (25.39) in Eq. (25.34), we get total minimum power,

$$IP_{min} = 2 \times \left(\frac{n}{n-1}\right) \dot{m}_a R T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] \quad ...(25.42)$$

$= 2 \times$ power required for one stage

In terms of overall pressure ratio

$$\frac{p_2}{p_1} = \frac{\sqrt{p_1 p_3}}{p_1} = \sqrt{\frac{p_3}{p_1}} = \left(\frac{p_3}{p_1}\right)^{\frac{1}{2}}$$

Total minimum power;

$$IP_{min} = 2 \times \left(\frac{n}{n-1}\right) \dot{m}_a R T_1 \left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}} - 1\right] \quad ...(25.43)$$

This expression can be extended to z stages of compression.Total minimum power;

$$IP_{min} = z \times \left(\frac{n}{n-1}\right) \dot{m}_a R T_1 \left[\left(\frac{p_{(z+1)}}{p_1}\right)^{\frac{n-1}{zn}} - 1\right] \quad ...(25.44)$$

where the pressure ratio in each stage $= \left(\frac{p_{z+1}}{p_1}\right)^{\frac{1}{z}}$

Example 25.19 *Calculate the power required to compress 25 m^3/min atmospheric air at 101.3 kPa, 20°C to a pressure ratio of 7 in an LP cylinder. Air is then cooled at constant pressure to 25°C in an intercooler, before entering HP cylinder, where air is again compressed to a pressure ratio of 6. Assume polytropic compression with n = 1.3 and R = 0.287 kJ/kg·K.*

Solution

Given A two-stage reciprocating air compressor with imperfect inercooler;

$\dot{V}_1 = 25$ m^3/min $\quad p_1 = 101.3$ kPa $\quad T_1 = 20°C = 293$ K $\quad T_3 = 25°C = 298$ K

$\frac{p_2}{p_1} = 7.0 \quad \frac{p_3}{p_2} = 6.0$

$n = 1.3 \quad R = 0.287$ kJ/kg·K

To find Power input to compressor.

Analysis The mass of air compressed per minute, using perfect gas equation

$$\dot{m}_a = \frac{p_1 \dot{V}_1}{R T_1} = \frac{(101.3 \text{ kPa}) \times (25 \text{ m}^3/\text{min})}{(0.287 \text{ kJ/kg·K}) \times (293 \text{ K})} = 30.11 \text{ kg/min}$$

Temperature of air after first-stage compression;

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = (293\text{K}) \times \left(\frac{7}{1}\right)^{\frac{1.3-1}{1.3}} = 459.08 \text{ K}$$

The work input in LP cylinder compressor,

$$IP_{LP} = \frac{n}{n-1} \dot{m}_a R (T_2 - T_1) = \frac{1.3}{1.3-1} \times 30.11 \times 0.287 \times (459.08 - 300)$$

$= 6219.24$ kJ/min or **103.65 kW**

Temperature of air after second-stage compression;

$$T_4 = T_3 \left(\frac{p_4}{p_3}\right)^{\frac{n-1}{n}} = (298\text{K}) \times \left(\frac{6}{1}\right)^{\frac{1.3-1}{1.3}} = 450.59 \text{ K}$$

The work input in HP cylinder of compressor,

$$IP_{HP} = \frac{n}{n-1} \dot{m}_a R (T_4 - T_3)$$

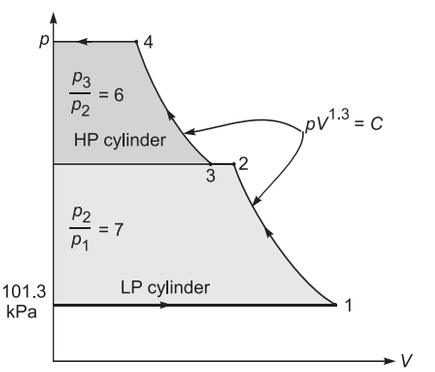

Fig. 25.21 *Two-stage compression*

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$$= \frac{1.3}{1.3-1} \times 30.11 \times 0.287 \times (450.59 - 298)$$

$= 5714$ kJ/min or **95.23 kW**

Total power input to the compressor

$IP = IP_{LP} + IP_{HP}$

$= 103.65$ kW + 95.23 kW = **198.88 kW**

Example 25.20 *The LP cylinder of a two-stage double-acting reciprocating air compressor running at 120 rpm has a 50-cm diameter and 75-cm stroke. It draws air at a pressure of 1 bar and 20°C and compresses it adiabatically to a pressure of 3 bar. The air is then delivered to the intercooler, where it is cooled at constant pressure to 35°C and is then further compressed polytropically (index n = 1.3) to 10 bar in HP cylinder. Determine the power required to drive the compressor. The mechanical efficiency of the compressor is 90% and motor effieincy is 86%.*

Solution

Given A two-stage, double-acting reciprocating air compressor with imperfect intercooling;

$N = 120$ rpm	$d_1 = 50$ cm = 0.5 m
$L = 75$ cm = 0.75 m	$p_1 = 1$ bar
$T_1 = 20°C = 293$ K	$p_2 = 3$ bar
$p_1V_1^{\gamma} = p_2V_2^{\gamma}$	$p_2 = p_3 = 3$ bar
$T_3 = 35°C = 308$ K	$p_4 = 10$ bar
$p_3V_3^n = p_4V_4^n$ with $n = 1.3$	$k = 2$
$\eta_{mech} = 0.9$	$\eta_{Motor} = 0.86$

To find Motor power input to drive the compressor.

Assumptions

(i) The effect of the piston rod is negligible on the cylinder volume.

(ii) For air: $R = 0.287$ kJ/kg·K, $C_p = 1.005$ kJ/kg·K and $\gamma = 1.4$.

Analysis The volume-flow rate of air to LP cylinder

$$\dot{V}_1 = \frac{\pi}{4} d_{LP}^2 L \frac{N}{60} k$$

$$= \frac{\pi}{4} \times (0.5\text{ m})^2 \times (0.7\text{ 5m}) \times \left(\frac{120}{60}\text{ rps}\right) \times 2$$

$= 0.589$ m^3/s

The density of incoming air

$$\rho_1 = \frac{p_1}{RT_1} = \frac{(100\text{ kPa})}{(0.287\text{ kJ/kg}\cdot\text{K}) \times (293\text{ K})}$$

$= 1.189$ kg/m^3

The mass-flow rate of air into LP cylinder

$\dot{m}_a = \rho_1 \dot{V}_1 = 1.189 \times 0.589 = 0.7$ kg/s

Temperature of air after first-stage compression;

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (293\text{K}) \times \left(\frac{3}{1}\right)^{\frac{1.4-1}{1.4}}$$

$= 401.04$ K

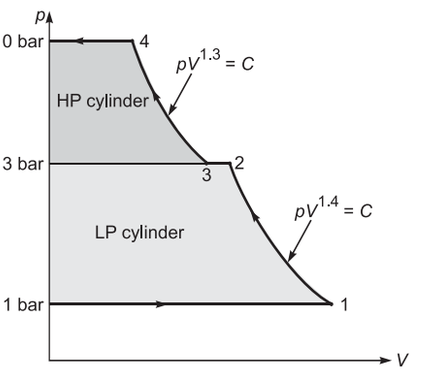

Fig. 25.22 *Two stage compression*

The *IP* input to LP cylinder,

$$IP_{LP} = \frac{\gamma}{\gamma-1} \dot{m}_a R (T_2 - T_1)$$

$$= \frac{1.4}{1.4-1} \times 0.7 \times 0.287 \times (401.04 - 293)$$

$= 75.97$ kW

Temperature of air after second-stage compression;

$$T_4 = T_3\left(\frac{p_4}{p_3}\right)^{\frac{n-1}{n}} = (308\text{K}) \times \left(\frac{10}{3}\right)^{\frac{1.3-1}{1.3}}$$

$= 406.64$ K

The work input in HP cylinder of compressor,

$$IP_{HP} = \frac{n}{n-1} \dot{m}_a R (T_4 - T_3)$$

$$= \frac{1.3}{1.3-1} \times 0.7 \times 0.287 \times (406.64 - 308)$$

$= 85.87$ kW

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Total power input to the compressor

$$IP = IP_{LP} + IP_{HP}$$
$$= 75.97 \text{ kW} + 85.87 \text{ kW} = 161.85 \text{ kW}$$

Motor power input

$$= \frac{IP}{\eta_{mech}\,\eta_{Motor}} = \frac{161.85}{0.9 \times 0.86}$$
$$= \mathbf{209.1 \text{ kW}}$$

Example 25.21 *Find the percentage saving in work input by compressing air in two stages from 1 bar to 7 bar instead of one stage. Assume a compression index of 1.35 in both the cases and optimum pressure and complete intercooling in a two-stage compressor.*

Solution

Given A two-stage air compressor with perfect intercooling

$p_1 =$ 1 r ba $\quad p_3 = 7$ bar $\quad n = 1.35$

To find Saving in work in comparison with single-stage compression.

Assumptions

(i) Single-acting reciprocating air compressor.
(ii) Compressions and expansions are reversible processes.
(iii) No effect of valve opening and closing on induction and delivery processes.

Analysis *Power required to drive compressor*

The minimum power input for two-stage compression with perfect intercooling;

$$IP_{multi} = 2 \times \left(\frac{n}{n-1}\right) \dot{m}_a\, RT_1 \left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}} - 1\right]$$

$$= 2 \times \left(\frac{1.35}{1.35-1}\right) \times \dot{m}_a RT \times \left[\left(\frac{7}{1}\right)^{\frac{1.35-1}{2\times 1.35}} - 1\right]$$

$$= 2.213\ \dot{m}\, RT$$

Power input with single-stage compression from 1 bar to 7 bar;

$$IP_{single} = \frac{n}{n-1} \dot{m}_a\, RT_1 \left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1.35}{1.35-1} \times \dot{m}_a RT \times \left[\left(\frac{7}{1}\right)^{\frac{1.35-1}{1.35}} - 1\right]$$

$$= 2.53\ \dot{m}\, RT$$

Percentage saving in power due to multistage compression

$$= \frac{IP_{single} - IP_{multi}}{IP_{single}} \times 100 = \frac{2.53 - 2.213}{2.53} \times 100$$
$$= \mathbf{12.56\%}$$

Example 25.22 *2 kg/s of air enters the LP cylinder of a two-stage, reciprocating air compressor. The overall pressure ratio is 9. The air at inlet to compressor is at 100 kPa and 35°C. The index of compression in each cylinder is 1.3. Find the intercooler pressure for perfect intercooling. Also, find the minimum power required for compression, and percentage saving over single-stage compression.*

Solution

Given A two-stage, single-acting, reciprocating air compressor with perfect intercooling

$\dot{m}_a = 2$ kg/s $\quad p_1 = 100$ kPa
$T_1 = 35°\text{C} = 308$ K $\quad p_3 = 9$ bar
$n = 1.3$

To find

(i) Intermediate pressure,
(ii) Power required to drive the compressor, and
(iii) Percentage saving in work in comparison with single-stage compression.

Assumptions

(i) For air; $R = 0.287$ kJ/kg·K and $C_p = 1$ kJ/kg·K.
(ii) No effect of valve opening and closing on induction and delivery processes.

Analysis

(i) *For perfect intercooling*

or $\quad p_2 = \sqrt{p_1 \times p_3} = \sqrt{1 \times 9} = 3$ bar

or $\quad \frac{p_2}{p_1} = 3$

(ii) *Power required to drive the compressor*

The minimum power input for two-stage compression with perfect intercooling;

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$$IP_{multi} = 2\times\left(\frac{n}{n-1}\right)\dot{m}_a\,RT_1\left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}}-1\right]$$

$$=2\times\left(\frac{1.3}{1.3-1}\right)\times2\times0.287\times308\times\left[\left(\frac{9}{1}\right)^{\frac{1.3-1}{2\times1.3}}-1\right]$$

$$= \mathbf{442.1\,kW}$$

(iii) *Percentage saving in work of comparison with single-stage compression*

Power input with single stage compression from 1 bar to 9 bar;

$$IP_{single}=\frac{n}{n-1}\dot{m}_a\,RT_1\left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{n}}-1\right]$$

$$=\frac{1.3}{1.3-1}\times2\times0.287\times308\times\left[\left(\frac{9}{1}\right)^{\frac{1.3-1}{1.3}}-1\right]$$

$$= \quad 505.92\text{ kW}$$

Saving in power due to multistage compression

$$= IP_{single} - IP_{multi} = 505.92 - 442.1$$

$$= 63.82\text{ kW}$$

Per cent saving

$$=\frac{IP_{multi}-IP_{single}}{IP_{single}}\times100=\frac{63.82}{505.92}\times100$$

$$= \quad \mathbf{12.61\%}$$

Example 25.23 *A two-stage, single-acting, reciprocating air compressor takes in air at 1 bar and 300 K. Air is discharged at 10 bar. The intermediate pressure is ideal for minimum work and perfect intercooling. The law of compression is $pV^{1.3}$ = constant. The rate of discharge is 0.1 kg/s. Calculate*

(a) power required to drive the compressor,
(b) saving in work in comparison with single stage compression,
(c) isothermal efficiency,
(d) heat transferred in intercooler.

Take $R = 0.287$ kJ/kg·K and $C_p = 1$ kJ/kg·K.

Solution

Given A two-stage, single-acting, reciprocating air compressor with perfect intercooling

$\dot{m}_a = 0.1$ kg/s $\qquad p_1 = 1$ bar $= 100$ kPa

$T_1 = 300$ K $\qquad p_3 = 10$ bar

$n = 1.3$ $\qquad R = 0.287$ kJ/kg·K

$C_p = 1$ kJ/kg·K $\qquad k = 1$

To find

(i) Power required to drive the compressor,
(ii) Saving in work in comparison with single-stage compression,
(iii) Isothermal efficiency, and
(iv) Heat transferred in intercooler.

Assumptions

(i) Given conditions leads to perfect intercooling.
(ii) Compressions and expansions are reversible processes.
(iii) No effect of valve opening and closing on induction and delivery processes.

Analysis For perfect intercooling

or $\qquad p_2 = \sqrt{p_1\times p_3} = \sqrt{1\times10} = 3.162$ bar

or $\qquad \dfrac{p_2}{p_1} = 3.162$

(i) *Power required to drive compressor*

The minimum power input for two-stage compression with perfect intercooling;

$$IP_{multi} = 2\times\left(\frac{n}{n-1}\right)\dot{m}_a\,RT_1\left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}}-1\right]$$

$$=2\times\left(\frac{1.3}{1.3-1}\right)\times0.1\times0.287\times300$$

$$\times\left[\left(\frac{10}{1}\right)^{\frac{1.3-1}{2\times1.3}}-1\right]$$

$$= \mathbf{22.7\,kW}$$

Power input with single-stage compression from 1 bar to 10 bar;

$$IP_{single} = \frac{n}{n-1}\dot{m}_a\,RT_1\left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{n}}-1\right]$$

$$=\frac{1.3}{1.3-1}\times0.1\times0.287\times300$$

$$\times\left[\left(\frac{10}{1}\right)^{\frac{1.3-1}{1.3}}-1\right]$$

$$= 26.16\text{ kW}$$

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(ii) Saving in power due to multistage compression

$= IP_{single} - IP_{multi} = 26.16 - 22.7$

$= \mathbf{3.46\,kW}$

Isothermal power input,

$$IP_{iso} = \dot{m}_a RT_1 \ln\left(\frac{p_3}{p_1}\right)$$

$$= 0.1 \times 0.287 \times 300 \times \ln\left(\frac{10}{1}\right)$$

$= 19.825\,\text{kW}$

(iii) Isothermal efficiency;

$$\eta_{iso} = \frac{IP_{iso}}{IP_{act}} = \frac{19.825}{22.7}$$

$= 0.8733$ or **87.33%**

Temperature after first-stage compression

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = (300\,\text{K}) \times \left(\frac{3.162}{1}\right)^{\frac{1.3-1}{1.3}}$$

$= 391.3$

(iv) Heat rejection rate in the intercooler

$\dot{Q} = \dot{m}_a C_p (T_2 - T_1)$

$= \times 1.1 \times (391.3 - 300)$

$= \mathbf{9.13\,kW}$

Example 25.24 *In a three-stage compressor, air is compressed from 98 kPa to 20 bar. Calculate for 1 m^3 of air per second,*

(a) Work under ideal condition for n = 1.3,

(b) Isothermal work,

(c) Saving in work due to multi staging,

(d) Isothermal efficiency.

Solution

Given An ideal three-stage reciprocating air compressor

$p_1 = 98$ kPa $\quad p_2 = 20$ bar $= 2000$ kPa

$n = 1.3 \quad \dot{V}_1 = 1\ \text{m}^3/\text{s}$

To find

(i) Power input to compressor,

(ii) Isothermal work,

(iii) Saving in work due to multi-staging, and

(iv) Isothermal efficiency.

Assumptions

(i) Neglecting clearance volume.

(ii) Perfect inetercooling.

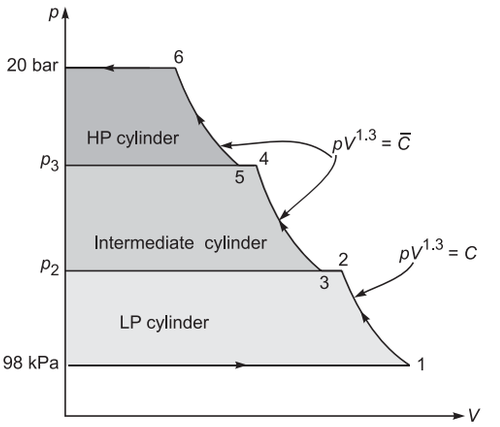

Fig. 25.23

Analysis For perfect intercooling

$$\frac{p_2}{p_1} = \frac{p_3}{p_2} = \frac{p_4}{p_3} = \left(\frac{p_4}{p_1}\right)^{\frac{1}{3}} = \left(\frac{2000}{98}\right)^{\frac{1}{3}} = 2.732$$

(i) *Power input to compressor*

$$IP_{3,stage} = 3 \times \left(\frac{n}{n-1}\right) p_1 \dot{V}_1 \left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{3n}} - 1\right]$$

$$= 3 \times \left(\frac{1.3}{1.3-1}\right) \times (98\,\text{kPa})$$

$$\times (1\ \text{m}^3/\text{s}) \times \left[\left(\frac{2000}{98}\right)^{\frac{1.3-1}{3\times 1.3}} - 1\right]$$

$= \mathbf{332.62\,kW}$

(ii) *Isothermal work*

$$IP_{iso} = p_1 \dot{V}_1 \ln\left(\frac{p_4}{p_1}\right)$$

$$= (98\,\text{kPa}) \times (1\,\text{m}^3/\text{s}) \times \ln\left(\frac{2000}{98}\right)$$

$= \mathbf{295.56\,kW}$

Power required in single-stage compression,

$$IP_{1,stage} = \left(\frac{n}{n-1}\right) p_1 \dot{V}_1 \left[\left(\frac{p_4}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

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$$= \left(\frac{1.3}{1.3-1}\right) \times 98 \times 1 \times \left[\left(\frac{2000}{98}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$= 427.08 \text{ kW}$$

(iii) *Saving in work due to multistaging*

$$= IP_{1,stage} - IP_{3,stage}$$

$$= 427.08 \text{ kW} - 332.62 \text{ kW} = \mathbf{94.42\ kW}$$

Percentage saving;

$$= \frac{\text{Saving}}{IP_{1,stage}} \times 100 = \frac{94.42}{427.08} \times 100$$

$$= \mathbf{22.1\%}$$

(iv) *Isothermal efficiency*

$$\eta_{iso} = \frac{IP_{iso}}{IP_{act}} \times 100 = \frac{295.56}{332.62} \times 100$$

$$= \mathbf{88.85\%}$$

Example 25.25 *A two-stage, single-acting reciprocating air compressor has an LP cylinder bore and stroke of 250 mm each. The clearance volume of a low-pressure cylinder is 5% of the stroke volume of the cylinder. The intake pressure and temperature are 1 bar and 17°C, respectively. Delivery pressure is 9 bar and the compressor runs at 300 rpm. The polytropic index is 1.3 throughout. The intercooling is complete and intermediate pressure is 3 bar. The overall efficiency of the plant including electric driving motor is 70%. Calculate*

(a) the mass-flow rate through the compressor, and

(b) energy input to electric motor.

Solution

Given Two-stage, single-acting reciprocating air compressor

LP cylinder: $d_1 = 250 \text{ mm} = 0.25 \text{ m}$

$L_1 = 250 \text{ mm} = 0.25 \text{ m}$

$c_1 = 0.05\ V_s$ or $c = 0.05$

$p_1 = 1 \text{ bar} = 100 \text{ kPa}$

$T_1 = 17°\text{C} = 290 \text{ K}$

$p_2 = 3 \text{ bar}$ $\quad p_3 = 9 \text{ bar}$

$n = 1.3$ $\quad k = 1$

$N = 300 \text{ rpm}$ $\quad \eta_{Overall} = 0.7$

To find

(i) The mass-flow rate of air through the compressor,

(ii) Power input to electric motor.

Assumptions

(i) Air as an ideal gas, with $R = 0.287 \text{ kJ/kg} \cdot \text{K}$.

(ii) Compression and expansion are reversible polytropic.

Analysis The stroke (swept) volume of LP cylinder

$$V_s = \left(\frac{\pi}{4}\right) d_1^2 L_1 = \left(\frac{\pi}{4}\right) \times (0.25 \text{ m})^2 \times (0.25 \text{ m})$$

$$= 0.01227 \text{ m}^3$$

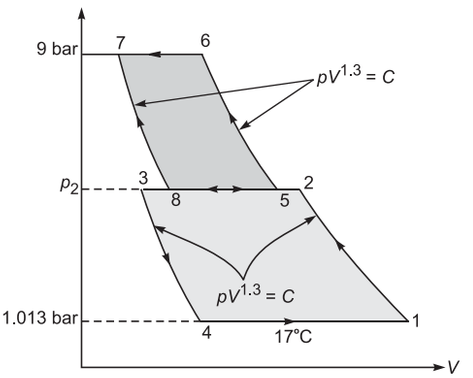

Fig. 25.24 *Two-stage compression with clearance*

The volumetric efficiency of LP cylinder is given by

$$\eta_{vol,\,LP} = 1 + c_1 - c_1 \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$= 1 + 0.05 - 0.05 \times \left(\frac{3}{1}\right)^{\frac{1}{1.3}} = 0.933$$

It is also expressed as

$$\eta_{vol,\,LP} = \frac{V_1 - V_4}{V_s}$$

or $$V_1 - V_4 = \eta_{vol,\,LP}\, V_s = 0.933 \times 0.01227$$

$$= 0.01145 \text{ m}^3$$

The mass of air inducted per cycle into LP cylinder

$$m_a = \frac{p_1 (V_1 - V_4)}{R T_1}$$

or $$m_a = \frac{(100 \text{ kPa}) \times (0.01145 \text{ m}^3)}{(0.287 \text{ kJ/kg} \cdot \text{K}) \times (290 \text{ K})}$$

$$= 0.0137 \text{ kg/cycle}$$

(i) *Mass-flow rate of air*

The mass flow rate of air per minute

$$\dot{m}_a = m_a N k$$

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$= 0.0137 \times 300 \times 1 = 4.13$ kg/min

$= \mathbf{0.069}$ **kg/s**

(ii) *Motor power input*

The indicated power input to the compressor with

$$\frac{p_2}{p_1} = \left(\frac{p_3}{p_1}\right)^{1/2}$$

$$IP = \frac{2n}{n-1}\ \dot{m}_a\ RT\left[\left(\frac{p_2}{p_2}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{2 \times 1.3}{1.3-1} \times 0.069 \times 0.287 \times 290 \times \left[\left(\frac{3}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$= 14.32$ kW

Motor power

$$= \frac{IP}{\eta_{Overall}} = \frac{14.32}{0.7} = \mathbf{20.46\ kW}$$

25.13 CYLINDER DIMENSIONS OF A MULTISTAGE COMPRESSOR

In an air compressor, the mass of air inducted through each cylinder is same. Therefore,

$$m_1 = m_2 = m_3 = \ldots$$

or $$\rho_1(V_1 - V_4) = \rho_2(V_5 - V_8) = \rho_3(V_9 - V_{12}) = \ldots \quad ...(25.45)$$

where $(V_1 - V_4)$, $(V_5 - V_8)$ and $(V_9 - V_{12})$ represent effective suction volume per cycle taken in LP, intermediate, and HP cylinders, respectively and ρ_1, ρ_2, ρ_3 are corresponding densities of air. The density of air can be expressed as

$$\rho = \frac{p}{RT}$$

$$\therefore \quad \frac{p_1}{RT_1}(V_1 - V_4) = \frac{p_2}{RT_3}(V_5 - V_8) = \frac{p_3}{RT_5}(V_9 - V_{12}) = \ldots$$

For perfect intercooling, isothermal conditions prevail, i.e.,

$$T_1 = T_3 = T_5 = \ldots$$

$$\therefore \quad p_1(V_1 - V_4) = p_2(V_5 - V_8) = p_3(V_9 - V_{12}) = \ldots \quad ...(25.46)$$

Introducing volumetric efficiency of respective cylinders;

$$p_1\eta_{vol_1}V_{s_1} = p_2\eta_{vol_2}V_{s_2} = p_3\eta_{vol_3}V_{s_3} = \ldots \quad ...(25.47)$$

Piston displacement volume as

$$V_s = \left(\frac{\pi}{4}\right)d^2 L\ , \text{ then}$$

$$p_1\eta_{vol_1}\left(\frac{\pi}{4}\right)d_1^2 L_1 = p_2\eta_{vol_2}\left(\frac{\pi}{4}\right)d_2^2 L_2 = p_3\eta_{vol_3}\left(\frac{\pi}{4}\right)d_3^2 L_3 = \ldots \quad ...(25.48)$$

Usually, the stroke length for all cylinders is same, i.e.,

$$L_1 = L_2 = L_3 = \ldots$$

$$\therefore \quad p_1\eta_{vol_1}d_1^2 = p_2\eta_{vol_2}d_2^2 = p_3\eta_{vol_3}d_3^2 = \ldots \quad ...(25.49)$$

If all cylinders have same clearance ratio, then

$$\eta_{vol_1} = \eta_{vol_2} = \eta_{vol_3} = \ldots$$

and $$p_1d_1^2 = p_2d_2^2 = p_3d_3^2 = \ldots \quad ...(25.50)$$

Equations (25.49) and (25.50) are used to calculate the cylinder dimension for multi-stage compressors.

Example 25.26 *A two-stage, single-acting, reciprocating air compressor with complete intercooling receives atmospheric air at 1 bar and 15°C, compresses it polytropically (n = 1.3) to 30 bar. If both cylinders have the same stroke, calculate the diameter of the HP cylinder, if the diameter of the LP cylinder is 300 mm.*

Solution

Given Two-stage, single-acting reciprocating air compressor

LP cylinder: $d_1 = 300$ mm, $p_1 = 1$ bar, $T_1 = 15°C = 288$ K, $T_3 = 15°C = 288$ K, $p_3 = 30$ bar, $n = 1.3$, $k = 1$

To find Diameter of HP cylinder.

Assumptions

(i) Compression in both cylinders is reversible.

(ii) Negligible clearance in both cylinders.

(iii) No effect of valve opening and closing on induction and delivery processes.

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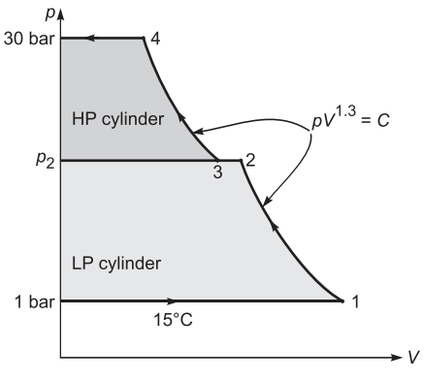

Fig. 25.25

Analysis For perfect intercooling, the pressure ratio per stage

$$\frac{p_2}{p_1} = \frac{p_3}{p_2} = \sqrt{p_1 \times p_3} = \sqrt{1 \times 30} = 5.477$$

$$p_2 = 5.477\, p_1 = 5.477 \times 1 = 5.477 \text{ bar}$$

The total volume of LP cylinder

$$V_1 = V_{s,LP} = \left(\frac{\pi}{4}\right) d_1^2 L$$

The total volume of HP cylinder

$$V_3 = V_{s,HP} = \left(\frac{\pi}{4}\right) d_2^2 L$$

Equation (25.46), for perfect intercooling, without clearance leads to

$$p_1 V_1 = p_2 V_3$$

or
$$V_{s,HP} = V_{s,LP}\left(\frac{p_1}{p_2}\right) = V_{s,LP} \times \frac{1}{5.477}$$

$$= 0.1825\, V_{s,LP}$$

Without clearance, with same pressure ratio and with perfect intercooling, the volumetric efficiency will be same. Expressing the above equation in terms of cylinder diameters

$$\left(\frac{\pi}{4}\right) d_{HP}^2 L = 0.1825 \times \left(\frac{\pi}{4}\right) \times (300 \text{ mm})^2 \times L$$

or
$$d_{HP} = \sqrt{0.1825 \times (300 \text{ mm}^2)}$$

$$= \mathbf{128.18\ mm}$$

Example 25.27 *A two-stage, single-acting, reciprocating air compressor receives atmospheric air at 15°C, compresses it isentropically in the LP cylinder to intermediate pressure, where air cools to its initial temperature and then again compresses polytropically (n = 1.3) in the HP cylinder. The clearance volume and pressure ratio in both cylinders are 5% of the swept volume and 2, respectively. Determine the stroke volume of HP cylinder for 60 litre swept volume of LP cylinder.*

Solution

Given Two-stage, single-acting reciprocating air compressor

$V_{s,LP} = 60$ litres $\qquad V_3 = 0.05\, V_{s,LP} = 3$ litres

$T_1 = 15°C = 288\,K$

$T_3 = 15°C = 288\,K$

$\frac{p_2}{p_1} = 2.0 \qquad \frac{p_3}{p_2} = 2.0$

$V_7 = 0.05\, V_{s,HP} \qquad n = 1.3 \qquad k = 1$

To find Stroke volume $(V_5 - V_7)$ of HP cylinder.

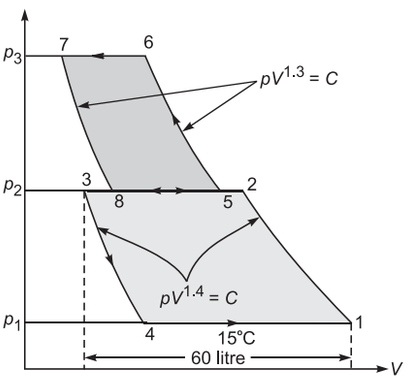

Fig. 25.26

Assumptions

(i) For air $\gamma = 1.4$.

(ii) For LP cylinder, the re-expansion of air is isentropic.

(iii) For HP cylinder, the re-expansion of air is polytropic.

Analysis The total volume of LP cylinder

$$V_1 = V_{s,LP} + V_3 = 60 \text{ lit} + 3 \text{ lit} = 63 \text{ litres}$$

The volume of air after first-stage compression

$$V_2 = V_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}}$$

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$$= (63\text{ lit}) \times \left(\frac{1}{2}\right)^{\frac{1}{1.4}} = 38.4\text{ litres}$$

The temperature of air after first-stage compression

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (288\text{ K}) \times \left(\frac{2}{1}\right)^{\frac{1.4-1}{1.4}}$$
$$= 351\text{ K}$$

The total volume of HP cylinder

$$V_5 = V_{c,HP} + V_{s,HP} = V_{c,HP} + 20\,V_{c,HP}$$
$$= V_7 + 20\,V_7 = 21\,V_7$$

The volume of air after re-expansion in HP cylinder

$$V_8 = V_7\left(\frac{p_7}{p_8}\right)^{\frac{1}{n}} = V_7 \times \left(\frac{2}{1}\right)^{\frac{1}{1.3}} = 1.704\,V_7$$

In intercooling, air is cooled to initial temperature, and volume of air reduces from $(V_2 - V_3)$ to $(V_5 - V_8)$ at constant pressure. Therefore,

$$\frac{V_2 - V_3}{T_2} = \frac{V_5 - V_8}{T_1}$$

or
$$V_5 - V_8 = (V_2 - V_3)\frac{T_1}{T_2} = (38.4 - 3) \times \frac{288}{351}$$
$$= 29.046\text{ litres}$$

or $21\,V_7 - 1.704\,V_7 = 29.046$ litres

or
$$V_7 = \frac{29.046}{21 - 1.704} = 1.505$$

or
$$V_5 = 21 \times 1.505 = 31.61\text{ litres}$$

Stroke volume of HP cylinder

$$V_5 - V_7 = 31.61 - 1.505 = \mathbf{30.10\text{ litres}}$$

Example 25.28 *In a trial on a two-stage, single-acting, reciprocating air compressor, following data were recorded:*

Free air delivery per min = *6 m³*
Free air conditions = *1 bar, 27°C*
Delivery pressure = *30 bar*
Compressor speed = *300 rpm*
Intermediate pressure = *6 bar*
Temperature at the inlet of HP cylinder = *27°C*
Law of compression = $pV^{1.3}$
Mechanical efficiency = *85%*
Stroke to bore ratio for LP cylinder = *1:2*
Stroke of HP cylinder = *Stroke of LP cylinder*

Calculate

(a) Cylinder diameters,
(b) Power input, neglecting clearance volume.

Solution

Given Two-stage, single-acting reciprocating air compressor $k = 1$

$FAD = \dot{V}_1 = 6\text{ m}^3/\text{min}$ $\quad p_1 = 1\text{ bar} = 100\text{ kPa}$
$T_1 = 27°\text{C} = 300\text{ K}$ $\quad p_2 = 6\text{ bar}$
$p_3 = 30\text{ bar}$ $\quad T_3 = 27°\text{C} = 300\text{ K}$
$n = 1.3$ $\quad N = 300\text{ rpm}$
$\eta_{mech} = 0.85$

LP cylinder: $L_1/d_1 = 1.2$
and $L_2 = L_1$
and Negligible clearance

To find

(i) Cylinder dimensions,
(ii) Power input to compressor.

Assumptions

(i) Air as an ideal gas, with $R = 0.287$ kJ/kg·K.
(ii) Compression and expansion are reversible polytropic.
(iii) Volumetric efficiency of both cylinders as 100%.

Analysis

(i) *Cylinder diameters*

The stroke (swept) volume rate of LP cylinder with $\eta_{vol} = 1.0$

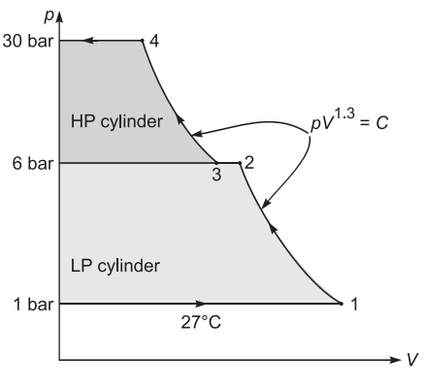

Fig. 25.27

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$$\dot{V}_1 = FAD = \left(\frac{\pi}{4}\right) d_1^2 L_1 N k$$

or $6\ \text{m}^3/\text{min} = \left(\frac{\pi}{4}\right) \times d_1^2 \times (1.2 d_1) \times 300 \times 1$

or $d_1 = 0.276\ \text{m}$ or **276 mm**

For same stroke length and with $\eta_{vol} = 1.0$, the diameters of *LP* and *HP* cylinders are related as

$$p_1 d_1^2 = p_2 d_2^2$$

$$1 \times (276)^2 = 6 \times d_2^2$$

or $d_2 = \mathbf{112.67\ mm}$

(ii) *Power input to compressor*

The mass-flow rate of air into compressor

$$\dot{m}_a = \frac{p_1 \dot{V}_1}{R T_1} = \frac{(100\ \text{kPa}) \times (6\ \text{m}^3/\text{min})}{(0.287\ \text{kJ/kg}\cdot\text{K}) \times (300\ \text{K})}$$

$= 6.9686\ \text{kg/min}$ or $0.116\ \text{kg/s}$

The indicated power input to two-stage, single-acting air compressor

$$IP = \frac{n}{n-1} \dot{m}_a R T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 2\right]$$

$$= \frac{1.3}{1.3-1} \times 0.116 \times 0.287 \times 300 \times \left[\left(\frac{6}{1}\right)^{\frac{1.3-1}{1.3}} + \left(\frac{30}{6}\right)^{\frac{1.3-1}{1.3}} - 2\right]$$

$= 84.9\ \text{kW}$

Brake power input

$$= \frac{IP}{\eta_{mech}} = \frac{84.9\ \text{kW}}{0.85} = \mathbf{99.88\ kW}$$

Example 25.29 *In a single-acting, two-stage reciprocating air compressor handles 4.5 kg of air per minute, and compresses it from 1.013 bar 17°C through a pressure ratio of 9. The index of compression and expansion in both stages is 1.3. If the intercooling is complete,find the minimum indicated power and cylinder swept volumes required. Assume the clearance volume of both the stages are 5% of their respective stroke volumes and the compressor runs at 300 rpm. Take R = 0.287 kJ/kg·K.*

Solution

Given A single-acting, two-stage reciprocating air compressor with perfect intercooling

$\dot{m}_a = 4.5\ \text{kg/min} = 0.075\ \text{kg/s}$

$k = 1$

$p_1 = 1.013\ \text{bar} = 101.3\ \text{kPa}$

$T_1 = 17°\text{C} = 290\ \text{K}$

$\frac{p_3}{p_1} = 9$ $\qquad n = 1.3$

$c_1 = c_2 = 0.05\, V_s$

$N = 300\ \text{rpm}$ $\qquad R = 0.287\ \text{kJ/kg}\cdot\text{K}$

To find

(i) Minimum indicated power, and

(ii) Swept volumes of LP and HP cylinders.

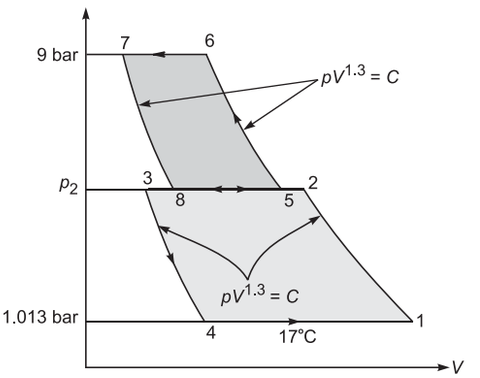

Fig. 25.28

Analysis

(i) *Minimum indicated power*

For perfect intercooling

$$p_2 = \sqrt{p_1 \times p_3} = \sqrt{p_1 \times 9 p_1} = 3 p_1$$

or $\frac{p_2}{p_1} = 3$

The minimum power input, Eq. (25.42)

$$IP_{min} = 2 \times \left(\frac{n}{n-1}\right) \dot{m}_a R T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= 2 \times \left(\frac{1.3}{1.3-1}\right) \times 0.075 \times 0.287 \times 290 \times \left[\left(\frac{3}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right] = \mathbf{15.61\ kW}$$

(ii) *The stroke (swept) volume of LP cylinder*

The mass of air inducted per cycle into LP

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cylinder

$$m_a = \frac{\dot{m}_a}{N\,k} = \frac{4.5 \text{ kg/min}}{(300 \text{ rotation/min}) \times 1}$$

$$= 0.015 \text{ kg/cycle}$$

Efffective swept volume of LP cylinder;

$$(V_1 - V_4) = \frac{m_a R T_1}{p_1} = \frac{0.015 \times 0.287 \times 290}{101.3}$$

$$= 0.0123 \text{ m}^3\text{/cycle}$$

The volumetric efficiency of LP and HP cylinders

$$\eta_{vol} = 1 + c_1 - c_1 \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$= 1 + 0.05 - 0.05 \times \left(\frac{3}{1}\right)^{\frac{1}{1.3}} = 0.933$$

It is also expressed as

$$\eta_{vol} = \frac{V_1 - V_4}{V_{s,\,LP}}$$

or $$V_{s,LP} = \frac{V_1 - V_4}{\eta_{vol}} = \frac{0.0123}{0.933}$$

$$= \mathbf{0.0131\ m^3/cycle}$$

The swept volume of LP cylinder is 0.0131 m^3/cycle.

Using Eq. (25.46) for ratio of LP and HP cylinders,

$$p_1(V_1 - V_4) = p_2(V_5 - V_8)$$

$$V_5 - V_8 = \frac{p_1}{p_2}(V_1 - V_4) = \frac{1}{3} \times (0.0123)$$

$$= 0.0041 \text{ m}^3\text{/cycle}$$

Then swept volume of HP cylinder

$$V_{s,HP} = \frac{V_5 - V_8}{\eta_{vol}} = \frac{0.0041}{0.933} = \mathbf{0.00440\ m^3}$$

Example 25.30 *A single-acting, two-stage reciprocating air compressor with complete intercooling delivers 10.5 kg/min of air at 16 bar. The compressor takes in air at 1 bar and 27°C. The compression and expansion follow the law $pV^{1.3}$ = Const. Calculate*

(a) Power required to drive the compressor
(b) Isothermal efficiency
(c) Free air delivery
(d) Heat transferred in intercooler

If the compressor runs at 440 rpm, the clearance ratios for LP and HP cylinders are 0.04 and 0.06, respectively, calculate the swept and clearance volumes for each cylinder.

Solution

Given A single-acting, two-stage reciprocating air compressor with perfect intercooling

$\dot{m}_a$ = 10.5 kg/min = 0.175 kg/s
p_1 = 1 bar = 100 kPa T_1 = 27°C = 300 K
p_3 = 16 bar n = 1.3
c_1 = 0.04 c_2 = 0.06
N = 440 rpm k = 1

To find

(i) Indicated power,
(ii) Isothermal efficiency,
(iii) FAD,
(iv) Heat transfer in intercooler, and
(v) Swept volumes of LP and HP cylinders.

Assumptions

(i) Air an ideal gas, R = 0.287 kJ/kg·K,
(ii) Compressions and expansions are reversible processes, and
(iii) Suction takes place at free air conditions.

Analysis For perfect intercooling, the pressure ratio per stage

$$\frac{p_2}{p_1} = \frac{p_3}{p_2} = \sqrt{p_1 \times p_3} = \sqrt{1 \times 16} = 4$$

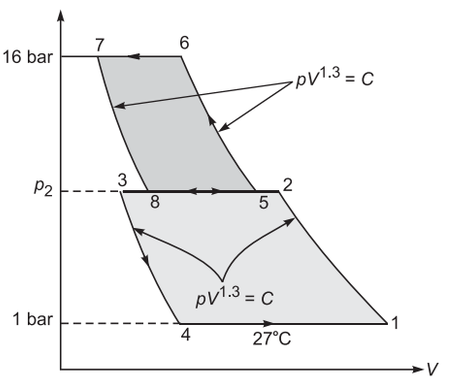

Fig. 25.29

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(i) *Indicated power input for two-stage compressor*

$$IP_{min} = 2\times\left(\frac{n}{n-1}\right)\dot{m}_a R T_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= 2\times\left(\frac{1.3}{1.3-1}\right)\times 0.175\times 0.287 \times 300\times\left[\left(\frac{4}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$= \mathbf{49.23\ kW}$$

(ii) *Isothermal efficiency*

Power input for isothermal compression from 1 bar to 16 bar pressure

$$IP_{iso} = \dot{m}_a R T_1 \ln\left(\frac{p_3}{p_1}\right)$$

$$= 0.175\times 0.287\times 300\times\ln\left(\frac{16}{1}\right)$$

$$= 41.77\ \text{kW}$$

$$\eta_{iso} = \frac{\text{Isothermal power}}{\text{Actual power}} = \frac{41.77\ \text{kW}}{49.23\ \text{kW}}$$

$$= 0.848 \text{ or } \mathbf{84.8\%}$$

(iii) *Free air delivery* (*FAD*)

$$\dot{V}_f = \dot{V}_1 - \dot{V}_4 = \frac{\dot{m}_a R T_1}{p_1}$$

$$= \frac{(10.5\ \text{kg/min})\times(0.287\ \text{kJ/kg}\cdot\text{K})\times(300\ \text{K})}{(100\ \text{kPa})}$$

$$= \mathbf{9.04\ m^3/min}$$

(iv) *Heat transferred in intercooler*

Temperature after compression in each stage

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = 300\times\left(\frac{4}{1}\right)^{\frac{1.3-1}{1.3}}$$

$$= 413.1\ \text{K}$$

Heat transfer rate in intercooler

$$\dot{Q} = \dot{m}_a C_p (T_2 - T_1)$$

$$= 10.5\times 1.005\times(413.1 - 300)$$

$$= 1193.5\ \text{kJ/min or} = \mathbf{19.89\ kW}$$

(v) *Swept volumes*

The volumetric efficiency of *LP* cylinder

$$\eta_{vol,LP} = 1 + c_1 - c_1\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$= 1 + 0.04 - 0.04\times\left(\frac{4}{1}\right)^{\frac{1}{1.3}}$$

$$= 0.923 \text{ or } 92.3\%$$

The volume handled per cycle by *LP* cylinder $(V_1 - V_4)$

$$= \frac{FAD}{\text{No. of cycle per minute}} = \frac{\dot{V}_f}{N\,k} = \frac{9.04}{440\times 1}$$

$$= 0.0205\ \text{m}^3\text{/cycle}$$

Further, it can also be expressed as

$$\eta_{vol,LP} = \frac{V_1 - V_4}{V_{s,LP}}$$

or $$V_{s,LP} = \frac{V_1 - V_4}{\eta_{vol,LP}} = \frac{0.0205}{0.923}$$

$$= \mathbf{0.0222\ m^3/cycle}$$

For *HP* cylinder

$$\eta_{vol,HP} = 1 + c_2 - c_2\left(\frac{p_3}{p_2}\right)^{\frac{1}{n}}$$

$$= 1 + 0.06 - 0.06\times\left(\frac{4}{1}\right)^{\frac{1}{1.3}}$$

$$= 0.885 \text{ or } 85.5\%$$

The effective swept volume handled by *HP* cylinder with perfect intercooling

$$p_2(V_5 - V_8) = p_1(V_1 - V_4)$$

or $$(V_5 - V_8) = \frac{p_1}{p_2}\times(V_1 - V_4)$$

$$= \frac{1}{4}\times 0.0205 = 0.005125\ \text{m}^3\text{/cycle}$$

The piston displacement volume of *HP* cylinder;

$$V_{s,HP} = \frac{V_5 - V_8}{\eta_{vol,HP}} = \frac{0.005125}{0.885}$$

$$= \mathbf{0.0058\ m^3/cycle}$$

Example 25.31 *A two-stage, double-acting, reciprocating air compressor operating at 300 rpm, receives air at 1 bar and 27°C. The bore of LP cylinder is 360 mm and its stroke is 400 mm. Both cylinders have equal stroke and equal clearance of 4%. The LP cylinder discharges air at a pressure of 5 bar. The air then passes through an intercooler to cool air to its initial temperature. Pressure drops in intercooler to 4.75 kPa. Finally air is discharged from HP cylinder at 15 bar. The index*

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of compression and expansion in both cylinders is 1.3. C_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K.

Calculate

(a) Heat rejected in the intercooler,

(b) Diameter of HP cylinder,

(c) Power required to drive HP cylinder.

Solution

Given Two-stage, double-acting reciprocating air compressor

k	= 2	N	= 300 rpm
p_1	= 1 bar = 100 kPa	T_1	= 27°C = 300 K
p_2	= 5 bar	p_5	= 4.75 bar
T_3	= 27°C = 300 K	p_6	= 15 bar
n	= 1.3	d_1	= 360 mm
L_1	= 400 mm	c_1	= c_2 = 0.04
C_p	= 1.005 kJ/kg·K	R	= 0.287 kJ/kg·K
L_2	= L_1		

To find

(i) Heat rejected in the intercooler,

(ii) Diameter of HP cylinder, and

(iii) Power required to drive HP cylinder.

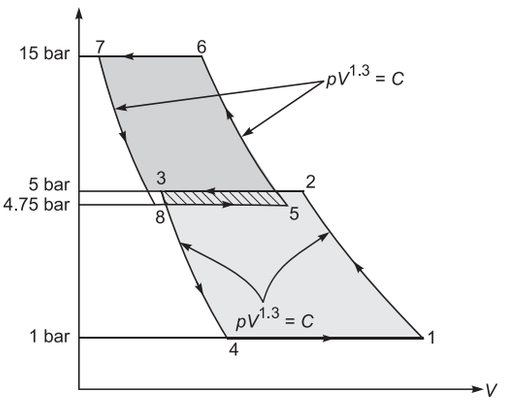

Fig. 25.30

Analysis The stroke (swept) volume per minute of *LP* cylinder

$$\dot{V}_{s,LP} = \dot{V}_1 - \dot{V}_3 = \left(\frac{\pi}{4}\right) d_1^2 L_1 N k$$

$$= \left(\frac{\pi}{4}\right) \times (0.3\ 6\text{m})^2 \times (0.4\ \text{m}) \times 300 \times 2$$

$$= 24.42\ \text{m}^3/\text{min}$$

The pressure ratio for *LP* cylinder;

$$\frac{p_2}{p_1} = 5$$

The volumetric efficiency of *LP* cylinder;

$$\eta_{vol,LP} = 1 + c_1 - c_1 \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$= 1 + 0.04 - 0.04 \times \left(\frac{5}{1}\right)^{\frac{1}{1.3}} = 0.902$$

It is also expressed using effective stroke volume rate $(\dot{V}_1 - \dot{V}_4)$ of *LP* cylinder as

$$\eta_{vol,\,LP} = \frac{\dot{V}_1 - \dot{V}_4}{\dot{V}_{s,\,LP}}$$

or $\dot{V}_1 - \dot{V}_4 = \dot{V}_{s,LP} \times \eta_{vol,LP}$

$$= (24.42\ \text{kg/min}) \times 0.902 = 22.02\ \text{m}^3/\text{min}$$

The mass flow rate of air into the *LP* cylinder;

$$\dot{m}_a = \frac{p_1(\dot{V}_1 - \dot{V}_4)}{RT_1} = \frac{(100\ \text{kPa}) \times (22.02\ \text{m}^3/\text{min})}{(0.287\ \text{kJ/kg·K}) \times (300\ \text{K})}$$

$$= 25.584\ \text{kg/min} \quad \text{or} \quad 0.426\ \text{kg/s}$$

Temperature after compression in each stage

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = 300 \times \left(\frac{5}{1}\right)^{\frac{1.3-1}{1.3}}$$

$$= 434.93\ \text{K}$$

(i) *Heat transfer rate in the intercooler*

$$\dot{Q} = \dot{m}_a C_p (T_2 - T_1)$$

$$= 25.584 \times 1.005 \times (434.93 - 300)$$

$$= 3469.33\ \text{kJ/min} \quad \text{or} \quad \mathbf{57.82\ kW}$$

(ii) *Diameter of HP cylinder*

The effective swept volume rate of *HP* cylinder

$$\dot{V}_5 - \dot{V}_8 = \frac{\dot{m}_a R T_1}{p_5}$$

$$= \frac{25.584 \times 0.287 \times 300}{475}$$

$$= 4.637\ \text{m}^3/\text{min}$$

The volumetric efficiency of *HP* cylinder;

$$\eta_{vol,HP} = 1 + c_2 - c_2 \left(\frac{p_3}{p_5}\right)^{\frac{1}{n}}$$

$$= 1 + 0.04 - 0.04 \times \left(\frac{15}{4.75}\right)^{\frac{1}{1.3}} = 0.943$$

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Further, the volumetric efficiency can be expressed in terms of piston displacement volume rate of *HP* cylinder as

$$\eta_{vol,\,HP} = \frac{\dot{V}_5 - \dot{V}_8}{\dot{V}_{s,\,HP}}$$

$$\text{or } \dot{V}_{s,HP} = \frac{\dot{V}_5 - \dot{V}_8}{\eta_{vol,HP}} = \frac{4.637}{0.943} = 4.917 \text{ m}^3/\text{min}$$

which can be further, expressed as

$$\dot{V}_{s,HP} = \left(\frac{\pi}{4}\right) d_2^2 L_1 N k$$

$$\text{or } 4.917 = \left(\frac{\pi}{4}\right) \times d_2^2 \times 0.4 \times 300 \times 2$$

or $d_2 = 0.1615$ m or **161.5 mm**

(iii) *Power required to drive HP cylinder*

$$IP = \frac{n}{n-1} p_5 (\dot{V}_5 - \dot{V}_8) \left[\left(\frac{p_3}{p_5}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1.3}{1.3-1} \times 4.75 \times 10^2 \times \left(\frac{4.637}{60}\right) \times \left[\left(\frac{15}{4.75}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$= \mathbf{48.34\ kW}$$

Example 25.32 *A three-stage, double-acting, reciprocating air compressor operating at 300 rpm, receives air at 1 bar and 27°C. The bore of LP cylinder is 360 mm and its stroke is 400 mm. Intermediate cylinder and HP cylinder have same stroke as LP cylinder. The clearance volume in each cylinder is 4 % of the stroke volume . The LP cylinder dischrges air at a pressure of 5 bar, the intermediate cylinder discharges at 20 bar and air is finally discharged by the HP cylinder at 75 bar. The air is cooled in intercoolers to initial temperature after each stage of compression. A Pressure drop of 0.2 bar takes place in intercooler after each stage. The index of compression and expansion for an LP cylinder is 1.3, for intermediate cylinder is 1.32 and for HP cylinder is 1.35. Neglect the effect of piston rod and assume C_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K.*

Calculate

(a) Heat rejected in each stages in intercooler and during compression,

(b) Heat rejected in after-cooler, if delivered air is cooled to initial temperature,

(c) Diameter of intermediate and HP cylinders,

(d) Power required to drive compressor, if its mechanical efficiency is 85%.

Solution

Given Three-stage, double-acting reciprocating air compressor

k	= 2	N	= 300 rpm
p_1	= 1 bar = 100 kPa	T_1	= 27°C = 300 K
p_2	= 5 bar	p_5	= 4.8 bar
T_3	= 27°C = 300 K	p_6	= 20 bar
p_9	= 19.8 bar	p_{10}	= 75 bar
n_1	= 1.3	n_2	= 1.32
n_3	= 1.35	d_1	= 360 mm
L_1	= 400 mm	$c_1 = c_2 = c_3$	= 0.04
C_p	= 1.005 kJ/kg·K	R	= 0.287 kJ/kg·K
$L_3 = L_2 = L_1$		η_{mech}	= 0.85

To find

(i) Heat rejected in each stage in intercooler and during compression,

(ii) Heat rejected in after-cooler,

(iii) Diameter of intermediate and HP cylinders,

(iv) Power required to drive compressor.

Analysis The stroke (swept) volume per minute of *LP* cylinder

$$\dot{V}_{s,LP} = \dot{V}_1 - \dot{V}_3 = \left(\frac{\pi}{4}\right) d_1^2 L_1 N k$$

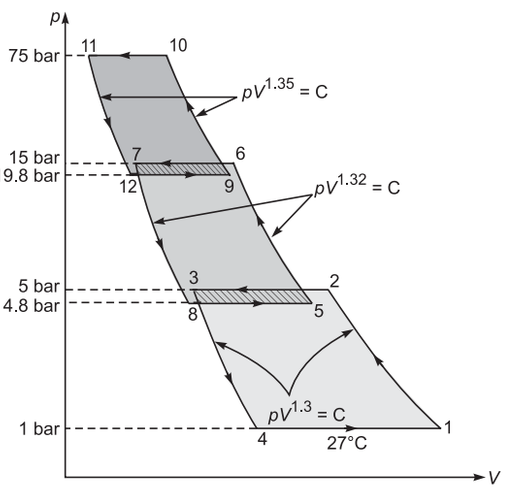

Fig. 25.31

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$$= \left(\frac{\pi}{4}\right) \times (0.3\ 6\text{m})^2 \times (0.4\ \text{m}) \times 300 \times 2$$

$$= 24.42\ \text{m}^3/\text{min}$$

The volumetric efficiencies of *LP*, intermediate and *HP* cylinders;

$$\eta_{vol,LP} = 1 + c_1 - c_1 \left(\frac{p_2}{p_1}\right)^{\frac{1}{n_1}}$$

$$= 1 + 0.04 - 0.04 \times \left(\frac{5}{1}\right)^{\frac{1}{1.3}} = 0.902$$

For *IP* cylinder

$$\eta_{vol,IP} = 1 + c_2 - c_2 \left(\frac{p_6}{p_5}\right)^{\frac{1}{n_2}}$$

$$= 1 + 0.04 - 0.04 \times \left(\frac{20}{4.8}\right)^{\frac{1}{1.32}} = 0.922$$

For *HP* cylinder

$$\eta_{vol,IP} = 1 + c_3 - c_3 \left(\frac{p_{10}}{p_9}\right)^{\frac{1}{n_3}}$$

$$= 1 + 0.04 - 0.04 \times \left(\frac{75}{19.8}\right)^{\frac{1}{1.35}} = 0.9327$$

The effective stroke volume $(\dot{V}_1 - \dot{V}_4)$ of air per minute in *LP* cylinder;

$$\dot{V}_1 - \dot{V}_4 = \dot{V}_{s,LP} \times \eta_{vol,LP}$$

$$= (24.42\ \text{m}^3/\text{min}) \times 0.902 = 22.02\ \text{m}^3/\text{min}$$

The mass flow rate of air in the compressor;

$$\dot{m}_a = \frac{p_1(\dot{V}_1 - \dot{V}_4)}{RT_1} = \frac{(100\ \text{kPa}) \times (22.02\ \text{m}^3/\text{min})}{(0.287\ \text{kJ/kg}\cdot\text{K}) \times (300\ \text{K})}$$

$$= 25.584\ \text{kg/min} \quad \text{or} \quad 0.426\ \text{kg/s}$$

Temperature after compression in each stage

After first stage,

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{n_1 - 1}{n_1}}$$

$$= 300 \times \left(\frac{5}{1}\right)^{\frac{1.3-1}{1.3}} = 434.93\ \text{K}$$

After second stage,

$$T_6 = T_5 \left(\frac{p_6}{p_5}\right)^{\frac{n_2 - 1}{n_2}}$$

$$= 300 \times \left(\frac{20}{4.8}\right)^{\frac{1.32-1}{1.32}} = 424.00\ \text{K}$$

After third stage

$$T_{10} = T_9 \left(\frac{p_{10}}{p_9}\right)^{\frac{n_3 - 1}{n_3}}$$

$$= 300 \times \left(\frac{75}{19.8}\right)^{\frac{1.35-1}{1.35}} = 423.71\ \text{K}$$

Specific heat at constant volume;

$$C_v = \frac{R}{\gamma - 1} = \frac{0.287}{1.4 - 1} = 0.717\ \text{kJ/kg}\cdot\text{K}$$

(i) *Heat rejection rate in each stage*

$$\dot{Q}_{comp} = \dot{m}_a \frac{(\gamma - n)}{n - 1} C_v (T_2 - T_1)$$

$$\dot{Q}_{intercooler} = \dot{m}_a C_p (T_2 - T_1)$$

Heat rejcetion rate in **first** stage (during compression and in intercooler);

$$\dot{Q}_1 = \dot{m}_a \left[\frac{(\gamma - n_1)}{n_1 - 1} C_v + C_p\right] (T_2 - T_1)$$

$$= 25.584 \times \left[\frac{(1.4 - 1.3)}{1.3 - 1} \times 0.717 + 1.005\right] \times (434.93 - 300)$$

= 4295 kJ/min

Heat rejcetion rate in **second** stage (during compression and in intercooler);

$$\dot{Q}_2 = \dot{m}_a \left[\frac{(\gamma - n_2)}{n_2 - 1} C_v + C_p\right] (T_6 - T_5)$$

$$= 25.584 \times \left[\frac{(1.4 - 1.32)}{1.32 - 1} \times 0.717 + 1.005\right] \times (424 - 300)$$

= 3757 kJ/min

Heat rejction rate in **third** stage compression;

$$\dot{Q}_3 = \dot{m}_a \left(\frac{\gamma - n_3}{n_3 - 1}\right) C_v (T_{10} - T_9)$$

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$$= 25.584 \times \left(\frac{1.4 - 1.3\,5}{1.35 - 1}\right) \times 0.717 \times (423.71 - 300)$$

$$= \mathbf{324.18\ kJ/min}$$

Heat rejection rate in after-cooler

$$\dot{Q}_3 = \dot{m}_a C_p (T_{10} - T_1)$$
$$= 25.584 \times 1.005 \times (423.71 - 300)$$
$$= \mathbf{3180.82\ kJ/min}$$

(ii) *Diameter of IP and HP cylinders*

The effective swept volume rate of *IP* cylinder

$$\dot{V}_5 - \dot{V}_8 = \frac{\dot{m}_a R T_1}{p_5} = \frac{25.584 \times 0.287 \times 300\ \text{K}}{480}$$
$$= 4.588\ \text{m}^3/\text{min}$$

Further, the volumetric efficiency can be expressed using piston displacement volume rate of *IP* cylinder as

$$\eta_{vol,IP} = \frac{\dot{V}_5 - \dot{V}_8}{\dot{V}_{s,\ IP}}$$

$$\therefore \dot{V}_{s,IP} = \frac{\dot{V}_5 - \dot{V}_8}{\eta_{vol,IP}} = \frac{4.588}{0.922} = 4.976\ \text{m}^3/\text{min}$$

which can be further, expressed as

$$\dot{V}_{s,IP} = \left(\frac{\pi}{4}\right) {d_2}^2 L_1 N k$$

$$\text{or } 4.976 = \left(\frac{\pi}{4}\right) \times d_2^2 \times 0.4 \times 300 \times 2$$

or $d_2 = 0.1624$ m or **162.4 mm**

Similarly, for *HP* cylinder;

The effective swept volume rate of *HP* cylinder

$$\dot{V}_9 - \dot{V}_{12} = \frac{\dot{m}_a R T_9}{p_9} = \frac{25.584 \times 0.287 \times 300\ \text{K}}{1980}$$
$$= 1.1125\ \text{m}^3/\text{min}$$

$$\dot{V}_{s,HP} = \frac{\dot{V}_9 - \dot{V}_{12}}{\eta_{vol,HP}} = \frac{1.1125}{0.9327} = 1.192\ \text{m}^3/\text{min}$$

which can be further, expressed as

$$1.192 = \left(\frac{\pi}{4}\right) \times d_3^2 \times 0.4 \times 300 \times 2$$

or $d_3 = 0.07954$ m or **79.54 mm**

The power input to compressor

Since $T_1 = T_5 = T_9$,

$$IP = \dot{m}_a R T_1 \left[\frac{n_1}{n_1 - 1} \left\{ \left(\frac{p_2}{p_1}\right)^{\frac{n_1 - 1}{n_1}} - 1 \right\} + \frac{n_2}{n_2 - 1} \left\{ \left(\frac{p_6}{p_5}\right)^{\frac{n_2 - 1}{n_2}} - 1 \right\} + \frac{n_3}{n_3 - 1} \left\{ \left(\frac{p_{10}}{p_9}\right)^{\frac{n_3 - 1}{n_3}} - 1 \right\} \right]$$

$$= 25.584 \times 0.287 \times 300 \times \left[\frac{1.3}{1.3 - 1} \left\{ \left(\frac{5}{1}\right)^{\frac{1.3 - 1}{1.3}} - 1 \right\} + \frac{1.32}{1.32 - 1} \left\{ \left(\frac{20}{4.8}\right)^{\frac{1.32 - 1}{1.32}} - 1 \right\} + \frac{1.35}{1.35 - 1} \left\{ \left(\frac{75}{19.8}\right)^{\frac{1.35 - 1}{1.35}} - 1 \right\} \right]$$

$$= 2202.78 \times (1.949 + 1.705 + 1.59)$$
$$= 11{,}551.4\ \text{kJ/min} \quad \text{or} \quad \mathbf{192.34\ kW}$$

Summary

- An air compressor is a machine that decreases the volume and increases the pressure of a quantity of air by mechanical means. The reciprocating compressor handles a small quantity of gas and produces very high pressure, while rotary compressors are used to handle a large volume of gas and to produce low and medium pressures.
- The volume flow rate to a single-acting, single-cylinder reciprocating compressor is

$$\dot{V} = \frac{\pi}{4} d^2 L \frac{N}{60}$$

- For the *double-acting reciprocating compressor*, the induction takes place on both sides of the piston for each revolution. Thus

$$\dot{V} = \frac{\pi}{4} d^2 L \left(\frac{2N}{60} \right)$$

where N is the speed of compressor in rotations per minute.

- *The capacity of a compressor* is the actual quantity of air delivered per unit time at atmospheric conditions. *Free Air delivery* (*FAD*) is the discharge volume of the compressor corresponding to ambient conditions.
- Piston speed is the linear speed of the piston measured in m/min. It is expressed as

$$\mathcal{V}_{piston} = 2LN$$

- The indicated work input to a single-stage, single-acting reciprocating compressor without clearance is

$$W_{in} = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \text{ (kJ/cycle)}$$

- The clearance volume is provided in the cylinder to accomodate valves. The clearance ratio c is the ratio of clearance volume to the swept volume. The clearance ratio for a reciprocating air compressor is usually 2 to 10%. The work input to the compressor with clearance ratio c is

$$W_{in} = \frac{n_c}{n_c - 1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n_c - 1}{n_c}} - 1 \right] - \frac{n_e}{n_e - 1} p_1 V_4 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n_e - 1}{n_e}} - 1 \right]$$

where n_c = index of compression and n_e = index of expansion.

If both indices are same, i.e., $n_c = n_e$ then

$$W_{in} = \frac{n}{n-1} p_1 (V_1 - V_4) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

- Isothermal efficiency of a compressor is defined as the ratio of isothermal work input to actual work input.
- The indicated power required by an air compressor is given as

$$IP = W_{in} \text{ per cycle} \times \text{No of compression per unit time} = W_{in} \left(\frac{Nk}{60} \right)$$

- From an indicated diagram, the indicated power is obtained in terms of indicated mean effective pressure, p_m as

$$IP = \frac{p_m L A N k}{60} \text{ (kW)}$$

where $k = 1$ for single acting and $k = 2$ for double acting reciprocating compressor.

- The mechanical efficiency of the compressor is given by

$$\eta_{mech} = \frac{\text{Indicated power}}{\text{Brake power}}$$

- The brake power is derived from a driving motor or engine. The input of the driving motor can be expressed as

Motor power

$$= \frac{\text{Shaft power (or Brake power)}}{\text{Mechanical efficiency of motor and drive}}$$

- The volumetric efficiency of a reciprocating air compressor is

$$\eta_{vol} = \frac{\text{Actual Mass sucked}}{\text{Mass corresponds to swept volume at atmospheric pressure and temperature}} = 1 + c - c \left(\frac{p_2}{p_1} \right)^{\frac{1}{n_e}}$$

- Multistage compression with intercooling reduces the compression work.
- For a two-stage reciprocating air compressor with intercooler, the work input per cycle is

$$W_{in} = \frac{n}{n-1} m_a R T_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2} \right)^{\frac{n-1}{n}} - 2 \right]$$

where p_2 and T_2 are intermediate pressure and temperature, respectively.

- The heat rejected with intercooler is

$$Q_{cooling} = m_a C_p (T_2 - T_1) \text{ (kJ)}$$

and heat rejected during polytropic compression is

$$Q_{comp} = m_a \frac{(\gamma - n)}{n - 1} C_v (T_2 - T_1)$$

- The compression work in a reciprocating compressor would be minimum when a stage pressure ratio is

$$\frac{p_2}{p_1} = \frac{p_3}{p_2}$$

and the minimum compression power for a two-stage compressor

$$IP_{min} = 2 \times \left(\frac{n}{n-1} \right) \dot{m}_a R T_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

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Glossary

Reciprocating compressor A reciprocating machine, used to compress the air during each stroke of piston

Rotary compressor A machine which compresses the air by dynamic action

Single-acting compressor A compressor in which all actions take place only one side of the piston during a cycle

Double-acting compressor A compressor in which suction, compression and delivery of gas take place on both sides of the piston

Single-stage compressor A compressor in which the compression of gas to final delivery pressure is carried out in one cylinder only

Multistage compressor A compressor which compresses the gas to the final pressure in more than one cylinder in series

Pressure ratio The ratio of absolute discharge pressure to absolute suction pressure

Free air Air that exists under atmospheric condition

Free Air delivery (FAD) Discharge volume of compressor corresponding to ambient conditions

Compressor Capacity Quantity of air delivered per unit time at atmospheric conditions

Inter-stage coolers Used to cool the air in between stages of compression

After coolers Used to remove the moisture in the air by cooling it

Air-dryers Removes traces of moisture after after-cooler is used

Moisture drain traps used for removal of moisture in the compressed air

Air receiver Storage tank used to store the compressed air

Review Questions

1. What is an air compressor? Why is it an important machine?
2. Write the uses of compressed air.
3. Classify the air compressors.
4. How do the suction and delivery valve activate in reciprocating air compressor?
5. State the main parts of reciprocating air compressor.
6. Differentiate between
   (i) Single-acting and double-acting compressors
   (ii) Single-stage and multistage compressors.
7. Why is a cooling arrangement provided with all compressors?
8. Define swept volume, and deduce it for single-cylinder, single-acting and double-acting compressor having bore *d,* stroke *L*, and speed *N* rpm.
9. Write the construction of a single-acting, single-stage reciprocating air compressor.
10. Explain the working of a single-acting reciprocating air compressor.
11. Explain the working of double-acting reciprocating air compressor.
12. Derive an expression for indicated work of a reciprocating air compressor by neglecting clearance.
13. Why is the clearance volume provided in each reciprocating compressor? Is it desirable to have a high clearance volume in a compressor?
14. What is clearance ratio? Write the effect of clearance volume on performance of a reciprocating compressor.
15. Derive an expression for indicated work of a reciprocating air compressor by considering its clearance.
16. Define volumetric efficiency and prove that

$$\eta_{vol} = 1 + c - c\left(\frac{p_2}{p_1}\right)^{\frac{1}{n_e}}$$

    where each term has its usual meaning.
17. Define volumetric efficiency. How is it affected by (i) pressure ratio, (ii) speed of compressor, and

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(iii) throttling across the valves? Explain in brief.

18. What are the advantages of multistage compression over single-stage compression?
19. Why is the intercooler provided between stages?
20. Prove that in a reciprocating air compressor, with perfect intercooling, the work done for compressing the air is equal to heat rejected by the air.
21. What is an after-cooler? Why is it provided with an air compressor.
22. Prove that for complete intercooling between two stages, the compression work would be minimum when intermediate pressure

$$p_2 = \sqrt{p_1 \times p_3}$$

23. Define overall volumetric efficiency. Discuss the parameters in brief, which affect it.
24. Show the effect of increase in compression ratio in single-stage reciprocating compressor on a p–V diagram and give its physical explanation.
25. Draw the indicator diagram for single-stage, double-acting reciprocating air compressor on a p–V diagram.
26. What are the advantages of using an after-cooler with an air compressor, when air under pressure has to be stored over long periods?
27. What is the effect of intake temperature and pressure on output of an air compressor?

Problems

1. Calculate the bore of the cylinder for a double-acting, single-stage reciprocating air compressor runs at 100 rpm with average piston speed of 150 m/min. The indicated power input is 50 kW. It receives air at 1 bar and 15°C and compresses it according to $pV^{1.2}$ = constant to 6 bar.

[349 mm]

2. A single-acting, single-cylinder reciprocating air compressor has a cylinder diameter of 300 mm and a stroke of 400 mm. It runs at 100 rpm. Air enters the cylinder at 1 bar; 20°C. It is then compressed to 5 bar. Calculate the mean effective pressure and indicated power input to compressor, when compression is

(a) isothermal,
(b) according to the law $pV^{1.2}$ = constant,
(c) adiabatic.

Calculate isothermal efficiency for each case. Neglect clearance.

[(a) 1.61 bar, 7.58 kW, 100% (b) 1.85 bar, 8.7 kW, 87.2% (c) 2.043 bar, 9.63 kW, 78.8%]

3. An air compressor takes in air at 100 kPa, 300 K. The air delivers at 400 kPa, 200°C at the rate of 2 kg/s. Determine minimum compressor work input. *[312.7 kW]*

4. A single-acting, single-cylinder reciprocating air compressor receives 30 m^3 of atmopheric air per hour at 1 bar and 15°C . It runs at 450 rpm and discharges air at 6.5 bar. It has a mechanical efficiency of 80% and a clearance ratio of 8.9%. Calculate

(a) the volumetric efficiency,
(b) mean effcetive pressure,
(c) brake power.

[(a) 75% (b) 1.06 bar (c) 1.48 kW]

5. Calculate the power required to drive a single-stage, single-acting reciprocating air compressor to compress 8 m^3/min of air, receiving at 1 bar, 20°C to 7 bar. The index of compression is 1.3.

Also, calculate the percentage saving in indicated power by compressing the same mass of air

(a) in two stages with optimum intercooler pressure and perfect intercooling,
(b) in two stages with imperfect intercooling to 27°C, intercooler pressure remaining the same as in case in (a),
(c) in three stages with optimum intercooler and perfect cooling.

[32.8 kW (a) 11.3% (b) 10.2% (c) 14.63%]

6. A power cylinder of 0.5 m^3 capacity is charged with compressed air without after-cooling it at 170 bar from a four-stage compressor with perfect intercooling between stages and working in best conditions. What are the most economical intermediate pressure?

[3.611bar 13.04 bar and 47.08 bar]

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7. The free air delivered by a single-stage, double-acting reciprocating air compressor, measured at 1 bar and 15°C is 16 m^3/min. The suction takes place at 96 kPa and 30°C and delivery pressure is 6 bar. The clearance volume is 4% of swept volume and mean piston speed is limited to 300 m/min. Determine

(a) power input to compressor, if mechanical efficiency is 90% and compression efficiency is 85%

(b) Bore and stroke if compressor runs at 500 rpm

Assume index of compression and expansion = 1.3.

[(a) 83.6 kW (b) 290 mm and 300 mm]

8. A double-acting, single-stage reciprocating air compressor has a bore of 330 mm, stroke of 350 mm, clearance of 5%, and runs at 300 rpm. It receives air at 95 kPa and 25°C . The delivery pressure is 4.5 bar and the index of compression is 1.25. The free air conditions are 1.013 bar and 20°C. Determine

(a) FAD,

(b) heat rejected during the compression, and

(c) power input to compressor, if its mechanical efficiency is 80%.

[(a) 14.51 m^3/min (b) 817.4 kJ/min. (c) 56.82 kW]

9. A Four cylinder, double acting reciprocating air compressor is used to compress 30 m^3/min of air at 1 bar and 27°C to a pressure of 16 bar. Calculate the size of motor required and cylinder dimensions for the following data:

speed of compressor = 320 rpm, clearance ratio 4%, stroke to bore ratio 1.2, η_{mech} = 82%, index of compression and expansion, n = 1.3.

Assume air gets healed by 12°C during suction.

[241 kW, 263 mm, 315.6 mm]

10. A single-acting, single-cylinder air compressor running at 300 rev/min is driven by an electric motor. Using the data given below, and assuming that the bore is equal to the stroke, calculate

(a) free air delivery,

(b) volumetric efficiency,

(c) bore and stroke.

Data: Air inlet conditions = 1.013 bar and 15°C; delivery pressure = 8 bar; clearance volume = 7% of swept volume; index of compression and re-expansion = 1.3; mechanical efficiency of the drive between motor and compressor = 87%; motor power output = 23 kW

(4.47 m^3/min; 72.7% 297 mm)

11. The LP cylinder of a two-stage, double-acting reciprocating air compressor running at 120 rpm has a 50-cm diameter and 75-mm stroke. It receives air at 1 bar and 20°C and compresses it adiabatically to 3 bar. Air is then delivered to an intercooler, where it is cooled at constant pressure to 35°C and then furter compressed to 10 bar in HP cylinder. Determine the power required of an electric motor to drive a compressor. Assume the mechanical efficiency of the compressor as 90% and of the motor as 86%. *[212.9 kW]*

12. A reciprocating air compressor takes in air at 40°C and 1.013 bar in the daytime.

(a) Calculate the percentage increase of mass output in the night, if the night temperature is 10°C.

(b) If the compressor is shifted to a hill station, where the barometric pressure is 0.92 bar, calculate percentage decrease in output, assuming suction temperature to be same at two places.

(c) Calculate the pressure ratio of the compressor at two places, if the law of compression is $pV^{1.25}$ = constant, if delivery gauge pressure is 7 bar at both places.

[(a) 10.61% (b) 9.18% (c) at first place 4.81% and second place 5.24]

13. A three-stage single-acting reciprocating air compressor has perfect intercooling. The pressure and temperature at the end of the suction stroke in an LP cylinder are 1.013 bar and 15°C, respetively. If 8.4 m^3 of free air is delivered by the compressor at 70 bar per minute and work done is minimum, calculate

(a) LP and IP cylinder delivery pressures,

(b) ratio of cylinder volumes,

(c) total indicated power.

Neglect clearance and assume n = 1.2.

[(a)4.16 bar, 17.05 bar (b) 2.02 (c) 676.8 kW]

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Objective Questions

1. For isothermal compression in a compressor, the compressor should run at
   (a) very high speed (b) very slow speed
   (c) constant speed (d) none of above
2. A reciprocating compressor handles
   (a) large volume for high pressure ratio
   (b) large volume for low pressure ratio
   (c) small volume for high pressure ratio
   (d) small volume for low pressure ratio
3. Usually, the index of actual compression is
   (a) near to 1 (b) 1.3 to 1.4
   (c) 1.1 to 1.3 (d) 1.4 to 1.6
4. Which of the following process takes place in an air compressor?
   (a) Specific volume of air decreases
   (b) Pressure of air increases
   (c) Mechanical energy is supplied
   (d) All of above
5. For which one of the following applications, the compressed air is not used?
   (a) Driving air motors
   (b) Oil and gas transmission
   (c) Starting of I.C. engines
   (d) Transmission of electrical energy
6. Reciprocating compressor is
   (a) a positive displacement machine
   (b) a negative displacement machine
   (c) a dynamic action machine
   (d) none of above
7. Air dryers are used in an air compressor
   (a) before air entry into cylinder
   (b) before entering air receiver
   (c) between two stages
   (d) after leaving air receiver
8. Air receiver used in an air compressor is used to
   (a) cool the air after compression
   (b) eliminate the pulsation
   (c) supply the air to utility
   (d) to separate the moisture
9. In a reciprocating air compressor, inlet and delivery valves actuate
   (a) by separate cam mechanism
   (b) by pressure difference
   (c) by use of compressed air
   (d) none of the above
10. In a reciprocating air compressor, the work input is minimum when compression is
    (a) isentropic (b) polytropic
    (c) isothermal (d) isobaric
11. What is the sequence of processes in a reciprocating air compressor?
    (a) Compression, expansion, and constant volume discharge
    (b) Induction, compression and constant pressure discharge
    (c) Induction, expansion and constant pressure discharge
    (d) Induction, compression and constant volume discharge
12. Work input in a reciprocating air compressor is given by
    (a) $\frac{n-1}{n} p_1 v_1 \left[1+\left(\frac{p_2}{p_1}\right)^{\frac{n}{n-1}}\right]$
    (b) $\frac{n-1}{n} p_1 v_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n}{n-1}}-1\right]$
    (c) $\frac{n-1}{n} p_1 v_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}-1\right]$
    (d) $\frac{n-1}{n} p_1 v_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}+1\right]$
13. The isothermal efficiency of a reciprocating air compressor is given by

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(a) $\dfrac{\text{Indicated power}}{\text{Isothermal power}}$

(b) $\dfrac{\text{Isothermal power}}{\text{Indicated power}}$

(c) $\dfrac{\text{Isothermal power}}{\text{Brake power}}$

(d) $\dfrac{\text{Brake power}}{\text{Isothermal power}}$

14. The compressor efficiency of a reciprocating air compressor is given by

(a) $\dfrac{\text{Indicated power}}{\text{Isothermal power}}$

(b) $\dfrac{\text{Isothermal power}}{\text{Indicated power}}$

(c) $\dfrac{\text{Isothermal power}}{\text{Brake power}}$

(d) $\dfrac{\text{Brake power}}{\text{Isothermal power}}$

15. The isothermal efficiency of a reciprocating air compressor can be improved by use of
   (a) water jacketing (b) external fins
   (c) intercooler (d) all of the above

16. The clearance volume in a reciprocating air compressor
   (a) reduces work input
   (b) reduces suction capacity
   (c) reduces discharge pressure
   (d) all of the above

17. The volumetric efficiency of a reciprocating air compressor is defined as

(a) $1 + c - c\left(\dfrac{p_2}{p_1}\right)^{\frac{n-1}{n}}$

(b) $1 - c + c\left(\dfrac{p_2}{p_1}\right)^{\frac{n-1}{n}}$

(c) $1 + c - c\left(\dfrac{p_2}{p_1}\right)^{\frac{1}{n}}$

(d) $1 - c + c\left(\dfrac{p_2}{p_1}\right)^{\frac{1}{n}}$

18. The maximum pressure ratio in a single-stage reciprocating air compressor is limited to
   (a) 2 (b) 4
   (c) 7 (d) 10

19. Multistage compression in a reciprocating air compressor improves
   (a) isothermal efficiency
   (b) volumetric efficiency
   (c) mechanical balance
   (d) all of above

20. Ideal intermediate pressure p_2 for two-stage reciprocating air compressor is given by

(a) $p_1 \times p_3$ (b) $\sqrt{p_1 \times p_3}$

(c) $\sqrt{\dfrac{p_3}{p_1}}$ (d) $\sqrt{\dfrac{p_3}{p_1}}$

21. Heat rejection rate per stage of air with perfect intercooler is given by

(a) $\dot{m}_a \left[\dfrac{(\gamma - n)}{n-1} C_v + C_p\right](T_2 - T_1)$

(b) $\dot{m}_a \left[C_p - \dfrac{(\gamma - n)}{n-1} C_v\right](T_2 - T_1)$

(c) $\dot{m}_a \left[C_p - \dfrac{(n-1)}{\gamma - 1} C_v\right](T_2 - T_1)$

(d) $\dot{m}_a \left[C_p + \dfrac{(n-1)}{\gamma - 1} C_v\right](T_2 - T_1)$

Answers

1. (b) 2. (c) 3. (c) 4. (d) 5. (d) 6. (a) 7. (b) 8. (b)
9. (b) 10. (c) 11. (b) 12. (c) 13. (c) 14. (b) 15. (d) 16. (b)
17. (c) 18. (c) 19. (d) 20. (b) 21. (a)

26

Rotary Compressor

Introduction

Rotary compressors are used to supply continuous pulsation free compressed air. They have rotor(s) and casing in place of piston cylinder arrangement. They are compact, well balanced, and high speed compressors. They have low starting torque thus they are directly coupled with prime-mover. They handle large mass of gas and are suitable for low and medium pressure ratios.

The special features of rotary compressors are:

- √ Designed to provide pulsation-free air,
- √ 100% continuous duty,
- √ Quiet operation,
- √ Energy efficient at full load,
- √ Extended service intervals,
- √ Reliable long life,
- √ Improved air quality,
- √ Low starting torque.

Fans and blowers are also rotary machines, which are used for supplying air or gas. These machines are differentiated by the method used to move the air, and by the system pressure, at which they are operating. The ratio of the discharge pressure to the suction pressure is used to define the fans, blowers and compressors as shown in Table 26.1.

Table 26.1 *Difference between fans, blowers and compressors*

Equipment	*Pressure ratio*	*Pressure rise in mm of water*
Fan	up to 1.11	1136
Blower	1.11 to 1.20	1136 - 2066
Compressor	more than 1.20	more than 2066

Some compressors are suitable only for low pressure ratio work such as for scavenging and supercharging of engines and for various applications of exhausting and vacuum pumping. For a pressure ratio above 9 bar, vane-type rotary machines can be used to boost the pressure.

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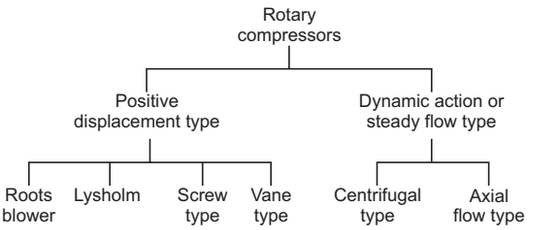
26.1 CLASSIFICATION OF ROTARY COMPRESSORS

The air compressors are either reciprocating types or rotary types. The rotary air compressors can broadly be classified as



In a rotary *positive displacement type* of air compressor, the air is compressed by being trapped in the reduced space formed by two sets of engaging surfaces. In a *non-positive displacement* or steady flow type of compressor, the air flows continuously through them and pressure is increased due to dynamic action.

The rotary compressors have adiabatic compression. They have high speed and no cooling arrangement is provided during compression.

26.2 ROOTS BLOWER COMPRESSOR

Roots blower is a positive dispacement compressor. It is also called *lobe compressor.* The roots blower

Comparison between Reciprocating and Rotary Compressors

S. No.	*Aspect*	*Reciprocating compressor*	*Rotary compressor*
1.	Pressure ratio	Discharge pressure of air is high. The pressure ratio per stage may be in order of 4 to 7.	Discharge pressure of air is low. The pressure ratio per stage may be in order of 3 to 5.
2.	Handled volume	Quantity of air handled is low and is limited to 50 m^3/s.	Larger quantity of air can be handled and it is about 500 m^3/s.
3.	Speed of compressor	Low speed of compressor.	High speed of compressor.
4.	Vibrational Problem	Due to reciprocating action, greater vibrational problem, the parts of machine are poorly balanced.	Rotary parts of machine, thus it has less vibrational problems. The machine parts are fairly balanced.
5.	Size	Size of compressor is bulky for given discharge volume.	Compressor size is small for given discharge volume.
6.	Air supply	Air supply is intermittent.	Air supply is steady and continuous.
7.	Purity of compressed air	Air delivered from the compressor is dirty, since it comes in contact with lubricating oil and cylinder surfaces.	Air delivered from the compressor is clean and free from dirt.
8.	Compression efficiency	Higher with pressure ratio more than 2.	Higher with compression ratio less than 2.
9.	Maintanance	Higher due to reciprocating parts.	Lower due to balanced rotary parts.
10.	Mechanical efficiency	Lower due to several sliding parts.	Higher due to less sliding parts.
11.	Lubrication	Complicated lubrication system.	Simple lubrication system.
12.	Initial cost	Higher.	Lower.
13.	Flexibility	Greater flexibility in capacity and pressure range.	No flexibility in capacity and pressure range.
14.	Suitability	For medium and high pressure ratio and low and medium gas volume.	For low and medium pressures and large volumes.

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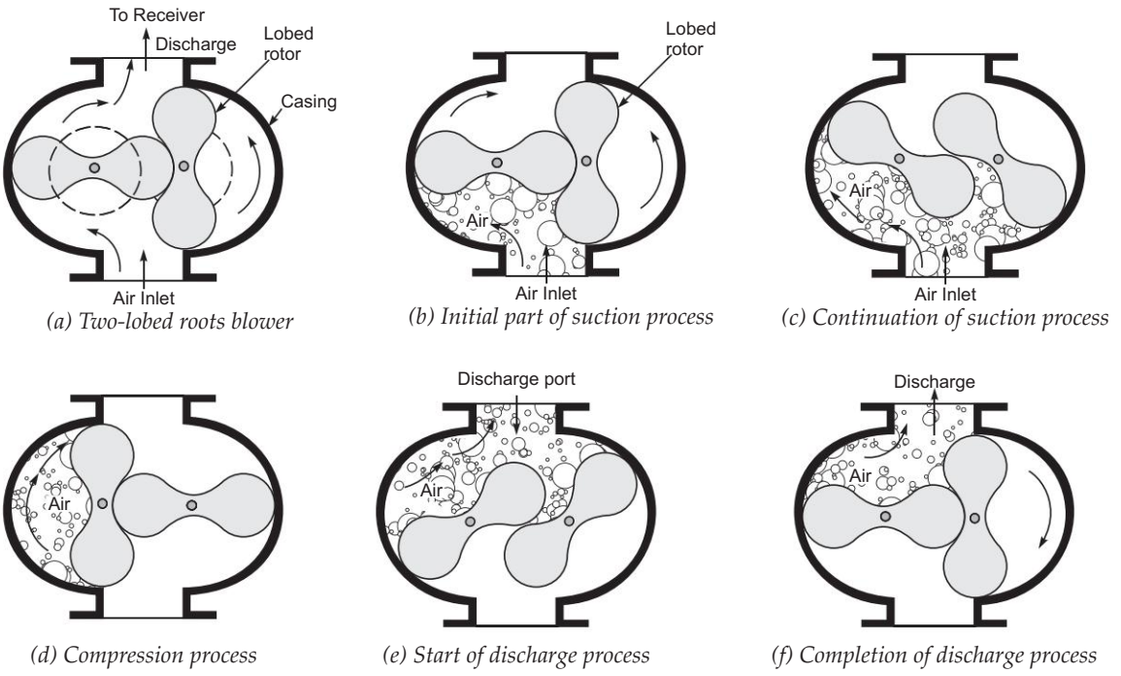

(a) Two-lobed roots blower *(b) Initial part of suction process* *(c) Continuation of suction process*

(d) Compression process *(e) Start of discharge process* *(f) Completion of discharge process*

Fig. 26.1

is essentially a low-pressure blower and is limited to a discharge pressure of 1 bar in single-stage design and up to 2.2 bar in a two-stage design. Its discharge capacity is limited to 1500 m^3/min and it can run up to 7000 rpm.

Construction This type of rotary compressor consists of two or more lobed rotors and casing with inlet and outlet passage of air. The lobed rotors rotate in an air tight casing with the help of gears in external housing. The compressor inlet is open to atmospheric air at one side and it is open to delivery side at the other side. The two lobes of the roots blower is shown in Fig. 26.1(a).

One of the rotors is connected to drive. The second rotor is gear driven from the first. Thus, both rotors rotate with the same speed. The profile of the lobes is made cycloidal or involute in order to seal the inlet side from the delivery side.

Working The rotation of rotors creates space in the casing at the entry port as shown in Fig. 26.1(b). The air is drawn into the casing to fill the space. The flow of gas in the casing space continues till both rotors change their position as shown in Fig. 26(c).

With further movement of the lobed rotor, the air is trapped between one rotor, when its tip tuches the casing as shown in Fig. 26(d). This part of the blower is not open to suction port. But the air flows into the space created by rotation of other rotor. This rotor is also carrying out the same cycle as first rotor after 90°.

The trapped volume of air is not internally compressed, it is only displaced at high speed from suction side to delivery side. Continued rotation of lobes opens the discharge port as shown in Fig.26 (e).

Since the compressed air at higher pressure is present at the delivery side, when the rotor lobe uncovers the exit port, some pressurised air enters into the space between the rotor and casing of the compressor. This flow of air is called *back flow* of

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air. This back flow of air continues until the pressure in the blower gets equalised. After back flow, the air is compressed irreversibly at constant volume. Finally, at higher pressure, the air is delivered from the blower to receiver as shown in Fig. 26.1(f). The process of compression can be represented by constant volume line on *p–V* plane as shown in Fig. 26.2.

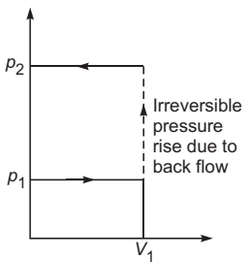

Fig. 26.2 *p-V diagram for roots blower*

Consider a volume of V_1 is trapped between the lobe and casing at atmospheric pressure p_1. The air is compressed to delivery pressure p_2. The actual work done on air;

$$W_{act} = -\int V dp = V_1(p_2 - p_1) \qquad ...(26.1)$$

The ideal work input for compression is isentropic work input. The theoretical work input to compress air from atmospheric pressure p_1 to delivery pressure p_2.

$$W_{isen} = \frac{\gamma}{\gamma - 1} p_1 V_1 \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} \qquad ...(26.2)$$

The efficiency of the roots blower can be defined as the ratio between adiabatic work to the actual work input. Mathematically;

$$\eta_{roots} = \frac{\text{Adiabatic work input}}{\text{Actual work input}}$$

$$= \frac{\frac{\gamma}{\gamma - 1} p_1 V_1 \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\}}{V_1(p_2 - p_1)}$$

Let $\frac{p_2}{p_1} = r_p$, pressure ratio. Dividing numerator and denominator by $p_1 V_1$, we get

$$\eta_{roots} = \frac{\frac{\gamma}{\gamma - 1} \left\{ \left(r_p \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\}}{(r_p - 1)}$$

$$= \frac{\gamma}{\gamma - 1} \times \frac{\left\{ \left(r_p \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\}}{(r_p - 1)} \qquad ...(26.3)$$

The efficiency of roots blower decreases with increase in pressure ratio. However, the compressor is suitable to give a pressure ratio between range of 1 to 3.6.

Applications

1. The root blowers are used for scavenging and supercharging of two-stroke internal combustion engines.
2. It is also used for low-pressure supply of air in steel furnaces, sewage disposal plants, low-pressure gas boosters, and for blower service in general.

Example 26.1 *A root blower compresses 1 m^3 of air per second from a pressure of 1.01325 bar to 1.8 bar. Find the power required to run the compressor and its efficiency.*

Solution

Given A root blower with

$\dot{V}$ = 1 m^3/s

p_1 = 1.01325 bar = 101.325 kPa

p_2 = 1.8 bar = 180 kPa

To find

(i) Power required to run the compressor, and

(ii) Compressor efficiency.

Assumptions

(i) Compression of air at constant volume.

(ii) Compression is reversible.

(iii) Index of isentropic compression be 1.4.

Analysis

(i) The actual power required to run the compressor

$$P = \dot{V}(p_2 - p_1)$$

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$= (1 \text{ m}^3/\text{s}) \times (180 - 101.325) \text{ (kPa)}$

$= \mathbf{78.675 \text{ kW}}$

Ideal power required to compress the air

$$P_{ise} = \frac{\gamma}{\gamma - 1} p_1 \dot{V}_1 \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right\}$$

$$= \frac{1.4}{1.4 - 1} \times 101.325 \times 1 \left\{ \left(\frac{1.8}{1.01325} \right)^{\frac{1.4 - 1}{1.4}} - 1 \right\}$$

$= 63.28$ kW

(ii) Compressor efficiency;

$$\eta_{roots} = \frac{\text{Adiabatic work input}}{\text{Actual work input}}$$

$$= \frac{63.28 \text{ kW}}{78.675 \text{ kW}} = 0.8043 \quad \text{or} \quad \mathbf{80.43\%}$$

Example 26.2 *1 kg of air per second is taken into a root blower compressor at 1 bar and 27°C. The delivery pressure of air is 1.5 bar. Calculate the motor power required to run the compressor, if mechanical efficiency is 80%.*

Solution

Given A root blower with

$\dot{m} = 1$ kg/s $\qquad p_1 = 1$ bar $= 100$ kPa

$T_1 = 27°\text{C} = 300$ K $\qquad p_2 = 1.5$ bar $= 150$ kPa

$\eta_{mech} = 0.8$

To find Power required to run the compressor.

Assumptions

(i) Compression of air at constant volume.

(ii) Compression is reversible.

(iii) Specific gas constnat for air as 0.287 kJ/kg · K.

Analysis The volume flow rate of air taken into the compressor

$$\dot{V} = \frac{\dot{m} R T_1}{p_1}$$

$$= \frac{(1 \text{ kg/s}) \times (0.287 \text{ kJ/kg} \cdot \text{K}) \times (300 \text{ K})}{(100 \text{ kPa})}$$

$= 0.861 \text{ m}^3/\text{s}$

The power required to run the compressor

$IP = \dot{V}(p_2 - p_1)$

$= (0.861 \text{ m}^3/\text{s}) \times (150 - 100) \text{ (kPa)}$

$= 43.05$ kW

The mechanical efficiency is given by

$$\eta_{mech} = \frac{IP}{BP}$$

or Brake power,

$$BP = \frac{IP}{\eta_{mech}} = \frac{43.05}{0.8} = \mathbf{53.81 \text{ kW}}$$

26.3 VANE-TYPE COMPRESSOR

Construction An arrangement of a typical vane-type compressor is shown in Fig. 26.3. It consists of an air-tight circular casing, in which a drum rotates about an ecentric centre of casing. The drum consists of a set of spring-loaded vanes. The slots are cut in the drum to accomodate the vanes. The drum rotates in anticlockwise direction. During the rotation of the drum, the vanes remain in contact with the casing. Size of inlet passage is larger than the size of outlet in the compressor.

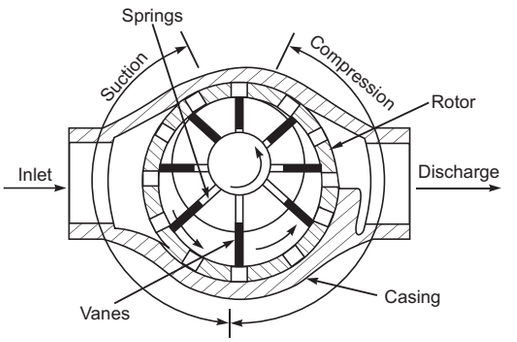

Fig. 26.3 *Vane-type compressor*

Working As the drum rotates, the volume of air V_1 at atmospheric pressure p_1 is trapped between the vanes, drum and casing. Air gets compressed due to two operations performed on air. First the compression begins due to decreasing volume between the drum and casing. The volume is reduced to V_2 and pressure increases to p_2. Secondly, the air is compressed due to back flow of compressed air in the receiver. Then the air is compressed at constant volume to a pressure p_3. The first part of compression follows adiabatic compression

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process and the second part follows constant-volume process. The process of compression is shown on the *p–V* diagram in Fig. 26.4

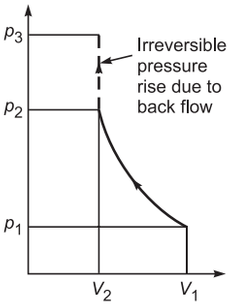

Fig. 26.4 *p–V diagram for Vane type compressor*

Work input for adiabatic compression;

$$W_{12} = \frac{\gamma}{\gamma - 1} p_1 V_1 \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\}$$

Work input for constant volume compression;

$$W_{23} = V_2(p_3 - p_2)$$

Total work input for compression within a vane;

$$W_{vane} = \frac{\gamma}{\gamma - 1} p_1 V_1 \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} + V_2(p_3 - p_2)$$

If there are N vanes within the drum, then total work input;

$$W_{N\,vane} = N \left[\frac{\gamma}{\gamma - 1} p_1 V_1 \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} + V_2 (p_3 - p_2) \right] \quad ...(26.4)$$

The efficiency of a vane compressor can be expressed as

$$\eta_{vane,\,comp} = \frac{\text{Work input for constant-volume compression}}{\text{Total work inputf or compression}}$$

$$= \frac{V_2(p_3 - p_2)}{\frac{\gamma}{\gamma - 1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] + V_2(p_3 - p_2)} \quad ...(26.5)$$

1. The vane-type compressor requires less work input compared to roots blower for same capacity and pressure ratio.
2. Vane-type compressors are commonly used to deliver air up to 150 m^3/min at a pressure ratio up to 8.5.
3. Vane-type compressors can run up to 3000 rpm.
4. Vane-type compressors are used for supercharging of IC engines and supply of air to cupola.
5. These are portable compressors used for construction purpose.

Example 26.3 *Calculate the power required to run the vane compressor and its efficiency, when it handles 6 m^3 of air per minute from 1 bar to 2.2. bar. The pressure rise due to compression in the compressor is limited to 1.6 bar. Take the mechanical efficiency of compressor as 80%.*

Solution

Given A vane compressor with

$\dot{V}$ = 6 m^3/min = 0.1 m^3/s p_1 = 1 bar = 100 kPa

p_2 = 1.6 bar = 160 kPa p_3 = 2.2 bar = 220 kPa

η_{mech} = 0.8

To find

(i) Brake power required to run the compressor,

(ii) Efficiency of the compressor.

Assumptions

(i) Compression is reversible.

(ii) Specific heat ratio for air as 1.4.

Analysis The isentropic compression power from state1 to state 2;

$$IP_1 = \frac{\gamma}{\gamma - 1} p_1 \dot{V}_1 \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\}$$

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$$= \frac{1.4}{1.4-1} \times (100 \text{ kPa}) \times (0.1 \text{ m}^3/\text{s}) \times \left\{ \left(\frac{1.6}{1} \right)^{\frac{1.4-1}{1.4}} - 1 \right\}$$

$= 5.03$ kW

The volume of air after compression to 1.6 bar;

$$\dot{V}_2 = \dot{V}_1 \left(\frac{p_1}{p_2} \right)^{\frac{1}{\gamma}} = (0.1 \text{ m}^3/\text{s}) \times \left(\frac{1}{1.6} \right)^{\frac{1}{1.4}}$$

$= 0.07148$ m^3/s

Work done due to back flow from the state 2 to state 3

$$IP_2 = \dot{V}_2 (p_3 - p_3)$$

$= (0.07148 \text{ m}^3/\text{s}) \times (220 - 160)$ (kPa)

$= 4.29$ kW

Total power input for compression of air

$$IP = IP_1 + IP_2 = 5.03 + 4.29 = 9.32 \text{ kW}$$

The mechanical efficiency is given by

$$\eta_{mech} = \frac{IP}{BP}$$

(i) The Brake power required to run the motor or shaft power

$$BP = \frac{IP}{\eta_{mech}} = \frac{9.32}{0.8} = \mathbf{11.65 \text{ kW}}$$

(ii) Efficiency of the vane compressor, Eq. (26.5);

$$\eta_{vane\ comp} = \frac{\text{Work input for constant volume compression}}{\text{Total work inputf or compression}}$$

$$= \frac{4.29 \text{ kW}}{9.32 \text{ kW}} = 0.46 \quad \text{or} \quad \mathbf{46\%}$$

Example 26.4 *Compare the work required for compression in a root blower and vane blower compressors for the following particulars:*

Intake volume = 0.05 m^3 per revolution

Inlet pressure = 1.013 bar

Pressure ratio = 1.5

For the vane-type compressor, internal compression takes place through half the pressure range.

Solution

Given Compression with

$V_1 = 0.05$ m^3/rev

$p_1 = 1.013$ bar $= 101.3$ kPa

Pressure ratio $= 1.5$

To find

(i) Work input to root blower and vane-type compressor.

Assumptions

(i) Compression is reversible.

(ii) Specific heat ratio for air as 1.4.

Analysis

(i) For a root blower compressor,

Pressure ratio, $\frac{p_2}{p_1} = 1.5$

or $p_2 = 1.5\, p_1 = 1.5 \times 1.013$

$= 1.5195$ bar or 151.95 kPa

work input per revolution

$$W_{root} = V_1 (p_2 - p_1) = 0.05 \times (151.95 - 101.3)$$

$= \mathbf{2.53 \text{ kJ}}$

(ii) For vane-type compressor:

Pressure ratio, $\frac{p_3}{p_1} = 1.5$

or $p_3 = 1.5\, p_1 = 1.5 \times 1.013$

$= 1.5195$ bar or 151.95 kPa

Intermediate pressure;

$$p_2 = p_1 + 0.5\,(p_3 - p_1)$$

$= 101.3 + 0.5 \times (151.95 - 101.3)$

$= 126.625$ kPa

The volume of air after intermediate compression

$$V_2 = V_1 \left(\frac{p_1}{p_2} \right)^{\frac{1}{\gamma}} = (0.05 \text{ m}^3) \times \left(\frac{101.3}{126.625} \right)^{\frac{1}{1.4}}$$

$= 0.0426$ m^3/rev

Work input per revolution for vane-type compressor

$$W_{Vane} = \frac{\gamma}{\gamma - 1} p_1 V_1 \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} + V_2 (p_2 - p_1) = 0$$

$$= \frac{1.4}{1.4-1} \times 101.3 \times 0.05 \times \left\{ \left(\frac{126.625}{101.3} \right)^{\frac{1.4-1}{1.4}} - 1 \right\} + 0.0426 \times (151.95 - 126.625)$$

$= 1.167 + 1.079 = \mathbf{2.246 \text{ kJ/rev}}$

Comment The vane-type compressor requires less work input for compression of given volume of air for same pressure ratio.

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26.4 LYSHOLM COMPRESSOR—A SCREW COMPRESSOR

The *screw compressors* are most commonly used rotary air compressor. They are single stage helical or spiral lobe, oil flooded screw air compressor. They are simple in design and have few wearing parts, and thus easy to install, maintain and operate.

There are available in wide range of pressure ratio and capacity. Lubricated types are available in sizes ranging from 200 to 2000 m^3 per hour with discharge pressure up to 10 bar.

The oil free rotary screw air compressor uses specially designed machine. It compresses air without oil in the compression chamber. These compressors are air or water cooled machines. They deliver oil-free air and are available in sizes up to 30,000 m^3 per hour and pressure upto 15 bar.

Alf Lysholm had developed the modern screw compressor, which is known as *Lysholm compressor*. Its construction and working are discussed below.

26.4.1 Construction

Screw compressors are equipped with two mating helical grooved rotors housed within a cylindrical casing equipped with inlet and discharge ports as shown in Fig. 26.5.

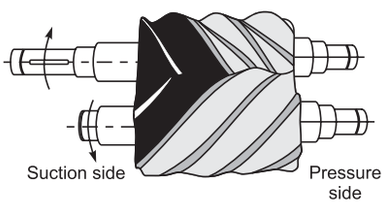

Fig. 26.5 *Screw compressor*

The main (male) rotor is normally driven by either an electric motor or an engine and transforms about 85–90% of the energy received at the coupling into pressure and heat energy. The number of lobes (or valleys) on the rotors will vary from one compressor manufacturer to another. Usually, the male rotor is made with four lobes along the length of the rotor that meshes with similarly formed correspondingly six helical flute on the auxiliary (female) rotor.

The auxiliary rotors seal the working space between the suction and pressure side. In the course of rotation, main and auxiliary rotors generate a *V*-shaped space for the air drawn in, which becomes smaller and smaller right up to the end, between the rotor lobes and the cylinder walls.

Because of the number of male lobes, there are four compression cycles per revolution which means that the resulting compressed air has small pulsations compared to a reciprocating compressor.

26.4.2 Working

As rotors rotate, the air is drawn through the inlet port to fill the space between the male lobe and female flute. As rotors continue to rotate, the air is moved past the suction port and sealed in the interlobe space. The trapped air is moved axially and radially and is compressed by direct volume reduction as enmeshing of lobes of compressor progressively reduces the space occupied by the gas with increase in pressure. Simultaneously, with this process, the oil is injected into the system. The oil seals the internal clearances and it absorbs the heat energy generated during compression. The compression of air continues until the interlobe space communicates with discharge port in the casing. The compressed air leaves the casing through the discharge port.

The working parts of compressor never get severe operating temperatures, since the cooling takes place right inside the compressor. The oil is separated out in the oil separator and cooled down in an oil cooler and is returned back to compressor through an oil filter.

The *internal volume ratio* of a screw compressor is defined as the ratio of the volume of flute at the start of compression process to volume of the same flute as it begins to open to discharge port.

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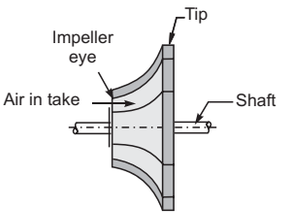
26.5 CENTRIFUGAL COMPRESSOR

The centrifugal compressors are dynamic action compressors. These compressors have appreciably different characteristics as compared to reciprocating machines. A small change in compression ratio produces a marked change in compressor output and efficiency. Centrifugal machines are better suited for applications requiring very high capacities, typically above 3000 m^3/min and a moderate pressure ratio of 4 to 6. They are preferred due to their simplicity, light weight and ruggedness.

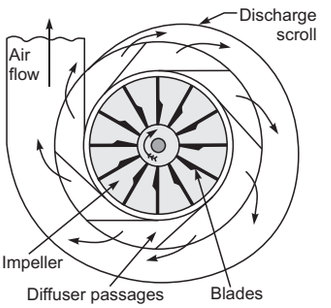
The centrifugal air compressor is an oil-free compressor by design. The oil-lubricated running gear is separated from the air by shaft seals and atmospheric vents. It is a continuous duty compressor, with few moving parts, and is particularly suited to high volume applications, especially where oil-free air is required.

26.5.1 Construction

The basic components of a typical centrifugal compressor includes an impeller, diffuser and casing as shown in Fig. 26.6. The impeller is a radial disc with a series of radial blades (vanes).



(a) Impeller eye



(b) Sectional view of centrifugal compressor

Fig. 26.6 *Centrifugal compressor*

The impeller rotates inside the casing. The impeller is usually forged or die casting of aluminium alloy.

The centre of the impeller is called the *eye*. The eye of the impeller is connected with the drive shaft. The casing of the compressor has a volute shape. A diffuser ring is housed in the radial portion of the casing.

26.5.2 Working

As the impeller rotates, the air enters radially into the impeller eye with low velocity $\mathcal{V}_1$ at atmospheric pressure p_1. Due to centrifugal action of the impeller, the air comes radially out and during its movement, it is guided by the blades within the impeller.

The high velocity of the impeller increases the momentum of air, causing rise in static pressure, temperature and kinetic energy of air. The pressure, temperature and velocity of air leaving the impeller are p_2, T_2 and $\mathcal{V}_2$, respectively.

The air leaving the outside edge of the impeller enters into the diffuser ring where its kinetic energy is converted into pressure energy. Thus, the static pressure of air is further increased. The air is then collected in the casing and discharged from the compressor. The change in pressure and velocity of air passing the impeller and diffuser passage are shown in Fig. 26.7.

26.5.3 Velocity Diagram

In a centrifugal air compressor, the air enters the impeller radially and leaves axially. Hence the

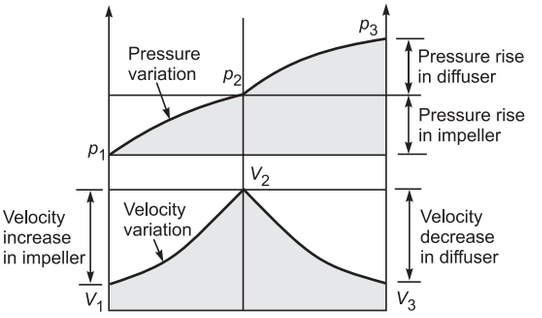


Fig. 26.7 *Velocity and pressure variation of air passing through impeller and diffuser*

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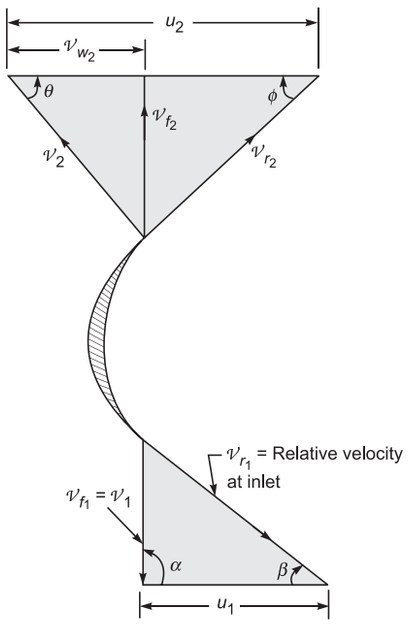

Fig. 26.8 *Velocity diagram at inlet and outlet of impeller blade*

blades are designed in such a way that the air enters and leaves the blades without shock.

Using usual notation, let

u_1 = Blade velocity at inlet

v_{r_1} = Relative velocity of blades at inlet

v_1 = Absolute velocity of inlet air

v_{f_1} = Inlet flow velocity

α = Air inlet angle

β = Blade angle at inlet

u_2, v_{r_2}, v_2, v_{f_2}, θ, and ϕ are the corresponding values at outlet.

Work input per kg of air

$$w = u_1 v_{w_1} + u_2 v_{w_2}$$

Since the working fluid enters radially, i.e., $\alpha = 90°$, thus $v_{w_1} = 0$

Hence work input by blade per kg of air

$$w = u_2 v_{w_2} \text{ (J/kg)} \qquad ...(26.6)$$

The above equation is known as *Euler's equation* or *Euler's work.*

If $\dot{m}$ is the mass flow rate of air in kg/s, then power input to compressor

$$P = \dot{m}\, u_2 v_{w_2} \text{ (Joules)} \qquad ...(26.7)$$

The work transfer per kg of air to centrifugal compressor can also be obtained by using steady-flow energy equation

$$q - w = (h_2 - h_1) + \left(\frac{v_2^2}{2} - \frac{v_1^2}{2}\right) + \Delta pe$$

For centrifugal air compressor, $q = 0$,

Change in potential energy, $\Delta pe = 0$;

Omitting the negative sign for work input, the compresson work per kg of air

$$w = \left(h_2 + \frac{v_2^2}{2}\right) - \left(h_1 + \frac{v_1^2}{2}\right)$$

or
$$w = (h_{02} - h_{01}) = C_p(T_{02} - T_{01})$$

$$= C_p T_{01}\left(\frac{T_{02}}{T_{01}} - 1\right) \qquad ...(26.8)$$

The static state and stagnation state are shown in Fig. 26.9. The stagnation temperatures T_{01} and T_{02} can be expressed as

$$T_{01} = T_1\left(\frac{p_{01}}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_{02} = T_2\left(\frac{p_{02}}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$$

and
$$\frac{T_{02}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}$$

Hence the work input to compressor per kg of air can be expressed as

$$w = C_p T_{01}\left(\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right) \qquad ...(26.9)$$

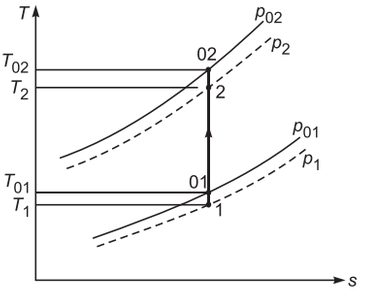

Fig. 26.9 *Static and stagnation properties during adiabatic compression*

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If $\mathcal{V}_1 = \mathcal{V}_2$, then stagnation pressure, $p_{01} = p_1$(static pressure) and $p_{02} = p_2$, $T_{01} = T_1$;

$$w = C_p T_1 \left(\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right) \qquad ...(26.10)$$

26.5.4 Width of Blades of Impeller and Diffuser

For constant mass flow rate of air, the width of the blades of impeller can be calculated as follows:

$$\dot{m} = \frac{A\mathcal{V}_1}{v_1} \text{ (Continuity equation)}$$

where v_1 = specific volume air at inlet

$A = \pi D_1 B_1$, with D_1, diameter of impeller at inlet and B_1 as width of impeller blades at inlet.

As air trapped radially, $\mathcal{V}_1 = \mathcal{V}_{f_1}$

$$\therefore \quad \dot{m} = \frac{\pi D_1 B_1}{v_1} \mathcal{V}_{f_1} \qquad ...(26.11)$$

$$\text{or} \quad B_1 = \frac{\dot{m} v_1}{\pi D_1 \mathcal{V}_{f_1}} \qquad ...(26.12)$$

Similarly, the width of impeller blade at the outlet can be obtained by using the suffix 2 in Eq. (26.12)

$$B_2 = \frac{\dot{m} v_2}{\pi D_2 \mathcal{V}_{f_2}} \qquad ...(26.13)$$

If the number and thickness of blades are considered then the effective area of the blade will be

$$A = \pi D - nt$$

where n = number of blades and t = thickness of blade.

$$\text{Then} \quad B_1 = \frac{\dot{m} v_1}{(\pi D_1 - nt)\mathcal{V}_{f_1}} \qquad ...(26.14)$$

$$\text{and} \quad B_2 = \frac{\dot{m} v_2}{(\pi D_2 - nt)\mathcal{V}_{f_2}} \qquad ...(26.15)$$

26.5.5 Degree of Reaction

The *degree of reaction* is defined as the ratio of static pressure rise in the impeller to the total static pressure rise in the compressor. The pressure rise in the impeller is equal to change in kinetic energy of air in the impeller, i.e.,

$$\Delta ke = \frac{\mathcal{V}_{r_1}^2 - \mathcal{V}_{r_2}^2}{2} + \frac{u_2^2 - u_1^2}{2} \qquad ...(26.16)$$

The *first* term above Eq. (26.16) indicates the pressure rise in the impeller due to diffusion action, and the *second* term represents the pressure rise in the compressor due to centrifugal action of the impeller.

Total pressure rise in the compressor is equal to work input to the compressor.

$\therefore$ degree of reaction

$$R_d = \frac{\text{Pressure rise in the impeller}}{\text{Pressure rise in the compressor}}$$

$$= \frac{\frac{\mathcal{V}_{r_1}^2 - \mathcal{V}_{r_2}^2}{2} + \frac{u_2^2 - u_1^2}{2}}{u_2 \mathcal{V}_{w_2}} = \frac{(\mathcal{V}_{r_1}^2 - \mathcal{V}_{r_2}^2) + (u_2^2 - u_1^2)}{2u_2 \mathcal{V}_{w_2}}$$

$$\text{or} \quad R_d = \frac{(u_2^2 - \mathcal{V}_{r_2}^2) + (\mathcal{V}_{r_1}^2 - u_1^2)}{2u_2 \mathcal{V}_{w_2}} \qquad ...(26.17)$$

From the inlet velocity triangle of Fig. 26.8;

$$\mathcal{V}_{r_1}^2 - u_1^2 = \mathcal{V}_{f_1}^2 = \mathcal{V}_{f_2}^2 \text{ (since } \mathcal{V}_{f_1} = \mathcal{V}_{f_2}) \qquad ...(26.18)$$

From outlet velocity trinagle of Fig. 26.8;

$$\mathcal{V}_{f_2}^2 + (u_2 - \mathcal{V}_{w_2})^2 = \mathcal{V}_{r_2}^2$$

$$\text{or} \quad \mathcal{V}_{r_2}^2 = \mathcal{V}_{f_2}^2 + u_2^2 + \mathcal{V}_{w_2}^2 - 2u_2 \mathcal{V}_{w_2}$$

$$\text{or} \quad u_2^2 - \mathcal{V}_{r_2}^2 = 2u_2 \mathcal{V}_{w_2} - \mathcal{V}_{f_2}^2 - \mathcal{V}_{w_2}^2 \qquad ...(26.19)$$

Using Eqs. (26.18) and (26.19) in Eq. (26.17), we get

$$R_d = \frac{2u_2 \mathcal{V}_{w_2} - \mathcal{V}_{f_2}^2 - \mathcal{V}_{w_2}^2 + \mathcal{V}_{f_2}^2}{2u_2 \mathcal{V}_{w_2}}$$

$$\text{or} \quad R_d = 1 - \frac{\mathcal{V}_{w_2}}{2u_2} \qquad ...(26.20)$$

26.5.6 Combined Velocity Diagram

Since the flow velocity remains constant at inlet and outlet, therefore, the inlet and exit velocity triangles can be drawn on the common base. Since air enters at right angles to the impeller blade, thus the air inlet angle at the impeller is equal to 90°, i.e., α = 90°. Combined velocity triangle for centrifugal compressor is shown in Fig. 26.10.

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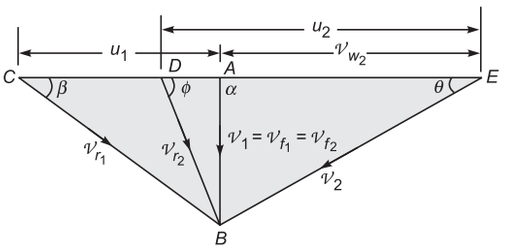

Fig. 26.10 *Combined velocity triangle for centrifugal compressor*

Procedure

1. Draw a vertical line *AB* to represent the flow velocity and it remains constant at inlet and exit.
2. The horizontal line *CA* represents the blade velocity u_1 at inlet.
3. The line *CB* inclined at the blade angle β represents relative velocity $\mathcal{V}_{r_1}$ of blade at the inlet.
4. The line *DB* inclined at blade angle ϕ represents the relative velocity $\mathcal{V}_{r_2}$ of the blade at the outlet.
5. The line *DE* represents the blade velocity u_2 of impeller at outlet.
6. Join the line *EB*. It represents absolute velocity $\mathcal{V}_2$ of air at the outlet inclined at angle θ with respect to the horizontal.

From the combined velocity triangle, the work input to the compressor per kg of air

$$w = u_2 \mathcal{V}_{w_2} \text{ (J/kg)}$$

Power input; $P = \dot{m} w = \dot{m} u_2 \mathcal{V}_{w_2}$ (W)

If the air flow through the impeller blade is radial (ideal case), the velocity diagram at the outlet takes the shape as shown in Fig. 26.11.

In this case, the blade velocity at the outlet becomes equal to whirl velocity at the outlet, i.e., $u_2 = \mathcal{V}_{w_2}$. The work input per kg of air is

$$w = u_2^2 \text{ (J/kg)} \qquad ...(26.21)$$

The exit whirl velocity $\mathcal{V}_{w_2}$ of air cannot be greater than the blade tip velocity. Thus, it is the limiting case and it is the *maximum work supplied* to air per kg.

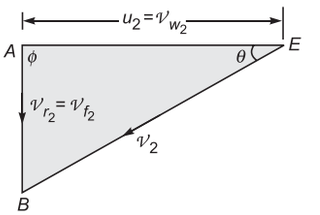

Fig. 26.11 *Velocity triangle at outlet for ideal radial air flow*

26.5.7 Losses and Efficiency of Centrifugal Compressor

The following losses occur in a centrifugal compressor, when air flows through the impeller:

1. Friction between moving air layers and impeller blades and friction between air layers moving with relative velocities,
2. Shock at entry, and
3. Turbulence caused in air.

These losses cause an increase in enthalpy of air without increasing the pressure of air. Therefore, the actual temperature of air coming out of the compressor is more than the temperature of air at the inlet. The actual work input for the same pressure ratio is more due to irreversibilities. The actual and isentropic compression for pressure ratio is shown in Fig. 26.12.

Since the cooling arrangement is not provided in dynamic compressors, the ideal compression

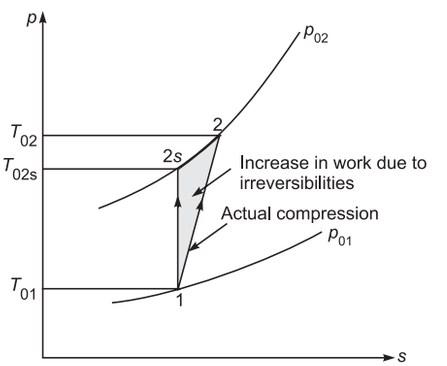

Fig. 26.12 *Actual and isentropic compression on a T–s diagram*

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process in dynamic compressors is *isentropic compression*. But the actual work input for compression is always more than isentropic work input for compression through same pressure ratio. The *isentropic efficiency* (η_{isen}) of a dynamic compressor is given by

$$\eta_{isen} = \frac{\text{Isentropic work input}}{\text{Actual work input}} = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}} \quad ...(26.22)$$

If specific heat C_p of air remains constant then

$$\eta_{isen} = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}} \quad ...(26.23)$$

If inlet velocity of air is equal to air exit velocity, i.e., $\mathcal{V}_1 = \mathcal{V}_2$ then

$$\eta_{isen} = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{\text{Isentropic temperature rise}}{\text{Actual temperature rise}} \quad ...(26.24)$$

where suffix 1 indicates the inlet state, 2s indicates the state after isentropic expansion, 2; the state after actual expansion, and the properties with suffix 0 indicates corresponding stagnation properties.

26.5.8 Slip and Slip Factor

A centrifugal compressor has maximum work input when $u_2 = \mathcal{V}_{w_2}$. In actual practice, the whirl velocity $\mathcal{V}_{w_2}$ is always less than the blade tip velocity u_2. The difference between blade tip velocity u_2 and whirl velocity $\mathcal{V}_{w_2}$ is known as *slip*, i.e., slip $= u_2 - \mathcal{V}_{w_2}$.

The *slip factor* is defined as the ratio of whirl velocity to blade tip velocity. It is designated as ϕ_s and expressed as

$$\text{Slip factor,} \quad \phi_s = \frac{\mathcal{V}_{w_2}}{u_2} \quad ...(26.25)$$

26.5.9 Work Factor and Pressure coefficient

The actual work done per kg of air by compressor is always greater than the impeller work $u_2\mathcal{V}_{w_2}$ due to fluid friction and windage losses. *Work factor* or *power input factor* is defined as ratio of actual work input to Euler work input. It is designated as ϕ_w and is given as

$$\phi_w = \frac{\text{Actual work input}}{\text{Euler work input}} = \frac{C_p(T_{02} - T_{01})}{u_2\mathcal{V}_{w_2}} \quad ...(26.26)$$

Pressure coefficient is defined as the ratio of isentropic work to Euler work. It is designated as ϕ_p.

$$\phi_p = \frac{\text{Isentropic work input}}{\text{Euler work input}} = \frac{C_p(T_{02s} - T_{01})}{u_2\mathcal{V}_{w_2}} \quad ...(26.27)$$

Using Eq. (26.23), and assuming radial vanes of impeller ($u_2 = \mathcal{V}_{w_2}$), then

$$\phi_p = \frac{\eta_{isen}C_p(T_{02} - T_{01})}{u_2^2} \quad ...(26.28)$$

From Eq. (26.26), we get

$$C_p(T_{02} - T_{01}) = \phi_w u_2 \mathcal{V}_{w_2}$$

Substituting the $\mathcal{V}_{w_2}$ as $\phi_s u_2$; from Eq. (26.25), we get

$$C_p(T_{02} - T_{01}) = \phi_w \phi_s u_2^2$$

Using in Eq. (26.28), we get;

$$\phi_p = \frac{\eta_{isen}\phi_w\phi_s u_2^2}{u_2^2} = \phi_w\, \phi_s\, \eta_{isen} \quad ...(26.29)$$

26.5.10 Effect of Impeller Blade Shape on Compressor Performance

There are usually three types of impeller blade shapes used in a centrifugal compressor. These are

1. Backward curved blades ($\theta < 90°$)
2. Radial blades ($\theta = 90°$)
3. Forward curved blades ($\theta > 90°$)

Figure 26.13 shows the geometry of backward, radial and forward curved vanes and performance of these vanes. The centrifugal action on the curved vanes creates bending moment and induces bending stresses.

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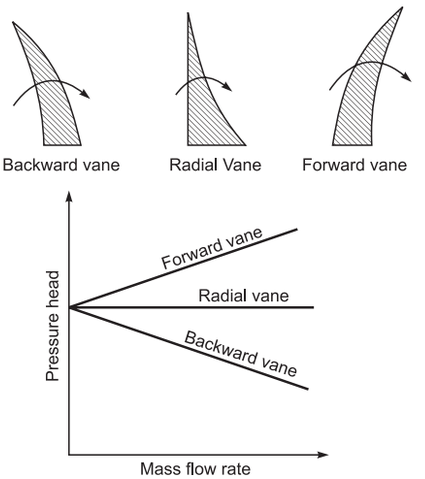

Fig. 26.13 *Charecteristic of backward, radial and forward curved vanes*

The pressure head of delivered air decreases with increase in mass-flow rate for backward curved vanes, while the pressure head increases for forward curved vanes. But for radial curved vanes, the pressure head of delivered air remains constant with mass-flow rate.

The backward vanes are normally used with $\theta = 20°$ to $25°$, except for delivery of air at high head. The radial vane is the compromise between backward and forward curved vanes. Therefore, the radial vane impeller is most commonly used in a centrifugal compressor due to the following reasons:

1. Radial-vane geometry is simple, thus vanes can be manufactures easily.
2. Radial vanes have lowest bending stress for given diameter and speed as compared to forward and backward curved vanes.
3. Radial vanes have a constant pressure head in the impeller as well as in the diffuser.
4. A radial-vane impeller has good efficiency and high pressure head.

26.5.11 Diffuser System

In a centrifugal compressor, the diffuser converts kinetic energy of air into static pressure head. For a radial vane impeller, the diffuser contributes about one half of the overall static pressure rise.

A diffuser consists of curved vanes, which are used to minimize the whirl of high speed air with smallest possible flow path and diameter. The flow of air through the diffuser vanes may be approximated as *logarithmic spiral path.* For fast diffusion, the axis of vanes is straight and tangential to spiral. Usually, the number of diffuser passages are less than the impeller passages for more uniform and smooth flow.

Figure 26.14 shows a typical curved vanes diffuser along with an impeller. The clearance provided between the impeller and diffuser rings acts as a vaneless diffuser and it functions to

1. smooth out velocity variation betweem the impeller tip and diffuser vanes
2. reduce circumferential pressure gradient at the impeller tip
3. reduce the velocity at the entry of vanes

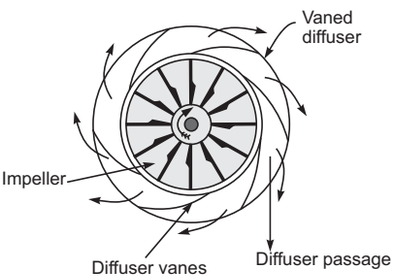

Fig. 26.14 *Typical vaned diffuser*

26.5.12 Choking and Surging

Choking At constant impeller speed, the decrease in pressure ratio leads to an increase in mass-flow rate and hence the density of compressed air is decreased. Consequently, the radial velocity of air increases, which increases the absolute velocity of air at impeller exit and incidence angle at diffuser vane tip. The slope of the characteristic curve decreases and finally the point A is reached as shown in Fig. 26.15. The mass-flow rate of fluid cannot be increased beyond the point A. This point is called *choking state.*

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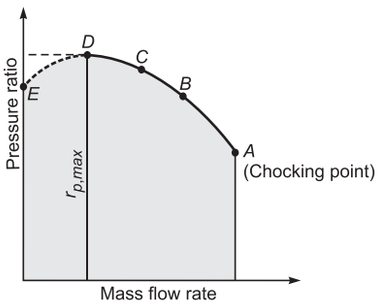

Fig. 26.15 *Typical characteristic of compressor at constant speed*

Surging Consider a compressor is running at constant speed and the full valve is open on delivery side at a pressure ratio located by the point *B* on the characteristic curve shown in Fig. 26.15. If any resistance is placed in the delivery path of a compressor, or by partial closing of the valve, the mass-flow rate decreases with increase in pressure ratio, and the operating point will shift towards left on characteristic curve, say at the point *C* from the point *B*.

If resistance is further increased in the delivery path, the mass-flow rate of air through the compressor decreases and the compressor operating point will shift toward left till it reaches the point *D*, the operating point for maximum pressure ratio.

If resistance is further increased, the mass-flow rate will decrease and reach the zone *D–E* on the curve, with decrease in pressure ratio also. In this situation, the pressure in downstream line will be more than the pressure of air at the actual delivery of the compressor. This situation leads to a stop of fluid flow and sometimes, flow of fluid in reverse direction.

Within a short interval of time, the pressure is built up within the compressor due to accumulation of mass; and the compressor may again start to deliver the fluid. If downstream conditions remain unchanged then fluid flow will again break down, as pressure of delivered air is subsidized and the cycle will be repeated with high frequency. This situation of instability is known as *surging* or *pumping*.

Surging in the line causes unstable compressor operation. *Surging* occurs when the operating points of the compressor get into the unstable area of the operating curve. During *surging,* the compressor shows cyclic flow and back-flow of the compressed air resulting into high vibrations, pressure shocks and over heating. The breakdown of flow due to persistent surging may lead to heavy damages.

Effect of Surging Consequences of surging can include

1. Rapid flow and pressure oscillations causing process instabilities
2. Rising temperatures inside the compressor
3. Tripping of the compressor
4. Mechanical damage

   Mechanical damage can include

   - √ Radial bearing load during the initial phase of surging
   - √ Thrust bearing load due to loading and unloading
   - √ Seal rubbing
   - √ Stationary and rotating part contact, if thrust bearing is overloaded

26.5.13 Comparison between Reciprocating and Centrifugal Compressors

Sr. No.	*Reciprocating Compressor*	*Centrifugal Compressor*
1.	Geater noise and vibrations	Compratively salient operation
2.	Poor mechanical efficiency due to large sliding parts	Better mechanical efficiency due to absence of sliding parts
3.	Installation cost is higher	Installation cost is lower
4.	Pressure ratio up to 5 to 8	Pressure ratio up to 4

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5.	Higher pressure ratio up to 500 atm. is possible with multistaging of compressor	It is not suitable for multistaging
6.	It runs intermittantly and delivers pulsating air	It runs continuously and delivers steady and pulsating free air
7.	Less volume is handled	Large volume is handled
8.	More maintenance is required	Less maintenance is required
9.	Weight of compressor is more	Comparatively less weight
10.	It operates at low speed	It operates at high speed
11.	Isothermal efficiency should be better	Isentropic efficiency should be better
12.	Higher compression efficiency at pressure ratio more than 2	Higher compression efficiency, if pressure ratio less than 2
13.	Suitable for low discharge and high pressure ratio	Suitable for high discharge and low pressure ratio

Example 26.5 *In a centrifugal compressor, the air enters at 27°C and leaves at 105°C. The air is compressed through a pressure ratio of 2. Calculate the isentropic efficiency and power required by the compressor, if 30 kg of air is compressed per minute. Take C_p = 1.00 kJ/kg·K and C_v = 0.716 kJ/kg·K.*

Solution

Given A centrifugal compressor with

$\dot{m}$ = 30 kg/min = 0.5 kg/s

$\frac{p_2}{p_1}$ = 2.0

T_1 = 27°C = 300 K

T_2 = 105°C = 378 K

C_p = 1.00 kJ/kg·K

C_v = 0.716 kJ/kg·K

To find

(i) Isentropic efficiency of the compressor, and

(ii) Power required to run the compressor.

Analysis The ratio of two specific heats

$$\gamma = \frac{C_p}{C_v} = \frac{1.00}{0.716} = 1.4$$

The temperature of air after isentropic compression

$$T_{2s} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (300\text{ K}) \times \left(\frac{2}{1}\right)^{\frac{1.4-1}{1.4}}$$

$$= 365.7\text{ K}$$

(i) Isentropic efficiency is given by;

$$\eta_{isen} = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{\text{Isentropic temperature rise}}{\text{Actual temperature rise}}$$

$$= \frac{365.7 - 300}{378 - 300} = 0.8423 \quad \text{or} \quad \mathbf{84.23\%}$$

(ii) The power input for compression

$$P = \dot{m}C_p(T_2 - T_1) = 0.5 \times 1.0 \times (378 - 300)$$

$$= \mathbf{39\ kW}$$

Example 26.6 *A centrifugal compressor compresses air from 1 bar at 15°C to 2.15 bar, 95°C. The mass of air delivered is 2.2 kg/s and no heat is added to the air from external sources during compression. Find the efficiency of the compressor relative to ideal adiabatic compression and estimate the power absorbed. Also, find the change in entropy of air during compression.*

Solution

Given A centrifugal compressor with

$\dot{m}$ = 2.2 kg/s $\qquad$ p_1 = 1 bar

T_1 = 15°C = 288 K $\qquad$ p_2 = 2.15 bar

T_2 = 95°C = 368 K $\qquad$ Q = 0

To find

(i) Isentropic efficiency of the compressor,

(ii) Power required to run the compressor,

(iii) Entropy change during compression process.

Assumptions

For air C_p = 1.00 kJ/kg·K

γ = 1.4 and R = 0.287 kJ/kg·K

Analysis The temperature of air after isentropic compression

$$T_{2s} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (288\text{ K}) \times \left(\frac{2.15}{1}\right)^{\frac{1.4-1}{1.4}}$$

$$= 358.4\text{ K}$$

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(i) *Isentropic efficiency* is given by

$$\eta_{isen} = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{358.4 - 288}{368 - 280}$$
$$= 0.880 \quad \text{or} \quad \mathbf{80\%}$$

(ii) *The power input for compression*

In absence of change in kinetic and potential energies and heat transfer, the steady-flow energy equation for compressor leads to

$$w_{in} = h_2 - h_1 = C_p(T_2 - T_1)$$
$$= 1.005 \times (358.4 - 288)$$
$$= 70.75 \text{ kJ/kg}$$

The power input

$$P = \dot{m} w_{in}$$
$$= 2.2 \times 70.75 = \mathbf{155.65\ kW}$$

(iii) *Entropy change during compression process:*

The entropy change during a process can be obtained as

$$\Delta S = \dot{m}\left[C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)\right]$$
$$= 2.2 \times \left[1.005 \times \ln\left(\frac{368}{288}\right) - 0.287 \times \ln\left(\frac{2.15}{1}\right)\right]$$
$$= \mathbf{0.586\ kJ/K}$$

Example 26.7 *A centrifugal compressor running at 2000 rpm has internal and external diameters of the impeller as 300 mm and 500 mm, respectively. The blade angles at inlet and outlet are 22° and 40°, respectively. The air enters the impeller radially. Determine the work done by the compressor per kg of air and degree of reaction.*

Solution

Given A centrifugal compressor with

$N = 2000$ rpm $\qquad D_1 = 300$ mm $= 0.3$ m

$\beta = 22°$ $\qquad D_2 = 500$ mm $= 0.5$ m

$\phi = 40°$

To find

(i) Work input to the compressor,

(ii) Degree of reaction.

Analysis The linear velocity of impeller at inlet

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.3 \times 2000}{60}$$
$$= 31.416 \text{ m/s}$$

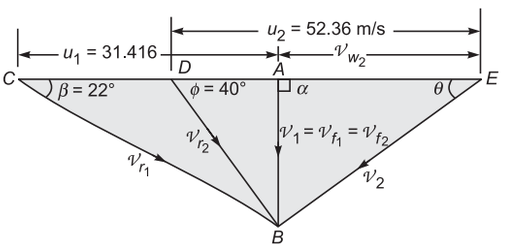

Fig. 26.16 *Combined velocity triangle for centrifugal compressor*

The linear velocity of impeller at the outlet

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 2000}{60} = 52.36 \text{ m/s}$$

With the use of blade velocities and blade angles at the inlet and outlet, the velocity triangle can be constructed as follows:

(i) The horizontal line *CA* represents the blade velocity $u_1 = 31.416$ m/s at inlet.

(ii) Draw line *CB* inclined at 22°.

(iii) From the point *D*, draw line *DB* inclined at angle $\phi = 40°$; which cuts line *CB* at point *B*.

(iv) Draw a vertical line *AB* to represent the flow velocity.

(v) Draw the line *DE* to represent blade velocity $u_2 = 52.36$ m/s.

(vi) Join the line *EB* to represent exit velocity of air $\mathcal{V}_2$.

From the velocity triangles, the measurements give

$\mathcal{V}_{w_2}$ = Length of *AE* = 37.3 m/s,

$\mathcal{V}_{r_1}$ = Length of *CB* = 34.1 m/s,

$\mathcal{V}_{r_2}$ = Length of *DB* = 19.9 m/s.

(i) Work input per kg of air

$$w = \mathcal{V}_{w_2} u_2 = 37.3 \times 52.36 = 1953.02 \text{ J/kg}$$
$$= \mathbf{1.953\ kJ/kg}$$

(ii) Degree of reaction;

$$R_d = \frac{\text{Pressure rise in the impeller}}{\text{Pressure rise in the compresso}}$$
$$= \frac{(u_2^2 - \mathcal{V}_{r_2}^2) + (\mathcal{V}_{r_1}^2 - u_1^2)}{2u_2\mathcal{V}_{w_2}}$$
$$= \frac{(52.36^2 - 19.9^2) + (34.1^2 - 31.416^2)}{2 \times 52.36 \times 37.3}$$
$$= 0.6455 \quad \text{or} \quad \mathbf{64.55\%}$$

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Example 26.8 *A centrifugal compressor running at 1440 rpm, handles air at 101 kPa and 20°C and compresses it to a pressure of 6 bar isentropically. The inner and outer diameters of the impeller are 14 cm and 28 cm, respectively. The width of the blade at the inlet is 2.5 cm. The blade angles are 16° and 40° at entry and exit. Calculate mass-flow rate of air, degree of reaction, power input and width of blades at outlet.*

Solution

Given A centrifugal compressor with

$N = 1440$ rpm $\quad D_1 = 14$ cm $= 0.14$ m

$\beta = 16°$ $\quad D_2 = 28$ cm $= 0.28$ m

$\phi = 40°$ $\quad p_1 = 101$ kPa

$p_2 = 6$ bar $= 600$ kPa $\quad T_1 = 20°C = 293$ K

$B_1 = 2.5$ cm $= 0.025$ m

To find

(i) Mass flow rate of air,

(ii) Power input to the compressor,

(iii) Degree of reaction, and

(iv) Width of blade at outlet.

Assumption The specific gas constant as 0.287 kJ/kg·K.

Analysis The linear velocity of impeller at the inlet

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.14 \times 1440}{60} = 10.56 \text{ m/s}$$

The linear velocity of impeller at the outlet

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.28 \times 1440}{60} = 21.12 \text{ m/s}$$

With the use of blade velocities and blade angles at inlet and outlet, the velocity triangle can be constructed as follows:

From the velocity triangles, the measurements give

$\mathcal{V}_{w_2}$ = Length of $AE = 17.5$ m/s

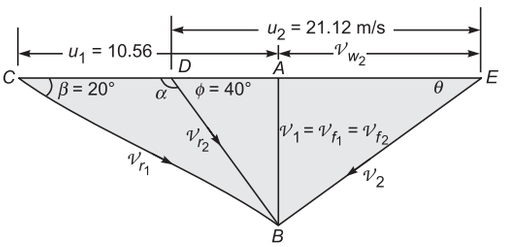

Fig. 26.17 *Combined velocity triangle for centrifugal compressor*

$\mathcal{V}_{r_1}$ = Length of $CB = 11$ m/s

$\mathcal{V}_{r_2}$ = Length of $DB = 4.7$ m/s

$\mathcal{V}_{f_1} = 3$ m/s

Work input per kg of air

$$w = \mathcal{V}_{w_2} u_2 = 17.5 \times 21.12 = 369.6 \text{ J/kg}$$

(i) *Mass-flow rate of air*

$$\dot{m} = \rho_1 A_1 \mathcal{V}_{f_1} = \rho_1 \pi D_1 B_1 \mathcal{V}_{f_1}$$

Density of air

$$\rho = \frac{p_1}{RT_1} = \frac{101}{0.287 \times 293}$$

$$= 1.201 \text{ kg/m}^3$$

$$\therefore \dot{m} = 1.201 \times \pi \times 0.14 \times 0.025 \times 3$$

$$= \mathbf{0.0396 \text{ kg/s}}$$

(ii) *Power input to run the compressor*

$$P = \dot{m} w = 0.0396 \times 369.6 = \mathbf{14.61 \text{ W}}$$

(iii) *Degree of reaction*

$$R_d = \frac{(u_2^2 - \mathcal{V}_{r_2}^2) + (\mathcal{V}_{r_1}^2 - u_1^2)}{2 u_2 \mathcal{V}_{w_2}}$$

$$= \frac{(21.12^2 - 4.7^2) + (11^2 - 10.56^2)}{2 \times 17.5 \times 21.12}$$

$$= 0.5864 \quad \text{or} \quad \mathbf{58.64\%}$$

(iv) *Width of blade at outlet*

Flow velocity at outlet

$$\mathcal{V}_{f_2} = \mathcal{V}_{f_1} = 3.0 \text{ m/s}$$

Temperature of air at the outlet

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} = 293 \times \left(\frac{600}{101}\right)^{\frac{1.4-1}{1.4}}$$

$$= 487.48 \text{ K}$$

Density of air at the outlet

$$\rho_2 = \frac{p_2}{RT_2} = \frac{600}{0.287 \times 487.48}$$

$$= 4.288 \text{ kg/m}^3$$

From continuity equation;

$$\dot{m} = \rho_2 \pi D_2 B_2 \mathcal{V}_{f_2}$$

or $$B_2 = \frac{\dot{m}}{\rho_2 \pi D_2 \mathcal{V}_{f_2}} = \frac{0.0396}{4.288 \times \pi \times 0.28 \times 3}$$

$$= 0.0035 \text{ m} = \mathbf{3.5 \text{ mm}}$$

Example 26.9 *A centrifugal compressor runing at 12000 rpm delivers 600 m^3/min of free air. The air is compressed from 1 bar and 27°C to a pressure ratio of 4 with an isentropic efficiency of 85%. The blades are radial at the impeller outlet and flow velocity of 60 m/s*

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may be assumed throughout constant. The outer radius of the impeller is twice the inner one and slip factor is 0.9. Calculate

(a) Final temperature of air
(b) Power input to compressor
(c) Impeller diameter at inlet and outlet
(d) Width of impeller at inlet

Solution

Given A centrifugal compressor with

$N = 12000$ rpm $\quad r_2 = 2\,r_1$

$p_1 = 1$ bar $= 100$ kPa $\quad T_1 = 27°C = 300$ K

$\frac{p_2}{p_1} = 4 \quad \mathcal{V}_{f_1} = 60$ m/s

$\eta_{isen} = 0.85$

$\dot{V} = 600$ m^3/min $= 10$ m^3/s

$\phi_s = 0.9$

To find

(i) Final temperature of air,
(ii) Power input to the compressor,
(iii) Impeller diameter at inlet and outlet, and
(iv) Width of impeller at inlet.

Assumption The specific heat of air as 1005 J/kg·K and $\gamma = 1.4$

Analysis

(i) *Final temperature of air*

$$T_{2s} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 300 \times (4)^{\frac{1.4-1}{1.4}} = 445.79 \text{ K}$$

The isentropic efficiency is given by

$$\eta_{isen} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

Final temperature of air is given by

$$T_2 = \frac{T_{2s} - T_1}{\eta_{isen}} + T_1$$

$$= \frac{445.79 - 300}{0.85} + 300$$

$$= 471.52 \text{ K} \quad \text{or} \quad \mathbf{198.52°C}$$

The mass flow rate of air

$$\dot{m} = \frac{p\dot{V}}{RT} = \frac{100 \times 10}{0.287 \times 300} = 11.61 \text{ kg/s}$$

(ii) *Power input to compressor*

Work input per kg of air

$$w = C_p(T_2 - T_1) = 1.005 \times (471.52 - 300)$$

$$= 172.37 \text{ kJ/kg}$$

Power input

$$P = \dot{m}w = 11.61 \times 172.37 = \mathbf{2001.3\ kW}$$

(iii) *Impeller diameters at inlet and outlet*

For radial blades, the work input to compressor with slip is given by

$$w = \phi_s u_2^2 \text{ (J/kg)}$$

Using numerical values and equating with work obtained above

$$0.9 \times u_2^2 = 172.37 \times 10^3 \text{ (J/kg)}$$

It gives $\quad u_2 = 437.63$ m/s

The linear blade velocity at impeller tip is given by

$$u_2 = \frac{\pi D_2 N}{60}$$

It gives impeller diameter at outlet

$$D_2 = \frac{437.63 \times 60}{\pi \times 12000}$$

$$= 0.6965 \text{ m} \quad \text{or} \quad \mathbf{69.65\ cm}$$

Impeller diameter at inlet

$$D_1 = \frac{D_2}{2} = \frac{69.65}{2}$$

$$= \mathbf{34.825\ cm}$$

(iv) Width of impeller at inlet, Eq. (26.12)

$$B_1 = \frac{\dot{m}v_1}{\pi D_1 \mathcal{V}_{f_1}} = \frac{\dot{V}}{\pi D_1 \mathcal{V}_{f_1}}$$

$$= \frac{10 \text{ m}^3/\text{s}}{\pi \times 0.34825 \times 60}$$

$$= 0.1523 \text{ m} \quad \text{or} \quad \mathbf{15.23\ cm}$$

Example 26.10 *A centrifugal compressor handles 600 kg/min of air.The ambient air conditions are 1 bar and 27°C. The compressor runs at 18000 rpm with an isentropic efficiency of 80%. The air is compressed in the compressor from 1 bar static pressure to 4 bar total pressure. The air enters the impeller eye with a velocity of 150 m/s with no prewhirl. Take the ratio of whirl speed to tip speed as 0.9. Calculate*

(a) rise in total temperature during compression if change in kinetic energy is negligible
(b) tip diameter of impeller
(c) power required
(d) Eye diameter, if hub diameter is 10 cm

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Solution

Given A centrifugal compressor with

$N = 18000$ rpm, $D_h = 10$ cm $= 0.1$ m

$p_1 = 1$ bar $= 100$ kPa, $T_1 = 27°C = 300$ K

$p_{02} = 4$ bar $= 400$ kPa, $\mathcal{V}_1 = 150$ m/s

$\dfrac{\mathcal{V}_{w2}}{u_2} = 0.9$, $\eta_{isen} = 0.8$

$\dot{m} = 600$ kg/min $= 10$ kg/s, $\mathcal{V}_{w_1} = 0$

To find

(i) Rise in total temperature of air,

(ii) Tip diameter of the impeller,

(iii) Power input, and

(iv) Eye diameter.

Assumption The specific heat of air as 1005 J/kg·K and $\gamma = 1.4$

Analysis

(i) *Rise in total temperature of air*

Stagnation temperature at the inlet of compressor

$$T_{01} = T_1 + \frac{\mathcal{V}_1^2}{2C_p} = 300 + \frac{150^2}{2\times 1005}$$

$$= 311.19 \text{ K}$$

The stagnation pressure at compressor inlet

$$p_{01} = p_1\left(\frac{T_{01}}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= (1 \text{ bar})\times\left(\frac{311.19}{300}\right)^{\frac{1.4}{1.4-1}}$$

$$= 1.137 \text{ bar}$$

Stagnation temperature after isentropic compression

$$T_{02s} = T_{01}\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 311.19\times\left(\frac{4}{1.137}\right)^{\frac{1.4-1}{1.4}}$$

$$= 445.79 \text{ K}$$

Isentropic rise in stagnation temperature

$$= T_{02s} - T_{01}$$

$$= 445.79 - 311.19 = 134.6°C$$

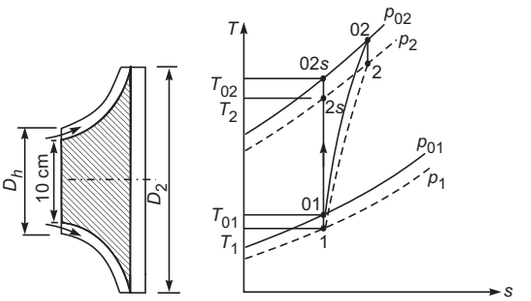

Fig. 26.18 *Schematic*

The isentropic efficiency in terms of stagnation temperatures may be given as

$$\eta_{isen} = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}}$$

Actual rise in stagnation temperature is given by

$$T_{02} - T_{01} = \frac{T_{02s} - T_{01}}{\eta_{isen}} = \frac{134.6}{0.8}$$

$$= \mathbf{168.25°C}$$

(ii) *Tip diameter of impeller, D_2*

Work input to compressor per kg of air

$$w = C_p(T_{02} - T_{01}) = 1.005 \times 168.25$$

$$= 169.09 \text{ kJ/kg}$$

Work input per kg of air to compressor is also given by

$$w = u_2\mathcal{V}_{w_2} \text{ (J/kg)}$$

or $169.09\times 10^3 = 0.9\, u_2^2$ $\left(\because \dfrac{\mathcal{V}_{w_2}}{u_2} = 0.9\right)$

or $u_2 = 433.45$ m/s

The blade velocity is given by

$$u_2 = \frac{\pi D_2 N}{60}$$

or $$D_2 = \frac{u_2\times 60}{\pi N} = \frac{433.45\times 60}{\pi\times 18000}$$

$$= 0.4599 \text{ m} \approx \mathbf{46 \text{ cm}}$$

(iii) *Power input to compressor*

$$P = \dot{m}w = 10\times 169.09 = \mathbf{1690.9 \text{ kW}}$$

(iv) *Eye diameter, D_1*

The density of air at compressor inlet is given by

$$\rho_1 = \frac{p_1}{RT_1} = \frac{100}{0.287\times 300}$$

$$= 1.161 \text{ kg/m}^3$$

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The mass-flow rate through impeller eye can be given by

$$\dot{m} = \frac{\pi}{4}\left(D_1^2 - D_h^2\right) \times \mathcal{V}_1 \times \rho_1$$

or $$10 = \frac{\pi}{4}\left(D_1^2 - 0.1^2\right) \times 150 \times 1.161$$

or $$D_1 = \sqrt{\frac{10 \times 4}{\pi \times 150 \times 1.161} + 0.1^2}$$

$$= 0.288 \text{ m} = \mathbf{28.8 \text{ cm}}$$

Example 26.11 *An aircraft engine is fitted with a single-sided centrifugal compressor. The aircraft flies with a speed of 900 km/h at an altitude, where the pressure is 0.23 bar and temperature is 217 K. The inlet duct of the impeller eye contains fixed vanes, which gives the air prewhirl of 25° at all radii. The inner and outer diameter of the eye are 180 and 330 mm, respectively. The diameter of the impeller tip is 540 mm and rotational speed is 15000 rpm. Estimate the stagnation pressure at compressor outlet when the mass flow rate is 210 kg/minute.*

Neglect losses in inlet duct and fixed vanes, and assume that the isentropic efficiency of the compressor is 80%. Take slip factor as 0.9 and power input factor as 1.04.

Solution

Given A single-sided centrifugal compressor of an airfraft with

$N = 15000$ rpm $\quad D_h = 180$ mm $= 0.15$ m

$D_1 = 330$ mm $= 0.33$ m $\quad D_2 = 540$ mm $= 0.54$ m

$p_1 = 0.23$ bar $= 23$ kPa $\quad T_1 = 217$ K

$\mathcal{V}_1 = 900$ km/h $\quad \eta_{isen} = 0.8$

$\dot{m} = 210$ kg/min $= 3.5$ kg/s

$$\phi_s = \frac{\mathcal{V}_{w_2}}{u_2} = 0.9 \qquad \phi_w = 1.04$$

To find Stagnation pressure at compressor outlet.

Assumption The specific heat of air as 1005 J/kg·K and $\gamma = 1.4$

Analysis The velocity of air with reference to aircraft

$$\mathcal{V}_1 = \frac{900 \times 1000}{3600} = 250 \text{ m/s}$$

Stagnation temperature at the inlet of compressor

$$T_{01} = T_1 + \frac{\mathcal{V}_1^2}{2C_p} = 217 + \frac{250^2}{2 \times 1005} = 248.1 \text{ K}$$

The stagnation pressure at compressor inlet

$$p_{01} = p_1\left(\frac{T_{01}}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= (0.23 \text{ bar}) \times \left(\frac{248.1}{217}\right)^{\frac{1.4}{1.4-1}}$$

$$= 0.3675 \text{ bar}$$

The blade velocity at exit is given by

$$u_2 = \frac{\pi D_2 N}{60}$$

or $$u_2 = \frac{\pi \times 0.54 \times 15000}{60} = 424.11 \text{ m/s}$$

The whirl velocity at exit

$$\mathcal{V}_{w_2} = \phi_s u_2 = 0.9 \times 424.11 = 381.7 \text{ m/s}$$

The power input factor is given as

$$\phi_w = \frac{C_p(T_{02} - T_{01})}{u_2 \mathcal{V}_{w_2}}$$

The stagnation temperature at exit

$$T_{02} = \frac{\phi_w u_2 \mathcal{V}_{w_2}}{C_p} + T_{01}$$

$$= \frac{1.04 \times 424.11 \times 381.7}{1005} + 248.1$$

$$= 415.62 \text{ K}$$

The isentropic efficiency in terms of stagnation temperatures may be given as

$$\eta_{isen} = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}}$$

Isentropic stagnation temperature at exit is given by

$$T_{02s} = \eta_{isen}(T_{02} - T_{01}) + T_{01}$$

$$= 0.8 \times (415.62 - 248.1) + 248.1$$

$$= 382.11 \text{ K}$$

Stagnation pressure after compression

$$p_{02} = p_{01}\left(\frac{T_{02s}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= 0.3675 \times \left(\frac{382.11}{248.1}\right)^{\frac{1.4}{1.4-1}}$$

$$= \mathbf{1.667 \text{ bar}}$$

26.6 AXIAL COMPRESSOR

Axial compressors are aerofoil (blade) based rotary compressors. The gas flows parallel to the axis of rotation in axial flow compressors and gas

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is continuously compressed. The several rows of aerofoil blades are used to achieve large pressure rise in the compressor.

The axial compressors are generally multi-stage machines; each stage can give a pressure ratio of 1.2 to 1.3. The axial flow compressors are suitable for higher pressure ratios and are generally more efficient than radial compressors.

Axial compressors are widely used in gas turbine plants and small power stations. They are also used in industrial applications such as blast-furnace air, large-volume air-separation plants, and propane dehydrogenation. Axial compressors are also used for supercharging. They are also used to boost the power of automotive reciprocating engines by compressing the intake air.

26.6.1 Construction

An axial air compressor consists of a large number of rotating blade rows, fixed on a rotating drum, and stator (fixed) blade rows fixed on the casing of the compressor as shown in Fig. 26.19. A pair of rotating and stationary blades is called a *stage*.

The moving blades act as a series of fans and the fixed blades act as guide vanes and diffuser. The moving blades are imparting energy into the fluid, and the fixed blades convert a part of kinetic energy of the fluid into pressure energy through diffusion-process and then guide and redirect the fluid onto the next stage of moving blades without shock.

The blades are made in aerofoil section to reduce the losses caused by shocks, turbulence and boundary separation. The annular area for air flow is gradually reduced from the inlet to the outlet of the compressor. The rotor of an axial compressor is made aerodynamic.

26.6.2 Working

The work input to a rotor shaft is transferred by moving blades to air, thus accelerating the air flow. The spaces between moving blades and casing form the diffuser passages, and thus the velocity of air decreases as air passes through them and results into increase in pressure and enthalpy. The air is then further diffused in stator blades which are also arranged to form diffuser passages. The fixed blades also guide the air to flow at an angle for smooth entry of next row of moving blades.

The temperature rise of air is almost same in moving as well as in fixed blades and axial velocity of air remains constant throughout the compressor.

26.6.3 Velocity Triangles for Axial Flow Compressor

The velocity triangles at inlet and outlet of moving blades of an axial-flow air compressor is

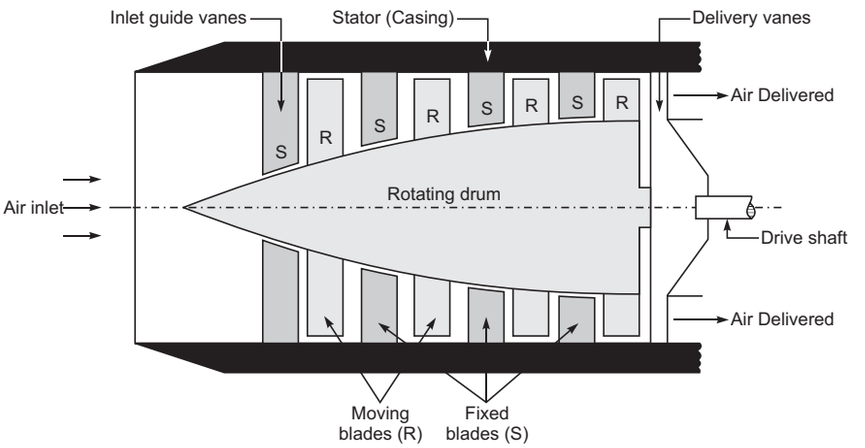

Fig. 26.19 *Axial-flow compressor*

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shown in Fig. 26.20. The following points should be considered for the construction of a velocity triangle for an axial-flow compressor.

1. The blade velocity remains same at inlet and outlet, i.e., $u_1 = u_2 = u$.
2. Flow velocity also remains constant, i.e., $\mathcal{V}_{f_1} = \mathcal{V}_{f_2}$.
3. Relative velocity at outlet is less than that at inlet, i.e., $\mathcal{V}_{r_2} < \mathcal{V}_{r_1}$.
4. Both the whirl velocity components lie in the same plane.

The work input per stage per kg of air

$$w = u(\mathcal{V}_{w_2} - \mathcal{V}_{w_1})$$
$$= C_p(T_{02} - T_{01}) \quad ...(26.30)$$

Power required to drive the compressor

$$P = \dot{m}\, w = \dot{m}\, u\, (\mathcal{V}_{w_2} - \mathcal{V}_{w_1}) \quad ...(26.31)$$

From inlet and outlet velocity triangles, we get

$$\mathcal{V}_{w_1} = \mathcal{V}_{f_1} \tan \alpha$$

and $$\mathcal{V}_{w_2} = \mathcal{V}_{f_2} \tan \theta \quad ...(26.32)$$

given that $\mathcal{V}_{f_1} = \mathcal{V}_{f_2} = \mathcal{V}_f$ (say)

Therefore, work input to axial flow compressor can also be given by

$$w = u\mathcal{V}_f(\tan \theta - \tan \alpha) \quad ...(26.33)$$

From Eqs. (26.30) and (26.33), we get

$$u(\mathcal{V}_{w_2} - \mathcal{V}_{w_1}) = u\mathcal{V}_f(\tan \theta - \tan \alpha) \quad ...(26.34)$$

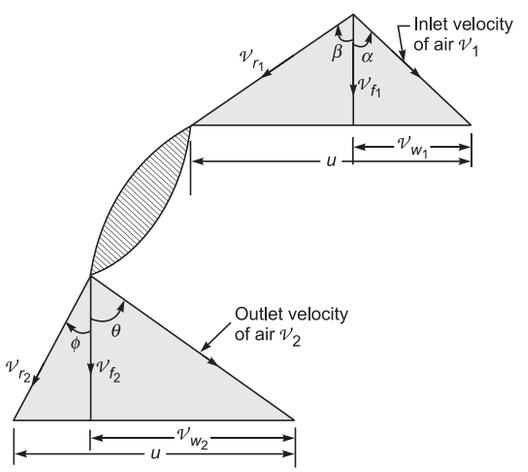

Fig. 26.20 *Inlet and out velocity triangles for axial compressor*

26.6.4 Combined Velocity Triangles for Axial Flow Compressor

Since the blade speed u and flow velocity $\mathcal{V}_f$ remain constant at the inlet and outlet, thus the combined velocity triangle can be constructed as below.

1. Draw the line AB to represent the blade velocity u.
2. Through the point A, draw a line at an inclination of $90° - \alpha$.
3. Through the point B, draw a line at an inclination of $90° - \beta$.
4. The above two lines intersect at the point C, line AC represents the air inlet velocity $\mathcal{V}_1$ and line BC represents relative velocity of air $\mathcal{V}_{r_1}$ at inlet.
5. The vertical line CD through the point C represents the velocity $\mathcal{V}_{f_1}$ at inlet.
6. Through the point B draw a line at an inclination of $90° - \phi$.
7. Along the inclined line, locate the point E with line EF, representing velocity $\mathcal{V}_{f_2} (= \mathcal{V}_{f_1})$ at the outlet.
8. Join the line AE to represent absolute velocity $\mathcal{V}_2$ of air at the outlet.

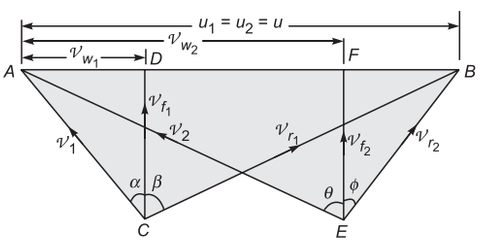

Fig. 26.21 *Combined velocity triangle for axial flow compressor*

26.6.5 Degree of Reaction

As discussed earlier, the degree of reaction is defined as the ratio of static pressure rise in rotor to the total static pressure rise in the compressor. Mathematically,

Degree of reaction

$$R_d = \frac{\text{Pressure rise in rotor blade}}{\text{Pressure rise in compressor}}$$

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$$\text{or} \qquad R_d = \frac{\frac{\mathcal{V}_{r_1}^2 - \mathcal{V}_{r_2}^2}{2}}{u(\mathcal{V}_{w_2} - \mathcal{V}_{w_1})}$$

$$= \frac{(\mathcal{V}_{r_1}^2 - \mathcal{V}_{r_2}^2)}{2u(\mathcal{V}_{w_2} - \mathcal{V}_{w_1})} \qquad ...(26.35)$$

From the inlet velocity triangle *CDB;*

$$\mathcal{V}_{r_1}^2 = \mathcal{V}_{f_1}^2 + (\mathcal{V}_{f_1} \tan \beta)^2$$

$$= \mathcal{V}_{f_1}^2 + \mathcal{V}_{f_1}^2 \tan^2 \beta \qquad ...(i)$$

Similarly, from the outlet velocity trinagle *BEF*;

$$\mathcal{V}_{r_2}^2 = \mathcal{V}_{f_2}^2 + \mathcal{V}_{f_2}^2 \tan^2 \phi \qquad ...(ii)$$

Since $\mathcal{V}_{f_1} = \mathcal{V}_{f_2} = \mathcal{V}_f$ (say)

Then $\mathcal{V}_{r_1}^2 - \mathcal{V}_{r_2}^2 = \mathcal{V}_f^2 (\tan^2 \beta - \tan^2 \phi)$ ...(26.36)

Substituting Eqs. (26.34) and (26.36) in Eq. (26.35), we get

$$R_d = \frac{\mathcal{V}_f^2 (\tan^2 \beta - \tan^2 \phi)}{2u \mathcal{V}_f (\tan \theta - \tan \alpha)} \qquad ...(26.37)$$

From symmetry of velocity triangles, we get

$$\beta = \theta \quad \text{and} \quad \alpha = \phi$$

Usually, the degree of reaction in axial flow compressor is taken 50%, i.e., $R_d = \frac{1}{2}$;

$$\therefore \qquad \frac{1}{2} = \frac{\mathcal{V}_f (\tan \beta + \tan \phi)}{2u}$$

$$\text{or} \qquad \frac{u}{\mathcal{V}_f} = \tan \beta + \tan \phi \qquad ...(26.38)$$

With 50% reaction blading, the axial compressor has symmetrical blades and losses in the compressor are drastically reduced.

26.6.6 Polytropic Work Input

The actual work input per kg of air is given by

$$w = C_p (T_{02} - T_{01})$$

which can be written as

$$w = C_p \left(\frac{T_{02} - T_{01}}{T_{02s} - T_{01}} \right) \times (T_{02s} - T_{01})$$

$$= C_{p\eta_{isen}} \times (T_{02s} - T_{01})$$

$$= C_p \eta_{isen} \times T_{01} \left(\frac{T_{02s}}{T_{01}} - 1 \right)$$

$$= C_p \, \eta_{isen} \times T_{01} \left(\left(\frac{p_{02}}{p_{01}} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right) \qquad ...(26.39)$$

Equation (26.39) demonstrate that for a given pressure ratio and isentropic efficiency, the work input to the compressor is directly proportional to the initial temperature of air in the stage. Thus, a compressor consisting of more than one stage of equal isentropic efficiency, will require more work input, because it receives fluid at increased temperature from preceeding stage. The two axial compressors of different pressure ratios will have different work input and overall efficiency.

26.6.7 Polytropic Efficiency

The *polytropic efficeincy* is the isentropic efficiency of one stage of a multistage, axial flow air compressor. The stage efficiency remains constant for all stages of the compressor.

Figure 26.22 shows a stage of multistage axial flow compressor. ΔT_{0s} is the isentropic stagnation temperture drop and ΔT_0 is the actual temperature drop during the stage. Thus, the polytropic effciency is defined as

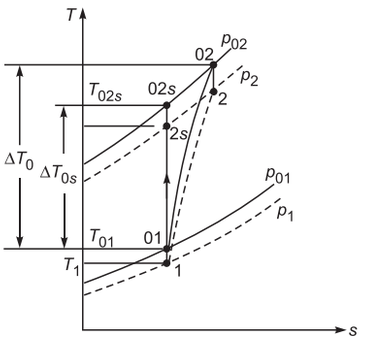

Fig. 26.22 *Polytropic compression process*

$$\eta_{poly} = \frac{\text{Isentropic tempature drop}}{\text{Actual temperature drop}} = \frac{dT_{0s}}{dT_0} \qquad ...(26.40)$$

For an irreversible polytropic process, the stagnation pressure and specific volume are related as

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$$p_0 v_0^n = Z \text{ (compressibility factor; a constant)}$$

or $$p_0 = Z\rho_0^n$$

Differentiating both sides, we get

$$dp_0 = nZ\rho_0^{n-1} d\rho_0 = n\frac{p_0}{\rho_0} d\rho_0 \qquad ...(26.41)$$

From characteristic gas equation, ρ_0 can be expressed as

$$\rho_0 = \frac{p_0}{RT_0}$$

On differentiation; we get

$$d\rho_0 = \frac{1}{R}\left[\frac{T_0 dp_0 - p_0 dT_0}{T_0^2}\right] \qquad ...(26.42)$$

Using $d\rho_0$ in Eq. (26.41);

$$dp_0 = n\frac{p_0}{\rho_0} \times \frac{1}{R}\left[\frac{T_0 dp_0 - p_0 dT_0}{T_0^2}\right]$$

$$= nRT_0 \times \frac{1}{R}\left[\frac{T_0 dp_0 - p_0 dT_0}{T_0^2}\right]$$

$$= n\left[\frac{T_0 dp_0 - p_0 dT_0}{T_0}\right]$$

or $$dp_0 = n\,dp_0 - np_0\frac{dT_0}{T_0}$$

$$np_0\frac{dT_0}{T_0} = ndp_0 - dp_0 = dp_0(n-1)$$

Actual stage temperature;

$$dT_0 = dp_0\left(\frac{n-1}{n}\right)\frac{T_0}{p_0} \qquad ...(26.43)$$

Similarly, for isentropic compression path, stagnation pressure and temperature are related as

$$dT_{0s} = dp_0\left(\frac{\gamma-1}{\gamma}\right)\frac{T_0}{p_0} \qquad ...(26.44)$$

Subsituting Eqs. (26.43) and (26.44) in Eq. (26.40) to express the polytropic efficiency;

$$\eta_{poly} = \frac{dp_0\left(\frac{\gamma-1}{\gamma}\right)\frac{T_0}{p_0}}{dp_0\left(\frac{n-1}{n}\right)\frac{T_0}{p_0}}$$

$$= \left(\frac{\gamma-1}{\gamma}\right)\left(\frac{n}{n-1}\right) \qquad ...(26.45)$$

Equation (26.45) expresses the polytropic efficiency in terms of polytropic and isentropic exponents.

Using $\left(\frac{n}{n-1}\right) = \frac{T_0}{p_0}\frac{dp_0}{dT_0}$ from Eq. (26.43), and subsituting in Eq. (26.45),

$$\eta_{poly} = \left(\frac{\gamma-1}{\gamma}\right)\frac{dp_0}{p_0} \times \frac{T_0}{dT_0}$$

or $$\eta_{poly}\left(\frac{dT_0}{T_0}\right) = \left(\frac{\gamma-1}{\gamma}\right)\frac{dp_0}{p_0}$$

Integrating between two states, we get

$$\eta_{poly}\ln\left(\frac{T_{02}}{T_{01}}\right) = \left(\frac{\gamma-1}{\gamma}\right)\ln\left(\frac{p_{02}}{p_{01}}\right)$$

or $$\eta_{poly} = \left(\frac{\gamma-1}{\gamma}\right)\frac{\ln\left(\frac{p_{02}}{p_{01}}\right)}{\ln\left(\frac{T_{02}}{T_{01}}\right)} \qquad ...(26.46)$$

Equation (26.46) expresses the polytropic efficiency in terms of stagnaton pressure ratio and stagnation temperature ratio.

26.6.8 Flow Coefficient, Work Coefficient, Pressure Coefficient

1. Flow Coefficient It is defined as the ratio of axial velocity ($\mathcal{V}_f$) to blade velocity (u) . It is designated as ϕ_f.

$$\phi_f = \frac{\mathcal{V}_f}{u}$$

From Eq. (26.38), $u = \mathcal{V}_f(\tan\alpha + \tan\beta)$

$$\therefore \quad \phi_f = \frac{\mathcal{V}_f}{\mathcal{V}_f(\tan\alpha + \tan\beta)}$$

$$= \frac{1}{\tan\alpha + \tan\beta} \qquad ...(26.47)$$

From symmetry of inlet and outlet velocity triangles for 50% reaction,

$$\beta = \theta \quad \text{and} \quad \alpha = \phi$$

The flow coefficient can also be expressed as

$$\phi_f = \frac{1}{\tan\phi + \tan\theta} \qquad ...(26.48)$$

2. Work Coefficient It is defined as the ratio of actual work input to kinetic energy corresponding to mean peripheral velocity. It is also called *head coefficient* and designated as ϕ_h.

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$$\phi_h = \frac{C_p \Delta T_{act}}{u^2/2} = \frac{u(\mathcal{V}_{w_2} - \mathcal{V}_{w_1})}{u^2/2} = \frac{2(\mathcal{V}_{w_2} - \mathcal{V}_{w_1})}{u}$$
$$= 2\left(\frac{\tan\beta - \tan\phi}{\tan\theta + \tan\phi}\right) \quad ...(26.49)$$

3. Pressure Coefficient It is defined as ratio of isentropic work input to kinetic energy corresponding to mean peripheral velocity. It is denoted by ϕ_p.

$$\phi_p = \frac{C_p \Delta T_{isen}}{u^2/2} = \eta_{isen}\, \phi_h \quad ...(26.50)$$

26.6.9 Losses in Axial-Flow Compressor

The losses in an axial-flow compressor can be divided in three groups:

1. Profile Losses The blade geometry of an axial compressor is two dimensional. The air flow along the profile of blade experiences skin friction. Further, the different streams of air are mixed after passing on blades. These losses lead to pressure loss of compressed air.

2. Annulus Losses When air flows through the annulus passage of the compressor, it experiences growth of boundary layer and skin friction. Therefore, there is loss of pressure of compressed air.

3. Secondary Losses In an axial-flow air compressor, the certain secondary flows are generated by combined effect of curvature of blade and growth of boundary layer in the annulus. The air is deflected by curvature of blades and bends in pipe, etc. It causes loss in pressure of compressed air.

26.6.10 Surging, Choking and Stalling of Axial-flow Compressor

Surging Surging is defined as a *self-oscillation* of the discharge pressure and flow rate, including a flow reversal. Every centrifugal or axial compressor has a characteristic combination of maximum head and minimum flow. Beyond this point, surging will occur. During surging, a flow reversal is often accompanied by a pressure drop. Surging is caused when mass-flow rate of fluid is reduced to a value which is less than the predetermined value. Thus, the flow becomes unsteady, periodic reversal. The surging state (from *M* to *U* on characteristic curve of Fig. 26.23) is dangerous for operation of a compressor of axial flow compressor also.

Choking When the velocity of fluid in the compressor reaches sonic velocity, the mass-flow rate through the compressor reaches a maximum value. This situation is called choking. At choking condition, the pressure ratio in the compressor becomes unity, i.e., there is no compression. Choking means constant mass flow irrespective of pressure ratio.

Figure 26.23 shows the characteristic curve for an axial flow compressor. The choking occurs at the point *C*. In region *M* to *C* on the curve, the flow is stable. Decrease in mass flow rate will result into increased pressure rise.

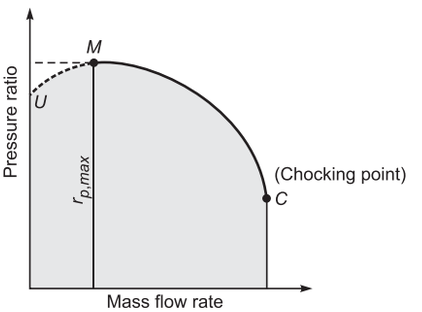

Fig. 26.23 *Characteristic curve of axial flow compressor*

Stalling A *compressor stalling* is a situation of abnormal air flow resulting from a reduction in the lift coefficient generated by an airfoil within the compressor. The stalling is the separation of flow from blade surface at low flow rates. At the large value of incidence, the flow separation occurs at suction side of the blade, which is referred as *positive stalling*. *Negative stalling* is due to separation of flow occuring on the delivery side of the blade due to large value of negative incidence.

Compressor stalls hamper the compressor performance, which can differ in severity from a momentary compression loss to a complete loss of compression.

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26.7 DIFFERENCE BETWEEN CENTRIFUGAL AND AXIAL FLOW COMPRESSORS

Sr. No.	*Centrifugal Compressor*	*Axial-flow Compressor*
1.	Air flows radially in the compressor.	Air flows parallel to axis of shaft.
2.	Low maintenance and running cost.	High maintenance and running cost.
3.	Low starting torque required.	Requires high starting torque.
4.	Not suitable for multistaging.	Suitable for only multistaging.
5.	Suitable for low pressure ratios up to 4.	Suitable up to a pressure ratio of 10.
6.	For given mass-flow rate, it requires, larger frontal area.	It requires less frontal area.
7.	Isentropic efficiency 80 to 82%.	Isentropic efficiency 86 to 88%.
8.	Better performance at part load.	Poor performance at part load.

Example 26.12 *An axial-flow compressor having 10 stages works with 50% degree of reaction. It compresses air with a pressure ratio of 5. The inlet conditions of air are 27°C and 100 kPa. The air enters the compressor with a velocity of 110 m/s. The mean speed of the rotor blade is 220 m/s. The isentropic efficiency of the compressor is 85%. Calculate the work input per kg of air and blade angles.*

Solution

Given An axial-flow air compressor with

$R_d = 0.5$ $\quad N_{stages} = 10$

$p_1 = 100$ kPa $\quad T_1 = 27°C = 300$ K

$p_2 = 500$ kPa $\quad \mathcal{V}_1 = 110$ m/s

$\eta_{isen} = 0.85$

$u = 220$ m/s

To find

(i) Power input, and

(ii) Blade angles.

Assumption The specific heat of air as 1005 J/kg·K and $\gamma = 1.4$

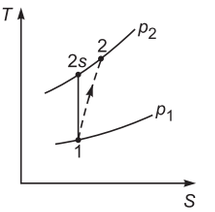

Fig. 26.24

Analysis The temperature after isentropic compression

$$T_{2s} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 300 \times (5)^{\frac{1.4-1}{1.4}}$$

$$= 475.14 \text{ K}$$

The isentropic efficiency is given by

$$\eta_{isen} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

Final temperature of air is given by

$$T_2 = \frac{T_{2s} - T_1}{\eta_{isen}} + T_1 = \frac{475.14 - 300}{0.85} + 300$$

$$= 506 \text{ K}$$

(i) Work input to compressor per kg of air

$$w = C_p(T_2 - T_1) = 1.005 \times (506 - 300)$$

$$= \mathbf{207 \text{ kJ/kg}}$$

Work input per kg is also given by Eq. (26.33),

$$w = u\mathcal{V}_f(\tan\theta - \tan\alpha) \times \text{No. of stages}$$

Using symmetry of velocity triangles, $\theta = \beta$ and numerical values;

$$207 \times 10^3 = 220 \times 110 \times (\tan\beta - \tan\alpha) \times 10$$

$$\tan\beta - \tan\alpha = 0.855 \qquad \text{(i)}$$

Further form Eq. (26.38) with $\theta = \beta$;

$$\frac{u}{\mathcal{V}_f} = \tan\beta + \tan\alpha$$

or $$\frac{220}{110} = \tan\beta + \tan\alpha$$

or $$\tan\beta + \tan\alpha = 2 \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$2\tan\beta = 2.855$$

or $$\beta = \mathbf{55°}$$

and $$\alpha = \mathbf{29.79°}$$

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Example 26.13 *An axial-flow compressor draws air at 20°C and delivers it at 50°C. Assuming 50% reaction, calculate the velocity of flow, if blade velocity is 100 m/s, work factor is 0.85. Take C_p = 1 kJ/kg·K, Assume α = 10°, and β = 40°. Find the number of stages.*

Solution

Given An axial-flow air compressor with

$R_d = 0.5$ $\quad T_1 = 20°C = 293$ K

$T_2 = 50°C = 323$ K $\quad \alpha = 10°$

$\beta = 40°$ $\quad u = 100$ m/s

$\phi_h = 0.85$ $\quad C_p = 1$ kJ/kg·K

To find

(i) Flow velocity, and

(ii) Number of stages.

Analysis The blade velocity and flow velocity are related as

$$\frac{u}{\mathcal{V}_f} = \tan\beta + \tan\alpha$$

or $$\frac{100}{\mathcal{V}_f} = \tan 40° + \tan 10°$$

or $$100 = 1.015\mathcal{V}_f$$

or $$\mathcal{V}_f = \mathbf{98.48\ m/s}$$

Further,

$$\mathcal{V}_{w_1} = \mathcal{V}_f \tan\alpha = 98.48 \times \tan 10°$$
$$= 17.36 \text{ m/s}$$

and $$\mathcal{V}_{w_2} = \mathcal{V}_f \tan\theta = \mathcal{V}_f \tan\beta = 98.48 \times \tan 40°$$
$$= 82.63 \text{ m/s}$$

Work done per stage

$$w_1 = u(\mathcal{V}_{w_2} - \mathcal{V}_{w_1})\phi_h$$
$$= 100 \times (82.63 - 17.36) \times 0.85$$
$$= 5548.33 \text{ J/kg}$$

Theoretical work required for a compressor

$$w = C_p(T_2 - T_1) = 1005 \times (323 - 293)$$
$$= 30150 \text{ J/kg}$$

Number of stages

$$= \frac{\text{Theoretical work}}{\text{Work input per stage}} = \frac{30150}{5548.33}$$
$$= 5.43 \approx \mathbf{6\ stages}$$

Example 26.14 *In an axial flow compressor, overall stagnation pressure ratio achieved is 4 and overall stagnation isentropic efficiency is 85%. The inlet stagnation pressure and temperature are 1 bar and 300 K, respectively. The mean blade velocity is 180 m/s. The degree of reaction is 50% at mean radius with relative air angles of 12° and 32° at rotor inlet and outlet, respectively. The work done factor is 0.9. Calculate*

(a) Stagnation polytropic efficiency,

(b) Inlet temperature and pressure,

(c) Number of stages,

(d) Blade height in first stage, if ratio of hub-to-tip diameter is 0.42, mass-flow rate is 19.5 kg/s.

Solution

Given An axial-flow air compressor with

$R_d = 0.5$ $\quad p_{01} = 100$ kPa

$T_{01} = 300$ K $\quad \dot{m} = 19.5$ kg/s

$\frac{p_{02}}{p_{01}} = 4$ $\quad u = 180$ m/s

$\eta_{isen} = 0.85$ $\quad \phi_h = 0.9$

$\alpha = \phi = 12°$ $\quad \beta = \theta = 32°$

$r_h = 0.42 r_1$

To find

(i) Stagnation Polytropic efficiency,

(ii) Inlet temperature and pressure,

(iii) Number of stages, and

(iv) Blade height in first stage.

Assumption The specific heat of air as 1005 J/kg·K and $\gamma = 1.4$.

Analysis

(i) *Stagnation polytropic efficiency:*

Stagnnation temperature after isentropic compression

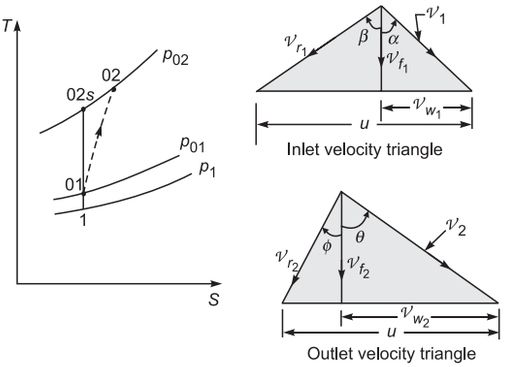

Fig. 26.25

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$$T_{02} = T_{01}\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} = 300 \times (4)^{\frac{1.4-1}{1.4}}$$

$$= 445.8 \text{ K}$$

The isentropic efficiency is given by

$$\eta_{isen} = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}}$$

Final stagnation temperature of air

$$T_{02} = \frac{T_{02s} - T_{01}}{\eta_{isen}} + T_{01}$$

$$= \frac{445.8 - 300}{0.85} + 300$$

$$= 471.52 \text{ K}$$

The stagnation polytropic efficiency

$$\eta_{poly} = \left(\frac{\gamma-1}{\gamma}\right)\frac{\ln\left(\frac{p_{02}}{p_{01}}\right)}{\ln\left(\frac{T_{02}}{T_{01}}\right)}$$

$$= \left(\frac{1.4-1}{1.4}\right)\frac{\ln\left(\frac{4}{1}\right)}{\ln\left(\frac{471.52}{300}\right)}$$

$$= 0.8759 \quad \text{or} \quad \mathbf{87.59\%}$$

(ii) *Inlet temperature and pressure:*

The blade velocity-to-flow velocity ratio is given by Eq. (26.38);

$$\frac{u}{\mathcal{V}_f} = \tan\alpha + \tan\beta = \tan 12° + \tan 32°$$

$$= 0.8374$$

or $$\mathcal{V}_f = \frac{180}{0.8374} = 214.95 \text{ m/s}$$

Inlet air velocity;

$$\mathcal{V}_1 = \frac{\mathcal{V}_f}{\cos\alpha} = \frac{214.95}{\cos 12°} = 219.75 \text{ m/s}$$

Whirl velocity at inlet;

$$\mathcal{V}_{w_1} = \mathcal{V}_f \tan\alpha = 214.95 \times \tan 12°$$

$$= 45.69 \text{ m/s}$$

Whirl velocity at outlet;

$$\mathcal{V}_{w_2} = \mathcal{V}_f \tan\theta = 214.95 \times \tan 32°$$

$$= 134.31 \text{ m/s}$$

Work input per kg of air per stage

$$w_{stage} = u(\mathcal{V}_{w_2} - \mathcal{V}_{w_1})\,\phi_h$$

$$= 180 \times (134.31 - 45.69) \times 0.9$$

$$= 14357.46 \text{ J/kg}$$

The stganation temperature is given by

$$T_{01} = T_1 + \frac{\mathcal{V}_1^2}{2C_p}$$

The static temperature at inlet

$$T_1 = T_{01} - \frac{\mathcal{V}_1^2}{2C_p} = 300 - \frac{219.75^2}{2 \times 1005}$$

$$= \mathbf{276 \text{ K}}$$

Inlet pressure can be expressed as

$$p_1 = p_{01}\left(\frac{T_1}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} = 1 \times \left(\frac{276}{300}\right)^{\frac{1.4}{1.4-1}}$$

$$= \mathbf{0.746 \text{ bar}}$$

(iii) *Number of stages*

Total work input to compressor per kg of air in all stages;

$$w_T = C_p(T_{02} - T_{01})$$

$$= 1005 \times (471.52 - 300)$$

$$= 172385 \text{ J/kg}$$

No. of stages

$$= \frac{w_T}{w_{stage}} = \frac{172385}{14356.44} \approx \mathbf{12}$$

(iv) *Blade height in first stage*

Density of air entering the first stage

$$\rho = \frac{p_1}{RT_1} = \frac{0.746 \times 100}{0.287 \times 276} = 0.9425 \text{ kg/m}^3$$

The mass-flow rate is given by

$$\dot{m} = \rho A_c \mathcal{V}_f = \rho\pi(r_1^2 - r_h^2)\mathcal{V}_f$$

or $$19.5 = 0.9425 \times \pi \times r_1^2(1 - 0.42^2) \times 214.95$$

$$(\because\ r_h = 0.42 r_1)$$

or $$r_1^2 = 0.03720$$

Itgi ves $r_1 = 0.1928$ m

and $$r_h = 0.42\, r_1 = 0.081 \text{ m}$$

Height of blade in first stage

$$= r_1 - r_h = 0.1928 - 0.081$$

$$= 0.1118 \text{ m or } \mathbf{11.18 \text{ cm}}$$

Example 26.15 *An axial-flow compressor has a constant axial velocity of 150 m/s and 50% reaction . The mean daimeter of the blade ring is 35 cm and speed is 15,000 rpm.. The exit angle of the blade is 27°. Calculate blade angle at inlet and work done per kg of air.*

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Solution

Given An axial flow air compressor with

$R_d = 0.5$ $\quad\quad$ $\mathcal{V}_{f_1} = 150$ m/s

$D = 35$ cm $= 0.35$ m $\quad\quad$ $\alpha = \phi = 27°$

$N = 15{,}000$ rpm

To find

(i) Blade angle β at inlet, and

(ii) Work input per kg of air.

Analysis The mean velocity of the blade ring

$$u = \frac{\pi DN}{60} = \frac{\pi \times 0.35 \times 15000}{60}$$

$= 274.89$ m/s

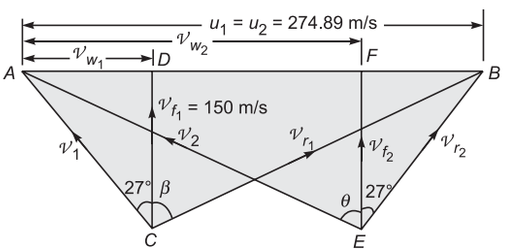

Fig. 26.26

Construction of velocity triangles

(i) Draw a horizontal line AB to represent the blade velocity $u = 274.89$ m/s.

(ii) Through the point A, draw an inclined line at angle $90° - \alpha = 63°$.

(iii) Mark the intersection point C and draw a vertical line CD representing flow velocity of 150 m/s at inlet.

(iv) Through the point B, draw an inclined line BE at angle $90° - 27° = 63°$.

(v) Mark the intersection point E and draw a vertical line EF representing flow velocity of 150 m/s at outlet.

(vi) Join the point E with A and the point C with B.

(vii) Line CB represents relative velocity $\mathcal{V}_{r_1}$ at the inlet and the line EA represents relative velocity $\mathcal{V}_{r_2}$ at the outlet.

(viii) Measure the $\angle AEF$ or $\angle DCB = 53°$.

(ix) Measure $AD = \mathcal{V}_{w_1} = 76.5$ m/s, and $AF = \mathcal{V}_{w_2} = 198.5$ m/s.

Work input per kg of air

$$w = u(\mathcal{V}_{w_2} - \mathcal{V}_{w_1})$$

$$= 274.89 \times (198.5 - 76.5)$$

$$= 33536.5 \text{ J/kg} \quad \text{or} \quad \mathbf{33.53 \text{ kJ/kg}}$$

Example 26.16 *An axial-flow compressor of 50% reaction has a blade outlet angle of 30°. The flow velocity is 0.5 times the mean blade velocity. The speed of the rotor is 7500 rpm. The stagnation condition of air at the entry is 1.013 bar and 5°C and the static pressure at this section is 0.91 bar. Draw the velocity triangle and find the power required to run the compressor, mass-flow rate and mean diameter of rotor. The mean flow area is 0.35 m^2.*

Solution

Given An axial-flow air compressor with

$R_d = 0.5$ $\quad\quad$ $\mathcal{V}_{f_1} = 0.5\,u$

$p_{01} = 1.013$ bar $\quad\quad$ $T_{01} = 5°\text{C} = 278$ K

$p_1 = 0.91$ bar $\quad\quad$ $\alpha = \phi = 30°$

$N = 7500$ rpm $\quad\quad$ $A = 0.35$ m^2

To find

(i) Mean diameter of rotor,

(ii) Power input per kg of air, and

(iii) Mass flow rate of air.

Assumption The specific heat of air as 1005 J/kg·K and $\gamma = 1.4$

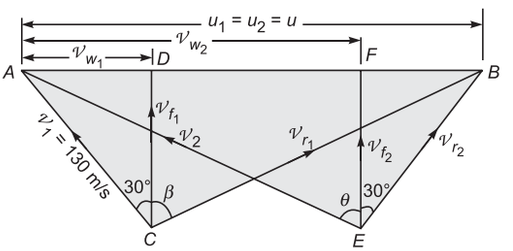

Fig. 26.27

Analysis The static temperature

$$T_1 = T_{01}\left(\frac{p_1}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} = 278 \times \left(\frac{0.91}{1.013}\right)^{\frac{1.4-1}{1.4}}$$

$= 269.6$ K

The stagnation temperature is given by

$$T_{01} = T_1 + \frac{\mathcal{V}_1^2}{2C_p}$$

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It gives

$$\mathcal{V}_1 = \sqrt{2C_p(T_{01} - T_1)} = \sqrt{2 \times 1005 \times (278 - 269.6)}$$
$$= 130 \text{ m/s}$$

Flow velocity

$$\mathcal{V}_{f_1} = \mathcal{V}_{f_2} = \mathcal{V}_1 \cos \alpha$$
$$= 130 \times \cos 30° = 112.5 \text{ m/s}$$

Blade velocity, $u = \dfrac{\mathcal{V}_f}{0.5} = \dfrac{112.5}{0.5} = 225$ m/s

For 50% reaction in axial flow compressor

$\alpha = \phi$, and $\beta = \theta$

Construct the velocity triangle as follows:

(i) Draw an inclined line *AC* to represent air velocity, $\mathcal{V}_1 = 130$ m/s at an angle (90°–30°) = 60°.

(ii) Draw a vertical line *CD*, which represents axial velocity of air. Its length is equivalent to $\mathcal{V}_{f_1}$ 112.5 m/s.

(iii) The line segment *AD*, represents whirl velocity at the inlet. Its length is equivalent to $\mathcal{V}_{w_1} = 65$ m/s.

(iv) Extend the line *AD* to the point *B* to represent u = 225 m/s.

(v) From the point *B,* draw a line *BE* inlined at angle (90° – 30°) = 60°.

(vi) Locate the point *E* on the line *BE* by drawing a vertical line $EF = \mathcal{V}_{f_2} = \mathcal{V}_{f_1} = 112.5$ m/s.

(vii) Length of line *BE* represents relative velocity at outlet $\mathcal{V}_{r_2} = \mathcal{V}_1$.

(viii) From the point *B,* meet the point C by a line *BC* to represent $\mathcal{V}_{r_1} = \mathcal{V}_2 = 195.5$ m/s.

(ix) Length of line *AF* represents $\mathcal{V}_{w_2} = 160$ m/s.

Mean diameter of rotor:

The mean velocity of blade ring,

$$u = \frac{\pi DN}{60}$$

or $$D = \frac{60u}{\pi N} = \frac{60 \times 225}{\pi \times 7500} = \mathbf{0.573\ m}$$

Power input per kg of air

$$w = u(\mathcal{V}_{w_2} - \mathcal{V}_{w_1})$$
$$= 225 \times (160 - 65)$$
$$= 21375 \text{ J/kg} \quad \text{or} \quad \mathbf{21.37\ kJ/kg}$$

Mass-flow rate of air

Density of air $\rho = \dfrac{p_1}{RT_1} = \dfrac{0.91}{0.287 \times 278}$

$= 1.176$ kg/m^3

Mass-flow rate, $\dot{m} = \rho A \mathcal{V}_{f_1} = 1.176 \times 0.35 \times 112.5$

$= \mathbf{46.3\ kg/s}$

Summary

- Rotary compressors have rotors in place of pistons and give a continuous, pulsation-free compressed air. The rotary compressors are mainly classified as rotary positive-displacement type air compressor and steady-flow type compressor.
- In a positive-displacement type compressor, the air is compressed by being trapped in the reduced space formed by two sets of engaging surfaces. In a non-positive dispacement or steady-flow type compressor, the air flows continuously through them and pressure is increased due to dynamic action.
- The *roots blower* is essentially a low-pressure blower and is limited to a discharge pressure of 1 bar in single-stage design and up to 2.2 bar in a two-stage design.
- *The Lysholm compressor* is a patented screw compressor, a single-stage helical lobe, oil flooded air compressor.
- The *centrifugal compressors* are dynamic action compressors. The centrifugal air compressor is an oil free compressor by design. Centrifugal machines are better suited due to their simplicity, light weight and ruggedness.
- *Axial compressors* are dynamic action, rotating, aerofoil blade compressors. Axial flow compressors produce a continuous flow of compressed gas, and have the benefits of high efficiencies and large mass flow capacity, particularly in relation to their cross-section.

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Glossary

Degree of Reaction Ratio of static pressure rise in impeller to the total static pressure rise in the compressor

Slip factor Ratio of whirl velocity to blade tip velocity

Euler's work Product of blade velocity and whirl velocity ($w = u_2 \mathcal{V}_{w_2}$)

Work factor Ratio of actual work input to Euler work input

Pressure coefficient Ratio of isentropic work to Euler work

Choking State of maximum mass-flow rate

Surging State of alternatively forward and backward flow of fluid

Stalling Separation of flow from blade surface

Polytropic Efficiency The isentropic efficiency of one stage of a multistage axial flow air compressor

Review Questions

1. Define rotary compressor. Classify them.
2. Differentiate between positive displacement and negative displacement compressors.
3. Compare reciprocating compressor with a rotary compressor.
4. Explain construction and working of a roots blower.
5. Explain construction and working of a vane-type compressor.
6. Explain working and construction of a screw compressor.
7. Describe the principle of operation, construction and working of centrifugal compressor.
8. Discuss the effect of impeller blade shape on performance of centrifugal compressor.
9. Define slip, slip factor and pressure coefficient.
10. Explain the construction and working of a diffuser in a centrifugal compressor.
11. Explain the phenomenon of surging and its effects in the compressor.
12. Prove that the work input per kg of air in an axial flow compressor is $w = u\mathcal{V}_w(\tan\beta - \tan\alpha)$
13. Compare axial flow compressor with centrifugal one.
14. What is stalling in an axial flow compressor?

Problems

1. Compare the work inputs required for a roots blower and a vane-type compressor having same volume inducted of 0.3 m^3/rev. The inlet pressure is 1.013 bar and pressure ratio is 1.5 in a compressor. For vane-type compressor, assume half the compression takes place through half the pressure range. *[1.52 kJ, 1.352 kJ]*

2. A roots blower compresses 0.08 m^3 of air from 100 kPa to 150 kPa per revolution. Calculate compressor efficiency. *[85.95%]*

3. A centrifugal compressor running at 2000 rpm has internal and external diameters of impeller as 300 mm and 500 mm, respectively. The vane angle at inlet and outlet are 22° and 40°, respectively. The air enters the impeller radially. Determine the work done by a compressor per kg of air. Also calculate the degree of reaction. *[1.95 kJ/kg, 64.5%]*

4. A rotary compressor handles 3 kg of air per second, runing at 2400 rpm. The internal and external diameters of the impeller are 120 mm and 240 mm, respectively. The impeller angle at the exit is 35°. The air enters the impeller radially with 7 m/s. Calculate the vane angle at inlet. Also calculate the power required to drive the compressor. *[25°, 1.83 kW]*

5. A centrifugal compressor runs at 8000 rpm, handles 4.8 m^3/s from 1 bar and 20°C to

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The index of compression is 1.5. The flow velocity is 65 m/s, same at the inlet and outlet of compressor. The inlet and outlet impeller diameters are 320 mm and 620 mm, respectively. Calculate (a) blade angle at inlet and outlet, (b) absolute angle at tip of impeller, and (c) width of blade at inlet and outlet.

[(a) $\beta = 25.88°$, $\phi = 34.2°$, (b) $\theta = 21.6°$, (c) 7.34 cb and 2.89 cm]

6. A centrifugal compressor runs at 1440 rpm, compresses air from 101 kPa , 20° to a pressure of 6 bar isentropically. The inner and outer diameters of the impeller are 140 mm and 280 mm, respectively. The width of blades at inlet is 25 mm. The blade angles are 16° and 40° at entry and exit. Determine the mass-flow rate of air, degree of reaction, power developed and width of blade at outlet.

[0.4 kg/s, 14.78 W, 58.6%, 3.5 mm]

7. A centrifugal compressor handles 16.5 kg/s of air with total pressure ratio of 4. The speed of the compressor is 15000 rpm. Inlet stagnation temperature is 20°, slip factor is 0.9, power input factor is 1.04 and isentropic efficiency as 80%. Calculate diameter of impeller and power input to the compressor. *[55.67 cm, 2951.7 kW]*

8. An axial-flow compressor stage has a mean diameter of 600 mm and runs at 15000 rpm. The mass-flow rate through the compressor is 50 kg/s. Calculate the power required to drive the compressor and degree of reaction, if the inlet angle is 12°. The blade angle at the inlet and exit are 35° and 27°, respectively. *[2318 kW, 66.55]*

9. An axial flow compressor, with a compression ratio of 4, draws air at 20°C and delivers it at 97°C. The blade velocity and flow velocity are constant throughout the compressor. The blade velocity is 300 m/s. Air enters the blade at an angle of 12°. Calculate the flow velocity, work done per kg of air and degree of reaction. Take the inlet stagnation temperature of 305 K.

[152 m/s, 77.38 kg/s, 46.2%]

10. An axial flow compressor with 50% degree of reaction has blades with inlet and outlet angles of 45° and 10°, respectively. The pressure ratio is 6 and isentropic efficiency is 85%, when the air inlet temperature is 40°C. The blade velocity is 200 m/s. The blade velocity and axial velocity are constant throughout the compressor. Calculate the number of stages required, when work factor is (a) unity, and (b) 0.89 for all stages.

[(a) 9 (b)10]

11. Air from a quiescent atmosphere, at pressure of 1 bar and 300 K enters a centrifugal compressor, fitted with radial vanes and the air leaves the diffuser with negligible velocity. The tip diameter of the impeller is 450 mm and the compressor rotates at 18000 rpm. Neglect all losses and calculate the temperature and pressure of air as it leaves the compressor. Take $\gamma = 1.4$, $C_p = 1.005$ kJ/kg·K, and $\phi_s = 0.9$. *[188°C, 4.5 bar]*

Objective Questions

1. Which one of the following is a non-positive type rotary compressor?
   (a) Vane blower
   (b) roots blower
   (c) Centrifugal compressor
   (d) Lysholm compressor
2. A machine is called a compressor when it has a pressure ratio
   (a) up to 1.11 (b) up to 1.2
   (c) more than 1.2 (d) none of the above
3. In a roots blower, the compression process can be represented by
   (a) isothermal line
   (b) isentropic line
   (c) constant-volume line
   (d) constant-pressure line
4. In a roots blower, the pressure is increased due to
   (a) rotation of lobes
   (b) increase in mass

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(c) back flow of air
(d) reduction in volume of air

5. In a centrifugal compressor, the increase in pressure is due to
   (a) back flow of air
   (b) dynamic action
   (c) intermittent flow
   (d) reduction in volume
6. The efficiency of a roots blower is given as
   (a) $\frac{\text{Isentropic work input}}{\text{Actual work input}}$
   (b) $\frac{\text{Actual work input}}{\text{Isentropic work input}}$
   (c) $\frac{\text{Actual work input}}{\text{Isothermal work input}}$
   (d) $\frac{\text{Isothermal work input}}{\text{Actual work input}}$
7. The stages are arranged in a multisatge compressor in
   (a) parallel (b) series
   (c) cross connected (d) none of the above
8. In a centrifugal compressor, the pressure rise takes place in
   (a) impeller only
   (b) diffuser only
   (c) casing only
   (d) impeller and diffuser both
9. The degree of reaction in a centrifugal compressor is defined as
   (a) $\frac{\text{Pressure rise in diffuser}}{\text{Pressure rise in impeller}}$
   (b) $\frac{\text{Pressure rise in impeller}}{\text{Pressure rise in compressor}}$
   (c) $\frac{\text{Pressure rise in diffuser}}{\text{Pressure rise in compressor}}$
   (d) none of the above
10. Pressure coefficient is defined as
   (a) $\frac{\text{Actual work input}}{\text{Euler work input}}$
   (b) $\frac{\text{Blade velocity}}{\text{Whirl velocity}}$
   (c) $\frac{\text{Isentropic work input}}{\text{Euler work input}}$
   (d) none of the above
11. Forward curved impeller vane has a blade exit angle of
   (a) $< 90°$ (b) $> 90°$
   (c) $= 90°$ (d) none of the above
12. At the state of choking in compressor,
   (a) mass-flow rate reaches a minimum value
   (b) mass-flow rate reaches a maximum value
   (c) pressure ratio reaches a maximum value
   (d) pressure ratio reaches a minimum value

Answers

1. (c) 2. (c) 3. (c) 4. (c) 5. (b) 6. (a) 7. (b) 8. (d)
9. (b) 10. (c) 11. (b) 12. (b)

27

Gas Turbine Plant

Introduction

Gas turbine is a rotary internal combustion engine. It consists of a compressor, a combustion chamber and a turbine. The gas turbine uses continuous gas flow as the working medium by which heat energy is partially converted into mechanical energy. Gas is produced in the engine by the combustion of certain fuels. Stationary nozzles discharge jets of this gas against the blades of a turbine wheel. The impulse force of the jets causes the shaft to spin.

A simple-cycle gas turbine is shown in Fig. 27.1. Atmospheric air is drawn into an axial compressor, where it is compressed to a higher pressure and temperature. This hot compressed air is passed through a combustion chamber, where the fuel in gaseous or liquid-spray form is injected and combustion takes place with the help of burner. The resulting combustion products expand through a turbine to produce power. The exhaust gases leaving the turbine are finally discharged into the atmosphere. Most of the turbine power is used to drive the compressor and auxilliary devices and the remainder is the net plant output, which is available to generate electricity, to propel an aircraft or for other purpose.

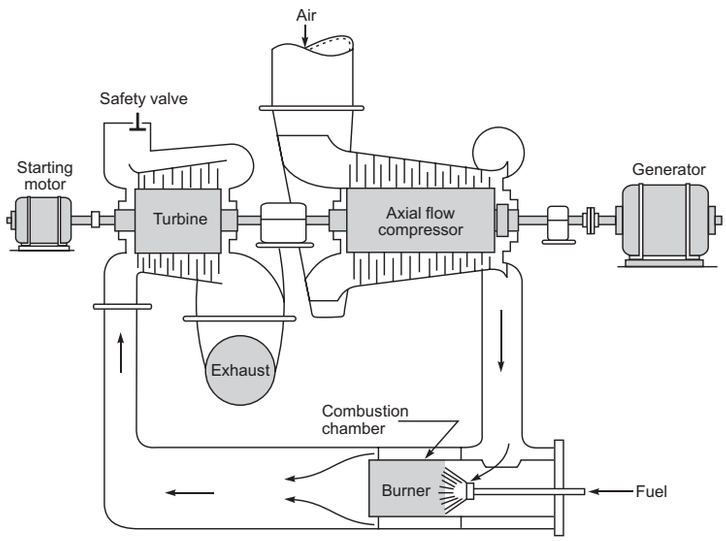

Fig. 27.1 *A simple gas turbine plant*

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27.1 APPLICATIONS OF GAS TURBINES

The gas turbine plants have their applications in the following fields:

1. They are widely used in the aircraft field, i.e., for propulsion of *turbo jet* and *turbo propeller* engines.
2. With availability of natural gas fuel, the gas turbines are increasingly being used for *electric power generation*.
3. These are also used in marine propulsion.
4. The gas turbines are used in conjunction with thermal power plants as *cogeneration* for power production.
5. The gas turbines are used in *high-speed racing cars.*
6. The gas turbines are also used to run the railway locomotives.
7. These are used to operate hovercrafts.
8. These are used to run turbo (high speed) pumps.
9. These are used to run the rotary compressors.
10. The gas turbines are also used in cross country pipe and charging stations.

27.1.1 Comparison of Gas Turbine and IC Engines

The gas turbines have the following merits and demerits over reciprocating IC engines.

Merits

1. Since the gas turbine is a rotary engine, its mechanical efficiency approaches 95%, quite high compared with IC engines.
2. The torque on the shaft of a gas turbine is continuous and uniform, thus a gas turbine does not require a flywheel, while without flywheel, IC engine cannot run smoothly.
3. The weight of the gas turbine per kW power is less than the IC engine.
4. Power developed by a gas turbine per kg of air is more than of IC engine.
5. The speed of the gas turbines is higher than the IC engines.
6. The gas turbines use cheaper fuel such as paraffin type, oil residues, or powdered coal, while special-grade oils are used in IC engines to avoid knocking in the engine.
7. The combustion is complete in a gas turbine, thus exhaust is less polluting as compared to IC engines.
8. In gas turbines, the ignition and lubrication systems are much simpler.
9. Since the gas turbine parts are not subjected to large pressure, so they can be made lighter.
10. Due to low specific weight, the gas turbines are particularly suitable for use in aircrafts.

Demerits

1. Thermal efficiency of the gas turbine power plant is lower (15 to 20%) as compared with IC engines (25 to 30%).
2. It is difficult to start a gas turbine as compared to an IC engine.
3. The gas turbine blades require a special cooling system.
4. The manufacturing of gas turbine blades is difficult and costly.
5. For the same power output, the gas turbine produces exhaust gas approximately five times more than that of an IC engine.

27.1.2 Comparison of Gas Turbine and Steam Turbines

The brief comparison between a gas turbine plant and a steam turbine power plant is given below:

1. Steam turbines use steam as working fluid. Steam is the cheapest medium and can be generated with the help of easily available fuels. The gas turbines are using costly fuel which can burn efficiently in its combustion chamber.
2. Steam is prepared outside the turbine, thus it is an external combustion engine, while the combustion chamber is an integral part of

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gas turbine, thus it is an internal combustion engine.

3. The steam turbine is in highly developed stage after long experience and modifications, thus it has higher thermal efficiency than gas turbines.
4. A steam power plant includes boiler plant, turbine, condensing plant including cooling tower and continuous supply of water. All the equipments of steam, power plant require large space and are bulky. A gas turbine plant includes less number of parts, i.e., compressor, combustion chamber and turbine. The combustion chamber is also compact. Thus, gas turbine is lighter engine and frequently used on aircrafts.
5. A gas turbine uses lower pressure ratio of the working fluid as compared to steam pressure in a steam turbine.
6. Except blade cooling, no cooling is required in gas turbine plants, where a large amount of cooling water is required in the condenser of steam power plants.
7. Gas turbines run at higher speed than steam turbines.
8. A gas turbine plant can easily be started or stopped as compared to a steam power plant.
9. Due to absence of components, such as condenser, feed-water treatment plant, cooling turbine, etc., the auxiliary equipment in a gas turbine are small and lighter as compared to steam turbine.
10. The number of skilled man power required to operate the gas turbine plant is comparatively much less than for steam power plant.

27.2 CLASSIFICATION OF GAS TURBINES

The gas turbines are rotary engines and can be classified as follows:

(i) According to Cycle of Operation

(*a*) *Open-cycle Gas Turbines* The fresh charge is supplied to the compressor in each cycle. The compressed air is mixed with fuel and combustion occurs in the the combustion chamber. The combustion gases leaving the turbine after expansion are expelled to the atmosphere. Thus, the cycle becomes an open cycle as shown in Fig. 27.2.

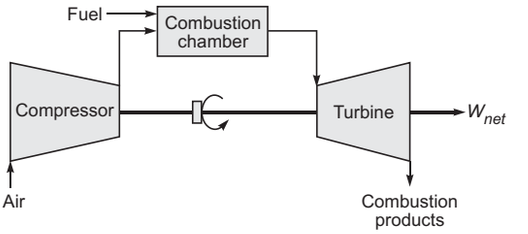

Fig. 27.2 *Open-cycle hgas turbine*

(*b*) *Closed-cycle Gas Turbine* In a closed-cycle gas turbine, the working substance air undergoes the cycle, repeatedly. The combustion process is replaced by heat addition from external source and exhaust process is replaced by heat rejection process to the surroundings.

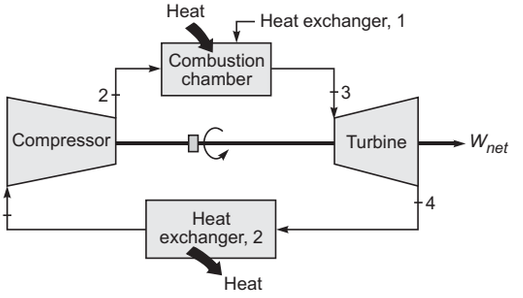

Fig. 27.3 *Closed-cycle gas turbine*

(ii) According to Combustion Process

(*a*) *Constant-pressure Combustion Gas Turbine* Theoretically, it operates on air standard Brayton cycle (Fig. 27.5) in which the combustion takes place at constant pressure by slow burning fuel.

(*b*) *Constant-volume (Explosive) Gas Turbine* It is also called *explosive gas turbine* and theoretically, it operates on the Atkinson power cycle.

The atmospheric air is compressed in a compressor from the pressure p_1 to the pressure

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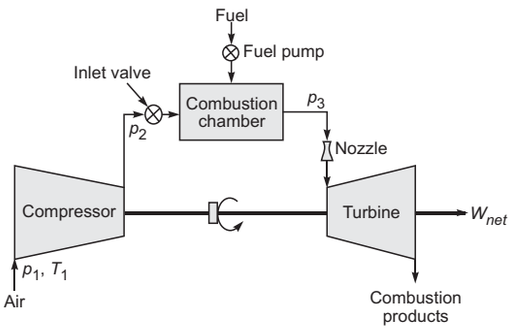

Fig. 27.4 *Schematic of constant-volume gas turbine*

p_2. The compressed air is supplied to a combustion chamber when the inlet valve opens. Fuel is injected through a fuel pump and burnt with the help of a spark plug at constant volume, thus pressure and temperature further increase to p_3 and T_3, respectively.

The hot combustion products expand in a nozzle partially to increase the velocity of fluid and then expand in a turbine to develop mechanical power.

27.3 COMPARISON BETWEEN CLOSE-CYCLE GAS TURBINE AND OPEN-CYCLE GAS TURBINE

Differences between closed-cycle and open-cycle gas turbines are tabulated below.

Sr. No.	*Criterion*	*Closed-cycle gas turbine*	*Open-cycle gas turbine*
1.	Cycle of operation	It works on closed cycle. The working fluid is recirculated again and again. It is a clean cycle.	It works on open cycle. The fresh charge is supplied to each cycle and after combustion and expansion, it is discharged to the atmosphere.
2.	Working fluid	The gases other than air like helium or helium–CO_2 mixture can be used, which has more favourable properties.	Air–fuel mixture is used which leads to lower thermal efficiency.
3.	Manner of heat input	The heat is transferred indirectly through a heat exchanger.	Direct heat supply. It is generated in the combustion chamber itself.
4.	Quality of heat input	The heat can be supplied from any source like waste heat from some process, nuclear heat and solar heat using a concentrator.	It requires high-grade heat energy for generation of power in a gas turbine.
5.	Type of fuel used	Since heat is transferred externally, so any type of fuel; solid, liquid or gaseous or a combination of these can be used for generation of heat.	Since combustion is an integral part of the system, thus it requires high-quality liquid or gaseous fuel for burning in a combustion chamber.
6.	Efficiency	High thermal efficiency for given lower and upper temperature limits.	Lower thermal efficiency for same temperature limits.
7.	Part load efficiency	Better.	Comparatively less.
8.	Size of the plant	Reduced size per MWh of power output.	Comparative large size for same power output.
9.	Blade life	Since combustion products do not come in direct contact of turbine blade, thus there is no blade fouling and longer blade life.	Direct contact with combustion products, the blades are subjected to higher thermal stresses and fouling and hence shorter blade life.
10.	Control	Better control on power production.	Poor control on power production.
11.	Cost	Plant is complex and costly.	Comparatively simple plant and less costly.

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27.4 MODELLING A GAS TURBINE PLANT

The gas turbines usually operate on open cycle as shown Fig. 27.2. The open-cycle gas turbine cycle can be modelled as a closed-cycle gas turbine, as shown in Fig. 27.3 by using air-standard assumptions.

Air is compressed in a compressor, heated in a heat exchanger, expanded in a turbine and cooled again to initial state before re-entering the compressor. This modified cycle operates on air standard *Brayton Cycle,* shown in Fig. 27.5, explained in Article 11.14.

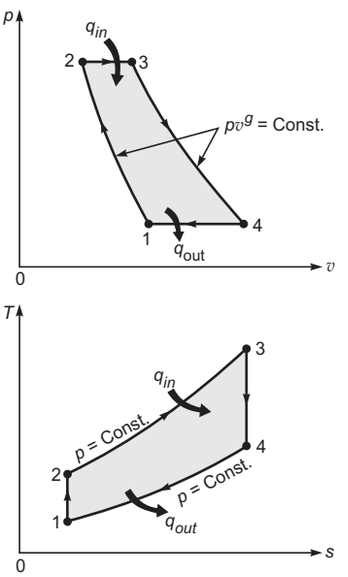
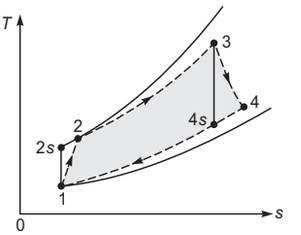


Fig. 27.5 *Air standard Brayton cycle*

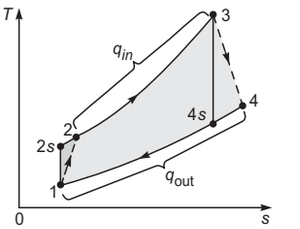
27.5 DEVIATION OF ACTUAL GAS TURBINE CYCLE FROM BRAYTON CYCLE

The actual gas turbine cycle in Fig. 27.6(a) differs from an ideal Brayton cycle in the following manner.

1. Due to *frictional effects*, the working fluid experiences a *pressure drop* during heat addition and heat rejection processes taking place in a heat exchanger. However, this pressure drop is less significant, and is thus neglected for simplicity of analysis as shown in Fig. 27.6(b).
2. Because of the *frictional effects* within the compressor and turbine,
   (a) the actual work input into the compressor is more than the isentropic work, and
   (b) the actual work output of the gas turbine is less than the isentropic work. It results in a marked decrease in the net work output of the gas turbine plant.
3. The specific heat of the working substance increases as temperature increases.



(a) Actual gas turbine cycle all processes are irreversible



(b) Gas turbine cycle with irreversible compression and expansion

Fig. 27.6 *Effects of irreversiblities on air-standard gas turbine cycle*

In absence of any changes in kinetic and potential energies; the steady-flow energy equation on a unit mass basis, as

$$q - w = \Delta h = h_{exit} - h_{inlet}$$

Assuming constant specific heat of air, the heat supplied to and rejected from air are

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$$q_{in} = h_3 - h_2 = C_p(T_3 - T_2)$$
$$q_{out} = h_4 - h_1 = C_p(T_4 - T_1)$$

Work developed per kg by turbine

$$w_T = h_3 - h_4 = C_p(T_3 - T_4) \quad ...(27.1)$$

Work input to compressor

$$w_C = h_2 - h_1 = C_p(T_2 - T_1) \quad ...(27.2)$$

Net work of the gas turbine plant

$$w_{net} = \Sigma q \quad \text{or} \quad \Sigma w$$
$$= q_{in} - q_{out} \quad \text{or} \quad w_T - w_C$$

Thermal efficiency of gas turbine cycle

$$\eta_{Thermal} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{C_p(T_4 - T_1)}{C_p(T_3 - T_2)} \quad ...(27.3)$$

In order to obtain an appreciable net work from the plant, the compressor and turbine must be highly efficient. The isentropic efficiencies of the compressor and gas turbine are defined as

$$\eta_T = \frac{\text{Actual work output}}{\text{isentropic work output}}$$

$$= \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{C_p(T_3 - T_4)}{C_p(T_3 - T_{4s})} = \frac{T_3 - T_4}{T_3 - T_{4s}} \quad ...(27.4)$$

$$\eta_C = \frac{\text{Isentropic work input}}{\text{Actual work input}}$$

$$= \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{C_p(T_{2s} - T_1)}{C_p(T_2 - T_1)} = \frac{T_{2s} - T_1}{T_2 - T_1} \quad ...(27.5)$$

where the states designated without suffix represent actual states of working fluid and states designated with suffix *s* denote the state after isentropic process.

Example 27.1 *Air enters the compressor of a gas turbine plant operating on air-standard cycle at 100 kPa and 300 K with a volumetric flow rate of 5 m^3/s. The compressor pressure ratio is 10. The turbine inlet temperature is 1400 K. The turbine and compressor each has an isentropic efficiency of 80%. Calculate*

(a) the thermal efficiency of the cycle,
(b) the back work ratio,
(c) the net power developed in kW.

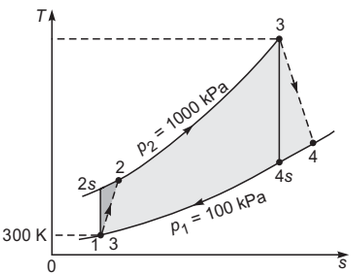

Fig. 27.7 *An actual gas turbine cycle*

Solution

Given An air-standard gas turbine cycle with

$p_1 = 100$ kPa, $T_1 = 300$ K
$r_p = 10$, $T_3 = 1400$
$\eta_T = 0.8$, $\eta_C = 0.8$
$\dot{V} = 5$ m^3/s

To find

(i) Thermal efficiency of the cycle,
(ii) The back work ratio, and
(iii) The net power developed in kW.

Assumptions

(i) Each device operates in a steady flow manner.
(ii) Compression and expansion are adiabatic.
(iii) No pressure drop during flow of gas through heat exchangers.
(iv) Kinetic and potential energy effects are negligible.
(v) The working fluid is air as an ideal gas with $C_p = 1.005$ kJ/kg·K, $\gamma = 1.4$.

Analysis The *T–s* diagram for given data of a gas turbine plant is shown in Fig. 27.7. Analysing each device in a steady flow manner.

Compressor The temperature after isentropic compression

$$T_{2s} = T_1(r_p)^{\frac{\gamma - 1}{\gamma}} = 300 \times (10)^{\frac{1.4-1}{1.4}} = 579.2 \text{ K}$$

The isentropic compressor work per kg of air,

$$w_C = h_{2s} - h_1 = C_p(T_{2s} - T_1)$$
$$= 1.005 \times (579.2 - 300) = 280.6 \text{ kJ/kg}$$

The compressor efficiency is given by

$$\eta_C = \frac{\text{Insentropic work input}}{\text{Actual work input}} = \frac{w_C}{w_{in}}$$

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Actual work input;

$$w_{in} = \frac{w_C}{\eta_C} = \frac{280.6}{0.8} = 350.75 \text{ kJ/kg}$$

Actual work input per kg to the compressor can also be expressed as

$$w_{in} = h_2 - h_1 = C_p(T_2 - T_1)$$

or $$T_2 = T_1 + \frac{w_{in}}{C_p} = 300 + \frac{350.75}{1.005} = 649 \text{ K}$$

Heat supplied The heat supplied to air in heat exchanger

$$q_{in} = h_3 - h_2 = C_p(T_3 - T_2)$$
$$= 1.005 \times (1400 - 649) = 754.75 \text{ kJ/kg}$$

Gas Turbine The temperature T_{4s}, after isentropic expansion

$$T_{4s} = \frac{T_3}{(r_p)^{\frac{\gamma-1}{\gamma}}} = \frac{1400}{(10)^{\frac{1.4-1}{1.4}}} = 725 \text{ K}$$

The isentropic work done per kg of air by gas turbine

$$w_T = h_3 - h_{4s} = C_p(T_3 - T_{4s})$$
$$= 1.005 \times (1400 - 725) = 678.25 \text{ kJ/kg}$$

The turbine efficiency is given as

$$\eta_T = \frac{\text{Actual work}}{\text{Isentropic work}} = \frac{w_{out}}{w_T}$$

Actual turbine work

$$w_{out} = \eta_T w_T = 0.8 \times 678.25 = 542.6 \text{ kJ/kg}$$

The net work output of the plant

$$w_{net} = w_{out} - w_{in}$$
$$= 542.6 - 350.75 = 191.85 \text{ kJ/kg}$$

(i) *Thermal efficiency of the cycle*

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{191.85}{754.75} = 0.2542 = \mathbf{25.42\%}$$

(ii) *The back work ratio*

$$b_{wr} = \frac{\text{Compression work}}{\text{Turbine work}} = \frac{350.75}{542.75}$$
$$= \mathbf{0.646\ (64.6\%)}$$

(iii) *Power output of the plant*

The density of incoming air,

$$\rho = \frac{p_1}{RT_1} = \frac{100 \text{ kPa}}{0.287 \times 300} = 1.16 \text{ kg/m}^3$$

The mass-flow rate of air,

$$\dot{m}_a = \rho \dot{V} = 1.161 \times 5 = 5.80 \text{ kg/s}$$

The power output

$$P = \dot{m}_a w_{net} = 5.80 \times 191.85$$
$$= \mathbf{1114.1 \text{ kW}}$$

Example 27.2 *A gas turbine unit has a pressure ratio of 6 and maximum cycle temperature of 610°C. The isentropic efficiency of the turbine and compressor are 0.82 and 0.8, respectively. Calculate the power output in kW of an electric generator, geared to the turbine, when air enters the compressors at 15°C at a rate of 16 kg/s.*

Take C_p = 1.005 kJ/kg·K and γ = 1.4 for compression process and C_p = 1.11 kJ/kg·K and γ = 1.333 for expansion process.

Solution

Given A gas turbine unit with

$r_p = 6$ $\quad \eta_T = 0.82$

$\eta_C = 0.80$ $\quad T_3 = 610°\text{C} = 883 \text{ K}$

$T_1 = 15°\text{C} = 288 \text{ K}$ $\quad \dot{m}_a = 16 \text{ kg/s}$

For compression

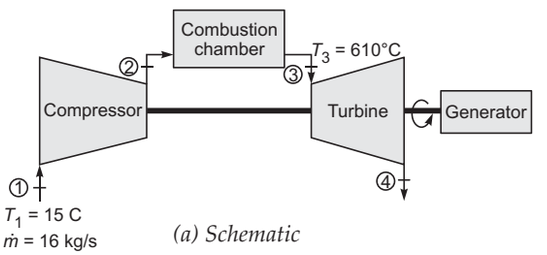
$C_p = 1.005 \text{ kJ/kg·K}$ $\quad \gamma = 1.4$

For expansion

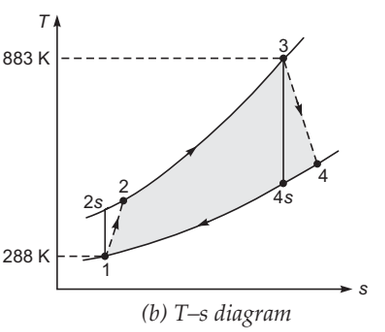
$C_{pg} = 1.11 \text{ kJ/kg·K}$ $\quad \gamma_g = 1.333$

To find The power output of the gas turbine plant.

Schematic



(a) Schematic



(b) T–s diagram

Fig. 27.8

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Assumptions

(i) Compression and expansion are adiabatic.
(ii) Each device operates in steady flow manner.
(iii) No pressure drop during flow of gas through combustion chamber.

Analysis The schematic and *T–s* diagram for a gas turbine unit are shown in Fig. 27.8. Analysing each device in separately.

Compressor

The temperature after isentropic compression,

$$T_{2s} = T_1 (r_p)^{\frac{\gamma-1}{\gamma}} = 288 \times (6)^{\frac{1.4-1}{1.4}} = 480.5\,\text{K}$$

The isentropic compression work per kg of air

$$w_C = h_{2s} - h_1 = C_p(T_{2s} - T_1)$$
$$= 1.005 \times (480.5 - 288) = 193.5\ \text{kJ/kg}$$

The isentropic efficiency of the compressor is given by

$$\eta_C = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{w_C}{w_{in}}$$

or
$$w_{in} = \frac{w_C}{\eta_C} = \frac{193.5}{0.80} = 241.87\ \text{kJ/kg}$$

The actual work input to compressor w_{in} can also be expressed as

$$w_{in} = h_2 - h_1 = C_p(T_2 - T_1)$$

or
$$T_2 = \frac{241.87}{1.005} + 288 = 528.66\ \text{K}$$

Turbine The temperature after isentropic expansion with $\gamma_g = 1.333$

$$T_{4s} = \frac{T_3}{(r_p)^{\frac{\gamma_g - 1}{\gamma_g}}} = \frac{883}{(6)^{\frac{1.333-1}{1.333}}} = 564.37\ \text{K}$$

Isentropic turbine work per kg of air

$$w_T = C_{pg}(T_3 - T_{4s})$$
$$= 1.11 \times (883 - 564.37) = 353.67\ \text{kJ/kg}$$

The isentropic efficiency of the turbine is given as

$$\eta_T = \frac{\text{Actual work output}}{\text{Isentropic turbine work}} = \frac{w_{out}}{w_T}$$

Actual work of turbine,

$$w_{out} = 0.82 \times 353.67 = 290\ \text{kJ/kg}$$

Net work output per kg of the plant

$$w_{net} = w_{out} - w_{in}$$
$$= 290 - 241.87 = 48.14\ \text{kJ/kg}$$

The power output of the plant

$$P = \dot{m}_a w_{net} = 16 \times 48.14 = 770.26\ \text{kW}$$

Example 27.3 *In a gas turbine plant, air enters the compressor at 15°C and it is compressed through a pressure ratio of 4 with isentropic efficiency of 85%. The air–fuel ratio is 80 and the calorific value of fuel is 42000 kJ/kg.*

The turbine inlet temperature is 1000 K and the isentropic efficiency of the turbine is 82%. Calculate the overall efficiency and air intake for a power output of 260 kW. Take the mass of fuel in account.

Solution

Given A gas turbine unit with

$T_1 = 15°\text{C} = 288\,\text{K}$ $\quad \eta_T = 0.82$
$\eta_C = 0.85$ $\quad T_3 = 1000\ \text{K}$
$r_p = 4$ $\quad \frac{\dot{m}_a}{\dot{m}_f} = 80$
$P = 260\ \text{kW}$ $\quad CV = 42000\ \text{kJ/kg}$

To find

(i) Overall efficiency of the plant,
(ii) Mass of air intake.

Assumptions

(i) Compression and expansion are adiabatic.
(ii) Each device operates in a steady flow manner.
(iii) No pressure drop during flow of gas through combustion chamber.

Schematic

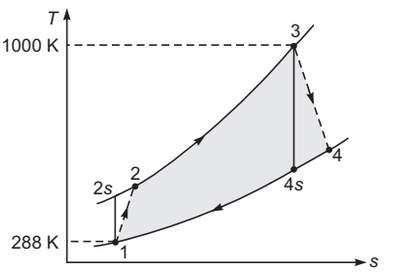

Fig. 27.9 *T–s diagram*

Analysis The *T–s* diagram for gas turbine unit is shown in Fig. 27.9. Analysing each device separately.

Compressor

The temperature after isentropic compression

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$$T_{2s} = T_1 (r_p)^{\frac{\gamma-1}{\gamma}} = 288 \times (4)^{\frac{1.4-1}{1.4}} = 428\,\text{K}$$

The isentropic efficiency of the compressor is given by

$$\eta_C = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

or $$T_2 = 288 + \frac{428 - 288}{0.85} = 452.7\,\text{K}$$

The actual work input to compressor w_{in} can also be expressed as

$$w_{in} = h_2 - h_1 = C_p(T_2 - T_1)$$
$$= 1.005 \times (452.7 - 288) = 165.52\ \text{kJ/kg}$$

Turbine The temperature T_{4s} after isentropic expansion;

$$T_{4s} = \frac{T_3}{(r_p)^{\frac{\gamma-1}{\gamma}}} = \frac{1000}{(4)^{\frac{1.4-1}{1.4}}} = 672.95\,\text{K}$$

The isentropic efficiency of the turbine is given as

$$\eta_T = \frac{\text{Actual work output}}{\text{Isentropic work output}} = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

Actual temperature after expansion in turbine,

$$T_4 = 1000 - 0.82 \times (1000 - 672.95)$$
$$= 731.82\ \text{K}$$

Turbine work output per kg of air

$$w_{out} = h_3 - h_4 = C_p(T_3 - T_4)$$
$$= 1.005 \times (1000 - 731.82)$$
$$= 269.52\ \text{kJ/kg}$$

Net work output per kg of the plant

$$w_{net} = w_{out} - w_{in}$$
$$= 269.52 - 165.52 = 104\ \text{kJ/kg of air}$$

Heat supplied per kg of air

$$Q_{in} = \frac{\dot{m}_f}{\dot{m}_a} \times CV \text{ of fuel}$$
$$= \frac{1}{80} \times 42000 = 525\ \text{kJ/kg of air}$$

(i) *Overall efficiency of plant*

$$\eta_{overall} = \frac{\text{Net work output per kg of air}}{\text{Heat supplied per kg of air}} \times 100$$
$$= \frac{104}{525} \times 100 = \mathbf{19.8\%}$$

(ii) *Mass-flow rate of air*

Net power out ot the plant is given by

$$P = (\dot{m}_a + \dot{m}_f) \times \text{Net work output per kg}$$
$$= \dot{m}_a \left(1 + \frac{\dot{m}_f}{\dot{m}_a}\right) \times w_{net}$$

$$260\ \text{kW} = \dot{m}_a \left(1 + \frac{1}{80}\right) \times 104$$

or $$\dot{m}_a = \mathbf{2.47\ kg/s}$$

Example 27.4 *Calculate the required air–fuel ratio in a gas turbine plant, whose turbine and compressor efficiencies are 85 and 80%, respectively. Maximum cycle temperature is 875°C. The working fluid can be taken as air (C_p = 1.0 kJ/kg·K, γ = 1.4), which enters the compressor at 1 bar and 27°C. The pressure ratio is 4. The fuel used has calorific value of 42000 kJ/kg. There is loss of 10% of calorific value in combustion chamber.*

Solution

Given A gas turbine plant with

$p_1 = 1$ bar K	$T_1 = 27°\text{C} = 300\,\text{K}$
$r_p = 4$	$\eta_T = 0.85$
$\eta_C = 0.80$	$T_3 = 875°\text{C} = 1148\,\text{K}$
$CV = 42000$ kJ/kg	$C_p = 1.0$ kJ/kg·K
$\gamma = 1.4$	

To find Air–fuel ratio of plant.

Assumptions

(i) Compression and expansion are adiabatic.
(ii) Each device operates in steady flow manner.
(iii) No pressure drop during flow of gas through combustion chamber.

Analysis The *T–s* diagram for a gas turbine unit is shown in Fig. 27.10. Analysing each device separately.

Compressor

The temperature after isentropic compression

$$T_{2s} = T_1 (r_p)^{\frac{\gamma-1}{\gamma}} = 300 \times (4)^{\frac{1.4-1}{1.4}} = 445.8\,\text{K}$$

The isentropic efficiency of the compressor is given by

$$\eta_C = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

or $$T_2 = 300 + \frac{445.8 - 300}{0.80} = 482.24\,\text{K}$$

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Schematic

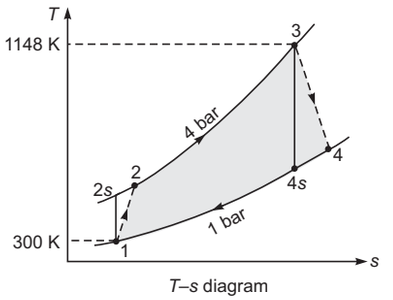

Fig. 27.10

The heat supplied in a combustion chamber can be analysed as

Heat supplied by fuel = Heat absorbed by burning fuel

$$(1 - 0.1)\ \dot{m}_f CV = (\dot{m}_a + \dot{m}_f)\, C_p (T_3 - T_2)$$

$$0.9 \times 42000\ \dot{m}_f = 1.0 \times (1148 - 482.24)\ (\dot{m}_a + \dot{m}_f)$$

or $$\frac{\dot{m}_a + \dot{m}_f}{\dot{m}_f} = \frac{0.9 \times 42000}{1.0 \times (1148 - 482.24)} = 56.777$$

or $$\frac{\dot{m}_a}{\dot{m}_f} = 56.777 - 1 = \textbf{55.777 kg/kg of fuel}$$

Example 27.5 *In a constant-pressure open-cycle gas turbine , air enters at 1 bar and 20°C and leaves the compressor at 5 bar. Using the following data:*

Temperature of gases entersing the turbine = 680°C
Pressure loss in combustion chamber = 0.1 bar
Isentropic efficiency of compressor = 85%
Isentropic efficiency of turbine = 80%
Combustion efficiency = 85%
For air; C_p = 1.024 kJ/kg·K, γ = 1.4

Find

(a) the quantity of air circulation, if plant develops 1065 kW,
(b) heat supplied per kg of air circulation,
(c) thermal efficiency of cycle,
Mass of fuel may be neglected.

Solution

Given A gas turbine plant with

$p_1 = 1$ bar $\quad T_1 = 20°C = 293$ K
$p_2 = 5$ bar $\quad r_p = 5$
$\eta_T = 0.80$ $\quad \eta_C = 0.85$
$\eta_{Comb} = 0.85$ $\quad T_3 = 680°C = 953$ K
$\Delta p_{2\text{-}3} = 0.1$ bar $\quad P = 1065$ kW
$C_p = 1.024$ kJ/kg·K $\quad \gamma = 1.4$

To find

(i) Air circulation rate
(ii) Heat supplied per kg of air, and
(iii) Thermal efficiency of the cycle

Assumptions

(i) Compression and expansion are adiabatic.
(ii) Each device operates in steady flow manner.

Analysis The *T–s* diagram for gas turbine unit is shown in Fig. 27.11. Analysing each device separately.

Compressor

The temperature after isentropic compression

$$T_{2s} = T_1 (r_p)^{\frac{\gamma-1}{\gamma}} = 293 \times (5)^{\frac{1.4-1}{1.4}} = 464.06\,\text{K}$$

The isentropic efficiency of the compressor is given by

$$\eta_C = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

or $$T_2 = 293 + \frac{464.06 - 293}{0.85} = 494.24\,\text{K}$$

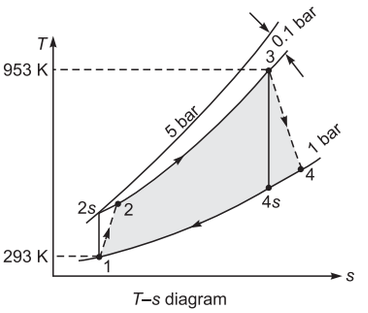

Fig. 27.11

The actual work input to compressor w_{in} can also be expressed as

$$w_{in} = h_2 - h_1 = C_p (T_2 - T_1)$$
$$= 1.024 \times (494.24 - 293) = 206.07\ \text{kJ/kg}$$

Turbine The pressure at inlet of turbine is $p_3 = p_2 - 0.1$ bar = 4.9 bar

The temperature T_{4s} after isentropic expansion;

$$T_{4s} = \frac{T_3}{\left(\frac{p_3}{p_1}\right)^{\frac{\gamma-1}{\gamma}}} = \frac{953}{\left(\frac{4.9}{1}\right)^{\frac{1.4-1}{1.4}}} = 605.19\,\text{K}$$

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The isentropic efficiency of the turbine is given as

$$\eta_T = \frac{\text{Actual work output}}{\text{Isentropic work output}} = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

Actual temperature after expansion in turbine,

$$T_4 = 953 - 0.8 \times (953 - 605.19) = 674.75\,\text{K}$$

Turbine work output per kg of air

$$w_{out} = h_3 - h_4 = C_p(T_3 - T_4)$$
$$= 1.024 \times (953 - 674.75) = 284.92\ \text{kJ/kg}$$

Net work output per kg of the plant

$$w_{net} = w_{out} - w_{in}$$
$$= 284.92 - 206.07 = 78.85\ \text{kJ/kg of air}$$

Power developed by plant is given by

$$P = \dot{m}_a w_{net}$$

(i) The mass flow rate of air,

$$\dot{m}_a = \frac{1065}{78.85} = \mathbf{13.5\ kg/s}$$

(ii) Actual heat supplied per kg of air,

$$q_{in} = \frac{C_p(T_3 - T_2)}{\eta_{Comb}} = \frac{1.024 \times (953 - 494.24)}{0.85}$$
$$= \mathbf{552.67\ kJ/kg}$$

(iii) Thermal efficiency of cycle,

$$\eta_{Th} = \frac{w_{net}}{q_{in}} = \frac{78.85}{552.67} = 0.1426 \quad \text{or} \quad \mathbf{14.26\%}$$

Example 27.6 *A gas turbine plant consists of two turbines. One turbine drives the compressor and the other develops the power output. Both turbines have their own combustion chambers which are served by air directly from the compressor. Air enters the compressor at 1 bar and 15°C and is compressed to 8 bar with an isentropic efficiency of 80%. Due to heat addition in the combustion chamber, the inlet temperature of gas to both the turbines reaches to 900°C. The isentropic efficiency of the turbine is 85%. The mass-flow rate of air at the compressor is 20 kg/s. The calorific value of fuel is 42000 kJ/kg. Calculate the output of plant and thermal efficiency, if mechanical efficiency is 96% and the generator efficiency is 95%. Take C_p = 1.005 kJ/kg·K and γ = 1.4 for air and C_p = 1.128 kJ/kg·K and γ = 1.34 for gases. Neglect the mass of fuel.*

Solution

Given A gas turbine plant with two turbines:

$p_1 = 1$ bar, $T_1 = 15°\text{C} = 288$ K, $r_p = 8$, $T_3 = 900°\text{C} = 1173$ K, $\eta_C = 0.80$, $\eta_T = 0.85$, $\dot{m}_a = 20$ kg/s, $CV = 42000$ kJ/kg, $\eta_{mech} = 0.96$, $\eta_{gen} = 0.95$

For air $C_p = 1.005$ kJ/kg·K, $\gamma = 1.4$

For gases $C_{pg} = 1.128$ kJ/kg·K and $\gamma_g = 1.34$

To find

(i) Output of plant, and

(ii) Thermal efficiency.

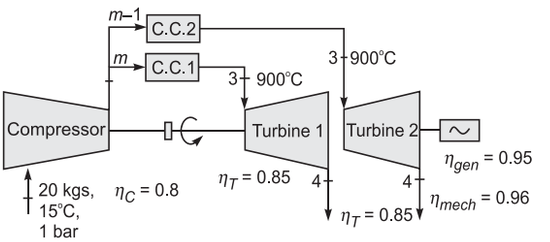
Assumptions

(i) Compression and expansion are adiabatic.

(ii) Each device operates in a steady flow manner.

(iii) No pressure drop during flow of gas through combustion chamber.

Schematic



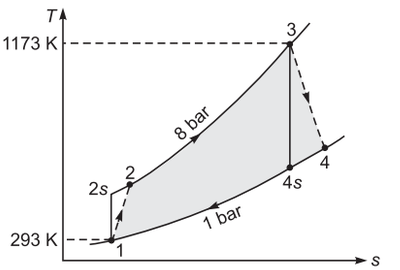


Fig. 27.12 *Schematic and T–s diagram*

Analysis The schematic and *T–s* diagram are shown in Fig. 27.12. Analysing each device separately.

Compressor

The temperature after isentropic compression

$$T_{2s} = T_1(r_p)^{\frac{\gamma-1}{\gamma}} = 288 \times (8)^{\frac{1.4-1}{1.4}} = 521.7\,\text{K}$$

The isentropic efficiency of the compressor is given by

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$$\eta_C = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

or $$T_2 = 288 + \frac{521.7 - 288}{0.8} = 580.12\,\text{K}$$

The actual work input to compressor w_{in} can also be expressed as

$$w_{in} = C_p(T_2 - T_1) = 1.005 \times (580.12 - 288)$$
$$= 293.58\ \text{kJ/kg}$$

Turbine The temperature T_{4s} after isentropic expansion of gases;

$$T_{4s} = \frac{T_3}{\left(\frac{p_3}{p_1}\right)^{\frac{\gamma_g - 1}{\gamma_g}}} = \frac{1173}{\left(\frac{8}{1}\right)^{\frac{1.34-1}{1.34}}} = 692.08\,\text{K}$$

The isentropic efficiency of the turbine is given as

$$\eta_T = \frac{\text{Actual work output}}{\text{Isentropic work output}} = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

Actual temperature after expansion in turbine,

$$T_4 = 1173 - 0.85 \times (1173 - 692.08)$$
$$= 764.21\ \text{K}$$

Turbine work output of each turbine per kg of air

$$w_T = C_{pg}(T_3 - T_4)$$
$$= 1.128 \times (1173 - 764.21) = 461.1\ \text{kJ/kg}$$

(i) *Power output of plant*

Let m kg mass flow through the turbine 1, which drives the compressor. Thus,

$$w_{in} = m_1 w_T$$

or $$293.58 = 461.1\, m_1$$

or $$m_1 = \frac{293.58}{461.1} = 0.6366\ \text{kg/kg of air}$$

The mass of air-flow rate through the turbine 2 (neglecting the mass of fuel)

$$\dot{m}_2 = \dot{m}_a\,(1 - m_1) = 20 \times (1 - m)$$
$$= 20 \times (1 - 0.6366)$$
$$= 7.266\ \text{kg/s}$$

Power output of the turbine 2

$$\dot{W}_T = \dot{m}_2 w_T = 7.266 \times 461.1$$
$$= 3350.35\ \text{kW}$$

Power output of the plant (generator)

$$P = \dot{W}_T\, \eta_{Mech}\, \eta_{gen}$$
$$= 3350.35 \times 0.95 \times 0.96$$
$$= \mathbf{3055.74\ kW}$$

(ii) *Thermal efficiency*

Heat supplied to combustion chambers

$$\dot{Q}_{in} = \dot{m}_a C_{pg}(T_3 - T_2)$$
$$= 20 \times 1.128 \times (1173 - 580.12)$$
$$= 13339.8\ \text{kW}$$

Thermal efficiency of the plant

$$\eta_{th} = \frac{\dot{W}_T}{\dot{Q}_{in}} = \frac{3350.35}{13339.8}$$
$$= 0.2511 \quad \text{or} \quad \mathbf{25.11\%}$$

27.6 METHODS FOR IMPROVEMENT OF THERMAL EFFICIENCY OF GAS TURBINE PLANT

The network output of a gas turbine plant can be increased by either decreasing compression work or by increasing the turbine work or both. The thermal efficiency of the cycle can be improved by saving the heat input for the same network output. The improvement is possible by

1. Regeneration by preheating the air leaving the compressor with turbine exhaust gases, thus saving the heat input.
2. Reheating of gases after each stage of expansion to obtain more work from the turbine.
3. Intercooling during compression stages to reduce the work input to the compressor.

27.6.1 Gas Turbine Cycle with Regeneration

The temperature of exhaust gases leaving the turbine of a gas turbine engine is considerably higher than the temperature of air delivered by the compressor. Therefore, high-pressure air leaving the compressor can be heated by hot exhaust gases.

The device, a counterflow heat exchanger used for transfer of heat from hot gases to air, is known as regenerator. A simple open-cycle gas turbine with a regenerator is shown in Fig. 27.13.

As illustrated in Fig. 27.14(b) *T–s* diagram, the compressed air at the state 2 enters the regenerator (a heat exchanger) in which it is heated to the

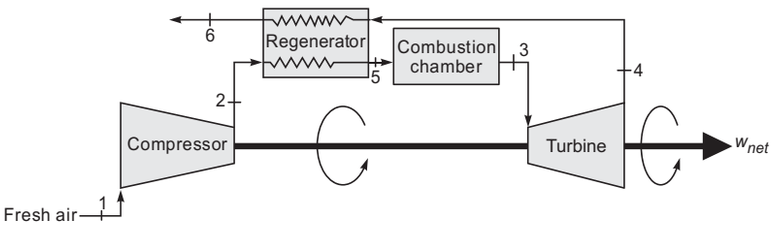

Fig. 27.13 *A gas turbine cycle with regenerator*

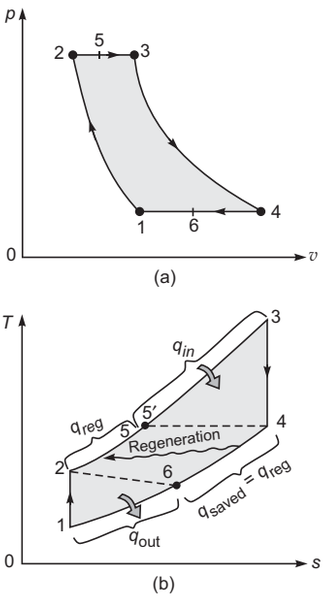

Fig. 27.14 *(a) p–v and (b) T–s diagrams for regenerative gas turbine cycle*

state 5. The hot exhaust gases leaving the turbine at the state 4, are used to preheat the air entering the combustion chamber. Hence, a heat transfer q_{in}, from an external source is required only to increase the air temperature from the state 5 to the state 3. This, in turn, decreases the heat input in the combustion chamber for same network output, and thus thermal efficiency of gas turbine cycle increases.

A certain temperature difference is necessary to transfer heat from one fluid to another. Therefore, the air cannot be preheated to a temperature equal to the turbine exhaust temperature T_4. Assuming a well-insulated heat exchanger, without any change in kinetic and potential energy changes, actual heat transfer in regenerator

$$q_{gen} = h_5 - h_2 \qquad ...(27.6)$$

Maximum possible heat transfer in regenerator

$$q_{max} = h_4 - h_2 \qquad ...(27.7)$$

The effectiveness of regenerator

$$\varepsilon = \frac{q_{gen}}{q_{max}} = \frac{h_5 - h_2}{h_4 - h_2} \qquad ...(27.8)$$

If specific heat C_p is assumed constant, then

$$\varepsilon = \frac{T_5 - T_2}{T_4 - T_2} \qquad ...(27.9)$$

The thermal efficiency of gas turbine cycle with regenerator can be found as

$$\eta_{th} = \frac{w_{net}}{q_{in}}$$

$$w_{net} = w_T - w_C = (h_3 - h_4) - (h_2 - h_1)$$

$$q_{in} = h_3 - h_5 = C_p(T_3 - T_5)$$

$$\eta_{th,\,gen} = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_5}$$

$$= \frac{(T_3 - T_4) - (T_2 - T_1)}{T_3 - T_5} \qquad ...(27.10)$$

Under ideal conditions $T_4 = T_5$ for $\varepsilon = 1$, then

$$\eta_{th,\,reg} = 1 - \frac{T_2 - T_1}{T_3 - T_4} = 1 - \frac{T_1\left(\frac{T_2}{T_1} - 1\right)}{T_3\left(1 - \frac{T_4}{T_3}\right)}$$

$$= 1 - \frac{T_1}{T_3}\left\{\frac{\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1}{1 - \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}}\right\}$$

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$$= 1 - \frac{T_1}{T_3}\left\{\frac{\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1}\right\} \times \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 1 - \frac{T_1}{T_3} \times (r_p)^{\frac{\gamma-1}{\gamma}} = 1 = \frac{T_{min}}{T_{max}} (r_p)^{\frac{\gamma-1}{\gamma}} \quad ...(27.11)$$

Example 27.7 *In a gas turbine plant, the compressor takes in air at a temperature of 15°C and compresses it to four times the initial pressure with an isentropic efficiency of 85%. The air is then passed through a heat exchanger, heated by the turbine exhaust before reaching the combustion chamber. The turbine inlet temperature is 600°C and its efficiency is 80%. Neglecting all losses except those mentioned, and treating the working fluid throughout the cycle to have the properties of air, calculate thermal efficiency and work ratio of the cycle if (a) heat exchanger is perfect, and (b) heat exchanger gives 85% of available heat to air.*

Take R = 0.287 kJ/kg·K, γ = 1.4, and constant specific heats throughout.

Solution

Given A gas turbine power plant with regeneration

$T = 15°C = 288\ K$ $\quad r_p = 4$

$\eta_C = 0.85$ $\quad \gamma = 1.4$

$T_3 = 600°C = 873\ K$ $\quad \eta_T = 0.8$

$R = 0.287\ kJ/kg \cdot K$

(i) $\varepsilon = 1.0$ (ii) $\varepsilon = 0.85$

To find

(i) Thermal efficiency of the cycle,

(ii) Work ratio.

Analysis Specific heat

$$C_p = \frac{\gamma R}{\gamma - 1} = \frac{1.4 \times 0.287}{1.4 - 1} = 1.0045\ kJ/kg \cdot K$$

Analysing each device in steady flow manner.

Compressor Temperature T_{2s} after isentropic compression

$$T_{2s} = T_1 (r_p)^{\frac{\gamma-1}{\gamma}}$$

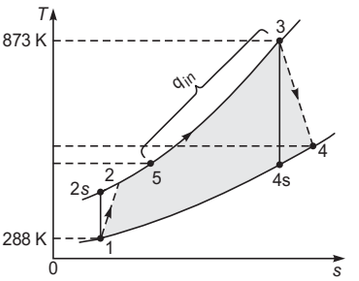

Fig. 27.15 *T–s diagram for a regenerative gas turbine*

$$= 288 \times (4)^{\frac{1.4-1}{1}} = 428\ K$$

The isentropic efficiency of the compressor is given by

$$\eta_C = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

or $$T_2 = 288 + \frac{428 - 288}{0.85} = 452.7\ K$$

The actual work input to compressor w_{in} can also be expressed as

$$w_{in} = h_2 - h_1 = C_p(T_2 - T_1)$$
$$= 1.0045 \times (452.7 - 288) = 165.44\ kJ/kg$$

Turbine The temperature T_{4s} after isentropic expansion

$$T_{4s} = \frac{T_3}{(r_p)^{\frac{\gamma-1}{\gamma}}} = \frac{873}{(4)^{\frac{1.4-1}{1.4}}} = 587.48\ K$$

The isentropic efficiency of the turbine is given as

$$\eta_T = \frac{\text{Actual work output}}{\text{Isentropic work output}} = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

Actual temperature after expansion in turbine,

$$T_4 = 873 - 0.80 \times (873 - 587.48) = 644.58\ K$$

Turbine work output per kg of air

$$w_{out} = h_3 - h_4 = C_p(T_3 - T_4)$$
$$= 1.0045 \times (873 - 644.58)$$
$$= 229.44\ kJ/kg$$

Net work output per kg of the plant

$$w_{net} = w_{out} - w_{in} = 229.44 - 165.44$$
$$= 64\ kJ/kg\ of\ air$$

Heat exchanger The effectiveness of heat exchanger is given by

$$\varepsilon = \frac{T_5 - T_2}{T_4 - T_2}$$

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Case (i) given that $\varepsilon = 1.0$,

$$1.0 = \frac{T_5 - 452.7}{644.58 - 452.7}$$

Thus the temperature T_5 of air leaving the heat exchanger

$$T_5 = 452.7 + (644.58 - 452.7) = 644.58$$

The heat supplied to air

$$q_{in} = C_p(T_3 - T_5) = 1.0045 \times (873 - 644.58)$$
$$= 229.44 \text{ kJ/kg}$$

(i) Thermal efficiency of the cycle

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{64}{229.44} = 0.2789 = \mathbf{27.89\%}$$

(ii) Work ratio,

$$r_w = \frac{\text{Net work output}}{\text{Turbine work}} = \frac{64}{229.44}$$
$$= 0.2789 \quad \mathbf{27.89\%}$$

Case (ii) given that $\varepsilon = 0.85$,

$$0.85 = \frac{T_5 - 452.7}{644.58 - 452.7}$$

Thus, the temperature T_5 of air leaving heat exchanger

or $$T_5 = 452.7 + 0.85 \times (644.58 - 452.7)$$
$$= 615.8 \text{ K}$$

The heat supplied to air

$$q_{in} = C_p(T_3 - T_5) = 1.0045 \times (873 - 615.8)$$
$$= 258.36 \text{ kJ/kg}$$

(i) Thermal efficiency of the cycle

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{64}{258.36} = 0.2277 = \mathbf{22.77\%}$$

(ii) The work ratio remains unaffected, thus r_w = **27.89%**

Example 27.8 *In a gas turbine plant, air at 10°C and atmospheric pressure is compressed through a pressure ratio of 4. In a heat exchanger and combustion chamber, air is heated to 700°C with a pressure drop of 0.14 bar. After expansion through the turbine, the air passes through a heat exchanger, which cools the air through 75% of maximum range possible and air is finally discharged to the atmosphere. The isentropic efficiency of compressor and turbine is 0.8 and 0.85 respectively. Calculate the thermal efficiency of the plant.*

Solution

Given A gas turbine power plant with regeneration

$T_1 = 10°\text{C} = 283 \text{ K}$ $\quad r_p = 4$

$\eta_C = 0.8$ $\quad \eta_T = 0.85$

$T_3 = 700°\text{C} = 973 \text{ K}$ $\quad \varepsilon = 0.75$

$\Delta p = 0.14 \text{ bar}$ $\quad \therefore \quad p_3 = 3.86 \text{ bar}$

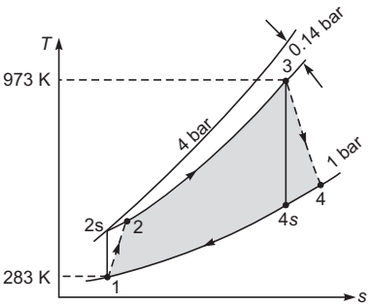

Fig. 27.16

To find Thermal efficiency of the cycle.

Assumptions For air;

$$C_p = 1.005 \text{ kJ/kg·K}, \quad \text{and} \quad \gamma = 1.4$$

Analysis The pressure at state 3 being

$$p_3 = p_4 - 0.14 \text{ bar} = 3.86 \text{ bar}$$

Analysing each device in steady flow manner.

Compressor Temperature T_{2s} after isentropic compression

$$T_{2s} = T_1(r_p)^{\frac{\gamma-1}{\gamma}} = 283 \times (4)^{\frac{1.4-1}{1}} = 420.53 \text{ K}$$

The isentropic efficiency of the compressor is given by

$$\eta_C = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

or $$T_2 = 283 + \frac{420.53 - 283}{0.8} = 454.92 \text{ K}$$

The actual work input to compressor w_{in} can also be expressed as

$$w_{in} = h_2 - h_1 = C_p(T_2 - T_1)$$
$$= 1.005 \times (454.92 - 283) = 172.78 \text{ kJ/kg}$$

Turbine The temperature T_{4s} after isentropic expansion from a pressure of 3.86 bar to 1 bar

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$$T_{4s} = \frac{T_3}{\left(\frac{p_3}{p_1}\right)^{\frac{\gamma-1}{\gamma}}} = \frac{973}{\left(\frac{3.86}{1}\right)^{\frac{1.4-1}{1.4}}} = 661.48\,\text{K}$$

The isentropic efficiency of the turbine is given as

$$\eta_T = \frac{\text{Actual work output}}{\text{Isentropic work output}} = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

Actual temperature after expansion in turbine,

$$T_4 = 973 - 0.85 \times (973 - 661.48)$$
$$= 708.2\ \text{K}$$

Turbine work output per kg of air

$$w_{out} = h_3 - h_4 = C_p(T_3 - T_4)$$
$$= 1.005 \times (973 - 708.2) = 266.11\ \text{kJ/kg}$$

Net work output per kg of the plant

$$w_{net} = w_{out} - w_{in} = 266.11 - 172.78$$
$$= 93.33\ \text{kJ/kg of air}$$

Heat exchanger

The effectiveness of heat exchanger is given by

$$\varepsilon = \frac{T_5 - T_2}{T_4 - T_2}$$

Given that $\varepsilon = 0.75$,

$$\therefore \quad 0.75 = \frac{T_5 - 454.92}{708.2 - 454.92}$$

Thus the temperature T_5 of air leaving heat exchanger

or $T_5 = 454.92 + 0.75 \times (708.2 - 454.92)$
$= 644.88\ \text{K}$

The heat supplied to air

$$q_{in} = C_p(T_3 - T_5) = 1.005 \times (973 - 644.88)$$
$$= 329.76\ \text{kJ/kg}$$

Thermal efficiency of the cycle

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{93.3}{329.76} = 0.28.29 \text{ or } \mathbf{28.29\%}$$

Example 27.9 *In a gas turbine plant, the compressor is driven by the high-pressure turbine. The exhaust from high pressure turbine enters into a low-pressure turbine, which runs the load. The air flow rate is 20 kg/s, and minimum and maximum temperatures in the cycle are 300 K and 1000 K, respectively. The compressor pressure ratio is 4. Calculate the pressure ratio of low-pressure turbine, temperature of exhaust gases from the unit and thermal efficiency of the plant. Compression and expansion are isentropic. C_p for air and exhaust gases can be taken as 1 kJ/kg·K and $\gamma = 1.4$.*

Solution

Given A gas turbine power plant with two stage turbine

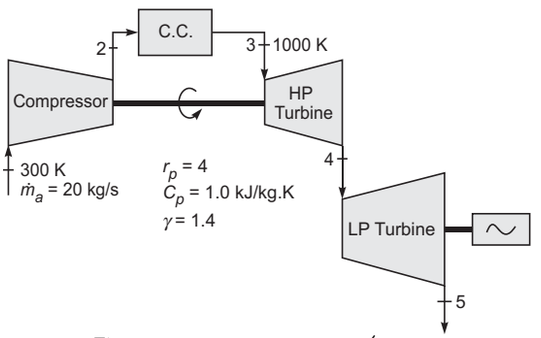
$T_1 = 300\ \text{K}$ $\quad r_p = 4$
$\dot{m}_a = 20\ \text{kg/s}$ $\quad T_3 = 1000\ \text{K}$
$C_p = 1.0\ \text{kJ/kg·K}$ $\quad \gamma = 1.4$
$w_C = w_{HP}$

To find

(i) Pressure ratio of low-pressure turbine,
(ii) Temperature of exhaust gases, and
(iii) Thermal efficiency of the plant.

Analysis Analysing each device in a steady flow manner.

Compressor Temperature T_2 after isentropic compression



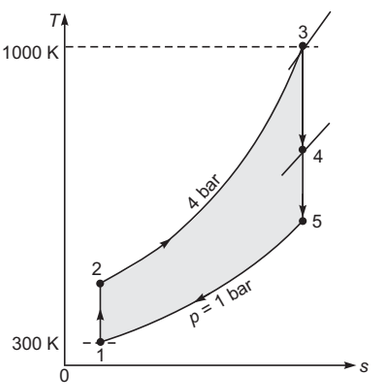


Fig. 27.17

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$$T_2 = T_1(r_p)^{\frac{\gamma-1}{\gamma}} = 300 \times (4)^{\frac{1.4-1}{1}}$$
$$= 445.8\,\text{K}$$

The work input to compressor

= Work output of high pressure turbine

$$w_C = w_{HP}$$
$$C_p(T_2 - T_1) = C_p(T_3 - T_4)$$
$$1.0 \times (445.8 - 300)$$
$$= 1.0 \times (1000 - T_4)$$

or $T_4 = 1000 - (445.8 - 300) = 854.2\,\text{K}$

(i) *Pressure ratio of low-pressure turbine*

The pressure ratio across the H.P. turbine can be given as

$$\frac{p_3}{p_4} = \left(\frac{T_3}{T_4}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{1000}{854.2}\right)^{\frac{1.4}{1.4-1}}$$
$$= 1.736$$

The pressure ratio $\frac{p_3}{p_4}$ can be expressed as

$$\frac{p_3}{p_4} = \frac{p_3}{p_5} \times \frac{p_5}{p_4} = \frac{p_2}{p_1} \times \frac{p_5}{p_4}$$

or $$1.736 = 4 \times \left(\frac{p_5}{p_4}\right)$$

or $$\frac{p_5}{p_4} = \frac{1.736}{4} = \mathbf{0.434}$$

(ii) *Temperature of exhaust gases*

The temperature T_5 of exhaust gases coming out from unit

$$\frac{T_5}{T_4} = \left(\frac{p_5}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = (0.434)^{\frac{1.4-1}{1.4}} = 0.7878$$

or $$T_5 = 0.788 \times 854.2 = \mathbf{672.95\,K}$$

(iii) *Thermal efficiency of the plant*

Net power output per kg of air of the plant

$$w_{net} = w_{Lp} = C_p(T_4 - T_5)$$
$$= 1.0 \times (854.2 - 672.95)$$
$$= 181.25\ \text{kJ/kg}$$

Heat supplied per kg of air

$$q_{in} = C_p(T_3 - T_2)$$
$$= 1.0 \times (1000 - 445.8)$$
$$= 554.2\ \text{kJ/kg}$$
$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{181.25}{554.2} = 0.327 \text{ or } \mathbf{32.7\%}$$

Example 27.10 *Air is drawn in a gas turbine unit at 15°C and 1.01 bar and the pressure ratio is 7:1. The compressor is driven by HP turbine and the LP turbine drives a separate power shaft. The isentropic efficiency of the compressor, HP and LP turbines are 82, 85 and 85%, respectively. If the maximum cycle temperature is 610°C, calculate*

(a) pressure and temperature of gases entering the power turbine

(b) net power developed by the unit per kg/s mass-flow rate

(c) the work ratio

(d) thermal efficiency of the unit

Neglect the mass of fuel and assume the following:

For compression process, C_{pa} = 1.005 kJ/kg·K and γ_a = 1.4,

For combustion and expansion processes, C_{pg} = 1.15 kJ/kg·K and γ_g = 1.33.

Solution

Given A gas turbine power plant with a two-stage turbine

$p_1 = 1.01$ bar, $T_1 = 15°\text{C} = 288\,\text{K}$

$r_p = 7$

$w_C = w_{HP}$, $T_3 = 610°\text{C} = 883\,\text{K}$

$\eta_C = 0.82$

$\eta_{T,HP} = 0.85$, $\eta_{T,LP} = 0.85$

For air $C_{pa} = 1.0$ kJ/kg·K and $\gamma_a = 1.4$

For gases $C_{pg} = 1.15$ kJ/kg·K and $\gamma_g = 1.33$

To find

(i) Pressure and temperature of gases entering low pressure turbine,

(ii) The power developed by the unit per kg/s of mass flow,

(iii) Work ratio, and

(iv) Thermal efficiency of the plant.

Analysis Analysing each device in a steady flow manner.

Compressor Temperature T_{2s} after isentropic compression

$$T_{2s} = T_1(r_p)^{\frac{\gamma-1}{\gamma}} = 288 \times (7)^{\frac{1.4-1}{1}}$$
$$= 502.16\ \text{K}$$

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Schamtic and T–s diagram

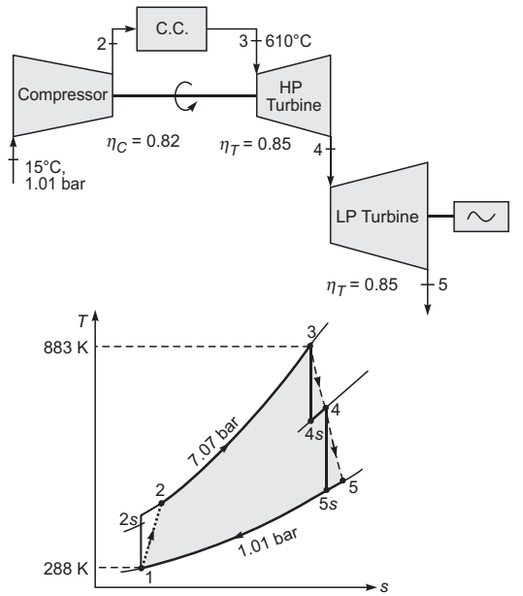

Fig. 27.18 *Schematic and T–s diagram*

The isentropic efficiency of the compressor is given by

$$\eta_C = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

or $$T_2 = 288 + \frac{502.16 - 288}{0.82} = 549.17\,\text{K}$$

(i) *Pressure and temperature entering the power turbine*

The actual work input to compressor w_{in} can also be expressed as

$$w_{in} = C_{pa}(T_2 - T_1)$$
$$= 1.005 \times (549.17 - 288)$$
$$= 262.47\ \text{kJ/kg}$$

Compression work = H.P. turbine work

or $$w_{in} = C_{pg}(T_3 - T_4)$$

$$262.47 = 1.15 \times (883 - T_4)$$

or $$T_4 = \mathbf{654.76\,K}$$

Temperature of gas entering the power turbine is **654.76 K**.

Using isentropic efficiency of HP turbine

$$\eta_{T,HP} = \frac{\text{Actual work output}}{\text{Isentropic work output}} = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

Isentropict emperature,

$$T_{4s} = T_3 - \frac{T_3 - T_4}{\eta_{T,HP}} = 883 - \frac{883 - 654.76}{0.85}$$
$$= 614.48\,\text{K}$$

The pressure ratio across H.P. turbine is given by

$$\frac{p_3}{p_4} = \left(\frac{T_3}{T_{4s}}\right)^{\frac{\gamma_g}{\gamma_g - 1}} = \left(\frac{883}{614.48}\right)^{\frac{1.33}{1.33-1}}$$
$$= 4.311$$

Pressure, $p_4 = \frac{p_3}{4.311} = \frac{7.07}{4.311} = \mathbf{1.64\ bar}$

Pressure of gases entering LP turbine is 1.64 bar.

(ii) *Net power developed per kg/s of mass flow*

The pressure ratio $\frac{p_4}{p_5}$ can be expressed as

$$\frac{p_4}{p_5} = \frac{p_4}{p_3} \times \frac{p_3}{p_5} = \frac{p_4}{p_3} \times \frac{p_2}{p_1} = \frac{7}{4.311}$$
$$= 1.623$$

Then temperature T_{5s} after isentropic expansion

$$\frac{T_4}{T_{5s}} = \left(\frac{p_4}{p_5}\right)^{\frac{\gamma_g - 1}{\gamma_g}} = \left(\frac{1.623}{1}\right)^{\frac{1.33-1}{1.33}}$$
$$= 1.127$$

or $$T_{5s} = \frac{654.76}{1.127} = 580.56\,\text{K}$$

For LP turbine, isentropic efficiency is given by

$$\eta_{T,LP} = \frac{T_4 - T_5}{T_4 - T_{5s}}$$

Actual temperature,

$$T_5 = T_4 - \eta_{T,LP}(T_4 - T_{5s})$$
$$= 654.76 - 0.85 \times (654.76 - 580.56)$$
$$= 591.64\ \text{K}$$

Net power output per kg/s of mass flow rate

$$w_{net} = w_{LP} = C_{pg}(T_4 - T_5)$$
$$= 1.15 \times (654.76 - 591.64)$$
$$= \mathbf{72.59\ kJ/kg}$$

(iii) *Work ratio*

$$\text{Work ratio} = \frac{\text{Net work output}}{\text{Total work produced}}$$

$$= \frac{72.59}{262.47 + 72.59} = \mathbf{0.2166}$$

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(iv) *Thermal efficiency of the plant*

Heat supplied per kg of air

$$q_{in} = C_{pg}(T_3 - T_2)$$
$$= 1.15 \times (883 - 549.17)$$
$$= 383.9 \text{ kJ/kg}$$
$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{72.59}{383.9}$$
$$= 0.189 \quad \text{or} \quad \mathbf{18.9\%}$$

27.6.2 Gas Turbine Cycle with Reheat

For metallurgical reasons, the temperature of the combustion products entering the turbine cannot be increased to a higher level. By introducing reheating during expansion, the turbine work and consequently, the network of the gas turbine cycle can be improved without changing compressor work or maximum temperature in the cycle. The expansion of combustion products takes place in two or more turbines with constant pressure heating before each expansion. This constant pressure heating between two turbine expansions is known as *reheating* and this modified cycle is known as *reheat cycle*.

The basic features of a two-stage gas turbine with reheat are brought out by considering an ideal air-standard Brayton cycle modified as shown in Fig. 27.19(a) After expansion from the state 3 to state 4 in the first turbine, the air is reheated at constant pressure from the state 4 to state 5. The expansion is then completed in the second turbine from the state 5 to the state 6. The ideal Brayton cycle without reheat is cycle 1–2–3–6′–1 and with reheat is cycle 1–2–3–4–5–6–1 as shown in Fig. 27.19(b).

Net work output per kg of air in a cycle

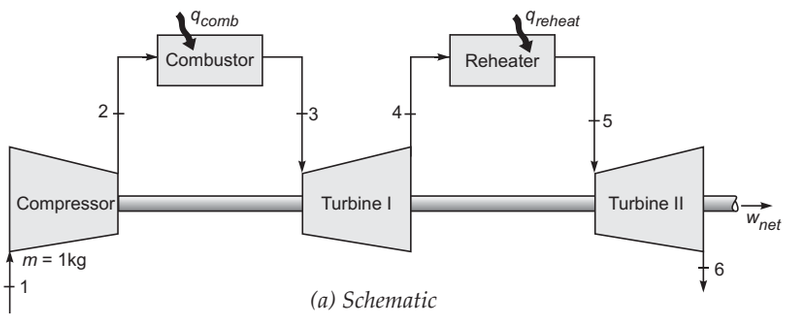
$$w_{net} = C_p(T_3 - T_4) + C_p(T_5 - T_6) - C_p(T_2 - T_1) \quad ...(27.12)$$

Heat supplied per kg of air in a cycle

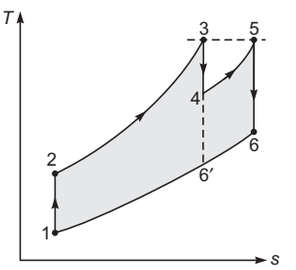
$$q_{in} = q_{comb} + q_{reheat}$$
$$= C_p(T_3 - T_2) + C_p(T_5 - T_4) \quad ...(27.13)$$

Thermal efficiency of reheat cycle can be expressed as

$$\eta_{reheat} = \frac{w_{net}}{q_{in}} = \frac{C_p(T_3 - T_4) + C_p(T_5 - T_6) - C_p(T_2 - T_1)}{C_p(T_3 - T_2) + C_p(T_5 - T_4)} \quad ...(27.14)$$



(a) Schematic



(b) T–s diagram

Fig. 27.19 *Ideal gas turbine with reheat*

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It can be analytically proved that the maximum net work output from the plant would be for

$$r_{p_1} = \sqrt{r_p} \qquad ...(27.15)$$

where $r_{p_1} = \dfrac{p_2}{p_4} = \dfrac{p_5}{p_6}$, intermediate pressure ratio, and

$$r_p = \frac{p_2}{p_1} = \frac{p_3}{p_6}, \text{ total pressure ratio.}$$

The total work of a two-stage turbine is greater than that of a single expansion from state 3 to state 6′. Thus, the network of reheat cycle is gerater than that of a cycle without reheat. Despite the increase in net work with reheat, the thermal efficiency would not necessarily improve because of an increase in total heat addition to the cycle. However, the temperature at the exit of the turbine is higher with reheat than without reheat, so potential for regeneration is enhanced.

Example 27.11 *Air enters the compressor of an ideal air-standard Brayton cycle at 100 kPa, 300 K and is compressed to 1000 kPa. The temperature at the inlet to the first turbine is 1400 K. The expansion takes place isentropically in two stages with reheat to 1400 K between the two stages at a constant pressure of 300 kPa. A regenerator having an effectiveness of 100% is also incorporated in the cycle. Determine the thermal efficiency of the cycle.*

Solution

Given An ideal air-standard Brayton cycle with reheat and regeneration:

$p_1 = 100$ kPa $\quad T_1 = 300$ K
$p_2 = 1000$ kPa $\quad T_3 = 1400$ K
$T_5 = 1400$ K $\quad p_4 = p_5 = 300$ kPa
$\varepsilon_{reg} = 1.0$

To find Thermal efficiency of the cycle.

Assumptions

(i) Each component in the cycle is analysed as control volume at steady state.
(ii) Compressor and turbine processes are isentropic.
(iii) No pressure drop for flow through the heat exchanger.
(iv) Working fluid is air with its standard properties.

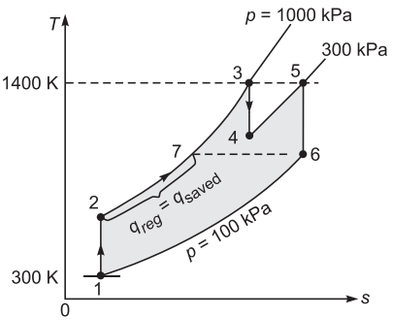

Fig. 27.20

Analysis The temperature after isentropic compression

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 300 \times \left(\frac{1000}{100}\right)^{\frac{1.4-1}{1.4}}$$
$$= 579.2 \text{ K}$$

Temperature after isentropic expansion 3–4;

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}}$$

or
$$T_4 = T_3\left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = 1400 \times \left(\frac{300}{1000}\right)^{\frac{1.4-1}{1.4}}$$
$$= 992.5 \text{ K}$$

Temperature after isentropic expansion 5–6;

$$T_6 = T_5\left(\frac{p_6}{p_5}\right)^{\frac{\gamma-1}{\gamma}} = 1400 \times \left(\frac{100}{300}\right)^{\frac{1.4-1}{1.4}}$$
$$= 1022.8 \text{ K}$$

Total turbine work;

$$w_T = C_p(T_3 - T_4) + C_p(T_5 - T_6)$$
$$= 1.005 \times [1400 - 992.5] + 1.005 \times [1400 - 1022.8]$$
$$= 409.53 + 379.08 = 788.6 \text{ kJ}$$

Compressor work;

$$w_C = C_p(T_2 - T_1) = 1.005 \times [579.2 - 300]$$
$$= 280.6 \text{ kJ}$$

Net work of the cycle

$$= w_T - w_C$$
$$= 788.6 - 280.6 = 508 \text{ kJ}$$

Total heat supplied in the cycle

$$q_{in} = C_p(T_3 - T_6) + C_p(T_5 - T_4)$$
$$= 1.005 \times (1400 - 1022.8) + 1.005 \times (1400 - 992.5)$$
$$= 379.08 + 409.53 = 788.6 \text{ kJ}$$

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The thermal efficiency of the cycle

$$\eta = \frac{\text{Network done/kg}}{\text{Total heat supplied/kg}}$$

$$= \frac{508}{788.6} = 0.644 = \mathbf{64.4\%}$$

Example 27.12 *The following data refers to a gas turbine plant:*

Power developed = 5 MW

Inlet pressure and temperature of air to compressor = 1 bar and 30°C

Pressure ratio of the cycle = 5

Isentropic efficiency of the compressor = 80%

Isentropic efficiency of turbines = 85%

Maximum temperature in the turbines = 550°C

Take for air, C_p = 1.0 kJ/kg·K, γ = 1.4 *and*

for gases, C_p = 1.15 kJ/kg·K, γ = 1.33.

If a reheater is used between two turbines at a pressure of 2.24 bar, calcualte the following:

(a) Mass flow rate of air,

(b) The overall efficiency,

Neglect the mass of fuel.

Solution

Given A gas turbine plant with reheating and regeneration:

p_1 = 1 bar T_1 = 30°C = 303 K

r_p = 5 $p_5 = p_6$ = 2.24 bar

p_2 = 5 bar T_3 = 550°C = 823 K

T_5 = 823 K η_C = 0.8

η_T = 0.85 P = 5 MW = 5000 kW

For air C_{pa} = 1.0 kJ/kg·K γ_a = 1.4 and

For gases C_{pg} = 1.15 kJ/kg·K γ_g = 1.33

To find

(i) Mass flow rate of air, and

(ii) Overall efficiency of the cycle.

Assumptions

(i) Compression and expansion are adiabatic.

(ii) Each device operates in steady flow manner.

(iii) No pressure drop during flow of gas through combustion chamber.

Analysis Analysing each device in steady flow manner.

Compressors The temperature after isentropic compression 1–2:

$$T_{2s} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 303 \times \left(\frac{5}{1}\right)^{\frac{1.4-1}{1.4}}$$

$$= 479.9 \text{ K}$$

The isentropic efficiency of the compressor is given by

$$\eta_C = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

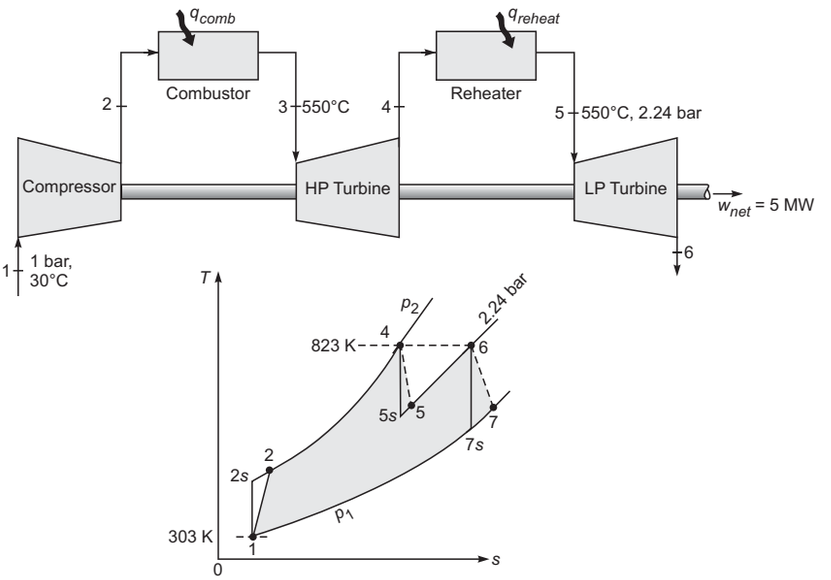

Fig. 27.21 *Schematic and T–s diagram*

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or $$T_2 = 303 + \frac{479.9 - 303}{0.8} = 524.12\,\text{K}$$

The actual work input to compressor w_{in} can also be expressed as

$$w_{in} = C_p(T_2 - T_1)$$
$$= 1.0 \times (524.12 - 303) = 221.12\ \text{kJ/kg}$$

HP Turbine The temperature T_{5s} after isentropic expansion in the HP turbine

$$T_{5s} = \frac{T_4}{\left(\frac{p_2}{p_5}\right)^{\frac{\gamma_g - 1}{\gamma_g}}} = \frac{823}{\left(\frac{5}{2.24}\right)^{\frac{1.33-1}{1.33}}}$$
$$= 674.33\ \text{K}$$

Using isentropic efficiency of HP turbine;

$$\eta_{T,HP} = \frac{T_4 - T_5}{T_4 - T_{5s}}$$

Actual temperature after expansion in HP turbine,

$$T_5 = 823 - 0.85 \times (823 - 674.33)$$
$$= 696.63\ \text{K}$$

LP Turbine The temperature T_{7s} after isentropic expansion in the LP turbine

$$T_{7s} = \frac{T_6}{\left(\frac{p_5}{p_1}\right)^{\frac{\gamma_g - 1}{\gamma_g}}} = \frac{823}{\left(\frac{2.24}{1}\right)^{\frac{1.33-1}{1.33}}}$$
$$= 673.74\ \text{K}$$

Using isentropic efficiency of LP turbine;

$$\eta_{T,LP} = \frac{T_6 - T_7}{T_6 - T_{7s}}$$

Actual temperature after expansion in LP turbine,

$$T_7 = 823 - 0.85 \times (823 - 673.74)$$
$$= 696.13\ \text{K}$$

Total turbine work output per kg of air

$$w_{out} = w_{HP} + w_{LP}$$
$$= C_{pg}(T_4 - T_5) + C_{pg}(T_6 - T_7)$$
$$= 1.15 \times (823 - 696.74) + 1.15 \times (823 - 696.13)$$
$$= 145.2 + 145.9 = 291\ \text{kJ/kg}$$

Net work output per kg of the plant

$$w_{net} = w_{out} - w_{in} = 291 - 221.12$$
$$= 69.88\ \text{kJ/kg of air}$$

(i) *Mass-flow rate of air*

Power developed by plant is given by

$$P = \dot{m}_a w_{net}$$

or $$5000 = 69.88\ \dot{m}_a$$

or $$\dot{m}_a = \mathbf{75.55\ kg/s}$$

(ii) *Overall efficiency of plant*

The heat supplied to air

$$q_{in} = C_{pg}(T_3 - T_2) + C_{pg}(T_6 - T_5)$$
$$= 1.15 \times (823 - 524.12) + 1.15$$
$$\times (823 - 696.63)$$
$$= 489\ \text{kJ/kg of air}$$

Overall efficiency of the cycle

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{69.88}{489} = 0.1428 \text{ or } \mathbf{14.28\%}$$

27.6.3 Compression with Intercooling

The net work output of a gas turbine can also be increased by reducing the compressor work input. It can be accomplished by use of multistage compression with intercooling, i.e., by an isothermal compression.

The isothermal cooling of gas during compression is difficult to achieve in practice. The gas is compressed in stages with cooling the gas between stages. The heat exchangers used for cooling of gas are known as *intercoolers*.

Figure 27.22 illustrates the effect of intercooling between two compression stages. Process 1–*c* represents an isentropic compression from the state 1 to state *c*, where the intermediate pressure is p_i. The process *c*–*d* represents constant pressure gas cooling in a heat exchanger from temperature T_c to T_d. The process *d*–2 is further isentropic compression to state 2. The work input for compression is represented on a *p*–*v* diagram by area *a*–1–*c*–*d*–2–*b*–*a* which is less than the isentropic compression in a single stage from the state 1 to the state 2*s*, without intercooling, *viz.,* enclosed area *a*–1–2*s*–*b*–1. The hatched area *d*–*c*–2*s*–2–*d* on a *p*–*v* diagram represents reduction in the work input due to intercooling between two stages.

If the gas after first-stage compression is cooled to initial temperature T_1 of air at constant intermediate pressure p_i, the cooling is called *perfect intercooling.*

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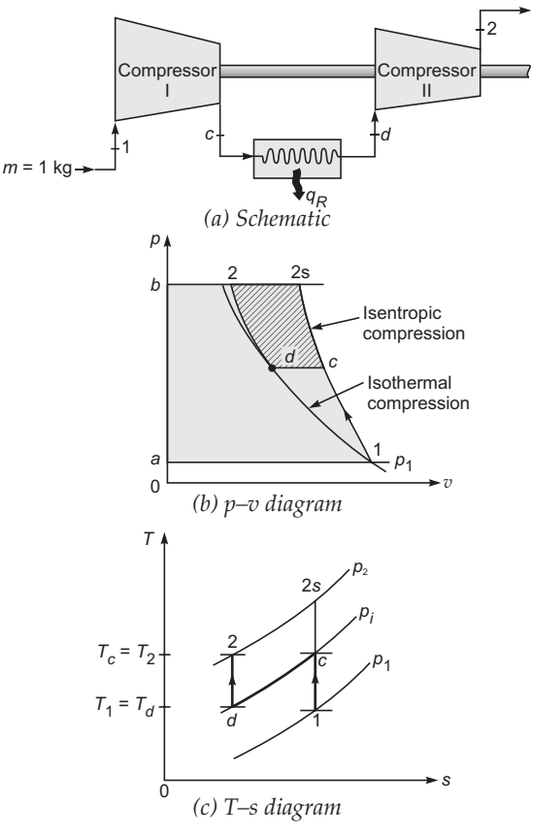

Fig. 27.22 *Two-stage compression with perfect intercooling*

Assuming isentropic steady flow in both compressors with negligible changes in kinetic and potential energies. For 1 kg of air flow into the system,

Work input to first stage compressor

$$w_{C_1} = C_p(T_c - T_1)$$

Work input to second stage compressor

$$w_{C_2} = C_p(T_2 - T_d)$$

Total work input for compression

$$w_C = w_{C_1} + w_{C_2}$$

$$= C_p(T_c - T_1) + C_p(T_2 - T_d) \quad ...(27.16)$$

$$= C_p T_1\left(\frac{T_c}{T_1} - 1\right) + C_p T_d\left(\frac{T_2}{T_d} - 1\right)$$

$$= C_p T_1\left[\left(\frac{p_i}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] + C_p T_d\left[\left(\frac{p_2}{p_i}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \quad ...(27.17)$$

For perfect intercoling between the stages $T_d = T_1$

$$\therefore \quad w_C = C_p T_1\left[\left(\frac{p_i}{p_1}\right)^{\frac{\gamma-1}{\gamma}} + \left(\frac{p_2}{p_i}\right)^{\frac{\gamma-1}{\gamma}} - 2\right]$$

Using $k = \dfrac{\gamma-1}{\gamma}$,

then $$w_C = C_p T_1\left[\left(\frac{p_i}{p_1}\right)^{k} + \left(\frac{p_2}{p_i}\right)^{k} - 2\right] \quad ...(27.18)$$

For minimum compression work or maximum output of the plant, applying the condition of maxima, i.e., differentiating Eq. (27.18) with respect to intermediate pressure p_i and equating it to zero,

$$\frac{dw_C}{dp_i} = C_p T_1\left[k\left(\frac{1}{p_1}\right)^{k}(p_i)^{k-1} - k(p_2)^k(p_i)^{-k-1}\right] = 0$$

or $$(p_i)^{k-1}(p_i)^{k+1} = p_1^k p_2^k$$

or $$(p_i)^{2k} = (p_1 p_2)^k$$

or $$p_i = \sqrt{p_1 p_2} \quad ...(27.19)$$

For minimum compression work, optimum intermediate pressure is $p_i = \sqrt{p_1 p_2}$. The optimization of the compression work also leads to equal pressure ratio across the each stage, i.e.,

$$\frac{p_i}{p_1} = \frac{p_2}{p_i} \quad ...(27.20)$$

The work input for the compression would be reduced to a minimum, if compression is carried out in several stages with perfect intercooling between

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stages, i.e., the compression process approaches to an isothermal compression. The optimization of compression work leads to equal pressure ratios across each stage.

Example 27.13 *Air is compressed from 100 kPa, 300 K to 1000 kPa in a two-stage compressor with intercooling between stages. The air is compressed to 300 kPa and is cooled back to 300 K in an intercooler before entering the second stage compressor. Each compression stage is isentropic. For steady state operation and negligible changes in kinetic and potential energy from inlet to exit, determine (a) temperature at exit of second compressor, (b) the total compressor work input per unit of mass flow, (c) reduction in work input by intercooling and two-stage compression, (d) isothermal compression work.*

Solution

Given Air is compressed at steady state in a two-stage compressor with an intercooler

$p_1 = 100$ kPa $\quad T_1 = 300$ K

$p_i = 300$ kPa $\quad T_d = 300$ K

$p_2 = 1000$ kPa

To find

(i) T_2, temperature at exit of the second compressor,

(ii) Total compressor work,

(iii) Reduction in work input due to mutlistage compression and intercooling, and

(iv) Isothermal compression work.

Assumptions

(i) Each compressor and intercooler are analysed as control volume at steady state.

(ii) There is no pressure drop for flow through the intercooler.

(iii) Working substance in compressor is air as an ideal gas with standard properties

Analysis

(i) Temperature T_2, at exit of second stage compressor;

$$\frac{T_2}{T_d} = \left(\frac{p_2}{p_d}\right)^{\frac{\gamma-1}{\gamma}}$$

or $$T_2 = 300 \times \left(\frac{1000}{300}\right)^{\frac{1.4-1}{1.4}}$$

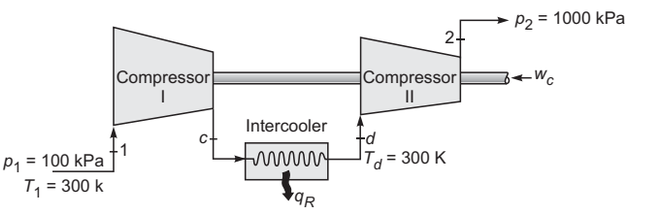
$$= 423\,\text{K} = \mathbf{150°C}$$

(ii) Total work input for compression per unit mass flow. For steady state operation,

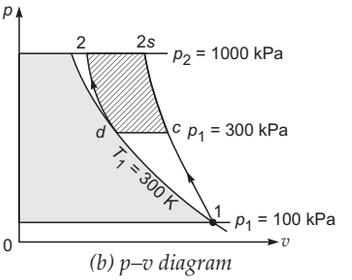
$$w_C = h_c - h_1 + h_2 - h_d$$

$$= C_p(T_c - T_1) + C_p(T_2 - T_d)$$

Schematic with given data



(a) Schematic



(b) p–v diagram

Fig. 27.23

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where $T_c = T_1\left(\frac{p_i}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 300 \times \left(\frac{300}{100}\right)^{\frac{1.4-1}{1.4}}$

$= 410.6$ K

Then $w_C = 1.005 \times (410.6 - 300) + 1.005 \times (423 - 300)$

$= 111.17 + 123.61$

$= \mathbf{234.76\ kJ/kg}$

(iii) Reduction in work input due to intercooler and two stage compression;

Temperature T_{2s} after single stage compression to pressure $p_2 = 1000$ kPa

$$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

or $T_{2s} = 300 \times \left(\frac{1000}{100}\right)^{\frac{1.4-1}{1.4}} = 579.2\,\text{K}$

Single-stage compression work

$w_s = h_{2s} - h_1 = C_p(T_{2s} - T_1)$

$= 1.005 \times (579.2 - 300)$

$= 280.6$ kJ/kg

Reduction in work input for compression

$w_s - w_C = 280.6 - 234.76 = \mathbf{45.84\ kJ/kg}$

(iv) Isothermal work input for compression;

$$w_{iso} = p_1 v_1 \ln\left(\frac{p_2}{p_1}\right) = RT_1 \ln\left(\frac{p_2}{p_1}\right)$$

$$= (0.287 \times 300) \times \ln\left(\frac{1000}{100}\right)$$

$$= \mathbf{198.25\ kJ/kg}$$

Example 27.14 *For a two-stage compressor with perfect intercooling operating at steady state, prove that the minimum total work input is required when the pressure ratio is same for each stage. Use cold air-standard analysis, assuming that compression process is isentropic, no pressure drop through the intercooler and temperature to each compressor stage is same. Kinetic and potential energy effects are negligble.*

Solution

Given Two-stage isentropic compression with perfect intercooling operates at steady state.

$$\frac{p_i}{p_1} = \frac{p_2}{p_i}$$

To find To prove for minimum compression work.

Schematic

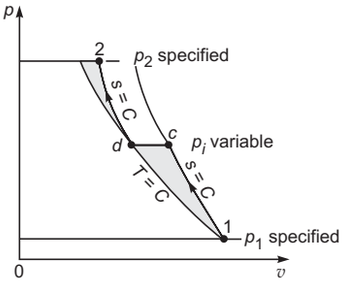

Fig. 27.24

Analysis Total compression work per kg for a two-stage compressor from pressure p_1 to p_2;

$$w_C = h_c - h_1 + h_2 - h_d$$

Since C_p is constant in cold air condition,

$$w_C = C_p(T_c - T_1) + C_p(T_2 - T_d)$$

For perfect intercooling $T_d = T_1$.

$$w_C = C_p(T_c + T_2 - 2T_1)$$

$$= C_p T_1\left[\frac{T_c}{T_1} + \frac{T_2}{T_1} - 2\right]$$

For isentropic compressions

$$\frac{T_c}{T_1} = \left(\frac{p_i}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \text{ and } \frac{T_2}{T_d} = \frac{T_2}{T_1} = \left(\frac{p_2}{p_i}\right)^{\frac{\gamma-1}{\gamma}}$$

Using $k = \frac{\gamma - 1}{\gamma}$

we get $w_C = C_p T_1\left[\left(\frac{p_i}{p_1}\right)^k + \left(\frac{p_2}{p_i}\right)^k - 2\right]$

For specified values of T_1, T_2, p_1, p_2 and constant C_p, the compressor work would be minimum, when

$$\frac{dw_C}{dp_i} = 0$$

or $$\frac{d}{dp_i}\left\{C_p T_1\left[\left(\frac{p_i}{p_1}\right)^k + \left(\frac{p_2}{p_i}\right)^k - 2\right]\right\} = 0$$

or $$C_p\,T_1\left\{k\,(p_i)^{k-1}\left(\frac{1}{p_1}\right)^k - k\,(p_2)^k\,(p_i)^{-k-1}\right\} = 0$$

or $$\left(\frac{p_i}{p_1}\right)^k \times \frac{1}{p_i} = \left(\frac{p_2}{p_i}\right)^k \times \frac{1}{p_i}$$

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or $$\left(\frac{p_i}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_2}{p_i}\right)^{\frac{\gamma-1}{\gamma}}$$

or $$\frac{p_i}{p_1} = \frac{p_2}{p_i}$$

It is the desired relation for minimum compression work input.

Example 27.15 *In a gas turbine plant, compression is carried out in two stages with perfect intercooling and expansion in a one-stage turbine. If maximum temperature T_{max} and minimum temperature T_{min} in the cycle remains constant, show that for maximum specific power output of the plant, the optimum overall pressure ratio is given by*

$$r_{opt} = \left(\eta_T \eta_C \frac{T_{max}}{T_{min}}\right)^{\frac{2\gamma}{3(\gamma-1)}}$$

Solution

Temperature T_{2s} after first stage isentropic compression

$$T_{2s} = T_1(r_{p_1})^{\frac{\gamma-1}{\gamma}}$$

For perfect intercooling,

$$r_{p_1} = (r_p)^{\frac{1}{2}}$$

$$\therefore \quad T_{2s} = T_{min}(r_p)^{\frac{\gamma-1}{2\gamma}}$$

The actual work input to compressor w_{in} in two stages can also be expressed as

$$w_C = 2\,C_p\,(T_2 - T_1)$$

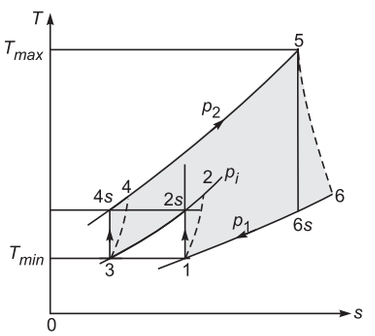

Fig. 27.25 *T–s diagram*

The isentropic efficiency of the compressor is given by

$$\eta_C = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$\therefore \quad w_C = 2\,C_p\,\frac{T_{2s} - T_1}{\eta_C} = \frac{2C_p}{\eta_C} T_{min}\left(r_p^{\frac{\gamma-1}{2\gamma}} - 1\right)$$

The temperature T_{6s} after isentropic expansion in the turbine

$$T_{6s} = \frac{T_5}{(r_p)^{\frac{\gamma-1}{\gamma}}} = \frac{T_{max}}{(r_p)^{\frac{\gamma-1}{\gamma}}}$$

Using isentropic efficiency of the turbine

$$\eta_T = \frac{T_5 - T_6}{T_5 - T_{6s}}$$

Turbine work output per kg of air

$$w_T = C_p(T_5 - T_6)$$

$$= C_p\eta_T\left(T_5 - T_{6s}\right) = C_p\eta_T\,T_{max}\left(1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}}\right)$$

Net work done per kg of air

$$w_{net} = w_T - w_C$$

$$= C_p\eta_T T_{max}\left(1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}}\right) - \frac{2C_pT_{min}}{\eta_C}\left(r_p^{\frac{\gamma-1}{2\gamma}} - 1\right)$$

For maximum work output

$$\frac{dw_{net}}{dr_p} = 0$$

$$-C_p\eta_T T_{max}\left(-\frac{\gamma-1}{\gamma}\right)\left(r_p^{-\left(\frac{\gamma-1}{\gamma}\right)-1}\right)$$

$$-\frac{2C_pT_{min}}{\eta_C}\left(\frac{\gamma-1}{2\gamma}\right)\left(r_p^{\frac{\gamma-1}{2\gamma}-1}\right) = 0$$

or $$\eta_T T_{max}\left(r_p^{-\left(\frac{\gamma-1}{\gamma}\right)-1}\right) - \frac{T_{min}}{\eta_C}\left(r_p^{\frac{\gamma-1}{2\gamma}-1}\right) = 0$$

or $$\eta_T\eta_C\frac{T_{max}}{T_{min}} = r_p^{\frac{3(\gamma-1)}{2\gamma}} = 0$$

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Hence optimum pressure ratio is

$$r_{opt} = \left[\eta_T \eta_C \frac{T_{max}}{T_{min}}\right]^{\frac{2\gamma}{3(\gamma-1)}}$$

Example 27.16 *The air supplied to a gas turbine plant is 10 kg/s. The pressure ratio is 6 and pressure at the inlet of compressor is 1 bar. The compressor is two-stage and is provided with perfect intercooling. The inlet temperature is 300 K and maximum temperature is limited to 1073 K. Take the following data:*

Isentropic efficiency of compressor at each stage = 80%

Isentropic efficiency of turbine = 85%

A regenerator is included in a plant whose effectiveness is 0.7. Neglecting the mass of fuel, determine the thermal efficiency of the plant.

Take C_p for air as 1.005 kJ/kg·K.

Solution

Given A gas turbine power plant with two-stage compression and regeneration

$p_1 = 1$ bar, $T_1 = 300$ K, $r_p = 6$, $\eta_C = 0.8$, $\eta_T = 0.85$, $T_6 = 1073$ K, $\varepsilon = 0.7$, $\dot{m}_a = 10$ kg/s, $C_p = 1.005$ kJ/kg·K

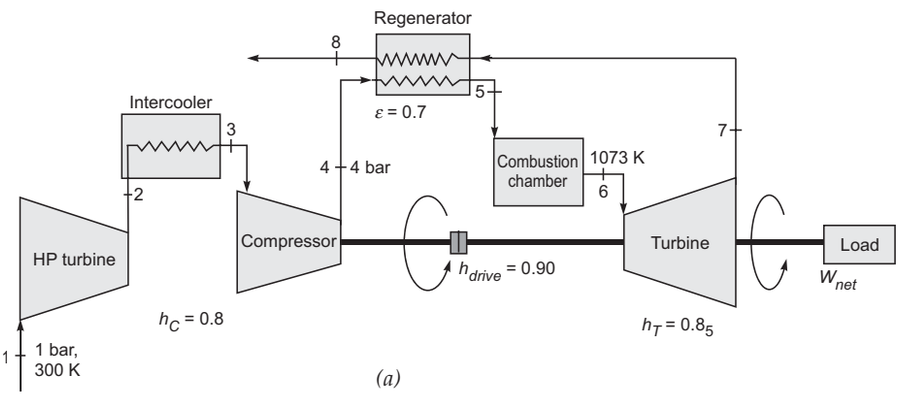
To find Thermal efficiency of the cycle.

Analysis Analysing each device in a steady flow manner.

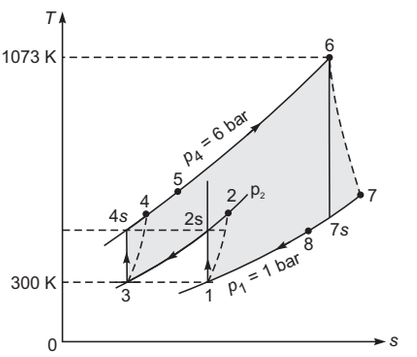
Compressors For perfect intercooling, the pressure ratio of a stage

$$r_{p_1} = \sqrt{p_1 p_3} = \sqrt{1 \times 6} = 2.45$$

Schematic



(a)



(b) T–s diagram

Fig. 27.26

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Temperature T_{2s} after isentropic compression

$$T_{2s} = T_1(r_{p_1})^{\frac{\gamma-1}{\gamma}} = 300 \times (2.45)^{\frac{1.4-1}{1.4}}$$
$$= 387.51\,\text{K}$$

The isentropic efficiency of LP compressor is given by

$$\eta_{C,LP} = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

or $$T_2 = 300 + \frac{387.51 - 300}{0.8} = 409.4\,\text{K}$$

The actual work input to LP compressor

$$w_{in,LP} = C_p(T_2 - T_1)$$
$$= 1.005 \times (409.4 - 300) = 109.93\ \text{kJ/kg}$$

For perfect intercooling,

$$T_3 = T_1\ ;\ r_{p_1} = \frac{p_2}{p_1} = \frac{p_4}{p_3} \text{ and } T_4 = T_2$$

$\therefore$ $$w_{in,LP} = w_{in,HP}$$

Therefore, the total work input of two stages

$$w_{in} \cong 2\,w_{in,LP} = 219.87\,\text{kJ/kg}$$

Turbine The temperature T_{7s} after isentropic expansion from a pressure of 6 bar to 1 bar

$$T_{7s} = \frac{T_6}{\left(\frac{p_4}{p_1}\right)^{\frac{\gamma-1}{\gamma}}} = \frac{1073}{\left(\frac{6}{1}\right)^{\frac{1.4-1}{1.4}}} = 643\,\text{K}$$

The isentropic efficiency of the turbine is given as

$$\eta_T = \frac{\text{Actual work output}}{\text{Isentropic work output}} = \frac{T_6 - T_7}{T_6 - T_{7s}}$$

Actual temperature after expansion in turbine,

$$T_7 = 1073 - 0.85 \times (1073 - 643)$$
$$= 707.57\ \text{K}$$

Turbine work output per kg of air

$$w_{out} = C_p(T_6 - T_7)$$
$$= 1.005 \times (1073 - 707.57)$$
$$= 367.25\ \text{kJ/kg}$$

Net work output per kg of the plant

$$w_{net} = w_{out} - w_{in} = 367.25 - 219.87$$
$$= 143.38\,\text{kJ/kg of air}$$

Heat exchanger

The effectiveness of a heat exchanger is given by

$$\varepsilon = \frac{T_5 - T_4}{T_7 - T_4}$$

Given that $\varepsilon = 0.7$,

$\therefore$ $$0.7 = \frac{T_5 - 409.4}{707.57 - 409.4}$$

Thus, the temperature T_5 of air leaving a heat exchanger

or $$T_5 = 409.4 + 0.7 \times (707.57 - 409.4)$$
$$= 618.11\ \text{K}$$

The heat supplied to air

$$q_{in} = C_p(T_6 - T_5) = 1.005 \times (1073 - 618.11)$$
$$= 457.15\ \text{kJ/kg}$$

Thermal efficiency of the cycle

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{143.38}{457.15} = 0.3136 \text{ or } \mathbf{31.36\%}$$

27.6.4 Gas Turbine with Reheating and Regeneration

When reheat and regeneration are used together, the thermal efficiency can be increased significantly. Figure 27.27 shows a schematic atrrangement of a

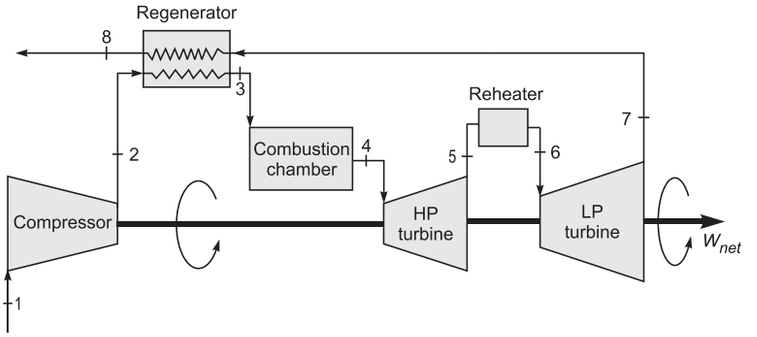

Fig. 27.27 *Gas turbine cycle with reheat and regeneration*

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gas turbine plant with reheat and regeneration, and Fig. 27.28 shows the corresponding *T–s* diagram. In this plant, the heat supply is reduced by the amount of heat recovered in the regenerator.

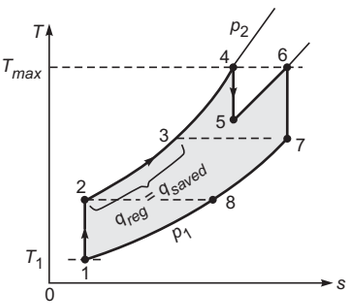

Fig. 27.28 *T–s diagram for reheat and regeneration cycle*

It can be analytically proved that the maximum net work output from the plant would be for

$$r_{p_1} = \sqrt{r_p} \qquad ...(27.21)$$

where $r_{p_1} = \frac{p_2}{p_5} = \frac{p_5}{p_1}$, intermediate pressure ratio, and

$$r_p = \frac{p_2}{p_1}, \text{ total pressure ratio.}$$

For a cycle with isentropic compression and isentropic expansion, ideal regeneration and best division of pressure between stages for reheat, the net work done and heat supplied are calculated as;

$$w_{net} = 2\,C_p(T_4 - T_5) - C_p(T_2 - T_1) \qquad ...(27.22)$$

Heat supplied per kg of air in a cycle

$$q_{in} = q_{comb} + q_{reheat}$$
$$= C_p(T_4 - T_3) + C_p(T_6 - T_5) \qquad ...(27.23)$$

Thermal efficiency of reheat cycle can be expressed as

$$\eta_{reheat} = \frac{w_{net}}{q_{in}}$$
$$= \frac{2C_p(T_4 - T_5) - C_p(T_2 - T_1)}{C_p(T_4 - T_3) + C_p(T_6 - T_5)} \qquad ...(27.24)$$

Example 27.17 *In a gas turbine plant of 6 MW capacity, air enters the compressor at 100 kPa, 300 K and is compressed to a pressure of 600 kPa in one stage. The temperature at the inlet to first turbine is 1000 K. The expansion takes place in two stages with reheat to 1000 K between the two stages . The isentropic efficiency of the compressor is 80% and that of both turbines is 85%. A regenerator having an effectiveness of 0.72 is also incorporated in the cycle to heat the compressed air before entering into combustion chamber. The calorific value of fuel is 18500 kJ/kg. Determine the following:*

(a) A/F ratio entering the first turbine,
(b) Thermal efficiency of the cycle,
(c) Air supply to plant,
(d) Fuel consumption of plant per hour.

Take for air C_p = 1.0 kJ/kg·K, γ = 1.4 and for gases, C_p = 1.15 kJ/kg·K, γ = 1.34

Solution

Given A gas turbine plant with reheating and regeneration

p_1 = 100 kPa T_1 = 300 K
p_2 = 600 kPa T_4 = 1000 K
T_6 = 1000 K η_C = 0.8
$\eta_{T,HP} = \eta_{T,LP}$ = 0.85 ε = 0.72
CV = 18500 kJ/kg

and P = 6 MW = 6000 kW

For air C_{pa} = 1.0 kJ/kg·K γ_a = 1.4 and

For gases C_{pg} = 1.15 kJ/kg·K γ_g = 1.34

To find

(i) A/F ratio entering the first turbine,
(ii) Thermal efficiency of the cycle,
(iii) Air supply to plant, and
(iv) Fuel consumption of plant per hour.

Analysis Analysing each device in a steady flow manner.

Compressors The temperature after isentropic compression 1–2:

$$T_{2s} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 300 \times \left(\frac{600}{100}\right)^{\frac{1.4-1}{1.4}}$$
$$= 500.55 \text{ K}$$

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Schematic and T–s diagram

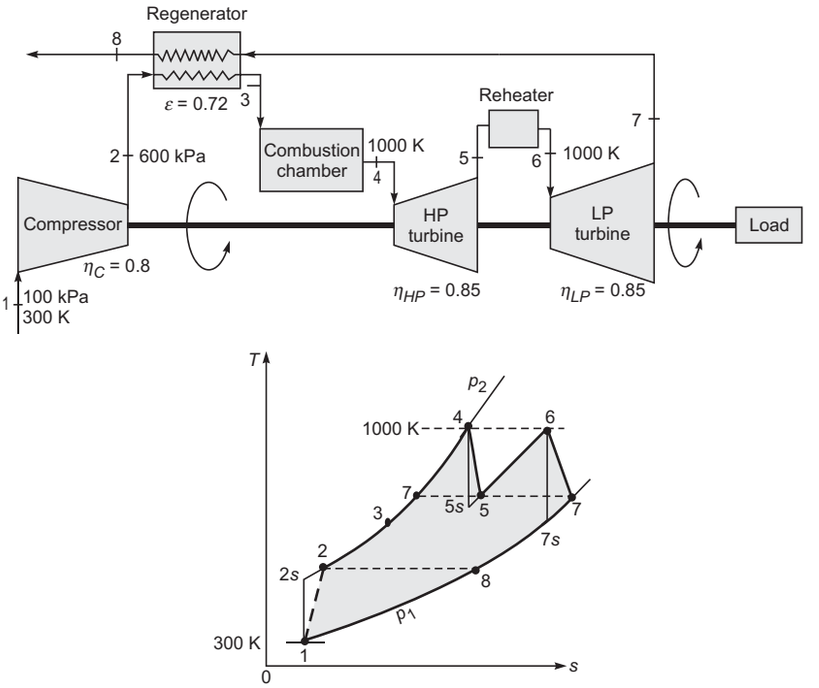

Fig. 27.29

The isentropic efficiency of the compressor is given by

$$\eta_C = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

or $$T_2 = 300 + \frac{500.55 - 300}{0.8} = 550.68\,\text{K}$$

The actual work input to compressor w_{in} can also be expressed as

$$w_{in} = C_p(T_2 - T_1)$$
$$= 1.0 \times (550.68 - 300) = 250.68\ \text{kJ/kg}$$

Turbines For perfect pressure division, the intermediate pressure between stages

$$p_5 = \sqrt{p_1 p_2} = \sqrt{100 \times 600} = 245\ \text{kPa}$$

The temperature T_{5s} after isentropic expansion in the HP turbine

$$T_{5s} = \frac{T_4}{\left(\frac{p_2}{p_3}\right)^{\frac{\gamma_g - 1}{\gamma_g}}} = \frac{1000}{\left(\frac{600}{245}\right)^{\frac{1.34-1}{1.34}}} = 796.7\,\text{K}$$

Using isentropic efficiency of the turbine

$$\eta_{T,HP} = \frac{T_4 - T_5}{T_4 - T_{5s}}$$

Actual temperature after expansion in HP turbine,

$$T_5 = 1000 - 0.85 \times (1000 - 796.7)$$
$$= 827.2\ \text{K}$$

Since $\eta_{T,HP} = \eta_{T,LP}$

$\therefore$ $$T_5 = T_7 = 827.2\ \text{K}$$

Heat exchanger

The effectiveness of a heat exchanger is given by

$$\varepsilon = \frac{T_3 - T_2}{T_7 - T_2}$$

Given that $\varepsilon = 0.72$,

$\therefore$ $$0.72 = \frac{T_3 - 550.68}{827.2 - 550.68}$$

Thus the temperature T_5 of air leaving heat exchanger

or $$T_3 = 550.68 + 0.72 \times (827.2 - 550.68)$$
$$= 749.7\ \text{K}$$

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(i) *A/F ratio supplied to HP turbine*

Let m_{f_1} = mass of fuel supplied per kg of air to main combustion chamber, then energy balance reveals that

$$m_{f_1} CV = (1 + m_{f_1}) C_{pg} (T_4 - T_3)$$

or $$m_{f_1} \times 18500 = (1 + m_{f_1}) \times 1.15 \times (1000 - 749.7)$$

$$18500\, m_{f_1} = 250.3 + 250.3\, m_{f_1}$$

It gives $$m_{f_1} = 0.0137 \text{ kg/kg of air}$$

Air fuel ratio $$\text{A/F} = \frac{1}{0.0137} = \mathbf{72.9}$$

(ii) *Thermal efficiency of a cycle*

Considering m_{f_2} is additional fuel added in a reheater then

$$m_{f_2} CV = (1 + m_{f_1} + m_{f_2}) C_{pg} (T_6 - T_5)$$

or $$m_{f_2} \times 18500 = (1 + m_{f_1} + m_{f_2}) \times 1.15 \times (1000 - 827.2)$$

$$18500\, m_{f_2} = 198.72 + 198.72 \times 0.0137 + 198.72\, m_{f_2}$$

It gives $$m_{f_2} = 0.011 \text{ kg/kg of air}$$

Total turbine work output per kg of air

$$w_{out} = w_{HP} + w_{LP}$$
$$= (1 + m_{f_1})\, C_{pg} (T_4 - T_5) + (1 + m_{f_1} + m_{f_2})\, C_{pg} (T_6 - T_7)$$
$$= (1 + 0.0137) \times 1.15 \times (1000 - 827.2) + (1 + 0.0137 + 0.011) \times 1.15 \times (1000 - 827.2)$$
$$= 201.14 + 203.62 = 404.76 \text{ kJ/kg}$$

Net work output per kg of the plant

$$w_{net} = w_{out} - w_{in} = 404.76 - 250.68$$
$$= 154.08 \text{ kJ/kg of air}$$

The heat supplied to air

$$q_{in} = (m_{f_1} + m_{f_2})\, CV$$
$$= (0.0137 + 0.011) \times 18500$$
$$= 456.95 \text{ kJ/kg of air}$$

Thermal efficiency of the cycle

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{154.08}{456.95}$$
$$= 0.337 \quad \text{or} \quad \mathbf{33.7\%}$$

(iii) *Air supplied to the plant*

Power developed by plant is given by

$$P = \dot{m}_a w_{net}$$

or $$6000 = 154.08\, \dot{m}_a$$

or $$\dot{m}_a = \mathbf{38.94 \text{ kg/s}}$$

(iv) *Fuel consumption of plant per hour*

$$\dot{m}_f = \dot{m}_a (m_{f_1} + m_{f_2}) \times 3600$$
$$= 38.94 \times (0.0137 + 0.011) \times 3600$$
$$= \mathbf{3462.62 \text{ kg/h}}$$

Example 27.18 *In a gas turbine plant, the air from the compressor passes through a heat exchanger, heated by exhaust gases coming from a low-pressure turbine. The air then enters the high-pressure combustion chamber. The high-pressure turbine drives the compressor only. The exhaust gases coming out of the high-pressure turbine is heated into a low-pressure combustion chamber and then enters the low-pressure turbine, which is coupled to external load. The following data refers to the plant:*

Pressure ratio in the compressor, 4:1

Isentropic efficiency of the compressor, 0.85

Isentropic efficiency of high-pressure turbine, 0.84

Isentropic efficiency of low-pressure turbine, 0.8

Effectiveness of heat exchanger, 0.75

Mechanical efficiency of the drive to the compressor, 0.90

Temperature of gases entering HP turbine, 660°C

Temperature of gases entering LP turbine, 625°C

Atmospheric pressure and temperature are 1 bar and 17°C.

Assume specific heat of gas and air as 1.005 kJ/kg·K and $\gamma = 1.4$.

Calculate the pressure of gases entering the low-pressure turbine and overall efficiency of the plant.

Solution

Given A gas turbine plant with reheat and regeneration

$p_1 = 1$ bar	$T_1 = 17°C = 290$ K
$p_2 = 4$ kPa	$T_4 = 660°C = 933$ K
$T_6 = 625°C = 898$ K	$\varepsilon = 0.75$
$\eta_C = 0.85$	$\eta_{HP} = 0.84$
$\eta_{LP} = 0.8$	$\eta_{drive} = 0.9$
$C_p = 1.005$ kJ/kg	$\gamma = 1.4$

To find

(i) Pressure of gas entering LP turbine, and

(ii) Thermal efficiency of the plant.

Assumptions

(i) Each component in the cycle is analysed as control volume at steady state.

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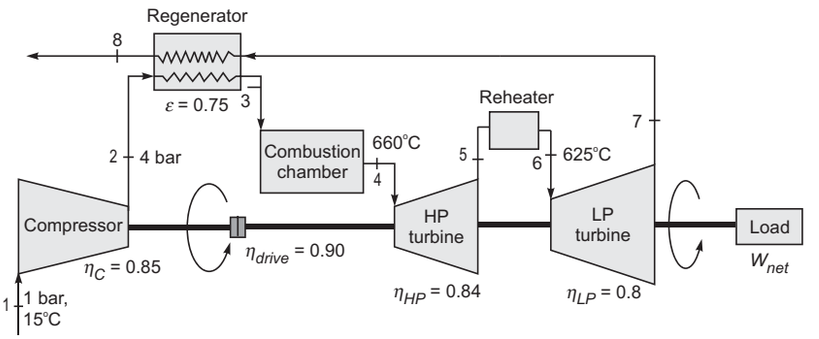
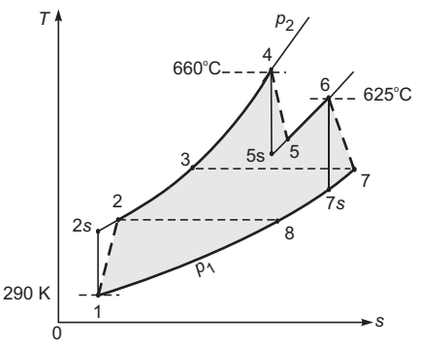
Schematic with given data



(a) Schematic



(b) T-s diagram

Fig. 27.30

(ii) No pressure drop for flow through the heat exchangers.

(iii) Kinetic and potential energy effects are negligible.

Analysis The temperature after isentropic compression 1–2:

$$T_{2s} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 290 \times \left(\frac{4}{1}\right)^{\frac{1.4-1}{1.4}}$$
$$= 430.94 \text{ K}$$

The isentropic efficiency of the compressor is given by

$$\eta_C = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

or
$$T_2 = 290 + \frac{430.94 - 290}{0.85} = 455.81 \text{ K}$$

The actual work input to compressor w_{in} can also be expressed as

$$w_{in} = h_2 - h_1 = C_p(T_2 - T_1)$$
$$= 1.005 \times (455.81 - 290)$$
$$= 166.64 \text{ kJ/kg}$$

Turbines

Work input to compressor

$$= \eta_{drive} \times \text{Work done by HP turbine}$$
$$w_{in} = \eta_{drive} \times C_p(T_4 - T_5)$$

or $166.64 = 0.9 \times 1.005 \times (933 - T_5)$

or
$$T_5 = 933 - \frac{166.94}{0.9 \times 1.005} = 748.43 \text{ K}$$

For HP turbine, the isentropic efficiency is given as

$$\eta_{HP} = \frac{\text{Actual work output}}{\text{Isentropic work output}} = \frac{T_4 - T_5}{T_4 - T_{5s}}$$

or
$$0.84 = \frac{933 - 748.43}{933 - T_{5s}}$$

or $T_{5s} = 736.65$ K

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(i) *Pressure of gas entering LP turbine*

The pressure ratio of HP turbine can be expressed as

$$\frac{p_4}{p_5} = \left(\frac{T_4}{T_{5s}}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{933}{736.65}\right)^{\frac{1.4}{1.4-1}}$$
$$= 2.286$$

or $$p_5 = \frac{4 \text{ bar}}{2.286} = \mathbf{1.75 \text{ bar}}$$

For LP turbine, the temperature ratio can be expressed as

$$\frac{T_6}{T_{7s}} = \left(\frac{p_6}{p_7}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1.75}{1}\right)^{\frac{1.4-1}{1.4}} = 1.173$$

or $$T_{7s} = \frac{898 \text{ K}}{1.173} = 765.55 \text{ K}$$

The isentropic efficiency of LP turbine is given as

$$\eta_{T,\,LP} = \frac{\text{Actual work output}}{\text{Isentropic work output}} = \frac{T_6 - T_7}{T_6 - T_{7s}}$$

Actual temperature, T_7 after expansion in turbine,

$$T_7 = 898 - 0.80 \times (898 - 765.55)$$
$$= 792.04 \text{ K}$$

Net work output per kg of air

= Output of LP turbine

$$w_{net} = h_6 - h_7 = C_p(T_6 - T_7)$$
$$= 1.005 \times (898 - 792.04)$$
$$= 106.49 \text{ kJ/kg}$$

Heat exchanger

The effectiveness of a heat exchanger is given by

$$\varepsilon = \frac{T_3 - T_2}{T_7 - T_2}$$

Given that $\varepsilon = 0.75$,

$$0.75 = \frac{T_3 - 455.81}{792.04 - 455.81}$$

Thus the temperature T_3 of air leaving heat exchanger

$$T_3 = 455.81 + 0.75 \times (792.04 - 455.81)$$
$$= 708 \text{ K}$$

The heat supplied to air

$$q_{in} = C_p(T_4 - T_3) + C_p(T_6 - T_5)$$
$$= 1.005 \times (898 - 708) + 1.005 \times (898 - 748.43)$$
$$= 432.72 \text{ kJ/kg}$$

(ii) Thermal efficiency of the cycle

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{106.49}{432.72}$$
$$= 0.2460 \text{ or } \mathbf{24.6\%}$$

27.6.5 Gas Turbine Cycle with Intercooling, Reheating and Regeneration

As discussed earlier, the turbine work increases by reheating between turbine stages and compression work reduces by intercooling between compressor stages. Further, exhaust gases leaving the turbine at relatively high temperature thus the potential for regeneration is also enhanced. Accordingly, when reheating and intercooling are used together with regeneration, a substancial improvement in the performance can be realised. Fig. 27.31 shows a schematic of physical arrangement incorporating intercooling, reheating and regeneration.

The air enters the first stage compressor at state 1, it is compressed isentropically to an intermediate pressure p_2, then it enters the intercooler, where it cools to state 3 ($T_3 = T_1$). Then air is again compressed isentropically to state p_4 in second stage compressor. At this state, the air enters the regenerator, where it is heated at constant pressure to the state 5. In an ideal case, the air leaving the regenerator at a temperature equal to turbine exhaust temperature T_9. Then heat is added to air in combustion chamber and it leaves at state 6 to enter the first stage turbine, where it expands isentropically to state 7, and the gases enter the reheater, and is heated at constant pressure to state 8 and then enter the second stage turbine to expand isentropically to the state 9. Finally, exhausted gases at T_9 enter the regenerator to preheat the compressed air at constant pressure, and leaves the regenerator at state 10.

Fig. 27.32(a) and (b) show p-v diagram and T–s diagrams respectively for an ideal gas turbine cycle with intercooling, reheating and regeneration, as arranged in Fig. 27.31.

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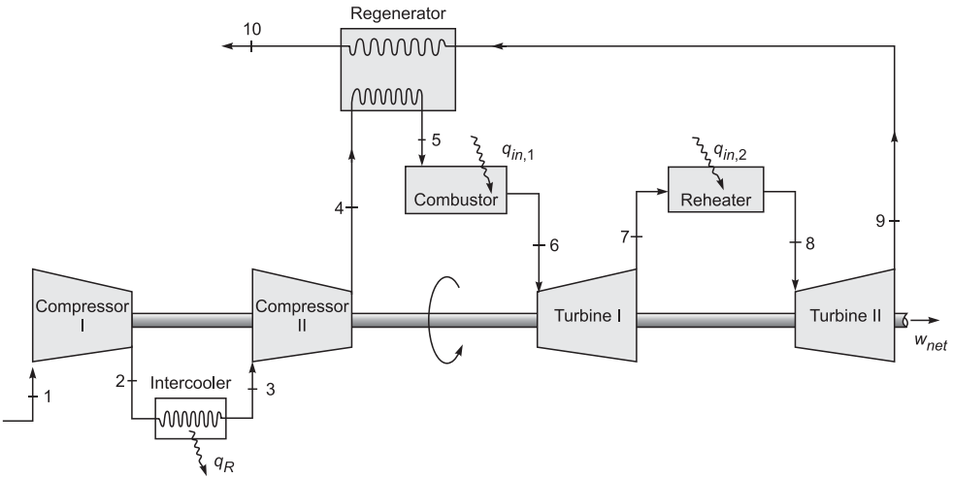


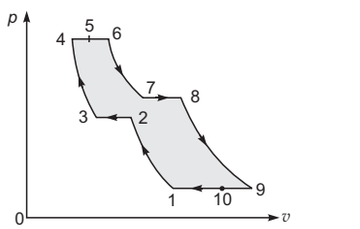
Fig. 27.31 *A gas-turbine engine with two-stage compression with intercooling, two stage expansion with reheating, and regeneration.*

It is shown with the help of Examples 27.13, and 27.14 that the work input to the two stages compression is minimum, when equal pressure ratio is maintained across each stage. Similarly, it can also be proved that the turbine work would be maximum, if same pressure is maintained between two turbine stages. Thus, for optimum operation

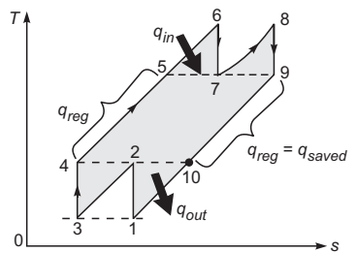
$$\frac{p_2}{p_1} = \frac{p_4}{p_3} \text{ and } \frac{p_6}{p_7} = \frac{p_8}{p_9}$$

In analysis of an actual gas turbine cycles, the irreversibilities are present within the compressor, turbine, and the regenerator as well as pressure drop in heat exchangers. The *T–s* diagram for an actual gas turbine cycle with intercooler, reheating and regeneration is shown in Fig. 27.33. In this figure pressure drop in heat exchanger is not considered.

If the number of compression stages with intercoolers and number of expansion stages with reheaters are increased, then the compression and expansion will occur almost isothermally and ideal gas turbine cycle will approach the Ericsson cycle as shown in Fig 27.34, and thermal efficiency of the cycle will reach theoretically to a maximum



(a) p–v diagram for an ideal gas turbine cycle with intercooling, reheating and regeneration



(b) T–s diagram for an ideal gas turbine cycle with intercooling, reheating and regeneration

Fig. 27.32

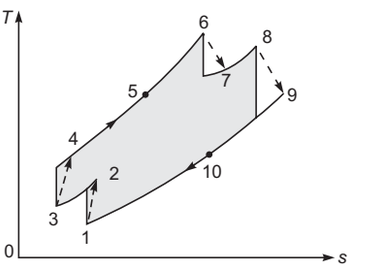


Fig. 27.33 *Actual regenerative gas turbine cycle with intercooling and reheating*

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value (the Carnot efficiency). However, the cost of multiple intercooler, reheaters are likely to exceed the cost of fuel saving, thus use of more than two or three stages cannot be justified economically.

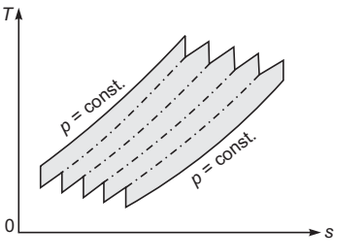

Fig. 27.34 *A gas turbine cycle with large number compression and expansion stages with intercooling, reheating and regeneration approaches the Ericsson cycle*

Example 27.19 *An ideal gas turbine cycle with two stages of compression and two stage of expansion has an overall pressure ratio of 8. Air enters the each stage of compressor at 300 K and each stage of turbine at 1300 K. Determine the back work ratio and thermal efficiency of the gas turbine cycle.*

Assuming an ideal generator with 100 per cent effectiveness.

Solution

Given An ideal gas turbine cycle with two stage of compression, expansion:

$$\frac{p_2}{p_1} = 8$$

$$T_1 = T_3 = T_{min} = 300\,\text{K}$$

$$T_6 = T_8 = T_{max} = 1300\,\text{K}$$

To find

(i) Back work ratio,

(ii) Thermal efficiency of the cycle.

Assumptions

(i) Each component in the cycle as steady flow model.

(ii) Constant specific heat.

(iii) Working substance as an ideal gas.

(iv) Compression and expansion are isentropic.

(v) No pressure drop in heat exchanges piping system etc.

(vi) Perfect intercooling.

T–s diagram

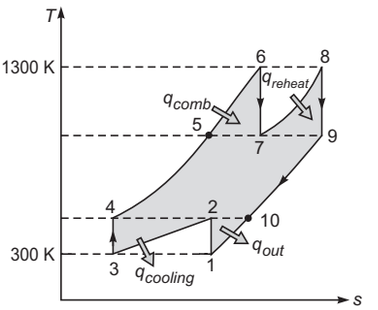

Fig. 27.35 *T–s diagram for gas turbine cycle with two stage compression with intercooling, two stage expansion with reheating, and regeneration*

Analysis For perfect intercooling and given overall pressure ratio

$$r_p = \frac{p_2}{p_1} = \frac{p_4}{p_3} = \sqrt{8} = 2.83$$

and $$\frac{p_6}{p_7} = \frac{p_8}{p_9} = 2.83$$

For an ideal case,

At inlet $T_1 = T_3 \quad \therefore \quad h_1 = h_3$

and $T_6 = T_8 \quad \therefore \quad h_6 = h_8$

At exit $T_2 = T_4 \quad \therefore \quad h_2 = h_4$

and $T_5 = T_7 = T_9 \quad \therefore \quad h_5 = h_7 = h_9$

Under these conditions, the work input of each compressor and work output of each turbine would be the same.

The temperature after first-stage and second-stage compression.

$$T_4 = T_2 = T_1(r_p)^{\frac{\gamma-1}{\gamma}} = 300 \times (2.83)^{0.286}$$

$$= 403.95\ \text{K}$$

Temperature after first-stage and second-stage expansion,

$$\frac{T_8}{T_9} = \frac{T_6}{T_7} = (r_p)^{\frac{\gamma-1}{\gamma}}$$

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or $$T_9 = T_7 = \frac{1300}{(2.83)^{\frac{1.4-1}{1.4}}} = 965.6\,\text{K}$$

Total compression work

$$\Rightarrow \quad w_C = 2\,(h_2 - h_1) = 2\,C_p(T_2 - T_1)$$
$$= 2 \times 1.005 \times (403.95 - 300)$$
$$= 208.94\ \text{kJ/kg}$$

Total expansion work

$$= 2w_T = 2\,C_p(T_6 - T_7)$$
$$= 2 \times 1.005 \times (1300 - 965.6)$$
$$= 672.12\ \text{kJ/kg}$$

$$w_{net} = w_T - w_C = 672.12 - 208.94$$
$$= 463.18\ \text{kJ/kg}$$

Heat supplied

$$q_{in} = q_{comb} + q_{reheat} = (h_6 - h_5) + (h_8 - h_7)$$
$$= C_p(T_6 - T_5) + C_p(T_8 - T_7)$$
$$= 2\,C_p(T_8 - T_7)$$
$$= 2 \times 1.005 \times (1300 - 965.6)$$
$$= 672.14\ \text{kJ/kg}$$

(i) Back work ratio

$$b_{wr} = \frac{w_C}{w_T} = \frac{208.94}{672.12} = \mathbf{0.31 = 31\%}$$

(ii) Thermal efficiency

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{463.18}{672.14}$$
$$= \mathbf{0.6891 \quad or \quad 68.91\%}$$

Summary

- The gas turbine is a rotary internal combustion engine. A simple gas turbine can be modelled as closed cycle, which operates on the Brayton cycle.
- The back work ratio of a gas turbine cycle is given as

$$b_{wr} = \frac{\text{Compressor work}}{\text{Turbine work}}$$

and work ratio $$r_w = \frac{\text{Net work}}{\text{Turbine work}}$$

- The maximum work from a gas turbine operated on ideal Brayton cycle can be obtained when the pressure ratio

$$r_p = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{2(\gamma-1)}}$$

- Actual compressor and turbine operations deviate from isentropic ones, and their isentropic efficiencies are expressed as

$$\eta_C = \frac{\text{Isentropic work input}}{\text{Actual work input}} = \frac{h_{2s} - h_1}{h_2 - h_1}$$

and $$\eta_T = \frac{\text{Actual work output}}{\text{Isentropic work output}} = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

where states 1 and 3 are inlet states, 2 and 4 are actual exit states and 2*s* and 4*s* are isentropic exit states from compressor and turbine, respectively.

- The regeneration in a gas turbine cycle is the heating of compressed air leaving the compressor with help of hot exhaust gases leaving the turbine in a counterflow heat exchanger. The effectiveness of the regenerator is expressed as

$$\varepsilon = \frac{q_{saved}}{q_{reg,max}} = \frac{h_5 - h_2}{h_4 - h_2}$$

- The thermal efficiency of a gas turbine cycle can also be increased by multistage compression with intercooling, regeneration and multistage expansion with reheating.

The gas turbine engines are widely used to propel the aircraft, because they are light, compact and have a high power-to-weight ratio

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Glossary

Gas turbine An internal combustion rotary engine using combustion gases as working fluid

Regenerator A counterflow heat exhanger, which transfers heat from hot gases to air

Regeneration Process of heat transfer from hot gases to air

Reheating Heating of gases after one stage expansion in turbine by burning of additional fuel

Intercooling Process of cooling of compressed air between stages of compression

Back work The work input to compressor is called back work

Review Questions

1. How is a gas turbine plant modeled as air standard Brayton cycle? Write the assumptions clearly.
2. What are the deviations in actual gas turbine cycle as compared with air standard cycle?
3. Define isentropic efficiency of a compressor and turbine.
4. Define isothermal efficiency of the compressor and prove that the isothermal work input to a compressor is always minimum.
5. What are the applications of the gas turbine plants?
6. Write the merits and demerits of gas turbine plant over internal combustion engines.
7. What are the different methods used to improve efficiency of a gas turbine plant? Expain any one method with a neat sketch.
8. Describe with a neat sketch, the working of a constant-pressure combustion gas turbine cycle.
9. Sketch the neat diagram of regenerative gas turbine plant and deduce an expression for its thermal efficiency.
10. What are advantages of cooling of compressed air between stages? State.
11. Explain the working of reheat gas turbine plant with the help of a *T–s* diagram.

Problems

1. In a gas turbine power plant, operating on Joule cycle, air is compressed from 1 bar and 15°C through a pressure ratio of 6. It is then heated to 727°C in the combustion chamber and expanded back to a pressure of 1 bar. Calculate the net work done, cycle efficiency and work ratio. Assume isentropic efficiencies of turbine and compressor are 90 and 85%, respectively.

[134.8 kJ/kg, 27.6%, 0.372]

2. Air enters the compressor of a simple gas turbine plant at 100 kPa, 20°C, at a rate of 2.2 kg/s. The compressor efficiency is 60%. The discharge pressure of the compressor is 450 kPa. Calculate the amount of heat (kJ/kg) that must be added to provide a turbine inlet temperature of 650°C.

[402 kJ/kg]

3. Air enters a gas turbine plant at 95 kPa, 5°C. The compression is adiabatic with an efficiency of 70% and pressure ratio of 5. The regenerative effectiveness is 60%. The turbine inlet conditions are 475 kPa, 850°C. The expansion in the turbine is also adiabatic with an efficiency of 70%. The power output of the plant is 1500 kW. Calculate

(a) mass flow rate of air through the plant.

(b) the irreversibility (kJ/kg) or the turbine expansion

[(a) 22.9 kg/s, (b) 44.4 kJ/kg]

4. A gas turbine takes in air at 27°C and 1 bar.The pressure ratio is 4. The maximum temperature of the cycle is 560°C. The efficiency of the compressor and turbine is 0.83 and 0.85,

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respectively. Find the overall efficiency, if the regenerator effectiveness is 0.75. *[21.24%]*

5. A gas turbine unit receives air at 1 bar and 300 K and compresses it adiabatically to 6.2 bar. The compressor efficiency is 88%. The fuel has a heating value of 44,186 kJ/kg and the fuel-air ratio is 0.017 kg of fuel per kg of air. The turbine internal efficiency is 90%. Calculate the work of turbine and compressor per kg of air and thermal efficiency of cycle.

Take for air C_p = 1.005 kJ/kg·K and γ = 1.4, and For product of combustion C_p = 1.147 kJ/kg·K and γ = 1.333.

[43.92 kJ/kg, 234.5 kJ/kg, 33.21%]

6. A gas turbine is supplied with a gas at 5 bar and 1000 K and expands isentropically to 1 bar. The mean specific heat at constant pressure and constant volume are 1.0425 and 0.7662 kJ/kg·K, respectively. Calculate the specific power developed in kJ/kg of gas and exhaust gas temperature. *[362 kW/kg, 653 K]*

7. Calculate the efficiency of a gas turbine plant fitted with a heat exchanger of 75% effectiveness. The pressure ratio is 4:1 and the compression is carried out in two stages of equal pressure ratio with intercooling back to initial temperature of 290 K. The maximum temperature is 925 K. The turbine isentropic efficiency is 88% and the isentropic efficiency of each compressor is 85%. For air, γ = 1.4 and C_p = 1.005 kJ/kg·K.

[32.8%]

8. A gas turbine has a pressure ratio of 6 and a maximum cycle temperature of 600°C. The isentropic efficiency of the compressor and turbine are 82 and 85%, respectively. Calculate the power input in kW to an electric generator geared to the turbine, with transmission efficiency of 95%. The air enters the compressor at 15°C at a rate of 15 kg/s. Take C_p = 1.005 kJ/kg·K and γ = 1.4 for compression process and C_p = 1.11 kJ/kg·K and γ = 1.333 for the expansion process.

[875 kW]

9. A gas turbine plant has an overall pressure ratio of 5 and a maximum temperature of 550°C. The turbine drives the compressor and an electric alternator, with transmission efficiency of 97%. The ambient temperature is 20°C and isentropic efficiency of the compressor and turbine are 80 and 83%, respectively. Calculate the power input to alternator for an air flow rate of 15 kg/s. Also, calculate the thermal efficiency and work ratio.

[655 kW, 12%, 0.168]

10. A gas turbine plant has a heat exchanger with 72% effectiveness. The turbine operates between pressures of 1.01 and 4.04 bar and the ambient temperature is 20°C. Isentropic efficiencies of the compressor and turbine are 80 and 85%, respectively. The pressure drop on each side of the heat exchanger is 0.05 bar and in the combustion chamber, 0.14 bar. Calorific value of the fuel is 41800 kJ/kg. Calculate the percentage increase in efficiency of the plant due to use of heat exchanger in comparison to a simple plant. Air–fuel ratio used in a simple cycle is 90 and the turbine entry temperature remains same in both cases.

Take C_p = 1.024 kJ/kg·K and γ = 1.4.

[5.3%]

Objective Questions

1. A gas turbine plant works on
   (a) Otto cycle (b) Diesel cycle
   (c) Dual cycle (d) Brayton cycle
2. In an ideal Brayton cycle, the heat is added at
   (a) constant volume
   (b) constant pressure
   (c) constant temperature
   (d) constant entropy
3. A simple gas turbine unit consists of the devices in the following order
   (a) Air compressor, gas turbine, combustion chamber and generator

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(b) Air compressor, combustion chamber, turbine and generator
(c) combustion chamber, gas compressor, turbine and generator
(d) Gas turbine, generator, compressor and combustion chamber

4. In a gas turbine plant, the intercooler is used in between
   (a) air compressor and regenerator
   (b) air compressor and combustion chamber
   (c) combustion chamber and turbine
   (d) LP compressor and HP compressor
5. The function of an inter cooler in a gas turbine plant is
   (a) to cool the exhaust gas from the turbine
   (b) to cool compressed air from compressor
   (c) to cool atmospheric air before inlet to compressor
   (d) to cool the compressed air in between the stages
6. The function of regenerator in a gas turbine plant is
   (a) to heat the compressed air from the compressor
   (b) to heat the gas before inlet to gas turbine
   (c) to exchange the heat from hot gases from combustion chamber to exhaust gases of the turbine
   (d) to heat the compressed air in between the stages
7. The compressor isentropic efficiency is defined an

   (a) $\frac{\text{Actual work}}{\text{Isothermal work}}$ (b) $\frac{\text{Actual work}}{\text{Isentropic work}}$

   (c) $\frac{\text{Isothermal work}}{\text{Actual work}}$ (d) $\frac{\text{Isentropic work}}{\text{Actual work}}$
8. Turbine isentropic efficiency is defined as

   (a) $\frac{\text{Actual work}}{\text{Isothermal work}}$ (b) $\frac{\text{Actual work}}{\text{Isentropic work}}$

   (c) $\frac{\text{Isothermal work}}{\text{Actual work}}$ (d) $\frac{\text{Isentropic work}}{\text{Actual work}}$

Answers

1. (d) 2. (b) 3. (b) 4. (d) 5. (d) 6. (a) 7. (d) 8. (b)

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28

Jet and Rocket Propulsions

Introduction

Jet and rocket engines are space vehicles. They are propelled by the reaction of a backward streaming jet of fluid. The jet engines are used in aircraft, while rocket engines are used in satellites. Satellites revolve around the earth at an altitude where the drag force is absent. Therefore, such satellites remain in orbit forever without any expenditure of energy.

The principle, construction, working and performance of jet engines are explained in the first part of the chapter. The operation of ram jet and pulse jet engines is integrated in brief. Rocket Propulsion and Propellants are also discussed at the end of this chapter.

28.1 JET PROPULSION

Jet engines are usually used as aircraft engines for jet aircraft. They are also used for cruise missiles and unmanned air vehicles. Jet engines have also been used to propel high-speed cars, particularly drag racers, with the all-time record held by a rocket car.

Jet engine designs are frequently modified to turn them into gas turbine engines which are used in a wide variety of industrial applications. These include electrical power generation, powering water, natural gas, or oil pumps, and providing propulsion for ships and locomotives.

28.1.1 Principle of Jet Propulsion

A *jet propulsion engine* is a reaction engine that discharges a fast-moving jet of fluid to generate thrust in the opposite direction of the jet to propell the aircraft in accordance with *Newton's second and third laws of motion*.

Whenever, the momentum is applied to a mass of fluid, a reaction occurs and it gives propulsive force in the opposite direction.

In a jet propulsion engine, the propulsive power is not produced by the gas turbine; instead it drives only the compressor and auxiliary equipment. Thus, the net work output of a jet propulsion cycle is zero. The exhaust gases that exist in the turbine at relatively higher pressure are subsequently accelerated in the nozzle and then are discharged to the surroundings to produce the thrust to propel the aircraft as shown in Fig. 28.1. A propeller discharges the gases at a very high velocity in a large mass of atmospheric air. Since the large mass of atmospheric air cannot be displaced by these gases, thus a reactive force is produced, which propels the aircraft forward.

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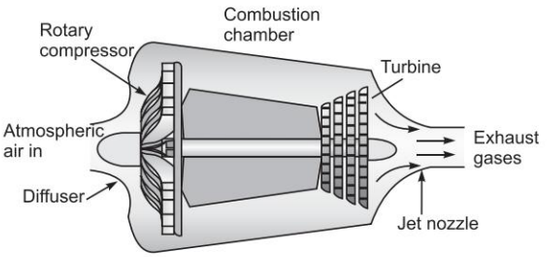

Fig. 28.1 *Principle of jet propulsion*

28.1.2 Classification of Propulsive Engines

These are classified as

1. Non-air-breathing jet engines called rockets.
2. Air-breathing jet engines operating with
   (a) Reciprocating IC engine
   (b) Ramming effect engines
   (c) Gas turbine engines

A *rocket engine* carries its own oxidizer for combustion of fuel and is therefore, independent of atmospheric air and altitude. Its specific fuel consumption is the maximum among all types of propulsive engines.

An *air-breathing engine* requires oxygen from the surroundings for combustion of the fuel. All aircraft engines are of air-breathing type. Air-breathing engines with reciprocating IC engines work on Otto cycle and those with gas turbines work on simple open-cycle Brayton cycle.

28.2 IC ENGINE-DRIVEN PROPULSIVE SYSTEM

The engine shown in Fig. 28.2 is called *engine-propelled driven aircraft* or simply as *engine prop*. It is an air cooled, multi cylinder, radial engine. The net power of the engine is used as shaft power to drive the propeller. With this engine, the aircraft can be used for short range flight with a speed limit of 700 km/h. It is most widely used to power propulsion of helicopters.

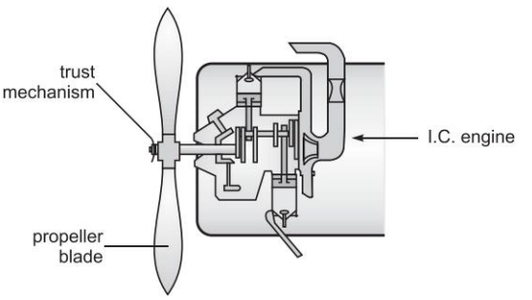

Fig. 28.2 *IC engine-driven propeller engine*

28.3 RAMMING EFFECT PROPULSION SYSTEMS

28.3.1 Ramjet Engine

A *ramjet engine*, is a form of jet engine using the engine's forward motion to compress incoming air, without a rotary compressor as shown in Fig. 28.3. Therefore, ramjet requires forward motion through the air to produce thrust.

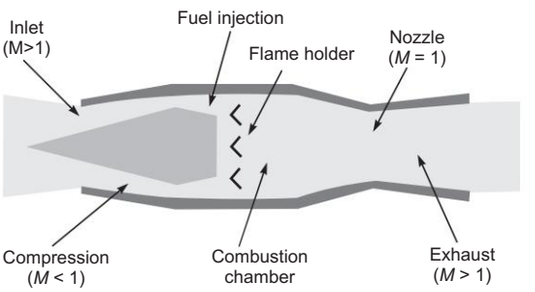

Fig. 28.3 *Ramjet engine*

Construction and Working A ramjet engine consists of supersonic and subsonic diffusers, a combustion chamber and a discharge nozzle. The atmospheric air first enters the supersonic diffuser and then subsonic diffuser, in which the kinetic energy of air entering is converted into static pressure, which is called *ram pressure*. The rammed air then enters into the combustion chamber, where fuel is injected and combustion takes place with the help of fuel burners. The air temperature is raised by continuous combustion of fuel. The

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hot combustion gases then expand in the exhaust nozzles with a velocity exceeding the velocity of air entering. Thus, due to change of momentum of working fluid, a thrust is developed in the direction of flight.

The ramjet engine cannot operate under static conditions, as there will be no pressure rise in the diffuser and it is not self-operating at zero flight velocity. The cycle pressure ratio of a ramjet engine depends on its flight velocity. Higher the flight velocity, larger the ram pressure and consequently, larger the thrust.

Ramjets require considerable forward speed to operate well, and as a class, work most efficiently at speeds around Mach 3, and this type of jet can operate up to speeds of Mach 5.

Ramjets can be particularly useful in applications requiring a small and simple engine for high speed use; such as missiles. They have also been used successfully, as tip jets on helicopter rotors.

Features The features of the ramjet engine are as follows:

1. Its fuel consumption is too high at low and moderate speeds.
2. Its fuel consumption decreases with flight speed and approaches reasonable value, when the flight Mach number is between 2 and 4. Thus, it is suitable for propelling supersonic missiles.
3. It has no moving part and hence is light in weight.
4. It is simple in construction and is adaptable to mass production at a relatively low cost.
5. It can operate with any type of liquid fuels and even with solid nuclear fuels.
6. It cannot be started on its own. It has to be accelerated to a certain flight velocity by some launching device.

28.3.2 Pulse Jet Engine

A *pulse jet engine* (or *pulsejet*) is a very simple form of internal-combustion engine based jet engine where combustion occurs in pulses. A pulse jet engine develops the thrust by a high velocity of jet of exhaust gases without the use of a compressor or turbine.

Ramjets are frequently confused with pulse-jet, which use an intermittent combustion, but ramjets use a continuous combustion process, and are a quite distinct type of jet engine.

A typical pulse jet engine shown in Fig. 28.4, comprises an air intake diffuser fitted with a one-way flap or reed valve, a combustion chamber, and an acoustically resonant exhaust nozzle. The valves are operated due to a pressure difference across them. Fuel in the form of a gas or liquid aerosol is either mixed with the air in the intake or injected into the combustion chamber.

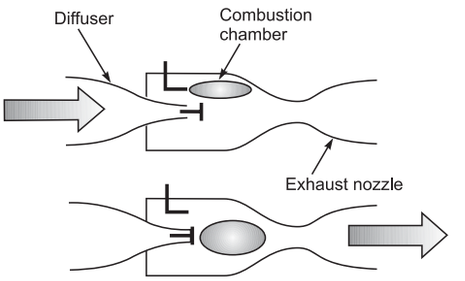

Fig. 28.4 *Pulse jet engine*

The incoming air is compressed by ram effect in the diffuser section and then passes through the passages which are opened and closed by non-return flap valves. The fuel is then injected into the combustion chamber. The combustion is then initiated by a spark plug. Once the engine is operating normally, the spark plug is turned off and it is ignited by residual heat from the previous cycle.

As combustion takes place, the pressure and temperature of combustion products exceed the ram pressure, thus non-return flap valves get closed. Consequently, the pressurized hot gases exit through the exhaust nozzle with a high velocity and produce forward thrust on the unit. With the escape of gases to the atmosphere, the pressure falls in the combustion chamber, the ram air forces the flap valves to open and fresh air enters in the combustion chamber for the next cycle.

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Starting the engine usually requires forced air and an ignition method such as a spark plug for the fuel–air mix. Once running, the engine only requires an input of fuel to maintain a self-sustaining combustion cycle.

The advantages and limitations of a pulse jet engine are listed below.

Advantages

1. It is very simple in construction and cheaper in comparison to turbojet engines.
2. It is free from moving parts like a compressor, turbine and propeller. Hence it is light and can carry higher payload.
3. It is capable to produce sufficient thrust at low speeds also.
4. It can be operated as a pilotless aircraft and is highly suitable for bombers and target missiles.

Limitations

1. It can be used for a short flight life of 30 to 60 minutes.
2. It produces very high noise.
3. Due to intermittend pulse, it produces severe vibrations.
4. It has high rate of fuel consumption and very low thermal efficiency of 2 to 3% only.
5. The operating altitude is limited due to atmospheric air density considerations.

28.3.3 Scramjet

A *scramjet* is a **s**upersonic **c**ombustion ramjet. It differs from a ramjet in which supersonic combustion takes place. At higher speeds, it is necessary to combust supersonically to maximize the efficiency of the combustion process. It operates at Mach numbers between 12 and 24.

Like a ramjet, a scramjet essentially consists of a restricted tube through which inlet air is compressed by the high speed of the vehicle, a combustion chamber where fuel is burned, and a nozzle through which the exhaust jet leaves at higher speed than the inlet air. Also like a ramjet, there are no moving parts. However, scramjets have weight and complexity issues that must be considered. A scramjet has very poor thrust to weight ratio (~2). It has extreme aerodynamic complexity, airframe difficulties and testing difficulties.

28.4 GAS TURBINE PROPULSION SYSTEMS

These engines are equipped with a gas turbine to drive the compressor and other auxiliary equipment on the aircraft. These are

1. Turbojet engines
2. Turboprop engines
3. Turbofan engines

28.5 TURBOJET ENGINE

28.5.1 Principle of Operation

The turbojet engine is propelled by the thrust produced due to acceleration of hot combustion gases through the exhaust nozzle. Therefore, at higher speed, the thrust developed is more and the turbojet engine gives higher propulsive efficiency.

28.5.2 Construction and Working

The cross-sectional view of a turbojet engine is shown in Fig. 28.5(a), and the basic components of a turbojet engine is shown in Fig. 28.5(b). It consists of three main sections—the diffuser, the gas generator, and the nozzle. The diffuser decelerates the incoming air relative to the engine and a part of the kinetic energy of the air stream is converted into pressure by ramming effect. The gas generator section consists of a compressor, combustor and a turbine, with same function as in a stationary gas turbine plant. In a turbojet engine, the power produced by the turbine is just sufficient to drive the compressor, fuel pump and other auxiliary equipment. The net power output of propulsive cycle is zero. The hot gases leaving the turbine, relatively at higher pressure are accelerated in a nozzle to a high velocity. The discharge of high velocity gases to surroundings,

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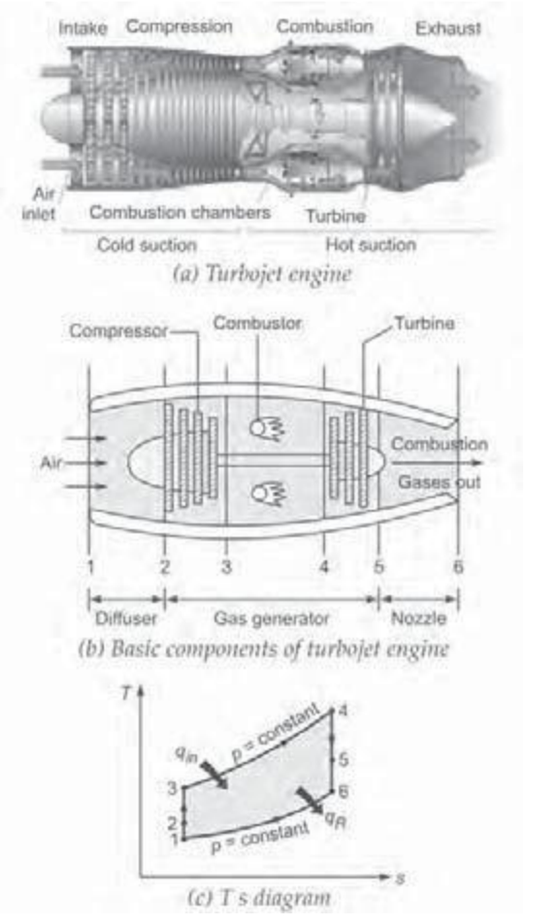

Fig. 28.5

produces forward thrust (propulsive force) on the aircraft to propel it in opposite direction, because the mass of atmospheric gases is large to displace in comparison to engine mass.

Figure 28.5(c) shows the *T*–*s* diagram for a turbojet engine. Process 1–2 represents isentropic pressure rise in the diffuser; Process 2–3—pressure rise in compressors Process 3–4 heat addition at constant pressure in the combustion chamber; Process 4–5—isentropic expansion in the turbine and Process 5–6—isentropic expansion in exhaust nozzle. In an actual turbojet engine, the working substance is not *re-circulated* through the process 6–1.

28.5.3 Applications

Turbojet engines are most suitable for aircrafts travelling above 800 km/h. They are used in pilot passengers, cargo long distance aircrafts, military aircrafts, guided missiles, etc.

28.5.4 Merits and Demerits over IC Engine Propelled Engine

Merits

1. Its construction is much simpler as compared to a multicylinder reciprocating IC engine for the same power output.
2. The engine runs smoothly without vibrations.
3. The engine speed is higher than a reciprocating engine.
4. Torque obtained is smooth and uninterrupted.
5. Engine weight to power-output ratio is lower as compared to a reciprocating engine.
6. These are suitable for longer flights at higher altitude and speed.
7. Fuel can burn over a large range of mixture strength.
8. Less maintenance is needed.

Demerits

1. Propulsive efficiency and thrust are lower at lower speed.
2. It becomes inefficient below a speed of 550 km/h.
3. It produces more noise than a reciprocating engine.
4. Its capital cost is high.
5. It requires longer runway for its take off and landing.
6. Thrust-specific fuel consumption is high.
7. It is not economical for short-distance flights.
8. Sudden decrease of acceleration and deceleration is difficult.

28.6 TURBOPROP ENGINE

28.6.1 Principle

A turboprop engine is also called *propjet*, and it is an intermediate device between a jet engine

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and a propeller driven by a reciprocating engine. The propeller is coupled to the turbine through a reduction gear that converts the high RPM, and low torque output to low RPM, and high torque.

28.6.2 Construction and Working

In its simplest form, a turboprop shown in Fig. 28.6, consists of an intake, compressor, combustion chamber, turbine and a propelling nozzle. Air is drawn into the intake and compressed by the compressor. Fuel is then added to the compressed air in the combustion chamber, where the fuel–air mixture burns. The hot combustion gases expand through the turbine. Some of the power generated by the turbine is used to drive the compressor. The rest is transmitted through the reduction gearing to the propeller. Further expansion of the gases occurs in the propelling nozzle, where the gases exhaust to atmospheric pressure. The propelling nozzle provides approximately 20 per cent thrust by velocity difference of gases leaving and air entering the aircraft, while the rest of thrust is provided by propeller.

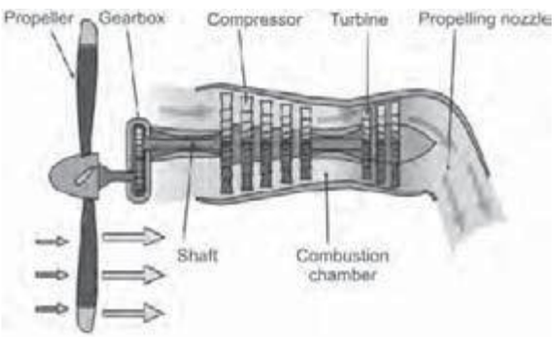

Fig. 28.6 *Turboprop engine*

Turboprops are very efficient at modest flight speeds of 600 km/h because the jet velocity of the propeller (and exhaust) is relatively low. Due to the high price of turboprop engines, they are mostly used where high-performance, short-takeoff and landing capability and efficiency at modest flight speeds are required.

Currently, turboprop engines are used on small subsonic aircraft, such as on small commuter aircraft, where their greater reliability as compared to reciprocating engines offsets their higher initial cost.

28.6.3 Advantages and Limitations of Turboprop

Advantages

1. It develops higher thrust at low speeds and hence take off rolling is short, requiring shorter runway.
2. Thrust-specific fuel consumption is low.
3. Propulsive efficiency is high.
4. Thrust reversal is easily achieved by varying the blade angle, and aircraft speed can be drastically decreased.
5. It is useful for short-range flights with a speed of 600 km/h.

Limitations

1. Turboprop engines can be used for low speeds, low altitude and short range of flights.
2. The engine is heavier and more complicated. It has higher weight per unit thrust.
3. At higher speeds, the propulsive efficiency decreases drastically.
4. With the use of a heavier propeller, compressor and turbine, the payload capacity is low.

28.7 TURBOFAN

A *turbofan* (shown in Fig. 28.7) is a type of aircraft engine consisting of a ducted fan which is powered by a gas turbine. A part of the airstream from the ducted fan passes through the gas turbine core, providing oxygen to burn fuel to create power, and remaining of the air flow is by-passed to the engine core, and is accelerated by the fan blades in the same manner as a propeller. The combination of thrust produced from the fan and the exhaust from

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the core is a more efficient process than other jet engine designs. It results into a comparatively low specific fuel consumption.

Turbofans have a net exhaust speed that is much lower than a turbojet. This makes them much more efficient at subsonic speeds than turbojets, and somewhat more efficient at supersonic speeds up to Mach 1.6, but they have also been found to be efficient when used with continuous afterburner at Mach 3 and above.

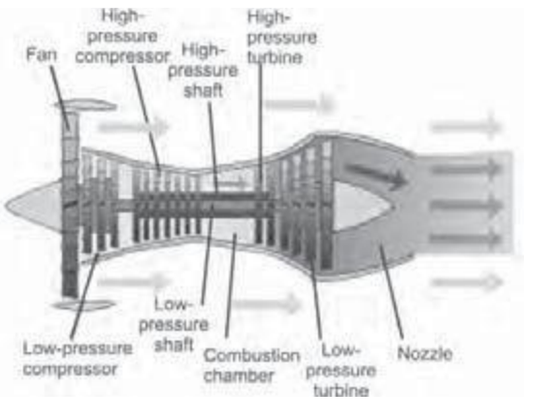

Fig. 28.7 *Turbofan*

All commercial jet aircraft are turbofans. They are used mainly because they are highly efficient and relatively quiet in operation. Turbofans are also used in many military jet aircraft.

28.8 TERMINOLOGY USED WITH TURBOJET ENGINE

28.8.1 Thrust

The *thrust* developed in a turbojet engine is the unbalance force, which is caused by the difference in the momentum of flow velocity of air entreing the engine and high velocity hot gases leaving the exhaust nozzle, and is given by Newton's second law of motion, i.e.,

$$F = (\dot{m}_a + \dot{m}_f)\,\mathcal{V}_{jet} - \dot{m}_a\,\mathcal{V}_a \qquad ...(28.1)$$

where $\dot{m}_a$ is the rate of flow of air through the engine

$\dot{m}_f$ is the rate of flow of fuel entering the engine

$\mathcal{V}_{jet}$ is the jet velocity of the gases leaving the nozzle

$\mathcal{V}_a$ is the velocity of air entering the engine

$(\dot{m}_a + \dot{m}_f)\,\mathcal{V}_{jet}$ represents the nozzle gross thrust

$\dot{m}_a\,\mathcal{V}_a$ represents the ram drag force of the flight

Since the air–fuel ratio used in the aircraft is very high, the mass flow rate of exhaust gases is almost same as of air entering the engine. If the mass of fuel to the gross thrust is ignored, the net thrust is

$$F = \dot{m}_a(\mathcal{V}_{jet} - \mathcal{V}_a) \qquad ...(28.2)$$

The speed of the jet must exceed the true air speed of the aircraft, if there is to be a net forward thrust on the air frame. For an aircraft cruising at steady speed, the thrust is used to overcome skin friction. The air at higher altitudes is thinner and it offers a smaller drag force on the aircraft. Therefore, commercial aircraft flying at the higher altitudes to save fuel.

Specific Thrust The specific thrust is defined as ratio of thrust deveoped to mass-flow rate of air.

$$\text{Specific thrust} = \frac{\text{Thrust}}{\text{Mass-flow rate}} = \frac{F}{\dot{m}_a} \qquad ...(28.3)$$

Specific Impulse The specific impulse is defined as the ratio of thrust developed to the weight of air, which passes through the engine;

$$I_{sp} = \frac{\text{Thrust}}{\text{Weight of propellant}} = \frac{F}{\dot{m}_a g} \qquad ...(28.4)$$

Thrust Power The power developed from thrust of the engine is called the *thrust power* $\dot{W}_p$, which is the product of propulsive force and flight velocity $\mathcal{V}_a$. That is

$$\dot{W}_{th} = F\,\mathcal{V}_a = \dot{m}_a(\mathcal{V}_{jet} - \mathcal{V}_a)\,\mathcal{V}_a \qquad ...(28.5)$$

Propulsive Power It is the gross thrust power. It is defined as change in momentum of mass-flow rate

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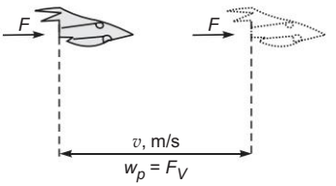

Fig. 28.8 *Propulsive power is the thrust acting on an aircraft through a distance per unit time*

of gases. It is the difference between rate of kinetic energy of air entering and gas leaving That is,

$$\dot{W}_{prop} = \Delta KE = \dot{m}_a \left(\frac{\mathcal{V}_{jet}^2 - \mathcal{V}_a^2}{2} \right) \text{ Watts} \quad ...(28.6)$$

Effective Speed Ratio It is ratio of flight velocity to jet velocity. It is designated as s and is given as

$$s = \frac{\mathcal{V}_a}{\mathcal{V}_{Jet}} \quad ...(28.7)$$

Propulsive Efficiency It is defined as the ratio of thrust power to propulsive power.

$$\eta_{prop} = \frac{\text{Thrust power}}{\text{Propulsive power}}$$

$$= \frac{\dot{m}_a \times (\mathcal{V}_{jet} - \mathcal{V}_a) \times \mathcal{V}_a}{\dot{m}_a \times (\mathcal{V}_{jet}^2 - \mathcal{V}_a^2)/2}$$

$$= \frac{2(\mathcal{V}_{jet} - \mathcal{V}_a) \times \mathcal{V}_a}{(\mathcal{V}_{jet}^2 - \mathcal{V}_a^2)}$$

or $$\eta_{prop} = \frac{2\mathcal{V}_a}{(\mathcal{V}_{jet} + \mathcal{V}_a)} \quad ...(28.8)$$

Thermal Efficiency The thermal efficiency of a turbojet engine is defined as

$$\eta_{th} = \frac{\text{Propulsive power}}{\text{Heat supply rate}} = \frac{\dot{W}_{prop}}{\dot{Q}_{in}} \quad ...(28.9)$$

where $\dot{Q}_{in} = \dot{m}_f CV$;

$\dot{m}_f$ is the rate of mass of fuel burning and CV is calorific value of fuel.

Overall Efficiency The overall efficiency is defined as a product of thermal efficiency and propulsive efficiency and is given as

$$\eta_{overall} = \eta_{th} \times \eta_{prop} \quad ...(28.10)$$

Thrust-Specific Fuel Consumption It is defined on the basis of thrust and it is one of the important parameters of an aircraft engine. It is measured in kg per kg of air-N thrust. It is given by

$$\text{Thrust } sfc = \frac{\text{Fuel flow rate}}{\text{Thrust produced}}$$

$$= \frac{\dot{m}_f}{\dot{m}_a \times (\mathcal{V}_{jet} - \mathcal{V}_a)} \quad ...(28.11)$$

28.9 ANALYSIS OF TURBOJET CYCLE

A turbojet engine consists of a diffuser, a compressor, a combustion chamber, a turbine and a jet nozzle. All of these devices operate in a steady flow manner. The thermodynamic cycle on *T-s* diagram for a turbojet engine is shown in Fig. 28.9.

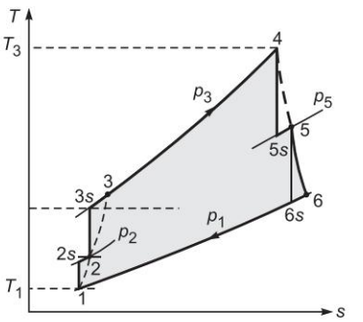

Fig. 28.9 *T-s diagram for turbojet engine*

28.9.1 Diffuser

Air enters the diffuser from the surrounding atmosphere at temperature T_1 and pressure p_1 at flight velocity $\mathcal{V}_a$. The air is slowed down in the diffuser and kinetic energy of air is converted into static pressure. Actual diffusion process 1–2 and isentropic diffusion process 1–2*s*, are shown in Fig. 28.10 (a) on a *T–s* diagram.

The stagnation enthalpy after isentropic diffusion process is expressed as

$$h_{2s} = h_1 + \frac{\mathcal{V}_a^2}{2} \quad (28.12)$$

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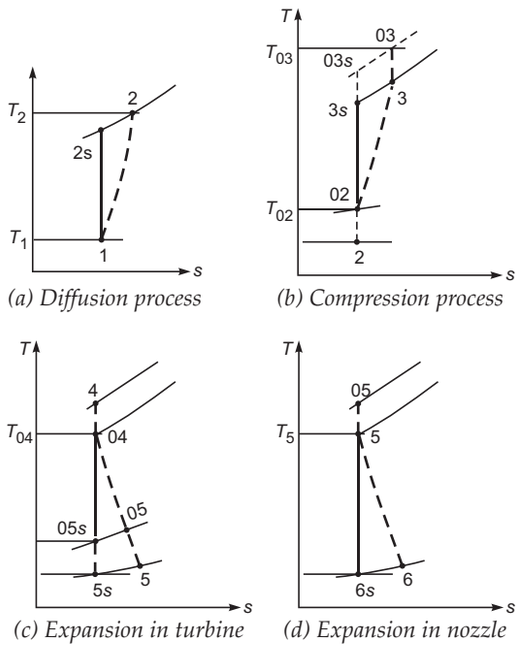

Fig. 28.10

The stagnation temperature T_{2s} after isentropic diffusion is given by

$$T_{2s} = T_1 + \frac{v_a^2}{2C_p} \qquad (28.13)$$

Pressure after isentropic diffusion is given by

$$p_2 = p_1\left(\frac{T_{2s}}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \qquad (28.14)$$

For small pressure rise in subsonic flow, diffuser efficiency is given by

$$\eta_{diffuser} = \frac{\text{Enthalpy rise in isentropic diffusion}}{\text{Enthalpy rise in actual diffusion}} = \frac{h_{2s} - h_1}{h_2 - h_1} \qquad ...(28.15)$$

For constant specific heat of air,

$$\eta_{diffuser} = \frac{\text{Isentropic temp. rise}}{\text{Actual temp. rise}} = \frac{T_{2s} - T_1}{T_2 - T_1} \qquad ...(28.16)$$

28.9.2 Compressor

The diffused air at pressure p_2 and temperature T_2 enters the compressor and is compressed to pressure p_3.

Isentropic compression work

$$w_C = h_3 - h_2 = C_p(T_{3s} - T_2)$$

Actual work input to compressor

$$w_{in} = h_3 - h_2 = C_p(T_3 - T_2) \qquad ...(28.17)$$

The isentropic efficiency of the compressor is given by

$$\eta_C = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{T_{3s} - T_2}{T_3 - T_2}$$

The actual compression work input w_{in} can also be expressed as

$$w_{in} = \frac{\text{Isentropic work}}{\eta_C} = \frac{C_p(T_{3s} - T_2)}{\eta_C} \qquad ...(28.18)$$

The pressures and temperatures at two states are related as

$$\frac{T_{3s}}{T_2} = \left(\frac{p_{3s}}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \qquad ...(28.19)$$

28.9.3 Combustion Chamber

Let $\dot{m}$ be mass of combustion gases generated in the combustion chamber;

$\dot{m}$ = Mass rate of air + Mass Rate of fuel

$= \dot{m}_a + \dot{m}_f$

The heat supplied in combustion chamber

$$\dot{Q}_{comb} = (\dot{m}_a + \dot{m}_f)h_4 - \dot{m}_a h_3$$

The combustion efficiency is defined as

$$\eta_{Comb} = \frac{\text{Actual rise in enthalpy of gases}}{\text{Energy sypplied by fuel}} = \frac{(\dot{m}_a + \dot{m}_f)h_4 - \dot{m}_a h_3}{\dot{m}_f CV} \qquad ...(28.20)$$

28.9.4 Turbine

The hot combustion gases generated in the combustion chamber expand partially in the turbine

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from pressure p_4 and temperature T_4 to pressure p_5 and temperature T_5.

Isentropic expansion work in the turbine

$$w_T = (h_4 - h_{5s}) = C_p(T_4 - T_{5s})$$

Actual work output of turbine;

$$w_{out} = h_4 - h_5$$
$$= C_p(T_4 - T_5) \quad ...(28.21)$$

The isentropic efficiency of the turbine is given as

$$\eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}}$$
$$= \frac{h_4 - h_5}{h_4 - h_{5s}}$$

For constant specific heat, the isentropic efficiency of the turbine

$$\eta_T = \frac{T_4 - T_5}{T_4 - T_{5s}} \quad ...(28.22)$$

The pressures and temperatures at two states are related as

$$\frac{T_4}{T_{5s}} = \left(\frac{p_4}{p_5}\right)^{\frac{\gamma-1}{\gamma}} \quad ...(28.23)$$

Further, for a turbojet engine

Turbine work = Compressor work

28.9.5 Jet Nozzle

After partial expansion of combustion gases in the turbine, the gases enter the nozzle and expand adiabatically from the state 5 to the state 6.

$$h_5 = h_6$$

The nozzle efficiency is given by

$$\eta_{Jet} = \frac{\text{Actual enthalpy drop}}{\text{Isenthalpic enthalpy drop}} = \frac{h_5 - h_6}{h_5 - h_{6s}}$$
$$= \frac{T_5 - T_6}{T_5 - T_{6s}} \quad ...(28.24)$$

where $h_5 = h_6 + \frac{\mathcal{V}_{jet}^2}{2}$

and $T_5 = T_6 + \frac{\mathcal{V}_{jet}^2}{2C_p}$ ...(28.25)

The jet velocity can be obtained as

$$\mathcal{V}_{jet} = \sqrt{2(h_5 - h_6)} = \sqrt{2C_p(T_5 - T_6)} \quad ...(28.23)$$

If the gases are leaving the turbine and entering the nozzle with certain velocity, then total energy input to nozzle

E_{in} = Isentropic enthalpy drop + KE carried over the turbine

Then isentropic efficiency of the nozzle jet can be defined as

$$\eta_{Jet} = \frac{\text{Final KE of jet}}{\text{Isentropic enthalpy drop + KE carried over turbine}}$$
$$= \frac{\frac{\mathcal{V}_{Jet}^2}{2}}{C_{pg}(T_5 - T_{6s}) + \frac{\mathcal{V}_{duct}^2}{2}} \quad ...(28.27)$$

28.10 THRUST AUGMENTATION IN TURBOJET ENGINES

Several modification to turbojet engines have been made to cater to the special performance requirement of various aircraft. One of them is thrust augmentation of turbojet engine for short duration for better take-off performance, higher rate of climb and increased performance at higher altitude. The thrust of a turbojet engine is given by

$$F = (\dot{m}_a + \dot{m}_f)\mathcal{V}_{jet} - \dot{m}_a\mathcal{V}_a$$

In the above equation;

1. $\mathcal{V}_{jet}$ is the function of maximum temperature in the cycle, as maximum temperature in the cycle increases, the exit velocity $\mathcal{V}_{jet}$ also increases, and hence thrust increases.
2. The thrust can also be increased by increasing the mass-flow rate of air $\dot{m}_a$.

Thus, the thrust of a turbojet engine can be increased by

1. Installing reheater or after burner before gases entering the exhaust nozzle
2. Water methanol injection system
3. Air bleed system

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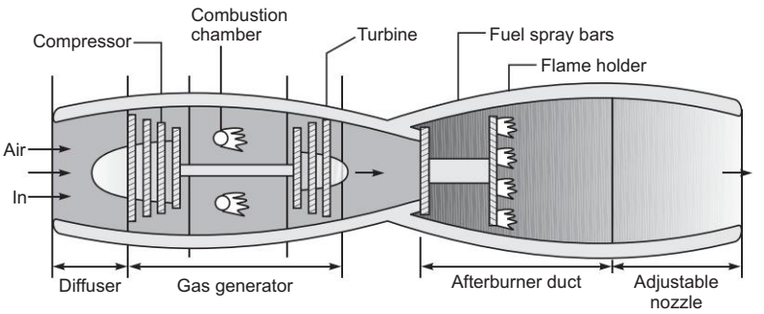

Fig. 28.11 *Schematic of turbojet with afterburner*

28.10.1 Reheater or Afterburner

An afterburner, or *reheat jet pipe* is a device added to the rear of the jet engine. It provides a means of spraying fuel directly into the hot exhaust gases, where it ignites and boosts available thrust significantly. Its purpose is to provide a temporary increase in thrust, both for supersonic flight and for take off.

Afterburners are used mostly on military aircraft. On military aircraft, the extra thrust is needed for combat situations. This is achieved by injecting additional fuel into the jet pipe downstream after the turbine.

The advantage of afterburning is the significantly increased thrust; the disadvantage is its very high fuel consumption and inefficiency, though this is often regarded as acceptable for the short periods during which it is generally used.

28.10.2 Water Methanol Injection

In some turbojet engines, a mixture of water and methanol is injected in the combustion chamber. The evaporation of water increases the mass-flow rate of gases. Heat absorbed by water is compensated by the burning methanol, thus the working temperature remains same.

When water is injected into the combustion chamber, the fresh air flow rate is reduced by an amount corresponding to water injected. A decrease in air flow in the compressor results into higher pressure ratio for same speed of turbine.

The water injection causes a lower mass-flow rate of air and higher pressure ratio in the compressor and a higher total mass flow through the turbine. Thus, more thrust is produced. Thrust augmentation of about 30% is possible by water injection.

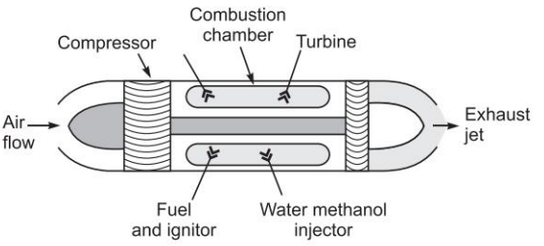

Fig. 28.12 *Turbojet with water methanol injection*

28.10.3 Air Bleeding System

Since excess air is already present in a gas turbine plant, thus a small quantity of compressed air is bled to an auxiliary combustion chamber by by-passing the turbine. In the auxiliary combustion

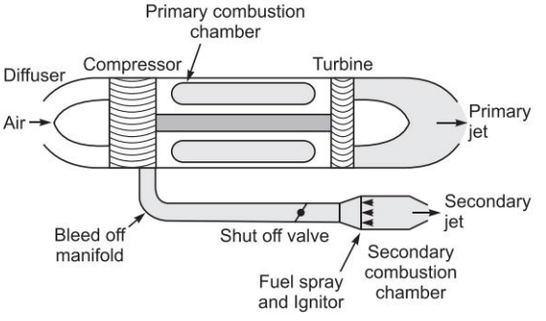

Fig. 28.13 *Turbojet with air bled system*

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chamber, the bled air is heated by burning of an additional fuel. The high-temperature gases are then discharged to an additional exhaust jet to bring thrust augmentation. A shut-off valve is used to bring the engine in normal position.

Example 28.1 *The diameter of the propeller of an aircraft is 2.5 m. It flies at a speed of 540 km/h at an elevation of 8000 m, where air density is 0.525 kg/m^3. The flight to jet speed ratio is 0.75. Calculate: (a) the air-flow rate through the propeller, (b) thrust produced, (c) specific thrust, (d) specific impulse, (e) thrust power.*

Solution

Given A propeller aircraft:

$\mathcal{V}_a = 540$ km/h $\qquad \rho = 0.525$ kg/m^3

$s = \dfrac{\mathcal{V}_a}{\mathcal{V}_{jet}} = 0.75 \qquad d = 2.5$ m

$z = 8000$ m

To find

(i) Air flow rate through the propeller,
(ii) Thrust produced,
(iii) Specific thrust,
(iv) Specific impulse, and
(v) Thrust power.

Assumption Air enters the propeller with flight velocity.

Analysis The flight velocity is

$$\mathcal{V}_a = \frac{(540 \times 1000 \text{ m})}{(3600 \text{ s})} = 150 \text{ m/s}$$

Exit jet velocity;

$$\mathcal{V}_{jet} = \frac{\mathcal{V}_a}{s} = \frac{150 \text{ m}}{0.75} = 200 \text{ m/s}$$

(i) *Air-flow rate through the propeller*

The air-flow rate through the propeller

$\dot{m}_a$ = density × cross-sectional area of propeller × flight velocity

$$= 0.525 \times \left(\frac{\pi}{4} \times 2.5^2\right) \times 150$$

$$= \mathbf{386.56 \text{ kg/s}}$$

(ii) *Thrust produced*

$$F = \dot{m}_a(\mathcal{V}_{jet} - \mathcal{V}_a)$$

$$= 386.56 \times (200 - 150)$$

$$= \mathbf{19328.15 \text{ N}}$$

(iii) *Specific thrust* $= \dfrac{F}{\dot{m}_a} = \dfrac{19328.15}{386.56} = \mathbf{50 \text{ N/kg}}$

(iv) *Thrust power* $= \dfrac{F\mathcal{V}_a}{1000} = \dfrac{19328.15 \times 150}{1000}$

$$= \mathbf{2899.22 \text{ kW}}$$

(v) *Specific impulse* $= \dfrac{F}{\dot{m}_a g} = \dfrac{19328.15}{386.56 \times 9.81}$

$$= \mathbf{5.09 \text{ s}}$$

Example 28.2 *For the combustion of 420 kg of petrol, a flying missile has a range of 240 km; an average velocity of 576 km/h and a propulsive force of 2700 N. The maximum temperature rise in the combustion chamber is 815°C. The diameter of the discharge nozzle is 30 cm. The altitude of the flight is 610 m, where the atmospheric pressure is 0.944 bar. The calorific value of the fuel is 42000 kJ/kg. C_p for exhaust gases can be taken as 1.16 kJ/kg·K. Calculate (a) air-fuel ratio, (b) exhaust gas temperature and their velocity relative to missile, (c) propulsive efficiency, and (d) overall efficiency of the unit.*

Solution

Given A flying missile

$\mathcal{V}_a = 576$ km/h $\qquad m_f = 420$ kg

Distance $= 240$ km $\qquad F = 2700$ N

$\Delta T_{comb} = 815°\text{C} = 815$ K $\qquad d_N = 0.3$ m

$p = 0.944$ bar $= 94.4$ kPa $\qquad z = 610$ m

$CV = 42000$ kJ/kg $\qquad C_p = 1.16$ kJ/kg·K

To find

(i) Air–fuel ratio,
(ii) Exhaust gas temperature,
(iii) Velocity of exhaust gases,
(iv) Propulsive efficiency, and
(v) Overall efficiency of unit.

Assumption Specific gas constant for exhaust gases as 0.287 kJkg·K.

Analysis The flight velocity is

$$\mathcal{V}_a = \frac{(576 \times 1000 \text{ m})}{(3600 \text{ s})} = 160 \text{ m/s}$$

$$\text{Flight duration, } \Delta t = \frac{\text{Distance range}}{\text{Flight velocity}} = \frac{(240 \times 1000 \text{ m})}{(160 \text{ m/s})}$$

$$= 1500 \text{ s or } 25 \text{ min.}$$

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(i) *Air–fuel ratio*

Fuel consumption rate;

$$\dot{m}_f = \frac{\text{Mass of fuel}}{\text{Duration of flight}} = \frac{(420\text{ kg})}{(1500\text{ s})}$$
$$= 0.28\text{ kg/s}$$

Let $\dot{m}_a$ be mass-flow rate of air with fuel. Then energy balance reveals that

$$(\dot{m}_a + \dot{m}_f) C_p (\Delta T)_{comb} = \dot{m}_f CV$$
$$(\dot{m}_a + 0.28) \times 1.16 \times 815 = 0.28 \times 42000$$

or $\dot{m}_a = 12.44$ kg/s

$$\text{A/F} = \frac{\dot{m}_a}{\dot{m}_f} = \frac{(12.44\text{ kg}/s)}{(0.28\text{ kg}/s)}$$
$$= \mathbf{44.42}$$

(ii) *Velocity of exhaust gases*

Propulsive force is given by

$$F = (\dot{m}_a + \dot{m}_f) \mathcal{V}_{jet} - \dot{m}_a \mathcal{V}_a$$

or $2700 = (12.44 + 0.28)\, \mathcal{V}_{Jet} - 12.44 \times 160$

It gives $\mathcal{V}_{Jet} = \mathbf{368.74\text{ m/s}}$

(iii) *Temperature of exhaust gases*

Applying continuity equation for flow of exhaust gases

$$\dot{m}_a + \dot{m}_f = \frac{A\mathcal{V}_{jet}}{v}$$

It gives specific volume of exhaust gases

$$v = \frac{A\mathcal{V}_{jet}}{\dot{m}_a + \dot{m}_f} = \frac{(\pi/4) \times (0.3^2) \times 368.74}{12.44 + 0.28}$$
$$= 2.04\text{ m}^3/\text{kg}$$

The charecteristic gas equation gives

$$pv = RT$$

or $94.4 \times 2.04 = 0.287T$

It gives $T = 671$ K or **398°C**

(iv) *Propulsive efficiency*

It is given by

$$\eta_{prop} = \frac{2\mathcal{V}_a}{\mathcal{V}_a + \mathcal{V}_{jet}} = \frac{2 \times 160}{160 + 368.74}$$
$$= 0.605 \quad \text{or} \quad \mathbf{60.5\%}$$

(v) *Overall efficiency:*

The rate of heat supply to missile

$$\dot{Q}_{in} = \dot{m}_f CV = 0.28 \times 42000$$
$$= 11760\text{ kW}$$

The propulsive power

$$\dot{W}_{prop} = \text{Propulsive force} \times \text{flight velocity}$$
$$= 2700 \times 160$$
$$= 432000\text{ W or } 432\text{ kW}$$

Thermal efficiency

$$\eta_{th} = \frac{\dot{W}_{prop}}{\dot{Q}_{in}} = \frac{432}{11760}$$
$$= 0.0367 \text{ or } 3.67\%$$

Overall efficiency;

$$\eta_{overall} = \eta_{th} \times \eta_{Prop} = 0.0367 \times 0.605$$
$$= 0.0222 \quad \text{or} \quad \mathbf{2.22\%}$$

Example 28.3 *A jet propelled engine having two jets and working on turbojet has a velocity of 210 m/s, when flying at an altitude of 12000 m. The density of air at this altitude is 0.172 kg/m^3.The resistance or drag of the plane is 6670.8 N and propulsive efficiency of the jet is 50%. The overall efficiency of the unit is 18%. Calorific value of the fuel is 4.895×10^4 kJ/kg. Calculate*

(a) Absolute velocity of jet,
(b) Quantity of air compressed per minute,
(c) Diameter of jet,
(d) Net power output of the plant,
(e) Thrust specific fuel consumption,
(f) Air–fuel ratio.

Solution

Given A jet propelled engine of a turbojet

$\mathcal{V}_a = 210$ m/s $\quad N_{jet} = 2$ Nos
$F_D = 6670.8$ N $\quad \rho = 0.172$ kg/m^3
$z = 12000$ m $\quad \eta_{prop} = 0.5$
$\eta_{Overall} = 0.18$ $\quad CV = 48950$ kJ/kg

To find

(i) Absolute velocity of jet,
(ii) Quantity of air compressed per minute,
(iii) Diameter of jet,
(iv) Net power output of the plant,
(v) Thrust specific fuel consumption, and
(vi) Air fuel ratio.

Analysis

(i) *Absolute velocity of jet*

The propulsive efficiency is given by

$$\eta_{prop} = \frac{2\mathcal{V}_a}{\mathcal{V}_a + \mathcal{V}_{jet}}$$

or $0.5 = \dfrac{2 \times 210}{210 + \mathcal{V}_{jet}}$

or $\mathcal{V}_{jet} = 630$ m/s

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Absolute velocity of jet

$$= \mathcal{V}_{jet} - \mathcal{V}_a = 630 - 210$$
$$= \mathbf{420\ m/s}$$

(ii) *Quantity of air compressed per minute*

The thrust force can be approximated as

$$F = \dot{m}_a (\mathcal{V}_{jet} - \mathcal{V}_a)$$

It is equal to drag force, thus

$$6670.8 = \dot{m}_a \times 420$$

or $\dot{m}_a = 15.88$ kg/s

The volume of air compressed per minute

$$\dot{V}_a = \frac{\dot{m}_a}{\rho} \times 60 = \frac{15.88}{0.172} \times 60$$
$$= \mathbf{5539.5\ m^3/min}$$

(iii) *Diameter of jet*

The mass-flow rate per jet

$$\dot{m}_1 = \frac{\dot{m}_a}{N_{jet}} = \frac{15.88}{2} = 7.94 \text{ kg/s}$$

Applying continuity equation to obtain the jet diameter

$$\dot{m}_1 = \rho \left(\frac{\pi}{4} d_{jet}^2 \right) \mathcal{V}_{jet}$$

or $$7.94 = 0.172 \times \left(\frac{\pi}{4} d_{jet}^2 \right) \times 630$$

or $d_{jet} = 0.305$ m or **30.5 cm**

(iv) *Net power output of the unit*

Thrust power = Thrust × Flight velocity
= 6670.8 × 210
= 1400868 W or 1400.868 kW

Propulsive Power

$$\dot{W}_{prop} = \frac{\text{Thrust power}}{\text{Propulsive efficiency}} = \frac{1400.868}{0.5}$$
$$= \mathbf{2801.74\ kW}$$

(v) *Air–fuel ratio*

Overall efficiency is given by

$$\eta_{overall} = \frac{\text{Power output}}{\text{Heat supply rate}} = \frac{\text{Propulsive power}}{\dot{m}_f \times CV}$$

or $$0.18 = \frac{2801.74\ kW}{\dot{m}_f \times 48950}$$

It gives $\dot{m}_f = 0.318$ kg/s

$$\text{A/F} = \frac{\dot{m}_a}{\dot{m}_f} = \frac{15.88}{0.318}$$
$$= \mathbf{49.94}\text{ kg of air/kg of fuel}$$

(vi) *Thrust–specific fuel consumption*

$$\text{Thrust } sfc = \frac{\text{Fuel-flow rate}}{\text{Thrust produced}}$$
$$= \frac{0.318}{6670.8}$$
$$= \mathbf{4.767 \times 10^{-5}\ kg/N\ thrust/s}$$

Example 28.4 *A turbojet engine flying at a speed of 990 km/h consumes air at a rate of 54.5 kg/s. Calculate*

(a) Exit velocity of jet, when enthalpy changes for the nozzle is 200 kJ/kg and velocity coefficient is 0.97,
(b) Fuel-flow rate in kg/s, when air-fuel ratio is 75 : 1
(c) Thrust-specific fuel consumption
(d) Thermal efficiency of the plant, when combustion efficiency is 93% and calorific value of fuel is 45000 kJ/kg
(e) Propulsive power
(f) Propulsive efficiency
(g) Overall efficiency

Solution

Given A turbojet engine

$\mathcal{V}_a = 990$ km/h
$\Delta h_N = 200$ kJ/kg
A/F = 75 : 1
$\dot{m}_a = 54.5$ kg/s
$k = 0.97$
$CV = 45000$ kJ/kg
$\eta_{comb} = 0.93$

To find

(i) Exit velocity of jet,
(ii) Fuel flow rate in kg/s,
(iii) Thrust specific fuel consumption,
(iv) Thermal efficiency of the plant,
(v) Propulsive power,
(vi) Propulsive efficiency, and
(vii) Overall efficiency.

Analysis

Flight velocity

$$\mathcal{V}_a = \frac{990 \times 1000}{3600} = 275 \text{ m/s}$$

(i) *Exit velocity of jet*

$$\mathcal{V}_{Jet} = k\sqrt{2\Delta h_N}$$
$$= 0.97 \times \sqrt{2 \times (200 \times 1000\ \text{J/kg})}$$
$$= \mathbf{613.48\ m/s}$$

(ii) *Fuel-flow rate*

Given that $\dot{m}_a = 54.5$ kg/s and A/F = 75

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Further, the Air-fuel ratio is given by

$$\text{A/F} = \frac{\text{Mass flow rate of air}}{\text{Mass flow rate of fuel}} = \frac{\dot{m}_a}{\dot{m}_f}$$

or $$\dot{m}_f = \frac{54.5 \text{ kg/s}}{75} = \mathbf{0.7267 \text{ kg/s}}$$

(iii) *Thrust specific fuel consumption*

Thrust produced

$$= \dot{m}_a(\mathcal{V}_{jet} - \mathcal{V}_a)$$
$$= 54.5 \times (613.48 - 275)$$
$$= 18447.16 \text{ N}$$

$$\text{Thrust } sfc = \frac{\text{Fuel-flow rate}}{\text{Thrust produced}}$$
$$= \frac{0.7267}{18447.16}$$
$$= \mathbf{3.94 \times 10^{-5} \text{ kg/N thrust/s}}$$

(iv) *Propulsive power*

$$\dot{W}_{prop} = \dot{m}_a \left(\frac{\mathcal{V}_{jet}^2 - \mathcal{V}_a^2}{2000} \right) \text{(kW)}$$
$$= 54.5 \times \left(\frac{613.48^2 - 275^2}{2000} \right)$$
$$= \mathbf{8194.96 \text{ kW}}$$

Thrust power;

$$\dot{W}_{th} = \text{Thrust} \times \mathcal{V}_a = 18447.16 \times 275$$
$$= 5072969 \text{ W} = 5072.97 \text{ kW}$$

(v) *Propulsive efficiency*

$$\eta_{prop} = \frac{\text{Thrust power}}{\text{Propulsive Power}} = \frac{5072.97}{8194.96}$$
$$= 0.619 \quad \text{or} \quad \mathbf{61.9\%}$$

It can also be obtained by using

$$\eta_{prop} = \frac{2\mathcal{V}_a}{(\mathcal{V}_{jet} + \mathcal{V}_a)}$$

(vi) *Thermal efficiency*

Heat supply rate;

$$\dot{Q}_{in} = \dot{m}_f \, CV = 0.7267 \times 45000$$
$$= 32701.5 \text{ kW}$$

$$\eta_{th} = \frac{\dot{W}_{prop}}{\dot{Q}_{in}} = \frac{8194.96}{32701.5}$$
$$= 0.2505 \quad \text{or} \quad \mathbf{25.05\%}$$

(vii) *Overall efficiency*

$$\eta_{overall} = \eta_{th} \times \eta_{prop}$$
$$= 0.2505 \times 0.619$$
$$= 0.1551 \quad \text{or} \quad \mathbf{15.51\%}$$

Example 28.5 *Air enters a turbojet engine at 80 kPa, 240 K and an inlet velocity of 280 m/s. The pressure ratio across the compressor is 8. The turbine inlet temperature is 1200 K and the pressure at the nozzle exist is 80 kPa. The work developed by the turbine equals the compressor work input. The diffuser, compressor, turbine and nozzle processes are isentropic, and there is no pressure drop for flow through the combustor. For operation at steady state, determine the velocity at the nozzle exit and the pressure at each principal state. Neglect kinetic energy at the exit of all components except the nozzle and neglect potential energy throughout.*

Solution

Given An ideal turbojet engine operates at steady state with

$p_1 = 80$ kPa, $T_1 = 240$ K
$\mathcal{V}_1 = 280$ m/s, $r_p = 8$
$p_6 = p_1 = 80$ kPa, $T_4 = 1200$ K
$W_T = W_{in}$

To find

Velocity at the nozzle exit and pressure at each principal states

Assumptions

(i) Each component is analysed as a control volume at steady state.

(ii) The working fluid is air modelled as an ideal gas with $C_p = 1005$ J/kg·K, $\gamma = 1.4$

Analysis Analysing each device separately.

Diffuser

Temperature T_2, after diffusion process,

$$T_2 = T_1 + \frac{\mathcal{V}_1^2}{2C_p} = 240 + \frac{280^2}{2 \times 1005} = 279 \text{ K}$$

$$p_2 = p_1 \left(\frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma - 1}} = 80 \times \left(\frac{279}{240} \right)^{3.5}$$
$$= 135.5 \text{ kPa}$$
$$p_3 = r_p \times p_2 = 8 \times 135.5 = 1084 \text{ kPa}$$

Compressor

$$T_3 = T_2 \left(\frac{p_3}{p_2} \right)^{\frac{\gamma - 1}{\gamma}} = 279 \times (8)^{\frac{1.4-1}{1.4}}$$
$$= 505.4 \text{ K}$$
$$p_4 = p_3 = 1084 \text{ kPa}$$

Schematic with given data

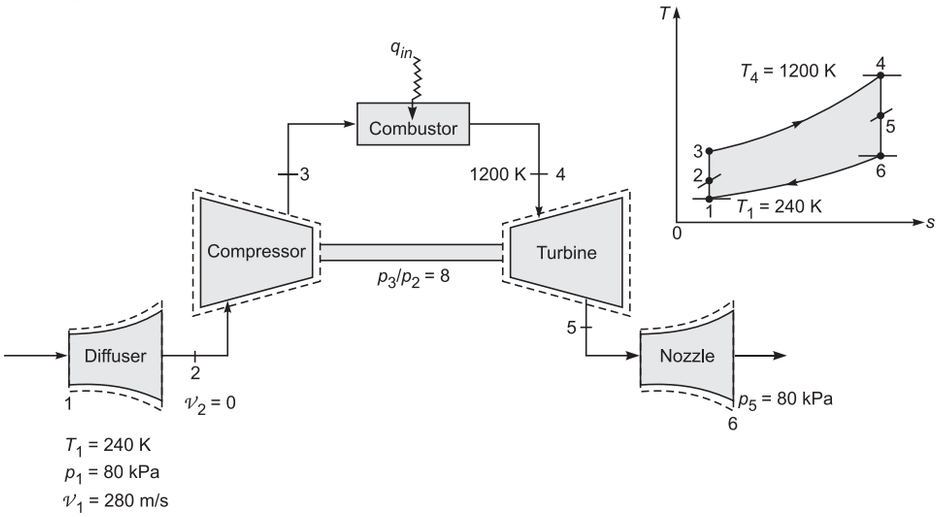

Fig. 28.14

$T_4 = 1200$ K (given)

$q_{2-3} = 0$

Compression work

$w_{in} = C_p (T_3 - T_2) + 0 + 0$

or $w_{2-3} = 1005 \times (505.4 - 279) = 227.53$ kJ/kg

Turbine: Given, compressor work is equal to turbine work,

$W_T = W_{in}$

or $W_{in} = C_p(T_4 - T_5)$

or $227.53 = 1.005 \times (1200 - T_5)$

or $T_5 = 1200 - \dfrac{227.54}{1.005} = 973.6$ K

$$p_5 = p_4\left(\frac{T_5}{T_4}\right)^{\frac{\gamma}{\gamma-1}} = 1084 \times \left(\frac{973.6}{1200}\right)^{3.5}$$

$= 521.46$ kPa

Nozzle: Expansion through nozzle

Jet velocity; $\mathcal{V}_{jet} = \sqrt{2C_p (T_5 - T_6)}$

where $T_6 = T_5\left(\dfrac{p_6}{p_5}\right)^{\frac{\gamma-1}{\gamma}} = 973.6 \times \left(\dfrac{80}{521.46}\right)^{\frac{1.4-1}{1.4}}$

$= 569.86$ K

Thus $\mathcal{V}_{jet} = \sqrt{2 \times 1005\,(973.6 - 569.86)}$

$= \mathbf{900.8\ m/s}$

Example 28.6 *A turbojet engine aircraft flies with a velocity of 260 m/s at an altitude where the air is at 35 kPa and – 40°C. The compressor has a pressure ratio of 10 and the temperature of the gases at the turbine inlet is 1095°C. The air enters the compressor at a rate of 8.5 kg/s. Using the cold air-standard assumptions, determine (a) temperature and the pressure of the gases at the turbine exit, (b) velocity of the gases at the nozzle exit, and (c) propulsive efficiency of the cycle.*

Solution

Given An air standard jet propulsive cycle with

$p_1 = 35$ kPa $\quad \mathcal{V}_a = 260$ m/s

$T_1 = -40°C = 233$ K $\quad r_p = p_3/p_2 = 10$

$T_4 = 1095°C = 1368$ K $\quad \dot{m} = 8.5$ kg/s

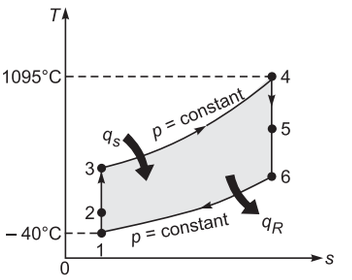

Fig. 28.15 *T-s diagram for the turbojet cycle*

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To find

(i) Temperature and pressure of the gases at the turbine exit,
(ii) Velocity of gases at the nozzle exit, and
(iii) Propulsive efficiency.

Assumptions

(i) Each component in the cycle is analysed as a control volume at steady state.
(ii) Diffuser, compressor, turbine and nozzle operations are isentropic.
(iii) The combustion process is replaced by heat addition.
(iv) There is no pressure drop in combustion chamber.
(v) Network output of the turbojet engine is zero i.e., turbine work is equal to compressor work.
(vi) Potential energy changes are negligble.
(vii) Except at inlet and exit the kinetic energy effects are ignored.
(viii) The working substance is air as an ideal gas with $C_p = 1.005$ kJ/kg·K and $\gamma = 1.4$.

Analysis

(i) *Temperature and pressure at the turbine exit*

Process 1–2 Raming effect: Isentropic compression

$$T_2 = T_1 + \frac{\mathcal{V}_a^2}{2C_p}$$

$$= 233 + \frac{260^2}{2 \times 1005}$$

$$= 233 + 33.63 = 266.63$$

and pressure p_2 after isentropic compression

$$p_2 = p_1 \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= 35 \times \left(\frac{266.63}{233}\right)^{\frac{1.4}{1.4-1}}$$

$$= 56.1 \text{ kPa}$$

Process 2–3 Isentropic compression in a compressor;

$$p_3 = r_p\, p_2 = 10 \times 56.1 = 561 \text{ kPa}$$

$$T_3 = T_2 \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 266.63 \times (10)^{\frac{1.4-1}{1.4}}$$

$$= 514.78 \text{ K}$$

Process 4–5 Isentropic expansion in the turbine;

$$w_{4-5} = w_{2-3}$$

$$h_4 - h_5 = h_3 - h_2$$

$$C_p(T_4 - T_5) = C_p(T_3 - T_2)$$

or $$T_5 = T_4 - T_3 + T_2$$

$$= 1368 - 514.78 + 266.63 = \mathbf{1120\ K}$$

and pressure at turbine exit

$$p_5 = p_4 \left(\frac{T_5}{T_4}\right)^{\frac{\gamma}{\gamma-1}} = p_3 \left(\frac{T_5}{T_4}\right)^{\frac{1-4}{1.4-1}}$$

$$= 561 \times \left(\frac{1120}{1368}\right)^{3.5} = \mathbf{278.43\ kPa}$$

(ii) *Process 5–6* Isentropic expansion of gases in the nozzle

$$T_6 = T_5 \left(\frac{p_6}{p_5}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 1120 \times \left(\frac{35}{278.43}\right)^{\frac{1.4-1}{1.4}}$$

$$= 619.3$$

(iii) *Velocity of Gases at nozzle exit*

$$\mathcal{V}_{jet} = \sqrt{2C_p(T_5 - T_6)}$$

$$= \sqrt{2 \times 1005 \times (1120 - 619.3)}$$

$$= 1003.2 \text{ m/s}$$

(iv) *Propulsive efficiency*

$$\eta_p = \frac{\text{Propulsive power}}{\text{Heat supply rate}} = \frac{\dot{m}(\mathcal{V}_{jet} - \mathcal{V}_a)\mathcal{V}_a}{\dot{m}C_p(T_4 - T_3)}$$

$$= \frac{(1003.2 - 260) \times 260}{1005 \times (1368 - 514.78)} = \mathbf{0.225 \quad or \quad 22.5\%}$$

Example 28.7 *A turbojet aircraft is flying at 800 km/h at 10700 m altitude where pressure and temperature of the atmosphere are 24 kPa and –50°C, respectively. The pressure ratio in the compressor is 10 and the maximum cycle temperature is 820°C. Calculate the thrust developed and specific fuel consumption, using the following information; entry duct efficiency 0.9; isentropic efficiency of the compressor 0.9; Stagnation pressure drop in the combustion chamber 0.14 bar; calorific value of the fuel 43,300 kJ/kg; combustion efficiency 98%; isentropic efficiency of the turbine 0.92; mechanical efficiency of the drive 0.98, jet pipe efficiency 0.92; nozzle outlet area 0.08 m^2; C_p and γ for compression process 1.005 kJ/kg·K and 1.4; C_p and γ_p for combustion and expansion precess 1.15 kJ/kg·K and 1.333, respectively. Assume that the nozzle is convergent.*

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Solution

Given A turbojet engine operates on a jet propulsion cycle

$\mathcal{V}_a = 800$ km/h = 222.22 m/s, $z = 10700$ m

$p_1 = 24$ kPa $\quad T_1 = -50°C = 223$ K

$r_p = \dfrac{p_3}{p_2} = 10 \quad T_4 = 820°$ C = 1093 K

$\eta_{diffuser} = 0.9 \quad \eta_C = 0.9,$

$p_4 - p_3 = 0.14$ bar $\quad CV = 43{,}300$ kJ/kg

$\eta_{comb} = 0.98 \quad \eta_T = 0.92$

$\eta_{mech} = 0.98 \quad \eta_{nozzle} = 0.92$

$A_6 = 0.08$ m^2

For compression and ramming

$C_{pa} = 1.005$ kJ/kg·K $\quad \gamma = 1.4$

For combustion and expansion

$C_{pg} = 1.15$ kJ/kg·K $\quad \gamma = 1.333$

To find

(i) Thrust developed, and

(ii) Specific fuel consumption.

Schematic with given data

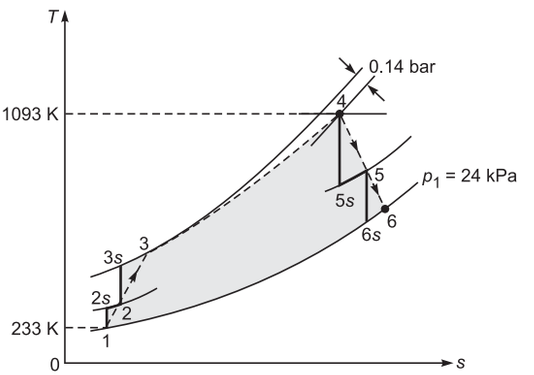

Fig. 28.16

Assumptions

(i) Each component in the cycle is analysed as a control volume at steady state.

(ii) Kinetic energy is negligible except at diffuser inlet and nozzle exit.

(iii) Negligible potential energy change in the systems.

Analysis Stagnation temperature after ramming

$$T_{2s} = T_1 + \frac{\mathcal{V}_a^2}{2C_P} = 223 + \frac{(222.22)^2}{2\times 1005}$$

$$= 247.56 \text{ K}$$

Now diffuser efficiency

$$\eta_{diffuser} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$T_2 = \frac{247.56 - 223}{0.9} + 233 = 250.3 \text{ K}$$

$$p_2 = p_1\left(\frac{T_{2s}}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = 24 \text{ kPa} \times \left(\frac{247.56}{223}\right)^{3.5}$$

$$= 29.67 \text{ kPa}$$

For isentropic compression 2–3

$$\frac{T_{3s}}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = (r_p)^{\frac{\gamma-1}{\gamma}}$$

$$= (10)^{\frac{1.4-1}{1.4}} = 1.93$$

Temperature after isentropic compression is

$$T_{3s} = 1.93 \times 250.3 = 483.25 \text{ K}$$

Using compressor efficiency

$$\eta_C = \frac{h_{3s} - h_2}{h_3 - h_2} = \frac{T_{3s} - T_2}{T_3 - T_2}$$

or $$T_3 = 250.3 + \frac{483.25 - 250.3}{0.9} = 509.13 \text{ K}$$

Pressure after compression;

$$p_3 = r_p p_2 = 10 \times 29.67 = 296.7 \text{ kPa}$$

Pressure after combustion

$$p_4 = p_3 - \Delta p = 296.7 - 0.14 \times 100 = 282.7 \text{ kPa}$$

Compressor work,

$$w_{comp} = h_3 - h_2 = C_p(T_3 - T_2)$$

$$= 1.005 \times (509.13 - 250.3) = 260.12 \text{ kJ/kg}$$

The actual turbine work required, just to drive the compressor and to overcome the mechanical losses

$$w_T = \frac{w_{comp}}{\eta_{mech}} = \frac{260.12}{0.98} = 265.43 \text{ kJ/kg}$$

Further $$w_T = C_{pg}(T_4 - T_5)$$

or $$T_5 = 1093 - \frac{265.43}{1.15} = 862.18 \text{ K}$$

Using isentropic efficiency of the turbine

$$\eta_T = \frac{h_4 - h_5}{h_4 - h_{5s}} = \frac{T_4 - T_5}{T_4 - T_{5s}} = 0.92$$

or $$T_{5s} = 1093 - \frac{1093 - 862.18}{0.92} = 842.1 \text{ K}$$

The pressure after turbine expansion

$$p_5 = p_4\left(\frac{T_{5s}}{T_4}\right)^{\frac{\gamma}{\gamma-1}} = 282.7 \times \left(\frac{842.1}{1093}\right)^{\frac{1.333}{1.333-1}}$$

$$= 99.5 \text{ kPa}$$

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For flow through nozzle (refer compressible fluid flow)

For chocked flow, the critical pressure ratio is given by Eq. (10.26)

$$\frac{p^*}{p_5} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{2}{1.333+1}\right)^{\frac{1.333}{1.333-1}}$$

$$= 0.54$$

and $p^* = 99.5 \times 0.54 = 53.7$ kPa

The atmospheric pressure is 24 kPa, it means mass flow rate through the nozzle is maximum and velocity of fluid at the nozzle exit would be sonic velocity.

Temperature at the nozzle exit: Eq. (10.25)

$$\frac{T_{6s}}{T_5} = \frac{2}{\gamma+1} = \frac{2}{1.333+1} = 0.857$$

and $T_{6s} = 0.857 \times 862.18 = 739.1$ K

Jet pipe or nozzle efficiency

$$\eta_{nozzle} = \frac{h_5 - h_6}{h_5 - h_{6s}} = \frac{T_5 - T_6}{T_5 - T_{6s}}$$

$$T_6 = T_5 - \eta_{nozzle} \times (T_5 - T_{6s})$$

$$= 862.18 - 0.92 \times (862.18 - 739.1)$$

$$= 749 \text{ K}$$

$$p_6 = p_5 \left(\frac{T_{6s}}{T_5}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= 99.5 \times \left(\frac{739.1}{862.18}\right)^{\frac{1.333}{0.333}}$$

$$= 53.7 \text{ kPa}$$

Specific gas constant

$$R = \frac{C_p(\gamma-1)}{\gamma} = \frac{1.15 \times (1.333-1)}{1.333}$$

$$= 0.287 \text{ kJ/kg K}$$

The specific volume at state 6

$$v_6 = \frac{RT_6}{p_6} = \frac{0.287 \times 749}{53.5} = 4.02 \text{ m}^3/\text{kg}$$

The velocity (sonic) at the nozzle exit

$$\mathcal{V}_{exit} = a = \sqrt{\gamma R T_6}$$

$$= \sqrt{1.333 \times 0.287 \times 10^3 \times 749}$$

$$= 535.3 \text{ m/s}$$

The mass flow rate of air through the nozzle

$$\dot{m} = \frac{A \cdot \mathcal{V}_{exit}}{v_6} = \frac{0.08 \times 535.3}{4.02} = 10.62 \text{ kg/s}$$

(i) The momentum thrust

$$F = \dot{m}(\mathcal{V}_{exit} - \mathcal{V}_a)$$

$$= 10.62 \times (535.3 - 222.22)$$

$$= 3335.1 \text{ N}$$

Since the pressure at the nozzle exit is greater than the atmospheric pressure, thus the pressure thrust will also act

Pressure thrust $= (p_6 - p_1)A$

$$= (53.7 - 24) \times 10^3 \times 0.08$$

$$= 2376 \text{ N}$$

Total thrust (propulsive force)

$$= 3335.1 + 2376$$

$$= \mathbf{5711.1\ N}$$

(ii) Ideal Heat supplied in the combustion chamber

$$= \dot{m} C_{pg}(T_4 - T_3)$$

$$= 10.62 \times 1.15 \times (1093 - 509.13)$$

$$= 7130.8 \text{ kJ/s}$$

Actual heat supplied

$$Q_{act} = \frac{\text{Ideal heat supply}}{\eta_{comb}}$$

$$= \frac{7130.8}{0.98} = 7276.3 \text{ kW}$$

It is also expressed as

$$Q_{act} = \dot{m}_f \times CV$$

or $7276.3 = \dot{m}_f \times 43300$

or $\dot{m}_f = 0.168$ kg/s

Specific fuel consumption rate

$$\text{Thrust } sfc = \frac{\dot{m}_f}{\text{Total thrust}} = \frac{0.168}{5711.1}$$

$$= \mathbf{0.029\ kg/kNs}$$

Remark:

(i) The equations used for critical flow through the nozzle are taken from compressible fluid flow given in Chapter 10.

(ii) Since air fuel ratio used in jet propulsion cycle is very low (= 0.029 kg/kNs), therefore, approximation of mass flow rate of air equal to mass flow rate of gases in the turbine and nozzle is proper.

Example 28.8 *A turboprop aircraft is flying at a speed of 720 km/h at an altitude where the temperature is –18°C. Determine specific power output and thermal*

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efficiency. Given specifications are

Compressor pressure ratio = 9
Maximum cycle temperature = 800°C
Intake duct efficiency = 90%
Isentropic efficiency of compressor = 86%
Isentropic efficiency of turbine = 90%
Mechanical Efficiency = 92%

Neglect the pressure loss in the combustion chamber. Assume that the exhaust gases leave the aircraft at 720 km/h relative to aircraft. For compression, C_p = 1.005 kJ/kg·K, γ = 1.4; and for expansion, C_p = 1.15 kJ/kg·K, γ = 1.35.

Solution

Given A turboprop engine

$\mathcal{V}_a = 720$ km/h $\quad T_1 = -18°C = 255$ K

$\frac{p_3}{p_2} = 9 \quad T_4 = 800°C = 1073$ K

$\eta_{diffuser} = 0.9 \quad \eta_C = 0.86$

$\eta_T = 0.9 \quad \eta_{mech} = 0.92$

For air; $C_{pa} = 1.005$ kJ/kg·K $\quad \gamma_a = 1.4$

For gas; $C_{pg} = 1.15$ kJ/kg·K $\quad \gamma_g = 1.35$

To find

(i) Specific power output, and

(ii) Thermal efficiency of the plant.

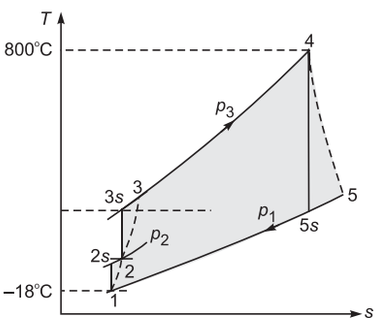

Fig. 28.17

Analysis The flight velocity

$$\mathcal{V}_a = \frac{720 \times 1000}{3600} = 200 \text{ m/s}$$

Thus the flow of air is subsonic.

Analysing each device separately;

Intake duct: Diffuser

Temperature T_{2s} after isentropic diffusion process,

$$T_{2s} = T_1 + \frac{\mathcal{V}_a^2}{2C_p} = 255 + \frac{200^2}{2 \times 1005} = 274.9 \text{ K}$$

The pressure ratio across the diffuser

$$\frac{p_2}{p_1} = \left(\frac{T_{2s}}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{274.9}{255}\right)^{\frac{1.4}{1.4-1}} = 1.3$$

For subsonic flow, the diffuser efficiency is given by

$$\eta_{diffuser} = \frac{\text{Isentropic temp. rise}}{\text{Actual temp. rise}} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

or $$T_2 = 255 + \frac{274.9 - 255}{0.9} = 277.11 \text{ K}$$

Compressor

Temperature T_{3s} after isentropic compression

$$T_{3s} = T_2\left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = 277.11 \times \left(\frac{9}{1}\right)^{\frac{1.4-1}{1.4}}$$

$$= 519.15 \text{ K}$$

The isentropic efficiency of the compressor is given by

$$\eta_C = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{T_{3s} - T_2}{T_3 - T_2}$$

or $$T_3 = 277.11 + \frac{519.15 - 277.11}{0.86} = 558.55 \text{ K}$$

The actual work input to compressor w_{in} is expressed as

$$w_{in} = C_p(T_3 - T_2)$$
$$= 1.005 \times (558.55 - 277.11)$$
$$= 282.85 \text{ kJ/kg}$$

Combustion chamber

The heat supplied to air

$$q_{in} = C_{pg}(T_4 - T_3)$$
$$= 1.15 \times (1073 - 558.55)$$
$$= 591.61 \text{ kJ/kg of air}$$

Turbine

The pressure ratio across the turbine is given by

$$\frac{p_4}{p_5} = \frac{p_3}{p_1} = \frac{p_3}{p_2} \times \frac{p_2}{p_1} = 9 \times 1.3 = 11.70$$

The temperature T_{5s} after isentropic expansion in the turbine;

$$T_{5s} = \frac{T_4}{\left(\frac{p_4}{p_5}\right)^{\frac{\gamma_g - 1}{\gamma_g}}} = \frac{1073}{\left(\frac{11.70}{1}\right)^{\frac{1.35-1}{1.35}}} = 567 \text{ K}$$

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Using isentropic efficiency of turbine;

$$\eta_T = \frac{T_4 - T_5}{T_4 - T_{5s}}$$

Actual temperature after expansion in turbine,

$$T_5 = 1073 - 0.9 \times (1073 - 567) = 617.6 \text{ K}$$

(i) *Specific work output*

Turbine work output per kg of air

$$w_{out} = C_{pg}(T_4 - T_5)$$
$$= 1.15 \times (1073 - 617.6)$$
$$= 523.7 \text{ kJ/kg}$$

Net work output per kg of the plant

$$w_{net} = w_{out} - w_{in} = 523.7 - 282.85$$
$$= \mathbf{240.85 \text{ kJ/kg of air}}$$

It is the specific work output of plant, which is supplied to propeller of turboprop.

(ii) *Thermal efficiency*

Thermal efficiency of the cycle

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{240.85}{591.61}$$
$$= 0.4071 \quad \text{or} \quad \mathbf{40.67\%}$$

Example 28.9 *A simple turbojet unit operates with a turbine inlet temperature of 1040°C. The following data refers to the design conditions:*

Compressor pressure ratio = 7.5
Compressor efficiency = 84%
Turbine efficiency = 84%
Nozzle efficiency = 98%
Pressure drop in combustion chamber = 0.2 bar
Mass flow rate = 25 kg/s
Atmospheric pressure and temperature = 1 bar, 27°C

Neglect the mass of the fuel and mechanical losses.

Calculate the design thrust, pressure and temperature at the inlet of the jet nozzle, temperature, velocity and Mach number at the nozzle exit.

Solution

Given A simple turbojet unit

$p_1 = 1$ bar, $T_1 = 27°C = 300$ K
$\frac{p_2}{p_1} = 7.5$, $T_3 = 1040°C = 1313$ K
$\eta_C = 0.84$, $\eta_T = 0.84$
$\eta_{Nozzle} = 0.98$, $\dot{m}_a = 25$ kg/s
$\dot{m}_f \approx 0$, $p_3 = p_2 - 0.2$ bar

To find

(i) Thrust developed,
(ii) Pressure and temperature at inlet of jet nozzle,
(iii) Temperature, velocity and Mach number at nozzle exit, and
(iv) Specific fuel consumption.

T–s diagram

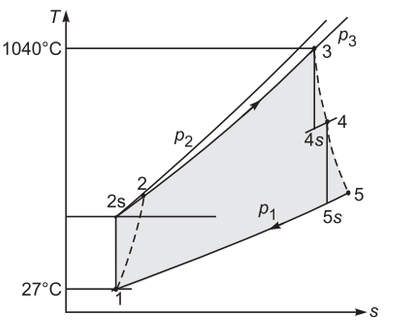

Fig. 28.18

Assumptions

(i) Each component in the cycle operates at steady state.
(ii) Kinetic energy effects are considered at nozzle only.
(iii) Pressure at nozzle exit is atmopheric pressure.
(iv) Assuming flight at ground level, i.e., $\mathcal{V}_a = 0$.
(v) Specific heat of air at constant pressure as 1.005 kJ/kg·K and $\gamma = 1.4$.

Analysis Analysing each device of the cycle separately in steady state.

Compressor Temperature T_{2s} after isentropic compression

$$T_{2s} = T_1(r_p)^{\frac{\gamma-1}{\gamma}}$$
$$= 300 \times \left(\frac{7.5}{1}\right)^{\frac{1.4-1}{1.4}} = 533.5 \text{ K}$$

The isentropic efficiency of the compressor is given by

$$\eta_C = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

or $$T_2 = 300 + \frac{533.5 - 300}{0.84} = 577.98 \text{ K}$$

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The actual work input to compressor w_{in}

$$w_{in} = C_p(T_2 - T_1)$$
$$= 1.005 \times (577.98 - 300) = 279.37 \text{ kJ/kg}$$

Turbine For a turbojet engine,

$$w_{out} = w_{in}$$
or $$w_{in} = C_p(T_3 - T_4)$$
or $$279.37 = 1.005 \times (1313 - T_4)$$
or $$T_4 = \mathbf{1035\ K}$$

It is the temperature at nozzle entry.

The isentropic efficiency of the turbine is given as

$$\eta_T = \frac{\text{Actual work output}}{\text{Isentropic work output}} = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

or $$0.84 = \frac{1313 - 1035}{1313 - T_{4s}}$$

or $$T_{4s} = 982.05 \text{ K},$$

Pressure p_3 before turbine entry

$$p_3 = p_2 - 0.2 \text{ bar} = 7.5 - 0.2 = 7.3 \text{ bar}$$

The pressure p_4 after isentropic expansion in the turbine can be expressed as

$$\frac{p_3}{p_4} = \left(\frac{T_3}{T_{4s}}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{1313}{982.05}\right)^{\frac{1.4}{1.4-1}} = 2.763$$

Pressure p_4 at nozzle entry

$$p_4 = \frac{p_3}{2.763} = \frac{7.3 \text{ bar}}{2.763} = \mathbf{2.64\ bar}$$

Isentropic expansion through nozzle

$$\frac{T_4}{T_{5s}} = \left(\frac{p_4}{p_5}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{2.64}{1}\right)^{\frac{1.4-1}{1.4}} = 1.32$$

or $$T_{5s} = \frac{1035}{1.32} = 784.3 \text{ K}$$

The isentropic efficiency of the nozzle is given as

$$\eta_{Nozzle} = \frac{\text{Actual enthalpy drop}}{\text{Isentropic enthalpy drop}} = \frac{T_4 - T_5}{T_4 - T_{5s}}$$

Actual temperature after expansion in nozzle,

$$T_5 = 1035 - 0.98 \times (1035 - 784.3)$$
$$= \mathbf{789.3\ K}$$

It is the temperature at nozzle exit.

Exit jet velocity from nozzle

$$\mathcal{V}_{jet} = \sqrt{2C_p(T_4 - T_5)}$$
$$= \sqrt{2 \times 1005 \times (1035 - 789.3)}$$
$$= \mathbf{702.75\ m/s}$$

Mach number at nozzle exit

Sonic velocity,

$$a = \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 789.3}$$
$$= 563.1 \text{ m/s}$$

Mach number,

$$M = \frac{\mathcal{V}_{jet}}{a} = \frac{702.75}{563.1} = \mathbf{1.25}$$

Thrust developed

$$F = \dot{m}_a(\mathcal{V}_{jet} - \mathcal{V}_a) = 25 \times (702.75 - 0)$$
$$= \mathbf{17568.75\ N}$$

Example 28.10 *A turbojet engine develops a thrust power of 750 kW, when flying at an altitude of 9200 m at a velocity of 220 m/s. The following data refers to the design conditions:*

Compressor pressure ratio = 5
Compressor efficiency = 85%
Turbine efficiency = 85%
Nozzle efficiency = 90%
Inlet pressure and temperature = 0.306 bar, –45.5°C
Temperature of gas leaving the combustion chamber = 670°C
Calorific value of fuel = 42500 kJ/kg
Velocity in ducts = 200 m/s
For air; C_p = 1.005 kJ/kg·K γ = 1.4
For combustion gases C_p = 1.087 kJ/kg·K γ = 1.33

Calculate

(a) air–fuel ratio
(b) overall thermal efficiency of unit
(c) rate of air consumption
(d) power developed by the turbine
(e) outlet area of the jet
(f) specific fuel consumption

Solution

Given A turbojet unit

$p_1 = 0.306$ bar
$T_1 = -45.5°C = 227.5$ K
$T_3 = 670°C = 943$ K
$\eta_T = 0.85$
$F\mathcal{V}_a = 750$ kW
$\mathcal{V}_{duct} = 200$ m/s
$\mathcal{V}_a = 220$ m/s
$\frac{p_2}{p_1} = 5$
$\eta_C = 0.85$
$\eta_{Nozzle} = 0.9$
$CV = 42500$ kJ/kg
$\Delta z = 9200$ m

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For air; $C_{pa} = 1.005$ kJ/kg·K $\quad \gamma_a = 1.4$
For gas; $C_{pg} = 1.087$ kJ/kg·K $\quad \gamma_g = 1.33$

To find

(i) Air–fuel ratio,
(ii) Overall thermal efficiency of unit,
(iii) Rate of air consumption,
(iv) Power developed by the turbine,
(v) Outlet area of the jet, and
(vi) Specific fuel consumption.

Assumptions

(i) Each component in the cycle operates at steady state.
(ii) Kinetic energy effect is considered in fluid flow through nozzle only.
(iii) Pressure at nozzle exit is atmopheric pressure.
(iv) No pressure drop in combustion chamber.
(v) Specific gas constant for exhaust gases as $R = 0.287$ kJ/kg·K.

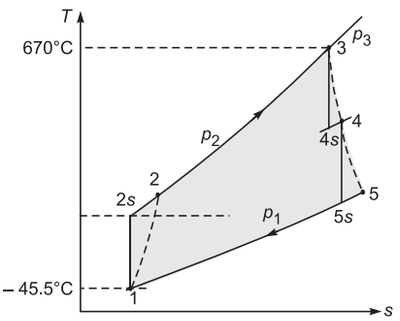

Fig. 28.19 *T–s diagram*

Analysis Analysing each device of the cycle separately in steady state.

Compressor Temperature T_{2s} after isentropic compression

$$T_{2s} = T_1\left(r_p\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 227.5 \times \left(\frac{5}{1}\right)^{\frac{1.4-1}{1.4}} = 360.3 \text{ K}$$

The isentropic efficiency of the compressor is given by;

$$\eta_C = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

or $$T_2 = 227.5 + \frac{360.3 - 227.5}{0.85} = 383.75 \text{ K}$$

The actual work input to compressor w_{in}

$$w_{in} = C_p(T_2 - T_1)$$
$$= 1.005 \times (383.75 - 227.5) = 157.03 \text{ kJ/kg}$$

Turbine For a turbojet engine;

$$w_{in} = w_T = C_{pg}(T_3 - T_4)$$

or $$157.03 = 1.087 \times (943 - T_4)$$

The actual temperature T_4 after turbine expansion

$$T_4 = 798.53 \text{ K}$$

The isentropic efficiency of the turbine is given as

$$\eta_T = \frac{\text{Actual work output}}{\text{Isentropic work output}} = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

or $$0.85 = \frac{943 - 798.53}{943 - T_{4s}}$$

or $$T_{4s} = 773.04 \text{ K},$$

The pressure p_4 after isentropic expansion in the turbine can be expressed as

$$\frac{p_3}{p_4} = \left(\frac{T_3}{T_{4s}}\right)^{\frac{\gamma_g}{\gamma_g - 1}} = \left(\frac{943}{773.04}\right)^{\frac{1.33}{1.33-1}} = 2.227$$

Pressure ratio $\frac{p_4}{p_5}$ across nozzle is given by

$$\frac{p_4}{p_5} = \frac{p_4}{p_3} \times \frac{p_3}{p_1} = \frac{p_4}{p_3} \times \frac{p_2}{p_1} = \frac{1}{2.227} \times 5$$
$$= 2.244$$

Isentropic expansion through nozzle

$$\frac{T_4}{T_{5s}} = \left(\frac{p_4}{p_5}\right)^{\frac{\gamma_g - 1}{\gamma_g}} = \left(\frac{2.244}{1}\right)^{\frac{1.33-1}{1.33}} = 1.222$$

or $$T_{5s} = \frac{798.53}{1.222} = 653.38 \text{ K}$$

The isentropic efficiency of the nozzle jet is given by Eq. (28.24);

$$\eta_{Nozzle} = \frac{\text{Final K.E. of jet}}{\text{Isentropic enthalpy drop + K.E. carried over turbine}}$$

$$= \frac{\frac{\mathcal{V}_{Jet}^2}{2}}{C_{pg}(T_4 - T_{5s}) + \frac{\mathcal{V}_{duct}^2}{2}}$$

or $$0.9 = \frac{\mathcal{V}_{Jet}^2}{2 \times 1087 \times (798.53 - 653.38) + 200^2}$$

or $$\mathcal{V}_{jet} = 565.68 \text{ m/s}$$

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The energy balance in nozzle is given by

Change in enthalpy = Change in KE

$$C_p(T_4 - T_5) = \frac{\mathcal{V}_{jet}^2 - \mathcal{V}_{duct}^2}{2}$$

$$1087 \times (798.53 - T_5) = \frac{565.68^2 - 200^2}{2}$$

Actual temperature after expansion in nozzle,

$T_5 = 669.73$ K

Combustion chamber

Heat supplied per kg of air is given by

$$q_{in} = \dot{m}_f CV = (1 + \dot{m}_f) C_{pg}(T_3 - T_2)$$

or $m_f \times 42500 = (1 + m_f) \times 1.087 \times (943 - 383.75)$

$$42500\, m_f = 607.9 + 607.9\, \dot{m}_f$$

or $m_f = 0.0145$ kg/kg of air

(i) *Air–fuel ratio* $= \dfrac{m_a}{m_f} = \dfrac{1}{0.0145} =$ **68.91**

(ii) *Overall thermal efficiency of unit*

$$\eta_{overall} = \frac{\left[(\dot{m}_a + \dot{m}_f)\mathcal{V}_{jet} - \dot{m}_a \mathcal{V}_a\right]\mathcal{V}_a}{\dot{m}_f CV}$$

$$= \frac{\left[\left(1 + \dfrac{m_f}{m_a}\right)\mathcal{V}_{jet} - \mathcal{V}_a\right]\mathcal{V}_a}{\dfrac{m_f}{\dot{m}_a} \times CV}$$

$$= \frac{\left[(1 + 0.0145) \times 565.68 - 220\right] \times 220}{0.0145 \times (42500 \times 1000 \text{ J/kg})}$$

$= 0.1263$ or **12.63%**

(iii) *Mass-flow rate of air*

Thrust power is given as

Thrustpo wer = Thrust × Flight Velocity

$= [(\dot{m}_a + \dot{m}_f)\, \mathcal{V}_{jet} - \dot{m}_a \mathcal{V}_a]\mathcal{V}_a$

750×1000 W

$= [(1 + 0.0145) \times 565.68 - 220]\, \dot{m}_a \times 220$

or $\dot{m}_a =$ **9.838 kg/s**

(iv) *Power developed by the turbine*

Turbine output

$= \dot{m}_a (1 + m_f)\, C_{pg}(T_3 - T_4)$

$= 9.838 \times (1 + 0.0145) \times 1.087 \times (943 - 798.53)$

$=$ **1567.4 kW**

(v) *Outlet area of the duct*

Density of exhaust gas

$$\rho_5 = \frac{p_5}{RT_5} = \frac{(0.306 \times 100 \text{ kPa})}{0.287 \times 669.73}$$

$= 0.159$ kg/m^3

Nozzle discharge area can be obtained from continuity equation

$\dot{m}_a(1 + m_f) = \rho_5 A_{jet} \mathcal{V}_{jet}$

or $A_{jet} = \dfrac{9.838 \times (1 + 0.0145)}{0.159 \times 565.68} =$ **0.11 m^2**

(vi) *Specific fuel consumption*

$$F = \frac{\text{Thrust Power}}{\text{Flight Velocity}} = \frac{750 \times 1000 \text{ W}}{220 \text{ m/s}}$$

$= 3409.1$ N

Mass-flow rate of fuel

$\dot{m}_f = \dot{m}_a \times m_a = 9.838 \times 0.0145$

$= 0.1426$ kg/s

Thrust *sfc* $= \dfrac{\dot{m}_f}{F} = \dfrac{0.1426}{3409.1} \times 3600$

$=$ **0.1506 kg/kN-h**

28.11 ROCKET PROPULSION

28.11.1 Principle

A *rocket* or *rocket vehicle* (shown in Fig. 28.20) is a missile, which obtains thrust by the reaction from

Fig. 28.20 *Rocket vehicle*

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the ejection of fast-moving exhaust fluid from a rocket engine. They are not suitable for low-speed use. As compared to other propulsion systems, they are very light in weight and powerful, capable of generating large accelerations and of attaining extremely high speeds with reasonable efficiency.

28.11.2 Construction and Working

A rocket is a non-air-breathing engine and it has a few moving parts. It consists of a combustion chamber and exhaust nozzle. It carries fuel and oxidiser (such as liquid oxygen) on the board of the craft. The fuel and oxidiser react chemically in the combustion chamber and then high-pressure combustion gases act as rocket propellant. These gases expand through the nozzles, and are accelerated to extremely high speed to exert a large reactive thrust on the rocket (since every action has an equal and opposite reaction).

28.11.3 Applications

Rockets are mostly used to take the space vehicle outside the earth's atmosphere, to launch artificial satellites in the space, human space flight and for exploration of other planets. Rockets are also used for fireworks and weaponry ejection seats.

Chemical rockets store a large amount of energy in an easily released form, and can be very dangerous. However, careful design, testing, construction, and use minimizes risks.

28.11.4 Features

1. It is a self-contained, non-air-breathing system.
2. Rockets are highly efficient at very high speed (> Mach 5.0 or so).
3. It develops high thrust per unit area. Thrust-to-weight ratio is over 100.
4. It has simple air inlet and high compression ratio.
5. It needs lots of propellant and has very low specific impulse; typically 100–450 seconds.
6. It offers extreme thermal stresses of combustion chamber.
7. Carrying oxidiser on-board makes the rocket a very risky vehicle.
8. It is extremely noisy.

28.11.5 Classification of Rockets

A rocket vehicle can be classified as following:

1. According to Type of Propellent Used

(a) Solid propellent rocket
(b) Liquid propellent rocket
(c) Hybrid propellent rockets

2. According to Fuel Used

(a) Chemical fuel rocket
(b) Nuclear fuel rocket
(c) Solar rocket
(d) Electrical rocket

3. According to Number of Stages

(a) Single-stage rocket (with one motor)
(b) Multi stage rocket (with more than one motor)

4. Based on Range

(a) Short-range rocket
(b) Medium-range rocket
(c) Long-range rocket

5. Based on Applications

(a) Weather-forcasting rockets
(b) Military rockets
(c) Space explorer rockets
(d) Booster rockets used in multistage rockets to elevate the main rocket
(e) Retro rockets used to reduce the speed of the main rocket and are fired in opposite direction of the main rocket

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28.11.6 Difference between Jet Engine and Rocket Engine

Sr. No.	*Jet Engine*	*Rocket engine*
1.	It is an air-breathing engine.	It is a non-air-breathing engine.
2.	Oxygen supply depends on atmospheric conditions. It carries only fuel.	It carries oxidiser on board, thus oxygen supply is not affected by outside atmosphere.
3.	Intake air supplies the oxygen required for combustion.	Oxidiser supplies the oxygen for combustion.
3.	It cannot be operated in vacuum.	It can be operated in vacuum efficiently. Space tavel is possible.
4.	Thrust produced depends on altitude and flight speed. Thrust decreases with altitude.	Production of thrust does not depend on altitude.
5.	Skin friction increases with flight speed.	It offers no surface drag. No gravitational effect. Rate of climb increases with altitude.
6.	Flight velocity is aways less than jet velocity.	Flight speed is unlimited, even greater than jet velocity.
7.	It has reasonable thermal efficiency and reasonable flight duration.	Low efficiency, except at extremely high speed.

28.12 SOLID PROPELLANT ROCKET

28.12.1 Construction and Working

The rockets, which use solid fuels and oxidisers are known as solid propellant rockets. A solid propellant rocket consists of a star-shaped combustion chamber, propellant and an expansion nozzle. The solid fuel and oxidiser are mixed in a single propellant and is packed inside the shell as shown in Fig. 28.21. A liner is provided between shell and propellant in order to protect the shell against high temperature.

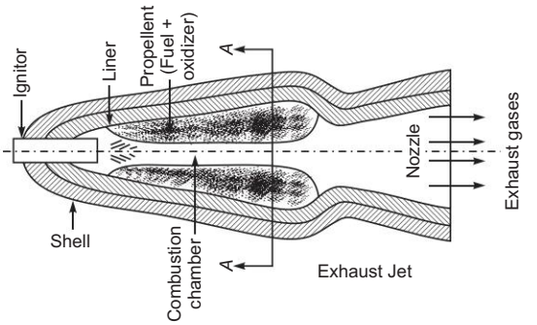

Fig. 28.21 *Solid propellant rocket*

An ignitor, which is a detonator or fuse containing a highly reactive explosive material is used to start the burning of the propellant. The burning occurs with rapid decomposition of propellant and heat release. The generated combustion products are discharged through the nozzle with very high velocity. Thus, a reactive thrust is developed and the rocket is propelled in forward direction.

Once the heat is produced, it sustains the combustion without any ignitor. The burning rate depends upon the pressure of the chamber and incrseases with an increase in pressure.

28.12.2 Applications

The solid propellant rockets are used in the following applications:

1. Assisted take off of missiles and projectiles
2. Small range rocket missiles

28.13 LIQUID PROPELLANT ROCKETS

Rockets which use liquid fuels along with oxidizers are called liquid propellant rockets. A liquid propellant rocket consists of a fuel tank, an oxidizer tank, fuel pump, oxidizer pump, injector, steam

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turbine, preheater, a combustion chamber, various control valves and an expanding nozzle as shown in Fig. 28.22. The fuel and oxidizer are stored at very low temperature.

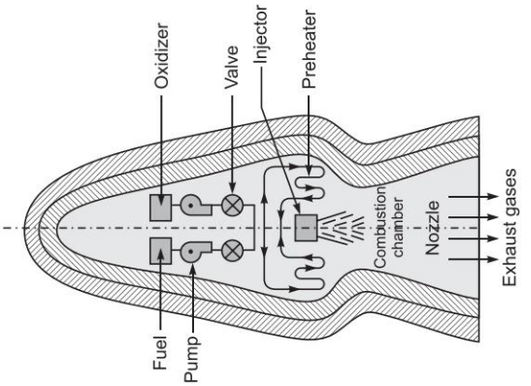

Fig. 28.22 *Liquid propellant rocket*

The fuel and oxidizer are pumped separately into the mixing chamber through valves, where the mixture is preheated to a suitable temperature. Then the preheated fuel oxidizer mixture is injected into the combustion chmber, where the mixture is ignited by an electric torch. The pumps are driven by a steam turbine. The steam is produced by mixing a very high concentrated hydrogen paraoxide with potassium permanganate.

The high-pressure and high-temperature combustion products are discharged through the nozzle with a high supersonic exit velocity. The high velcity exhaust propels the rocket in the forward direction.

28.14 HYBRID PROPELLANT ROCKET

A *hybrid rocket* is a rocket with a rocket engine which uses propellants in two different states of matter—one solid and the other, either gas or liquid. In most of the cases, solid fuel along with liquid oxidizer is used. Fig. 28.23 shows the schematic arrangement of hybrid propellant rocket.

The solid fuel is packed in the combustion chamber. The liquid oxidizer is stored in a separate tank. Liquid oxidizer is pumped through a valve and

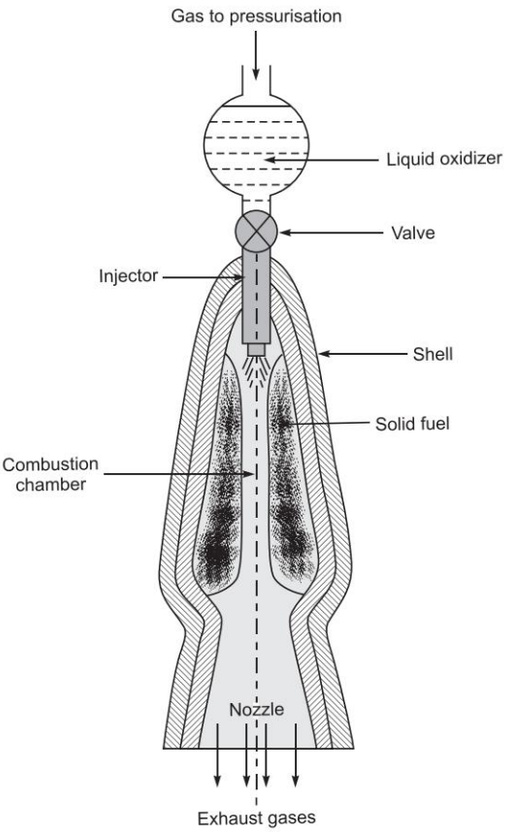

Fig. 28.23 *Hybrid propellant rocket*

injected on the fuel in the combustion chamber. The oxygen reacts with the fuel and combustion starts autimatically. The high-temperature combustion products escape with a very high velocity through the nozzle, thus propelling the rocket in forward direction.

28.15 NUCLEAR PROLELLANT ROCKET

Nuclear rocket uses nuclear fuel. The nuclear fuel undergoes a fusion reaction for generation of energy. The principal components of a nuclear propellant system are shown in Fig. 28.24. It consists of a propellant tank, pump, thrust chamber, nozzle and a pressure shell. The thrust chamber consists of nuclear reactor, a source of energy. The nuclear reactor consists of fuel element uranium

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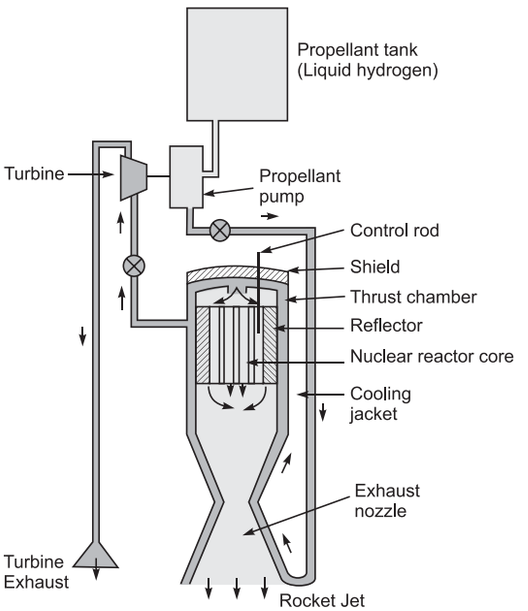

Fig. 28.24 *Nuclear propellant rocket*

235, a moderator to slow down the fast-moving neutrons, a reflector for conserving the neutrons and control rods to control the reaction rate.

The propellant is the working fluid. It is liquid hydrogen. A hydrogen pump driven by a gas turbine, transports the propellant from the tank to the thrust chamber. The hot gases from the reactor expand in the turbine to generate the power.

The heat generated in the reactor is transferred to hydrogen, and subsequently it is expanded in the nozzle to a high ejection velocity, and consequently, the thrust is produced to propel the rocket in the forward direction.

The merits and demerits of nuclear propellant rocket are as following.

Merits

1. Nuclear rocket produces high specific impulse. Compared to other type of rockets, a nuclear rocket produces maximum specific impulse.
2. Hydrogen is a light-weight gas, thus the self-weight of the rocket is relatively less.

Demerits

1. Nuclear reaction releases the heat energy at a very high rate. Thus it requires a heavy radiation shield.
2. A nuclear reactor emits large radiaton rays, which are harmful to material and persons.

28.16 PROPELLANTS

The *propellant* is a combination of fuel and oxidiser (such as liquid oxygen) on the board of a craft. There are

1. Solid propellent rockets
2. Liquid propellent rockets
3. Hybrid propellent rockets

28.16.1 Solid Propellants

Two types of solid propellants are used in rockets. These are the following.

Homogeneous Propellants These are mixture of organic substances like nitroglycereine, cellulose, nitrates, etc. The compounds act as a plastic and give a fuel of colloidal charecteristic. Additives are added to impart stability of combustion and freedom from ageing.

Characteristics of a homogeneous propellant are as follows:

1. These are plastic in nature.
2. They contain very high viscosity.
3. They appear smooth and waxy.
4. These can be made in required shape by casting and extrusion.

Hetrogeneous, Propellant These are also called composite propellants. For hetrogeneous solid propellants, nitrates, potassium and perchlorates are used as oxidisers and polymers, polyvinyl chloride (PVC) are used as fuel along with some additives to regulate the burning rate.

Characteristics of a heterogeneous propellants are as follows:

1. They are difficult to cast.

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2. They have high oxidiser content.
3. The exhaust has high density.
4. Mechanical properties of a propellant depends on the nature of the binder.

Desirable Properties of a Solid Propellant

1. The chemical reaction between fuel and oxidser during combustion must release a large amount of heat energy, giving higher combustion temperature and specific impulse.
2. The physical and chemical properties should not change during combustion.
3. The propellant must have lower molecular weight, higher density.
4. It should be non-corrosive, non-poisonous and hazardous.
5. The propellant should be chemically inert.
6. The product of combustion should be smokeless and colourless.
7. It should be easily available.

Advantages and Disadvantages of Solid Propellants

Advantages

1. These are simple in design and easy to manufacture.
2. They do not require mechanical feed system.
3. Density of fuel is high, the design is compact.
4. Due to absence of moving and sliding parts, the vibrations are almost absent.
5. Maintenance problem is less.
6. They are suitable for short-range applications.

Disadvantages

1. Once combustion begins, it cannot be stopped or controlled.
2. Since it contains oxidiser with it, the storage and transportation require utmost care.
3. Malfuctioning or accident leads to an abandon of the project.
4. It produces low specific impulse and cannot be reused.
5. It erodes the nozzle since it cannot be cooled.

28.16.2 Liquid Propellants

Liquid propellants are carried under the pressure as cryogenic liquids. These are classified as

(a) Monopropellants, and
(b) Bi-propellants

Monopropellant When fuel and oxidizer are present in a single chemical compound or solution, then the propellant is called monopropellant. The commonly used monopropellants are acetylene, hydrazine, ethylene oxide, and hydrogen peroxide. The monopropellants are suitable for auxiliary and turbopump plants in a rocket engine. These rocket engines are smaller and simple in construction.

Bi-propellant When the fuel and oxidizer are different from each other in its chemical composition then the propellant is called a bi-propellant. The fuel and oxidiser are mixed in a combustion chamber. This propellant is used for long-range and long-duration flights.

Commonly used bi-propellants (liquid fuel and oxidizer combination) are liquid hydrogen, + N_2O_3, demethyl hydrazine + hydrogen peroxide, gasolene + nitric acid, ethanol + liquid O_2, etc.

Advantages and Disadvantages of Liquid Propellants

Advantages

1. More flexible and greater control over the propellant supplied and thus control on thrust developed.
2. Propellants are not stored in a combustion chamber.
3. It gives high specific impulse.
4. It is much easier to stop the operation to avoid a catastrophe.
5. Auxiliary power plant can easily be operated.

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6. Liquid propellant rockets are more economical for long-range space and military operations.

Disadvantages

1. Liquid propellant rockets consist of a large number of parts and thus the design of liquid propellant rockets is more complicated.
2. The liquid propellants are stored at very low temperature with very heavy insulation.
3. Feed pump and insulated pressurised tanks increase the total weight of the rocket.
4. Due to low density of liquid propellants as compared to solid propellants, the store space required is more.

Requirement of Good Liquid Propellants

A good liquid propellant should have the following characteristics.

1. It should have very high calorific value.
2. It should have very high density, so that it can be stored in a small space.
3. It should be easy to store and handle.
4. It should be non-corrosive and stable.
5. Its combustion should be smooth and uniform.
6. It should have minimum change in its viscosity with temperature change.

28.16.3 HyBrid Propellants

Beryllium, hydride, lithium, polythene are the fuels used for hybrid propellant rockets and chloro triflourine, nitrogen tetraoxide are oxidisers. The hybrid propellants have the following advantages and disadvantages.

Hybrid rocket engines exhibit some advantages over liquid-fuel rockets and solid rockets. A brief summary of some of these is given below:

Advantages Compared with Bi-propellant Liquid Rockets

1. These are mechanically simpler, require only a single liquid propellant resulting in less plumbing, fewer valves, and simpler operations.
2. Hybrid fuels are in the solid phase, generally have higher density than those in the liquid phase.
3. High-energy metal additives, such as aluminum, magnesium, lithium or beryllium can be easily included in the fuel grain for increasing specific impulse (I_{sp}) .

Advantages Compared with Solid Rockets

1. Higher theoretical specific impulse (I_{sp}) and the rocket can carry high payload.
2. Less explosion hazard—Propellant grain has more tolerance for processing errors such as cracks.
3. Easy controllable—start/stop/restart and throttling are all achievable by regulating the supply of oxidizer.
4. Safe and non-toxic oxidizers such as liquid oxygen and nitrous oxide can be used.
5. The propellant can be recharged and reused.

Disadvantages of Hybrid Rockets

1. Oxidizer-to-fuel ratio shift (O/F shift) with a constant oxidizer flow-rate, the ratio of fuel production rate to oxidizer flow rate will change as a grain regresses. This leads to off-peak operation from a chemical performance point of view.
2. Low regression-rate (rate at which the solid phase recedes) fuels often drives multiport fuel grains. Multiport fuel grains have poor volumetric efficiency and, often, structural deficiencies.
3. Hybrid propellant erode the nozzle at the faster rate.
4. If chlorine trifluorine is used as oxidant, it harms the ozone layer in the stratosphere.

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28.17 ANALYSIS OF ROCKET PROPULSION

Thrust The *thrust* developed in a rocket engine is sum of momentum thrust and pressure thrust.

F = Momentum thrust + Pressure thrust

The momentum thrust is caused due to difference in flow velocity of propellant entering the combustion chamber and high velocity hot gases leaving the exhaust nozzle and is given by Newton's second law of motion, i.e.,

$$F_{mom} = \dot{m}(\mathcal{V}_{jet} - \mathcal{V}_i)$$

where $\dot{m} = \dot{m}_{O_2} + \dot{m}_f$ (mass rate of fuel and oxidizer). Since oxygen and fuel are stored within rocket itself, thus its entry velocity relative to the aircraft is zero ($\mathcal{V}_i = 0$). Thus, the momentum thrust is

$$F_{mom} = \dot{m}\mathcal{V}_{jet}$$

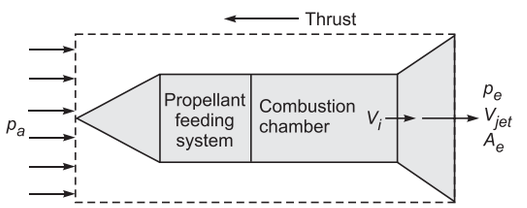

Fig. 28.25 *Rocket launching*

The ambient pressure acts over the entire surface of the rocket. Let p_e be the exit pressure from nozzle, that is higher than the pressure acting on frontal area. Then the pressure thrust acting in forward direction

$$F_{Pr} = (p_e - p_a)\, A_e$$

The total thrust produced in a rocket engine

$$F = \dot{m}\mathcal{V}_{jet} + (p_e - p_a)\, A_e \qquad ...(28.25)$$

In rocket propulsion, the thrust is independent of flight velocity. The nozzle exit pressure remains constant. Therefore, the thrust increases with altitude on account of decrease in ambient pressure p_a up to certain altitude. Then the ambient pressure variation becomes negligible and thrust becomes constant as shown in Fig. 28.26.

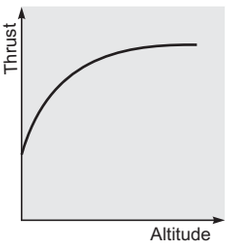

Fig. 28.26 *Variation of thrust with altitude*

Specific Thrust The specific thrust is given as ratio of thrust deveoped to mass-flow rate of propellant.

$$\text{Specific thrust} = \frac{\text{Thrust}}{\text{Mass flow rate}} = \frac{F}{\dot{m}} \qquad ...(28.26)$$

Specific Impulse The specific impulse is defined as the ratio of thrust developed to weight of propellant, which passes through the rocket engine;

$$I_{sp} = \frac{\text{Thrust}}{\text{Weight of propellant}} = \frac{\mathcal{V}_{jet}}{g} \qquad ...(28.27)$$

Total Impulse It is the product of specific impulse and weight flow rate of propellant used. It is given by

$$I_{total} = I_{sp}(\dot{m}\, g)$$

$$= \frac{\mathcal{V}_{Jet}}{g} \times \dot{m}g = \dot{m}\mathcal{V}_{Jet} = F \qquad ...(28.28)$$

It is equal to thrust developed.

Thrust Power The power developed from thrust of the engine is called the *thrust power* $\dot{W}_{th}$, which is the product of propulsive force and aircraft velocity $\mathcal{V}_a$. That is,

$$\dot{W}_{th} = F\mathcal{V}_a = \dot{m}\mathcal{V}_{jet}\mathcal{V}_a \qquad ...(28.29)$$

Power Developed by Rocket Engine Theoretical power developed by the rocket engine is the sum of thrust power and power loss in exhaust gases

Propulsive Power

= Thrust power + Power loss in exhaust

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Power lost due to energy carried by exhaust

$$= \frac{1}{2}\dot{m}(\mathcal{V}_{jet} - \mathcal{V}_a)^2$$

$\therefore$ Propulsive Power

$$= \dot{m}\mathcal{V}_{jet}\mathcal{V}_a + \frac{1}{2}\dot{m}(\mathcal{V}_{Jet} - \mathcal{V}_a)^2$$

$$= \dot{m}\mathcal{V}_{jet}\mathcal{V}_a + \frac{1}{2}\dot{m}(\mathcal{V}_{Jet}^2 + \mathcal{V}_a^2 - 2\mathcal{V}_{Jet}\mathcal{V}_a)$$

$$= \frac{1}{2}\dot{m}(\mathcal{V}_{Jet}^2 + \mathcal{V}_a^2) \quad ...(28.30)$$

Thrust Coefficient It is defined as the ratio of total thrust developed F to pressure force in combustion chamber.

$$C_F = \frac{\text{Total Thrust}}{\text{Pressure Force}} = \frac{F}{p_0 A^*} \quad ...(28.31)$$

where

p_0 = pressure in combustion chamber,

and A^* = area of throat of nozzle.

Propulsive Efficiency It is defined as the ratio of thrust power to propulsive power.

$$\eta_{prop} = \frac{\text{Thrust power}}{\text{Propulsive power}}$$

$$= \frac{\dot{m}_a \times \mathcal{V}_{jet} \times \mathcal{V}_a}{\frac{1}{2}\dot{m}_a \times (\mathcal{V}_{jet}^2 + \mathcal{V}_a^2)} = \frac{2\left(\frac{\mathcal{V}_a}{\mathcal{V}_{jet}}\right)}{1 + \left(\frac{\mathcal{V}_a^2}{\mathcal{V}_{Jet}^2}\right)}$$

or $$\eta_{prop} = \frac{2s}{1 + s^2} \quad ...(28.32)$$

where s = speed ratio = $\frac{\mathcal{V}_a}{\mathcal{V}_{jet}}$

The variation of propulsive efficiency with speed ratio is shown in Fig. 28.27. The propulsive efficiency reaches maxmimum, when speed ratio is unity and for other cases: For

$s = 0$; $\eta_{prop} = 0$,

$s < 1$; η_{prop} increases with increase in speed ratio.

$s > 1$; η_{prop} decreases with increase in speed ratio from 1.

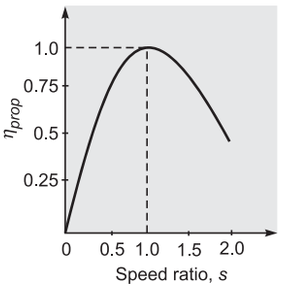

Fig. 28.27 *Variation of propulsive efficiency with speed ratio*

Thermal Efficiency The thermal efficiency of a rocket engine is defined as

$$\eta_{th} = \frac{\text{Propulsive}}{\text{Heat supply rate}}$$

$$= \frac{\frac{1}{2}\dot{m}(\mathcal{V}_{Jet}^2 + \mathcal{V}_a^2)}{\dot{m}CV} = \frac{\mathcal{V}_{Jet}^2 + \mathcal{V}_a^2}{2CV} \quad ...(28.33)$$

Specific Propellant Consumption It is expressed as weight of propellant used to produce a thrust of 1 Newton. It is denoted as *spc* and mathematically given as;

$$spc = \frac{\text{Weight of propellant}}{\text{Thrust produced}}$$

$$= \frac{mg}{F} = \frac{1}{I_{sc}} \quad ...(28.34)$$

Example 28.11 *The effective jet velocity from a rocket engine is 3000 m/s. The forward velocity is 1500 m/s and propellant consumption is 80 kg/s. Calculate the thrust, thrust power and propulsive efficency.*

Solution

Given A rocket engine

$\mathcal{V}_{Jet}$ = 3000 m/s

$\mathcal{V}_a$ = 1500 m/s

$\dot{m}$ = 80 kg/s

To find

(i) Thrust developed,

(ii) Thrust power, and

(iii) Propulsive efficiency.

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Analysis

(i) The thrust produced

$$F = \dot{m}\mathcal{V}_{jet} = 80 \times 3000 = 240{,}000 \text{ N}$$
$$= \mathbf{240\ kN}$$

(ii) *Thrust power*

Thrust power $= F\mathcal{V}_a = 240 \times 1500$
$= 360{,}000$ kW or **360 MW**

(iii) *Propulsive efficiency*

$$\eta_{prop} = \frac{2s}{1+s^2}$$

where $s = \dfrac{\mathcal{V}_a}{\mathcal{V}_{Jet}} = \dfrac{1500}{3000} = 0.5$

$$\therefore \quad \eta_{prop} = \frac{2 \times 0.5}{1+0.5^2} = 0.8 \text{ or } \mathbf{80\%}$$

Example 28.12 *The following data refer to a rocket engine. Propellant flow rate is 200 kg/s. The thrust chamber pressure is 37 bar and its temperature is 3000 K. The nozzle exit diameter is 0.6 m, nozzle exit pressure is 1.1 bar, ambient pressure is 0.60 bar, and thrust produced is 380 kN. Calculate effective jet velocity, actual jet velocity, specific impulse and specific propellant consumption.*

Solution

Given A rocket engine

$\dot{m} = 200$ kg/s $\quad p_2 = 37$ bar
$T_2 = 3000$ K $\quad d = 0.6$ m
$F = 380$ kN $\quad p_e = 1.1$ bar
$p_a = 0.6$ bar

To find

(i) Effective jet velocity,
(ii) Actual jet velocity,
(iii) Specific impulse, and
(iv) Specific propellant consumption.

Analysis

(i) *Effective jet velocity*

When combustion gases expands in the nozzle up to ambient pressure, without pressure thrust, resulting into effective jet velocity.

$$F = \dot{m}\mathcal{V}_{jet}$$
or $380 \times 10^3 = 200 \times \mathcal{V}_{jet}$
or $\mathcal{V}_{jet} = \mathbf{1900\ m/s}$

(ii) *Actual jet velocity*

Since nozzle exit pressure p_e is greater than ambient pressure p_a. Thus pressure thrust will also act on nozzle

Pressure thrust,

$$F_{pr} = A_e(p_e - p_a) = \frac{\pi}{4}d^2(p_e - p_a)$$
$$= \frac{\pi}{4} \times (0.6)^2 \times (1.1 - 0.6) \times 1\,0^3$$
$$= 141.37 \text{ kN}$$

Momentum thrust,

$$F_{mom} = F - F_{pr} = 380 - 141.37$$
$$= 238.63 \text{ kN} = 238.63 \times 10^3 \text{ N}$$

Actual jet velocity;

$$\mathcal{V}_{exit} = \frac{F_{mom}}{\dot{m}} = \frac{238.63 \times 10^3}{200}$$
$$= \mathbf{1193.15\ m/s}$$

(iii) *Specific impulse*

$$I_{sc} = \frac{F}{\dot{m}g} = \frac{380 \times 10^3}{200 \times 9.81} = \mathbf{193.68\ s}$$

(iv) *Specific propellant consumption*

$$spc = \frac{1}{I_{sc}} = \frac{1}{193.68} = \mathbf{5.16\ s^{-1}}$$

Example 28.13 *A rocket engine refer to following data:*

Jet velocity $= 1600$ *m/s*
Flight to jet speed ratio $= 0.7$
Oxidizer flow rate $= 4$ *kg/s*
Fuel flow rate $= 1$ *kg/s*
Heat of reaction per kg of exhaust gase $= 2500$ *kJ/kg*

Calculate the thrust, specific impulse, propulsive efficiency, thermal and overall efficiency of rocket engine.

Solution

Given A rocket engine

$\mathcal{V}_{Jet} = 1600$ m/s $\quad s = 0.7$
$\dot{m}_{O_2} = 4$ kg/s $\quad \dot{m}_f = 1$ kg/s
$\Delta h_R = 2500$ kJ/kg

To find

(i) Thrust developed,
(ii) Specific impulse,
(iii) Propulsive efficiency,

(iv) Thermal efficiency, and
(v) Overall efficiency.

Analysis Flight velocity

$$\mathcal{V}_a = s\,\mathcal{V}_{jet} = 0.7 \times 1600 = 1120 \text{ m/s}$$

(i) *Thrust produced*

$$F = \dot{m}\,\mathcal{V}_{jet} = (m_{O_2} + \dot{m}_f)\,\mathcal{V}_{jet}$$
$$= (4 + 1) \times 1600 = 8000 \text{ N}$$
$$= \mathbf{8.0\ kN}$$

(ii) *Specific impulse*

$$I_{sc} = \frac{F}{(\dot{m}_{O_2} + \dot{m}_f)g} = \frac{8000}{5 \times 9.81}$$
$$= \mathbf{163.1\ s}$$

(iii) *Propulsive efficiency*

$$\eta_{prop} = \frac{2s}{1+s^2}$$
$$= \frac{2 \times 0.7}{1 + 0.7^2} = 0.9396 \text{ or } \mathbf{93.96\%}$$

(iv) *Thermal efficiency*

$$\eta_{th} = \frac{\text{Propulsive power developed}}{\text{Heat Supplied}}$$

$$\text{Propulsive power} = \frac{1}{2}\dot{m}(\mathcal{V}_{Jet}^2 + \mathcal{V}_a^2)$$
$$= \frac{1}{2} \times 5 \times (1600^2 + 1120^2)$$
$$= 9536000 \text{ W} = 9536 \text{ kW}$$

$$\text{Heat supplied} = \dot{m}\,\Delta h_R = 5 \times 2500 = 12500 \text{ kW}$$

$$\therefore \quad \eta_{th} = \frac{9536}{12500} = 0.7628 \quad \text{or} \quad \mathbf{76.28\%}$$

(v) Overall efficiency

$$\eta_{overall} = \eta_{prop} \times \eta_{th} = 0.9396 \times 0.7628$$
$$= 0.6885 \quad \text{or} \quad \mathbf{68.85\%}$$

Summary

- A *jet propulsion engine* is a reaction engine that discharges a fast-moving jet of fluid which generates thrust in the opposite direction of a jet to propell the aircraft in accordance with *Newton's second and third laws of motion*.
- A *ramjet engine* uses the engine's forward motion to compress incoming air without a rotary compressor. Fuel burns to heat the air and hot gases expand through the nozzle to produce thrust.
- In a *pulsejet engine* combustion occurs in pulses. A pulsejet engine develops the thrust by a high velocity of jet of exhaust gases without use of compressor or turbine.
- A *scramjet* is a **s**upersonic **c**ombustion ramjet. It operates with supersonic speed at Mach numbers, 12 to 24.
- The *turbojet engine* is propelled by the thrust produced due to acceleration of hot combustion gases through the exhaust nozzle.
- In a *turboprop engine*, the propelling nozzle provides approximately 20 per cent thrust by velocity difference of gases leaving and air entering the aircraft, while the rest of the thrust is provided by the propeller.
- A *turbofan engine* is the combination of turbojet and turboprob engines. It consists of a ducted fan which is powered by a gas turbine. The combination of thrust produced from the fan and the exhaust from the core engine is more efficient than other jet engine designs.
- For a turbojet engine, absolute velocity of gas at exit is given by

$$= (\mathcal{V}_{jet} - \mathcal{V}_a) \text{ m/s}$$
$$\text{Thrust} = \dot{m}_a(\mathcal{V}_{jet} - \mathcal{V}_a) \text{ N}$$
$$\text{Propulsive power} = \dot{m}_a\left(\frac{\mathcal{V}_{jet}^2 - \mathcal{V}_a^2}{2}\right)$$
$$\eta_{prop} = \frac{2\mathcal{V}_a}{\mathcal{V}_{jet} - \mathcal{V}_a}$$
$$\eta_{th} = \frac{\text{Propulsive work}}{\text{Heat supplied}}$$
$$\eta_{overall} = \eta_{th} \times \eta_{prop}$$

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- For a *rocket engine*, the thrust produced is the sum of momentum thrust and pressure thrust.

$$F = \dot{m}\mathcal{V}_{jet} + (p_e - p_a)A_e$$

where $F_{mom} = \dot{m}\mathcal{V}_{jet}$

$$\dot{m} = \dot{m}_{O_2} + \dot{m}_f$$

$$F_{Pr} = (p_e - p_a)A_e$$

$$\text{Specific thrust} = \frac{\text{Thrust}}{\text{Mass-flow rate}} = \frac{F}{\dot{m}}$$

Specific impulse;

$$I_{sp} = \frac{\text{Thrust}}{\text{Weight of propellant}} = \frac{\mathcal{V}_{jet}}{g}$$

Theoretical power developed by the rocket engine is the sum of propulsive power and power loss in exhaust gases

PropulsivePo wer

= Thrust power + Power loss in exhaust

$$\therefore \quad \text{Engine Power} = \frac{1}{2}\dot{m}(\mathcal{V}_{Jet}^2 + \mathcal{V}_a^2)$$

Propulsive efficiency of a rocket engine is given by

$$\eta_{prop} = \frac{2s}{1+s^2}$$

where s = speed ratio = $\dfrac{\mathcal{V}_a}{\mathcal{V}_{jet}}$

Glossary

Jet propulsion Propelling of a vehicle due to thrust produced by fast-moving gases at the rear end

Ramjet engine An engine which uses engine's forward motion to compress the incoming air

Pulse jet engine An internal combustion jet engine in which combustion occurs in pulses

Turbojet engine An engine in which thrust is produced due to acceleration of hot combustion gases through the exhaust nozzle

Turboprop engine A turbojet engine in which a propeller is coupled to the turbojet engine

Review Questions

1. What are the principles of jet and rocket propulsion?
2. Explain the working of a ramjet engine with the help of a sketch. What are its advantages, disadvantage and applications?
3. Explain the working of pulsejet engine with the help of a sketch.
4. Explain the working of a turboprop engine with the help of a sketch.
5. Explain the working of a turbofan engine with the help of a sketch.
6. Explain the working of a turbojet engine with the help of a sketch. What are its advantages, disadvantages and applications?
7. State the difference between jet propulsion and rocket propulsion.
8. State the difference between air breathing and non-air breathing propulsion systems.
9. What is the difference between a propeller engine and jet engine.
10. Why are propeller engines not commonly used nowdays in aircrafts?
11. Define the following terms as applied to jet propulsion: specific thrust, specific impulse, thrust-specific fuel consumption, thrust power and propulsive power.
12. Why is thrust augmentation necessary? What are the methods for thrust augmentation in a turbojet engine?
13. Why does a ramjet engine not require a compressor or turbine?
14. Prove that the propulsion efficiency of a rocket motor is obtained as

$$\eta_{prop} = \frac{2s}{1+s^2}$$

where s = speed ratio = $\dfrac{\mathcal{V}_a}{\mathcal{V}_{jet}}$.

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15. Explain briefly with a sketch the working principle of a rocket.
16. List out the applications of rockets.
17. What is a propellant? How are propellants classified?
18. What are composite and homogeneous solid propellant? How do they work? State their merits and demerits.
19. What are desirable properties of a liquid propellant for a rocket engine?
20. What are the advantages of using nuclear thermal rocket for space propulsion?

Problems

1. The diameter of an aircraft is 4 m. The speed ratio is 0.8 at a flight speed of 450 kmph. If the ambient conditions of air at flight altitude are 0.54 bar and 256 K. Calculate (a) propulsive efficiency, (b) thrust, (c) thrust power.

[(a) 0.888, (b) 40.5 N, (c) 5073 N]

2. A turbojet is flying with a speed of 850 kmph at an altitude, where air density is 0.17 kg/m^3. The propulsive and overall efficiencies are 55% and 17% respectively. If the drag on aircraft is 6100 N, calculate the exit velocity of jet , diameter of jet and propulsive power.

[622.5 m/s, 29.24 cm, 1.44 MW]

3. An aircraft flying at 241 m/s at an altitude, at which pressure is 0.46 bar and temperature is –30°C. Calorific value of fuel is 41820 kJ/kg. The diffuser isentropic efficiency is 100%. The isentropic efficiency of compressor is 80%. Compressor pressure ratio is 5. Calculate the air–fuel ratio, if maximum temperature must not exceed 874°C. Assume C_p = 1.005 kJ/kg·K.

[61.4]

4. In a jet propulsion unit, the compressor pressure ratio is 3.5. The temperature rise during compression is 1.2 times that of isentropic compression. The maximum temperature of the cycle is 753 K. The gases are expanded in the nozzle to 283 K and 1.01325 bar. Calculate the power required to drive the compressor per kg of air, thrust developed, air–fuel ratio, if calorific value of fuel is 43000 kJ/kg.

[146.73 kJ/kg, 402 N, 132]

5. In a jet propulsion engine, the air is compressed from 100 kPa, 288K to 400 kPa. The isentropic efficiency of the compressor is 82%. The maximum temperature of the cycle is 1023 K. The isentropic efficiency of the turbine is 88%. The turbine is used to drive the compressor. A nozzle having isentropic efficiency 90% is used to expand the gases up to a pressure of 100 kPa. Calculate the power developed by the turbine to run the compressor per kg of air. Neglect the mass of fuel used.

Also calculate air–fuel ratio, if calorific value is 42000 kJ/kg, pressure of gases leaving the turbine and thrust developed.

[171.54 kJ/kg, 74, 172 kPa,466 N]

6. A twin jet aircraft travels at 200 m/s at an altitude where, the density of atmospheric air is 0.1712 kg/m^3. The aircraft has to overcome a total drag force of 6376 N. If propulsion efficiency is 58%, find the diameter of jet. *[40.8 cm]*

7. A turbojet has a speed of 750 km/h, while flying at an altitude of 10,000 m, where density of air is 0.173 kg/m^3. The propulsive efficiency of the jet is 50% and overall efficiency of the unit is 16%. The drag on the plane is 6200 N. Calorific value of the fuel is 48000 kJ/kg. Calculate

(a) Absolute velocity of jet,
(b) Quantity of air compressed per minute,
(c) Diameter of jet,
(d) Net power output of the unit in kW,
(e) Air–fuel ratio.

[(a) 417.3 m/s, (b) 5160 m^3/min, (c) 41.5 cm, (d) 2500 kW, (e) 46]

8. A turbojet flies with a speed of 800 km/h at an ambient pressure of 60 kPa. The properties of

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gas entering the nozzle are 300 kPa and 200°C. The mass-flow rate is 20 kg/s. assuming air as working fluid. Find the thrust developed, thrust power and propulsive efficiency. Take $\gamma = 1.4$ and $R = 0.287$ kJ/kg · K.

[7.39 kN, 1643.5 kW, 54.6%]

9. The diameter of jet of a turbojet is 2.5 m and it flies at 500 km/h at an elevation of 8 km for flight to jet speed 0.75. The density of air is 0.525 kg/m^3. Calculate (a) mass flow rate of air, (b) thrust, (c) specific thrust, (d) specific impulse, and (e) thrust power.

[(a) 477.2 kg/s, (b) 22.1 kN, (c) 3.07 kW, (d) 46.3 N/kg/s, (e) 4.72 s]

10. A turbojet engine flies with a speed of 880 km/h. Mass-flow rate of air is 50 kg/s. The isentropic change in enthalpy for a nozzle is 188 kJ/kg and its velocity as constant as 0.96. The fuel–air ratio is 1.2%. Calorific value of the fuel is 44 MJ/kg. Determine (a) thermal efficiency, (b) fuel flow rate, (c) propulsive efficiency, and (d) overall efficiency.

[(a) 27.5%, (b) 0.6 kg/s, (c) 58.7%, (d)16.13%]

11. The following data are applicable to a jet unit flight:

Speed of airplane = 950 km/h
Stagnation pressure and temperature at turbine exit = 180 kN/m^2 and 800 K
Atmospheric pressure = 60 kN/m^2
Nozzle efficiency = 98%
Mass flow rate of air = 25 kg/s
Fuel air ratio = 0.018
Lower calorific value of fuel = 40000 kJ/kg

Calculate
(a) gross and net thrust,
(b) jet equivalent velocity, and
(c) propulsive, thermal and overall efficiencies.

12. A rocket flies with a velocity of 2800 m/s with an effective jet velocity of 1400 m/s and propellant flow rate of 5 kg/s. If the heat of reaction of propellant is 6500 kJ/kg, calculate propulsive power, propulsive efficiency, engine output, thermal efficiency and overall efficiency.

[19.6 MW, 80%, 24.5 MW, 75.4%, 60.3%]

Objective Questions

1. A jet engine has
   (a) no propeller (b) propeller at front
   (c) propeller at back (d) propeller on top
2. The efficiency of a jet engine is higher at
   (a) low speed (b) high speed
   (c) low altitude (d) high altitude
3. The thrust is calculated as
   (a) $\mathcal{V}_{jet} - \mathcal{V}_a$
   (b) $\dot{m}_a(\mathcal{V}_{jet} - \mathcal{V}_a)$
   (c) $\dfrac{2\mathcal{V}_a}{\mathcal{V}_{jet} + \mathcal{V}_a}$
   (d) $\dot{m}_a(\mathcal{V}_{jet} - \mathcal{V}_a)\,\mathcal{V}_a$
4. The thrust of a jet engine can be increased by
   (a) Injecting water into the compressor
   (b) reheating gas after turbine
   (c) Injecting methanol into compressor
   (d) All of the above
5. A turboprop is preferred to a turbojet engine because
   (a) it has high propulsive efficiency at high speed
   (b) it can fly at supersonic speed
   (c) it can fly at high altitude
   (d) it has high power for take-off
6. In a jet propulsion unit, the product of combustion after passing through the gas turbine are discharged into
   (a) atmosphere
   (b) Vacuum
   (c) discharge nozle
   (d) back to compressor for next cycle

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7. In a turbojet engine used in military aircraft, the exhaust gas from the turbine is reheated by burning of additional fuel to increase the
   (a) thrust and range of aircraft
   (b) efficiency of the engine
   (c) both (a) and (b)
   (d) none of the above
8. In an ideal Turbojet engine after heat addition to compressed air, the working substance is expanded in
   (a) exit nozzle at constant pressure
   (b) exit nozzle at isentropic manner
   (c) turbine blades at constant temperature
   (d) turbine blades in isentropic manner
9. A jet engine uses oxygen for combustion of its fuel from
   (a) surrounding air (b) oxidizer
   (c) propellent (d) none of the above
10. In a turbojet engine, the diffuser is fitted at
   (a) nose (b) after compressor
   (c) after turbine (d) before turbine
11. The propulsive efficiency is defined as
   (a) $\dfrac{\mathcal{V}_{jet} - \mathcal{V}_a^2}{2\mathcal{V}_a}$ (b) $\dfrac{2\mathcal{V}_a}{\mathcal{V}_{jet} + \mathcal{V}_a}$
   (c) $\dfrac{2\mathcal{V}_a}{\mathcal{V}_{jet} - \mathcal{V}_a}$ (d) $\dfrac{\mathcal{V}_{jet} + \mathcal{V}_a}{2\mathcal{V}_a}$
12. A rocket engine receives oxygen for combustion of fuel from
   (a) surrounding air
   (b) compressed atmospheric air
   (c) oxidizer on board
   (d) none of the above
13. A rocket engine can be propelled to space, because
   (a) it generates very high thrust
   (b) it has high propulsive efficiency
   (c) it can work on several fuels
   (d) it is not air breathing engine
14. A rocket engine operates only in
   (a) ratified atmosphere (b) vacuum
   (c) atmosphere (d) all of the above
15. When a rocket is used as weapon with a warhead as payload, then it is called a
   (a) jet
   (b) turbojet
   (c) missile
   (d) turbo propeller unit
16. The propellent for a rocket engine is a
   (a) jet carried by the rocket only
   (b) Oxidizer carried by the rocket only
   (c) fuel and oxidizer carried by the rocket only
   (d) none of the above
17. Thrust in a rocket is generated by a jet of
   (a) air taken from the atmosphere
   (b) exhaust gases coming out at high velocity at the rear
   (c) compressed air
   (d) none of the above

Answers

1. (a) 2. (b) 3. (b) 4. (d) 5. (d) 6. (c) 7. (a) 8. (d)
9. (a) 10. (a) 11. (b) 12. (c) 13. (d) 14. (d) 15. (c) 16. (c)
17. (b)

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29

Air-conditioning

Introduction

Air-conditioning is the process of treating air in an internal environment to achieve and maintain required standards of temperature, humidity, cleanliness and motion of air, regardless of surrounding conditions. However, in popular usage, the term *air conditioning* is often improperly referred for providing a cool environment. The conditioning of air is controlled by

1. *Temperature* Air temperature is controlled by heating or cooling of air.
2. *Humidity* The humidity is controlled by adding or removing the water vapour to or from the air.
3. *Purity* The quality of air is controlled by filtration, removal of undesirable contaminants from air.
4. *Motion* Air velocity in the conditioned space is controlled by appropriate air distribution equipment.

29.1 APPLICATIONS OF AIR-CONDITIONING

Most of the air-conditioning systems are used to

(a) promote human health and comfort,
(b) improve human efficiency and working conditions,
(c) keep materials and the products in the most natural state, while in shortage or under production or manufacturing processes.

Air-conditioning is nowadays becoming more and more popular and is widely used in the following fields.

(*a*) *Comfort Air-conditioning*

It is used in

(i) residential buildings,
(ii) consumer stores, hotels and restaurants,
(iii) transportation of goods and human beings,
(iv) operation theaters, ICU and hospitals,
(v) offices, libraries, threaters and auditoriums.

(*b*) *Industrial Applications*

It is used in

(i) laboratories where precision measurements are carried out
(ii) printing and textile industries
(iii) pharmaceuticals companies during processing of medicines
(iv) production and preservation of photographic materials
(v) preservation of archeological and important documents
(vi) electronic data-processing units
(vii) control rooms

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29.2 COMFORT AIR-CONDITIONING

When the heat loss from the body is within certain limits, it results in a feeling of comfort. If the rate of heat loss is too large, as in a cold ambient, one feels cold. If the rate of heat dissipation is too low, one feels hot. The human body requires an environment for comfort which is neither very hot, nor very cold, neither very humid nor very dry.

Human comfort also depends on both psychological and physiolocal conditions of the human mind. Therefore, when designing air-conditioning systems for human beings, it is essential to understand the thermodynamic aspect of a human body. The factors which affect human comfort conditions are the following:

29.2.1 Heat Production and Regulation in the Human Body

The human body can be viewed as a heat engine with thermal efficiency about 20%. The source of the heat engine is the energy input as food, and its sink is the surrounding environment. The human engine generates heat when the combustion of food starts by taking in oxygen from inhaled air. The process of combustion in a human body is called *metabolism*. The human body converts about 20% of heat input into useful work and the remaining heat generated must be rejected to the environment to regulate the body temperature. Otherwise, the accumulation of heat in the body leads to discomfort.

The rate of heat generation in the body depends on individual health and the level of activity. On an average, an adult generates 87 W when sleeping, 115 W when doing office work or resting, 230 W when bowling and 440 W when doing heavy physical work. The corresponding numbers in a female adult are about 15 per cent less.

29.2.2 Heat and Moisture Losses from Human Body

The processes by which a body loses heat to the surroundings are convection, radiation and evaporation.

The body convects heat to air immediately around it self. The warm air moves away naturally and cooler air comes in which in turn receives heat. In *radiation*, the body transmits heat through space directly to the nearby walls and windows. A body can warm up in front of fire, even though the air between the fire and the body is very cold. Similarly, in a warm room, one may feel chilly, if the ceiling or the wall surfaces are at a considerable lower temperature. This is due to direct heat transfer between the body and its surroundings.

The human body is also cooled by *evaporation* of water on the skin (perspiration) which absorbs heat from the body and cools it. But this perspiration does not help when humidity of air is close to 100%.

The heat loss from a human body mainly depend on

1. Air (dry bulb) temperature
2. Air humidity
3. Air motion
4. Air purity
5. Air stratification

1. Temperature It is necessary to reject the heat at the same rate at which it is generated in the body in order to keep the body temperature constant. Thus, the environment temperature plays very important role in feeling of comfort. Most people feel a sense of comfort when the environment temperature is 22°C to 25°C.

2. Air Humidity Relative humidity affects the heat-transfer rate through evaporation. High humidity slows down the heat dissipation rate and low humidity speeds it up. Most people feel comfort with a relative humidity of 40 to 60%.

3. Air Motion Even if the temperature and humidity are within comfort range, a certain air movement is essential for effective heat transfer from the body. A high air motion increases the heat loss from the body by convection and evaporation. It removes the warm moist air that gathers around the body and replaces it with fresh air. Most people feel comfort with an air speed of 5 to 8 m/min.

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4. Air Purity People cannot feel comfort when breathing in contaminated air, even if other parameters are satisfactory. Therefore, proper filtration, cleanness and purification of air is necessary to keep it free from dust, dirt and other impurities. Other factors that affect human comfort are odour, noise and radiation effects.

5. Air Stratification When heat is rejected from a human body, the surrounding air gets heated. When air is heated, the density of air gets reduced and it rises to the ceiling of conditioned space. Hence within a confined space, there may be considerable temperature variation between the floor and ceiling. The movement of air from floor to ceiling is referred as *air stratification.*

29.3 EFFECTIVE TEMPERATURE

The effective temperature is defined as the index which correlates the combined effects of air temperature, relative humidity and air velocity on the human body. The numerical value of effective temperature is equal to the temperature of saturated air (ϕ = 100%) moving with a velocity of 5 to 8 m/min, which produces the same sensation of warmth and coldness as produced under given conditions.

Extensive tests have been conducted by the American Society of Heating, Refrigeration and Air-Conditioning Engineers (ASHRAE) on different kinds of people to test the level of comfort feeling by varying the environmental temperature, relative humidity and air movement. The values were obtained on the feelings of a mass percentage of people and the results are presented in the form of a comfort chart. The recommended effective temperature and relative humidity with air velocity of 5 to 8 m/min, for Indian conditions are presented in Table 29.1.

29.3.1 Factors Affecting Optimum Effective Temperature

The factors which affect the *optimum effective temperature* for comfort are the following:

Table 29.1 *Recommended effective temperature and relative humidity*

Sl. No.	*Climate*	*Range of effective temperature*	*Equivalent room conditions*
1.	Hot and dry	21.1 – 26.7°C	23.9°C and 50% RH
2.	Hot and humid	22.0 – 25.6°C	24.2°C and 50% RH
3.	Raining season	20.5°C – 24.0°C	23.0°C and 50% RH
4.	Winter	19.7 – 22.2°C	22.0°C and 45% RH

1. Climate and Seasonal Variation The people living in a warmer region may feel comfort at higher effective temperature than those living in a colder region.

2. Clothing People wearing light clothing need higher optimum effective temperature than a person wearing heavy clothing.

3. Age and Sex Women of all ages require higher effective temperature than men. Children also need higher effective temperature than adults.

4. Duration of Stay For shorter stay, higher effective temperature is needed than for long stay.

5. Kind of Activity For heavy activities like dancing or labour work in a factory, a low effective temperature is required.

6. Density of Occupation In densely occupied space like auditorium, the heat is radiated person to person, thus required a slight lower effective temperature.

29.3.2 Comfort Chart

The comfort chart shown in Fig. 29.1, is a graphical representation of a range of effective temperature for comfort feeling for summer and winter seasons. It shows dry-bulb temperature on the horizontal axis, and wet-bulb temperature as the ordinate. Relative humidity lines are replotted

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from a psychrometric chart. The comfort chart reveals several combinations of wet and dry-bulb temperatures with different relative humidity, which will produce the same effect of comfort. By looking at a comfort chart, one can decide the effective temperature, relative humidity and other physical aspects of the climate.

29.3.3 Requirement of Comfort Air-Conditioning

A person can feel comfort in an air-conditioned space if the following conditions are satisfied.

1. Continuous Supply of Oxygen and Removal of Carbon Dioxide A normal person inhales 0.65 m^3 of oxygen per hour, while producing 0.2 m^3 of CO_2 under normal conditions. The presence of excessive CO_2 yields to discomfort and even unconsciousness, if it exceeds 10%. Hence, fresh air supply in a conditioned space should be maintained.

2. Removal of Heat Continuously All the occupants and equipment release heat continuously in a conditioned space. This heat must be removed from the conditioned space at the same rate at which it is

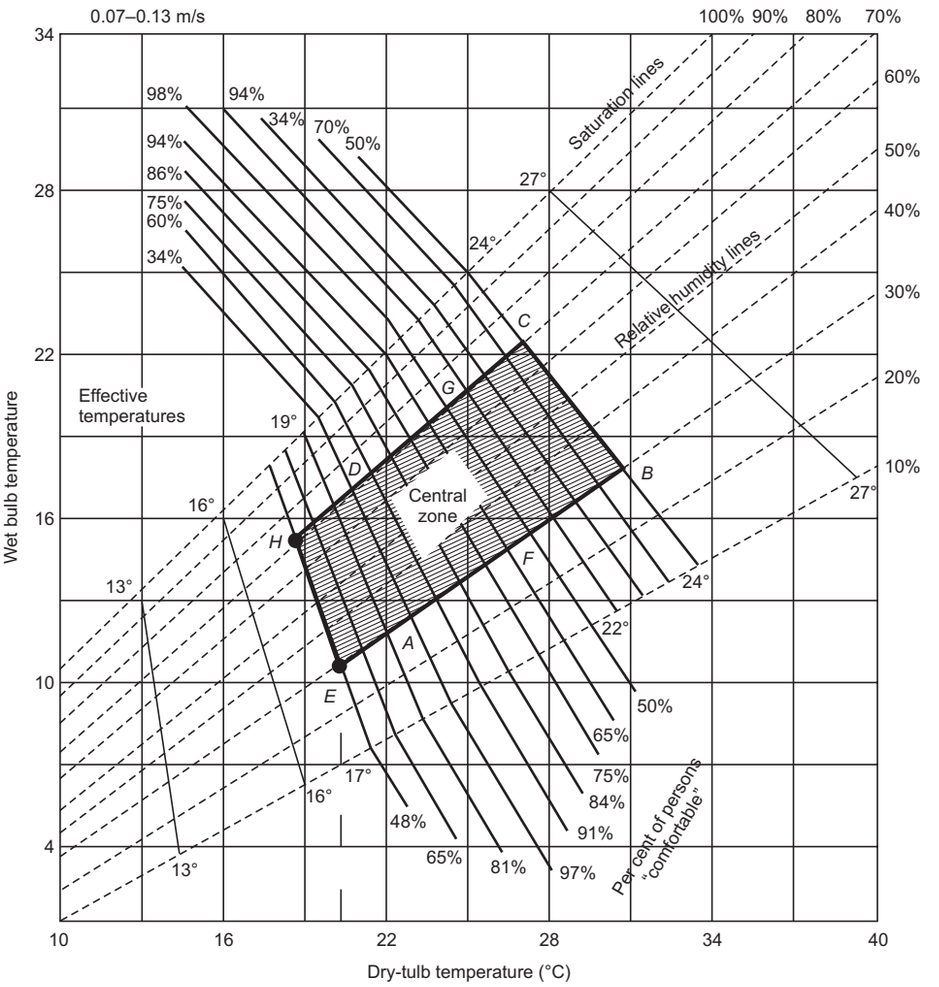

Fig. 29.1 Typical *comfort chart*

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generated in order to maintaine the temperature of the conditioned space as constant.

3. Removal of Moisture Content of Air Human beings also dissipate some water vapour from their bodies as a result of evaporation. These water vapours increase the humidity level in the conditioned space. High humidity in air reduces comfort level. Therefore, the relative humidity of the conditioned space should be kept constant by removing the moisture, which is added by the occupants.

29.4 AIR-CONDITIONING SYSTEMS

An air-conditioning system may provide heating, cooling or both. An air-conditioning system controls the conditions of a confined space to produce a comforting effect over occupants. The size and complexity may range from a window unit for a small room to a large system for a building complex, but the basic principle is same. The air-conditioning systems can be classified into various types based on the following heads.

1. *According to Season*
   (a) Summer air-conditioning system
   (b) Winter air-conditioning system
   (c) Year-round air-conditioning
2. *According to Purpose*
   (a) Comfort air-conditioning system
   (b) Industrial air-conditioning system
3. *According to Arrangement of Equipment*
   (a) Unitary air-conditioning system
   (b) Central air-conditioning system

29.5 AIR-CONDITIONING CYCLE

An air-conditioning cycle as shown in Fig. 29.2, consists of fans, filters, a refrigerating plant, ducting and control units. The working of an air-conditioning cycle can be divided into air circuit and refrigeration circuit separately.

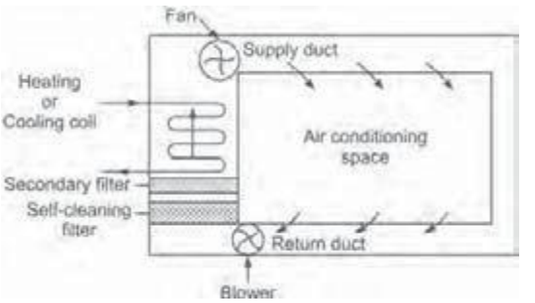

Fig. 29.2 *Air-conditioning cycle*

29.5.1 Air Circuit

The air circuit involves two air-flow ducts as

1. A supply duct for circulation of treated air inside the house
2. A return duct for flow of air through a condenser outside the house

The fan forces the conditioned air into the supply duct, which is connected to an opening in the room. The duct directs the air to the room through its outlets. The air enters the room and creates either a heating or cooling effect as required.

After use, the air is drawn by a blower from the room through the return duct and passes over the self-cleaning filter and secondary filter (electrostatic or fabric type) to remove the dust particles, and then passes over the evaporator (or heating coil).

The hot air comes in contact of the cooled fins and tubes of the evaporator, gets cooled, and the refrigerant in the evaporator vaporizes. The clean cooled air is then again sent to the air-conditioned space through the supply ducts for the next cycle.

29.5.2 Refrigeration Circuit

The refrigeration cycle circulates the refrigerant between the inside and outside units.The refrigerant vapour coming out of the evaporator enters the compressor, where it is compressed and delivered to the condenser. Another blower forces the atmospheric air over the condenser; thus the refrigerant is cooled and condensed. The hot air is exhausted outside and the condensed refrigerant passes through the expansion device to the evaporator.

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29.6 SUMMER AIR-CONDITIONING SYSTEM

In summer, the comfort conditions are achieved by cooling and dehumidification of air. The dry bulb temperature of air entering the conditioner is quite high and thus it is essentially cooled by using cooling coils. The hot air also has high moisture contents; therefore, the removal of moisture is achieved by a dehumidifier.

A summer conditioning system is shown in Fig. 29.3. The fresh air from the surrounding atmosphere is mixed with re-circulated air in suitable proportion and then the mixture is passed through the filter to remove dust, dirt and other unwanted materials from air to enter the system. The air is then cooled sensibly with the help of a cooling coil. This cooled air is passed through a perforated membrane to remove the moisture from air. The restricted passage of the perforated membrane condenses the water vapour into liquid water and is collected in the swap. The air is then passed over heating coil for mild heating before entering into conditioned space by using a circulating fan.

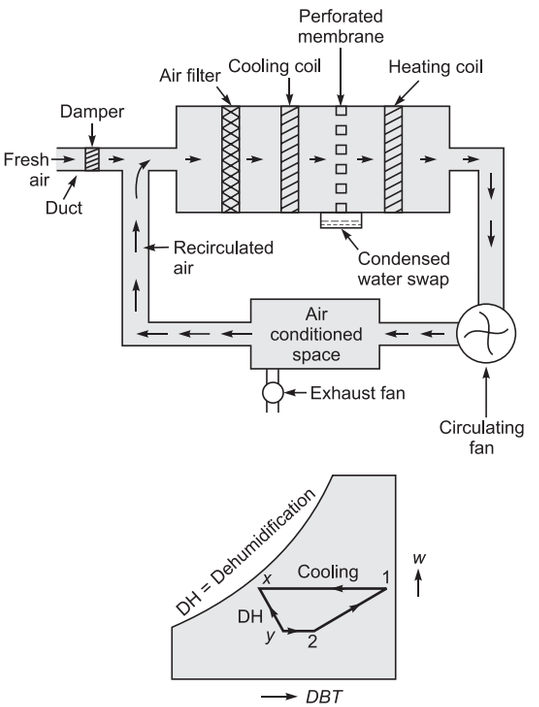

Fig. 29.3 *Summer air-conditioning system and its processes on psychrometric plot*

The conditioned air provides required comfort conditions within confined space. A portion of air is exhausted to the atmosphere and the remaining air is used for recirculation with fresh air for the next cycle.

The processes of cooling, dehumidification and heating are shown on the psychrometic plot in Fig. 29.3(b).

29.7 WINTER AIR-CONDITIONING SYSTEM

In winter, comfort conditions are achieved by heating and humidification of air. The dry-bulb temperature of air is quite low, and thus it is essentially heated by using heating coils. The cold air is usually dry and has very low moisture content; therefore, moisure is added to improve relative humidity.

A winter air-conditioning system is shown in Fig. 29.4. The fresh air from the surrounding atmosphere is mixed with re-circulated air in suitable proportion and then the mixture is passed through the filters to remove dust, dirt and other unwanted materials from air about to enter the system. The air is then passed over a preheater, which heats the air sensibly to remove residual moisture from the fresh air.

The preheated air is then passed through the humidifier in which the required moisture is added to improve relative humidity of air. During flow of air through the humidifier, the air temperature gets reduced; therefore, the air is again heated to suitable temperature with the help of re-heater.

The air is then passed to the air-conditioned space through the ducting system to provide required comfort conditions within the confined space. A portion of the air is exhausted to the atmosphere using a suitable ventilator or exhaust fan and the remaining air is used for recirculation with fresh air for the next cycle.

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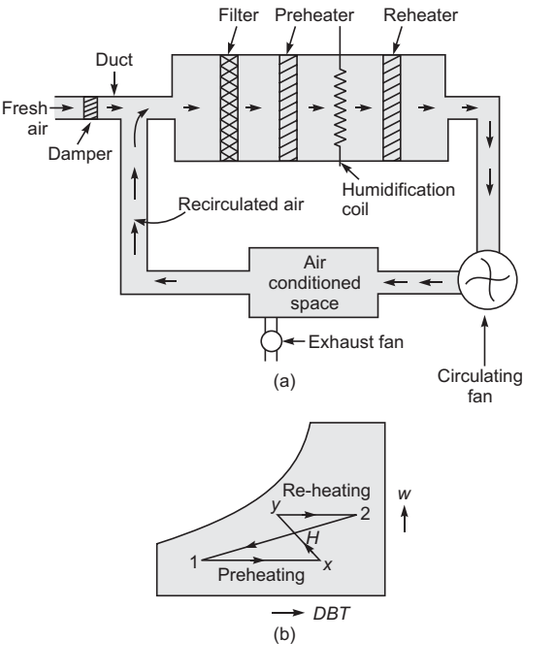

Fig. 29.4 *Winter air-conditioning system along with its processes on a psychrometric plot*

The processes of preheating, humidification and reheating are shown on the psychrometic plot in Fig. 29.3(b).

29.8 YEAR-ROUND AIR CONDITIONING SYSTEM

In the tropical countries, the air-conditioning system design is primarily concerned with year-round cooling and humidification/dehumidification.

Thus, the year-round air-conditioning system is the combination of both summer and winter air-conditioning systems. The essential components of a year round air-conditioning system are shown in Fig. 29.5. The air-conditioner has a cooling coil, humidifier and heating coil.

The fresh air taken from the surrounding atmosphere is mixed with re-circulated air in suitable proportion and then the mixture is passed through the air filters to remove dust, dirt and other unwanted materials from air about to enter the system.

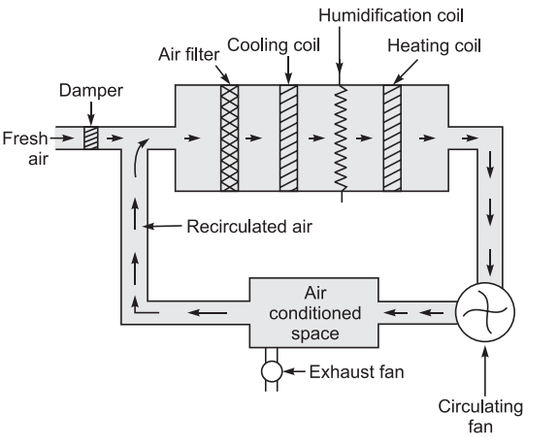

Fig. 29.5 *Year-round air-conditioning system*

For summer air-conditioning, the humidifier and heating coil are made inactive. The air is passed over a cooling coil. The temperature of the cooling coil is slightly kept higher than a summer air-conditioning system in order to avoid condensation of water particles.

During the winter season, the heating coil and humidifier are made active. The heating coil heats the air and the humidifier adds the humidity to air, if it is dry. Thus, the year-round air-conditioning system can be used in any kind of ambient condition.

Types of Air-Conditioners

Air-conditioning systems may be classified as

(a) Unitary systems
(b) Central station systems

29.9 UNITARY SYSTEM

The unitary systems are usually factory-assembled units including internal wiring, controls and piping and are available in wide ranges capacities. These units are installed within or near the conditioned space. These systems offer the advantage of ease of selection, low initial cost, ease of installation and removal. These systems are available as

1. Self-contained or single-package unit (window air-conditioner)

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2. Split system or split-package unit
3. Roof-top air-conditioner

29.9.1 Window Air-Conditioner

It is a simple type of air-conditioner unit made as an enclosed assembly as shown in Fig. 29.6. It is designed as a unit for mounting in a window or through a wall. The function of a window-mounted air-conditioner is to provide comfort to the occupants in room by circulating clean, cool air. Their capacity is such that one unit is adequate to condition one room. Roughly 0.08 to 0.1 TR capacity is required for cooling of 1 m^2 room area. It does not require ducts for the free delivery of conditioned air to a space. The unit is divided into two parts:

(a) Indoor part
(b) Outdoor part

The indoor part includes a filter, evaporator, a motor-driven fan or a blower and an expansion device. The outdoor portion includes a hermetical sealed motor-driven compressor unit, a condenser and a fan.

A fan is used to force the outside air over the condenser coil to remove the heat from the compressed refrigerant. In order to draw air through the filter and force it over the evaporator coil to cool the atmospheric air, another fan is provided in the indoor portion. For driving the two fans, either the same motor or separate motors can be used for driving two fans.

A window air-conditioner works on the principle of vapour-compression refrigeration system.

29.9.2 Split Air-Conditioner

A split air-conditioner is also known as a *remote-mounted air-conditioner*. It is basically an air-conditioning system built in two distinct units: indoor unit and outdoor unit. The two units are connected by refrigerant piping lines.

The indoor unit consists of a fan and cooling coil. It is located in the space to be conditioned. It is a well-designed single casing, well insulated on the inside housing the evaporator coil, twin blower system with a motor, capillary tubes for refrigerant expansion, electronic controls and condensate drain provision.

The outdoor unit consists of a compressor, condenser coil and propeller fan with motor. The outdoor unit is connected to an indoor unit by extended suction and liquid pipelines.

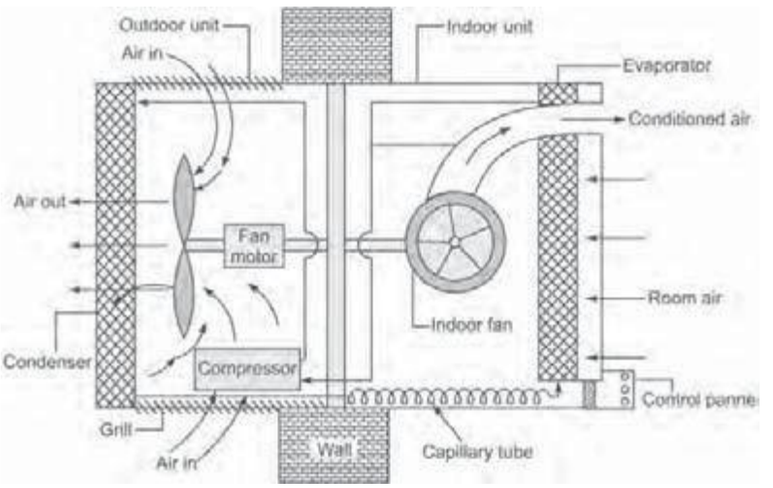

Fig. 29.6 *Window air-conditioner*

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Split air-conditioners are nowadays becoming popular because

1. They require less space in the room
2. They require no wall opening necessary
3. They have better air circulations inside, due to flexibility of proper location
4. The condenser, compressor and fan package can be located at any convenient place, where its noise is less objectionable and more accessible for maintainance.
5. They have flexibility of multiple evaporators in different rooms using single condenser–compressor unit.

29.9.3 Rooftop Unit

The rooftop unit is designed to be located on the roof of the building. The refrigeration, cooling and air-handling equipment are available in sections that are assembled together. The compressor and condenser are located remotely.

The rooftop units are used with duct work and air outlets. These units do not use valuable building space and they are relatively low in cost.

29.10 CENTRAL AIR-CONDITIONING SYSTEM

A central air-conditioning system provides the coolant from one central location to different parts of a building. In this system, all the components are grouped together in one central room and conditioned air is distributed from the central room to the required places through insulated ducts as shown in Fig. 29.7. The central air-conditioner system is generally used for a refrigerating load above 25 tonns and 2500 m^3/min of conditioned air.

The central air-conditioning systems provide fully controlled heating, cooling and ventilation. They are widely used in theaters, stores, restaurants and other public buildings. The system is complex and generally installed when the building is constructed.

The central air-conditioning system has the following advantages:

1. Automatic central control point
2. Better air distribution
3. Easier maintenance, single cooling and heating equipments are installed in one location.

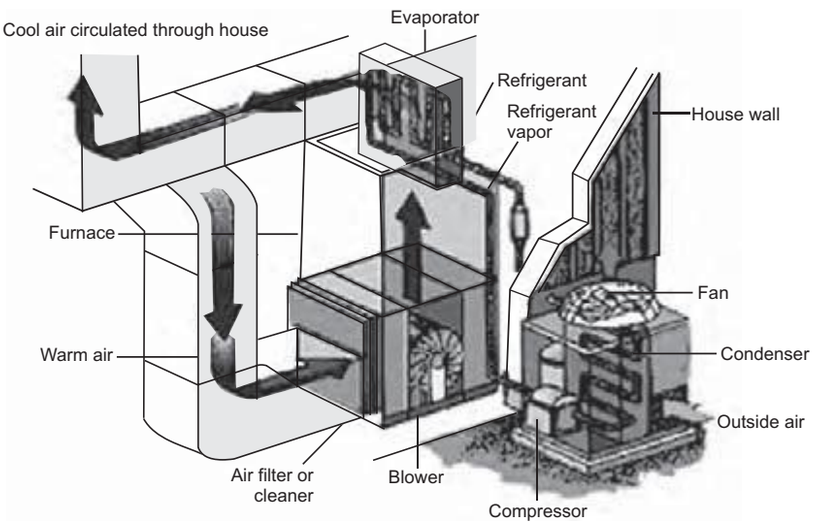

Fig. 29.7 *Central air-condition system*

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29.11 CLASSIFICATIONS OF CENTRAL AIR-CONDITIONING SYSTEM

The central station air-conditioning systems are classified according to cooling medium used to transport thermal energy to or from the air-conditioned space. The main types of cooling medium used in a central air-conditioning system are

1. All-air system
2. All-water system
3. Air–water system

29.11.1 All-Air System

In an all-air system, the sensible and latent cooling requirement of various parts of a building and equipments is provided by the conditioned cold air supplied by the central air-handling unit. The circulated air is then returned back to the air-handling unit, where it is further treated for the next cycle. The schematic arrangement of an all-air system is shown in Fig. 29.8.

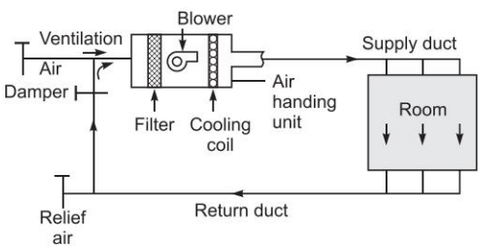

Fig. 29.8 *All-air-system central air-conditioning system*

29.11.2 All-Water System

In an all-water system, both sensible and latent space cooling are achieved by chilled water obtained from central refrigeration system through the cooling coils in terminal units located in various spaces in the building. Water has high latent heat, and thus it has the capacity to transport large thermal energy with lowest mass-flow rate. This characteristic of water lowers the size of piping work. It offers lowest initial and operating cost.

29.11.3 Air–Water System

In an air–water system, both air and water are distributed to each air–conditioned space to create a cooling effect. In this system, the temperature of same space can be better regulated by varying the temperature of air or water (or both) during all seasons of the year. It requires less quantity of air circulation as compared to an all-air system. The reduced quantity of air can be supplied at high velocity to the conditioned space.

29.12 RATING OF AIR-CONDITIONING

Air-conditioning units are rated in terms of effective cooling capacity, which should be properly expressed in kilowatt units. But manufacturers of air-conditioners still rate them in terms of tonnes of refrigeration, which is equivalent to 3.5 kW. Window air-conditioners are available in 1 tonne, 1.5 tonne, 2 tonne capacity.

The indicative TR load required for air-conditioning is presented below:

- Small office = 0.1 TR/m^2
- Medium size of office (10-30 persons occupancy) = 0.06 TR/m^2
- Large multistoried office/complex = 0.04 TR/m^2.

29.13 COOLING AND HEATING LOAD CALCULATIONS

The desired conditions inside a conditioned room can economically be maintained by properly designing heating and cooling systems. The design of heating, ventilation and air-conditioning (HVAC) systems need a close examination of thermal charecteristic of the building and the study of rate of heat transfer from occupants, equipment, infiltration. The heating load calculations require consideration of the following parameters:

29.13.1 Internal Heat Load

The sources of internal heat gain are lighting system, equipment and occupants within the space.

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(a) Heat Load from Lighting System and Equipments

$\dot{Q}_{electrical}$ = Sum of ratings (watts) of equipments and lighting lamps

(a) Heat Gain from Human Body

The human body dissipates heat by convection, sweating and evaporation to surroundings.

$\dot{Q}_{body}$ = Nos. of occupants × heat gain rate per person

Table 29.2 shows the heat loads from occupants as functions of their activity.

Sensible heat load from occupants

= Heat gain per person from Table 29.2 × No. of persons inside the room × percentage sensible heat gain from Table 29.2

Latent heat load from occupants

= Heat gain per person from Table 29.2 × No. of persons inside room × (1– percentage sensible heat gain from Table 29.2)

Table 29.2 *Heat-gain rate from occupants*

Activity	*Heat gain in W per person*	*Sensible heat gain %*
Sleeping	70	75
Seated, quiet	100	60
Standing	150	50
Walking	305	35
Office work	150	55
Teaching work	175	50
Industrial work	300–600	35

29.13.2 Wall Heat Load

Heat loss or gain by thermal transmission through a building wall can be calculated as

$$\dot{Q}_{Wall} = UA(\Delta T)_{Wall} = UA(T_o - T_i) \quad ...(29.1)$$

where

U = Overall heat transfer coefficient, W/m^2.K

A = Surface area, m^2

$(\Delta T)_{Wall}$ = Temperature difference between outside T_o and inside T_i, °C

Table 29.3 gives the value of overall heat transfer coefficient for some common cross-sections.

Table 29.3 *Overall heat-transfer coefficients*

Cross section	U (W/m^2.K)
Common brick wall plaster on both sides	1.7
Concrete 10 cm thick with plaster	2.8
Thermocole 5 cm thick with plywood on both sides	1.12
Single glass	5.5
Double glass with 6 mm air gap	3.4

29.13.3 Infilatration and Ventilation Load

Infiltration is defined as the uncontrolled entry of outside air directly into the conditioned space due to wind and buoyancy effects arising out of temperature difference between inside and outside environments.

Ventilation is defined as the fresh air intentionally mixed with recalculated air to meet the oxygen requirement of the occupants.

The entry of infiltration and fresh air into air-conditioned space change the temperature and humidity level of indoor air. Heat gain by room air due to entry of infiltration air can be calculated as

$$\dot{Q}_{in,\,sensible} = \dot{m}_a(h_3 - h_2) \quad ...(29.2)$$

and $$\dot{Q}_{in,latent} = \dot{m}_a(h_1 - h_3) \quad ...(29.3)$$

where $\dot{m}_a$ = mass-flow rate of infiltrated air (kg/s)

h_3 = Enthalpy at intersection of horizontal and vertical lines from room and outside air as shown in Fig. 29.9

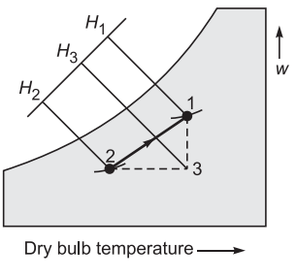

Fig. 29.9 *Enthalpy of infiltrated air on psychrometric plot*

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h_2 = Enthalpy of room air, kJ/kg

h_1 = Enthalpy of outside air, kJ/kg

Ventilation also imposes a considerable heat (or cooling) load and the choice of ventilation rates should be considered carefully.

29.13.4 Solar Heat Gain through Glass Windows and Walls

Direct solar heat gain through glass windows can be calculated as wall heat-transfer rate by replacing T_1 by equivalent outside air temeprature T_e, increased by few degrees to account for direct solar radiation.

$$\dot{Q}_{Solar} = UA(T_e - T_i) \qquad ...(29.4)$$

29.14 SENSIBLE HEAT FACTOR

The sensible heat factor is the ratio between the total sensible heat to total heat. The total heat is the sum of total sensible heat and total latent heat. It is designated as *SHF* and expressed as

$$SHF = \frac{\text{Total Sensible Heat}}{\text{Total Sensible Heat + Total Latent Heat}} \qquad ...(29.5)$$

Example 29.1 *An air conditioner is installed in an auditorium hall. Air supply rate is 5 m^3/s. The auditorium is maintained at 20°C DBT and 60% RH. The atmospheric air at 35°C DBT and 55% RH is treated by the air-conditioner. Calculate the sensible heat and latent heat removal rate. Also, calculate the sensible heat factor.*

Solution

Given An air-conditioner for an auditorium hall

Outdoor conditions

$T_1 = 35°C$ $\quad \phi_1 = 55\%$ $\quad \dot{V} = 5\,m^3/s$

Room conditions

$T_2 = 20°C$ $\quad \phi_2 = 60\%$

To find

(i) Seansible heat and latent heat heat removal rate,

(ii) Sensible heat factor.

Psychrometric plot with given data

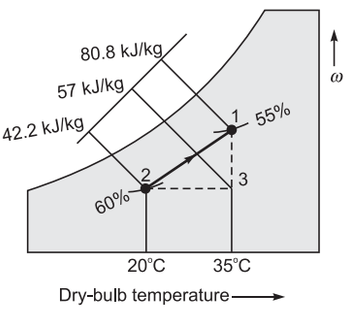

Fig. 29.10

Analysis The outside and room conditions are represented on a psychrometric plot with given data. The point 1 is marked at coordinates 35°C *DBT* and 55% *RH* corresponding to outside conditions and the point 2 corresponds to room conditions, 20°C *DBT* and 60% *RH*. The point 3 represents the intersection point of horizontal and vertical lines.

State 1: 35°C *DBT* and 55% *RH*

$h_1 = 80.8$ kJ/kg, $\qquad v_1 = 0.9\ m^3/kg$

State 2: 20°C *DBT* and 60% *RH*

$h_2 = 42.2$ kJ/kg

Intersection state 3 $\quad h_3 = 57$ kJ/kg

The mass-flow rate of air

$$\dot{m}_a = \frac{\dot{V}}{v_{s_1}} = \frac{(5\ m^3/s)}{(0.9\ m^3/kg)} = 5.555\ kg/s$$

Sensible heat removal rate

$$\dot{Q}_{SH} = \dot{m}_a (h_3 - h_2)$$
$$= 5.555 \times (57 - 42.2) = \mathbf{82.22\ kW}$$

Latent heat removal rate

$$\dot{Q}_{LH} = \dot{m}_a (h_1 - h_3)$$
$$= 5.555 \times (80.8 - 57) = \mathbf{132.2\ kW}$$

Total heat removal rate,

$$= \dot{Q}_{SH} + \dot{Q}_{LH} = 82.22 + 132.2$$
$$= 214.43\ kW$$

Sensible heat factor

$$SHF = \frac{\text{Sensible heat}}{\text{Total Heat}} = \frac{82.22}{214.43} = \mathbf{0.383}$$

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Example 29.2 *Air at the rate of 300 kg/min is supplied into an air-conditioned space maintained at 20°C DBT and 60% RH. The atmospheric air is taken at 34°C DBT and 28°C WBT. Calculate the sensible heat factor if 30 kW of sensible heat and 10 kW latent heat are already present in the room.*

Solution

Given An air-conditioner for an auditorium hall

Outdoor conditions

$T_1 = 34°C$ $T_{wb_1} = 28°C$

$\dot{m}_s = 300$ kg/min $= 5$ kg/s

Room conditions

$T_2 = 20°C$ $\phi_2 = 60\%$

Residual heats;

$\dot{Q}_{SH_1} = 30$ kJ/s $\dot{Q}_{LH_1} = 10$ kJ/s

To find Sensible heat factor.

Psychrometric plot with given data

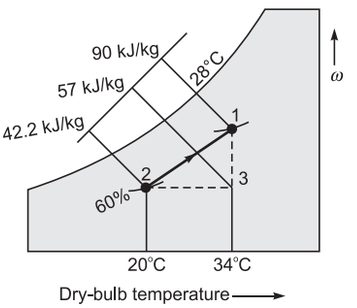

Fig. 29.11

Analysis The outside and room conditions are represented on the psychrometric plot with given data. The point 1 is marked at coordinates: 34°C *DBT* and 28°C *WBT* corresponding to outside conditions, and the point 2 corresponds to room conditions, 20°C *DBT* and 60% *RH*. The point 3 represents the intersection point of horizontal and vertical lines.

State 1: 34°C *DBT* and 28 *WBT*

$h_1 = 90$ kJ/kg

State 2: 20°C *DBT* and 60% *RH*

$h_2 = 42.2$ kJ/kg

Intersection state 3

$h_3 = 57$ kJ/kg

Sensible heat removal rate,

$$\dot{Q}_{SH_2} = \dot{m}_a(h_3 - h_2) = 5 \times (57 - 42.2) = 74 \text{ kW}$$

Latent heat removal rate,

$$\dot{Q}_{LH_2} = \dot{m}_a(h_1 - h_3) = 5 \times (90 - 57) = 165 \text{ kW}$$

Total sensible heat rate,

$$\dot{Q}_{SH} = \dot{Q}_{SH_1} + \dot{Q}_{SH_2} = 30 + 74 = 104 \text{ kW}$$

Total latent heat rate,

$$\dot{Q}_{LH} = \dot{Q}_{LH_1} + \dot{Q}_{LH_2} = 10 + 165 = 175 \text{ kW}$$

Total heat rate,

$$\dot{Q} = \dot{Q}_{SH} + \dot{Q}_{LH} = 104 + 175 = 279 \text{ kW}$$

Sensible heat factor,

$$SHF = \frac{\text{Sensible heat}}{\text{Total Heat}} = \frac{104}{279} = \mathbf{0.3727}$$

Example 29.3 *An air-conditioner plant is required to supply 50 m^3 of air per minute at a DBT of 22°C and 50% RH. The atmospheric condition is 32°C with 65% RH. Calculate the mass of moisture removed and capacity of cooling coil if the required effect is obtained by dehumidification and sensible cooling process. Also, calculate the sensible heat factor.*

Solution

Given An air-conditioner plant

Outdoor conditions

$T_1 = T_{db_1} = 32°C$ $\phi_1 = 65\%$

Room conditions

$T_2 = T_{db_2} = 22°C$ $\phi_2 = 50\%$

$\dot{V} = 50$ m^3/min

To find

(i) Mass of moisture removed,

(ii) Capacity of cooling coil, and

(iii) Sensible heat factor.

Psychrometric plot with given data

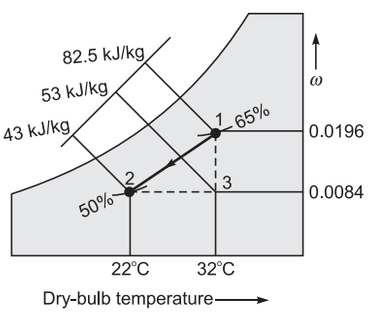

Fig. 29.12

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Analysis The outside and room conditions are represented on a psychrometric plot with given data. The point 1 represents outside conditions and the point 2 represents room conditions. The point 3 represents the intersection point of horizontal and vertical lines.

State 1: 32°C *DBT* and 65% *RH*

h_1 = 82.5 kJ/kg

ω_1 = 0.0196 kg/kg of air

v_1 = 0.845 m^3/kg

State 2: 22°C *DBT* and 50% *RH*

h_2 = 43 kJ/kg

ω_2 = 0.0084 kg/kg of air

Intersection state 3

h_3 = 53 kJ/kg

(i) *Mass of moisture removed*

The mass-flow rate of air

$$\dot{m}_a = \frac{\dot{V}}{v_1} = \frac{(50 \text{ m}^3/\text{min})}{(0.845 \text{ m}^3/\text{kg})}$$

$$= 59.17 \text{ kg/min}$$

Mass of moisture removed

$$= \dot{m}_a(\omega_2 - \omega_1)$$

$$= 59.17 \times (0.0196 - 0.0084)$$

$$= 0.662 \text{ kg/min} \quad \text{or} \quad \mathbf{39.76 \text{ kg/h}}$$

(ii) *Capacity of cooling coil*

The heat transfer

$$\dot{Q} = \dot{m}_a(h_1 - h_2)$$

$$= 59.17 \times (82.5 - 43)$$

$$= 2337.21 \text{ kJ/min}$$

$$\text{Coilc apacity} = \frac{\dot{Q} \text{ (kJ/min)}}{(210 \text{ kJ/min/Ton})} = \frac{2337.21}{210}$$

$$= \mathbf{11.13 \text{ Ton}}$$

(iii) *Sensible heat factor*

$$SHF = \frac{\text{Sensible heat}}{\text{Total heat}} = \frac{h_3 - h_2}{h_1 - h_2} = \frac{53 - 43}{82.5 - 43}$$

$$= \mathbf{0.2523}$$

Example 29.4 *An air-conditioner plant is to be designed for a small office for winter conditions:*

Outdoor conditions 10°C DBT and 8°C WBT

Indoor conditions 20°C DBT and 60%

Amount of free air circulation 0.3 m^3/min/person

Seating capacity of office = 50 persons

The required conditions are achieved first by heating and then by adiabatic humidifying. Find (a) heating capacity of coil in kW and surface temperature, if the by-pass factor of the coil is 0.32, and (b) capacity of the humidifier.

Solution

Given An air-conditioner plant

Room conditions

T_3 = 20°C ϕ_3 = 60%

Office capacity = 50 persons

Outdoorc onditions

T_1 = 10°C T_{wb_1} = 8°C

BPF = 0.32

Air circulation = 0.3 m^3/min/person

To find

(i) Heating capacity of coil in kW,

(ii) Surface temperature of heating coil, and

(iii) Capacity of humidifier.

Psychrometric plot with given data

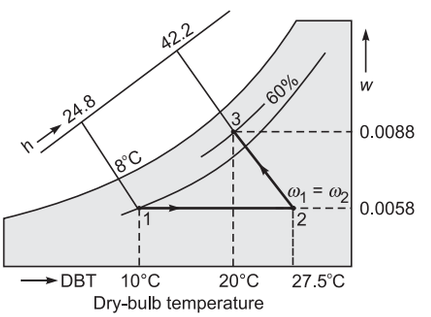

Fig. 29.13

Analysis Mark the point 1 corresponding to outdoor conditions 10°C *DBT* and 8°C *WBT* on a psychrometric chart and draw a horizontal line. Now mark the point 3 corresponding to indoor conditions to 20°C *DBT* and ϕ = 60% and draw a line along the constant WBT line till it intersects horizontal line at the point 2. Process 1–2 represents sensible heating and process 2–3 represents adiabatic humidification. From psychrometric chart,

State 1: *Atmospheric air* 10°C *DBT* and 8°C WBT

h_1 = 24.8 kJ/kg ω_1 = 0.0058 kg/kg of air

v_{s_1} = 0.81 m^3/kg

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State 2: *Intermediate state* $T_{wt_2} = T_{wt_3} = 27.5°C$

$h_2 = 42.2$ kJ/kg $\quad \omega_2 = 0.0058$ kg/kg of air

State 3: *Indoor conditions* 20°C *DBT* and $\phi = 60\%$

$h_3 = 42.2$ kJ/kg, $\quad \omega_3 = 0.0088$ kg/kg of air

Volume flow rate into office;

$$\dot{V} = 50 \text{ persons} \times 0.3 \text{ m}^3\text{/min/person} = 15 \text{ m}^3\text{/min}$$

The mass flow rate into office;

$$\dot{m}_a = \frac{\dot{V}}{v_{s_1}} = \frac{(15 \text{ m}^3\text{/min})}{(0.81 \text{ m}^3\text{/kg})} = 18.52 \text{ kg/min}$$

(i) *Capacity of heating coils*

Heat-transfer rate from the heating coil;

$$\dot{Q} = \dot{m}_a(h_2 - h_1) = 18.52 \times (42.2 - 24.8) = \mathbf{322.25 \text{ kW}}$$

(ii) *Surface temperature of heating coil*

The bypass factor is given by Eq. (15.33)

$$BPF = \frac{T_c - T_{exit}}{T_c - T_{inlet}} = \frac{T_c - T_2}{T_c - T_1}$$

$$0.32 = \frac{T_c - 27.5}{T_c - 10}$$

$$\Rightarrow \quad T_c = \mathbf{35.73°C}$$

(ii) *Capacity of humidifier*

$$\dot{m}_w = \dot{m}_a(\omega_3 - \omega_2) = 18.52 \times (0.0088 - 0.0058) = 0.0555 \text{ kg/s} \quad \text{or} \quad \mathbf{3.33 \text{ kg/h}}$$

Example 29.5 *A restaurant having a capacity of 100 seats is to be air-conditioned, when outdoor conditions are 30°C DBT and 70% RH. Desired inside conditions are 23°C DBT and 55% RH. The quantity of outdoor air supplied is 0.5 m³/min/person.*

The desired conditions are achieved by cooling, dehumidifying and then heating. Calculate

(a) Capacity of cooling coil,

(b) Capacity of heating coil,

(c) Amount of water removed by dehumidifier,

(d) By pass factor of heating coil, if its surface temperature is 35°C.

Solution

Given An air-conditioner for a restaurant

Outdoor conditions

$T_1 = 30°C \quad \phi_1 = 70\%$

Air circulation = 0.5 m³/min/person

Room conditions

$T_4 = 23°C \quad \phi_4 = 55\%$

Office capacity = 100 persons

Temperature of coil, $T_c = 35°C$

To find

(i) Capacity of cooling coil,

(ii) Capacity of heating coil,

(iii) Amount of water removed by dehumidifier,

(iv) By-pass factor.

Psychrometric plot with given data

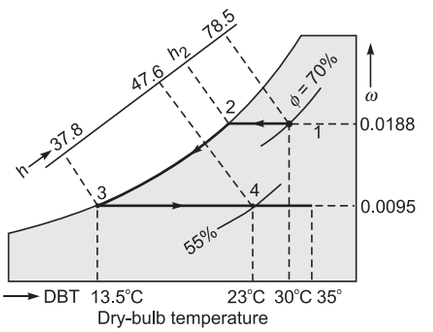

Fig. 29.14

Analysis Mark the point 1 corresponding to outdoor conditions of 30°C *DBT* and 70% *RH* on psychrometric chart and draw a horizontal line. Now mark point 4 corresponding to indoor conditions to 23°C *DBT* and ϕ = 55%. Draw sensible cooling process as line 1–2 and curve 2–3 as dehumidification and line 3–4 as heating process. From the psychrometric chart:

State 1: *Outdoor conditions* 30°C *DBT* and 70% RH

$h_1 = 78.5$ kJ/kg $\quad \omega_1 = 0.0188$ kg/kg of air

$v_{s_1} = 0.885$ m³/kg

State 4: *Indoor conditions* 23°C *DBT* and $\phi = 55\%$

$h_4 = 47.6$ kJ/kg $\quad \omega_4 = 0.0095$ kg/kg of air

State 2: *Intermediate states before and after humidification*

$h_2 = 72.5$ kJ/kg $\quad h_3 = 37.8$ kJ/kg

$T_2 = 24°C \quad T_3 = 13.5°C$

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Volume flow rate into office;

$$\dot{V} = 100 \text{ persons} \times 0.5 \text{ m}^3/\text{min/person} = 50 \text{ m}^3/\text{min}$$

The mass flow rate into office;

$$\dot{m}_a = \frac{\dot{V}}{v_{s_1}} = \frac{(50 \text{ m}^3/\text{min})}{(0.885 \text{ m}^3/\text{kg})} = 56.5 \text{ kg/min}$$

(i) *Capacity of cooling coils*

Heat transfer rate to cooling coil;

$$\dot{Q}_1 = \dot{m}_a(h_1 - h_3) = 56.5 \times (78.5 - 37.8) = 2300 \text{ kJ/min}$$

$$\text{Coil capacity} = \frac{\dot{Q}_1 \text{ (kJ/min)}}{(210 \text{ kJ/min/Ton})} = \frac{2300}{210} = \mathbf{10.95\ TR}$$

(ii) *Capacity of heating coil*

Heat transfer rate from heating coil;

$$\dot{Q}_1 = \dot{m}_a(h_4 - h_3) = 56.5 \times (47.6 - 37.8) = 554 \text{ kJ/min} \quad \text{or} \quad 9.23 \textbf{ kW}$$

(iii) *Amount of water removed by dehumdifier*

$$\dot{m}_w = \dot{m}_a(\omega_1 - \omega_4) = 56.5 \times (0.0188 - 0.0095) = 0.525 \text{ kg/min} \quad \text{or} \quad \mathbf{31.5\ kg/h}$$

(iv) *By-pass factor of heating coil*

$$BPF = \frac{T_c - T_{exit}}{T_c - T_{inlet}} = \frac{35 - 23}{35 - 13.5} = \mathbf{0.558}$$

Example 29.6 *A theater of 1000-person capacity is to be air conditioned for winter conditions:*

Outdoor conditions: 11°C DBT and 55% RH

Indoor conditions: 21°C DBT and 60% RH

Amount of free air supplied per person = 0.5 m³/min

The required copndition is achieved by heating, humidifying and then again heating. If the air coming out of the humifier has 80% RH, calculate the heating capacity of both heating coils and the mass of water added in the humidifier.

Solution

Given An air-conditioner for a theater

Outdoor conditions

$T_1 = 11°C$ $\phi_1 = 55\%$

Air circulation = 0.5 m³/min/person

Indoor conditions

$T_4 = 21°C$ $\phi_4 = 60\%$

Theater capacity = 1000 persons

Intermedite state, $\phi_3 = 80\%$

To find

(i) Capacity of heating coils, and

(ii) Amount of water added in humidifier.

Psychrometric plot with given data

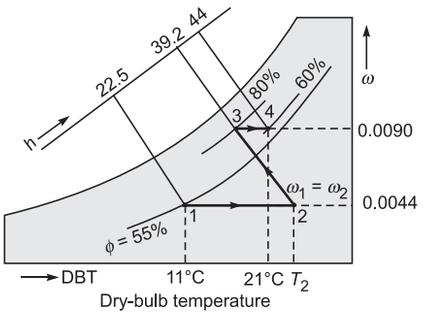

Fig. 29.15

Analysis Mark the point 1 corresponding to outdoor conditions of 11°C *DBT* and 55% *RH* on the psychromtric chart and draw a horizontal line 1–2. Now mark the point 4 corresponding to indoor conditions to 21°C *DBT* and $\phi = 60\%$. Draw a horizontal line 3–4 till it cuts 80% *RH* curve. Line 2–3 along constant *WBT* represents humidification. Process 1–2 and process 3–4 represent heating processes. From psychrometric chart:

State 1: *Outdoor conditions* 11°C *DBT* and 55% RH

$h_1 = 22.5.5$ kJ/kg $\omega_1 = 0.0044$ kg/kg of air

$v_{s_1} = 0.81$ m³/kg

State 4: *Indoor conditions* 21°C *DBT* and $\phi = 60\%$

$h_4 = 44$ kJ/kg $\omega_4 = 0.0090$ kg/kg of air

State 2: *Intermediate states before and after humidification*

$h_2 = h_3 = 39.2$ kJ/kg $T_2 = 27.8°C$

$T_3 = 16.2°C$

Volume flow rate into office;

$$\dot{V} = 1000 \text{ persons} \times 0.5 \text{ m}^3/\text{min/person} = 500 \text{ m}^3/\text{min}$$

The mass flow rate into office;

$$\dot{m}_a = \frac{\dot{V}}{v_{s_1}} = \frac{(500 \text{ m}^3/\text{min})}{(0.81 \text{ m}^3/\text{kg}) \times (60 \text{ s/min})} = 10.29 \text{ kg/s}$$

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(i) *Capacity of heating coils*

Capacity (heat-transfer rate from) of first heating coil;

$$\dot{Q}_1 = \dot{m}_a(h_2 - h_1)$$
$$= 10.29 \times (39.2 - 22.5) = \mathbf{171.81\ kW}$$

Capacity (heat-transfer rate from) of second heating coil;

$$\dot{Q}_2 = \dot{m}_a(h_4 - h_3)$$
$$= 10.29 \times (44 - 39.2) = \mathbf{49.4\ kW}$$

(ii) *Capacity of humdifier:*

$$\dot{m}_w = \dot{m}_a(\omega_4 - \omega_1)$$
$$= 10.29 \times (0.0090 - 0.0044)$$
$$= 0.0473 \text{ kg/s} \quad \text{or} \quad \mathbf{170.4\ kg/h}$$

29.15 WATER COOLERS

Water coolers use refrigeration cycle for cooling of water. They are widely used in schools, offices restaurants, factories and stores. They have two different circuits:

(i) Refrigertion circuit
(ii) Water circuit

29.15.1 Refrigeration Circuit

It is exactly a vapour compression cycle as shown in Fig. 29.16. Low-pressure refrigerant vapour enters the compressor and it is compressed and discharged as high-temperature, high-pressure refrigerant vapour into the condenser, where it is desurperheated and condensed and sometimes undercooled also. The high-pressure liquid refrigerant coming out of the condenser then passes through the drier to the capillary tube. The long capillary tube lowers the pressure of the liquid refrigerant as well as temperature due to resistance of small diameter of the tube. The low-temperature refrigerant then enters the cooling coils wrapped around the water-storage tank. The refrigerant evaporates by extracting the heat of water through the storage tank. Thus, water cools and the liquid refrigerant converts into refrigerant gas, which again enters the compressor for the next cycle.

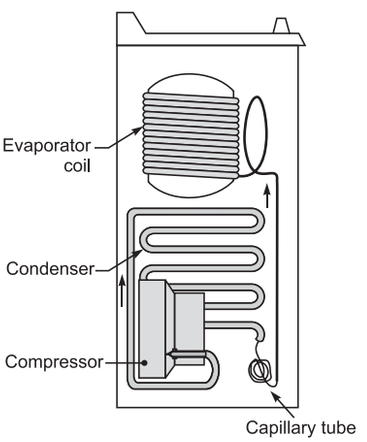

Fig. 29.16 *Schematic arrangement of a water cooler*

29.15.2 Water Circuit

The drinking water enters the water tank surrounded by evaporating coils. The liquid refrigerant extracts its latent heat of evaporation from water, and the water cools. The cooled water can be used through a water tap. The waste water from the basin comes out through the drain line. The float valve maintains a constant level of water in the tank, and a thermostatic valve regulates the temperature of water in the tank.

29.16 ICE PLANT

Ice manufacturing is one of the major applications of refrigeration. Ice is the cheapest means for short-time preservation of food. An ice plant uses a vapour-compression refrigeration cycle as shown in Fig. 29.17. The evaporator is made in the form of a tank. The cooling coils are mounted inside the inner wall tank. The tank is filled with secondary refrigerant. The secondary refrigerant used is brine solution: salt in water, which lowers the freezing temperature of water. With sodium chloride (NaCl), the freezing temperature of water may decrease to –14°C. With calcium chloride $CaCl_2$, the freezing temperature of water may decrease to –21°C.

The GI ice cans of any convenient shape and size are placed in the brine solution.

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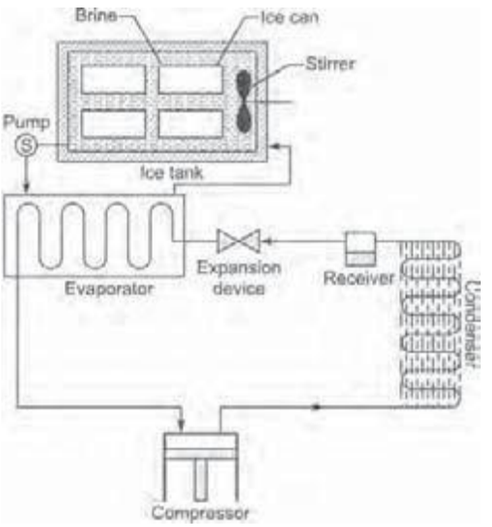

Fig. 29.17 *Schematic of an ice plant compressor*

The brine solution, a secondary refrigerant, acts as a heat carrier. It absorbs the heat from water in the ice cans and transfers it to cooling coils of the evaporator.

Since the brine solution is maintained at a temperature which is lower than 0°C, thus water freezes in the ice cans and ice is produced.

29.17 AIR COOLERS

Air coolers, as shown in Fig. 29.18, are also called *desert coolers*. They work on the principle of evaporative cooling. Evaporative cooling is one of the most ancient and energy-efficient methods for cooling a house. The air coolers use evaporation of water for cooling of air. The sensible heat of hot atmospheric air is transferred continuously to water in the form of latent heat for its evaporation, when the air makes physical contact with falling water on the filter pads. Since at lower temperatures, the latent heat of water is much higher, a large quantity of air can be cooled by evaporation of a small quantity of water.

Air coolers are considered environmentally *safe* since the process typically uses no ozone-depleting chemicals, and demands only one-fourth as much energy in comparison to an air-conditioning system

Fig. 29.18 *Air coolers*

during the peak cooling months of the year.

Small size air coolers can be installed in a window, blowing cooled air directly into a room. Larger units can supply air into a central location, from where the air can travel through a duct work to individual rooms.

29.17.1 Construction and Working of Desert Cooler

They consist of a blower, a water circulation pump, filter pads and a water sump. The blower draws in outside air and forces it through the wetted filter pads. As hot dry air moves through the filter pads, water evaporates, cooling and humidifying the air. The cool air is then supplied to the room. Since water is continuously lost through evaporation, thus the make-up water is required from time to time, when the water level gets lowered in the sump. Under normal conditions, a desert cooler can use 3 to 15 buckets of water a day.

Small water-distribution lines are provided at the top of the pads. The water falls down drop by drop on the pads. The pads soak water. Some of the water evaporates from the pads as air passes through them. The remaining water from the pads is collected into the sump at the bottom of the cooler. A small recirculation water pump sends the collected water back to the top of the pads.

The pads can be made of wood shavings. Grass—wood from aspen trees is a traditional choice—or other materials that absorb and hold moisture while

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resisting mild dew. Aspen wood pads, also called *excelsiors*. The wood pads are generally replaced after every season or two.

These coolers usually have 2 or 3 fan speeds, and they actually cool more efficiently at lower speeds.

29.17.2 How a Desert Cooler Works

A fan draws hot atmospheric air through the moist pads, where evaporation of water drops the temperature of air by approximately 20 degrees.

The effective cooling of incoming air in coolers depends on the moisture content of air, air velocity, surface area of wetted pads and time of contact between air and water.

1. The cooling capacity of evaporative coolers is best when the air is drawn slowly over the wetted pads, which allows maximum evaporation of water. Coolers with larger total porous areas will provide better cooling of air because the air can flow more slowly over the porous pads.
2. Evaporative coolers can perform better when the outside air is dry, desert. With increase in moisture content of air, the capacity of evaporative coolers to cool the air effectively decreases.
3. Performance of all evaporative coolers increases as atmospheric temperature rises and the air becomes dry.

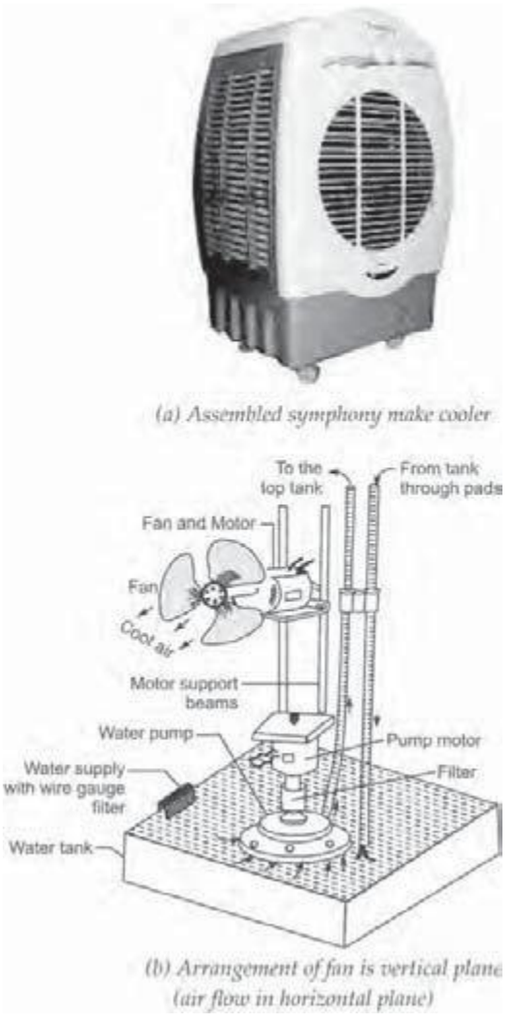
29.17.3 Classification of Coolers

Desert coolers are classified according to

(i) *Position of fans*
  (a) Fan in vertical plane
  (b) Fan in horizontal plane
  (c) Drum-type fan

(ii) *Types of pads used*
  (a) Fibre-pad coolers
  (b) Rigid-Sheet-pad coolers

(iii) *Independent ducted system*



(a) Assembled symphony make cooler

(b) Arrangement of fan is vertical plane (air flow in horizontal plane)

Fig. 29.19 *Cooler with fan in vertical plane*

Cooler with Fan in Vertical Plane The arrangement of a fan and water pump is shown in Fig. 29.19(b). It uses separate motors for the pump and-fan, so they can be run independently. It is provided with three vertical pads of cellulose fiber. The water pump delivers water from the cooler sump to the top of the pad. As water runs down the porous pads, it is absorbed by the pads and they become wet. When the fan draws atmospheric air through the porous pads on three sides, water evaporates by absorbing its latent heat from the incoming air.

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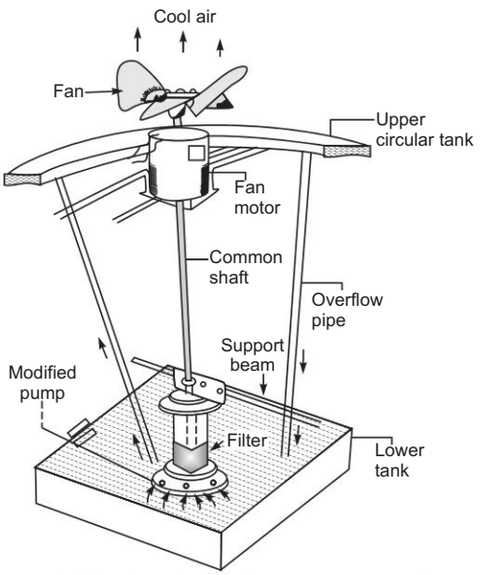
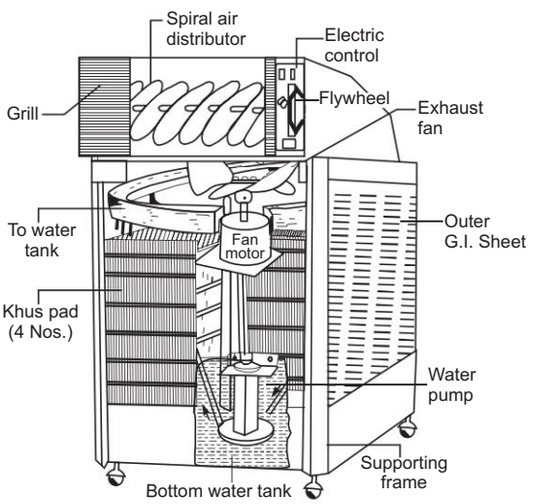
Thus air moving into the room becomes cooler than the outdoor air.

Cooler with Fan in Horizontal Plane The arrangement of components are shown in Fig. 29.20(a). The pump and fan are mounted on the same vertical shaft and run by a common motor. Therefore, the fan and pump cannot be run independently. It consists of four vertical fibre pads with water supply arrangement and water falls drop by drop on the pads. The fan draws air through the pads and is first moved in the vertical direction and then through the spiral distributors in horizontal planes.



(a) Box-type (cooler inner arrangement)



(b) Front cut-section view of box-type desent cooler

Fig. 29.20

It has certain unique features. Therefore, it ensures maximum cooling in minimum time.

1. It consists of four pads, and thus there is more exposure area for air. Therefore, the cooler has more cooling capacity for the same volumetric capacity.
2. Spiral directional air flow provides better air distribution in the room.
3. Noise level is not objectionable.

Cooler Without Water Pump The components of a cooler are shown in Fig. 29.21. It consists of a drum-type evaporator with rubberized coir, which is used for getting cooling effect instead of wetted pads. The evaporator dips in the water during its rotation and becomes wet. It uses an exhaust fan instead of conventional fan which provides maximum air thrust.

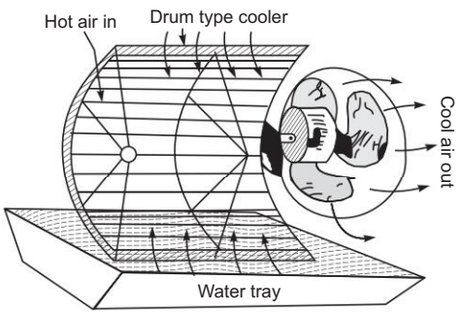


Fig. 29.21 *Ducted cooler*

Fibre-Pad Cooler The fibre pads are khus straws or shredded wood fibres packed in a plastic or metallic net. These pads are 1 to 2 inches thick. The quality and cost vary considerably. The thinnest pads are usually least expensive and more effective. Generally two thin pads are used to improve

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saturation effectiveness. These pads are replaced with new ones every 1–2 years.

Rigid-Sheet Pad Coolers The rigid-sheet pad is a stack of corrugated sheet material that allows air to move through it at higher velocities instead of fibre pads. These pads are generally 8 or 12 inches thick.

Independent Ducted System It is an evaporative cooler which blows cooled air through a single diffuser in the hall ceiling or into a distributed duct system in the ceiling space. It uses four vertical filter pads and separate motors for fan and water circulation pump. The fan is driven by a belt drive. It also consists of a float valve in the path of water supply which ensures a constant level of water in the sump. The pump transfers water through the piping to the upper header from where water is distributed to pads.

29.18 DIFFERENCE BETWEEN AIR COOLER AND AIR-CONDITIONER

An air-conditioner works on the refrigerating principle, where the air is cooled, when it passes over chilled tubes through which the refrigerant is evaporating. An evaporative cooler is based on the principle that when water comes in contact of hot moving air, the hot air is cooled immediately through evaporation of water. Although there is no substitute for an air-conditioner's cooling comfort in hot and humid climates. In dry climates, like in Rajasthan, Gujarat, MP and most of central India, a desert cooler is a cheaper and comfortable option.

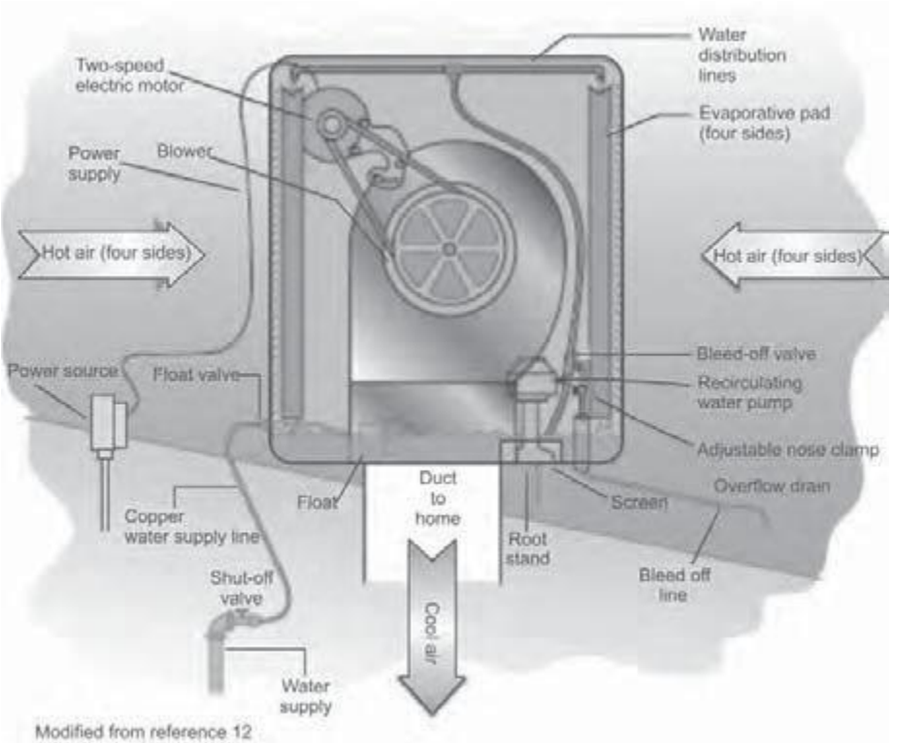

Fig. 29.22 *Ducted air-cooling system*

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Summary

- *Air-conditioning* is the process of treating air in an internal environment to achieve and maintain required standards of temperature, humidity, cleanliness and motion of air, regardless of surrounding conditions.
- A human body dissipates heat by convection and radiation and it generates heat energy as *metabolism*. The human body converts about 20% of heat input into useful work and the remaining of heat generated must be rejected to the environment to regulate the body temperature.
- The *effective temperature* is the index which correlates the combined effects of air temperature, relative humidity and air velocity on the human body. A person can feel comfort in air-conditioned space, if he/she gets
  1. continuous supply of oxygen and removal of carbon dioxide,
  2. removal of heat continuously, and
  3. removal of Moisture content of air.
- An air-conditioning system may provide heating, cooling or both. An air-conditioning system controls the conditions of a confined space to produce a comfort effect over occupants. The indicative TR load required for air-conditioning is presented below:
  - Small office = 0.1 TR/m^2
  - Medium size of office (10–30 persons occupancy) = 0.06 TR/m^2
  - Large multistoried office/complex = 0.04 TR/m^2.
- In summer, the comfort conditions are achieved by cooling and dehumidification of air. In winter, the comfort conditions are achieved by heating and humidification of air and the year-round air conditioning system is the combination of both summer and winter air-conditioning systems.
- Water coolers use refrigeration cycle for cooling of water.
- An ice plant uses vapour-compression refrigeration cycle and it produces ice. The brine solution, a second refrigerant, acts as a heat carrier in the ice plant.
- Air coolers work on the principle of evaporative cooling. The sensible heat of hot atmospheric air is transferred continuously to water in the form of latent heat for its evaporation, when air makes physical contact with falling water on filter pads.

Glossary

Air-conditioning Process of maintaining air at controlled temperature, humidity, purity and motion

Air cooler Device which cools the air with the help of evaporation of water

Air refrigeration A refrigeration system in which air is the working medium

Air–conditioner Machine which creates air-conditioning

DBT Temperature indicated by an ordinary thermometer

Dehumidiciation Reduction of water vapour from moist air

DPT Temperature at which vapour starts to condense

Humidification Addition of water vapour in air

Ice plant Machine used for producing ice

Psychrometry Study of moist air

Refrigerant It is the working substance used in the refrigeration system

Refrigeration effect Cooling effect

Refrigeration Maintaining a space at low temperature than its surroundings

Relative humidity Ratio of partial pressure of vapour to total pressure of mixture

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Sensible heating/cooling Heat transfer with only change in DBT

Specific humidity Mass of water vapour per kg of dry air

Water cooler Machine which cools the water using refrigeration system

WBT Temperature indicated by thermometer, when its bulb is covered by wet cotton wick

Review Questions

1. What is sensible heating or cooling?
2. How does humidity affect human comfort?
3. State the applications of air conditioning.
4. State the human comfort conditions. What are the factors which affect the human comfort?
5. What is an effective temperature?
6. What is a comfort chart? What are its applications?
7. What are the requirements of comfort air conditioning?
8. Classify the air-conditioning systems.
9. Explain summer air-conditioning systems with the help of a schematic.
10. Explain winter air-conditioning systems with the help of a schematic.
11. Explain year-round air conditioning systems with the help of a schematic.
12. What is a unitary air-conditioning system? Explain a window air-conditioner.
13. Explain a central air-conditioning unit.
14. What are sources of heating loads in a retaurant? List them.
15. How does the human body maintain its temperature? Why does the human body need air-conditioning?
16. Explain the construction and working of a split air-conditioning system.
17. What are infilatration and ventilation? How do they contribute to heating load?
18. Explain the working of an ice plant with the help of a neat sketch.
19. What do you mean by refrigeration and air conditioning. Draw a schematic layout of an air-conditioner and explain its working.
20. Draw a schematic of desert cooler and explain its construction and working.

Problems

1. The indoor conditions of a building are 21°C and 40% RH, when outdoor conditions are 28°C and 50%. The sensible heat gain to a room is 12 kW and the latent heat gain is 3 kW. There is no recirculation and fresh air is cooled and dehumidified and then heated before entering the room. The cooling coil by-pass factor is 0.2 and the volume-flow rate of fresh air is 5 m^3/s. Calculate the temperature of air leaving the coil, capacity of cooling coil and heat supplied in the heater. *[6.5°C, 220 kW, 65 kW]*

2. A cinema hall of 1500 seating capacity is to be air-conditioned with following data:

Outdoor conditions 40°C DBT and 20°C WBT,

Indoor conditions 20°C, and 60%

Fresh air circulation 0.3 m^3/min/person,

If the required condition is achieved first by adiabatic humidifying and then by cooling, calculate

(a) Capacity of cooling coil and its surface temperature, if by-pass factor is 0.25

(b) Capacity of humidifier, and its efficiency

[118.8 kW, 15.3°C, 72 kg/h, 29%]

3. An office of 25 people in Mumbai is to be air-conditioned when outdoor conditions are 29°C DBT and 73% RH. The required comfort conditions are 21°C DBT and 59% RH with 0.5 m^3/min/person fresh air supply. Air is first cooled, dehumidified and then heated. Calculate the cooling coil capacity, heating coil capacity and capacity of the dehumidifier.

[3.2 TR, 2.1 kW, 9.2 kg/h]

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4. The atmospheric air at 30°C DBT with 75% RH enters a cooling coil at the rate of 200 m^3/min. The dew point temperature of cooling coil is 14°C and by-pass factor is 0.25. Calculate the temperature of air leaving the cooling coil, capacity of cooling coil, amount of water removed and sensible heat factor.

[18°C, 41.47 TR, 124.86 kg/h, 0.3766]

5. A small office of 50 seating capacity is to be air-conditioned at 20°C DBT, 60% RH. The outdoor conditions are 10°C and 8°C WBT. The rate of fresh air circulation is 0.3 m^3/min/person. The required conditions are achieved by heating, adiabatic humidifying. Calculate the heating capacity of coil and capacity of humidifier. Also, calcaulate the surface temperature of heating coil, if by-pass factor is 0.28.

[5.5 kW, 3.33 kg/h, 34.3°C]

6. A living room is to be maintained at 35°C DBT and 22.5 WBT during a winter season. The atmospheric conditions are 10°C with 90% RH. If the humidified air coming out of the air washer is at 90% RH, calculate the temperature of air entering the air washer and efficiency of air washer, RH of air leaving the washer and quantity of moisture added.

[17.5°C, 92.7%, 35%, 0.0054 kg/kg of air]

Objective Questions

1. During cooling and humidification process in air-conditioning
   (a) DBT decrease (b) RH decrease
   (c) DBT increases (d) None of above
2. Air at 20°C is heated to 25°C using a heater with a surface temperature of 30°C. The by-pass factor is
   (a) 0.15 (b) 0.5
   (c) 0.25 (d) 0.2
3. The by-pass factor of cooling coil is 0.3 when its coil surface temperature is 5°C and air enters at 40°C. The exit air temperature is
   (a) 10°C (b) 12.5°C
   (c) 15.5°C (d) 20°C
4. Infiltration air in an air-conditioned space
   (a) reduces heat load
   (b) leaks through the gaps
   (c) increases heat load
   (d) both (b) and (c)
5. Sensible heat factor is the ratio of total sensible heat to
   (a) total latent heat
   (b) total heat
   (c) capacity of cooling coil
   (d) capacity of heating coil
6. Capacity of cooling coil decreases with
   (a) decrease in mass-flow rate of air
   (b) decrease in by-pass factor
   (c) decrease in entry temperature of air
   (d) all of the above
7. Air-conditioning means control of
   (a) DBT
   (b) RH
   (c) velocity and purity of air
   (d) all of the above
8. In summer air-conditioning, the air is
   (a) cooled
   (b) cooled and dehumidified
   (c) heated and humidified
   (d) cooled and humidified
9. In winter air-condioning, the air is
   (a) heated
   (b) heated and dehumidified
   (c) heated and humidified
   (d) cooled and humidified
10. In summer air-conditioning, RH of conditioned space is generally kept
    (a) 40% (b) 50%
    (c) 60% (d) 100%

Answers

1. (a) 2. (b) 3. (c) 4. (d) 5. (b) 6. (a) 7. (d) 8. (b)
9. (c) 10. (c)

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30

Elements of Heat Transfer

Introduction

Heat transfer is a branch of thermal science which deals with analysis of rate of heat transfer and temperature distribution taking place in a system as well the nature of heat transfer. The design of boilers, condensers, evaporators, heaters, refrigerators and heat exchangers, requires considerations of the amount of heat to be transmitted as well as the rate at which heat is to be transferred. A heat transfer analysis must also be accounted in the design of electronic components, electric machines, transformers, and bearings to avoid the overheating and damage of equipments.

30.1 MODES OF HEAT TRANSFER

When temperature gradient exists in a medium which may be a solid, fluid or gas, then there is an energy transfer from high temperature region to low temperature region. This energy transfer as heat is called *heat conduction*. In contrast, *heat convection* refers to heat transfer that will occur between a surface and the adjacent moving medium, liquid or gas, when they are at different temperatures. It involves the combined effects of *conduction* and *fluid motion*. If there is no fluid motion, then the heat is transferred between a solid and its adjacent fluid by pure conduction. The third mode of heat transfer is *thermal radiation.* All surfaces at finite temperature emit energy in the form of electromagnetic waves or (photons) as result of the changes in electron configuration of the atoms or molecules. This mode of heat transfer does not require the presence of a material medium. The energy transfer by radiation is fastest and it can also travel in vacuum.

30.2 FOURIER LAW OF HEAT CONDUCTION

The rate of heat conduction through a medium depends on its geometry, thickness and material of the medium as well as temperature difference. The *Fourier law* states that the rate of heat conduction per unit area (heat flux) is directly proportional to temperature gradient.

$$\frac{\dot{Q}}{A} \propto \frac{dT}{dx}$$

or
$$q = \frac{\dot{Q}}{A} = -k\frac{dT}{dx} \qquad \text{...(30.1)}$$

$$\dot{Q} = -kA\frac{dT}{dx}$$

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where q = heat flux (W/m^2)

$\dot{Q}$ = rate of heat transfer, W

A = area normal to direction of heat flow, m^2

$\frac{dT}{dx}$ = temperature gradient °C/m, slope of temperature curve on T–x diagram

k = constant of proportionality, called *thermal conductivity* of material, W/m °C or W/m · K

The minus sign is inserted to make natural heat flow a positive quantity. According to the second law of thermodynamics, heat always flows in the direction of decreasing temperature. Thus the temperature gradient dT/dx becomes negative.

30.3 THERMAL CONDUCTIVITY

The *thermal conductivity* is a property of a material and is defined as the ability of the material to conduct heat through it. It can also be defined as *the rate of heat transfer through a unit thickness of material per unit area per unit temperature difference*. The thermal conductivity of a material is a measure of how fast heat will flow in that material. A large value of thermal conductivity indicates that the material is a good heat conductor and a low value indicates that the material is a poor heat conductor or an insulator.

The thermal conductivity is measured in watts per meter per degree Celsius or Watt metre per kelvin, when heat flow rate is expressed in watts. The thermal conductivity of some typical substances is given Table 30.1.

Table 30.1 *Typical Value of Thermal Conductivities*

Material	*Thermal Conductivity k (W/m.K)*
Metals	
Copper (pure)	399
Aluminium (pure)	227
Iron (Pure)	73
Carbon Steel, 1% C	43
Non-Metallic Solids	
Window glass	0.72
Glass wool	0.038
Asbestos	0.149
Cork	0.045
Fluids	
Water	0.556
Ethylene glycol	0.249
Ammonia	0.54
Air	0.024
Steam	0.0206
Carbon dioxide	0.0146

30.4 CONVECTION HEAT TRANSFER: NEWTON'S LAW OF COOLING

The convection heat transfer comprises of two mechanisms. The first is the transfer of energy due to random molecular motion (diffusion), and the second is the energy transfer by bulk motion of the fluid. The molecules of fluid are moving collectively or as aggregates and thus carry energy from a high-temperature region to a low-temperature region. If the fluid motion is artificially induced by a pump, fan or a blower that forces the fluid over a surface to flow, the heat transfer is said to be *forced convection*. If the fluid motion is set up by buoyancy effects resulting from density difference caused by temperature difference in the fluid, the heat transfer is said to be by *free (or natural) convection*.

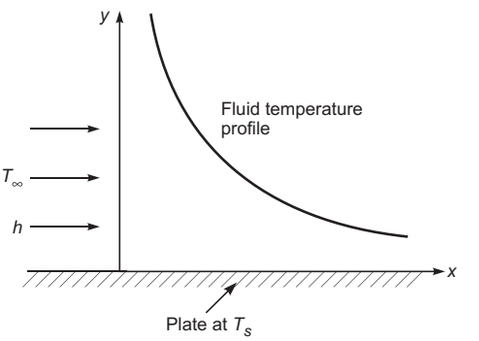

Fig. 30.1 *Temperature profile in convection*

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The *Newton's law of cooling* is the governing equation of convection heat transfer. It states that the rate of heat transfer is directly proportional to temperature difference between a surface and fluid or mathematically

$$\frac{\dot{Q}}{A}\ (\text{W/m}^2) \propto (T_s - T_\infty)\ (°\text{C})$$

or
$$\frac{\dot{Q}}{A} = h(T_s - T_\infty) \qquad ...(30.2)$$

where, T_s = surface temperature, °C
T_∞ = fluid temperature, °C
h = constant of proportionality, is called the heat transfer coefficient.

The heat transfer coefficient is measured in W/m^2 K or W/m^2 °C. The value of the heat transfer coefficient depends on the properties of fluid as well as fluid flow conditions.

30.5 RADIATION HEAT TRANSFER: STEFAN–BOLTZMANN LAW

When energy propagates in the form of electromagnetic waves from a high-temperature region to a low-temperature region, the form of energy transfer is referred as *thermal radiation. Stefan–Boltzmann law* governs the radiation heat transfer. It states that the rate of radiation heat transfer per unit area from a black surface is directly proportional to fourth power of the absolute temperature of the surface and is given by

$$\frac{\dot{Q}}{A} \propto (T^4)$$

or
$$\frac{\dot{Q}}{A} = \sigma T_s^4 \qquad ...(30.3)$$

where T_s = absolute temperature of surface K
σ = constant of proportionality, called *Stefan Boltzmann Constant* and has value of 5.67×10^{-8} W/m^2 K^4

The heat flux emitted by a real surface is less than that of black surface and is given by

$$\frac{\dot{Q}}{A} = \sigma \varepsilon (T_s^4) \qquad ...(30.4)$$

where, ε = a radiative property of the surface is called the *emissivity*

The net rate of radiation heat exchange between a real surface and its surrounding is

$$\frac{\dot{Q}}{A} = \sigma \varepsilon (T_s^4 - T_\infty^4) \qquad ...(30.5)$$

where, T_∞ = surrounding temperature K
T_s = surface temperature, K

The three other radiation laws, Planck's law, Wein's law and Kirchhoff's law, are also used in radiation heat transfer.

30.6 STEADY-STATE HEAT CONDUCTION IN SOLIDS

30.6.1 Plane Wall

Consider a plane wall of thickness L as shown in Fig. 30.2. Its left face at $x = 0$, is at a temperature T_1 and right face at $x = L$ is at temperature T_2. The wall has a constant thermal conductivity k.

Fourier equation for elemental strip of plane wall

$$\frac{\dot{Q}}{A} = -k\frac{dT}{dx}$$

Rearranging, we get;

$$\frac{\dot{Q}}{A}dx = -k\,dT$$

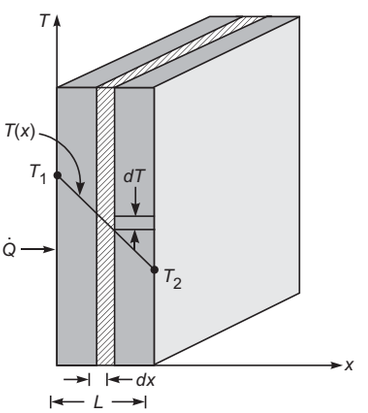

Fig. 30.2 *Heat conduction through a plane wall*

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Integrating, within limits,

$$\frac{\dot{Q}}{A}\int_0^L dx = -k\int_{T_1}^{T_2} dT$$

or $$\frac{\dot{Q}}{A}L = -k(T_2 - T_1)$$

The total heat flow rate $\dot{Q}$, through an area A normal to direction of heat flow,

$$\dot{Q} = kA\left(\frac{T_1 - T_2}{L}\right) \qquad ...(30.6)$$

30.6.2 Hollow Cylinder

Consider a hollow cylinder as shown in Fig. 30.3.

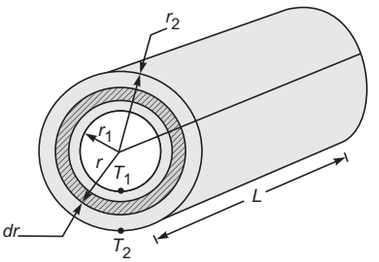

Fig. 30.3 *Heat conduction through a hollow cylinder*

For an elemental strip of cylinder,

$$\frac{\dot{Q}}{A} = -k\frac{dT}{dr}$$

where $A = 2\pi rL$, area of elemental strip

Rearranging, we get

$$\frac{\dot{Q}}{2\pi L}\cdot\frac{dr}{r} = -k\,dT$$

Integrating both sides,

$$\frac{\dot{Q}}{2\pi L}\int_{r_1}^{r_2}\frac{dr}{r} = -k\int_{T_1}^{T_2} dT$$

$$\frac{\dot{Q}}{2\pi L}\cdot\ln\left(\frac{r_2}{r_1}\right) = -k(T_2 - T_1)$$

or $$\dot{Q} = \frac{2\pi Lk(T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)} \qquad ...(30.7)$$

30.6.3 Hollow Sphere

Consider a hollow sphere as shown in Fig. 30.4.

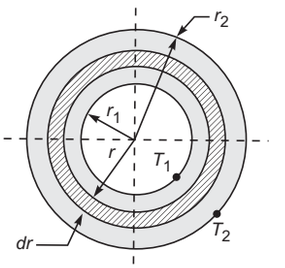

Fig. 30.4 *Heat conduction through a hollow sphere*

Applying Fourier law of heat conduction to elemental strip of thickness dr of hollow sphere,

$$\frac{\dot{Q}}{A} = -k\frac{dT}{dr}$$

Here, $A = 4\pi r^2$, area of elemental strip

Then $$\frac{\dot{Q}}{4\pi}\int_{r_1}^{r_2}\frac{dr}{r^2} = -k\int_{T_1}^{T_2} dT$$

$$\frac{\dot{Q}}{4\pi}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = -k(T_2 - T_1)$$

or $$\dot{Q} = \frac{4\pi r_1 r_2\, k(T_1 - T_2)}{r_2 - r_1} \qquad ...(30.8)$$

30.6.4 Electrical Analogy for Plane Wall

Rewriting the Eq. (30.6) in the form,

$$\dot{Q} = \frac{T_1 - T_2}{\frac{L}{kA}} = \frac{\Delta T}{R_{slab}} \qquad ...(30.9)$$

Comparing this equation with Ohm's law for electrical network,

current, $I = \frac{\Delta V}{R} = \frac{\text{Potential difference}}{\text{Resistance}}$

There is analogy between these two equations and

ΔT = Thermal potential difference, °C,

$\frac{L}{kA}$ = Thermal resistance (R_{slab}), °C/W,

$\dot{Q}$ = Heat flow rate or heat current in W.

Its equivalent thermal circuit is shown in Fig. 30.5.

The inverse of thermal resistance is called thermal conductance $K = \frac{kA}{L}$ (W/K)

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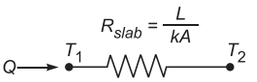


Fig. 30.5 *Equivalent thermal resistance*

Further, the thermal resistance may also be associated with convection heat transfer at a surface as shown in Fig. 30.6. Equation (30.2) is rewritten as

$$\dot{Q} = hA(T_s - T_\infty)$$

or
$$\dot{Q} = \frac{T_s - T_\infty}{\frac{1}{hA}} = \frac{T_s - T_\infty}{R_{conv}} \qquad ...(30.10)$$

where $R_{conv} = \frac{1}{hA}$ = Convective resistance,

$T_s - T_\infty$ = Thermal potential difference at the surface.

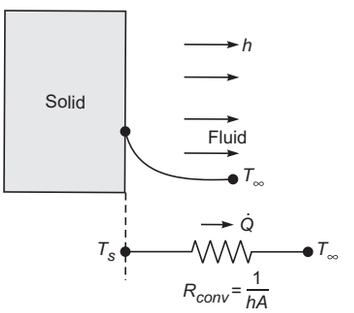


Fig. 30.6 *Convection resistance network*

30.7 COMBINED MODES OF HEAT TRANSFER

The three modes of heat transfer have been explained separately earlier. In actual situations, it is very rare that only one mechanism is involved in transfer of heat energy, instead a combination of mechanisms are involved. The thermal circuit representation provides a useful tool for the analysis of such heat transfer problems.

30.7.1 Plane Wall

Consider a plane wall of thickness L, exposed on both sides to two different temperatures environments. The steady-state heat-transfer rate through the wall, when its two surfaces are maintained at constant temperatures T_1 and T_2, can be expressed as

$$\dot{Q}_1 = \frac{kA(T_1 - T_2)}{L} \qquad ...(30.11)$$

The left face and right face involve convection heat transfer due to temperature difference between the surfaces and their surroundings. From Eq. (30.2);

The convection heat transfer at the left face,

$$\dot{Q}_2 = h_1 A(T_{\infty_1} - T_1) \qquad \ldots(30.12)$$

The convection heat transfer at the right face,

$$\dot{Q}_3 = h_2 A(T_2 - T_{\infty_2}) \qquad \ldots(30.13)$$

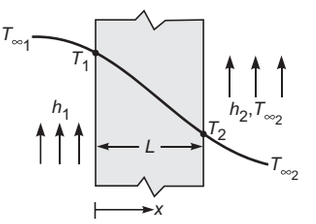
In steady-state condition,

$$\dot{Q} = \dot{Q}_1 = \dot{Q}_2 = \dot{Q}_3$$

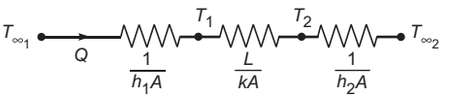
Hence
$$\dot{Q} = h_1 A(T_{\infty_1} - T_1)$$
$$= \frac{k\,A(T_1 \quad T_2)}{L}$$
$$= h_2 A(T_2 - T_{\infty_2}) \qquad \ldots(30.14)$$

The equivalent thermal resistances for a plane wall with convection on both *sides as shown in* Fig. 30.7 may be analysed by considering each element in circuit separately.

The concept of thermal circuit may also be introduced for a composite wall. Such a wall may involve any number of series and parallel thermal resistances due to a layer of different materials.



(a) Schematic



(b) Equivalent thermal resistances

Fig. 30.7 *Plane wall subjected to convection boundaries*

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30.7.2 Multilayer Plane Wall

Consider a composite wall with three layers and convection heat transfer on both boundary surfaces as shown in Fig. (30.8).

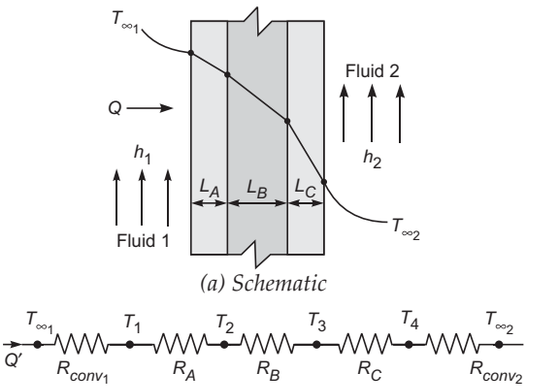

Fig. 30.8 *Composite slab and its equivalent network*

Here, heat transfer rate can be expressed as

$$\dot{Q} = \frac{(\Delta T)_{overall}}{\Sigma R_{th}}$$

where, $(\Delta T)_{overall} = T_{\infty_1} - T_{\infty_2}$

$$\Sigma R_{th} = R_{conv_1} + R_A + R_B + R_C + R_{conv_2}$$

$$R_{conv_1} = \frac{1}{h_1 A}$$

$$R_A = \frac{L_A}{k_A A}, \qquad R_B = \frac{L_B}{k_B A}$$

$$R_C = \frac{L_C}{k_C A}, \quad R_{conv_2} = \frac{1}{h_2 A}$$

Thus $$\Sigma R_{th} = \frac{1}{h_1 A} + \frac{L_A}{k_A A} + \frac{L_B}{k_B A} + \frac{L_C}{k_C A} + \frac{1}{h_2 A}$$

A = Area normal to heat transfer

Then, $$\dot{Q} = \frac{T_{\infty_1} - T_{\infty_2}}{\frac{1}{h_1 A} + \frac{L_A}{k_A A} + \frac{L_B}{k_B A} + \frac{L_C}{k_C A} + \frac{1}{h_2 A}} \quad ...(30.15)$$

Alternatively, the heat-transfer rate associated with each layer in the composite wall can also be expressed as

$$\dot{Q} = \frac{T_{\infty_1} - T_1}{\frac{1}{h_1 A}} = \frac{T_1 - T_2}{\frac{L_A}{k_A A}} = \frac{T_2 - T_3}{\frac{L_B}{k_B A}} = \frac{T_3 - T_4}{\frac{L_C}{k_C A}} = \frac{T_4 - T_{\infty_2}}{\frac{1}{h_2 A}} \quad ...(30.16)$$

30.7.3 Electrical Analogy for Hollow Cylinders

The electrical analogy can also be applied for a hollow cylinder by rearranging Eq. (30.7)

$$\dot{Q} = \frac{T_1 - T_2}{\frac{\ln(r_2/r_1)}{2\pi L k}} = \frac{\Delta T}{R_{cyl}} \quad ...(30.17)$$

where, $$R_{cyl} = \frac{\ln(r_2/r_1)}{2\pi L k}$$

where R_{cyl} = thermal resistance to heat flow through a hollow cylinder.

r_1 = inner radius

r_2 = outer radius

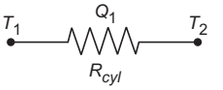

Fig. 30.9 *Equivalent thermal resistance for hollow cylinder*

Now, consider steady-state heat conduction through a hollow cylinder that is subjected to convection heat transfer on both sides to fluids at temperatures T_{∞_1} and T_{∞_2} with heat transfer coefficients h_1 and h_2, respectively as shown in Fig. 30.10. The thermal resistance network in this case consists of one conduction resistance and two convection resistances, and the rate of heat transfer can be expressed as

$$\dot{Q} = \frac{T_{\infty_1} - T_{\infty_2}}{\Sigma R_{th}}$$

where $$\Sigma R_{th} = R_{conv_1} + R_{cyl} + R_{conv_2}$$

$$= \frac{1}{(2\pi r_1 L) h_1} + \frac{\ln(r_2/r_1)}{2\pi L k} + \frac{1}{(2\pi r_2 L) h_2} \quad ...(30.18)$$

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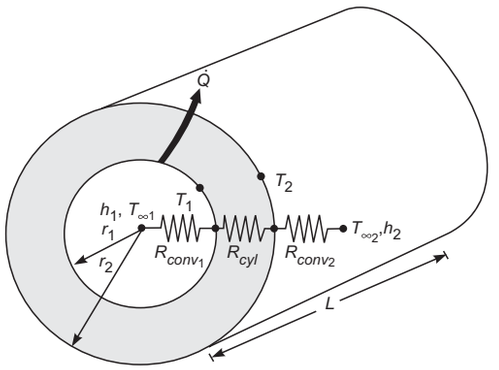

Fig. 30.10 *Thermal resistances network for a hollow cylinder exposed to convection on both sides*

30.7.4 Multi-layered Cylinders

Consider composite system of three hollow cylinders (thermal conductivity k_A, k_B, k_C respectively) as show in Fig. 30.11, with heat convection on inner and outer surfaces. Recalling the treatment given to composite wall, the expression for heat flow can be written as

$$\dot{Q} = \frac{(\Delta T)_{overall}}{\Sigma R_{th}}$$

$$= \frac{T_{\infty_1} - T_{\infty_2}}{\frac{1}{h_1 A_1} + \frac{\ln(r_2/r_1)}{2\pi L k_A} + \frac{\ln(r_3/r_2)}{2\pi L k_B} + \frac{\ln(r_4/r_3)}{2\pi L k_C} + \frac{1}{h_2 A_2}}$$

where, $A_1 = 2\pi r_1 L$ and $A_2 = 2\pi r_4 L$

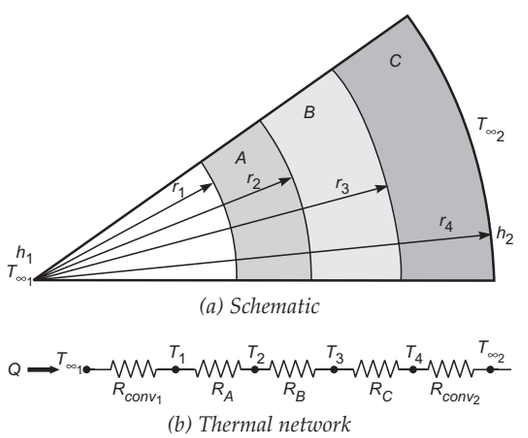

Fig. 30.11 *Thermal resistance network for a composite cylinder subjected to convection on both sides.*

Total resistance

$$\Sigma R_{th} = R_{conv_1} + R_A + R_B + R_C + R_{conv_2}$$

or $$\Sigma R_{th} = \frac{1}{2\pi r_1 L h_1} + \frac{\ln(r_2/r_1)}{2\pi L k_A} + \frac{\ln(r_3/r_2)}{2\pi L k_B} + \frac{\ln(r_4/r_3)}{2\pi L k_C} + \frac{1}{2\pi r_4 L h_2} \quad \ldots(30.19)$$

30.7.5 Electrical Analogy for Hollow Sphere

Similar to a plane wall or hollow cylinder, the use of electrical analogy can also be extended to hollow spheres. Rewriting Eq. (30.8) in the form;

$$\dot{Q} = \frac{T_1 - T_2}{\frac{r_2 - r_1}{4\pi r_1 r_2 k}} = \frac{\Delta T}{R_{sph}} \quad \ldots(30.20)$$

where $R_{sph} = \frac{r_2 - r_1}{4\pi r_1 r_2 k}$ thermal resistance of hollow sphere.

r_1 = inner radius of sphere, and

r_2 = outer radius of sphere

Now consider steady-state heat flow through a hollow sphere that is subjected to convection heat transfer on inner and outer sides to fluids at temperatures T_{∞_1} and T_{∞_2}, with heat transfer coefficients h_1 and h_2, respectively. The thermal resistance network for such a case, consists of one conduction resistance and two convection resistances as shown in Fig. 30.12, and the rate of

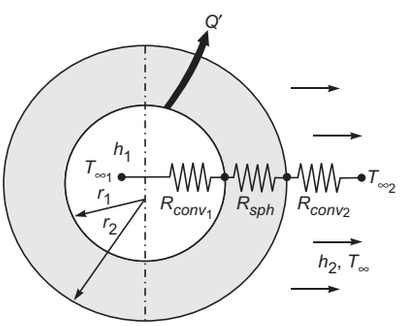

Fig. 30.12 *Thermal resistance network for a hollow sphere subjected to convection on inner and outer surfaces*

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heat transfer may be expressed as

$$\dot{Q} = \frac{T_{\infty_1} - T_{\infty_2}}{\Sigma R_{th}}$$

where $\Sigma R_{th} = R_{conv_1} + R_{sph} + R_{conv_2}$

$$= \frac{1}{4\pi r_1^2 h_1} + \frac{r_2 - r_1}{4\pi r_1 r_2 k} + \frac{1}{4\pi r_2^2 h_2} \quad \ldots(30.21)$$

30.7.6 Heat Conduction through Composite Spheres

The radial heat flow rate $\dot{Q}$ through multilayer sphere shown in figure (30.13) can be obtained by using thermal resistance concept to each later of sphere,

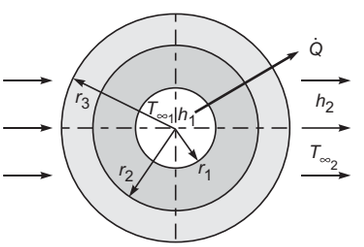
$$\dot{Q} = \frac{T_{\infty_1} - T_1}{R_{conv_1}} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_{\infty_2}}{R_{conv_2}} \quad \ldots(30.22)$$

Since all thermal resistances are in series as shown in Fig. (30.13b). Therefore,

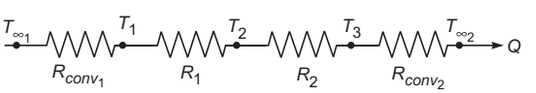
$$\dot{Q} = \frac{T_{\infty_1} - T_{\infty_2}}{R_{conv_1} + R_1 + R_2 + R_{conv_2}} = \frac{(\Delta T)_{overall}}{\Sigma R_{th}}$$

where various resistances are

$$R_{conv_1} = \frac{1}{4\pi r_1^2 h_1}, \qquad R_1 = \frac{r_2 - r_1}{4\pi r_1 r_2 k_1}$$



(a) Composite sphere



(b) Equivalent thermal resistences

Fig. 30.13

$$R_2 = \frac{r_3 - r_2}{4\pi r_2 r_3 k_2}, \; R_{conv_2} = \frac{1}{4\pi r_3^2 h_2} \quad \ldots(30.23)$$

and $\Sigma R_{th} = R_{conv_1} + R_1 + R_2 + R_{conv_2}$

It is total thermal resistance between the temperatures T_{∞_1} and T_{∞_2}.

30.8 OVERALL HEAT-TRANSFER COEFFICIENT

An overall heat-transfer coefficient can be evaluated as

$$\dot{Q} = UA(\Delta T)_{overall} = \frac{(\Delta T)_{overall}}{\Sigma R_{th}}$$

Therefore, $U = \dfrac{1}{A\Sigma R_{th}}$

For plane wall, from Eq. (30.15)

$$U = \frac{1}{\frac{1}{h_1} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_2}} \quad \ldots(30.24)$$

For a hollow cylinder the heat-transfer rate is expressed as

$$\dot{Q} = U_1 A_1 (\Delta T)_{overall} = U_2 A_2 (\Delta T)_{overall}$$

where $A_1 = 2\pi r_1 L$ and $A_2 = 2\pi r_2 L$.

Using Eq. (30.19) for composite cylinder exposed to convection on both sides;

$$U_1 = \frac{1}{2\pi r_1 L \Sigma R_{th}}$$

or $$U_1 = \frac{1}{2\pi r_1 L\left[\frac{1}{2\pi r_1 L h_1} + \frac{\ln(r_2/r_1)}{2\pi L k_A} + \frac{\ln(r_3/r_2)}{2\pi L k_B} + \frac{\ln(r_4/r_3)}{2\pi L k_C} + \frac{1}{2\pi r_4 L h}\right]}$$

$$U_1 = \frac{1}{\frac{1}{h} + \frac{r_1 \ln(r_2/r_1)}{k_A} + \frac{r_1 \ln(r_3/r_2)}{k_B} + \frac{r_1 \ln(r_4/r_3)}{k_C} + \frac{r_1}{r_4 h_2}} \quad \ldots(30.25)$$

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where U_1 = overall heat-transfer coefficient based on inner surface area A_1 ($=2\pi r_1 L$). It may also be defined in terms of any of the intermediate area providing

$$U_1 A_1 = U_2 A_2 = U_3 A_3 = U_4 A_4$$

The specific forms of U_2, U_3 and U_4 may be evaluated from Eq. (30.19).

The expression of overall heat transfer coefficient for hollow sphere can also be obtained in similar manner.

30.9 LOG MEAN AREA

Sometimes, it is convenient to use an expression for the heat flow through a hollow cylinder of the same form as that of a plane wall. Then the hollow cylinder can be replaced by a plane wall of thickness $(r_2 - r_1)$ and area by log mean area A_m.

Rewriting Eq. (30.17)

$$R_{cyl} = \frac{\ln (r_2/r_1)}{2\pi Lk}$$

Rearranging this equation as

$$R_{cyl} = \frac{(r_2 - r_1)}{(r_2 - r_1)} \times \frac{\ln\left[\dfrac{2\pi r_2 L}{2\pi r_1 L}\right]}{2\pi Lk}$$

$$= \frac{(r_2 - r_1)\ln (A_2/A_1)}{(A_2 - A_1)k}$$

or $$R_{cyl} = \frac{(r_2 - r_1)}{A_m k} \qquad ...(30.26)$$

where, $$A_m = \frac{A_2 - A_1}{\ln (A_2/A_1)} \qquad ...(30.27)$$

where, $A_2 = 2\pi r_2 L$ = area of outer surface of cylinder

$A_1 = 2\pi r_1 L$ = area of inner surface of cylinder

A_m = Logarithmic mean area or log mean area

$r_2 - r_1$ = Thickness of cylinder

R_{cyl} = Thermal resistance

Now, the heat flow for a hollow cylinder can be written as

$$\dot{Q} = \frac{A_m k (T_1 - T_2)}{r_2 - r_1} \qquad ...(30.28)$$

This approach can be used to transform a cylinder into an equivalent slab of thickness $(r_2 - r_1)$.

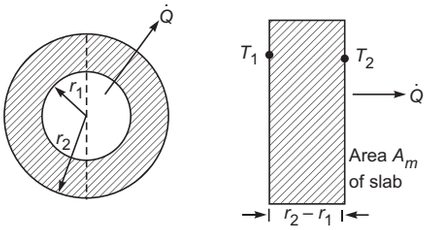

Fig. 30.14 *Concept of log mean area*

Example 30.1 *The wall of a furnace is constructed from 15 cm thick fire brick having constant thermal conductivity of 1.7 W/m.K. The two sides of the wall are maintained at 1400 K and 1150 K, respectively. What is the rate of heat loss through the wall which is 50 cm × 3 m on a side?*

Solution

Given The wall of a furnace

T_1 = 1400 K

A = 50 cm × 3 m = 0.5 × 3 = 1.5 m^2

k = 1.7 W/mK

T_2 = 1150 K

L = 15 cm = 0.15 m

To find Heat loss through the wall.

Assumptions

(i) Steady state conditions.

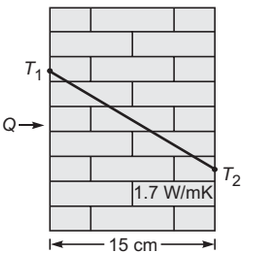

Fig. 30.15

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(ii) One dimensional heat conduction through the wall.
(iii) Constant properties.

Analysis According to Fourier law of heat conduction.

$$\dot{Q} = \frac{kA(T_1 - T_2)}{L}$$

Using numerical values

$$\dot{Q} = \frac{(1.7\,\text{W/m.K}) \times (1.5\,\text{m}^2) \times (1400\,\text{K} - 1150\,\text{K})}{0.15\,\text{m}}$$

$$= \mathbf{4250\ W}$$

Example 30.2 *Hot air at 150°C flows over a flat plate maintained at 50°C. The forced convection heat transfer coefficient is 75 W/m^2.°C. Calculate the heat gain rate by the plate through and area of 2 m^2.*

Solution

Given Flow of hot air over a plate

$T_\infty = 150°C$ $\quad T_s = 50°C$
$h = 75\ \text{W/m}^2.°C$
$A = 2\ \text{m}^2$

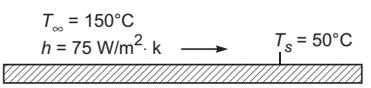

Fig. 30.16

To find Heat transfer rate by air to plate.

Assumptions

(i) Steady state conditions.
(ii) Constant properties.
(iii) Heat is transferred by forced convection only.

Analysis According to Newton's law of cooling.

$$Q = hA_s(T_\infty - T_s)$$
$$= (75\ \text{W/m}^2.\ \text{K}) \times (2\ \text{m}^2) \times (150 - 50)\ (°C)$$
$$= 15 \times 10^3\ \text{W} = \mathbf{15\ kW}$$

Example 30.3 *The thermal conductivity of plane wall varies as:*

$$k = k_0(1 + bT + cT^2)$$

If the wall thickness is L and surface temperatures are maintained at T_1 and T_2. Show that the heat flux q through the wall is given by

$$q = \frac{k_0(T_1 - T_2)}{L}\left\{1 + \frac{b}{2}[T_1 + T_2] + \frac{c}{3}\left[T_1^2 + T_1T_2 + T_2^2\right]\right\}$$

Solution

Given The relation for variable thermal conductivity as

$$k = k_0(1 + bT + cT^2)$$

Assumptions

(i) k_0 is constant.
(ii) Steady-state conditions.
(iii) One-dimensional heat conduction.

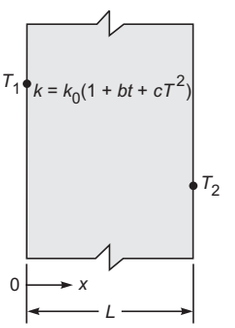

Fig. 30.17

Analysis According to Fourier law of heat conduction,

$$\frac{\dot{Q}}{A} = q = -k\frac{dT}{dx}$$

or $$q = -k_0(1 + bT + cT^2)\frac{dT}{dx}$$

or $$q\,dx = -k_0(1 + bT + cT^2)dT$$

Integrating both sides,

$$q\int_0^L dx = -k_0\int_{T_1}^{T_2}(1 + bT + cT^2)\,dT$$

or $$q\,(L - 0) = -k_0\left[T + b\frac{T^2}{2} + c\frac{T^3}{3}\right]_{T_1}^{T_2}$$

or $$q = -\frac{k_0}{L}\left\{(T_2 - T_1) + \frac{b}{2}(T_2^2 - T_1^2) + \frac{c}{3}(T_2^3 - T_1^3)\right\}$$

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or $$q = \frac{k_0 (T_1 - T_2)}{L}\left\{1 + \frac{b}{2}[T_1 + T_2] + \frac{c}{3}\left[T_2^2 + T_1 T_2 + T_2^2\right]\right\}$$

Example 30.4 *A thermopane window consists of two 5 mm thick glass ($k = 0.78$ W/m K) sheets separated by 10 mm stagnant air gap ($k = 0.025$ W/m K). The convection heat transfer coefficient for inner and outside air are 10 W/m^2 K and 50 W/m^2 K respectively.*

(a) Determine the rate of heat loss per m^2 of the glass surface for a temperature difference of 60°C between the inside and outside air.

(b) Compare the result with the heat loss, if the window has only a single sheet of glass of thickness 5 mm instead of thermopane.

(c) Compare the result with the heat flow, if window has made of sheet of glass 10 mm thick only.

Solution

Given A thermopane glass window,

$L_1 = L_3 = 0.5$ cm $= 0.005$ m

$L_2 = 10$ mm $= 0.01$ m

$k_1 = k_3 = 0.78$ W/m K

$k_2 = 0.025$ W/m K

$\Delta T = T_{\infty_1} - T_{\infty_2} = 60$°C

$h_1 = 10$ W/m^2 K

$h_2 = 50$ W/m^2 K

To find

(i) Heat flow rate through the glass per m^2.

(ii) Heat flow rate when window has only a glass sheet of 5 mm thickness.

(iii) When window has only a glass sheet of 10 mm thickness.

Assumptions

(i) One dimensional steady-state heat flow.

(ii) Constant properties.

Analysis The various specific thermal resistances (for 1 m^2) are shown and calculated as,

$$R_{conv_1} = \frac{1}{h_1 A} = \frac{1}{10 \times 1} = 0.10 \text{ K/W}$$

$$R_1 = R_3 = \frac{L_1}{k_1 A} = \frac{0.005}{0.78 \times 1} = 0.00641 \text{ K/W}$$

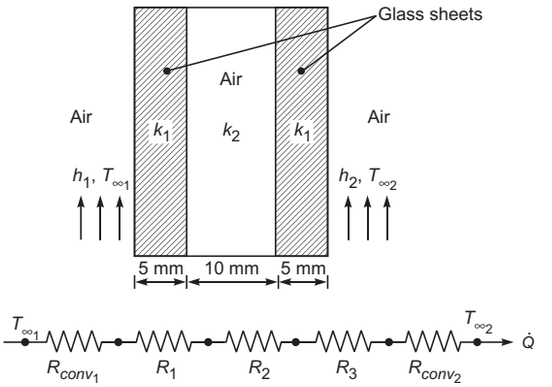

Fig. 30.18 *Schematic and thermal network*

$$R_2 = \frac{L_2}{k_2 A} = \frac{0.01}{0.025 \times 1} = 0.40 \text{ K/W}$$

$$R_{conv_2} = \frac{1}{h_2 A} = \frac{1}{50 \times 1} = 0.020 \text{ K/W}$$

(i) *The heat-flow rate through the glass air window*

All resistances are in series, thus the total resistance

$$\Sigma R_{th} = R_{conv_1} + R_1 + R_2 + R_3 + R_{conv_2}$$
$$= 0.1 + 0.00641 + 0.40 + 0.00641 + 0.020$$
$$= 0.53282 \text{ m}^2 \text{ K/W}$$

$$\dot{Q} = \frac{T_{\infty_1} - T_{\infty_2}}{\Sigma R_{th}}$$
$$= \frac{60}{0.53282} = \mathbf{112.60 \text{ W/m}^2}$$

(ii) If the window has a single sheet of glass of 5 mm thick, the total thermal resistance;

$$\Sigma R_{th} = R_{conv_1} + R_1 + R_{conv_2}$$
$$= 0.1 + 0.00641 + 0.02$$
$$= 0.12641 \text{ K/W}$$

$$\dot{Q} = \frac{60}{0.12641} = \mathbf{474.65 \text{ W/m}^2}$$

The heat loss is about four times that of previous case.

(iii) If the window has a glass sheet of 10 mm thick, then total resistance are;

$$\Sigma R_{th} = R_{conv_1} + 2R_2 + R_{conv_2}$$
$$= 0.1 + 2 \times 0.00641 + 0.02$$
$$= 0.13282 \text{ K/W}$$

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The heat loss per m^2;

$$\dot{Q} = \frac{60}{0.13282} = \mathbf{451.74\ W/m^2}$$

Example 30.5 *An exterior wall of a house consists of a 10.16 cm layer of common brick having thermal conductivity 0.7 W/mK. It is followed by a 3.8 cm layer of gypsum plaster with thermal conductivity of 0.48 W/mK. What thickness of loosely packed rockwool insulation (k = 0.065 W/m K) should be added to reduce the heat loss through the wall by 80%?*

Solution

Given

$L_1 = 10.16$ cm $= 0.1016$ m $\quad k_1 = 0.7$ W/mK
$L_2 = 3.8$ cm $= 0.038$ m $\quad k_2 = 0.48$ W/mK
$k_3 = 0.065$ W/mK

To find Thickness of rockwool insulation

Assumptions

(i) Steady-state heat conduction.
(ii) Heat conduction in one direction only.
(iii) Constant properties.
(iv) No contact thermal resistance at the interfaces.

Analysis Considering ΔT is the temperature difference across the composite wall, then heat flow per unit area or heat flux.

$$\frac{\dot{Q}_1}{A} = q_1 = \frac{\Delta T}{\frac{L_1}{k_1} + \frac{L_2}{k_2}}$$

$$= \frac{\Delta T}{\frac{0.1016}{0.7} + \frac{0.038}{0.48}} = 4.458\,\Delta T$$

After addition of insulation, heat loss is reduced by 80%, therefore, permissible heat flux will only be 20% of q_1

or $$q_2 = 0.2 \times \frac{q_1}{A} = 0.8196\,\Delta T$$

where q_2 = heat flux with rock wool insulation and it can be expressed as

$$q_2 = \frac{\Delta T}{\frac{L_1}{k_1} + \frac{L_2}{k_3} + \frac{L_3}{k_3}}$$

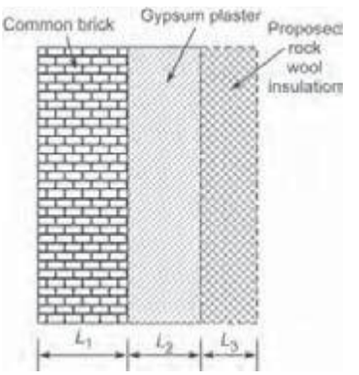

Fig. 30.19

using numerical values.

$$0.8196\,\Delta T = \frac{\Delta T}{\frac{0.1016}{0.7} + \frac{0.038}{0.48} + \frac{L_3}{0.065}}$$

or $$0.224 + \frac{L_3}{0.065} = 1.121$$

or $L_3 = 0.0583$ m = **5.83 cm,** thickness of rockwool insulation

Example 30.6 *A spherical thin-walled metallic container is used to store liquid nitrogen at 77 K. The container has a diameter of 0.5 m and is covered with an evacuated reflective insulation system composed of silica powder (k = 0.0017 W/m K). The insulation is 25 mm thick and its outer surface is exposed to ambient air at 300 K. The convective heat transfer coefficient is known to be 20 W/m^2.K. The Latent heat of vapourisation and density of liquid nitrogen are 2×10^5 J/kg and 804 kg/m^3, respectively.*

(a) What is the rate of heat transfer to the liquid nitrogen?
(b) What is the rate of liquid boil off?

Solution

Given A spherical metallic container filled with liquid nitrogen

$d = 0.5$ m or $r_1 = 0.25$ m $\quad h_{fg} = 2 \times 10^5$ J/kg
$r_2 = 0.25$ m + 25 mm $= 0.275$ m $\quad k = 0.0017$ W/mK
$h_o = 20$ W/m^2.K $\quad \rho = 804$ kg/m^3
$T_{\infty_1} = 77$ K $\quad T_{\infty_2} = 300$ K

To find

(i) The heat transfer rate to the nitrogen, and
(ii) The mass rate of nitrogen boil off.

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Assumptions

(i) Steady-state conditions.
(ii) One-dimensional heat transfer in radial direction only.
(iii) Negligible resistance between container wall and liquid nitrogen.
(iv) Constant properties.
(v) Negligible radiation heat loss.

Analysis The thermal circuit involves a conduction and convection resistance in series, therefore, total resistance,

$$\Sigma R_{th} = R_{sph} + R_{conv} = \frac{(r_2 - r_1)}{4\pi k r_1 r_2} + \frac{1}{4\pi r_2^2 h_o}$$

$$= \frac{0.275 - 0.25}{4\pi \times 0.0017 \times 0.25 \times 0.275} + \frac{1}{4\pi (0.275)^2 \times 20}$$

$$= 17.074 \text{ K/W}$$

The rate of heat transfer to liquid nitrogen,

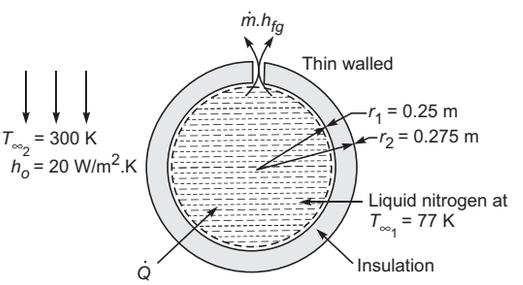
$$\dot{Q} = \frac{T_{\infty_2} - T_{\infty_1}}{\Sigma R_{th}} = \frac{300 - 77}{17.074} = \mathbf{13.06\ W}$$

The heat loss to nitrogen will cause evaporation of nitrogen

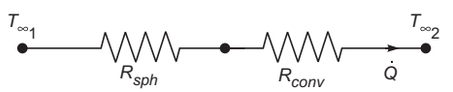
Thus, $\dot{Q} = \dot{m} h_{fg}$

or $\dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{13.06}{2 \times 10^5} = 6.53 \times 10^{-5}$ kg/sec

$= \mathbf{0.235\ kg/h}$



(a) Schematic of hollow sphere filled with nitrogen



(b) Thermal resistance network

Fig. 30.20

or on volumetric basis

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{0.235}{804}$$

$$= 2.923 \times 10^{-4} \text{ lit/h} \approx \mathbf{7\ lit/day}$$

30.10 PRINCIPLE OF HEAT CONVECTION

In the preceeding articles, we have discussed the heat transfer by conduction and have considered the convection only to the extent of boundary conditions. As discussed earlier, the convection is the mode of heat transfer, in which energy is transferred between a surface and a moving fluid.

The convection heat transfer comprises of two mechanisms. First is transfer of energy due to random molecular motion (diffusion), and second is the energy transfer by bulk or macroscopic motion of the fluid. The molecules of fluid are moving collectively or as aggregates and thus carry energy from a high-termperature region to a low-temperature region. Hence, the heat-transfer rate increases in presence of temperature gradient. The convection heat transfer is due to superposition of energy transfer by random motion of the molecules and by the bulk motion of the fluid.

Convection heat transfer may be classified according to the nature of fluid flow. If the fluid motion is artificially induced by a pump, fan or a blower, that forces the fluid over a surface to flow, the heat transfer is said to be *forced convection*.

If the fluid motion is set up by buoyancy effects resulting from density difference caused by temperature difference in the fluid, the heat transfer is said to be by *free* (*or natural*) *convection.* For an example, a hot plate vertically suspended in stagnant cool air, causes a motion in the air layer adjacent to the plate surface because of temperature difference in air, which gives rise to density gradient which in turn set-up the air motion.

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30.11 CONVECTION BOUNDARY LAYERS

30.11.1 Velocity Boundary Layer

When fluid flows over a surface, the fluid particles adjacent to the surface get zero velocity. These particles, then act to retard the motion of particles in the adjoining fluid layer, which acts to retard the motion of particles in the next layer and so on, until a distance $y = \delta$ from the surface reaches, where these effects become negligible. The region of flow over the surface, bounded by δ in which the effects of the viscous shearing forces caused by the fluid viscosity are felt is called *boundary layer or velocity boundary layer or hydrodynamic boundary layer. The thickness* of boundary layer δ is generally defined as a distance from the surface at which the local velocity $u = 0.99$ of free stream velocity u_∞.

The retardation of fluid motion in the boundary layer is due to the shear (viscous) stress acting in opposite direction. With increasing the distance y from the surface, shear stress decreases, the local velocity u increases until it approaches u_∞. With increasing the distance from the leading edge, the effect of viscosity penetrates further into the free stream and boundary layer thickness grows (δ increases with x) as shown in Fig. 30.21.

30.11.2 Thermal Boundary Layer

If the fluid flowing on a surface has a different temperature than that of the surface, the thermal layer boundary developed is similar to velocity boundary layer as shown in Fig. 30.22. The flow region over a surface in which the temperature variation in the direction normal to surface is significant is the *thermal boundary layer*. It is denoted by δ_{th} and is characterized by temperature gradients and heat transfer. The thickness of thermal boundary layer δ_{th} at any location along the surface is defined as the distance from the surface at which the temperature difference $(T - T_\infty)$ equals 0.99 $(T_s - T_\infty)$

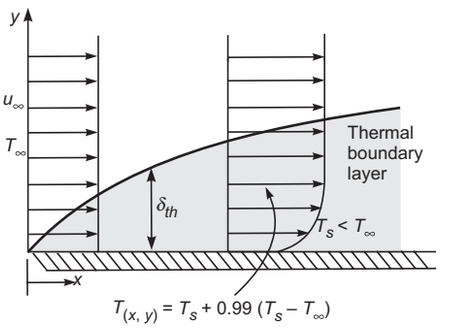

Fig. 30.22 *Thermal boundary layer for flow of hot fluid over a cold surface*

30.11.3 Significance of Boundary Layers

The analysis of convection problems require the knowledge of the type of boundary layer developed, whether it is laminar or turbulent. The skin friction and convection coefficient depend strongly on such conditions.

The developed boundary layer may consist of laminar boundary, transition region and turbulent boundary layer.

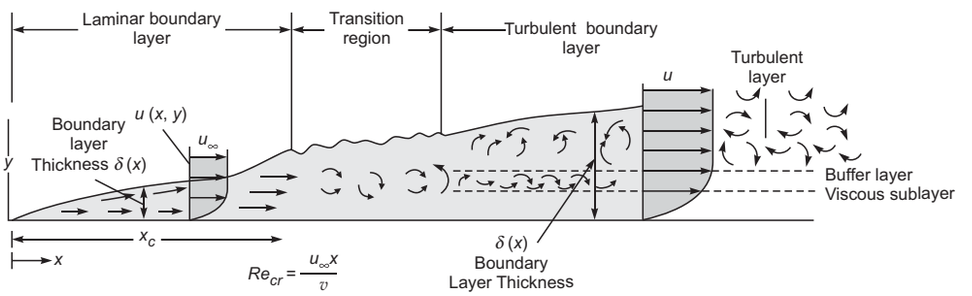

Fig. 30.21 *Boundary layer concept for flow along a flat plate*

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The velocity boundary layer $\delta(x)$ is characterized by the presence of velocity gradients and shear stresses. The thermal boundary layer $\delta_{th}(x)$ is characterized by temperature gradients and heat transfer.

The characteristic of fluid flow is governed by a dimensionless quantity called the *Reynolds number.*

30.12 PHYSICAL SIGNIFICANCE OF THE CONVECTION DIMENSIONLESS PARAMETERS

30.12.1 Reynolds Number

It is the *ratio of inertia forces to viscous forces* in the velocity boundary layer. It is used in forced convection and approximated as

$$Re = \frac{\text{Inertia forces}}{\text{Viscous forces}} = \frac{\rho u_\infty L_c}{\mu} = \frac{\mu_\infty L_c}{\nu} \quad \text{...(30.29)}$$

where, L_c = Characteristic length of the geometry, = distance from leading edge, in the flow direction for a flat plate and d for a cylinder or sphere (m)

u_∞ = free stream velocity (m/s)

ρ = fluid density (kg/m^3)

μ = dynamic viscosity (Ns/m^2)

ν = kinetic viscosity (m^2/s) or viscous diffusivity

The Reynolds number characterizes the type of flow, whether it is laminar of turbulent flow.

30.12.2 Critical Reynolds Number

It is the *value of Reynolds number, where boundary layer changes from laminar to turbulent nature. It is denoted by* Re_{cr}. The value of critical Reynolds number is different for different geometries. For flow over a flat plate, transition from laminar to turbulent boundary layer occurs roughly when critical Reynolds number is

$$Re_{cr} \geq 10^5 \quad \text{…(30.30)}$$

In fluid flow through tubes, the Reynolds number is also used to characterize the fluid flow. The transition from laminar to turbulent boundary layer occurs, when

$$Re_D = \frac{u_\infty d}{\nu} > 2300 \quad \text{…(30.31)}$$

30.12.3 Grashof Number

It is defined as the *ratio of the buoyancy forces to the viscous forces* acting in the fluid. It is used in free convection and its role is same as that of Reynolds number in forced convection. The Grashof number characterises the type of boundary layer developed in natural convection heat transfer. It is denoted by *Gr* and expressed as

$$Gr = \frac{\text{Buoyancy forces}}{\text{Viscous forces}} = \frac{g\Delta\rho V}{\rho\nu^2} = \frac{g\beta\Delta T L_c^3}{\nu^2} \quad \text{...(30.32)}$$

where, g = Acceleration due to gravity, m/s^2

β = Coefficient of volumetric expansion = $1/(T_{mf} + 273)$

ΔT = Temperature difference between surface and fluid, °C or K

ν = Kinetic viscosity of fluid, m^2/s

L_c = Significant length of the body, m

= Height, L for vertical plates and cylinders

= Diameter, d for horizontal cylinder and sphere

$$= \frac{\text{Surface area}}{\text{Perimeter}} = \frac{A_s}{P}$$

30.12.4 Nusselt Number

It is defined as the *ratio of convection heat flux to conduction heat flux* in fluid boundary layer or

$$Nu = \frac{\text{convection heat flux}}{\text{conduction heat flux}} = \frac{h\Delta T}{k_f \Delta T/L_c} = \frac{hL_c}{k_f}$$

where, L_c = Characteristic length or length in direction of flow, m

h = The heat transfer coefficient, W/m^2.K

k_f = Thermal conductivity of the fluid, W/m.K

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The low value of *Nu*, indicates the more conduction in the fluid as compared to convection in the boundary layer and large value of *Nu*, indicates large convection in the fluid.

30.12.5 Prandtl Number

It is defined as the *ratio of the momentum diffusivity* ν *to the thermal diffusivity* α

or $$Pr = \frac{\nu}{\alpha} = \frac{\mu}{\rho} \times \frac{\rho C_p}{k_f} = \frac{\mu C_p}{k_f} \quad ...(30.33)$$

The Prandtl number provides a measure of relative effectiveness of momentum and energy transfer in the velocity and thermal boundary layer, respectively. In other words, it compares the relative thickness of velocity and thermal boundary layers.

For gases $Pr \cong 1$

For liquid metal $Pr >> 1$

Further, the thickness of two boundaries can be expressed as

$$\frac{\delta_{th}}{\delta} \approx Pr^n$$

where, δ_{th} = thickness of thermal boundary layer

δ = thickness of velocity boundary layer

Pr = Prandtl number.

where n is the exponent.

30.12.6 Stanton Number

It is the *ratio of the Nusselt number to product of Reynolds number and Prandtl number* and is expressed as

$$St_x = \frac{Nu_x}{Re_x Pr} = \frac{h}{\rho C_p u_\infty}$$

or $$St_x = \frac{h\Delta T}{\rho C_p u_\infty \Delta T}$$

$$= \frac{\text{Heat flux to the fluid}}{\text{Heat transfer capacity of fluid}} \quad ...(30.34)$$

30.12.7 Peclet Number

It is the *product of Reynolds number and Prandtl number.* It is denoted by *Pe* and expressed as

$$Pe = Re\, Pr \quad ...(30.35)$$

30.12.8 Rayleigh Number

It is the *product of Grashof number and Prandtl number* in natural convection boundary layer. It is denoted by *Ra* and used to characterize the type of boundary layer in natural convection.

$$Ra = Gr Pr$$

For laminar boundary layer $10^4 \le Ra \le 10^8$

For turbulent boundary layer $Ra \ge 10^8$ ...(30.36)

The properties of the fluid are used at mean film temperature

$$T_f = \frac{T_s + T_\infty}{2}$$

30.13 DIMENSIONAL ANALYSIS

Dimensional analysis differs from conventional methods of approach in which certain equations are solved for a resulting equation. Instead, it combines several variables affecting a phenomenon in a dimensionless group, such as Nusselt number, which facilitates the interpretation and extends its application to experimental data.

The dimensional analysis does not provide any information about the nature of phenomenon, hence success or failure of the method depends on proper selection of affecting variables.

30.13.1 Buckingham π Theorem

According to this rule, the required number of independent groups that can be formed by combining physical variables, related to a phenomenon is equal to the total number of these physical quantities, minus the number of primary dimension m, required to express the dimensional formulae for n physical quantities or independent groups are $n–m$.

The independent dimensionless groups can be expressed as

$$f(\pi_1, \pi_2, \pi_3, \ldots) = 0$$

In a particular problem involving seven physical quantities which can be expressed in four primary dimensions then

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Dimensionless π terms $= n - m = 7 - 4 = 3$

Hence $f(\pi_1, \pi_2, \pi_3) = 0$

or the form $\pi_1 = f(\pi_2, \pi_3)$.

30.13.2 Dimensional Analysis for the Forced Convection

The force convection phenomenon can be influenced by the variables given in the table below:

Table 30.2

Sr. No.	*Parameters*	*Symbol and Unit*	*Primary Dimensions*
1.	Tube Diameter (Characteristic length)	D, m	L
2.	Fluid density	ρ, kg/m^3	ML^{-3}
3.	Fluid viscosity	μ, kg/m s	$ML^{-1}t^{-1}$
4.	Fluid velocity	u_∞, m/s	Lt^{-1}
5.	Fluid thermal conductivity	k_f, W/m·K	$MLt^{-3}T^{-1}$
6.	Heat transfer coefficient	h, W/m^2·K	$Mt^{-3}T^{-1}$
7.	Fluid specific heat	C_p, J/kg·K	$L^2t^{-2}T^{-1}$

These seven variables are expressed in four primary dimensions (M, L, T, t). Therefore, according to Buckingham π theorem, the independent dimensionless groups formed are

= No. of variable affecting the phenomenon – No. of primary dimensions used

or $7 - 4 = 3$(i.e., π_1, π_2, π_3)

Writing these three groups as,

$$\pi_1 = D^a, \rho^b, \mu^c, k_f^d, u_\infty$$
$$\pi_2 = D^e, \rho^f, \mu^g, k_f^h, C_p$$
$$\pi_3 = D^i, \rho^j, \mu^k, k_f^l, h$$

where, D, ρ, μ, k form a core group is called *repeating variables* and u_∞, C_p and h are as selected variables. The repeating variables are chosen arbitrarily such that they together involve all fundamental dimensions and they themselves do not form a dimensionless parameter. Since the groups π_1, π_2, π_3 are dimensionless, hence certain exponents are applied on the core group variable, which are to be determined,

(i) Expressing the Variable in their Primary Dimensions for π_1,

$$\pi_1 = M^0 L^0 T^0 t^0$$
$$= L^a (ML^{-3})^b (ML^{-1}t^{-1})^c (MLt^{-3} T^{-1})^d (Lt^{-1})$$

Separating the exponents for

M: $0 = b + c + d$

L: $0 = a - 3b - c + d + 1$

T: $0 = -d$

t: $0 = -c - 3d - 1$

Solving these simultaneous equations, we get

$d = 0, \quad c = -1$

$b = 1, \quad a = 1$

Hence, the dimensionless group formed is,

$$\pi_1 = \frac{D\rho u_\infty}{\mu} = Re \text{ (Reynolds number)}$$

(ii) Expressing the Primary Dimensions for Variables of π_2,

$$\pi_2 = M^0 L^0 T^0 t^0$$
$$= L^e (ML^{-3})^f (ML^{-1}t^{-1})^g (ML\, t^{-3}T^{-1})^h (L^2\, t^{-2}\, T^{-1})$$

Separating the exponents for

M: $0 = f + g + h$

L: $0 = e - 3f - g + h + 2$

T: $0 = -h - 1$

t: $0 = -g - 3h - 2$

Solving these simultaneous equations, we get

$h = -1, \quad g = 1$

$f = 0, \quad e = 0$

Hence, the dimensionless group formed is,

$$\pi_2 = \frac{\mu C_p}{k_f} = Pr \text{ (Prandtl Number)}$$

(iii) Expressing the Primary Dimensions for Variables of π_3,

$$\pi_3 = M^0 L^0 T^0 t^0$$
$$= L^i (ML^{-3})^j (ML^{-1}\, t^{-1})^k (ML\, t^{-3}\, T^{-1})^l (Mt^{-3}\, T^{-1})$$

Separating the exponents for

M: $0 = j + k + l + 1$

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L: $0 = i - 3j - k + l$

T: $0 = -l - 1$

t: $0 = -k - 3l - 3$

Solving these simultaneous equations, we get

$l = -1, \quad k = 0$

$j = 0, \quad i = 1$

Hence, the dimensionless group formed is,

$$\pi_3 = \frac{hD}{k_f} = Nu \text{ (Nusselt number)}$$

Hence for forced convection,

$$Nu = f(Re, Pr) \qquad ...(30.37)$$

30.14 SUMMARY OF DIMENSIONLESS PARAMETERS AND THEIR CORRELATIONS

Table 30.3 *Dimensional groups of heat transfer*

Groups	*Definition*	*Interpretation*
Coefficient of friction (C_f)	$\frac{\tau_s}{\rho u_\infty^2/2}$	Dimensionless surface shear stress
Friction factor (f)	$\frac{\Delta p}{(L/D)\rho u_\infty^2/2}$	Dimensionless pressure drop
Grashof number (Gr_L)	$\frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$	Ratio of buoyance forces to viscous forces
Nusselt number (Nu_L)	$\frac{hL_c}{k_f}$	Ratio of convection heat flux to conduction heat flux
Prandtl number (Pr)	$\frac{\mu C_p}{k_f}$	Ratio of momentum diffusivity to thermal diffusivity
Reynolds number (Re_L)	$\frac{u_\infty L_c}{\nu}$	Ratio of inertia forces to viscous forces

Table 30.4 *Summary of convection heat transfer correlations for external flow*

Correlation	*Geometry Condition*	*Remark*
$\delta = \frac{5.0x}{\sqrt{Re_x}}$	Flat plate, Laminar	Thickness of velocity of boundary layer
$C_{fx} = \frac{0.646}{\sqrt{Re_x}}$	Flat plate, Laminar	Local coefficient of friction
$Nu_x = 0.332\, Re_x^{1/2}\, Pr^{1/3}$	Flat Plate, Laminar	Local Nusselt number
$\delta_{th} = \delta Pr^{-1/3}$	Flat Plate, Laminar	Thickness of thermal boundary layer
$C_f = \frac{1.328}{\sqrt{Re_L}}$	Flat plate, Laminar	Average coefficient of friction
$Nu_L = 0.664\, Re_L^{1/2}\, Pr^{1/3}$	Flat plate, Laminar	Average Nusselt number
$C_{fx} = \frac{0.059}{\sqrt{Re_x^{1/5}}}$	Flat plate, Turbulent	Local coefficient of friction
$\delta = \frac{0.37x}{\sqrt{Re_x^{1/5}}}$	Flat plate, Turbulent	Thickness of velocity of boundary layer
$Nu_x = 0.296\, Re_x^{4/5}\, Pr^{1/3}$	Flat plate, Turbulent	Local Nusselt number
$Nu_d = CRe_d^m\, Pr^{1/3}$	Cylinder	Average Nusselt number, Table 30.5
$Nu_d = 2 + (0.4\, Re_d^{1/2} + 0.06\, Re_d^{2/3})\, Pr^{0.4}\, (\mu_\infty/\mu_s)$	Sphere	Average Nusselt number

Table 30.5 *Constant C and m to used for circular cylinder in cross flow*

Re_d	C	m
4 – 40	0.911	0.330
40 – 4000	0.683	0.466
4000 – 40000	0.193	0.681
40000 – 400000	0.027	0.805

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Example 30.8 *Air at 27°C and 1 atm flows over a heated plate with a velocity of 2 m/s. The plate is at uniform temperature of 60°C. Calculate the heat transferred (a) first 0.2 m of the plate and (b) first 0.4 m of the plate.*

Solution

Given The flow over a heated flat plate

$T_\infty = 27°C$ $\quad p = 1$ atm

$u_\infty = 2$ m/s $\quad T_s = 60°C$

$x_1 = 0.2$ m $\quad x_2 = 0.4$ m

To find

(i) Heat transfer from first 0.2 m, and

(ii) Heat transfer from first 0.4 m.

Assumptions

(i) No heat radiation exchange,

(ii) The unit depth in z direction,

(iii) Air and surface temperatures are different, taking the properties at mean film temperature.

Properties of air The mean film temperature

$$T_f = \frac{T_s + T_\infty}{2} = \frac{60 + 27}{2} = 43.5°C$$

The properties of air at 43.5°C (from properties of air in appendix, Table A-8)

$\nu = 17.36 \times 10^{-6}$ m^2/s. $\quad k_f = 0.02749$ W/m·K

$Pr = 0.7$ $\quad C_p = 1.006$ kJ/kg·K

Analysis The Reynolds number at $x = 0.2$ m

$$Re_{x_1} = \frac{u_\infty . x}{\nu} = \frac{(2\text{ m/s}) \times (0.2\text{ m})}{(17.36 \times 10^{-6}\text{ m}^2/\text{s})} = 23041$$

(i) *The heat transfer from first 0.2 m*

The local value of heat transfer coefficient can be calculated as

$$Nu_{x_1} = \frac{h_{x_1} x}{k_f} = 0.332\, Re_{x_1}^{1/2}\, Pr^{1/3}$$

or $$h_{x_1} = \frac{0.332 \times (0.02749\text{ W/m·K})}{(0.2\text{ m})} \times (23041)^{1/2}\,(0.7)^{1/3}$$

$$= 6.15\text{ W/m}^2\text{. K}$$

The average value of heat transfer coefficient

$$h_1 = 2h_{x_1} = 2 \times 6.15 = 12.3\text{ W/m}^2\text{. K}$$

The heat transfer rate upto $x = 0.2$ m

$$\dot{Q}_1 = \bar{h}_1 A_s (\Delta T)$$

$$\frac{\dot{Q}_1}{L} = (12.3\text{ W/m}^2\text{.K}) \times (0.2\text{ m}) \times (60 - 27)\text{ (K)}$$

$$= \mathbf{81.18\text{ W/m}}$$

(ii) *The heat transfer from 0.4 m*

Reynolds no.

$$Re_{x_2} = \frac{u_\infty x_2}{\nu} = \frac{2 \times 0.4}{17.36 \times 10^{6}} = 46082$$

The local value of heat transfer coefficient

$$Nu_{x_2} = 0.332\, Re_{x_2}^{1/2}\, Pr^{1/3}$$

or $$h_{x_2} = \frac{0.02749}{0.4} \times 0.332 \times (46082)^{1/2}\,(0.7)^{1/3}$$

$$= 4.35\text{ W/m}^2\text{. K}$$

Average heat transfer coefficient

$$\bar{h}_2 = 2h_{x_2} = 2 \times 4.35 = 8.7\text{ W/m}^2\text{·K}$$

The heat transfer

$$\frac{\dot{Q}_2}{L} = (8.7\text{ W/m}^2\text{. K}) \times (0.4\text{ m}) \times (60 - 27)\text{ (K)}$$

$$= \mathbf{114.8\text{ W/m}}$$

30.15 FLOW THROUGH DUCTS

The dimensionless parameters such as Reynolds number, Prandtl number and Nusselt number can also be used in forced convection through ducts. For flow through tubes the characteristic length L_c is set to the diameter of the tube d.

In majority of fluid flow through tubes, the nature of the flow is turbulent and the correlation used for approximation of friction factor and Nusselt number as

$$f = 0.316\, Re^{-1/4} \quad \text{for } Re < 2 \times 10^4$$

$$= 0.184\, Re^{-0.2} \quad \text{for } 2 \times 10^4 < Re < 3 \times 10^5$$

$$Nu = 0.023\, Re^{0.8}\, Pr^n$$

where $n = 0.4$ for heating

$= 0.3$ for cooling

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$$Nu = \frac{hd}{k_f}$$

$$Re = \frac{u_m d}{\nu}$$

$$Pr = \frac{\mu C_p}{k_f} = \frac{\nu}{\alpha}$$

For flow through non-circular ducts, the tube diameter d is replaced by the hydraulic diameter of a non-circular duct, defined as

$$D_h = \frac{4 A_c}{P} = \frac{4 \times \text{Cross-section area of duct}}{\text{wetted perimeter}}.$$

Example 30.9 *Air at 200 kPa and 200°C is heated as it flows through a tube with a diameter of 25 mm at a velocity of 10 m/s. Calculate the heat-transfer rate per unit length of the tube, if a constant heat-flux condition is maintained at the wall and the wall temperature is 20°C above the air temperature, all along the length of the tube. How much would the bulk temperature increase over 3 m length of the tube?*

Solution

Given Uniform heating of the tube

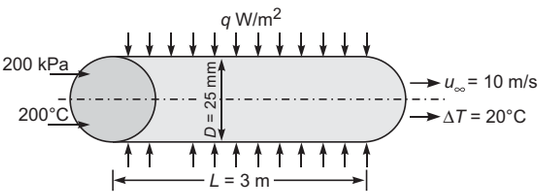

Fig. 30.23

To find

(i) Heat transfer rate per unit length of the tube,
(ii) Bulk temperature rise over 3 m length of the tube.

Assumptions

(i) Steady-state heat transfer conditions.
(ii) Fully developed flow through a tube.
(iii) Conduction and radiation effects are negligible.

Properties of the air. The properties of air at temperature of 200°C from Table A-8

$C_p = 1025$ J/kg·K $\quad k_f = 0.0386$ W/m.K
$\mu = 2.57 \times 10^{-5}$ kg/m-s $\quad Pr = 0.681$

Analysis The density of air at 200 kPa 200°C is

$$\rho = \frac{p}{RT} = \frac{200\,\text{kPa}}{(0.287\,\text{kJ/kg}\cdot\text{K})\,(473\,\text{K})} = 1.473\ \text{kg/m}^3$$

The Reynolds number is

$$Re_D = \frac{\rho u_m D}{\mu} = \frac{1.473 \times 10 \times 0.025}{2.57 \times 10^{-5}} = 14332$$

Since $Re_d > 2300$, hence the flow is turbulent.
Using correlation

$$Nu_d = \frac{hD}{k_f} = 0.023\ Re_d^{0.8}\ Pr^{0.4}$$

$$= 0.023 \times (14332)^{0.8}\ (0.681)^{0.4} = 41.69$$

or $$h = \frac{k_f}{D} Nu_d = \frac{0.0386}{0.025} \times 41.69$$

$$= 64.37\ \text{W/m}^2.\text{K}.$$

(i) *The heat transfer rate per metre length*

$$\frac{\dot{Q}}{L} = h\,(\pi D)\,(\Delta T)$$
$$= 64.37 \times (\pi \times 0.025) \times (20)$$
$$= \mathbf{101.1\ W/m}$$

(ii) *Bulk temperature rise*

Making the energy balance over 3 m length of the tube;
Heat supply rate
= Enthalpy rise rate of the fluid

$$\frac{\dot{Q}}{L} \times L = \dot{m} C_p (\Delta T_b)$$

when $\dot{m}$ is mass flow rate of the air and it can be calculated by continuity equation;

$$\dot{m} = \rho\, u_m\, A_c = \rho\, u_m \frac{\pi D^2}{4}$$

$$= (1.473) \times (10)\ \frac{\pi \times (0.025)^2}{4}$$

$$= 7.329 \times 10^{-3}\ \text{kg/s}$$

Using mass-flow rate in the energy balance

$$7.329 \times 10^{-3} \times (1025) \times (\Delta T_b) = 3 \times 101.1$$
$$\Delta T_b = \mathbf{40.37°C}$$

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30.16 FREE CONVECTION

When the fluid is heated, the density gradients are developed, and results into buoyancy force, which induces free convection. Such a situation is referred as *natural or free convection*. The buoyancy effect is developed due to the presence of fluid density gradients and body force (gravitational force).

In free convection, the fluid motions set up by buoyancy forces are much smaller than those associated with forced convection. Therefore, the heat transfer rate in natural convection is also smaller.

There are many situations, where the heat is transferred by free convection to the surrounding air. Heat transfer from a heater to heat a room, heat transfer from pipes, transmission line, condenser coil of a refrigerator, electric transformer, electric motors and electronic equipments are some typical examples.

30.17 EMPIRICAL RELATIONS FOR FREE CONVECTION

The empirical relations are presented in following functional form for variety of applications,

$$Nu_f = \frac{h L_c}{k_f} = C(Gr_f \cdot Pr_f)^m$$

where L_c is significant length depending on the position of flow.

C and m are constants and subscript f indicates that the properties (except for inclined plate) in the dimensionless group are evaluated at mean film temperature,

$$T_f = \frac{T_s + T_\infty}{2}$$

The product of Grashof number and Prandtl number is called Rayleigh number or

$$Ra = Gr\ Pr$$

Example 30.10 *Consider a rectangular plate of size 0.2 m × 0.4 m is maintained at a uniform temperature of 80°C. It is placed in atmospheric air at 24°C. Compare the heat transfer rates from the plate for the cases when the vertical height is (a) 0.2 m, and (b) 0.4 m.*

Solution

Given

$L_1 = 0.2$ m $\quad L_2 = 0.4$ m
$T_s = 80°C \quad T_\infty = 24°C$

To find Comparison of heat transfer rates when the vertical height is (i) 0.2 m, and (ii) 0.4 m

Properties of fluid The mean film temperature;

$$T_f = \frac{T_s + T_\infty}{2} = \frac{80 + 24}{2} = 52°C = 325\,K$$

The properties of air are:

$\nu = 1.822 \times 10^{-5}$ m^2/s, $Pr = 0.703$
$k_{air} = 0.02814$ W/m·K,
$\beta = 1/325 = 3.077 \times 10^{-3}$ K^{-1}

Analysis The Grashof number with appropriate significant length L_c of plate:

The Grashof no.,

$$Gr = \frac{g\beta\Delta T L_c^3}{\nu^2} = \frac{(9.81) \times (3.077 \times 10^{-3}) \times (80 - 24) L_c^3}{(1.822 \times 10^{-5})^2} = 5.092 \times 10^9\, L_c^3$$

The Reyleigh no.,

$$Ra = Gr\ Pr = (5.092 \times 10^9\, L_c^3) \times (0.703) = 3.579 \times 10^9 \times L_c^3$$

(i) When 0.2 m side is vertical:

$L_c = L_1 = 0.2$ m
$Ra_{L_1} = 3.579 \times 10^9 \times (0.2)^3 = 28.637 \times 10^6$

Thus, the flow is laminar, and relation from Table 30.6;

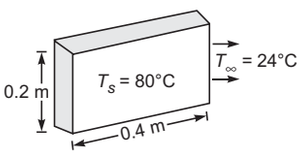
$$Nu_{L_1} = 0.59\,(Ra_{L_1})^{1/4} = 0.59 \times (28.637 \times 10^6)^{1/4} = 43.176$$

The average heat transfer coefficient

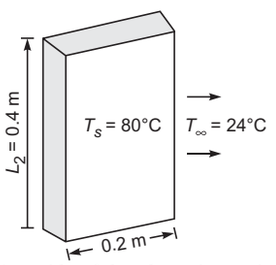
$$h_1 = Nu_{L_1} \frac{k_{air}}{L_1} = 43.176 \times \frac{0.02814}{0.2} = 6.075\ \text{W/m}^2\text{. K}$$

Table 30.6 *The relation for free convection over the bodies for constant wall temperature*

Sr. No.	*Geometry*	*Significant Length L_c*	*Types of Flow*	*Range of Gr Pr*	*Correlation Average, Nu =*
1. (a)	Vertical Planes and Cylinders	Height	Laminar Turbulent	$10^4 \le Ra \le 10^8$ $10^8 \le Ra \le 10^{12}$	$0.59\,(Ra)^{1/4}$ $0.13\,(Ra)^{1/3}$
(b)	Vertical Plates	Height	No restriction Entire range or Ra_L		$\left\{0.825 + \dfrac{0.387\,Ra^{1/6}}{[1+(0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}$
2.	Horizontal Cylinder	Diameter	Laminar Turbulent	$10^4 \le Ra \le 10^8$ $10^8 \le Ra \le 10^{12}$	$0.53\,(Ra)^{1/4}$ $0.13\,(Ra)^{1/3}$
3.	The horizontal Plates				
(i)	Heated surface facing down or Cold surface facing up	A_s/P	Laminar Turbulent	$10^4 \le Ra \le 10^7$ $10^7 \le Ra \le 10^{11}$	$0.54\,Ra^{1/4}$ $0.14\,Ra^{1/3}$
(ii)	Heated surface facing up or Cold surface facing down		Laminar	$10^5 \le Ra \le 10^{10}$	$0.27\,Ra^{1/4}$
4.	Spheres	Diameter	Laminar Turbulent	$1 \le Ra \le 10^5$ $10^5 \le Ra \le 10^8$	$2 + 0.43\,Ra^{1/4}$ $2 + 0.50\,Ra^{1/4}$
5.	Inclined hot surfaces			$10^5 \le Ra \le 10^{11}$ and $\theta < 88°$	$0.56\,(Ra\,.\cos\theta)^{1/4}$
(i)	Facing downward				
(ii)	Facing upward	Height		$10^5 \le Ra \le 10^{11}$ and $-15° > \theta > 75°$	$0.415\,\{Ra^{1/3} - Ra_c^{1/3}\} + 0.56\,(Ra_c \cos\theta)^{1/4}$



(a) 0.2 m side of the plate is vertical.



(b) 0.4 m side of the plates is vertical.

Fig. 30.24

The heat transfer rate:

$$Q_1 = h_1 A_s (T_s - T_\infty)$$
$$= 6.075 \times 0.2 \times 0.4 \times (80 - 24)$$
$$= 27.216 \text{ W}$$

(ii) For the different vertical orientation of the plate of $L_2 = 0.4$ m. The relevant Rayleigh no.

$$Ra_{L_2} = 3.579 \times 10^9 \times (0.4)^3$$
$$= 229.0 \times 10^6$$

The boundary layer is laminar, hence using relation

$$Nu_{L_2} = 0.59\,(Ra_{L_2})^{1/4}$$
$$= 0.59 \times (229.0 \times 10^6)^{1/4} = 72.58$$

and $$h_2 = Nu_{L_2} \frac{k_{air}}{L_2} = 72.58 \times \frac{0.02814}{0.4}$$
$$= 5.10 \text{ W/m}^2\text{. K}$$

The heat transfer rate:

$$Q_2 = h_2 A_s (T_s - T_\infty)$$
$$= 5.10 \times 0.2 \times 0.4 \times (80 - 24)$$
$$= 22.848 \text{ W}$$

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The percentage decrease in heat transfer

$$\frac{(Q_1 - Q_2)}{Q_1} = \frac{(27.216 - 22.848)}{27.216} = \mathbf{0.16}$$

The heat transfer rate is 16% lower when the vertical side is 0.4 m instead of 0.2 m.

30.18 RADIATION HEAT TRANSFER

Thermal radiation refers to the heat energy emitted by the bodies because of their temperatures. All bodies at a temperature above absolute zero temperature emit thermal radiation. For example, the energy emitted by sun travels through space and reaches the earth surface. The energy transfer by radiation does not require any medium between hot and cold surfaces. In fact, the radiation heat transfer is more effective in vacuum.

30.19 THEORIES OF RADIATION

The actual mechanism of radiation propagation is not fully understood till date. Still two theories are in use.

30.19.1 Maxwell's Theory

According to Maxwell's electromagnetic theory, the energy is transferred from a hot body to cold body in the form of the electromagnetic waves. All electromagnetic waves travel with the speed of light. This concept is useful in study for the prediction of the radiation properties of the surfaces and materials.

30.19.2 Max Planck's Theory

According to max Planck's concept, the propagation of thermal radiation takes place in the form of discrete quanta called *photons*, each quantum having an energy of

$$E = h\nu$$

where h is Planck's constant and ν is the frequency of photons. This theory is used to predict the magnitude of emitted energy by a body at a given temperature under ideal conditions.

Both concept are used in study of thermal radiation.

30.20 BLACK-BODY RADIATION

It is an ideal surface having the following properties:

1. A black body absorbs all incident radiation from all directions at all wavelengths.
2. At any temperature and wavelength, no body can emit energy more than a black body.
3. Although the radiation emitted by a black body depends upon wavelength and temperature, it is independent of direction.
4. A black body neither reflects nor transmits any amount of incident radiation.

Consider a radiation beam entering the cavity of an enclosure as shown in Fig. 30.25. It experiences many reflections within the enclosure and almost entire beam is absorbed by the cavity and the black body behaviour is experienced.

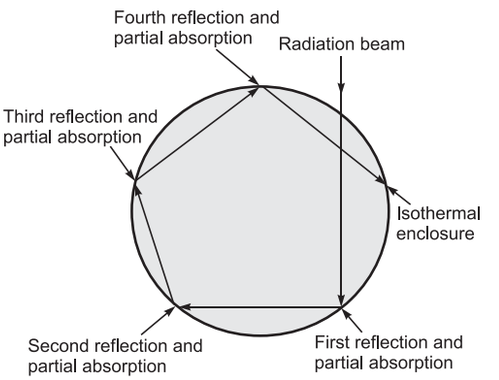

Fig. 30.25 *Characteristic of a black body*

30.20.1 Black Body Spectral Emissive Power

Max Planck's law is based on quantum theory, and he has explained that the radiation energy emitted by a black body into vacuum is related as

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$$E_{b\lambda}(T) = \frac{C_1}{\lambda^5 . \{\exp[C_2/(\lambda T)] - 1\}} \quad ...(30.38)$$

where, C_1 and C_2 are constants, and their magnitude

$C_1 = 3.743 \times 10^8$ W. $\mu m^4/m^2$

$C_2 = 1.4387$ µm.K

$T =$ Absolute temperature, K

$\lambda =$ Wavelength, µm

$E_{b\lambda} =$ Spectral blackbody emissive power, W/m^2 µm

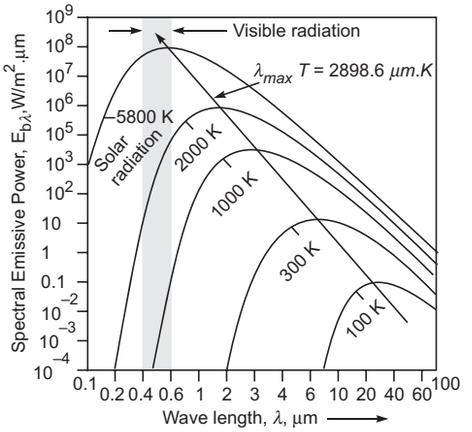

Fig. 30.26 *Spectral emissire power distribution of a black surface at various temperatures according to Plank's law*

When spectral emissive power $E_{b\lambda}(T)$ is plotted against wavelength λ against various temperature of a black body, it is observed that as temperature increases, rate of emission increases, and peak of spectral distribution shifts to shorter wavelength as shown in Fig. 30.26.

30.20.2 Wein's Displacement Law

Wein found that the product of wavelength and absolute temperature corresponds to the locus of all peaks is always constant and his analytical formula is

$$(\lambda T)_{max} = 2897.6 \text{ µm K}$$

This formula is valid over entire spectrum of wavelength for a black body.

30.20.3 Emissive Power of a Black Body

The total or hemispherical emissive power is the amount of radiation energy emitted by a black body at a temperature per unit area per unit time over entire spectrum of wavelength. It is measured in W/m^2.

The total emissive power of a black surface is calculated as

$$E_b = \int_0^\infty E_{b\lambda}\, d\lambda \quad (30.39)$$

30.20.4 Emissivity

It is the ratio of radiation heat flux emitted by a real surface at a temperature T, over all wavelengths into hemispherical space, to that which would have been emitted by a black body at same temperature. Mathematically

$$\varepsilon = \frac{E}{\int_0^\infty E_{b\lambda}\, d\lambda} = \frac{E}{E_b} \quad ...(30.40)$$

For real surfaces, the emissive power

$$E = \varepsilon E_b = \varepsilon \sigma T^4 \quad ...(30.41)$$

30.21 SURFACE ABSORPTION, REFLECTION AND TRANSMISSION

30.21.1 Monochromatic Irradiation

It is also called *spectral irradiation* G_λ(W/m^2. µm) and is defined as the radiant energy incident on a surface per unit wavelength about wavelength λ from all directions and expressed as

$$G_\lambda = \frac{dG}{d\lambda} \quad ...(30.42)$$

The *total irradiation* G(W/m^2) is the total radiation energy incident per unit area per unit time over the entire wavelength from all directions. It may be evaluated as

$$G = \int_0^\infty G_\lambda\, d\lambda \quad ...(30.43)$$

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30.21.2 Monochromatic Properties

In most situations, the spectral irradiation in form of a beam incidents on a body as shown in Fig. 30.27, may be absorbed, reflected and transmitted.

1. *Monochromatic absorptivity* α_λ is the fraction of monochromatic irradiation absorbed.
2. *Monochromatic reflectivity* ρ_λ is the fraction of monochromatic irradiation reflected.
3. *Monochromatic transmissivity* τ_λ is the fraction of monochromatic energy transmitted.

With this consideration $\alpha_\lambda + \rho_\lambda + \tau_\lambda = 1$ ...(30.44)

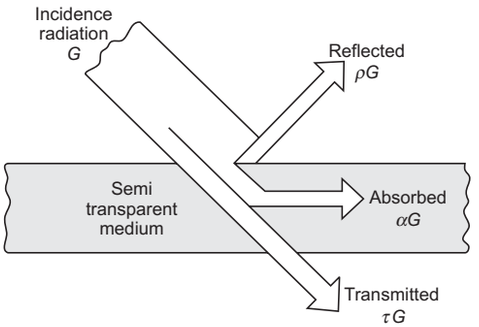

Fig. 30.27 *Reflection, Absorption and transmission of incident radiation*

30.21.3 Absorptivity

A black body absorbs all incident radiation, hence its absorptivity is considered unity. But real surfaces do not absorb all energy incident on it. The *total or average or hemispherical absorptivity* α *is defined as fraction of radiation energy incident on the surface from all directions, over entire wavelength spectrum, that is absorbed by the surface.* Mathematically,

$$\alpha = \frac{Ga}{G} \qquad ...(30.45)$$

where,

Ga = Energy absorbed by the surface, W/m^2

G = Irradiation, W/m^2

30.21.4 Reflectivity

When radiation beam is incident on a non-black surface, a fraction is reflected by the surface. The *reflectivity* ρ *of a surface is defined as the fraction of radiation energy incident on a surface from all directions over all wavelengths, that is reflected*. It is expressed as;

$$\rho = \frac{Gr}{G} \qquad ...(30.46)$$

If the surface is perfectly smooth and the angle of incident and reflected rays is equal, the reflection is called *specular reflection* as shown in Fig. 30.28(a).

If the surface has some roughness, the incident radiation is scattered in all directions after reflection. Such a reflection is called *diffuse reflection* as shown in Fig. 30.28(b).

The reflection from real surfaces is neither specular nor diffuse but combination of diffuse and specular behaviour as shown in Fig. 30.28(c).

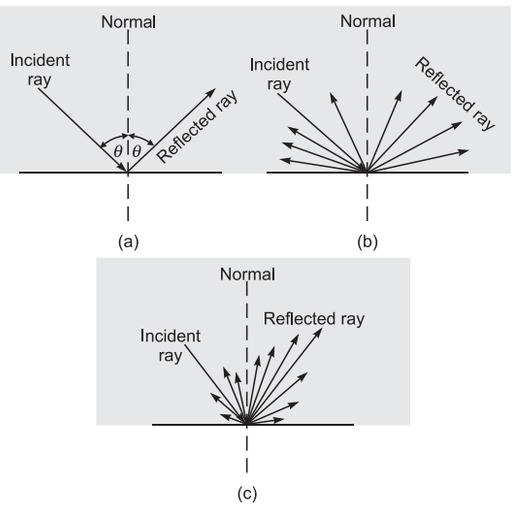

Fig. 30.28 *Different types of reflection from a surface: (a) Specular or Mirror like, (b) diffuse, (c) actual or irregular*

30.21.5 Transmissivity

When a radiation beam is incident on a semi-transparent surface, a part is reflected, a part is

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absorbed and the remaining is transmitted. Hence, *transmissivity is the fraction of incident energy transmitted through the surface*. Mathematically,

$$\tau = \frac{G\tau}{G} \qquad ...(30.47)$$

and for average properties:

$$\alpha + \rho + \tau = 1 \qquad ...(30.48)$$

White Body A body is called white body when it reflects almost all radiation incident upon it. For a white body,

$$\rho \equiv 1$$

Opaque Body For an opaque surface, there is no transmission; thus the reflectivity and absorptivity are

$$\alpha + \rho = 1 \qquad ...(30.49a)$$

and $$\alpha_\lambda + \rho_\lambda = 1 \qquad ...(30.49b)$$

Gray Surface The radiation properties of a real surface such as absorptivity, emissivity, etc., depend on the wavelength of radiation. The calculation of radiation heat transfer for all wavelength becomes very tedious. To overcome such difficulties, an uniform emissivity is assumed over the entire wavelength spectrum. Thus, a *gray surface* is defined as a surface for which the monochromatic emissivity ε_λ is independent of the wavelength.

30.21.6 Radiosity

It is defined as total radiation energy, which leaves a surface due to emission and reflections per unit time per unit area. It is denoted by J and measured in W/m^2

$$J = \varepsilon E_b + \rho G \qquad ...(30.50)$$

J = Radiation emitted by the surface per unit area + Radiation reflected by the surface per unit area.

30.22 KIRCHHOFF'S LAW

It states that at thermal equilibrium, the ratio of the spectral emissive power to spectral absorptivity for all bodies is constant or

$$\frac{E_{\lambda 1}}{\alpha_{\lambda 1}} = \frac{E_{\lambda 2}}{\alpha_{\lambda 2}} = \frac{E_{\lambda 3}}{\alpha_{\lambda 3}} = \frac{E_{\lambda b}}{\alpha_{\lambda b}} = C \qquad ...(30.51)$$

Since $\alpha_{\lambda b} = 1$

$$\frac{E_{\lambda 1}}{E_{\lambda b}} = \alpha_{\lambda 1}$$

or $$E_{\lambda 1} = \alpha_{\lambda 1} \qquad ...(30.52)$$

Similarly, for other bodies, it can be shown that at thermal equilibrium, the energy emitted by a surface must be equal to energy absorbed by the surface. Hence, spectral emissivity is equal to spectral absorptivity at thermal equilibrium. This law is applicable when the radiation properties are independent of wavelength (for graybodies) or when incident and emitted radiation have the spectral distribution.

Example 30.11 *A black surface is positioned in a vacuum container so that it absorbs incident solar radiant energy at the rate of 950 W/m^2. If the surface conducts no heat to its surroundings, determine its equilibrium temperature.*

Solution

Given $q = 950\ W/m^2$

To find To calculate the equilibrium temperature.

Assumptions

(i) Stefan Boltzmann constant $\sigma = 5.67 \times 10^{-8}$ $W/m^2.K^4$.

(ii) Not heat loss by conduction and convection.

Analysis The radiant heat flux for a black surface can be expressed as

$$q = \sigma \cdot T^4$$

Hence, $T = (q/\sigma)^{1/4} = [950/5.67 \times 10^{-8})]^{1/4} = 360\ K$

The equilibrium temperature of black surfaces will be, $T =$ **87°C**

Example 30.12 *A black body at 30°C is heated to 100°C. Calculated the increase in its emissive power.*

Solution

Given A black body emission

$T_1 = 30 + 273 = 303\ K$

$T_2 = 100 + 273 = 373\ K$

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To find The increase in emissive power.

Analysis The radiant heat flux or emissive power for a black surface can be expressed as

$$E_b = \sigma T^4$$

Where, σ = Stefan Boltzmann's constant, as 5.67×10^{-8} W/m$^2 \cdot$K^4.

Hence the increase in emissive power can be calculated as

$$E_{b2} - E_{b1} = \sigma (T_2^4 - T_1^4)$$

or $$E_{b2} - E_{b1} = (5.67 \times 10^{-8} \text{ W/m}^2\text{. K}^4) \{(373 \text{ K})^4 - (303 \text{ K})^4\}$$

$$= \mathbf{619.62 \text{ W/m}^2}$$

Example 30.13 *The surface temperature of a central heating radiator is 60°C. What is the net black body radiation heat transfer between radiator and surroundings at 20°C?*

Solution

Given Central heating radiator with

$T_s = 60°\text{C} = 333$ K

$T_\infty = 20°\text{C} = 293$ K

To find Radiation heat transfer.

Assumptions

(i) The Stefan Boltzmann constant

$\sigma = 5.67 \times 10^{-8}$ W/m^2.K^4

(ii) 1 m^2 surface area of radiator

Analysis The black-body radiation heat transfer per unit area is expressed as

$$q = \frac{Q}{A} = \sigma (T_s^4 - T_\infty^4)$$

$$= 5.67 \times 10^{-8} \times (333^4 - 293^4)$$

$$= 5.67 \times 10^{-8} \times 4.9263 \times 10^9$$

$$= \mathbf{279.32 \text{ W/m}^2}$$

Example 30.14 *A black body at 600 K emits radiation at a wavelength of 4 μm. Calculate its spectral emissive power.*

Solution

Given Black-body radiation

$T = 600$ K $\qquad \lambda = 4$ μm

To find Spectral emissive power.

Analysis According to Planck's law, Eq. (30.38)

$$E_{b\lambda}(T) = \frac{C_1}{\lambda^5 \left\{ \exp\left(\frac{C_2}{\lambda T}\right) - 1 \right\}}$$

Where, $C_1 = 3.743 \times 10^8$ W μm^4/m^2,

$C_2 = 1.4387 \times 10^4$ μm.K

Using numerical values,

$$E_{b\lambda}(T) = \frac{3.743 \times 10^8}{(4)^5 \left\{ \exp\left(\frac{1.4387 \times 10^4}{4 \times 600}\right) - 1 \right\}}$$

$$= \mathbf{913.25 \text{ W/(m}^2\text{. μm)}}$$

Example 30.15 *A black body emits energy at 2000 K. Determine the wavelength, at which the black-body spectral emissive power would be maximum.*

Solution

Given A black-body emission

$T = 2000$ K

To find Wavelength corresponds to maximum $E_{b\lambda}$.

Analysis Using Wien's displacement law, Eq. (30.49)

$$(\lambda T)_{max} = 2897.6 \text{ μm.K}$$

or $$\lambda = \frac{2897.6}{2000} = \mathbf{1.448 \text{ μm}}$$

Example 30.16 *A large spherical enclosure (black body) maintains its inner surface at 1000 K. It has a hole of 0.4 cm diameter. Calculate the rate of emission of radiation energy through this opening.*

Solution

Given Radiation through an opening of large spherical enclosure.

$T = 1000$ K $\qquad D = 0.4$ cm

To find Rate of emission of radiation energy through hole

Analysis The black-body emissive power

$$E_b(T) = \sigma T^4$$

$$= (5.67 \times 10^{-8}) \times (1000)^4 = 56700 \text{ W/m}^2$$

Hole area $$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times (0.4)^2 = 0.125 \text{ cm}^2$$

$$= 0.125 \times 10^{-4} \text{ m}^2$$

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Rate of emission of radiation energy through hole opening

$$Q = A\,E_b(T)$$
$$= 0.125 \times 10^{-4} \times 56700 = \mathbf{0.7125\ W}$$

Example 30.17 *An uninsulated steam pipe is passed through a room in which air and walls are at 25°C. The outer diameter of the pipe is 50 mm and surface temperature and emissivity are 500 K and 0.8 respectively. If the free convection heat-transfer coefficient is 15 W/m^2.K. What is the rate of heat loss from the surface per unit length of pipe?*

Solution

Given Flow of air over a hot cylinder

$T_\infty = T_w = 25°C = 298$ K
Surface temp. $T_\infty = 500$ K
Dia. of pipe $D = 50$ mm $= 0.05$ m
Emissivity of pipe surface $\varepsilon = 0.8$
Convection coefficient $h = 15$ W/m^2.K

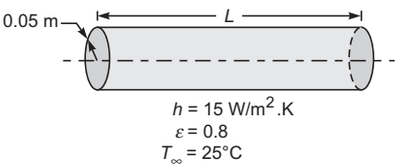

Fig. 30.29

To find Heat loss per unit length of pipe (Q/L).

Assumptions

(i) Steady state conditions.
(ii) Heat loss by radiation and convection heat transfer.
(iii) Constant properties.
(iv) Stefan Boltzmann constant, $\sigma = 5.67 \times 10^{-8}$ W/m$^2 \cdot$K^4.

Analysis

(i) Heat loss from the pipe by convection is given by

$$Q_{conv} = h\,A_s(T_s - T_\infty) = h\,(\pi DL)\,(T_s - T_\infty)$$

or $$\frac{Q_{conv}}{L} = 15 \times \pi \times 0.05 \times (500 - 298)$$
$$= 476 \text{ W/m}$$

(ii) Heat loss per unit length of pipe by radiation is given by

$$\frac{Q_{rad}}{L} = \sigma\,\varepsilon\,(\pi D)\,(T_s^4 - T_\infty^4)$$
$$= 5.67 \times 10^{-8} \times 0.8 \times (\pi \times 0.05)\,(500^4 - 298^4)$$
$$= 389.13 \text{ W/m}$$

Total heat loss from pipe surface per unit length,

$$\left(\frac{Q}{L}\right) = \frac{Q_{conv}}{L} + \frac{Q_{rad}}{L} = 476 + 389.13$$
$$= \mathbf{865.13\ W/m}$$

Table 30.7 *Glossary of radiative terms*

Terms	*Definition*
Absorption	The process of converting the radiation intercepted by the matter to internal thermal energy
Absorptivity	Fraction of the incident radiation absorbed by the matter
Black body	Ideal body which absorbs all incident radiation and emits maximum energy
Emission	The process of radiation production by the matter at a finite temperature
Emissive power	The rate of radiant energy emitted by a surface in all direction per unit area of the surface, E (W/m^2)
Emissivity	The ratio of the emissive power of a surface to the emissive power of the black body at the same temperature
Gray surface	A surface for which the spectral absorptivity and emissivity are independent of the wavelength over the spectral region of the surface irradiation and emission
Kirchhoff's Law	The emission is equal to absorption for all bodies at thermal equilibrium
Planck's law	It is associated with spectral distribution of emission from a black body
Radiation shield	It is made of low emissivity material and is used to reduce the net radiation transfer between the two surfaces

Contd.

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Table 30.7 *Contd.*

Reflection	The process of reflection of the radiation energy incident on a surface
Reflectivity	The fraction of incident radiation energy reflected by the matter
Semi-transparent	It is a medium in which radiation absorption is the volumetric process
Spectral	It refers to a single wavelength (monochromatic) radiation. The quantity is denoted by subscript λ
Specular	It refers to the surface for which the angle of reflected radiation is equal to the angle of incident radiation
Stefan–Boltzmann law	The emissive power of the black body is directly proportional to fourth power of the absolute temperature
Thermal radiation	It is the electromagnetic energy emitted by a matter at a finite temperature in the spectral region approximately from 0.1 to 100 μm
Transmission	It is precess of the thermal radiation passing through the matter
Transmissivity	It is the fraction of radiation energy transmitted by the matter

30.23 HEAT EXCHANGERS

The device used for the heat exchange between the two fluids that are at different temperatures is called *a heat exchanger*. For example, a car radiator, a refrigerator, steam condenser, air cooler, water cooler, etc. The heat exchangers are also used in space heating and air-conditioning, waste heat recovery and chemical precessing. Therefore, different types of heat exchangers are needed for different applications.

30.23.1 Classification of Heat Exchangers

Heat exchangers are designed in many sizes, types, configurations and flow arrangements and used for many purposes. These are classified according to heat-transfer process, flow arrangement and type of construction.

(A) According to Heat Transfer Process

(*i*) *Direct-contact Type* In this type of heat exchanger, the two immiscible fluids at different temperatures come in direct contact. For heat exchange between two fluids, one fluid is sprayed through the other. Cooling towers, jet condensers, and scrubbers are the best examples.

(*ii*) *Direct Transfer-type Heat Exchanger* In this type of heat exchanger, the cold and hot fluids flow simultaneously through the device and the heat is transferred through the wall separating them. These types of heat exchangers are most commonly used in almost all fields of engineering. These are further classified as

(a) *Regenerators* These are also called *storage type heat exchangers*, in which hot and cold fluid flow alternatively on the same surface. The hot fluid gives heat to the surface and cold fluid extracts heat from it. In many applications a rotating disc-type matrix is used, a continuous flow of both the hot and cold fluids are maintained. These are preheaters for steam power plants, blast furnaces, oxygen producers, etc.

(b) *Recuperators* These are also called transfer-type heat exchangers. In these heat exchangers, the hot and cold fluids are separated by a plane wall or tube surface, hence heat is indirectly transferred from hot fluid to cold fluid by convection and conduction.

(B) According to Constructional Features

(*i*) *Tubular Heat Exchanger* These are also called *tube-in-tube* or *concentric tube or double-pipe heat exchanger*. These are widely used in many sizes and different flow arrangements and types.

(*ii*) *Shell and Tube-type Heat Exchanger* These are also called surface condensers and are most commonly used for heating, cooling, condensation or evaporation applications. It consists of a shell

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and number of tubes housed in it. These have large surface area in small volume. A typical shell and tube-type heat exchanger in shown in Fig. 30.30.

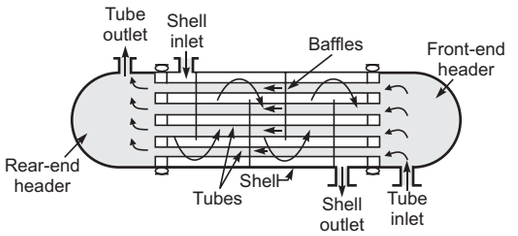


Fig. 30.30 *The schematic of a shell-and-tube heat exchanger (one-shell pass and one-tube pass)*

(*iii*) *Finned Tube Type* When a high-operating pressure or an enchanced heat-transfer rate is required, the extended surfaces are used on one side of the heat exchanger. These heat exchangers are used for liquid-to-gas heat exchange. Fins are always used on gas side. The tube fins are used in gas turbines, automobiles, aeroplanes, heat pumps, refrigeration, electronics, cryogenics, air-conditioning, systems, etc.

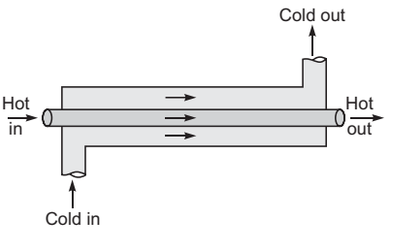
Compact Heat Exchanger These are a special class of heat exchanger in which the heat transfer area per unit volume is greater than 700 m^2/m^3. These heat exchangers have dense arrays of finned tubes or plates, when at least one of the fluid used is gas. For example, automobile radiators have an area density in order of 1100 m^2/m^3.

(*C*) *According to Flow Arrangement*

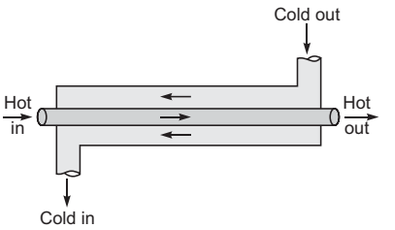
(*i*) *Parallel Flow* The hot and cold fluids enter at same end of the heat exchanger, flow through in same direction and leave at the same end. Fig. 30.31(a).

(*ii*) *Counter Flow* The hot and cold fluids enter at the opposite ends of heat exchanger, flow through in opposite direction and leave at opposite ends. Fig. 30.31(b).

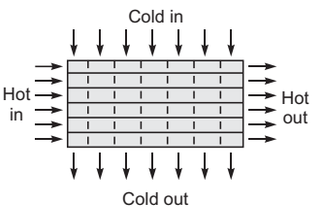
(*iii*) *Cross Flow* The two fluids flow at right angles to each other. In the cross flow heat exchanger



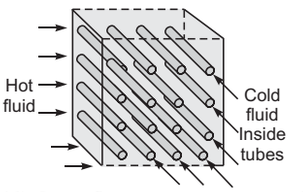
(a) Parallel flow arrangement



(b) Counter flow arrangement



(c) Cross flow arrangement with both fluid unmixed



(d) Cross flow arrangement one fluid mixed and one fluid unmixed

Fig. 30.31 *Heat exchanger with different flow arrangements*

arrangement, the fluid flow may be mixed or unmixed. If both the fluids flow through individual channels and are not free to move in transverse direction, the arrangement is called unmixed, Fig. 30.31(c). If any fluid flows on the surface and is free to move in the transverse direction, then this fluid stream is said to be mixed.

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30.24 HEAT EXCHANGER ANALYSIS

Use of Log Mean Temperature Difference In the thermal analysis of a heat exchanger, the total heat transfer rate between the hot and cold fluids can be calculated by using inlet and outlet fluids temperature, overall heat transfer coefficients and surface area as

$$\dot{Q} = UA(\Delta T)_{lm} \qquad ...(30.53)$$

where, U = Overall heat transfer coefficient

A = Surface area for heat exchange

$(\Delta T)_{lm}$ = Appropriate mean value of temperature difference

$$= \frac{\Delta T_1 - \Delta T_2}{\ln\left[\frac{\Delta T_1}{\Delta T_2}\right]} = \frac{\Delta T_2 - \Delta T_1}{\ln\left[\frac{\Delta T_2}{\Delta T_1}\right]} \qquad ...(30.54)$$

The $(\Delta T)_{lm}$ is called log mean temperature difference.

For Fig. 30.32,

$\Delta T_1 = T_{hi} - T_{ci}$ (Temperature difference at the inlet ($x = 0$))

$\Delta T_2 = T_{ho} - T_{co}$ (Temperature difference at the outlet ($x = L$))

For Fig. 30.33

$\Delta T_1 = T_{hi} - T_{co}$ Temperature difference at $x = 0$.

$\Delta T_2 = T_{ho} - T_{ci}$ Temperature difference at $x = L$.

While; $\dot{Q} = UA(\Delta T)_{lm}$

$= U_o A_o (\Delta T)_{lm}$

$= U_i A_i (\Delta T)_{lm}$ ...(30.55)

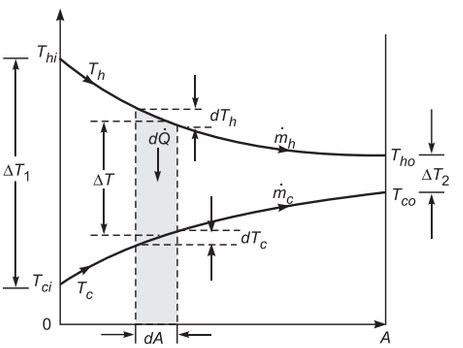

Fig. 30.32 *Temperature distribution for a parallel flow heat exchanger*

For Fig. 30.33,

$\Delta T_1 = T_{hi} - T_{co}$ $\qquad$ $\Delta T_2 = T_{ho} - T_{ci}$

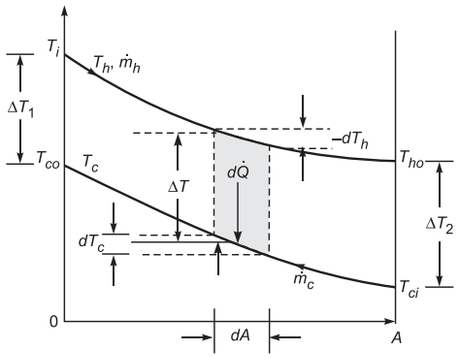

Fig. 30.33 *Temperature distribution for a counter flow heat exchanger*

30.24.1 Heat Exchanger Effectiveness

It is defined as the ratio of actual heat transfer rate $\dot{Q}_{act}$ by a heat exchanger to maximum possible heat transfer rate $\dot{Q}_{max}$. It is denoted by ε and expressed as

$$\varepsilon = \frac{\dot{Q}_{act}}{\dot{Q}_{max}} = \frac{\dot{m}_c C_{pc}(T_{co} \quad T_{ci})}{\dot{Q}_{max}}$$

$$= \frac{\dot{m}_h \cdot C_{ph}(T_{hi} \quad T_{ho})}{\dot{Q}_{max}} \qquad ...(30.56)$$

In any heat exchanger, the objective is either maximization of heating or cooling rate, i.e., to gain the maximum temperature difference and hence, the maximum heat transfer rate $\dot{Q}_{max}$ can be achieved in counter flow heat exchanger of infinite length. In such a heat exchanger, one fluid having minimum value of heat capacity $(\dot{m}C_p)$ can experience maximum possible temperature difference $(T_{hi} - T_{ci})$, because the energy balance requires that the heat given by one fluid should be equal to heat gain by other.

Therefore,

$$\dot{Q}_{max} = (\dot{m}C_p)_{min}(\Delta T)_{max}$$

$$= (\dot{m}C_p)_{min}(T_{hi} - T_{ci}) \qquad ...(30.57)$$

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30.24.2 NTU

It is number of transfer units and is dimensionless parameter, which is expressed as

$$NTU = \frac{UA}{C_{min}} = \frac{UA}{(\dot{m}C_p)_{min}}$$

$$= \frac{\text{Heat capacity of exchanger W/K}}{\text{Heat capacity of flow W/K}} \qquad ...(30.58)$$

where U and C_{min} are constant quantities for given flow conditions, hence

$$NTU \propto A$$

Therefore, NTU is the measure of the physical size (heat transfer area) of the heat exchanger. Higher the value of NTU, larger the physical size.

For any heat exchanger, the effectiveness is the function of NTU and heat capacity ratio.

$$\varepsilon = f\left(NTU, \frac{C_{min}}{C_{max}}\right) \qquad ...(30.59)$$

where $C_{max} = (\dot{m}C_p)_{max}$ among the two fluids

Example 30.18 *A thin-walled concentric tube heat exchanger is used to cool engine oil from 160°C to 60°C and water, which is available at 25°C acts as a coolant. The oil and water flow rates are each 2 kg/s each and the diameter of the inner tube is 0.5 m and the corresponding value of overall heat transfer coefficient is 250 W/m^2 K. How long must the heat exchanger be to accomplish the desired cooling?*

Take, C_p of water = 4.187 kJ/kg K

C_p of engine oil = 2.035 kJ/kg K.

Solution

Given A concentric tube heat exchanger with

T_{hi} = 160°C T_{ho} = 60°C

T_{ci} = 25°C C_{pc} = 4.187 kJ/kg K

C_{ph} = 2.035 kJ/kg K U = 250 W/m^2 K

$\dot{m}_h = \dot{m}_c$ = 2 kg/s D = 0.5 m

To find Length of the heat exchanger.

Assumption

(i) No heat loss to the surroundings.

(ii) No scaling on heat transfer surface.

Analysis The outlet temperature of water can be obtained by energy balance.

$$\dot{m}_h C_{ph}(T_{hi} - T_{ho}) = \dot{m}_c C_{pc}(T_{co} - T_{ci})$$

$$2 \times 2.035\,(160 - 60) = 2 \times 4.187\,(T_{co} - 25)$$

or $$T_{co} = \frac{2.035 \times 100}{4.187} + 25 = 73.6°C$$

Since $T_{co} > T_{ho}$, hence the heat exchanger is in *counterflow arrangement* and its *LMTD*,

$$(\Delta T)_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln\left[\frac{\Delta T_1}{\Delta T_2}\right]}$$

where, $\Delta T_1 = T_{ho} - T_{ci} = 60 - 25 = 35°C$

$\Delta T_2 = T_{hi} - T_{co} = 160 - 73.6 = 86.4°C$

and $$(\Delta T)_{lm} = \frac{35 - 86.4}{\ln\left[\frac{35}{86.4}\right]} = 56.88°C$$

The heat transfer rate,

$$\dot{Q} = \dot{m}_h C_{ph}(T_{hi} - T_{ho})$$

$$= UA(\Delta T)_{lm}$$

and $$L = \frac{\dot{m}_h C_{ph}(T_{hi} - T_{ho})}{U(\pi D)(\Delta T)_{lm}}$$

$$= \frac{(2\text{ kg/s})(2.035 \times 10^3\text{ J/kg K})(160 - 60)(\text{ C})}{(250\text{ W/m}^2\text{ K})(\pi \times 0.5\text{ m})(56.88°C)}$$

$$= \mathbf{18.22\ m}$$

Example 30.19 *Water at 225 kg/h is to be heated from 35°C to 95°C by means of concentric tube heat exchanger. Oil at 225 kg/h and 210°C with a specific heat of 2095 J/kg K is to be used as hot fluid. If the overall heat-transfer coefficient based on the outer diameter of inner tube is 550 W/m^2.K, determine the length of the heat exchanger, if the outer diameter is 100 mm.*

Solution

Given A concentric tube heat exchanger

T_{hi} = 210°C $\dot{m}_c$ = 225 kg/h

T_{ci} = 35°C T_{co} = 95°C

$\dot{m}_h$ = 225 kg/h C_{pc} = 4.18 kJ/kg K

C_{ph} = 2095 J/kg K U = 550 W/m^2 K

D = 100 mm = 0.1 m

To find Length of the heat exchanger.

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Assumptions

(i) No heat loss to the surroundings.
(ii) No scaling on heat transfer surfaces.
(iii) The specific heat of the water as 4180 J/kg K.

Analysis The heat transfer rate through the heat exchanger can be calculated as

$$\dot{Q} = \dot{m}_c C_{pc}(T_{co} - T_{ci})$$
$$= (225/3600)\ (\text{kg/s}) \times (4180\ \text{J/kg K}) \times (95 - 35)\ (\text{K})$$
$$= 15675\ \text{W}$$

The outlet temperature of oil,

$$\dot{Q} = \dot{m}_h C_{ph}(T_{hi} - T_{ho})$$

or
$$T_{ho} = T_{hi} - \frac{\dot{Q}}{\dot{m}_c C_{pc}} = 210 - \frac{15675}{(225/3600)\,(2095)}$$
$$= 90.28°\text{C}$$

The outlet temperature of hot fluid is less than cold fluid outlet temperature, hence used heat exchanger is in counterflow arrangement.

$$(\Delta T)_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

where, $\Delta T_1 = \Delta T_{ho} - \Delta T_{ci} = 90.28 - 35 = 55.28°\text{C}$
$\Delta T_2 = \Delta T_{hi} - \Delta T_{co} = 210 - 95 = 115°\text{C}$

and
$$(\Delta T)_{lm} = \frac{55.28 - 115}{\ln\left[\frac{55.28}{115}\right]} = 81.52°\text{C}$$

Then $\dot{Q} = UA\,(\Delta T)_{lm} = U(\pi\, dL)\,(\Delta T)_{lm}$

or
$$L = \frac{\dot{Q}}{U\,(\pi D)\,(\Delta T)_{lm}}$$
$$= \frac{15675\,\text{W}}{(550\,\text{W/m}^2\,\text{K})\,\pi\,(0.1\,\text{m})\,(8.1.52\ \text{C})}$$
$$= \mathbf{1.11\ m}$$

Summary

- Heat transfer is the branch of thermal science which deals with analysis of rate and nature of heat transfer as well as temperature distribution in the system.
- When temperature gradient exists in a material medium then heat is transferred by conduction. Heat convection takes place between heated surface and adjacent moving fluid. The thermal radiation takes place in the form of electromagnetic waves between radiating bodies.
- Fourier law of heat conduction is given by
$$\frac{\dot{Q}}{A} = -k\frac{dT}{dx}$$
where k is thermal conductivity, a property of material, measured in W/m · K.
- Newton's law of cooling is the governing equation for heat convection and it is given by
$$\dot{Q} = hA_s(T_s - T_\infty)$$
where h is the heat transfer coefficient a property of ambient conditions.
- The electrical analogy for conduction heat transfer demonstrates
$$\dot{Q} = \frac{(\Delta T)_{overall}}{\Sigma R_{th}}$$
- The overall heat-transfer coefficient is given by
$$U = \frac{1}{A\Sigma R_{th}}$$
- A black body is an imaginary body which absorbs all incident radiation and emits maximum energy at a given temperature. The Stefan–Boltzmann law is the fundamental law of radiation
$$\dot{Q} = \sigma A(T_1^4 - T_2^4)$$
where σ = Stefan–Boltzmann constant, W/m^2.K^4
- A heat exchanger is a device which exchanges heat energy between two fluids at two different temperatures. The log mean temperature difference is given by
$$(\Delta T)_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

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where ΔT_1 = Temperature difference at left end of heat exchanger, and

ΔT_2 = Temperature difference at right end of heat exchanger.

- The effectiveness of a heat exchanger is given as

$$\varepsilon = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}}$$

- The number of transfer units (NTU) is given by

$$\text{NTU} = \frac{\text{Heat capacity rate of heat exchanger}}{\text{Heat capacity rate of fluid}} = \frac{UA}{(\dot{m}C_p)_{min}}$$

Glossary

Heat flux Heat transfer rate per unit area

Conduction Heat transfer due to existance of temperature gradient in material medium

Free convection Heat transfer due to density difference induced by temperature difference in fluids

Forced convection Heat transfer due to velocity difference induced artificially

Radiation Heat transfer due to electromagnetic waves from surfaces

Thermal conductivity Ability of material to conduct the heat

Thermal potential Temperature difference; responsible for heat transfer

Thermal resistance Opposes the heat flow through the material medium

Heat transfer coefficient Property of ambient conditions

Black body An imaginary ideal body for radiation

Emisive power Radiation heat transfer per unit area

Emissivity Property of a radiating surface

LMTD Log mean temperature difference

NTU Number of transfer unit

Effectiveness Ratio of actual heat transfer rate to maximum possible heat transfer rate

Problems

1. Determine the heat flow across a plane wall of 10 cm thickness with a thermal conductivity of 8.5 W/m.K, when the surface temperatures are steady and at 200°C and 50°C. The wall area is 2 m^2. Also find the temperature gradient in flow direction. *[25500 W, 1500°C/m]*

2. Determine the heat transfer rate by convection over a surface of 1 m^2 if the surface at 100°C is exposed to a fluid at 40°C with convection coefficient of 25 $W/m^2.K$. *[1500 W]*

3. A surface at 200°C is exposed to surroundings at 60°C and convects and radiates heat to the surroundings. Calculate the heat transfer rate from surface to surroundings, if the convection coefficient is 80 $W/m^2.K$. Consider the black bodies for radiation heat transfer. Take $\sigma = 5.67 \times 10^{-8}$ $W/m^2\ K^4$. *[14.24 kW/m^2]*

4. Consider a furnace wall [k = 1 W/(m°C)] with the inside surface at 1000°C and the outside surface at 400°C. If the heat flow through the wall should not exceed 2000 W/m^2, what is the minimum wall thickness L? *[30 cm]*

5. A metal pipe of 10-cm *OD* is covered with a 2-cm thick insulation [k = 0.07 W/(m°C)]. The heat loss from the pipe is 100 W per meter of length when the pipe surface is at 100°C. What is the temperature of the outer surface of the insulation? *[23.5°C]*

6. A 6-cm-OD, 2-cm-thick copper hollow sphere [k = 386 W/(m.°C)] is uniformly heated at the inner surface at a rate of 150 W/m^2. The outer surface is cooled with air at 20°C with a heat-transfer coefficient of 10 $W/(m^2.°C)$. Calculate the temperature of the outer surface. *[21.7°C]*

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7. The wall of a building consists of 10 cm of brick [k = 0.69 W/(m.°C)], 1.25 cm of Celotex [k = 0.048 W/(m.°C)], 8 cm of glass wool [k = 0.038 W/(m.°C)], and 1.25 cm of asbestos cement board [k = 0.74 W/(m.°C)]. If the outside surface of the brick is at 5°C and the inside surface of the cement board is at 20°C, calculate the heat flow rate per square meter of wall surface.

[–5.94 W/m^2]

8. Consider a brass tube [k = 115 W/(m.°C)], with an outside radius of 4 cm and a thickness of 0.5. The inside surface of the tube is kept at uniform temperature, and the outside surface is covered with two layers of insulation each 1 cm thick, with thermal conductivities of 0.1 W/(m.°C) and 0.05 W/(m.°C) respectively. Calculate the overall heat-transfer coefficient based on the outside surface area of the outer insulation.

[2.83 W/m^2.°C]

9. A double glazed window is made of 2 glass panes of 6 mm thickness each with an air gap of 6 mm between them. Assuming that the layer is stagnant and only conduction is involved, determine the thermal resistance and the overall heat-transfer coefficient. The inside is exposed to convection with h = 1.5 W/m^2.K. Compare the values with that of a single glass of 12 mm thickness. The conductivity of the glass = 1.4 W/m.K and that for air is 0.025 W/m.K.

10. Atmospheric air at 27°C flows along a flat plate with a velocity of 8 m/s. The critical Reynold number at which transition from laminar to turbulent takes place is 5×10^5. Determine the distance from the leading edge of the plate at which the transition occurs.

11. Air at 24°C flows along a 4-m long flat plate with a velocity of 5 m/s. The plate is maintained at 130°C. Calculate the heat transfer coefficient over the entire length of the plate and the heat-transfer rate per metre width of the plate.

[9.73 W/m^2.K, 4120 W/m]

12. Air flows along a thin plate with a velocity of 2.5 m/s. The plate is 1 m long and 1 m wide. Estimate the boundary layer thickness at the trailing edge of the plate and the force necessary to hold the plate in the stream of air. The air has a viscosity of 0.86×10^{-5} kg/m.s and a density of 1.12 kg/m^3.

[8.1 mm, 0.0158 N]

13. Air flows along a thin flat plate 1 m wide and 1.5 m long, at a velocity of 1 m/s. The free stream temperature is 4°C. Calculate the amount of heat that must be supplied to plate in order to maintain it at a uniform temperature of 50°C.

[441.5 W]

14. Water at mean temperature of 60°C flows inside a 2.5 cm *ID*, 10 m long tube with a velocity of 6 m/s. The tube wall is maintained at a uniform temperature of 100°C by condensing steam. Determine the heat transfer rate to water. Assume an inlet temperature of 30°C. *[670 W]*

15. Estimate the coefficient of free convection for a wire, 2 mm in diameter immersed in water at 20°C, if the wire surface is maintained at 300°C.

[3366 W/m^2.K]

16. A flat electrical heater of 0.5 m × 0.5 m is placed vertically in still air at 20°C. The heat generated is 1200 W/m^2. Determine the value of natural convection coefficient and average temperature of the plate. *[33.02 W/m^2.K, 56.5°C]*

17. A vertical pipe of 10 cm diameter and 3 m length at a surface temperature of 100°C, is in a room where the air is at 20°C. What is the rate of heat loss per unit length of the pipe?

[119.7 W/m]

18. The heat-transfer rate per unit length due to free convection from a horizontal tube is 200 W/m, when its surface is maintained at 70°C in the ambient air at 20°C. Estimate the heat-transfer rate per unit length, when the tube surface is maintained at 145°C. Neglect the heat-transfer rate by radiation and any influence of temperature on thermophysical properties of air.

[625 W/m]

19. Calculate the heat flux emitted due to thermal radiation from a black surface at 6000°C. At what wavelength is the monochromatic emissive power maximum and what is the maximum value?

[87,798 kW/m^2; 0.462 μm; 1.25 × 10^{14} W/m^2]

20. Estimate the rate at which the sun emits the radiant energy. What fraction of this energy is

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absorbed by the earth and in what amount? If effective temperature of the sun is 5700 K and surface of the sun is treated black. The diameter of the sun is 1.39×10^6 km. The diameter of the earth is 1.27×10^4 km and the distance between sun and earth is 1.5×10^8 km.

[3.81×10^{26} W; 4.48×10^{-10}; 1.71×10^{17} W]

21. A double pipe heat exchanger is constructed of 0.287 cm thick steel tubing with 2.09 cm inner tube and 2.66 cm outer tube. The inside and the outside coefficients of heat transfer are 1135 W/m^2.K and 5677 W/m^2.K, respectively, and the fouling factor is 9.98×10^5 m^2.K/W. Calculate the overall coefficient of heat transfer.

[$U = 851.5$ W/m^2.K]

22. Water at the rate of 4080 kg/h is heated from 35°C to 75°C by an oil having a specific heat of 1900 J/kg/K. The exchanger is of a counterflow double-pipe design. The oil enters at 110°C and leaves at 75°C. Determine the area of the heat exchanger necessary to handle this load if the overall heat transfer coefficient is 320 W/m^2 K.

[$A = 15.82$ m^2]

Objective Questions

1. Heat transfer takes place from a high-temperature body to a low-temperature body according to
   (a) zeroth law of thermodynamics
   (b) first law of thermodynamics
   (c) second law of thermodynamics
   (d) third law of thermodynamics
2. Conduction heat transfer takes place in the medium due to
   (a) temperature difference
   (b) temperature gradient
   (c) thermal conductivity
   (d) surface area
3. Which one of the following represents Fourier equation?
   (a) $\dot{Q} = \frac{k}{A}\left(\frac{dT}{dx}\right)$ (b) $\dot{Q} = -\frac{k}{A}\left(\frac{dT}{dx}\right)$
   (c) $\dot{Q} = -kA\left(\frac{dT}{dx}\right)$ (d) $\dot{Q} = kA\left(\frac{dT}{dx}\right)$
4. Thermal conductivity of a medium may be defined as
   (a) amount of heat flow through 1 m^2 area for temperature gradient of 1 K
   (b) rate of heat transfer through 1 m^2 area for temperature gradient of 1°C
   (c) amount of heat flow through 1 m^2 area for temperature gradient of 1°C/m
   (d) rate of heat flow through 1 m^2 area for temperature gradient of 1°C/m
5. Heat is conducted through a pure metal due to
   (a) vibration of lattice structure
   (b) flow of free electrons
   (c) density difference
   (d) none of the above
6. Newton's law of cooling is expressed as
   (a) $\dot{Q} = -kA\left(\frac{dT}{dx}\right)$ (b) $\dot{Q} = -hA\left(\frac{dT}{dx}\right)$
   (c) $\dot{Q} = hA^2(T_s - T_\infty)$ (d) $\dot{Q} = hA(T_s - T_\infty)$
7. The value of heat transfer coefficient depends on
   (a) velocity of fluid and temperature difference
   (b) thermal conductivity and dynamic viscosity
   (c) geometry of surface and its ambient
   (d) all of the above
8. The overall heat-transfer coefficient with respect to surface 1 is expressed as
   (a) $U_1 = \frac{1}{A_1 \Sigma R_{th}}$ (b) $U_1 = \frac{\dot{Q}}{A_1(\Delta T)_{overall}}$
   (c) $U_1 = \frac{U_2 A_2}{A_1}$ (d) all of the above
9. The Reynolds number for a fluid flow is defined as
   (a) $\frac{\text{Bouyancy force}}{\text{Viscous force}}$

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(b) $\dfrac{\text{Inertia force}}{\text{Viscous force}}$

(c) $\dfrac{\text{Convection heat transfer}}{\text{Conduction heat transfer}}$

(d) $\dfrac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}}$

10. A gray body has one of the following properties:
    (a) It reflects all of the energy falling on it.
    (b) It transmits all of the energy falling on it.
    (c) It has constant emissivity.
    (d) It absorbs all of the energy falling on it.
11. Which one of the following heat exchanger is most efficient for a given surface area and temperature difference?
    (a) Parallel flow heat exchanger
    (b) Counter flow heat exchanger
    (c) Cross flow heat exchanger
    (d) Shell and tube type heat exchanger
12. Which one of the following is an expression for LMTD for a counterflow heat exchanger?

(a) $\dfrac{\Delta T_1 - \Delta T_2}{\ln\left(\dfrac{\Delta T_2}{\Delta T_1}\right)}$ (b) $\dfrac{\Delta T_1 - \Delta T_2}{\ln\left(\dfrac{\Delta T_1}{\Delta T_2}\right)}$

(c) $\dfrac{\ln\left(\dfrac{\Delta T_1}{\Delta T_2}\right)}{\Delta T_1 - \Delta T_2}$ (d) $\dfrac{\Delta T_1 - \Delta T_2}{\log_{10}\left(\dfrac{\Delta T_2}{\Delta T_1}\right)}$

Answers

1. (c) 2. (b) 3. (c) 4. (d) 5. (b) 6. (d) 7. (d) 8. (d)
9. (b) 10. (c) 11. (b) 12. (b)

A

Appendix

Table A.1 *Atomic or Molecular Weights and Critical Properties of selected Elements and Compounds*

Substance	*Chemical Formula*	$\mathcal{M}$ *(kg./kmol)*	R *kJ/kg-K*	ρ *kg/m^3*	T_c *(K)*	p_c *(bar)*	$Z_c = \frac{p_c v_c}{RT_c}$
Acetylene	C_2H_2	26.04	0.3193	–1.05	309	62.8	0.274
Air (equivalent)	–	28.97	0.287	1.169	133	37.7	0.284
Ammonia	NH_3	17.03	0.4882	0.694	406	112.8	0.242
Argon	Ar	39.94	0.2081	–1.613	151	48.6	0.290
Benzene	C_6H_6	78.11	0.1064	—	563	49.3	0.274
Butane	C_4H_{10}	58.12	0.1430	2.407	425	38.0	0.274
Carbon	C	12.01	—	—	—	—	—
Carbon dioxide	CO_2	44.01	0.1889	1.775	304	73.9	0.276
Carbon monoxide	CO	28.01	0.2968	1.13	133	35.0	0.294
Copper	Cu	63.54	—	—	—	—	—
Ethane	C_2H_6	30.07	0.2765	1.222	305	48.8	0.285
Ethyl alcohol	C_2H_5OH	46.07	0.1805	1.883	516	63.8	0.249
Ethylene	C_2H_4	28.05	0.2964	1.138	283	51.2	0.270
Helium	He	4.003	2.0771	0.1615	5.2	2.3	0.300
Hydrogen	H_2	2.016	4.1243	0.0813	33.2	13.0	0.304
Methane	CH_4	16.04	0.5183	0.648	191	46.4	0.290
Methyl alcohol	CH_3OH	32.04	0.2595	1.31	513	79.5	0.220
Nitrogen	N_2	28.01	0.2968	1.13	126	33.9	0.291
Octane	C_8H_{18}	114.22	0.07279	0.092	569	24.9	0.258
Oxygen	O_2	32.00	0.2598	1.292	154	50.5	0.290
Propane	C_3H_8	44.09	0.1886	1.808	370	42.7	0.276
Refrigerant 12	CCl_2F_2	120.92	0.06876	4.98	385	41.2	0.278
Refrigerant 22	$CHClF_2$	86.48	0.09616	3.54	369	49.8	0.267
Refrigerant 134a	CF_3CH_2F	102.03	0.08149	4.20	374	40.7	0.260
Sulfur dioxide	SO_2	64.06	0.1298	2.618	431	78.7	0.268
Water	H_2O	18.02	0.4613	1000	647.3	220.9	0.233

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Table A.2 *Properties of Selected Solids at 25°C*

Substance	ρ *kg/m³*	C_p *kJ/kg-K*
Asphalt	2120	0.92
Brick, common	1800	0.84
Carbon, dimond	3250	0.51
Carbon, graphite	2000-2500	0.61
Coal	1200-1500	1.26
Concrete	2200	0.88
Glass, plate	2500	0.80
Glass, wool	200	0.66
Granite	2750	0.89
Ice (0 C)	917	2.04
Paper	700	1.2
Plexiglas	1180	1.44
Polystyrene	920	2.3
Polyvinyl chloride	1380	0.96
Rubber, soft	1100	1.67
Salt, rock	2100-2500	0.92
Sand, dry	1500	0.8
Silicon	2330	0.70
Snow, firm	560	2.1
Wood, hard (oak)	720	1.26
Wood, soft (pine)	510	1.38
Wool	100	1.72
Metals		
Aluminum	2700	0.90
Copper, commercial	8300	0.42
Brass, 60-40	8400	0.38
Gold	19300	0.13
Iron, cast	7272	0.42
Iron. 304 St Steel	7820	0.46
Lead	11340	0.13
Megnesium, 2% Mn	1778	1.00
nickel. 10% Cr	8666	0.44
Silver, 99.9% Ag	10524	0.24
Sodium	971	1.21
Tim	7304	0.22
Tungsten	19300	0.13
Zinc	7144	0.39

Table A.3 *Properties of selected Liquids at 25°C*

Substance	ρ *kg/m³*	C_p *kJ/kg-K*
Ammonia	604	4.84
Benzene	879	1.72
Butane	556	2.47
CCL_4	1584	0.83
CC_2	680	2.9
Ethanol	783	2.46
Gasoline	750	2.08
Glycerine	1260	2.42
Kerosene	815	2.0
Methanol	787	2.55
n-Octane	692	2.23
Oil engine	885	1.9
Oil light	910	1.8
Propane	510	2.54
R-12	1310	0.97
R-22	1190	1.26
R-134a	1206	1.43
Water	997	4.18
Liquid Metals		
Bismuth, Bi	10040	0.14
Lead, Pb	10660	0.16
mercury, Hg	13580	0.14
Potassium, K	828	0.81
Sodium, Na	929	1.38
Tin, sn	6950	0.24
Zinc, Zn	6570	0.50
Nak (56/44)	887	1.13

Table A.4 *Thermochemical Properties of Selected Substances at 25°C and 1atm*

Substance	*Formula*	*molar mass, $\mathcal{M}$ (kg/kmol)*	*Enthalpy of Formation, $\overline{h}^\circ_f$ (kJ/kmol)*	*Gibbs Function of Formation, $\overline{g}^\circ_f$ (kJ/kmol)*	*Absolute Entropy, $\overline{s}^\circ$ (kJ/kmol-K)*	*Heating Values*	
						Higher, HCV (kJ/kg)	*Lower, LCV (kJ/kg)*
Carbon	C(s)	12.01	0	0	5.74	32,770	32,770
Hydrogen	H_2(g)	2.016	0	0	130.57	141,780	119,950
Nitrogen	N_2(g)	28.01	0	0	191.50	–	–
Oxygen	O_2(g)	32.00	0	0	205.03	–	–
Carbon monoxide	CO(g)	28.01	–110,530	–137,150	197.54	–	–
Carbon dioxide	CO_2(g)	44.01	–393,520	–394,380	213.69	–	–
Water	H_2O(g)	18.02	–241,820	228,590	188.72	–	–
Water	H_2O(1)	18.02	–285,830	–237,180	69.95	–	–
Hydrogen peroxide	H_2O_2(g)	34.02	–136,310	–105,600	232.63	–	–
Ammonia	NH_3(g)	17.03	–46,190	–16,590	192.33	–	–
Oxygen	O(g)	16.00	249,170	231,770	160.95	–	–
Hydrogen	H(g)	1.008	218,000	203,290	114.61	–	–
Nitrogen	N(g)	14.01	472,680	455,510	153.19	–	–
Hydroxyl	OH(g)	17.01	39,460	34,280	183.75	–	–
Methane	CH_4(g)	16.04	–74,850	–50,790	186.16	–	–
Acetylene	C_2H_2(g)	26.04	226,730	209,170	200.85	55,510	50,020
Ethylene	C_2H_2(g)	28.05	52,280	68,120	219.83	49,910	48,220
Ethane	C_2H_6(g)	30.07	–84,680	–32,890	229.49	50,300	47,160
Propylene	C_3H_6(g)	42.08	20,410	62,720	266.94	51,870	47,480
Propane	C_3H_8(g)	44.09	–103,850	–23,490	269.91	48,920	45,780
Butane	C_4H_{10}(g)	58.12	–126,150	–15,710	310.03	50,350	46,360
Pentane	C_5H_{12}(g)	72.15	–146,440	–8,200	348.40	49,500	45,720
Octane	C_8H_{18}(g)	114.22	–208,450	17,320	463.67	49,010	45,350
Octane	C_8H_{18}(1)	114.22	249,910	6,610	360.79	48,260	44,790
Benzene	C_6H_6(g)	78.11	82,930	129,660	269.20	47,900	44,430
Methyl alcohol	CH_3OH(g)	32.04	–200,890	–162,140	239.70	42,270	40,580
Methyl alcohol	CH_3OH(1)	32.04	– 238,810	–166,290	126.80	23,850	21,110
Ethyl alcohol	C_2H_5OH(g)	46.07	– 235,310	– 168,570	282.59	22,670	19,920
Ethyl alcohol	C_2H_5OH(1)	46.07	–277,690	174,890	160.70	29,670	26,800

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Table A.5 *Constants for the van der Waals, Redlich–Kwong, and Benedict–Webb–Rubin Equations of state*

1. van der Waals and Redlich–Kwong: Constants for pressure in bar, specific volume in m^3/kmol, and temperature

	van der Waals		*Redlich–Kwong*	
Substance	a $bar\left(\frac{m^3}{kmol}\right)^2$	b $\frac{m^3}{kmol}$	a $bar\left(\frac{m^3}{kmol}\right)^2 K^{1/2}$	b $\frac{m^3}{kmol}$
Air	1.368	0.0367	15.989	0.02541
Butane (C_4H_{10})	13.86	0.1162	289.55	0.08060
Carbon dioxide (CO_2)	3.647	0.0428	64.43	0.02963
Carbon monoxide (CO)	1.474	0.0395	17.22	0.02737
Methane (CH_4)	2.293	0.0428	32.11	0.02965
Nitrogen (N_2)	1.366	0.0386	15.53	0.02677
Oxygen (O_2)	1.369	0.0317	17.22	0.02197
Propane (C_3H_8)	9.349	0.0901	182.23	0.06242
Refrigerant 12	10.49	0.0971	208.59	0.06731
Sulfur dioxide (SO_2)	6.883	0.0569	144.80	0.03945
Water (H_2O)	5.531	0.0305	142.59	0.02111

Source: Calculated from critical data.

2. Benedict–Webb–Rubin: Constants for pressure in bar, specific volume in m^3/kmol, and temperature in **K**

Substance	*a*	*A*	*b*	*B*	*c*	*C*	α	γ *(gamma)*
C_4H_{10}	1.9073	10.218	0.039998	0.12436	3.206×10^5	1.006×10^6	1.101×10^{-3}	0.0340
CO_2	0.1386	2.7737	0.007210	0.04991	1.512×10^4	1.404×10^5	8.47×10^{-5}	0.00539
CO	0.0371	1.3590	0.002632	0.05454	1.054×10^3	8.676×10^3	1.350×10^{-4}	0.0060
CH_4	0.0501	1.8796	0.003380	0.04260	2.579×10^3	2.287×10^4	1.244×10^{-4}	0.0060
N_2	0.0254	1.0676	0.002328	0.04074	7.381×10^2	8.166×10^3	1.272×10^{-4}	0.0053

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Table A.6 *Variation of $\bar{C}_p$ With Temperature for Selected Ideal Gases*

$$\frac{\bar{C}_p}{R_u} = \alpha + \beta T + \gamma T^2 + \delta T^3 + \varepsilon T^4$$

T is in K, equations valid from 300 to 1000 K

Gas	α	$\beta \times 10^3$	$\gamma \times 10^6$	$\delta \times 10^9$	$\varepsilon \times 10^{12}$
CO	3.710	–1.619	3.692	–2.032	0.240
CO_2	2.401	8.735	–6.607	2.002	0
H_2	3.057	2.677	–5.810	5.521	–1.812
H_2O	4.070	–1.108	4.152	–2.964	0.807
O_2	3.626	–1.878	7.055	–6.764	2.156
N_2	3.675	–1.208	2.324	–0.632	0.226
Air	3.653	–1.337	3.294	–1.913	0.2763
SO_2	3.267	5.324	0.684	5.281	2.559
CH_4	3.826	–3.979	24.558	–22.733	6.963
C_2H_2	1.410	19.057	–24.501	16.391	– 4.135
C_2H_4	1.426	11.383	7.989	–16.254	6.749
Monatomic gases[α]	2.5	0	0	0	0

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Table A.7 ***Ideal Gas*** *Specific Heats of Some Common Gases (kJ/kg · K)*

Temp. K	C_p	C_v	k	C_p	C_v	k	C_p	C_v	k	*Temp. K*
	Air			*Nitrogen, N_2*			*Oxygen, O_2*			
250	1.003	0.716	1.401	1.039	0.742	1.400	0.913	0.653	1.398	250
300	1.005	0.718	1.400	1.039	0.743	1.400	0.918	0.658	1.395	300
350	1.008	0.721	1.398	1.041	0.744	1.399	0.928	0.668	1.389	350
400	1.013	0.726	1.395	1.044	0.747	1.397	0.941	0.681	1.382	400
450	1.020	0.733	1.391	1.049	0.752	1.395	0.956	0.696	1.373	450
500	1.029	0.742	1.387	1.056	0.759	1.391	0.972	0.712	1.365	500
550	1.040	0.753	1.381	1.065	0.768	1.387	0.988	0.728	1.358	550
600	1.051	0.764	1.376	1.075	0.778	1.382	1.003	0.743	1.350	600
650	1.063	0.776	1.370	1.086	0.789	1.376	1.017	0.758	1.343	650
700	1.075	0.788	1.364	1.098	0.801	1.371	1.031	0.771	1.337	700
750	1.087	0.800	1.359	1.110	0.813	1.365	1.043	0.783	1.332	750
800	1.099	0.812	1.354	1.121	0.825	1.360	1.054	0.794	1.327	800
900	1.121	0.834	1.344	1.145	0.849	1.349	1.074	0.814	1.319	900
1000	1.142	0.855	1.336	1.167	0.870	1.341	1.090	0.830	1.313	1000
Temp. K	*Carbon Dioxide, CO_2*			*Carbon Monoxide, CO*			*Hydrogen, H_2*			*Temp. K*
250	0.791	0.602	1.314	1.039	0.743	1.400	14.051	9.927	1.416	250
300	0.846	0.657	1.288	1.040	0.744	1.399	14.307	10.183	1.405	300
350	0.895	0.706	1.268	1.043	0.746	1.398	14.427	10.302	1.400	350
400	0.939	0.750	1.252	1.047	0.751	1.395	14.476	10.352	1.398	400
450	0.978	0.790	1.239	1.054	0.757	1.392	14.501	10.377	1.398	450
500	1.014	0.825	1.229	1.063	0.767	1.387	14.513	10.389	1.397	500
550	1.046	0.857	1.220	1.075	0.778	1.382	14.530	10.405	1.396	550
600	1.075	0.886	1.213	1.087	0.790	1.376	14.546	10.422	1.396	600
650	1.102	0.913	1.207	1.100	0.803	1.370	14.571	10.447	1.395	650
700	1.126	0.937	1.202	1.113	0.816	1.364	14.604	10.480	1.394	700
750	1.148	0.959	1.197	1.126	0.829	1.358	14.645	10.521	1.392	750
800	1.169	0.980	1.193	1.139	0.842	1.353	14.695	10.570	1.390	800
900	1.204	1.015	1.186	1.163	0.866	1.343	14.822	10.698	1.385	900
1000	1.234	1.045	1.181	1.185	0.888	1.335	14.983	10.859	1.380	1000

Table A.8 *Ideal Gas Properties of* ***Air***

T (K), h and u(kJ/kg), s° (kJ/kg.K)											
				when $\Delta s = 0^1$						*when* $\Delta s = 0$	
T	h	u	$s°$	p_r	v_r	T	h	u	$s°$	p_r	v_r
200	199.97	142.56	1.29559	0.3363	1707.0	450	451.80	322.62	2.11161	5.775	223.6
210	209.97	149.69	1.34444	0.3987	1512.0	460	462.02	329.97	2.13407	6.245	221.4
220	219.97	156.82	1.39105	0.4690	1346.0	470	472.24	337.32	2.15604	6.742	200.1
230	230.02	164.00	1.43557	0.5477	1205.0	480	482.49	344.70	2.17760	7.268	189.5
240	240.02	171.13	1.47824	0.6355	1084.0	490	492.74	352.08	2.19876	7.824	179.7
250	250.05	178.28	1.51917	0.7329	979.0	500	503.02	359.49	2.21952	8.411	170.6
260	260.09	185.45	1.55848	0.8405	887.8	510	513.32	366.92	2.23993	9.031	162.1
270	270.11	192.60	1.59634	0.9590	808.0	520	523.63	374.36	2.25997	9.684	154.1
280	280.13	199.75	1.63279	1.0889	738.0	530	533.98	381.84	2.27967	10.37	146.7
285	285.14	203.33	1.65055	1.1584	706.1	540	544.35	389.34	2.29906	11.10	139.7
290	290.16	206.91	1.66802	1.2311	676.1	550	554.74	396.86	2.31809	11.86	133.1
295	295.17	210.49	1.68515	1.3068	647.9	560	565.17	404.42	2.33685	12.66	127.0
300	300.19	214.07	1.70203	1.3860	621.2	570	575.59	411.97	2.35531	13.50	121.2
305	305.22	217.67	1.71865	1.4686	596.0	580	586.04	419.55	2.37348	14.38	115.7
310	310.24	221.25	1.73498	1.5546	572.3	590	596.52	427.15	2.39140	15.31	110.6
315	315.27	224.85	1.75106	1.6442	549.8	600	607.02	434.78	2.40902	16.28	105.8
320	320.29	228.42	1.76690	1.7375	528.6	610	617.53	442.42	2.42644	17.30	101.2
325	325.31	232.02	1.78249	1.8345	508.4	620	628.07	450.09	2.44356	18.36	96.92
330	330.34	235.61	1.79783	1.9352	489.4	630	638.63	457.78	2.46048	19.84	92.84
340	340.42	242.82	1.82790	2.149	454.1	640	649.22	465.50	2.47716	20.64	88.99
350	350.49	250.02	1.85708	2.379	422.2	650	659.84	473.25	2.49364	21.86	85.34
360	360.58	257.24	1.88543	2.626	393.4	660	670.47	481.01	2.50985	23.13	81.89
370	370.67	264.46	1.91313	2.892	367.2	670	681.14	488.81	2.52589	24.46	78.61
380	380.77	271.69	1.94001	3.176	343.4	680	691.82	496.62	2.54175	25.85	75.50
390	390.88	278.93	1.96633	3.481	321.5	690	702.52	504.45	2.55731	27.29	72.56
400	400.98	286.16	1.99194	3.806	301.6	700	713.27	512.33	2.57277	28.80	69.76
410	411.12	293.43	2.01699	4.153	283.3	710	724.04	520.23	2.58810	30.38	67.07
420	421.26	300.69	2.04142	4.522	266.6	720	734.82	528.14	2.60319	32.02	64.53
430	431.43	307.99	2.06533	4.915	251.1	730	745.62	536.07	2.61803	33.72	62.13
440	441.61	315.30	2.08870	5.332	236.8	740	756.44	544.02	2.63280	35.50	59.82

Contd.

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Table A.8 *(Continued)*

T (K), h and u(kJ/kg), s° (kJ/kg·K)											
				when Δs = 0						*when Δs = 0*	
T	*h*	*u*	*s°*	p_r	v_r	*T*	*h*	*u*	*s°*	p_r	v_r
750	767.29	551.99	2.64737	37.35	57.63	1300	1395.97	1022.82	3.27345	330.9	11.275
760	778.18	560.01	2.66176	39.27	55.54	1320	1419.76	1040.88	3.29160	352.5	10.747
770	789.11	568.07	2.67595	41.31	53.39	1340	1443.60	1058.94	3.30959	375.3	10.247
780	800.03	576.12	2.69013	43.35	51.64	1360	1467.49	1077.10	3.32724	399.1	9.780
790	810.99	584.21	2.70400	45.55	49.86	1380	1491.44	1095.26	3.34474	424.2	9.337
800	821.95	592.30	2.71787	47.75	48.08	1400	1515.42	1113.52	3.36200	450.5	8.919
820	843.98	608.59	2.74504	52.59	44.84	1420	1539.44	1131.77	3.37901	478.0	8.526
840	866.08	624.95	2.77170	57.60	41.85	1440	1563.51	1150.13	3.39586	506.9	8.153
860	888.27	641.40	2.79783	63.09	39.12	1460	1587.63	1168.49	3.41247	537.1	7.801
880	910.56	657.95	2.82344	68.98	36.61	1480	1611.79	1186.95	3.42892	568.8	7.468
900	932.93	674.58	2.84856	75.29	34.31	1500	1635.97	1205.41	3.44516	601.9	7.152
920	955.38	691.28	2.87324	82.05	32.18	1520	1660.23	1223.87	3.46120	636.5	6.854
940	977.92	708.08	2.89748	89.28	30.22	1540	1684.51	1242.43	3.477.12	672.8	6.569
960	1000.55	725.02	2.92128	97.00	28.40	1560	1708.82	1260.99	3.49276	710.5	6.301
980	1023.25	741.98	2.94468	105.2	26.73	1580	1733.17	1279.65	3.50829	750.0	6.046
1000	1046.04	758.94	2.96770	114.0	25.17	1600	1757.57	1298.30	3.52364	791.2	5.804
1020	1068.89	776.10	2.99034	123.4	23.72	1620	1782.00	1316.96	3.53879	834.1	5.574
1040	1091.85	793.36	3.01260	133.3	22.39	1640	1806.46	1335.72	3.55381	878.9	5.355
1060	1114.86	810.62	3.03449	143.9	21.14	1660	1830.96	1354.48	3.56867	925.6	5.147
1080	1137.89	827.88	3.05608	155.2	19.98	1680	1855.50	1373.24	3.58335	974.2	4.949
1100	1161.07	845.33	3.07732	167.1	18.896	1700	1880.1	1392.7	3.5979	1025	4.761
1120	1184.28	862.79	3.09825	179.7	17.886	1750	1941.6	1439.8	3.6336	1161	4.328
1140	1207.57	880.35	3.11883	193.1	16.946	1800	2003.3	1487.2	3.6684	1310	3.944
1160	1230.92	897.91	3.13916	207.2	16.064	1850	2065.3	1534.9	3.7023	1475	3.601
1180	1254.34	915.57	3.15916	222.2	15.241	1900	2127.4	1582.6	3.7354	1655	3.295
1200	1277.79	933.33	3.17888	238.0	14.470	1950	2189.7	1630.6	3.7677	1852	3.022
1220	1301.31	951.09	3.19834	254.7	13.747	2000	2252.1	1678.7	3.7994	2068	2.776
1240	1324.93	968.95	3.21751	272.3	13.069	2050	2314.6	1726.8	3.8303	2303	2.555
1260	1348.55	986.90	3.23638	290.8	12.435	2100	2377.4	1775.3	3.8605	2559	2.356
1280	1372.24	1004.76	3.25510	310.4	11.835	2150	2440.3	1823.8	3.8901	2837	2.175
						2200	2503.2	1872.4	3.9191	3138	2.012
						2250	2566.4	1921.3	3.9474	3464	1.864

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Table A.9 *Ideal Gas Properties of Selected Gases*

	T(K), $\overline{h}$ and $\overline{u}$ (kJ/mol), $\overline{s}^\circ$ (kJ/kmol·K)															
	Carbon Dioxide, CO_2 ($\overline{h}_{f^\circ} = -393{,}520$ kJ/kmol)			*Carbon Monoxide, CO ($\overline{h}_{f^\circ} = -110{,}530$ kJ/kmol)*			*Water Vapour, H_2O ($\overline{h}_{f^\circ} = -241{,}820$ kJ/kmol)*			*Oxygen, O_2 ($\overline{h}_{f^\circ} = 0$ kJ/kmol)*			*Nitrogen, N_2 ($\overline{h}_{f^\circ} = 0$ kJ/kmol)*			
T	$\overline{h}$	$\overline{u}$	$\overline{s}^\circ$	$\overline{h}$	$\overline{u}$	$\overline{s}^\circ$	$\overline{h}$	$\overline{u}$	$\overline{s}^\circ$	$\overline{h}$	$\overline{u}$	$\overline{s}^\circ$	$\overline{h}$	$\overline{u}$	$\overline{s}^\circ$	*T*
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
220	6,601	4,772	202.966	6,391	4,562	188.683	7,295	5,466	178.576	6,404	4,575	196.171	6,391	4,562	182.638	220
230	6,938	5,026	204.464	6,683	4,771	189.980	7,628	5,715	180.054	6,694	4,782	197.461	6,683	4,770	183.938	230
240	7,280	5,285	205.920	6,975	4,979	191.221	7,961	5,965	181.471	6,984	4,989	198.696	6,975	4,979	185.180	240
250	7,627	5,548	207.337	7,266	5,188	192.411	8,294	6,215	182.831	7,275	5,197	199.885	7,266	5,188	186.370	250
260	7,979	5,817	208.717	7,558	5,396	193.554	8,627	6,466	184.139	7,566	5,405	201.027	7,558	5,396	187.514	260
270	8,335	6,091	210.062	7,849	5,604	194.654	8,961	6,716	185.399	7,858	5,613	202.128	7,849	5,604	188.614	270
280	8,697	6,369	211.376	8,140	5,812	195.173	9,296	6,968	186.616	8,150	5,822	203.191	8,141	5,813	189.673	280
290	9,063	6,651	212.660	8,432	6,020	196.735	9,631	7,219	187.791	8,443	6,032	204.218	8,432	6,021	190.695	290
298	9,364	6,885	213.685	8,669	6,190	197.543	9,904	7,425	188.720	8,682	6,203	205.033	8,669	6,190	191.502	298
300	9,431	6,939	213.915	8,723	6,229	197.723	9,966	7,472	188.923	8,736	6,242	205.213	8,723	6,229	191.682	300
310	9,807	7,230	215.146	9,014	6,437	198.678	10,302	7,725	190.030	9,030	6,453	206.177	9,014	6,437	192.638	310
320	10,186	7,526	216.351	9,306	6,645	199.603	10,639	7,978	191.098	9,325	6,664	207.112	9,306	6,645	193.562	320
330	10,570	7,826	217.534	9,597	6,854	200.500	10,976	8,232	192.136	9,620	6,877	208.020	9,597	6,853	194.459	330
340	10,959	8,131	218.694	9,889	7,062	201.371	11,314	8,487	193.144	9,916	7,090	208.904	9,888	7,061	195.328	340
350	11,351	8,439	219.831	10,181	7,271	202.217	11,652	8,742	194.125	10,213	7,303	209.765	10,180	7,270	196.173	350
360	11,748	8,752	220.948	10,473	7,480	203.040	11,992	8,998	195.081	10,511	7,518	210.604	10,471	7,478	196.995	360
370	12,148	9,068	222.044	10,765	7,689	203.842	12,331	9,255	196.012	10,809	7,733	211.423	10,763	7,687	197.794	370
380	12,552	9,392	223.122	11,058	7,899	204.622	12,672	9,513	196.020	11,109	7,949	212.222	11,055	7,895	198.572	380
390	12,960	9,718	224.182	11,351	8,108	205.383	13,014	9,771	197.807	11,409	8,166	213.002	11,347	8,104	199.331	390
400	13,372	10,046	225.225	11,644	8,319	206.125	13,356	10,030	198.673	11,711	8,384	213.765	11,640	8,314	200.071	400
410	13,787	10,378	226.250	11,938	8,529	206.850	13,699	10,290	199.521	12,012	8,603	214.510	11,932	8,523	200.794	410
420	14,206	10,714	227.258	12,232	8,740	207.549	14,043	10,551	200.350	12,314	8,822	215.241	12,225	8,733	201.499	420
430	14,628	11,053	228.252	12,526	8,951	208.252	14,389	10,813	200.160	12,618	9,043	215.955	12,518	8,943	202.189	430
440	15,054	11,393	229.230	12,821	9,163	208.929	14,734	11,075	201.955	12,908	9,264	216.656	12,811	9,153	202.863	440
450	15,483	11,742	230.194	13,116	9,375	209.593	15,080	11,339	202.734	13,228	9,487	217.342	13,105	9,363	203.523	450
460	15,916	12,091	231.144	13,412	9,587	210.243	15,428	11,603	203.497	13,535	9,710	218.016	13,399	9,574	204.170	460

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The McGraw-Hill Companies

Table A.9 *Contd.*

$T(K)$, $\bar{h}$ and $\bar{u}$ (kJ/mol), $\bar{s}^\circ$ (kJ/kmol·K)																
	Carbon Dioxide, CO_2 ($\bar{h}_{f^\circ} = -393{,}520$ kJ/kmol)			*Carbon Monoxide, CO ($\bar{h}_{f^\circ} = -110{,}530$ kJ/kmol)*			*Water Vapour, H_2O ($\bar{h}_{f^\circ} = -241{,}820$ kJ/kmol)*			*Oxygen, O_2 ($\bar{h}_{f^\circ} = 0$ kJ/kmol)*			*Nitrogen, N_2 ($\bar{h}_{f^\circ} = 0$ kJ/kmol)*			
T	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	T
470	16,351	12,444	232.080	13,708	9,800	210.880	15,777	11,869	204.247	13,842	9,935	218.716	13,693	9,786	204.803	470
480	16,791	12,800	233.004	14,005	10,014	211.504	16,126	12,135	204.982	14,151	10,160	219.326	13,988	9,997	205.424	480
490	17,232	13,158	233.916	14,302	10,228	212.117	16,477	12,403	205.705	14,460	10,386	219.963	14,285	10,210	206.033	490
500	17,678	13,521	234.814	14,600	10,443	212.719	16,828	12,671	206.413	14,770	10,614	220.589	14,581	10,423	206.630	500
510	18,126	13,885	235.700	14,898	10,658	213.310	17,181	12,940	207.112	15,082	10,842	221.206	14,876	10,635	207.216	510
520	18,576	14,253	236.575	15,197	10,874	213.890	17,534	13,211	207.799	15,395	11,071	221.812	15,172	10,848	207.792	520
530	19,029	14,622	237.439	15,497	11,090	214.460	17,889	13,482	208.475	15,708	11,301	222.409	15,469	11,062	208.358	530
540	19,485	14,996	238.292	15,797	11,307	215.020	18,245	13,755	209.139	16,022	11,533	222.997	15,766	11,277	208.914	540
550	19,945	15,372	239.135	16,097	11,524	215.572	18,601	14,028	209.795	16,338	11,765	223.576	16,064	11,492	209.461	550
560	20,407	15,751	239.962	16,399	11,743	216.115	18,959	14,303	210.440	16,654	11,998	224.146	16,363	11,707	209.999	560
570	20,870	16,131	240.789	16,701	11,961	216.649	19,318	14,579	211.075	16,971	12,232	224.708	16,662	11,923	210.528	570
580	21,337	16,515	241.602	17,003	12,181	217.175	19,678	14,856	211.702	17,290	12,467	225.262	16,962	12,139	211.049	580
590	21,807	16,902	242.405	17,307	12,401	217.693	20,093	15,134	212.320	17,609	12,703	225.808	17,262	12,356	211.562	590
600	22,280	17,291	243.199	17,611	12,622	218.204	20,402	15,413	212.920	17,929	12,940	226.346	17,563	12,574	212.066	600
610	22,754	17,683	243.983	17,915	12,843	218.708	20,765	15,693	213.529	18,250	13,178	226.877	17,864	12,792	212.564	610
620	23,231	18,076	244.758	18,221	13,066	219.205	21,130	15,975	214.122	18,572	13,417	227.400	18,166	13,011	213.055	620
630	23,709	18,471	245.524	18,527	13,289	219.695	21,495	16,257	214.707	18,895	13,657	227.918	18,468	13,230	213.541	630
640	24,190	18,869	246.282	18,833	13,512	220.179	21,862	16,541	215.285	19,219	13,898	228.429	18,772	13,450	214.018	640
650	24,674	19,270	247.032	19,141	13,736	220.656	22,230	16,826	215.856	19,544	14,140	228.932	19,075	13,671	214.489	650
660	25,160	19,672	247.773	19,449	13,962	221.127	22,600	17,112	216.419	19,870	14,383	229.430	19,380	13,892	214.954	660
670	25,648	20,078	248.507	19,758	14,187	221.592	22,970	17,399	216.970	20,197	14,626	229.920	19,685	14,114	215.413	670
680	26,138	20,484	249.233	20,068	14,414	222.052	23,342	17,683	217.527	20,524	14,871	230.405	19,991	14,337	215.866	680
690	26,631	20,894	249.952	20,378	14,641	222.505	23,714	17,978	218.071	20,854	15,116	230.885	20,297	14,560	216.314	690
700	27,125	21,305	250.663	20,690	14,870	222.953	24,088	18,268	218.610	21,184	15,364	231.358	20,604	14,784	216.756	700
710	27,622	21,719	251.368	21,002	15,099	223.396	24,464	18,561	219.142	21,514	15,611	231.827	20,912	15,008	217.192	710
720	28,121	22,134	252.065	21,315	15,328	223.833	24,840	18,854	219.668	21,845	15,859	232.291	21,220	15,234	217.624	720
730	28,622	22,552	252.755	21,628	15,558	224.265	25,218	19,148	220.189	22,177	16,107	232.748	21,529	15,460	218.059	730
740	29,124	22,972	253.439	21,943	15,789	224.692	25,597	19,444	220.707	22,510	16,357	233.201	21,839	15,686	218.472	740

Contd.

The McGraw-Hill Companies

Table A.9 *Contd.*

T(K), $\bar{h}$ and $\bar{u}$ (kJ/mol), $\bar{s}^\circ$ (kJ/kmol·K)																
	Carbon Dioxide, CO_2 ($\bar{h}_{f^\circ} = -393{,}520$ kJ/kmol)			*Carbon Monoxide, CO* ($\bar{h}_{f^\circ} = -110{,}530$ kJ/kmol)			*Water Vapour, H_2O* ($\bar{h}_{f^\circ} = -241{,}820$ kJ/kmol)			*Oxygen, O_2* ($\bar{h}_{f^\circ} = 0$ kJ/kmol)			*Nitrogen, N_2* ($\bar{h}_{f^\circ} = 0$ kJ/kmol)			
T	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	T
750	29,629	23,393	254.117	22,258	16,022	225.115	25,977	19,741	221.215	22,844	16,607	233.649	22,149	15,913	218.889	750
760	30,135	23,817	254.787	22,573	16,255	225.533	26,358	20,039	221.720	23,178	16,859	234.091	22,460	16,141	219.301	760
770	30,644	24,242	255.452	22,890	16,488	225.947	26,741	20,339	222.221	23,513	17,111	234.528	22,772	16,370	219.709	770
780	31,154	24,669	256.110	23,208	16,723	226.357	27,125	20,639	222.717	23,850	17,364	234.960	23,085	16,599	220.113	780
790	31,665	25,097	256.762	23,526	16,957	226.762	27,510	20,941	223.207	24,186	17,618	235.387	23,398	16,830	220.512	790
800	32,179	25,527	257.408	23,844	17,193	227.162	27,896	21,245	223.693	24,523	17,872	235.810	23,714	17,061	220.907	800
810	32,694	25,959	258.048	24,164	17,429	227.559	28,284	21,549	224.174	24,861	18,126	236.230	24,027	17,292	221.298	810
820	33,212	26,394	258.682	24,483	17,665	227.952	28,672	21,855	224.651	25,199	18,382	236.644	24,342	17,524	221.684	820
830	33,730	26,829	259.311	24,803	17,902	228.339	29,062	22,162	225.123	25,537	18,637	237.055	24,658	17,757	222.067	830
840	34,251	27,267	259.934	25,124	18,140	228.724	29,454	22,470	225.592	25,877	18,893	237.462	24,974	17,990	222.447	840
850	34,773	27,706	260.551	25,446	18,379	229,106	29,846	22,779	226.057	26,218	19,150	237.864	25,292	18,224	222.822	850
860	35,296	28,125	261.164	25,768	18,617	229,482	30,240	23,090	226.517	26,599	19,666	238.264	25,610	18,459	223.194	860
870	35,821	28,588	261.770	26,091	18,858	229.856	30,635	23,402	226.973	26,899	19,666	238.660	25,928	18,695	223.562	870
880	36,347	29,031	262.371	26,415	19,099	230.227	31,032	23,715	227.426	27,242	19,925	239.051	26,248	18,931	223.927	880
890	36,876	29,476	262.968	26,740	19,341	230.593	31,429	24,029	227.875	27,584	20,185	239.439	26,568	19,168	224.288	890
900	37,405	29,922	263.559	27,066	19,583	230.957	31,828	24,345	228.321	27,928	20,445	239.823	26,890	19,407	224.647	900
910	37,935	30,369	264.146	27,392	19,826	231.317	32,228	24,662	228.763	28,272	20,706	240.203	27,210	19,644	225.002	910
920	38,467	30,818	264.728	27,719	20,070	231.674	32,629	24,980	229.202	28,616	20,967	240.580	27,532	19,883	225.353	920
930	39,000	31,268	265.304	28,046	20,314	232.028	33,032	25,300	229.637	28,960	21,228	240.953	27,854	20,122	225.701	930
940	39,535	31,719	265.877	28,375	20,559	232.397	33,436	25,621	230.070	29,306	21,491	241.323	28,178	20,362	226.047	940
950	40,070	32,171	266.444	28,703	20,805	232.727	33,841	25,943	230.499	29,652	21,754	241.689	28,501	20,603	226.389	950
960	40,607	32,625	267.007	29,033	21,051	233.072	34,247	26,265	230.924	29,999	22,017	242.052	28,826	20,844	226.728	960
970	41,145	33,081	267.566	29,362	21,298	233.413	34,653	26,588	231.347	30,345	22,280	242.411	29,151	21,086	227.064	970
980	41,685	33,537	268.119	29,693	21,545	233.752	35,061	26,913	231.767	30,692	22,544	242.768	29,476	21,328	227.398	980
990	42,226	33,995	268.670	30,024	21,793	234.088	35,472	27,240	232.184	31,041	22,809	243.120	29,803	21,571	227.728	990
1000	42,769	34,455	269.215	30,355	22,041	234.421	35,882	27,568	232.597	31,389	23,075	243.471	30,129	21,815	228.057	1000
1020	43,859	35,378	270.293	31,020	22,540	235.079	36,709	28,228	233.415	32,088	23,607	244.164	30,784	22,304	228.706	1020
1040	44,953	36,306	271.354	31,688	23,041	235.728	37,542	28,895	234.223	32,789	24,142	244.844	31,442	22,795	229.344	1040

Contd.

The McGraw-Hill Companies

Table A.9 *Contd.*

$T(K)$, $\bar{h}$ and $\bar{u}$ (kJ/mol), $\bar{s}^\circ$ (kJ/kmol·K)																
	Carbon Dioxide, CO_2 ($\bar{h}_{f^\circ} = -393{,}520$ kJ/kmol)			*Carbon Monoxide, CO ($\bar{h}_{f^\circ} = -110{,}530$ kJ/kmol)*			*Water Vapour, H_2O ($\bar{h}_{f^\circ} = -241{,}820$ kJ/kmol)*			*Oxygen, O_2 ($\bar{h}_{f^\circ} = 0$ kJ/kmol)*			*Nitrogen, N_2 ($\bar{h}_{f^\circ} = 0$ kJ/kmol)*			
T	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	T
1060	46,051	37,238	272.400	32,357	23,544	236.364	38,380	29,567	235.020	33,490	24,677	245.513	32,101	23,288	229.973	1060
1080	47,153	38,174	273.430	33,029	24,049	236.992	39,223	30,243	235.806	34,194	25,214	246.171	32,762	23,782	230.591	1080
1100	48,258	39,112	274.445	33,702	24,557	237.609	40,071	30,925	236.584	34,899	25,753	246.818	33,426	24,280	231.199	1100
1120	49,369	40,057	275.444	34,377	25,065	238.217	40,923	31,611	237.352	35,606	26,294	247.454	34,092	24,780	231.799	1120
1140	50,484	41,006	276.430	35,054	25,575	238.817	41,780	32,301	238.110	36,314	26,836	248.081	34,760	25,282	232.391	1140
1160	51,602	41,957	277.403	35,733	26,088	239.407	42,642	32,997	238.859	37,023	27,379	248.698	35,430	25,786	232.973	1160
1180	52,724	42,913	278.362	36,406	26,602	239.989	43,509	33,698	239.600	37,734	27,923	249.307	36,104	26,291	233.549	1180
1200	53,848	43,871	279.307	37,095	27,118	240.663	44,380	34,403	240.333	38,447	28,469	249.906	36,777	26,799	234.115	1200
1220	54,977	44,834	280.238	37,780	27,637	241.128	45,256	35,112	241.057	39,162	29,018	250.497	37,452	27,308	234.673	1220
1240	56,108	45,799	281.158	38,466	28,426	241.686	46,137	35,827	241.773	39,877	29,568	251.079	38,129	27,819	235.223	1240
1260	57,244	46,768	282.066	39,154	28,678	242.236	47,022	36,546	242.482	40,594	30,118	251.653	38,807	28,331	235.766	1260
1280	58,381	47,739	282.962	39,884	29,201	242.780	47,912	37,270	243.183	41,312	30,670	252.219	39,488	28,845	236.302	1280
1300	59,522	48,713	283.847	40,534	29,725	243.316	48,807	38,000	243.877	42,033	31,224	252.776	40,170	29,361	236.831	1300
1320	60,666	49,691	284.722	41,266	30,251	243.844	49,707	38,732	244.564	42,753	31,778	253.325	40,853	29,878	237.353	1320
1340	61,813	50,672	285.586	41,919	30,778	244.366	50,612	39,470	245.243	43,475	32,334	353.868	41,539	30,398	237.867	1340
1360	62,963	51,656	286.439	42,613	31,306	244.880	51,521	40,213	245.915	44,198	32,891	254.404	42,227	30,919	238.376	1360
1380	64,116	52,643	287.283	43,309	31,836	245.388	52,434	40,960	246.582	44,923	33,449	254.932	42,915	31,441	238.878	1380
1400	65,271	53,631	288.106	44,007	32,367	245.889	53,351	41,711	247.241	45,648	34,008	255.454	43,605	31,964	239.375	1400
1420	66,427	54,621	288.934	44,707	32,900	246.385	54,273	42,466	247.895	46,374	34,567	255.968	44,295	32,489	239.865	1420
1440	67,586	55,614	289.743	45,408	33,434	246.876	55,198	43,226	248.543	47,102	35,129	256.475	44,988	33,014	240.350	1440
1460	68,748	56,609	290.542	46,110	33,971	247.360	56,128	43,989	249.185	47,831	35,692	256.978	45,682	33,543	240.827	1460
1480	69,911	57,606	291.333	46,813	34,508	247.839	57,062	44,756	249.820	48,561	36,256	257.474	46,377	34,071	241.301	1480
1500	71,078	58,606	292.114	47,517	35,046	248.312	57,999	45,528	250.450	49,292	36,821	257.965	47,073	34,601	241.768	1500
1520	72,246	59,609	292.888	48,222	35,584	248.778	58,942	46,304	251.074	50,024	37,387	258.450	47,771	35,133	242.228	1520
1540	73,417	60,613	292.654	48,928	36,124	249.240	59,888	47,084	251.693	50,756	37,952	258.928	48,470	35,665	242.685	1540
1560	74,590	61,620	294.411	49,635	36,665	249.695	60,838	47,868	252.305	51,490	38,520	259.402	49,168	36,197	243.137	1560
1580	76,767	62,630	295.161	50,344	37,207	250.147	61,792	48,655	252.912	52,224	39,088	259.870	49,869	36,732	243.585	1580

Contd.

The McGraw-Hill Companies

Table A.9 *Contd.*

$T(K)$, $\bar{h}$ and $\bar{u}$ (kJ/mol), $\bar{s}^\circ$ (kJ/kmol·K)																
	Carbon Dioxide, CO_2 ($\bar{h}_{f^\circ} = -393{,}520$ kJ/kmol)			Carbon Monoxide, CO ($\bar{h}_{f^\circ} = -110{,}530$ kJ/kmol)			Water Vapour, H_2O ($\bar{h}_{f^\circ} = -241{,}820$ kJ/kmol)			Oxygen, O_2 ($\bar{h}_{f^\circ} = 0$ kJ/kmol)			Nitrogen, N_2 ($\bar{h}_{f^\circ} = 0$ kJ/kmol)			
T	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	T
1600	76,944	63,741	295.901	51,053	37,750	250.592	62,748	49,445	253.513	52,961	39,658	260.333	50,571	37,268	244.028	1600
1620	78,123	64,653	296.632	51,763	38,293	251.033	63,709	50,240	254.111	53,696	40,227	260.791	51,275	37,806	244.464	1620
1640	79,303	65,668	297.356	52,472	38,837	251.470	64,675	51,039	254.703	54,434	40,799	261.242	51,980	38,344	244.896	1640
1660	80,486	66,592	298.072	53,184	39,382	251.901	65,643	51,841	255.290	55,172	41,370	261.690	52,686	38,884	245.324	1660
1680	81,670	67,702	298.781	53,895	39,927	252.329	66,614	52,646	255.873	55,912	41,944	262.132	53,393	39,424	245.747	1680
1700	82,856	68,721	299.482	54,609	40,474	252.751	67,589	53,455	256.450	56,652	42,517	262.571	54,099	39,965	246.166	1700
1720	84,043	69,742	300.177	55,323	41,023	253.169	68,567	54,267	257.022	57,394	43,093	263.005	54,807	40,507	246.580	1720
1740	85,231	70,764	300.863	56,039	41,572	253.582	69,550	55,083	257.589	58,136	43.669	263.435	55,516	41,049	246.990	1740
1760	86,420	71,787	301.543	56,756	42,123	253.991	70,535	55,902	258.151	58,800	44,247	263.861	56,227	41,594	247.396	1760
1780	87,612	72,812	302.271	57,473	42,673	254.398	71,523	56,723	258.708	59,624	44,825	264.283	56,938	42,139	247.798	1780
1800	88,806	73,840	302.884	58,191	43,225	254.797	72,513	57,547	259.262	60,371	45,405	264.701	57,651	42,685	248.195	1800
1820	90,000	74,868	303.544	58,910	43,778	225.194	73,507	58,375	259.811	61,118	45,986	265.113	58,363	43,231	248.589	1820
1840	91,196	75,897	304.198	59,629	44,331	255.587	74,506	59,207	260.357	61,866	46,568	265.521	59,075	43,777	248.979	1840
1860	92,394	76,929	304.845	60,351	44,886	255.976	75,506	60,042	260.898	62,616	47,151	265.925	59,790	44,324	249.365	1860
1880	93,593	77,962	305.487	61,072	45,441	256.361	76,511	60,880	261.436	63,365	47,734	266.326	60,504	44,873	249.748	1880
1900	94,793	78,996	306.122	61,794	45,997	256.743	77,517	61,720	261.969	64,116	48,319	266.722	61,220	45,423	250.128	1900
1920	95,995	80,031	306.751	62,516	46,552	257.122	78,527	62,564	262.497	64,868	48,904	267.115	61,936	45,973	250.502	1920
1940	97,197	81,067	307.374	63,238	47,108	257.497	79,540	63,411	263.022	65,620	49,490	267.505	62,654	46,524	250.874	1940
1960	98,401	82,105	307.992	63,961	47,665	257.868	80,555	64,259	263.542	66,374	50,078	267.891	63,318	47,075	251.242	1860
1980	99,606	83,144	308.604	64,684	48,221	258.236	81,573	65,111	264.059	67,127	50,665	268.275	64,090	47,627	251.607	1980
2000	100,804	84,185	309.210	65,408	48,780	258.600	82,593	65,965	264.571	67,881	51,253	268.655	64,810	48,181	251.969	2000
2050	103,835	86,791	310.701	67,224	50,179	259.494	85,156	68,111	265.838	69,772	52,727	269.588	66,612	49,567	252.858	2050
2100	106,864	89,404	312.160	69,044	51,584	260.370	87,735	70,275	267.081	71,668	54,208	270.504	68,417	50,957	253.726	2100
2150	109,898	92,023	313.589	70,864	52,988	261.226	90,330	72,454	268.301	73,573	55,697	271.399	70,226	52,351	254.578	2150
2200	112,939	94,648	314.988	72,688	54,396	262.065	92,040	74,649	269.500	75,484	57,192	272.278	72,040	53,749	255.412	2200
2250	115,984	97,277	316.356	74,516	55,809	262.887	95,562	76,855	270.679	77,397	58,690	273.136	73,856	55,149	256.227	2250
2300	119,035	99,912	317.695	76,345	57,222	263.692	98,199	79,076	271.839	79,316	60,193	273.981	75,676	56,553	257.027	2300
2350	122,091	102,552	319.011	78,178	58,640	264.480	100,846	81,308	272.978	81,243	61,704	274.809	77,496	57,958	257.810	2350
2400	125,152	105,197	320.302	80,015	60,060	265.253	103,508	83,553	274.098	83,174	63,219	275.625	79,320	59,366	258.580	2400
2450	128,219	107,849	321.566	81,852	61,482	266.012	106,183	85,811	275.201	85,112	64,742	276.424	81,149	60,779	259.332	2450

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Table A.9 *Contd.*

$T(K)$, $\bar{h}$ and $\bar{u}$ (kJ/mol), $\bar{s}^\circ$ (kJ/kmol·K)																
	Carbon Dioxide, CO_2 ($\bar{h}_{f^\circ} = -393{,}520$ kJ/kmol)			*Carbon Monoxide, CO ($\bar{h}_{f^\circ} = -110{,}530$ kJ/kmol)*			*Water Vapour, H_2O ($\bar{h}_{f^\circ} = -241{,}820$ kJ/kmol)*			*Oxygen, O_2 ($\bar{h}_{f^\circ} = 0$ kJ/kmol)*			*Nitrogen, N_2 ($\bar{h}_{f^\circ} = 0$ kJ/kmol)*			
T	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	T
2500	131,290	110,504	322.808	83,692	62,906	266.755	108,868	88,082	276.286	87,057	66,271	277.207	82,981	62,195	260.073	2500
2550	134,368	113,166	324.026	85,537	64,335	267.485	111,565	90,364	277.354	89,004	67,802	277.979	84,814	63,613	260.799	2550
2600	137,449	115,832	325.222	87,383	65,766	268.202	114,273	92,656	278.407	90,956	69,339	278.738	86,650	65,033	261.512	2600
2650	140,533	118,500	326.396	89,230	67,197	268.905	116,991	94,958	279.441	92,916	70,883	279.485	88,488	66,455	262.213	2650
2700	143,620	121,172	327.549	91,077	68,628	269.596	119,717	97,269	280.462	94,881	72,433	280.219	90,328	67,880	262.902	2700
2750	146,713	123,849	328.684	92,930	70,066	270.285	122,453	99,588	281.464	96,852	73,987	280.942	92,171	69,306	263.577	2750
2800	149,808	126,528	329.800	94,784	71,504	270.943	125,198	101,917	282.453	98,826	75,546	281.654	94,014	70,734	264.241	2800
2850	152,908	129,212	330.896	96,639	72,945	271.602	127,952	104,256	283.429	100,80	77,112	282.357	95,859	72,163	264.895	2850
2900	156,009	131,898	331.975	98,495	74,383	272.249	130,717	106,605	284.390	102,793	78,682	283.048	97,705	73,593	265.538	2900
2950	159,117	134,589	333.037	100,352	75,825	272.884	133,486	106,959	285.338	104,785	80,258	283.728	99,556	75,028	266.170	2950
3000	162,226	137,283	334.084	102,210	77,267	273.508	136,264	111,321	286.273	106,780	81,837	284.399	101,407	76,464	266.793	3000
3050	165,341	139,982	335.114	104,073	78,715	274.123	139,051	113,692	287.194	108,778	83,419	285.060	103,260	77,902	267.404	3050
3100	168,456	142,681	336.126	105,939	80,164	274.730	141,846	116,072	288.102	110,784	85,009	285.713	105,115	79,341	268.007	3100
3150	171,576	145,385	337.124	107,802	81,612	275.326	144,648	118,458	288.999	112,795	86,601	286.355	106,972	80,782	268.601	3150
3200	174,695	148,089	338.109	109,667	83,061	275.914	147,457	120,851	289.884	114,809	88,203	286.989	108,830	82,224	269.186	3200
3250	177,822	150,801	339.069	111,534	84,513	276.494	150,272	123,250	290.756	116,827	89,804	287.614	110,690	83,668	269.763	3250

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Table A.10 *One Dimensional Isentropic* ***Compressible-Flow Functions*** *for an Ideal Gas with Constant Specific Heat and Molecular Weight and* $\gamma = 1.4$

M	M^*	A/A^*	p/p_0	ρ/ρ_0	T/T_0
0.0	0.00000	∞	1.00000	1.00000	1.00000
0.1	0.10944	5.82183	0.99303	0.99502	0.99800
0.2	0.21822	2.96352	0.97250	0.98028	0.99206
0.3	0.32572	2.03506	0.93947	0.95638	0.98232
0.4	0.43133	1.59014	0.89561	0.92427	0.96899
0.5	0.53452	1.33984	0.84302	0.88517	0.95238
0.6	0.63481	1.18820	0.78400	0.84045	0.93284
0.7	0.73179	1.09437	0.72093	0.79158	0.91075
0.8	0.82514	1.03823	0.65602	0.73999	0.88652
0.9	0.91460	1.00886	0.59126	0.68704	0.68059
1.0	1.0000	1.00000	0.52828	0.63394	0.83333
1.1	1.0812	1.00793	0.46835	0.58170	0.80515
1.2	1.1583	1.03044	0.41238	0.53114	0.77640
1.3	1.2311	1.06630	0.36091	0.48290	0.77640
1.4	1.2999	1.11493	0.31424	0.43742	0.71839
1.5	1.3646	1.17617	0.27240	0.39498	0.68966
1.6	1.4254	1.25023	0.23527	0.35573	0.66138
1.7	1.4825	1.33761	0.20259	0.31969	0.63371
1.8	1.360	1.43898	0.17404	0.28682	0.60680
1.9	1.5861	1.55526	0.14924	0.25699	0.58072
2.0	1.6330	1.68750	0.12780	0.23005	0.86059
2.1	1.6769	1.83694	0.10935	0.20580	0.53135
2.2	1.7179	2.00497	0.93522E-01	0.18405	0.50813
2.3	1.7563	2.19313	0.79973E-01	0.16458	0.48591
2.4	1.7922	2.40310	0.68399E-01	0.14720	0.46468
2.5	1.8257	2.63672	0.58528E-01	0.13169	0.44444
2.6	1.8571	2.89598	0.50115E-01	0.11787	0.42517
2.7	1.8865	3.18301	0.42950E-01	0.10557	0.40683
2.8	1.9140	3.50012	0.36848E-01	0.94626E-01	0.38941
2.9	1.9398	3.84977	0.31651E-01	0.84889E-01	0.37286
3.0	1.9640	4.23457	0.27224E-01	0.76226E-01	0.35714
3.5	2.0642	6.78962	0.13111E-01	0.45233E-01	0.28986
4.0	2.3181	10.7188	0.65861E-02	0.27662E-01	0.23810
4.5	2.1936	16.5622	0.34553E-02	0.17449E-01	0.19802
5.0	2.2361	25.0000	0.18900E-02	0.11340E-01	0.16667
6.0	2.2953	53.1798	0.63336E-02	0.51936E-02	0.12195
7.0	2.3333	104.143	0.24156E-03	0.26088E-02	0.09259
8.0	2.3591	190.109	0.10243E-03	0.14135E-03	0.07246
9.0	2.3772	237.189	0.47386E-04	0.81504E-03	0.05814
10.0	2.3905	535.938	0.23563E-04	0.49482E-03	0.04762
∞	2.4495	∞	0.0	0.0	0.0

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Table A.11 *One-Dimensional* ***Normal Shock Functions*** *for an Ideal Gas with Constant Specific Heat and Molecular Weight and* $\gamma = 1{,}4$

M_x	M_y	p_y/p_x	ρ_y/ρ_x	T_y/T	p_{0y}/p_{0x}	p_{0y}/p_x
1.00	1.00000	1.0000	1.0000	1.0000	1.00000	1.8928
1.05	0.95313	1.1196	1.0840	1.0328	0.99985	2.0083
1.10	0.91177	1.2450	1.1691	1.0642	0.99893	2.1328
1.15	0.87502	1.3763	1.2550	1.0966	0.99669	2.2661
1.20	0.84217	1.5133	1.3416	1.1280	0.99280	2.4075
1.25	0.81264	1.6563	1.4286	1.1594	0.98706	2.5568
1.30	0.78596	1.8050	1.5157	1.1909	0.97937	2.7136
1.35	0.76175	1.9596	1.6028	1.2226	0.96974	2.8778
1.40	0.73971	2.1200	1.6897	1.2547	0.95819	3.0492
1.45	0.71956	2.2863	1.7761	1.2872	0.94484	3.2278
1.50	0.70109	2.4583	1.8621	1.3202	0.92979	3.4133
1.55	0.68610	2.6362	1.9473	1.3538	0.91319	3.6057
1.60	0.66844	2.8200	2.0317	1.3880	0.89520	3.8050
1.65	0.65396	3.0096	2.1152	1.4228	0.87598	4.0110
1.70	0.64054	3.2050	2.1977	1.4583	0.85572	4.2238
1.75	0.62809	3.4063	2.2791	1.4946	0.83457	4.4433
1.80	0.61650	3.6133	2.3592	1.5316	0.81268	4.6695
1.85	0.60570	3.8263	2.4381	1.5693	0.79023	4.9023
1.90	0.59562	4.0450	2.5157	1.6079	0.76736	5.1418
1.95	0.58618	4.2696	2.5919	1.6473	0.74420	5.3878
2.00	0.57735	4.5000	2.6667	1.6875	0.72087	5.6404
2.05	0.56906	4.7362	2.7400	1.7285	0.69751	5.8996
2.10	0.56128	4.9783	2.8119	1.7705	0.67420	6.1654
2.15	0.55395	5.2263	2.8823	1.8132	0.65105	6.4377
2.20	0.54706	5.4800	2.9512	1.8569	0.62814	6.7165
2.25	0.54055	5.7396	3.0186	1.9014	0.60553	7.0018
2.30	0.53441	6.0050	3.0845	1.9468	0.58329	7.2937
2.35	0.52861	6.2762	3.1490	1.9931	0.56148	7.5920
2.40	0.52312	6.5533	3.2119	2.0403	0.54014	7.8969
2.45	0.51792	6.8363	3.2733	2.0885	0.51931	8.2083
2.50	0.51299	7.1250	3.3333	2.1375	0.49901	8.5261
2.55	0.50831	7.4196	3.3919	2.1875	0.47928	8.8505
2.60	0.50387	7.7200	3.4490	2.2383	0.46012	9.1813
2.70	0.49563	8.3383	3.5590	2.3429	0.42359	9.8624
2.80	0.48817	8.9800	3.6636	2.4512	0.38946	10.569
2.90	0.48138	9.6450	3.7629	2.5632	0.35773	11.302
3.00	0.47519	10.333	3.8571	2.6790	0.32834	12.061
4.00	0.43496	18.500	4.5714	4.0469	0.13876	21.068
5.00	0.41523	29.000	5.0000	5.8000	0.06172	32.653
10.00	0.38758	116.50	5.7143	20.387	0.00304	129.22

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Table A.12 *Properties of Saturated Water (Liquid–Vapour): Temperature Table*

Temp. °C	Sat. Presss. bar	Specific Volume m^3/kg		Internal Energy kJ/kg		Enthalpy kJ/kg			Entropy kJ/kg.K		Temp. °C
		Sat. Liquid $v_f \times 10^3$	Sat. Vapour v_g	Sat. Liquid u_f	Sat. Vapour u_g	Sat. Liquid h_f	Evap. h_{fg}	Sat. Vapour h_g	Sat. Liquid s_f	Sat. Vapour s_g	
0.01	0.00611	1.0002	206.136	0.00	2375.3	0.01	2501.3	2501.4	0.0000	9.1562	0.01
4	0.00813	1.0001	157.232	16.77	2380.9	16.78	2491.9	2508.7	0.0610	9.0514	4
5	0.00872	1.0001	147.120	20.97	2382.3	20.98	2489.6	2510.6	0.0761	9.0257	5
6	0.00935	1.0001	137.734	25.19	2383.6	25.20	2487.2	2512.4	0.0912	9.0003	6
8	0.01072	1.0002	120.917	33.59	2386.4	33.60	2482.5	2516.1	0.1212	8.9501	8
10	0.01228	1.0004	106.379	42.00	2389.2	42.01	2477.7	2519.8	0.1510	8.9008	10
11	0.01312	1.0004	99.857	46.20	2390.5	46.20	2475.4	2521.6	0.1658	8.8765	11
12	0.01402	1.0005	93.784	50.41	2391.9	50.41	2473.0	2523.4	0.1806	8.8524	12
13	0.01497	1.0007	88.124	54.60	2393.3	54.60	2470.7	2525.3	0.1953	8.8285	13
14	0.01598	1.0008	82.848	58.79	2394.7	58.80	2468.3	2527.1	0.2099	8.8048	14
15	0.01705	1.0009	77.926	62.99	2396.1	62.99	2465.9	2528.9	0.2245	8.7814	15
16	0.01818	1.0011	73.333	67.18	2397.4	67.19	2463.6	2530.8	0.2390	8.7582	18
17	0.01938	1.0012	69.044	71.38	2398.8	71.38	2461.2	2532.6	0.2535	8.7351	17
18	0.02064	1.0014	65.038	75.57	2400.2	75.58	2458.8	2534.4	0.2679	8.7123	18
19	0.02198	1.0016	61.293	79.76	2401.6	79.77	2456.5	2536.2	0.2823	8.6897	19
20	0.02339	1.0018	57.791	83.95	2402.9	83.96	2454.1	2538.1	0.2966	8.6672	20
21	0.02487	1.0020	54.514	88.14	2404.3	88.14	2451.8	2539.9	0.3109	8.6450	21
22	0.02645	1.0022	51.447	92.32	2405.7	92.33	2449.4	2541.7	0.3251	8.6229	22
23	0.02810	1.0024	48.574	96.51	2407.0	96.52	2447.0	2543.5	0.3393	8.6011	23
24	0.02985	1.0027	45.883	100.70	2408.4	100.70	2444.7	2545.4	0.3534	8.5794	24
25	0.03169	1.0029	43.360	104.88	2409.8	104.89	2442.3	2547.2	0.3674	8.5580	25
26	0.03363	1.0032	40.994	109.06	2411.1	109.07	2439.9	2549.0	0.3814	8.5367	26
27	0.03567	1.0035	38.774	113.25	2412.5	113.25	2437.6	2550.8	0.3954	8.5156	27
28	0.03782	1.0037	36.690	117.42	2413.9	117.43	2435.2	2552.6	0.4093	8.4946	28
29	0.04008	1.0040	34.733	121.60	2415.2	121.61	2432.8	2554.5	0.4231	8.4739	29
30	0.04246	1.0043	32.894	125.78	2416.6	125.79	2430.5	2556.3	0.4369	8.4533	30
31	0.04496	1.0046	31.165	129.96	2418.0	129.97	2428.1	2558.1	0.4507	8.4329	31
32	0.04759	1.0050	29.540	134.14	2419.3	134.15	2425.7	2559.9	0.4644	8.4127	32
33	0.05034	1.0053	28.011	138.32	2420.7	138.33	2423.4	2561.7	0.4781	8.3927	33
34	0.05324	1.0056	26.571	142.50	2422.0	142.50	2421.0	2563.5	0.4917	8.3728	34
35	0.05628	1.0060	25.216	146.67	2423.4	146.68	2418.6	2565.3	0.5053	8.3531	35
36	0.05947	1.0063	23.940	150.85	2424.7	150.86	2416.2	2567.1	0.5188	8.3336	36
38	0.06632	1.0071	21.602	159.20	2427.4	159.21	2411.5	2570.7	0.5458	8.2950	38
40	0.07384	1.0078	19.523	167.56	2430.1	167.57	2406.7	2574.3	0.5725	8.2570	40
45	0.09593	1.0099	15.258	188.44	2436.8	188.45	2394.8	2583.2	0.6387	8.1648	45
50	0.1235	1.0121	12.032	209.32	2443.5	209.33	2382.7	2592.1	0.7038	8.0763	50

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Table A.12 *Contd.*

		Specific Volume m^3/kg		*Internal Energy* kJ/kg		*Enthalpy* kJ/kg			*Entropy* kJ/kg.K		
Temp. °C	*Sat. Presss.* bar	*Sat. Liquid* $v_f \times 10^3$	*Sat. Vapour* v_g	*Sat. Liquid* u_f	*Sat. Vapour* u_g	*Sat. Liquid* h_f	*Evap.* h_{fg}	*Sat. Vapour* h_g	*Sat. Liquid* s_f	*Sat. Vapour* s_g	*Temp.* °C
55	0.1576	1.0146	9.568	230.21	2450.1	230.23	2370.7	2600.9	0.7679	7.9913	55
60	0.1994	1.0172	7.671	251.11	2456.6	251.13	2358.5	2609.6	0.8312	7.9096	60
65	0.2503	1.0199	6.197	272.02	2463.1	272.06	2346.2	2618.3	0.8935	7.8310	65
70	0.3119	1.0228	5.042	292.95	2469.6	292.98	2333.8	2626.8	0.9549	7.7553	70
75	0.3858	1.0259	4.131	313.90	2475.9	313.93	2321.4	2635.3	1.0155	7.6824	75
80	0.4739	1.0291	3.407	334.86	2482.2	334.91	2308.8	2643.7	1.0753	7.6122	80
85	0.5783	1.0325	2.828	355.84	2488.4	355.90	2296.0	2651.9	1.1343	7.5445	85
90	0.7014	1.0360	2.361	376.85	2494.5	376.92	2283.2	2660.1	1.1925	7.4791	90
95	0.8455	1.0397	1.982	397.88	2500.6	397.96	2270.2	2668.1	1.2500	7.4159	95
100	1.014	1.0435	1.673	418.94	2506.5	419.04	2257.0	2676.1	1.3069	7.3549	100
110	1.433	1.0516	1.210	461.14	2518.1	461.30	2230.2	2691.5	1.4185	7.2387	110
120	1.985	1.0603	0.8919	503.50	2529.3	503.71	2202.6	2706.3	1.5276	7.1296	120
130	2.701	1.0697	0.6685	546.02	2539.9	546.31	2174.2	2720.5	1.6344	7.0269	130
140	3.6153	1.0797	0.5089	588.74	2550.0	589.13	2144.7	2733.9	1.7391	6.9299	140
150	4.758	1.0905	0.3928	631.68	2559.5	632.20	2114.3	2746.5	1.8418	6.8379	150
160	6.178	1.1020	0.3071	674.86	2568.4	675.55	2082.6	2758.1	1.9427	6.7502	160
170	7.917	1.1143	0.2428	718.33	2576.5	719.21	2049.5	2768.7	2.0419	6.6663	170
180	10.02	1.1274	0.1941	762.09	2583.7	763.22	2015.0	2778.2	2.1396	6.5857	180
190	12.54	1.1414	0.1565	806.19	2590.0	807.62	1978.8	2786.4	2.2359	6.5079	190
200	15.54	1.1565	0.1274	850.65	2595.3	852.45	1940.7	2793.2	2.3309	6.4323	200
210	19.06	1.1726	0.1044	895.53	2599.5	897.76	1900.7	2798.5	2.4248	6.3585	210
220	23.18	1.1900	0.08619	940.87	2602.4	943.62	1858.5	2802.1	2.5178	6.2861	220
230	27.95	1.2088	0.07158	986.74	2603.9	990.12	1813.8	2804.0	2.6099	6.2146	230
240	33.44	1.2291	0.05976	1033.2	2604.0	1037.3	1766.5	2803.8	2.7015	6.1437	240
250	39.73	1.2512	0.05013	1080.4	2602.4	1085.4	1716.2	2801.5	2.7927	6.0730	250
260	46.88	1.2755	0.04221	1128.4	2599.0	1134.4	1662.5	2796.6	2.8838	6.0019	260
270	54.99	1.3023	0.03564	1177.4	2593.7	1184.5	1605.2	2789.7	2.9751	5.9301	270
280	64.12	1.3321	0.03017	1227.5	2586.1	1236.0	1543.6	2779.6	3.0668	5.8571	280
290	74.36	1.3656	0.02557	1278.9	2576.0	1289.1	1477.1	2766.2	3.1594	5.7821	290
300	85.81	1.4036	0.02167	1332.0	2563.0	1344.0	1404.9	2749.0	3.2534	5.7045	300
320	112.7	1.4988	0.01549	1444.6	2525.5	1461.5	1238.6	2700.1	3.4480	5.5362	320
340	145.9	1.6379	0.01080	1570.3	2464.6	1594.2	1027.9	2622.0	3.6594	5.3357	340
360	186.5	1.8925	0.006945	1725.2	2351.5	1760.5	720.5	2481.0	3.9147	5.0526	360
374.14	220.9	3.155	0.003155	2029.6	2029.6	2099.3	0	2099.3	4.4298	4.4298	374.14

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Table A.13 *Properties of* ***Saturated Water*** *(Liquid vapour): Pressure Table*

		Specific Volume m^3/kg		*Internal Energy* kJ/kg		*Enthalpy* kJ/kg			*Entropy* $kJ/kg \cdot K$		
Press. *bar*	*Sat. Temp.* °C	*Sat. Liquid* $v_f \times 10^3$	*Sat. Vapour* v_g	*Sat. Liquid* u_f	*Sat. Vapour* u_g	*Sat. Liquid* h_f	*Evap.* h_{fg}	*Sat. Vapour* h_g	*Sat. Liquid* s_f	*Sat. Vapour* s_g	*Press.* *bar*
0.04	28.96	1.0040	34.800	121.45	2415.2	121.46	2432.9	2554.4	0.4226	8.4746	0.04
0.06	36.16	1.0064	23.739	151.53	2425.0	151.53	2415.9	2567.4	0.5210	8.3304	0.06
0.08	41.51	1.0084	18.103	173.87	2432.2	173.88	2403.1	2577.0	0.5926	8.2287	0.18
0.10	45.81	1.0102	14.674	181.82	2437.9	191.83	2392.8	2584.7	0.6493	8.1502	0.10
0.20	60.06	1.0172	7.649	251.38	2456.7	251.40	2358.3	2609.7	0.8320	7.9085	0.20
0.30	69.10	1.0223	5.229	289.20	2468.4	289.23	2336.1	2625.3	0.9439	7.7686	0.30
0.40	75.87	1.0265	3.993	317.53	2477.0	317.58	2319.2	2636.8	1.0259	7.6700	0.40
0.50	81.33	1.0300	3.240	340.44	2483.9	340.49	2305.4	2645.9	1.0910	7.5939	0.50
0.60	85.94	1.0331	2.732	359.79	2489.6	359.86	2293.6	2653.5	1.1453	7.5320	0.60
0.70	89.95	1.0360	2.365	376.63	2494.5	376.70	2283.3	2660.0	1.1919	7.4797	0.70
0.80	93.50	1.0380	2.087	391.58	2498.8	391.66	2274.1	2665.8	1.2329	7.4346	0.80
0.90	96.71	1.0410	1.869	405.06	2502.6	405.15	2265.7	2670.9	1.2695	7.3949	0.90
1.00	99.63	1.0432	1.694	417.36	2506.1	417.46	2258.0	2675.5	1.3026	7.3594	1.00
1.50	111.4	1.0528	1.159	466.94	2519.7	467.11	2226.5	2693.6	1.4336	7.2233	1.50
2.00	120.2	1.0605	0.8857	504.49	2529.5	504.70	2201.9	2706.7	1.5301	7.1271	2.00
2.50	127.4	1.0672	0.7187	535.10	2537.2	535.37	2181.5	2716.9	1.6072	7.0527	2.50
3.00	133.6	1.0732	0.6058	561.15	2543.6	561.47	2163.8	2725.3	1.6718	6.9919	3.00
3.50	138.9	1.0786	0.5243	583.95	2546.9	584.33	2148.1	2732.4	1.7275	6.9405	3.50
4.00	143.6	1.0836	0.4625	604.31	2553.6	604.74	2133.8	2738.6	1.7766	6.8959	4.00
4.50	147.9	1.0882	0.4140	622.25	2557.6	623.25	2120.7	2743.9	1.8207	6.8565	4.50
5.00	151.9	1.0926	0.3749	639.68	2561.2	640.23	2108.5	2748.7	1.8607	6.8212	5.00
6.00	158.9	1.1006	0.3157	669.90	2567.4	670.56	2086.3	2756.8	1.9312	6.7600	6.00
7.00	165.0	1.1080	0.2729	696.44	2572.5	697.22	2066.3	2763.5	1.9922	6.7080	7.00
8.00	170.4	1.1148	0.2404	720.22	2576.8	721.11	2048.0	2769.1	2.0462	6.6628	8.00
9.00	175.4	1.1212	0.2150	741.83	2580.5	742.83	2031.1	2773.9	2.0946	6.6226	9.00
10.0	179.9	1.1273	0.1944	761.68	2583.6	762.81	2015.3	2778.1	2.1387	6.5863	10.0
15.0	198.3	1.1539	0.1318	843.16	2594.5	844.84	1947.3	2792.2	2.3150	6.4448	15.0
20.0	212.4	1.1767	0.09963	906.44	2600.3	908.79	1890.7	2799.5	2.4474	6.3409	20.0
25.0	224.0	1.1983	0.07998	959.11	2603.1	962.11	1841.0	2803.1	2.5547	6.2575	25.0
30.0	233.9	1.2165	0.06668	1004.8	2604.1	1008.4	1795.7	2804.2	2.6457	6.1869	30.0
35.0	242.6	1.2347	0.05707	1045.4	2603.7	1049.8	1753.7	2803.4	2.7253	6.1253	35.0
40.0	250.4	1.2522	0.04978	1082.3	2602.3	1087.3	1714.1	2801.4	2.7964	6.0701	40.0
45.0	257.5	1.2692	0.04406	1116.2	2600.1	1121.9	1676.4	2798.3	2.8610	6.0199	45.0
50.0	264.0	1.2859	0.03944	1147.8	2597.1	1154.2	1640.1	2794.3	2.9202	5.9734	50.0

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Table A.13 *Contd.*

		Specific Volume m^3/kg		*Internal Energy* kJ/kg		*Enthalpy* kJ/kg			*Entropy* $kJ/kg \cdot K$		
Press. *bar*	*Sat. Temp.* °C	*Sat. Liquid* $v_f \times 10^3$	*Sat. Vapour* v_g	*Sat. Liquid* u_f	*Sat. Vapour* u_g	*Sat. Liquid* h_f	*Evap.* h_{fg}	*Sat. Vapour* h_g	*Sat. Liquid* s_f	*Sat. Vapour* s_g	*Press.* *bar*
60.0	275.6	1.3187	0.03244	1205.4	2589.7	1213.4	1571.0	2784.3	3.0267	5.8892	60.0
70.0	285.9	1.3513	0.02737	1257.6	2580.5	1267.0	1505.1	2772.1	3.1211	5.8133	70.0
80.0	295.1	1.3842	0.02352	1305.6	2569.8	1316.6	1441.3	2758.0	3.2068	5.7432	80.0
90.0	303.4	1.4178	0.02048	1350.5	2557.8	1363.3	1378.9	2742.1	3.2858	5.6772	90.0
100.0	311.1	1.4524	0.01803	1393.0	2544.4	1407.6	1317.1	2724.7	3.3596	5.6141	100.0
110.0	318.2	1.4886	0.01599	1433.7	2529.8	1450.1	1255.5	2705.6	3.4295	5.5527	110.0
120.0	324.8	1.5267	0.01426	1473.0	2513.7	1491.3	1193.6	2684.9	3.4962	5.4924	120.0
130.0	330.9	1.5671	0.01278	1511.0	2496.1	1531.5	1130.7	2662.2	3.5606	5.4323	130.0
140.0	336.8	1.6107	0.01149	1548.6	2476.8	1571.1	1066.5	2637.6	3.6232	5.3717	140.0
150.0	342.2	1.6581	0.01034	1585.6	2455.5	1610.5	1000.0	2610.5	3.6848	5.3098	150.0
160.0	347.4	1.7107	0.009306	1622.7	2431.7	1650.1	930.6	2580.6	3.7481	5.2455	160.0
170.0	352.4	1.7702	0.008364	1660.2	2405.0	1690.3	856.9	2547.2	3.8079	5.1777	170.0
180.0	357.1	1.8397	0.007489	1698.9	2374.3	1732.0	777.1	2509.1	3.8715	5.1044	180.0
190.0	361.5	1.9243	0.006657	1739.9	2338.1	1776.5	688.0	2464.5	3.9388	5.0228	190.0
200.0	365.8	2.036	0.005834	1785.6	2293.0	1826.3	583.4	2409.7	4.0139	4.9269	200.0
220.9	374.1	3.155	0.003155	2029.6	2029.6	2099.3	0	2099.3	4.4298	4.4298	220.9

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Table A.14 *Properties of* ***Superheated Water Vapor***

T °C	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	$p = 0.06$ bar $= 0.006$ MPa ($T_{sat} = 36.16$°C)				$p = 0.35$ bar $= 0.035$ MPa ($T_{sat} = 72.69$°C)			
Sat.	23.739	2425.0	2567.4	8.3304	4.526	2473.0	2631.4	7.7158
80	27.132	2487.3	2650.1	8.5804	4.625	2483.7	2645.6	7.7564
120	30.219	2544.7	2726.0	8.7840	5.163	2542.4	2723.1	7.9644
160	33.302	2602.7	2802.5	8.9693	5.696	2601.2	2800.6	8.1519
200	36.383	2661.4	2879.7	9.1398	6.228	2660.4	2878.4	8.3237
240	39.462	2721.0	2957.8	9.2982	6.758	2720.3	2956.8	8.4828
280	42.540	2781.5	3036.8	9.4464	7.287	2780.9	3036.0	8.6314
320	45.618	2843.0	3116.7	9.5859	7.815	2842.5	3116.1	8.7712
360	48.696	2905.5	3197.7	9.7180	8.344	2905.1	3197.1	8.9034
400	51.774	2969.0	3279.6	9.8435	8.872	2968.6	3279.2	9.0291
440	54.851	3033.5	3362.6	9.9633	9.400	3033.2	3362.2	9.1490
500	59.467	3132.3	3489.1	10.1336	10.192	3132.1	3488.8	9.3194
	$p = 0.70$ bar $= 0.07$ MPa ($T_{sat} = 89.95$°C)				$p = 1.0$ bar $= 0.10$ MPa ($T_{sat} = 99.63$°C)			
Sat.	2.365	2494.5	2660.0	7.4797	1.694	2506.1	2675.5	7.3594
100	2.434	2509.7	2680.0	7.5341	1.696	2506.7	2676.2	7.3614
120	2.571	2539.7	2719.6	7.6375	1.793	2537.3	2716.6	7.4668
160	2.841	2599.4	2798.2	7.8279	1.984	2597.8	2796.2	7.6597
200	3.108	2659.1	2876.7	8.0012	2.172	2658.1	2875.3	7.8343
240	3.374	2719.3	2955.5	8.1611	2.359	2718.5	2954.5	7.9949
280	3.640	2780.2	3035.0	8.3162	2.546	2779.6	3034.2	8.1445
320	3.905	2842.0	3115.3	8.4504	2.732	2841.5	3114.6	8.2849
360	4.170	2904.6	3196.5	8.5828	2.917	2904.2	3195.9	8.4175
400	4.434	2968.2	3278.6	8.7086	3.103	2967.9	3278.2	8.5435
440	4.698	3032.9	3361.8	8.8286	3.288	3032.6	3361.4	8.6636
500	5.095	3131.8	3488.5	8.9991	3.565	3131.6	3488.1	8.8342
	$p = 1.5$ bar $= 0.15$ MPa ($T_{sat} = 111.37$°C)				$p = 3.0$ bar $= 0.30$ MPa ($T_{sat} = 133.55$°C)			
Sat.	1.159	2519.7	2693.6	7.2233	0.606	2543.6	2725.3	6.9919
120	1.188	2533.3	2711.4	7.2693				
160	1.317	2595.2	2792.8	7.4665	0.651	2587.1	2782.3	7.1276
200	1.444	2656.2	2872.9	7.6433	0.716	2650.7	2865.5	7.3115
240	1.570	2717.2	2952.7	7.8052	0.781	2713.1	2947.3	7.4774
280	1.695	2778.6	3032.8	7.9555	0.844	2775.4	3028.6	7.6299
320	1.819	2840.6	3113.5	8.0964	0.907	2838.1	3110.1	7.7722
360	1.943	2903.5	3195.0	8.2293	0.969	2901.4	3192.2	7.9061
400	2.067	2967.3	3277.4	8.3555	1.032	2965.6	3275.0	8.0330
440	2.191	3032.1	3360.7	8.4757	1.094	3030.6	3358.7	8.1538
500	2.376	3131.2	3487.6	8.6466	1.187	3130.0	3486.0	8.3251
600	2.685	3301.7	3704.3	8.9101	1.341	3300.8	3703.2	8.5892

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Table A.14 *Contd.*

T °C	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	$p = 5.0$ bar $= 0.50$ MPa ($T_{sat} = 151.86$°C)				$p = 7.0$ bar $= 0.70$ MPa ($T_{sat} = 164.97$°C)			
Sat.	0.3749	2561.2	2748.7	6.8213	0.2729	2572.5	2763.5	6.7080
180	0.4045	2609.7	2812.0	6.9656	0.2847	2599.8	2799.1	6.7880
200	0.4249	2642.9	2855.4	7.0592	0.2999	2634.8	2844.8	6.8865
240	0.4646	2707.6	2939.9	7.2307	0.3292	2701.8	2932.2	7.0641
280	0.5034	2771.2	3022.9	7.3865	0.3574	2766.9	3017.1	7.2233
320	0.5416	2834.7	3105.6	7.5308	0.3852	2831.3	3100.9	7.3697
360	0.5796	2898.7	3188.4	7.6660	0.4126	2895.8	3184.7	7.5063
400	0.6173	2963.2	3271.9	7.7938	0.4397	2960.9	3268.7	7.6350
440	0.6548	3028.6	3356.0	7.9152	0.4667	3026.6	3353.3	7.7571
500	0.7109	3128.4	3483.9	8.0873	0.5070	3126.8	3481.7	7.9299
600	0.8041	3299.6	3701.7	8.3522	0.5738	3298.5	3700.2	8.1956
700	0.8969	3477.5	3925.9	8.5952	0.6403	3476.6	3924.8	8.4391
	$p = 10.0$ bar $= 1.0$ MPa ($T_{sat} = 179.91$°C)				$p = 15.0$ bar $= 1.5$ MPa ($T_{sat} = 198.32$°C)			
Sat.	0.1944	2583.6	2778.1	6.5865	0.1318	2594.5	2792.2	6.4448
200	0.2060	2621.9	2827.9	6.6940	0.1325	2598.1	2796.8	6.4546
240	0.2275	2692.9	2920.4	6.8817	0.1483	2676.9	2899.3	6.6628
280	0.2480	2760.2	3008.2	7.0465	0.1627	2748.6	2992.7	6.8381
320	0.2678	2826.1	3093.9	7.1962	0.1765	2817.1	3081.9	6.9938
360	0.2873	2891.6	3178.9	7.3349	0.1899	2884.4	3169.2	7.1363
400	0.3066	2957.3	3263.9	7.4651	0.2030	2951.3	3255.8	7.2690
440	0.3257	3023.6	3349.3	7.5883	0.2160	3018.5	3342.5	7.3940
500	0.3541	3124.4	3478.5	7.7622	0.2352	3120.3	3473.1	7.5698
540	0.3729	3192.6	3565.6	7.8720	0.2478	3189.1	3560.9	7.6805
600	0.4011	3296.8	3697.9	8.0290	0.2668	3293.9	3694.0	7.8385
640	0.4198	3367.4	3787.2	8.1290	0.2793	3364.8	3783.8	7.9391
	$p = 20.0$ bar $= 2.0$ MPa ($T_{sat} = 212.42$°C)				$p = 30.0$ bar $= 3.0$ MPa ($T_{sat} = 233.90$°C)			
Sat.	0.0996	2600.3	2799.5	6.3409	0.0667	2604.1	2804.2	6.1869
240	0.1085	2659.6	2876.5	6.4952	0.0682	2619.7	2824.3	6.2265
280	0.1200	2736.4	2976.4	6.6828	0.0771	2709.9	2941.3	6.4462
320	0.1308	2807.9	3069.3	6.8452	0.0850	2788.4	3043.4	6.6245
360	0.1411	2877.0	3159.3	6.9917	0.0923	2861.7	3138.7	6.7801
400	0.1512	2945.2	3247.6	7.1271	0.0994	2932.8	3230.9	6.9212
440	0.1611	3013.4	3335.5	7.2540	0.1062	3002.9	3321.5	7.0520
500	0.1757	3116.2	3467.6	7.4317	0.1162	3108.0	3456.5	7.2338
540	0.1853	3185.6	3556.1	7.5434	0.1227	3178.4	3546.6	7.3474
600	0.1996	3290.9	3690.1	7.7024	0.1324	3285.0	3682.3	7.5085
640	0.2091	3362.2	2780.4	7.8035	0.1388	3357.0	3773.5	7.6106
700	0.2232	3470.9	3917.4	7.9487	0.1484	3466.5	3911.7	7.7571

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Table A.14 *Contd.*

T °C	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	p = 40 bar = 4.0 MPa (T_{sat} = 250.4°C)				p = 60 bar = 6.0 MPa (T_{sat} = 275.64°C)			
Sat.	0.04978	2602.3	2801.4	6.0701	0.03244	2589.7	2784.3	5.8892
280	0.05546	2680.0	2901.8	6.2568	0.03317	2605.2	2804.2	5.9252
320	0.06199	2767.4	3015.4	6.4553	0.03876	2720.0	2952.6	6.1846
360	0.06788	2845.7	3117.2	6.6215	0.04331	2811.2	3071.1	6.3782
400	0.07341	2919.9	3213.3	6.7690	0.04793	2892.9	3177.2	6.5408
440	0.07872	2992.2	3307.1	6.9041	0.05122	2970.0	3277.3	6.6853
500	0.08643	3099.5	3445.3	7.0901	0.05665	3082.2	3422.2	6.8803
540	0.09145	3171.1	3536.9	7.2056	0.06015	3156.1	3517.0	6.9999
600	0.09885	3279.1	3674.4	7.3688	0.06525	3266.9	3658.4	7.1677
640	0.1037	3351.8	3766.6	7.4720	0.06859	3341.0	3752.6	7.2731
700	0.1110	3462.1	3905.9	7.6198	0.07352	3453.1	3894.1	7.4234
740	0.1157	3536.6	3999.6	7.7141	0.07677	3528.3	3989.2	7.5190
	p = 80 bar = 8.0 MPa (T_{sat} = 295.06°C)				p = 100 bar = 10.0 MPa (T_{sat} = 311.06°C)			
Sat.	0.02352	2569.8	2758.0	5.7432	0.01803	2544.4	2724.7	5.6141
320	0.02682	2662.7	2877.2	5.9489	0.01925	2588.8	2781.3	5.7103
360	0.03089	2772.7	3019.8	6.1819	0.02331	2729.1	2962.1	6.0060
400	0.03432	2863.8	3138.3	6.3634	0.02641	2832.4	3096.5	6.2120
440	0.03742	2946.7	3246.1	6.5190	0.02911	2922.1	3213.2	6.3805
480	0.04034	3025.7	3348.4	6.6586	0.03160	3005.4	3321.4	6.5282
520	0.04313	3102.7	3447.7	6.7871	0.03394	3085.6	3425.1	6.6622
560	0.04582	3178.7	3545.3	6.9072	0.03619	3164.1	3526.0	6.7864
600	0.04845	3254.4	3642.0	7.0206	0.03837	3241.7	3625.3	6.9029
640	0.05102	3330.1	3738.3	7.1283	0.04048	3318.9	3723.7	7.0131
700	0.05481	3443.9	3882.4	7.2812	0.04358	3434.7	3870.5	7.1687
740	0.05729	3520.4	3978.7	7.3782	0.04560	3512.4	3968.1	7.2670
	p = 120 bar = 12.0 MPa (T_{sat} = 324.75°C)				p = 140 bar = 14.0 MPa (T_{sat} = 336.75°C)			
Sat.	0.01426	2513.7	2684.9	5.4924	0.01149	2476.8	2637.61	5.3717
360	0.01811	2678.4	2895.7	5.8361	0.01422	2617.4	2816.5	5.6602
400	0.02108	2798.3	3051.3	6.0747	0.01722	2760.9	3001.9	5.9448
440	0.02355	2896.1	3178.7	6.2586	0.01954	2868.6	3142.2	6.1474
480	0.02576	2984.4	3293.5	6.4154	0.02157	2962.5	3264.5	6.3143
520	0.02781	3068.0	3401.8	6.5555	0.02343	3049.8	3377.8	6.4610
560	0.02977	3149.0	3506.2	6.6840	0.02517	3133.6	3486.0	6.5941
600	0.03164	3228.7	3608.3	6.8037	0.02683	3215.4	3591.1	6.7172
640	0.03345	3307.5	3709.0	6.9164	0.02843	3296.0	3694.1	6.8326
700	0.03610	3425.2	3858.4	7.0749	0.03075	3415.7	3846.2	6.9939
740	0.03781	3503.7	3957.4	7.1746	0.03225	3495.2	3946.7	7.0952

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Table A.14 *Contd.*

T °C	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	p = 160 bar = 16.0 MPa (T_{sat} = 347.44°C)				p = 180 bar = 18.0 MPa (T_{sat} = 357.06°C)			
Sat.	0.00931	2431.7	2580.6	5.2455	0.00749	2374.3	2509.1	5.1044
360	0.01105	2539.0	2715.8	5.4614	0.00809	2418.9	2564.5	5.1922
400	0.01426	2719.4	2947.6	5.8175	0.01190	2672.8	2887.0	5.6887
440	0.01652	2839.4	3103.7	6.0429	0.01414	2808.2	3062.8	5.9428
480	0.01842	2939.7	3234.4	6.2215	0.01596	2915.9	3203.2	6.1345
520	0.02013	3031.1	3353.3	6.3752	0.01757	3011.8	3378.0	6.2960
560	0.02172	3117.8	3465.4	6.5132	0.01904	3101.7	3444.4	6.4392
600	0.02323	3201.8	3573.5	6.6399	0.02042	3188.0	3555.6	6.5696
640	0.02467	3284.2	3678.9	6.7580	0.02174	3272.3	3663.6	6.6905
700	0.02674	3406.0	3833.9	6.9224	0.02362	3396.3	3821.5	6.8580
740	0.02808	3486.7	3935.9	7.0251	0.02483	3478.0	3925.0	6.9623
	p = 200 bar = 20.0 MPa (T_{sat} = 365.81°C)				p = 240 bar = 24.0 MPa			
Sat.	0.00583	2293.0	2409.7	4.9269				
400	0.00994	2619.3	2818.1	5.5540	0.00673	2477.8	2639.4	5.2393
440	0.01222	2774.9	3019.4	5.8450	0.00929	2700.6	2923.4	5.6506
480	0.01399	2891.2	3170.8	6.0518	0.01100	2838.3	3102.3	5.8950
520	0.01551	2992.0	3302.2	6.2218	0.01241	2950.5	3248.5	6.0842
560	0.01689	3085.2	3423.0	6.3705	0.01366	3051.1	3379.0	6.2448
600	0.01818	3174.0	3537.6	6.5048	0.01481	3145.2	3500.7	6.3875
640	0.01940	3260.2	3648.1	6.6286	0.01588	3235.5	3616.7	6.5174
700	0.02113	3386.4	3809.0	6.7993	0.01739	3366.4	3783.8	6.6947
740	0.02224	3469.3	3914.1	6.9052	0.01835	3451.7	3892.1	6.8038
800	0.02385	3592.7	4069.7	7.0544	0.01974	3578.0	4051.6	6.9567
	p = 280 bar = 28.0 MPa				p = 320 bar = 32.0 MPa			
400	0.00383	2223.5	2330.7	4.7494	0.00236	1980.4	2055.9	4.3239
440	0.00712	2613.2	2812.6	5.4494	0.00544	2509.0	2683.0	5.2327
480	0.00885	2780.8	3028.5	5.7446	0.00722	2718.1	2949.2	5.5968
520	0.01020	2906.8	3192.3	5.9566	0.00853	2860.7	3133.7	5.8357
560	0.01136	3015.7	3333.7	6.1307	0.00963	2979.0	3287.2	6.0246
600	0.01241	3115.6	3463.0	6.2823	0.01061	3085.3	3424.6	6.1858
640	0.01338	3210.3	3584.8	6.4187	0.01150	3184.5	3552.5	6.3290
700	0.01473	3346.1	3758.4	6.6029	0.01273	3325.4	3732.8	6.5203
740	0.01558	3433.9	3870.0	6.7153	0.01350	3415.9	3847.8	6.6361
800	0.01680	3563.1	4033.4	6.8720	0.01460	3548.0	4015.1	6.7966
900	0.01873	3774.3	4298.8	7.1084	0.01633	3762.7	4285.1	7.0372

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Table A.15 *Properties of Compressed Liquid Water*

T °C	$v \times 10^3$ m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	$v \times 10^3$ m^3/kg	u kJ/kg	h Kj/kg	s kJ/kg · K
	p = 25 bar = 2.5 MPa (T_{sat} = 223.99°C)				p = 50 bar = 5.0 MPa (T_{sat} = 263.99°C)			
20	1.0006	83.80	86.30	0.2961	0.9995	83.65	88.65	0.2956
40	1.0067	167.25	169.77	0.5715	1.0056	166.95	171.97	0.5705
80	1.0280	334.29	336.86	1.0737	1.0268	333.72	338.85	1.0720
100	1.0423	418.24	420.85	1.3050	1.0410	417.52	422.72	1.3030
140	1.0784	587.82	590.52	1.7369	1.0768	586.76	592.15	1.7343
180	1.1261	761.16	763.97	2.1375	1.1240	759.63	765.25	2.1341
200	1.1555	849.9	852.8	2.3294	1.1530	848.1	853.9	2.3255
220	1.1898	940.7	943.7	2.5174	1.1866	938.4	944.4	2.5128
Sat.	1.1973	959.1	962.1	2.5546	1.2859	1147.8	1154.2	2.9202
	p = 75 bar = 7.5 MPa (T_{sat} = 290.59°C)				p = 100 bar = 10.0 MPa (T_{sat} = 311.06°C)			
20	0.9984	83.50	90.99	0.2950	0.9972	83.36	93.33	0.2945
40	1.0045	166.64	174.18	0.5696	1.0034	166.35	176.38	0.5686
80	1.0256	333.15	340.84	1.0704	1.0245	332.59	342.83	1.0688
100	1.0397	416.81	424.62	1.3011	1.0385	416.12	426.50	1.2992
140	1.0752	585.72	593.78	1.7317	1.0737	584.68	595.42	1.7292
180	1.1219	758.13	766.55	2.1308	1.1199	756.65	767.84	2.1275
220	1.1835	936.2	945.1	2.5083	1.1805	934.1	945.9	2.5039
260	1.2696	1124.4	1134.0	2.8763	1.2645	1121.1	1133.7	2.8699
Sat.	1.3677	1282.0	1292.2	3.1649	1.4524	1393.0	1407.6	3.3596
	p = 150 bar = 15.0 MPa (T_{sat} = 342.24°C)				p = 200 bar = 20.0 MPa (T_{sat} = 365.81°C)			
20	0.9950	83.06	97.99	0.2934	0.9928	82.77	102.62	0.2923
40	1.0013	165.76	180.78	0.5666	0.9992	165.17	185.16	0.5646
80	1.0222	331.48	346.81	1.0656	1.0199	330.40	350.80	1.0624
100	1.0361	414.74	430.28	1.2955	1.0337	413.39	434.06	1.2917
140	1.0707	582.66	598.72	1.7242	1.0678	580.69	602.04	1.7193
180	1.1159	753.76	770.50	2.1210	1.1120	750.95	773.20	2.1147
220	1.1748	929.9	947.5	2.4953	1.1693	925.9	949.3	2.4870
260	1.2550	1114.6	1133.4	2.8576	1.2462	1108.6	1133.5	2.8459
300	1.3770	1316.6	1337.3	3.2260	1.3596	1306.1	1333.3	3.2071
Sat.	1.6581	1585.6	1610.5	3.6848	2.036	1785.6	1826.3	4.0139
	p = 250 bar = 25 MPa				p = 300 bar = 30.0 MPa			
20	0.9907	82.47	107.24	0.2911	0.9886	82.17	111.84	0.2899
40	0.9971	164.60	189.52	0.5626	0.9951	164.04	193.89	0.5607
100	1.0313	412.08	437.85	1.2881	1.0290	410.78	441.66	1.2844
200	1.1344	834.5	862.8	2.2961	1.1302	831.4	865.3	2.2893
300	1.3442	1296.6	1330.2	3.1900	1.3304	1287.9	1327.8	3.1741

Table A.16 *Properties of* ***Saturated Water*** *(Solid–Vapour): Temperature Table*

		Specific Volume m^3/kg		*Internal Energy* kJ/kg			*Enthalpy* kJ/kg			*Entropy* $kJ/kg \cdot K$		
Temp. °C	*Sat. Pressure* kPa	*Sat. Solid* $v_i \times 10^3$	*Sat. Vapour* v_g	*Sat. Solid* u_i	*Subl.* u_{ig}	*Sat. Vapour* u_g	*Sat. Solid* h_i	*Subl.* h_{ig}	*Sat. Vapour* h_g	*Sat. Solid* s_i	*Subl.* s_{ig}	*Sat. Vapour* s_g
0.01	0.6113	1.0908	206.1	–333.40	2708.7	2375.3	–333.40	2834.8	2501.4	–1.221	10.378	9.156
0	0.6108	1.0908	206.3	–333.43	2708.8	2375.3	–333.43	2834.8	2501.3	–1.221	10.378	9.157
–2	0.5176	1.0904	241.7	–337.62	2710.2	2372.6	–337.62	2835.3	2497.7	–1.237	10.456	9.219
–4	0.4375	1.0901	283.8	–341.78	2711.6	2369.8	–341.78	2835.7	2494.0	–1.253	10.536	9.283
–6	0.3689	1.0898	334.2	–345.91	2712.9	2367.0	–345.91	2836.2	2490.3	–1.268	10.616	9.348
–8	0.3102	1.0894	394.4	–350.02	2714.2	2364.2	–350.02	2836.6	2486.6	–1.284	10.698	9.414
–10	0.2602	1.0891	466.7	–354.09	2715.5	2361.4	–354.09	2837.0	2482.9	–1.299	10.781	9.481
–12	0.2176	1.0888	553.7	–358.14	2716.8	2358.7	–358.14	2837.3	2479.2	–1.315	10.865	9.550
–14	0.1815	1.0884	658.8	–362.15	2718.0	2355.9	–362.15	2837.6	2475.5	–1.331	10.950	9.619
–16	0.1510	1.0881	786.0	–366.14	2719.2	2353.1	–366.14	2837.9	2471.8	–1.346	11.036	9.690
–18	0.1252	1.0878	940.5	–370.10	2720.4	2350.3	–370.10	2838.2	2468.1	–1.362	11.123	9.762
–20	0.1035	1.0874	1128.6	–374.03	2721.6	2347.5	–374.03	2838.4	2464.3	–1.377	11.212	9.835
– 22	0.0853	1.0871	1358.4	–377.93	2722.7	2344.7	–377.93	2838.6	2460.6	–1.393	11.302	9.909
– 24	0.0701	1.0868	1640.1	–381.80	2723.7	2342.0	–381.80	2838.7	2456.9	–1.408	11.394	9.985
–26	0.0574	1.0864	1986.4	–385.64	2724.8	2339.2	–385.64	2838.9	2453.2	–1.424	11.486	10.06
– 28	0.0469	1.0861	2413.7	–389.45	2725.8	2336.4	–389.45	2839.0	2449.5	–1.439	11.580	10.141
–30	0.0381	1.0858	2943	–393.23	2726.8	2333.6	–393.23	2839.0	2445.8	–1.455	11.676	10.221
–32	0.0309	1.0854	3600	–396.98	2727.8	2330.8	–396.98	2839.1	2442.1	–1.471	11.773	10.303
–34	0.0250	1.0851	4419	–400.71	2728.7	2328.0	–400.71	2839.1	2438.4	–1.486	11.872	10.386
–36	0.0201	1.0848	5444	–404.40	2729.6	2325.2	–404.40	2839.1	2434.7	–1.501	11.972	10.470
–38	0.0161	1.0844	6731	–408.06	2730.5	2322.4	–408.06	2839.0	2430.9	–1.517	12.073	10.556
–40	0.0129	1.0841	8354	–411.70	2731.3	2319.6	–411.70	2838.9	2427.2	–1.532	12.176	10.644

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B

Appendix

Table B.1 *Properties of Saturated* ***Refrigerant 22*** *(Liquid–Vapour): Temperature entry*

		Specific Volume m^3/kg		*Internal Energy* kJ/kg		*Enthalpy* kJ/kg			*Entropy* $kJ/kg \cdot K$		
	Sat.	*Sat.*	*Sat.*	*Sat.*	*Sat.*	*Sat.*	*Sat.*	*Sat.*	*Sat.*	*Sat.*	
Temp. °C	*Press.* bar	*Liquid* $v_f \times 10^3$	*Vapour* v_g	*Liquid* u_f	*Vapour* u_g	*Liquid* h_f	*Evap.* h_{fg}	*Vapour* h_g	*Liquid* s_f	*Vapour* s_g	*Temp.* °C
–60	0.3749	0.6833	0.5370	–21.57	203.67	–21.55	245.35	223.81	–0.0964	1.0547	–60
–50	0.6451	0.6966	0.3239	–10.89	207.70	–10.85	239.44	228.60	–0.0474	1.0256	–50
–45	0.8290	0.7037	0.2564	–5.50	209.70	–5.44	236.39	230.95	–0.0235	1.0126	–45
–40	1.0522	0.7109	0.2052	–0.07	211.68	0.00	233.27	233.27	0.0000	1.0005	–40
–36	1.2627	0.7169	0.1730	4.29	213.25	4.38	230.71	235.09	0.0186	0.9914	–36
–32	1.5049	0.7231	0.1468	8.68	214.80	8.79	228.10	236.89	0.0369	0.9828	–32
–30	1.6389	0.7262	0.1355	10.88	215.58	11.00	226.77	237.78	0.0460	0.9787	–30
–28	1.7819	0.7294	0.1252	13.09	216.34	13.22	225.43	238.66	0.0551	0.9746	–28
–26	1.9345	0.7327	0.1159	15.31	217.11	15.45	224.08	239.53	0.0641	0.9707	–26
–22	2.2698	0.7383	0.0997	19.76	218.62	19.92	221.32	241.24	0.0819	0.9631	–22
–20	2.4534	0.7427	0.0926	21.99	219.37	22.17	219.91	242.09	0.0908	0.9595	–20
–18	2.6482	0.7462	0.0861	24.23	220.11	24.43	218.49	242.92	0.0996	0.9559	–18
–16	2.8547	0.7497	0.0802	26.48	220.85	26.69	217.05	243.74	0.1084	0.9525	–16
–14	3.0733	0.7533	0.0748	28.73	221.58	28.97	215.59	244.56	0.1171	0.9490	–14
–12	3.3044	0.7569	0.0698	31.00	222.30	31.25	214.11	245.36	0.1258	0.9457	–12
–10	3.5485	0.7606	0.0652	33.27	223.02	33.54	212.62	246.15	0.1345	0.9424	–10
–8	3.8062	0.7644	0.0610	35.54	223.73	35.83	211.10	246.93	0.1431	0.9392	–8
–6	4.0777	0.7683	0.0571	37.83	224.43	38.14	209.56	247.70	0.1517	0.9361	–6
–4	4.3638	0.7722	0.0535	40.12	225.13	40.46	208.00	248.45	0.1602	0.9330	–4
–2	4.6647	0.7762	0.0501	42.42	225.82	42.78	206.41	249.20	0.1688	0.9300	–2
0	4.9811	0.7803	0.0470	44.73	226.50	45.12	204.81	249.92	0.1773	0.9271	0
2	5.3133	0.7844	0.0442	47.04	227.17	47.46	203.18	250.64	0.1858	0.9241	2
4	5.6619	0.7887	0.0415	49.37	227.83	49.82	201.52	251.34	0.1941	0.9213	4
6	6.0275	0.7930	0.0391	51.71	228.48	52.18	199.84	252.02	0.2025	0.9184	6
8	6.4105	0.7974	0.0368	54.05	229.13	54.56	198.14	252.70	0.2109	0.9157	8
10	6.8113	0.8020	0.0346	56.40	229.76	56.95	196.40	253.35	0.2193	0.9129	10
12	7.2307	0.8066	0.0326	58.77	230.38	59.35	194.64	253.99	0.2276	0.9102	12
16	8.1268	0.8162	0.0291	63.53	231.59	64.19	191.02	255.21	0.2442	0.9048	16
20	9.1030	0.8263	0.0259	68.33	232.76	69.09	187.28	256.37	0.2607	0.8996	20
24	10.164	0.8369	0.0232	73.19	233.87	74.04	183.40	257.44	0.2772	0.8944	24
28	11.313	0.8480	0.0208	78.09	234.92	79.05	179.37	258.43	0.2936	0.8893	28
32	12.556	0.8599	0.0186	83.06	235.91	84.14	175.18	259.32	0.3101	0.8842	32
36	13.897	0.8724	0.0168	88.08	236.83	89.29	170.82	260.11	0.3265	0.8790	36
40	15.341	0.8858	0.0151	93.18	237.66	94.53	166.25	260.79	0.3429	0.8738	40
45	17.298	0.9039	0.0132	99.65	238.59	101.21	160.24	261.46	0.3635	0.8672	45
50	19.433	0.9238	0.0116	106.26	239.34	108.06	153.84	261.90	0.3842	0.8603	50
60	24.281	0.9705	0.0089	120.00	240.24	122.35	139.61	261.96	0.4264	0.8455	60

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Table B.2 *Properties of Saturated* ***Refrigerant 22*** *(Liquid–Vapour): Pressure entry*

		Specific Volume m^3/kg		*Internal Energy* kJ/kg		*Enthalpy* kJ/kg			*Entropy* $kJ/kg \cdot K$		
Press. *bar*	*Sat. Temp.* *°C*	*Sat. Liquid* $v_f \times 10^3$	*Sat. Vapour* v_g	*Sat. Liquid* u_f	*Sat. Vapour* u_g	*Sat. Liquid* h_f	*Evap.* h_{fg}	*Sat. Vapour* h_g	*Sat. Liquid* s_f	*Sat. Vapour* s_g	*Press.* *bar*
0.40	–58.86	0.6847	0.5056	–20.36	204.13	–20.34	244.69	224.36	–0.0907	1.0512	0.40
0.50	–54.83	0.6901	0.4107	–16.07	205.76	–16.03	242.33	226.30	–0.0709	1.0391	0.50
0.60	–51.40	0.6947	0.3466	–12.39	207.14	–12.35	240.28	227.93	–0.0542	1.0294	0.60
0.70	–48.40	0.6989	0.3002	–9.17	208.34	–9.12	238.47	229.35	–0.0397	1.0213	0.70
0.80	–45.73	0.7026	0.2650	–6.28	209.41	–6.23	236.84	230.61	–0.0270	1.0144	0.80
0.90	–43.30	0.7061	0.2374	–3.66	210.37	–3.60	235.34	231.74	–0.0155	1.0084	0.90
1.00	–41.09	0.7093	0.2152	–1.26	211.25	–1.19	233.95	232.77	–0.0051	1.0031	1.00
1.25	–36.23	0.7166	0.1746	4.04	213.16	4.14	230.86	234.99	0.0175	0.9919	1.25
1.50	–32.08	0.7230	0.1472	8.60	214.77	8.70	228.15	236.86	0.0366	0.9830	1.50
1.75	–28.44	0.7287	0.1274	12.61	216.18	12.74	225.73	238.47	0.0531	0.9755	1.75
2.00	–25.18	0.7340	0.1123	16.22	217.42	16.37	223.52	239.88	0.0678	0.9691	2.00
2.25	–22.22	0.7389	0.1005	19.51	218.53	19.67	221.47	241.15	0.0809	0.9636	2.25
2.50	–19.51	0.7436	0.0910	22.54	219.55	22.72	219.57	242.29	0.0930	0.9586	2.50
2.75	–17.00	0.7479	0.0831	25.36	220.48	25.56	217.77	243.33	0.1040	0.9542	2.75
3.00	–14.66	0.7721	0.0765	27.99	221.34	28.22	216.07	244.29	0.1143	0.9502	3.00
3.25	–12.46	0.7561	0.0709	30.47	222.13	30.72	214.46	245.18	0.1238	0.9465	3.25
3.50	–10.39	0.7599	0.0661	32.82	222.88	33.09	212.91	246.00	0.1328	0.9431	3.50
3.75	–8.43	0.7636	0.0618	35.06	223.58	35.34	211.42	246.77	0.1423	0.9399	3.75
4.00	–6.56	0.7672	0.0581	37.18	224.24	37.49	209.99	247.48	0.1493	0.9370	4.00
4.25	–4.78	0.7706	0.0548	39.22	224.86	39.55	208.61	248.16	0.1569	0.9342	4.25
4.50	–3.08	0.7740	0.0519	41.17	225.45	41.52	207.27	248.80	0.1642	0.9316	4.50
4.75	–1.45	0.7773	0.0492	43.05	226.00	43.42	205.98	249.40	0.1711	0.9292	4.75
5.00	0.12	0.7805	0.0469	44.86	226.54	45.25	204.71	249.97	0.1777	0.9269	5.00
5.25	1.63	0.7836	0.0447	46.61	227.04	47.02	203.48	250.51	0.1841	0.9247	5.25
5.50	3.08	0.7867	0.0427	48.30	227.53	48.74	202.28	251.02	0.1903	0.9226	5.50
5.75	4.49	0.7897	0.0409	49.94	227.99	50.40	201.11	251.51	0.1962	0.9206	5.75
6.00	5.85	0.7927	0.0392	51.53	228.44	52.01	199.97	251.98	0.2019	0.9186	6.00
7.00	10.91	0.8041	0.0337	57.48	230.04	58.04	195.60	253.64	0.2231	0.9117	7.00
8.00	15.45	0.8149	0.2095	62.88	231.43	63.53	191.52	255.05	0.2419	0.9056	8.00
9.00	19.59	0.8252	0.0262	67.84	231.64	68.59	187.67	256.25	0.2591	0.9001	9.00
10.00	23.40	0.8352	0.0236	72.46	233.71	73.30	183.99	257.28	0.2748	0.8952	10.00
12.00	30.25	0.8546	0.0195	80.87	235.48	81.90	177.04	258.94	0.3029	0.8864	12.00
14.00	36.29	0.8734	0.0166	88.45	236.89	89.68	170.49	260.16	0.3277	0.8786	14.00
16.00	41.73	0.8919	0.0144	95.41	238.00	96.83	164.21	261.04	0.3500	0.8715	16.00
18.00	46.89	0.9104	0.0127	101.87	238.86	103.51	158.13	261.64	0.3705	0.8649	18.00
20.00	51.26	0.9291	0.0112	107.95	239.51	109.81	152.17	261.98	0.3895	0.8586	20.00
24.00	59.46	0.9677	0.0091	119.24	240.22	121.56	140.43	261.99	0.4241	0.8463	24.00

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Table B.3 *Properties of supreheated* ***Refrigerant 22*** *Vapour*

T °C	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m^3/kg	u kJ/kg	h kJkg	s kJ/kg·K
	$p = 0.4$ bar $= 0.04$ MPa ($T_{sat} = -58.86$ °C)				$p = 0.6$ bar $= 0.06$ MPa ($T_{sat} = -51.40$ °C)			
Sat	0.50559	204.13	224.36	1.0512	0.34656	207.14	227.93	1.0294
–55	0.51532	205.92	226.53	1.0612				
–50	0.52787	208.26	229.38	1.0741	0.34895	207.80	228.74	1.0330
–45	0.54037	210.63	232.24	1.0868	0.35747	210.20	231.65	1.0459
–40	0.55284	213.02	235.13	1.0993	0.36594	212.62	234.58	1.0586
–35	0.56526	215.43	238.05	1.1117	0.37437	215.06	237.52	1.0711
–30	0.57766	217.88	240.99	1.1239	0.38277	217.53	240.49	1.0835
–25	0.59002	220.35	243.95	1.1360	0.39114	220.02	243.49	1.0956
–20	0.60236	222.85	246.95	1.1479	0.39948	222.54	246.51	1.1077
–15	0.61468	225.38	249.97	1.1597	0.40779	225.08	249.55	1.1196
–10	0.62697	227.93	253.01	1.1714	0.41608	227.65	252.62	1.1314
–5	0.63925	230.52	256.09	1.1830	0.42436	230.25	255.71	1.1430
0	0.65151	233.13	259.19	1.1944	0.43261	232.88	258.83	1.1545
	$p = 0.8$ bar $= 0.08$ MPa ($T_{sat} = -45.73$°C)				$p = 1.0$ bar $= 0.10$ MPa ($T_{sat} = -41.9$°C)			
Sat	0.26503	209.41	230.61	1.0144	0.21518	211.25	232.77	1.0031
–45	0.26597	209.76	231.04	1.0163				
–40	0.27245	212.21	234.01	1.0292	0.21633	211.79	233.42	1.0059
–35	0.27890	214.68	236.99	1.0418	0.22158	214.29	236.44	1.0187
–30	0.28530	217.17	239.99	1.0543	0.22679	216.80	239.48	1.0313
–25	0.29167	219.68	243.02	1.0666	0.23197	219.34	242.54	1.0438
–20	0.29801	222.22	246.06	1.0788	0.23712	221.90	245.61	1.0560
–15	0.30433	224.78	249.13	1.0908	0.24334	224.48	248.70	1.0681
–10	0.31062	227.37	252.13	1.1026	0.24734	227.08	251.82	1.0801
–5	0.31690	229.98	255.34	1.1143	0.25241	229.71	254.95	1.0919
0	0.32315	232.62	258.47	1.1259	0.25747	232.36	258.11	1.1035
5	0.32939	235.29	261.64	1.1374	0.26251	235.04	281.29	1.1151
10	0.33561	237.98	264.83	1.1488	0.26753	237.74	264.50	1.1265
	$p = 1.5$ bar $= 0.15$ MPa ($T_{sat} = -32.08$°C)				$p = 2.0$ bar $= 0.20$ MPa ($T_{sat} = -25.18$°C)			
Sat	0.14721	214.77	236.86	0.9830	0.11232	217.42	239.88	0.9691
–30	0.14872	215.85	238.16	0.9883				
–25	0.15232	218.45	241.30	1.0011	0.11242	217.51	240.00	0.9696
–20	0.15588	221.07	244.45	1.0137	0.11520	220.19	243.23	0.9825
–15	0.15941	223.70	247.61	1.0260	0.11795	222.88	246.47	0.9952
–10	0.16292	226.35	250.78	1.0382	0.12067	225.48	249.72	1.0076
–5	0.16640	229.02	253.98	1.0502	0.12336	228.30	252.97	1.0199
0	0.16987	231.70	257.18	1.0621	0.12603	231.03	256.23	1.0310
5	0.17331	234.42	260.41	1.0738	0.12868	233.78	259.51	1.0438
10	0.17674	237.15	263.66	1.0854	0.13132	236.54	262.81	1.0555
15	0.18015	239.91	266.93	1.0968	0.13393	239.33	266.12	1.0671
20	0.18355	242.69	270.22	1.1081	0.13653	242.14	269.44	1.0786
25	0.18693	245.49	273.53	1.1193	0.13912	244.97	272.79	1.0899

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Table B.3 *Contd.*

T °C	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K
	$p = 2.5$ bar $= 0.25$ MPa ($T_{sat} = -19.51$°C)				$p = 3.0$ bar $= 0.30$ MPa ($T_{sat} = -14.66$°C)			
Sat	0.09097	219.55	242.29	0.9586	0.07651	221.34	244.29	0.9502
–15	0.09303	222.03	245.29	0.9703				
–10	0.09528	224.79	248.61	0.9831	0.07833	223.96	247.46	0.9623
–5	0.09751	227.55	251.93	0.9956	0.08025	226.78	250.86	0.9751
0	0.09971	230.33	255.26	1.0078	0.08214	229.61	254.25	0.9876
5	0.10189	233.12	258.59	1.0199	0.08400	232.44	257.64	0.9999
10	0.10405	235.92	261.93	1.0318	0.08585	235.28	261.04	1.0120
15	0.10619	238.74	265.29	1.0436	0.08767	238.14	264.44	1.0239
20	0.10831	241.58	268.66	1.0552	0.08949	241.01	267.85	1.0357
25	0.11043	244.44	272.04	1.0666	0.09128	243.89	271.28	1.0472
30	0.11253	247.31	275.44	1.0779	0.09307	246.80	274.72	1.0587
35	0.11461	250.21	278.86	1.0891	0.09484	249.72	278.17	1.0700
40	0.11669	253.13	282.30	1.1002	0.09660	252.66	281.64	1.0811
	$p = 3.5$ bar $= 0.35$ MPa ($T_{sat} = -10.39$°C)				$p = 4.0$ bar $= 0.40$ MPa ($T_{sat} = -6.56$°C)			
Sat	0.06605	222.88	246.00	0.9431	0.05812	224.24	247.48	0.9370
–10	0.06619	223.10	246.27	0.9441				
–5	0.06789	225.99	249.75	0.9572	0.05860	225.16	248.60	0.9411
0	0.06956	228.86	253.21	0.9700	0.06011	228.09	252.14	0.9542
5	0.07121	231.74	256.67	0.9825	0.06160	231.02	255.66	0.9670
10	0.07284	234.63	260.12	0.9948	0.06306	233.95	259.10	0.9795
15	0.07444	237.52	263.57	1.0069	0.06450	236.89	262.69	0.9918
20	0.07603	240.42	267.03	1.0188	0.06592	239.83	266.19	1.0039
25	0.07760	243.34	270.50	1.0305	0.06733	242.77	269.71	1.0158
30	0.07916	246.27	273.97	1.0421	0.06872	245.73	273.22	1.0274
35	0.08070	249.22	277.46	1.0535	0.07010	248.71	276.75	1.0390
40	0.08224	252.18	280.97	1.0648	0.07146	251.70	280.28	1.0504
45	0.08376	255.17	284.48	1.0759	0.07282	254.70	283.83	1.0616
	$p = 4.5$ bar $= 0.45$ MPa ($T_{sat} = -3.08$°C)				$p = 5.0$ bar $= 0.50$ MPa ($T_{sat} = -0.12$°C)			
Sat	0.05189	225.45	248.80	0.9316	0.04686	226.54	249.97	0.9269
0	0.05275	227.29	251.03	0.9399				
5	0.05411	230.28	254.63	0.9529	0.04810	229.52	253.57	0.9399
10	0.05545	233.26	258.21	0.9657	0.04934	232.55	257.22	0.9530
15	0.05676	236.24	261.78	0.9782	0.05056	235.57	260.85	0.9657
20	0.05805	239.22	265.34	0.9904	0.05175	238.59	264.47	0.9781
25	0.05933	242.20	268.90	1.0025	0.05293	241.61	268.07	0.9903
30	0.06059	245.19	272.46	1.0143	0.05409	244.63	271.68	1.0023
35	0.06184	248.19	276.02	1.0259	0.05523	247.66	275.28	1.0141
40	0.06308	251.20	279.59	1.0374	0.05636	250.70	282.50	1.0257
45	0.06430	254.23	283.17	1.0488	0.05748	253.76	282.50	1.0371
50	0.06552	257.28	286.76	1.0600	0.05859	256.82	286.12	1.0484
55	0.06672	260.34	290.36	1.0710	0.05969	259.90	289.75	1.0595

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Table B.3 *Contd.*

T °C	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m^3/kg	u kJ/kg	h kJkg	s kJ/kg·K
	p = 5.5 bar = 0.55 MPa (T_{sat} = 3.08°C)				*p = 6.0 bar = 0.60 MPa (T_{sat} = 5.8°C)*			
Sat.	0.04271	227.53	251.02	0.9226	0.03923	228.44	251.98	0.9186
5	0.04317	228.72	252.46	0.9278				
10	0.04433	231.81	256.20	0.9411	0.04015	231.05	255.14	0.9299
15	0.04547	234.89	259.90	0.9540	0.04122	234.18	258.91	0.9431
20	0.04658	237.95	263.57	0.9667	0.04227	237.29	262.65	0.9560
25	0.04768	241.01	267.23	0.9790	0.04330	240.39	266.37	0.9685
30	0.04875	244.07	270.88	0.9912	0.04431	243.49	270.07	0.9808
35	0.04982	247.13	274.53	1.0031	0.04530	246.58	273.76	0.9929
40	0.05086	250.20	278.17	1.0148	0.04628	249.68	277.45	1.0048
45	0.05190	253.27	281.82	1.0264	0.04724	252.78	281.13	1.0164
50	0.05293	256.36	285.47	1.0378	0.04820	255.90	284.82	1.0279
55	0.05394	259.46	289.13	1.0490	0.04914	259.02	288.51	1.0393
60	0.05495	262.58	292.80	1.0601	0.05008	262.15	292.20	1.0504
	p = 7.0 bar = 0.70 MPa (T_{sat} = 10.91°C)				*p = 8.0 bar = 0.80 MPa (T_{sat} = 15.45°C)*			
Sat.	0.03371	230.04	253.64	0.9117	0.02953	231.43	255.05	0.9056
15	0.03451	232.70	256.86	0.9229				
20	0.03547	235.92	260.75	0.9363	0.03033	234.47	258.74	0.9182
25	0.03639	239.12	264.59	0.9493	0.03118	237.76	262.70	0.9315
30	0.03730	242.29	268.40	0.9619	0.03202	241.04	266.66	0.9448
35	0.03819	245.46	272.19	0.9743	0.03283	244.28	270.54	0.9574
40	0.03906	248.62	275.96	0.9865	0.03363	247.52	274.42	0.9700
45	0.03992	251.78	279.72	0.9984	0.03440	250.74	278.26	0.9821
50	0.04076	254.94	283.48	1.0101	0.03517	253.96	282.10	0.9941
55	0.04160	258.11	287.23	1.0216	0.03592	257.18	285.92	1.0058
60	0.04242	261.29	290.99	1.0330	0.03667	260.40	289.74	1.0174
65	0.04324	264.48	294.75	1.0442	0.03741	263.64	293.56	1.0287
70	0.04405	267.68	298.51	1.0552	0.03814	266.87	297.38	1.0400
	p = 9.0 bar = 0.90 MPa (T_{sat} = 19.59°C)				*p = 10.0 bar = 1.00 MPa (T_{sat} = 23.40°C)*			
Sat.	0.02623	232.64	256.25	0.9001	0.02358	233.71	257.28	0.8952
20	0.02630	232.92	256.59	0.9013				
30	0.02789	239.73	264.83	0.9289	0.02457	238.34	262.91	0.9139
40	0.02939	246.37	272.82	0.9549	0.02598	245.18	271.17	0.9407
50	0.03082	252.95	280.68	0.9795	0.02732	251.90	279.22	0.9660
60	0.03219	259.49	288.46	1.0033	0.02860	258.56	287.15	0.9902
70	0.03353	266.04	296.21	1.0262	0.02984	265.19	295.03	1.0135
80	0.03483	272.62	303.96	1.0484	0.03104	271.84	302.88	1.0361
90	0.03611	279.23	311.73	1.0701	0.03221	278.52	310.74	1.0580
100	0.03736	285.90	319.53	1.0913	0.03337	285.24	318.61	1.0794
110	0.03860	292.63	327.37	1.1120	0.03450	292.02	326.52	1.1003
120	0.03982	299.42	335.26	1.1323	0.03562	298.85	334.46	1.1207
130	0.04103	306.28	343.21	1.1523	0.03672	305.74	342.46	1.1408
140	0.04323	313.21	351.22	1.1719	0.03781	312.70	350.51	1.1605
150	0.04342	320.21	359.29	1.1912	0.03889	319.74	358.63	1.1790

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Table B.3 *Contd.*

T °C	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K
	p = 12.0 bar = 1.20 MPa (T_{sat} = 30.25°C)				p = 14.0 bar = 1.40 MPa (T_{sat} = 36.29°C)			
Sat.	0.01955	235.48	258.94	0.8864	0.01662	236.89	260.16	0.8786
40	0.02083	242.63	267.62	0.9146	0.01708	239.78	263.70	0.8900
50	0.02204	249.69	276.14	0.9413	0.01823	247.29	272.81	0.9186
60	0.02319	256.60	284.43	0.9666	0.01929	254.52	281.53	0.9452
70	0.02428	263.44	292.58	0.9907	0.02029	261.60	290.01	0.9703
80	0.02534	270.25	300.66	1.0139	0.02125	268.60	298.34	1.9942
90	0.02636	277.07	308.70	1.0363	0.02217	275.56	306.60	1.0172
100	0.02736	283.90	316.73	1.0582	0.02306	282.52	314.80	1.0395
110	0.02834	290.77	324.78	1.0794	0.02393	289.49	323.00	1.0612
120	0.02930	297.69	332.85	1.1002	0.02478	296.50	331.19	1.0823
130	0.03024	304.65	340.95	1.1205	0.02562	303.55	339.41	1.1029
140	0.03118	311.68	349.09	1.1405	0.02644	310.64	347.65	1.1231
150	0.03210	318.77	357.29	1.1601	0.02725	317.79	355.94	1.1429
160	0.03301	325.92	365.54	1.1793	0.02805	324.99	364.26	1.1624
170	0.03392	333.14	373.84	1.1983	0.02884	332.26	372.64	1.1815
	p = 16.0 bar = 1.60 MPa (T_{sat} = 41.73°C)				p = 18.0 bar = 1.80 MPa (T_{sat} = 46.69°C)			
Sat.	0.01440	238.00	261.04	0.8715	0.01265	238.86	261.64	0.8649
50	0.01533	244.66	269.18	0.8971	0.01301	241.72	265.14	0.8484
60	0.01634	252.29	278.43	0.9252	0.01401	249.86	275.09	0.9061
70	0.01728	259.65	287.30	0.9515	0.01492	257.57	284.43	0.9337
80	0.01817	266.86	275.93	0.9762	0.01576	265.04	293.40	0.9595
90	0.01901	274.00	304.42	0.9999	0.01655	272.37	302.16	0.9839
100	0.01983	281.09	312.82	1.0228	0.01731	279.62	310.77	1.0073
110	0.02062	288.18	321.17	1.0448	0.01804	286.83	319.30	1.0299
120	0.02139	295.28	329.51	1.0663	0.01874	294.04	327.78	1.0517
130	0.02214	302.41	337.84	1.0872	0.01943	301.26	336.24	1.0730
140	0.02288	309.58	346.19	1.1077	0.02011	308.50	344.70	1.0937
150	0.02361	316.79	354.56	1.1277	0.02077	315.78	353.17	1.1139
160	0.02432	324.05	362.97	1.1473	0.02142	323.10	361.66	1.1338
170	0.02503	331.37	371.42	1.1666	0.02207	330.47	370.19	1.1532
	p = 16.0 bar = 2.00 MPa (T_{sat} = 51.26°C)				p = 24.0 bar = 2.4 MPa (T_{sat} = 59.46°C)			
Sat.	0.01124	239.51	261.98	0.8586	0.00907	240.22	261.99	0.8463
60	0.01212	247.20	271.43	0.8873	0.00913	240.78	262.68	0.8484
70	0.01300	255.35	281.36	0.9167	0.01006	250.30	274.43	0.8831
80	0.01381	263.12	290.74	0.9436	0.01085	258.89	284.93	0.9133
90	0.01457	270.67	299.80	0.9689	0.01156	267.01	294.75	0.9407
100	0.01528	278.09	308.65	0.9929	0.01222	274.85	304.18	0.9663
110	0.01596	285.44	317.37	1.0160	0.01284	282.53	313.35	0.9906
120	0.01663	292.76	326.01	1.0383	0.01343	290.11	322.35	1.0137
130	0.01727	300.08	334.61	1.0598	0.01400	297.64	331.25	1.0361
140	0.01789	307.40	343.19	1.0808	0.01456	305.14	340.08	1.0577
150	0.01850	314.75	351.76	1.1013	0.01509	312.64	348.87	1.0787
160	0.01910	322.14	360.34	1.1214	0.01562	320.16	357.64	1.0992
170	0.01969	329.56	368.95	1.1410	0.01613	327.70	366.41	1.1192
180	0.02027	337.03	377.58	1.1603	0.01663	336.27	375.20	1.1388

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Table B.4 *Thermodynamic Properities of* ***R-12***

Temp *C* *T*	*Press.* *kPa* *p*	*Specific Volume,* m^3/kg				*Internal Energy, kJ/kg*					*Temp* *°C* *T*
		Sat. Liquid v_f	*Sat. Vapour* v_g	*Sat. Liquid* u_f	*Sat. Vapour* u_g	*Sat. Liquid* h_f	*Evap.* h_{fg}	*Sat. Vapour* h_g	*Sat. Liquid* s_f	*Sat. Vapour* s_g	
–90	2.8	0.000608	4.41555	–43.29	133.91	–43.28	189.75	146.46	–0.2086	0.8273	–90
–80	6.2	0.000617	2.13835	–34.73	137.82	–34.72	285.74	151.02	–0.1631	0.7984	–80
–70	12.3	0.000627	1.12728	–26.14	141.81	–26.13	181.76	155.64	–0.1198	0.7749	–70
–60	22.6	0.000637	0.63791	–17.50	145.86	–17.49	177.77	160.29	–0.0783	0.7557	–60
–50	39.1	0.000648	0.38310	–8.80	149.95	–8.78	173.73	164.95	–0.0384	0.7401	–50
–45	50.4	0.000654	0.30268	–4.43	152.01	–4.40	171.68	167.28	–0.0190	0.7334	–45
–40	64.2	0.000659	0.24191	–0.04	154.07	0	169.59	169.59	0	0.7274	–40
–35	80.7	0.000666	0.19540	4.37	156.13	4.42	167.48	171.90	0.0187	0.7219	–35
–30	100.4	0.000672	0.15937	8.79	158.28	8.86	165.34	174.09	0.0371	0.7170	–30
–29.8	101.3	0.000673	0.15803	8.98	158.30	9.05	165.24	174.20	0.0379	0.7168	–29.8
–25	123.7	0.000679	0.13117	13.24	160.25	13.33	163.15	176.48	0.0552	0.7126	–25
–20	150.9	0.000685	0.10885	17.71	162.31	17.82	160.92	178.74	0.0731	0.7087	–20
–15	182.6	0.000693	0.09102	22.20	164.35	22.33	158.64	180.97	0.0906	0.7051	–15
–10	219.1	0.000700	0.07665	26.72	166.39	26.87	156.31	183.19	0.1080	0.7019	–10
–5	261.0	0.000708	0.06496	31.26	168.42	31.45	153.93	185.37	0.1251	0.6991	–5
0	308.6	0.000716	0.05539	35.83	170.44	36.05	151.48	187.53	0.1420	0.6965	0
5	362.6	0.000724	0.04749	40.43	172.44	40.69	148.96	189.65	0.1587	0.6942	5
10	423.3	0.000733	0.04091	45.06	174.42	45.37	146.37	191.74	0.1752	0.6921	10
15	491.4	0.000743	0.03541	49.73	176.38	50.10	143.68	193.78	0.1915	0.6902	15
20	567.3	0.000752	0.03078	54.45	178.32	54.87	140.91	195.78	0.2078	0.6884	20
25	651.6	0.000763	0.02685	59.21	180.23	59.70	138.03	197.73	0.2239	0.6868	25
30	744.9	0.000774	0.02351	64.02	182.11	64.59	135.03	199.62	0.2399	0.6853	30
35	847.7	0.000786	0.02064	68.88	183.95	69.55	131.90	201.45	0.2559	0.6839	35
40	960.7	0.000798	0.01817	73.82	185.74	74.59	128.61	203.20	0.2718	0.6825	40
45	1084.3	0.000811	0.01603	78.83	187.49	79.71	125.16	204.87	0.2877	0.6811	45
50	1219.3	0.000826	0.01417	83.93	189.17	84.94	121.51	206.45	0.3037	0.6797	50
55	1366.3	0.000841	0.01254	89.12	190.78	90.27	117.65	107.92	0.3197	0.6782	55
60	1525.9	0.000858	0.01111	94.43	192.31	95.74	113.52	209.26	0.3358	0.6765	60
65	1698.8	0.000877	0.00985	99.87	193.73	101.36	109.10	210.46	0.3521	0.6747	65
70	1885.8	0.000997	0.00873	105.46	195.03	107.15	104.33	211.48	0.3686	0.6726	70
75	2087.5	0.000920	0.00772	111.23	196.17	113.15	99.14	212.29	0.3854	0.6702	75
80	2304.6	0.000946	0.00682	117.21	197.11	120.39	93.44	212.83	0.4027	0.6672	80
85	2538.0	0.000976	0.00600	123.45	197.80	125.93	87.11	213.04	0.4204	0.6636	85
90	2788.5	0.001012	0.00526	130.02	198.14	132.84	79.96	212.80	0.4389	0.6590	90
95	3056.9	0.001056	0.00456	137.01	197.99	140.23	71.71	211.94	0.4583	0.6531	95
100	3344.1	0.001113	0.00390	144.59	197.07	148.31	61.81	210.12	0.4793	0.6449	100
105	3650.9	0.001197	0.00324	153.15	194.73	157.52	49.05	206.57	0.5028	0.6325	105
110	3978.5	0.001364	0.00246	164.12	188.20	169.55	28.44	197.99	0.5533	0.6076	110
120.0	4116.8	0.001792	0.00179	176.06	176.06	183.43	0	183.43	0.5689	0.5689	120.0

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Table B.5 *Properties of superheated Refrigerant,* ***R-12***

Temp. *C*	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg·K*	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg·K*	*v* m^3/kg	*h* *kJkg*	*s* *kJ/kg·K*
	25 kPa (– 58.26)			*50 kPa (– 45.18)*			*100 kPa (– 30.10)*		
Sat.	0.58130	161.10	0.7527	0.30515	167.19	0.7336	0.15999	174.15	0.7171
–30	0.66179	176.19	0.8187	0.32738	175.55	0.7691	0.16006	174.21	0.7174
–20	0.69001	181.74	0.8410	0.34186	181.17	0.7917	0.16770	179.99	0.7406
–10	0.71811	187.40	0.8630	0.35623	186.89	0.8139	0.17522	185.84	0.7633
0	0.74613	193.17	0.8844	0.37051	192.70	0.8356	0.18265	191.77	0.7854
10	0.77409	199.03	0.9055	0.38472	198.61	0.8568	0.18999	197.77	0.8070
20	0.80198	204.99	0.9262	0.39886	204.62	0.8776	0.19728	203.85	0.8281
30	0.82982	211.05	0.9465	0.41296	210.71	0.8981	0.20451	210.02	0.8488
40	0.85762	217.20	0.9665	0.42701	216.89	0.9181	0.21169	216.26	0.8691
50	0.88538	223.45	0.9861	0.44103	223.16	0.9378	0.21884	222.58	0.8889
60	0.91312	229.77	1.0054	0.45502	229.51	0.9572	0.22596	228.98	0.9084
70	0.94083	236.19	1.0244	0.46898	235.95	0.9762	0.23305	235.46	0.9276
80	0.96852	242.68	1.0430	0.48292	242.46	0.9949	0.24011	242.01	0.9464
90	0.99618	249.26	1.0614	0.49684	249.05	1.0133	0.24716	248.63	0.9649
100	1.02384	255.91	1.0795	0.51074	255.71	1.0314	0.25419	255.32	0.9831
110	1.05148	262.63	1.0972	0.52463	262.45	1.0493	0.26121	262.08	1.0009
120	1.07910	269.43	1.1148	0.53851	269.26	1.0668	0.26821	268.91	1.0185
	200 kPa (– 12.53)			*300 kPa (– 0.86)*			*400 kPa (– 8.15)*		
Sat.	0.08354	182.07	0.7035	0.05690	187.16	0.6969	0.04321	190.97	0.6928
0	0.08861	189.80	0.7325	0.05715	187.72	0.6989	—	—	—
10	0.09255	196.02	0.7548	0.05998	194.17	0.7222	0.04363	192.21	0.6972
20	0.09642	202.28	0.7766	0.06273	200.64	0.7446	0.04584	198.91	0.7204
30	0.10023	208.60	0.7978	0.06542	207.12	0.7663	0.04797	205.58	0.7428
40	0.10399	214.97	0.8184	0.06805	213.64	0.7875	0.05005	212.25	0.7645
50	0.10771	221.41	0.8387	0.07064	220.19	0.8081	0.05207	218.94	0.7855
60	0.11140	227.90	0.8585	0.07319	226.79	0.8282	0.05406	225.65	0.8060
70	0.11506	234.46	0.8779	0.07571	233.44	0.8479	0.05601	232.40	0.8259
80	0.11869	241.09	0.8969	0.07820	240.15	0.8671	0.05794	239.19	0.8454
90	0.12230	247.77	0.9156	0.08067	246.90	0.8860	0.05985	246.02	0.8645
100	0.12590	254.53	0.9339	0.08313	253.72	0.9045	0.06173	252.89	0.8831
110	0.12948	261.34	0.9519	0.08557	260.58	0.9226	0.06360	259.81	0.9015
120	0.13305	268.21	0.9696	0.08799	267.50	0.9405	0.06546	266.79	0.9194
130	0.13661	275.15	0.9870	0.09041	274.48	0.9580	0.06730	273.81	0.9370
140	0.14016	282.14	1.0042	0.09281	281.51	0.9752	0.06913	280.88	0.9544
150	0.14370	289.19	1.0210	0.09520	288.59	0.9922	0.07095	287.99	0.9714

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Table B.5 *Contd.*

Temp. *C*	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg·K*	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg·K*	*v* m^3/kg	*h* *kJkg*	*s* *kJ/kg·K*
	500 kPa (15.60)			*750 kPa (30.26)*			*1000 kPa (41.64)*		
Sat.	0.03482	194.03	0.6899	0.02335	199.72	0.6852	0.01744	203.76	0.6820
30	0.03746	203.96	0.7235	–	–	–	–	–	–
40	0.03921	210.81	0.7457	0.02467	206.91	0.7086	–	–	–
50	0.04091	217.64	0.7672	0.02595	214.18	0.7314	0.01837	210.32	0.7026
60	0.04257	224.68	0.7881	0.02718	221.37	0.7547	0.01941	217.97	0.7259
70	0.04418	231.33	0.8083	0.02837	228.52	0.7745	0.02040	225.49	0.7481
80	0.04577	238.21	0.8281	0.02952	235.65	0.7949	0.02134	232.91	0.7695
90	0.04734	245.11	0.8473	0.03064	242.76	0.8148	0.02225	240.28	0.7900
100	0.04889	242.05	0.8662	0.03174	249.89	0.8342	0.02313	247.61	0.8100
110	0.05041	259.03	0.8847	0.03282	257.03	0.8530	0.02399	254.93	0.8293
120	0.05193	266.06	0.9028	0.03388	264.19	0.8715	0.02483	262.25	0.8482
130	0.05343	273.12	0.9205	0.03493	271.38	0.8895	0.02566	269.57	0.8665
140	0.05492	280.23	0.9379	0.03596	278.59	0.9072	0.02647	276.90	0.8845
150	0.05640	287.39	0.9550	0.03699	285.84	0.9246	0.02728	284.26	0.9021
160	0.05788	294.59	0.9718	0.03801	293.13	0.9416	0.02885	299.04	0.9193
170	0.05934	301.83	0.9884	0.03902	300.45	0.9583	0.02885	299.04	0.9362
180	0.06080	309.12	1.0046	0.04002	307.81	0.9747	0.02963	306.47	0.9528
	500 kPa (59.22)			*2000 kPa (72.88)*			*4000 kPa (110.32)*		
Sat	0.01132	209.06	0.6768	0.00813	211.97	0.6713	0.00239	196.90	0.6046
80	0.01305	226.73	0.7284	0.00870	219.02	0.6914	–	–	–
90	0.01377	234.77	0.7508	0.00941	228.23	0.7171	–	–	–
100	0.01446	242.65	0.7722	0.01003	236.94	0.7408	–	–	–
110	0.01512	250.41	0.7928	0.01061	245.34	0.7630	–	–	–
120	0.01575	258.10	0.8126	0.01116	253.53	0.7841	0.00374	225.18	0.6777
130	0.01636	265.74	0.8318	0.01168	261.58	0.8043	0.00433	238.69	0.7116
140	0.01696	273.35	0.8504	0.01217	269.53	0.8238	0.00478	249.93	0.7392
150	0.01754	280.94	0.8686	0.01265	277.41	0.8426	0.00517	260.12	0.7636
160	0.01811	288.52	0.8863	0.01312	285.24	0.8609	0.00552	269.71	0.7860
170	0.01867	296.11	0.9036	0.01357	293.04	0.8787	0.00585	278.90	0.8069
180	0.01922	303.70	0.9205	0.01401	300.82	0.8961	0.00615	287.82	0.8269
190	0.01977	311.31	0.9371	0.01445	308.59	0.9131	0.00643	296.55	0.8459
200	0.02031	318.93	0.9534	0.01488	316.36	0.9297	0.00671	305.14	0.8642
210	0.02084	326.58	0.9694	0.01530	324.14	0.9459	0.00697	313.61	0.8820
220	0.02137	334.24	0.9851	0.01572	331.92	0.9619	0.00723	322.01	0.8992

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Table B.6 *Properties of* ***Saturated Refrigerant 134a*** *(Liquid-Vapour): Temperature Entry*

		Specific Volume m^3/kg		*Internal Energy* kJ/kg		*Enthalpy* kJ/kg			*Entropy* $kJ/kg \cdot K$		
Temp. °C	*Sat. Press.* bar	*Sat. Liquid* $v_f \times 10^3$	*Sat. Vapour* v_g	*Sat. Liquid* u_f	*Sat. Vapour* u_g	*Sat. Liquid* h_f	*Evap.* h_{fg}	*Sat. Vapour* h_g	*Sat. Liquid* s_f	*Sat. Vapour* s_g	*Temp.* °C
–40	0.5164	0.7055	0.3569	–0.04	204.45	0.00	222.88	222.88	0.0000	0.9560	–40
–36	0.6332	0.7113	0.2947	4.68	206.73	4.73	220.67	225.40	0.0201	0.9506	–36
–32	0.7704	0.7172	0.2451	9.47	209.01	9.52	218.37	227.90	0.0401	0.9456	–32
–28	0.9305	0.7233	0.2052	14.31	211.29	14.37	216.01	230.38	0.0600	0.9411	–28
–26	1.0199	0.7265	0.1882	16.75	212.43	16.82	214.80	231.62	0.0699	0.9390	–26
–24	1.1160	0.7296	0.1728	19.21	213.57	19.29	213.57	232.85	0.0798	0.9370	–24
–22	1.2192	0.7328	0.1590	21.68	214.70	21.77	212.32	234.08	0.0897	0.9351	–22
–20	1.3299	0.7361	0.1464	24.17	215.84	24.26	211.05	235.31	0.0996	0.9332	–20
–18	1.4483	0.7395	0.1350	26.67	216.97	26.77	209.76	236.53	0.1094	0.9315	–18
–16	1.5748	0.7428	0.1247	29.18	218.10	29.30	208.45	237,74	0.1192	0.9298	–16
–12	1.8540	0.7498	0.1068	34.25	220.36	34.39	205.77	240.15	0.1388	0.9267	–12
–8	2.1704	0.7569	0.0919	39.38	222.60	39.65	203.00	242.54	0.1583	0.9239	–8
–4	2.5274	0.7644	0.0794	44.56	224.84	44.75	200.15	244.90	0.1777	0.9213	–4
0	2.9282	0.7721	0.0689	49.79	227.06	50.02	197.21	247.23	0.1970	0.9190	0
4	3.3765	0.7801	0.0600	55.08	229.27	55.35	194.19	249.53	0.2162	0.9169	4
8	3.8756	0.7884	0.0525	60.43	231.46	60.73	191.07	251.80	0.2354	0.9150	8
12	4.4294	0.7971	0.0460	65.83	233.63	66.18	187.85	254.03	0.2505	0.9132	12
16	5.0416	0.8062	0.0405	71.29	235.78	71.69	184.52	256.22	0.2735	0.9116	16
20	5.7160	0.8157	0.0358	76.80	237.91	77.26	181.09	258.36	0.2924	0.9102	20
24	6.4566	0.8257	0.0317	82.37	240.01	82.90	177.55	260.45	0.3113	0.9089	24
26	6.8530	0.8309	0.0298	85.18	241.05	85.75	175.73	261.48	0.3208	0.9082	26
28	7.2675	0.8362	0.0281	88.00	242.08	88.61	173.89	262.50	0.3302	0.9076	28
30	7.7006	0.8417	0.0265	90.84	243.10	91.49	172.00	263.50	0.3396	0.9070	30
32	8.1528	0.8473	0.0250	93.70	244.12	94.39	170.09	264.48	0.3490	0.9064	32
34	8.6247	0.8530	0.0236	96.38	245.12	97.31	168.14	265.45	0.3584	0.9058	34
36	9.1168	0.8590	0.0223	99.47	246.11	100.25	166.15	266.40	0.3678	0.9053	36
38	9.6298	0.8651	0.0210	102.38	247.09	103.21	164.12	267.33	0.3772	0.9047	38
40	10.164	0.8714	0.0199	105.30	248.06	106.19	162.05	268.24	0.3866	0.9041	40
42	10.720	0.8780	0.0188	108.25	249.02	109.19	159.94	269.14	0.3960	0.9035	42
44	11.299	0.8847	0.0177	111.22	249.96	112.22	157.79	270.01	0.4054	0.9030	44
48	12.526	0.8989	0.0159	117.22	251.79	118.35	153.33	271.68	0.4243	0.9017	48
52	13.851	0.9142	0.0142	123.31	253.55	124.58	148.66	273.24	0.4432	0.9004	52
56	15.278	0.9308	0.0127	129.51	255.23	130.93	143.75	271.68	0.4622	0.8990	56
60	16.813	0.9488	0.0114	135.83	256.81	137.42	138.57	275.99	0.4814	0.8973	60
70	21.162	1.0026	0.0086	152.22	260.15	154.34	124.08	278.43	0.5302	0.8918	70
80	26.324	1.0766	0.0054	169.88	262.14	172.71	106.41	279.12	0.5814	0.8827	80
90	32.435	1.1949	0.0046	189.82	261.34	193.69	82.63	276.32	0.6380	0.8655	90
100	39.742	1.5443	0.0027	218.60	248.49	224.74	34.40	259.13	0.7196	0.8117	100

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Table B.7 *Properties of* ***Saturated refrigerant 134a*** *(Liquid–Vapour): Pressure Entry*

		Specific Volume m^3/kg		*Internal Energy* kJ/kg		*Enthalpy* kJ/kg			*Entropy* $kJ/kg \cdot K$		
Press. *bar*	*Sat. Temp.* °C	*Sat. Liquid* $v_f \times 10^3$	*Sat. Vapour* v_g	*Sat. Liquid* u_f	*Sat. Vapour* u_g	*Sat. Liquid* h_f	*Evap.* h_{fg}	*Sat. Vapour* h_g	*Sat. Liquid* s_f	*Sat. Vapour* s_g	*Press.* *bar*
0.6	–37.07	0.7097	0.3100	3.41	206.12	3.46	221.27	224.72	0.0147	0.9520	0.6
0.8	–31.21	0.7184	0.2366	10.41	209.46	10.47	217.92	228.39	0.0440	0.9447	0.8
1.0	–26.43	0.7258	0.1917	16.22	212.18	16.29	215.06	231.35	0.0678	0.9395	1.0
1.2	–22.36	0.7323	0.1614	21.23	214.50	21.32	212.54	233.86	0.0879	0.9354	1.2
1.4	–18.80	0.7381	0.1395	25.66	216.52	25.77	210.27	236.04	0.1055	0.9322	1.4
1.6	–15.62	0.7435	0.1229	29.66	218.32	29.78	208.19	237.97	0.1211	0.9295	1.6
1.8	–12.73	0.7485	0.1098	33.31	219.94	33.45	206.26	239.71	0.1352	0.9273	1.8
2.0	–10.09	0.7532	0.0993	36.69	221.43	36.84	204.46	241.30	0.1481	0.9253	2.0
2.4	–5.37	0.7618	0.0834	42.77	224.07	42.95	201.14	244.09	0.1710	0.9222	2.4
2.8	–1.23	0.7697	0.0719	48.18	226.38	48.39	198.13	246.52	0.1911	0.9197	2.8
3.2	2.48	0.7701	0.0632	53.06	228.43	53.31	195.35	248.66	0.2089	0.9177	3.2
3.6	5.84	0.7839	0.0564	57.54	230.28	57.82	192.76	250.58	0.2251	0.9160	3.6
4.0	8.93	0.7904	0.0509	61.69	231.97	62.00	190.32	252.32	0.2399	0.9145	4.0
5.0	15.74	0.8056	0.0409	70.93	235.64	71.33	184.74	256.07	0.2723	0.9117	5.0
6.0	21.58	0.8196	0.0341	78.99	238.74	79.48	179.71	259.19	0.2999	0.9097	6.0
7.0	26.72	0.8328	0.0292	86.19	241.42	86.78	175.07	261.85	0.3242	0.9080	7.0
8.0	31.33	0.8454	0.0255	92.75	243.78	93.42	170.73	264.15	0.3459	0.9066	8.0
9.0	35.53	0.8576	0.0226	98.79	245.88	99.56	166.62	266.18	0.3656	0.9054	9.0
10.0	39.39	0.8695	0.0202	104.42	247.77	105.29	162.68	267.97	0.3838	0.9043	10.0
12.0	46.32	0.8928	0.0166	114.69	251.03	115.76	155.23	270.89	0.4164	0.9023	12.0
14.0	52.43	0.9159	0.0140	123.98	253.74	125.26	148.14	273.40	0.4453	0.9003	14.0
16.0	57.92	0.9392	0.0121	132.52	256.00	134.02	141.31	275.53	0.4714	0.8982	16.0
18.0	62.91	0.9631	0.0105	140.49	257.88	142.22	134.60	276.83	0.4954	0.8959	18.0
20.0	67.49	0.9878	0.0093	148.02	259.41	149.99	127.95	277.94	0.5178	0.8934	20.0
25.0	77.59	1.0562	0.0069	165.48	261.84	168.22	111.06	279.17	0.5686	0.8854	25.0
30.0	86.22	1.1416	0.0053	181.88	262.16	185.30	92.71	278.01	0.6156	0.8735	30.0

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Table B.8 *Properties of* ***Superheated Refrigerant 134a*** *Vapour*

T °C	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m^3/kg	u kJ/kg	h kJkg	s kJ/kg·K
	p = 0.6 bar = 0.06 MPa (T_{sat} = – 37.07°C)				*p = 1.0 bar = 0.10 MPa* (T_{sat} = – 26.43°C)			
Sat.	0.31003	206.12	224.72	0.9520	0.19170	212.18	231.35	0.9395
–20	0.33536	217.86	237.98	1.0062	0.19770	216.77	236.54	0.9602
–10	0.34992	224.97	245.96	1.0371	0.20686	224.70	244.70	0.9918
0	0.36433	232.24	254.10	1.0675	0.21587	231.41	252.99	1.0227
10	0.37861	239.69	262.41	1.0973	0.22473	238.96	261.43	1.0531
20	0.39279	247.32	270.89	1.1267	0.23349	246.67	270.02	1.0829
30	0.40688	255.12	279.53	1.1557	0.24216	254.54	278.76	1.1122
40	0.42091	263.10	288.35	1.1844	0.25076	262.58	287.66	1.1411
50	0.43487	271.25	297.34	1.2126	0.25930	270.79	296.72	1.1696
60	0.44879	279.58	306.51	1.2405	0.26779	279.16	305.94	1.1977
70	0.46266	288.08	315.84	1.2681	0.27623	287.70	315.32	1.2254
80	0.47650	296.75	325.34	1.2954	0.28464	296.40	324.87	1.2528
90	0.49031	305.58	335.00	1.3224	0.29302	305.27	334.57	1.2799
	p = 1.4 bar = 0.14 MPa (T_{sat} = – 18.80°C)				*p = 1.8 bar = 0.18 MPa* (T_{sat} = – 12.75°C)			
Sat	0.13945	216.52	236.04	0.9322	0.10983	219.94	239.71	0.9273
–10	0.14549	223.03	243.40	0.9606	0.11135	222.02	242.06	0.9362
0	0.15219	230.55	251.86	0.9922	0.11678	229.67	250.69	0.9684
10	0.15875	238.21	260.43	1.0230	0.12207	237.44	259.41	0.9998
20	0.16520	246.01	269.13	1.0532	0.12723	244.33	268.23	1.0304
30	0.17155	253.96	277.97	1.0828	0.13230	253.36	277.17	1.0604
40	0.17783	262.06	286.96	1.1120	0.13730	261.53	286.24	1.0898
50	0.18404	270.32	296.09	1.1407	0.14222	269.85	295.45	1.1187
60	0.19020	278.74	305.37	1.1690	0.14710	278.31	304.79	1.1472
70	0.19633	287.32	314.80	1.1969	0.15183	286.93	314.28	1.1753
80	0.20241	296.06	324.39	1.2244	0.15672	295.71	323.92	1.2030
90	0.20846	304.95	334.14	1.2516	0.16148	304.63	333.70	1.2303
100	0.21449	314.01	344.04	1.2785	0.16622	313.72	343.63	1.2573
	p = 2 bar = 0.20 MPa (T_{sat} = – 5.42°C)				*p = 2.4 bar = 0.24 MPa* (T_{sat} = –5.42°C)			
Sat.	0.09933	221.43	241.30	0.9253	0.08343	224.07	244.09	0.9222
–10	0.09938	221.50	241.38	0.9256				
0	0.10438	229.23	250.10	0.9582	0.08574	228.31	248.89	0.9399
10	0.10922	237.05	258.89	0.9898	0.08993	236.26	257.84	0.9721
20	0.11394	244.99	267.78	1.0206	0.09399	244.30	266.85	1.0034
30	0.11856	253.06	276.77	1.0508	0.09794	252.45	275.95	1.0339
40	0.12311	261.26	285.88	1.0804	0.10181	260.72	285.16	1.0930
50	0.12758	269.61	295.12	1.1094	0.10562	269.12	294.47	1.0930
60	0.13201	278.10	304.50	1.1380	0.10937	277.87	303.91	1.1218
70	0.13639	286.74	314.02	1.1661	0.11307	286.35	313.49	1.1501
80	0.14073	295.53	323.68	1.1939	0.11674	295.18	323.19	1.1780
90	0.14504	304.47	333.48	1.2212	0.12037	304.15	333.04	1.2055
100	0.14932	313.57	343.43	1.2483	0.12398	313.27	343.03	1.2326

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Table B.8 *Contd.*

T °C	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m^3/kg	u kJ/kg	h kJkg	s kJ/kg·K
	p = 2.8 bar = 0.28 MPa (T_{sat} = – 1.23°C)				*p = 3.2 bar = 0.32 MPa* (T_{sat} = 2.48°C)			
Sat.	0.07193	226.38	246.52	0.9197	0.06322	228.43	248.66	0.9177
0	0.07240	227.37	247.64	0.9238				
10	0.07613	235.44	256.76	0.9566	0.06576	234.61	255.65	0.9427
20	0.07972	243.59	265.91	0.9883	0.06901	242.87	264.95	0.9749
30	0.08320	251.83	275.12	1.0192	0.07214	251.19	274.28	1.0062
40	0.08660	260.17	284.42	1.0494	0.07518	259.61	283.67	1.0367
50	0.08992	268.64	293.81	1.0789	0.07815	268.14	293.15	1.0665
60	0.09319	277.23	303.32	1.1079	0.08106	276.79	302.72	1.0957
70	0.09641	285.96	312.95	1.1364	0.08392	285.56	312.41	1.1243
80	0.09960	294.82	322.71	1.1644	0.08674	294.46	322.22	1.1525
90	0.10275	303.83	332.60	1.1920	0.08953	303.50	332.15	1.1802
100	0.10587	312.98	342.62	1.2193	0.09229	312.68	342.21	1.2076
110	0.10897	322.27	352.78	1.2461	0.09503	322.00	352.40	1.2345
120	0.11205	331.71	363.08	1.2727	0.09774	331.45	362.73	1.2611
	p = 4.0 bar = 0.40 MPa (T_{sat} = 8.93°C)				*p = 5.0 bar = 0.50 MPa* (T_{sat} = 15.74°C)			
Sat.	0.05089	231.97	252.32	0.9145	0.04086	235.64	256.07	0.9117
10	0.05119	232.87	253.35	0.9182				
20	0.05397	241.37	262.96	0.9515	0.04188	239.40	260.34	0.9264
30	0.05662	249.89	272.54	0.9837	0.04416	248.20	270.28	0.9597
40	0.05917	258.47	282.14	1.0148	0.04633	256.99	280.16	0.9918
50	0.06164	267.13	291.79	1.0452	0.04842	265.83	290.04	1.0229
60	0.06405	275.89	301.51	1.0748	0.05043	274.73	299.05	1.0531
70	0.06641	284.75	311.32	1.1038	0.05240	283.72	309.92	1.0825
80	0.06873	293.73	321.23	1.1322	0.05432	292.80	319.96	1.1114
90	0.07102	302.84	331.25	1.1602	0.05620	302.00	330.10	1.1397
100	0.07327	312.07	341.38	1.1878	0.05805	311.31	340.33	1.1675
110	0.07550	321.44	351.64	1.2149	0.05988	320.74	350.68	1.1949
120	0.07771	330.94	362.03	1.2417	0.06168	330.30	361.14	1.2218
130	0.07991	340.58	372.54	1.2681	0.06347	339.98	371.72	1.2484
140	0.08208	350.35	383.18	1.2941	0.06524	349.79	382.42	1.2746
	p = 6.0 bar = 0.60 MPa (T_{sat} = 21.58°C)				*p = 7.0 bar = 0.70 MPa* (T_{sat} = 26.72°C)			
Sat.	0.03408	238.74	259.19	0.9097	0.02918	241.42	261.85	0.9080
30	0.03581	246.41	267.89	0.9388	0.02979	244.51	265.37	0.9197
40	0.03774	255.45	378.90	0.9719	0.03157	253.83	275.93	0.9539
50	0.03958	264.48	288.23	1.0037	0.03324	263.08	286.35	0.9867
60	0.04134	273.54	298.35	1.0346	0.03482	272.31	296.69	1.0182
70	0.04304	282.66	208.48	1.0645	0.03634	281.57	307.01	1.0487
80	0.04469	291.86	318.67	1.0938	0.03781	290.88	317.35	1.0784
90	0.04631	301.14	328.93	1.1225	0.03924	300.27	327.74	1.1074
100	0.04790	310.53	339.27	1.1505	0.04064	309.74	338.19	1.1358
110	0.04946	320.03	349.70	1.1781	0.04201	319.31	348.71	1.1637
120	0.05099	329.64	360.24	1.2053	0.04335	328.98	359.33	1.1910
130	0.05251	339.38	370.88	1.2320	0.04468	338.76	370.04	1.2179
140	0.05402	349.23	381.64	1.2584	0.04599	348.66	380.86	1.2444
150	0.05550	359.21	392.52	1.2844	0.04729	358.68	391.79	1.2706
160	0.05698	369.32	403.51	1.3100	0.04857	368.82	402.82	1.2963

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Table B.8 *Contd.*

T °C	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m^3/kg	u kJ/kg	h kJkg	s kJ/kg·K
	p = 8 bar = 0.8 MPa (T_{sat} = 31.33°C)				*p = 9.0 bar = 0.90 MPa* (T_{sat} = 35.53°C)			
Sat.	0.02547	243.78	264.15	0.9066	0.02255	245.88	266.18	0.9054
40	0.02691	252.13	273.66	0.9374	0.02325	250.32	271.25	0.9217
50	0.02846	261.62	284.39	0.9711	0.02472	260.09	282.34	0.9566
60	0.02992	271.04	294.98	1.0034	0.02609	269.72	293.21	0.9897
70	0.03131	280.45	305.50	1.0345	0.02738	279.30	303.94	1.0214
80	0.03264	289.89	316.00	1.0647	0.02861	288.87	314.62	1.0521
90	0.03393	299.37	326.52	1.0940	0.02980	298.46	325.28	1.0819
100	0.03519	308.93	337.08	1.1227	0.03095	308.11	335.96	1.1109
110	0.03642	318.57	347.71	1.1508	0.03207	317.82	346.68	1.1392
120	0.03762	328.31	358.40	1.1784	0.03316	327.62	357.47	1.1670
130	0.03881	338.14	369.19	1.2055	0.03423	337.52	368.33	1.1943
140	0.03997	348.09	380.07	1.2321	0.03529	347.51	379.27	1.2211
150	0.04113	358.15	391.05	1.2584	0.03633	357.61	390.31	1.2475
160	0.04227	368.32	402.14	1.2843	0.03736	367.82	401.44	1.2735
170	0.04340	378.61	413.33	1.3098	0.03838	378.14	412.68	1.2992
180	0.04452	389.02	424.63	1.3351	0.03939	388.57	424.02	1.3245
	p = 10.0 bar = 1.00 MPa (T_{sat} = 39.39°C)				*p = 12.0 bar = 1.20 MPa* (T_{sat} = 46.32°C)			
Sat.	0.02020	247.77	267.97	0.9043	0.01663	251.03	270.99	0.9023
40	0.02029	248.39	268.68	0.9066				
50	0.02171	258.48	280.19	0.9428	0.01712	254.98	275.52	0.9164
60	0.02301	268.35	291.36	0.9768	0.01835	265.42	287.44	0.9527
70	0.02423	278.11	302.34	1.0093	0.01947	275.59	298.96	0.9868
80	0.02538	287.82	313.20	1.0405	0.02051	285.62	310.24	1.0192
90	0.02649	297.53	324.01	1.0707	0.02150	295.59	321.39	1.0503
100	0.02755	307.27	334.82	1.1000	0.02244	305.54	332.47	1.0804
110	0.02858	317.06	345.65	1.1286	0.02335	315.50	343.52	1.1096
120	0.02959	326.93	356.52	1.1567	0.02423	325.51	354.58	1.1381
130	0.03058	336.88	367.46	1.1841	0.02508	335.58	365.68	1.1600
140	0.03154	346.92	378.46	1.2111	0.02592	345.73	376.83	1.1933
150	0.03250	357.06	389.56	1.2376	0.02674	355.95	388.04	1.2201
160	0.03344	367.31	400.74	1.2638	0.02754	366.27	399.33	1.2465
170	0.03436	377.66	412.02	1.2895	0.02834	376.69	410.70	1.2724
180	0.03528	388.12	423.40	1.3149	0.02912	387.21	422.16	1.2980
	p = 14.0 bar = 1.40 MPa (T_{sat} = 52.43°C)				*p = 12.0 bar = 1.20 MPa* (T_{sat} = 57.92°C)			
Sat.	0.01405	253.74	273.40	0.9003	0.01208	256.00	275.33	0.8982
60	0.01495	262.17	283.10	0.9297	0.01233	258.48	278.20	0.9069
70	0.10603	272.87	295.31	0.9658	0.01340	269.89	291.33	0.9457
80	0.01701	283.29	307.10	0.9997	0.01435	280.78	303.74	0.9813
90	0.01792	293.55	318.63	1.0319	0.01521	291.39	315.72	1.0148
100	0.01878	303.73	330.02	1.0628	0.01601	301.84	327.46	1.0467
110	0.01960	313.88	341.32	1.0927	0.01677	312.20	339.04	1.0773
120	0.02039	324.05	352.59	1.1218	0.01750	322.53	350.53	1.1069
130	0.02115	334.25	363.86	1.1501	0.01820	332.87	361.99	1.1357
140	0.02189	344.50	375.15	1.1777	0.01887	343.24	373.44	1.1638
150	0.02262	354.82	386.49	1.2048	0.01953	353.66	384.91	1.1912
160	0.02333	365.22	397.89	1.2315	0.02017	364.15	396.43	1.2181
170	0.02403	375.71	409.36	1.2576	0.02080	374.71	407.99	1.2445
180	0.02472	386.29	420.90	1.2834	0.02142	385.35	419.62	1.2704
190	0.02541	396.96	432.53	1.3088	0.02203	396.08	431.33	1.2960
200	0.02608	407.73	444.24	1.3338	0.02263	406.90	443.11	1.3212

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Table B.9 *Properties of* ***Saturated Ammonia*** *(Liquid–Vapour): Temperature Entry*

Temp. °C	Press. bar	Specific Volume m^3/kg		Internal Energy kJ/kg		Enthalpy kJ/kg			Entropy kJ/kg · K		Temp. °C
		Sat. Liquid $v_f \times 10^3$	Sat. Vapour v_g	Sat. Liquid u_f	Sat. Vapour u_g	Sat. Liquid h_f	Sat. Evap. h_{fg}	Sat. Vapor h_g	Sat. Liquid s_f	Sat. Vapor s_g	
–50	0.4086	1.4245	2.6265	–43.94	1264.99	–43.88	1416.20	1372.32	–0.1922	6.1543	–50
–45	0.5453	1.4367	2.0060	–22.03	1271.19	–21.95	1402.52	1380.57	–0.0951	6.0523	–45
–40	0.7174	1.4493	1.5524	–0.10	1277.20	0.00	1388.56	1388.56	0.0000	5.9557	–40
–36	0.8850	1.4597	1.2757	17.47	1281.87	17.60	1377.17	1394.77	0.0747	5.8819	–36
–32	1.0832	1.4703	1.0561	35.09	1286.41	35.25	1365.55	1400.81	0.1484	5.8111	–32
–30	1.1950	1.4757	0.9634	43.93	1288.63	44.10	1359.65	1403.75	0.1849	5.7765	–30
–28	1.3159	1.4812	0.8803	52.78	1290.82	52.97	1353.68	1406.66	0.2212	5.7430	–28
–26	1.4465	1.4867	0.8056	61.65	1292.97	61.86	1347.65	1409.51	0.2572	5.7100	–26
–22	1.7390	1.4980	0.6780	79.46	1297.18	79.72	1335.36	1415.08	0.3287	5.6457	–22
–20	1.9019	1.5038	0.6233	88.40	1299.23	88.68	1329.10	1417.79	0.3642	5.6144	–20
–18	2.0769	1.5096	0.5739	97.36	1301.25	97.68	1322.77	1420.45	0.3994	5.5837	–18
–16	2.2644	1.5155	0.5291	106.36	1303.23	106.70	1316.35	1423.05	0.4346	5.5536	–16
–14	2.4652	1.5215	0.4885	115.37	1305.17	115.75	1309.86	1425.61	0.4695	5.5239	–14
–12	2.6798	1.5276	0.4516	124.42	1307.08	124.83	1303.28	1428.11	0.5043	5.4948	–12
–10	2.9089	1.5338	0.4180	133.50	1308.95	133.94	1296.61	1430.55	0.5389	5.4662	–10
–8	3.1532	1.5400	0.3874	142.60	1310.78	143.09	1289.86	1432.95	0.5734	5.4380	–8
–6	3.4134	1.5464	0.3595	151.74	1312.57	152.26	1283.02	1435.28	0.6077	5.4103	–6
–4	3.6901	1.5528	0.3340	160.88	1314.32	161.46	1276.10	1437.56	0.6418	5.3831	–4
–2	3.9842	1.5594	0.3106	170.07	1316.04	170.69	1269.08	1439.78	0.6759	5.3562	–2
0	4.2962	1.5660	0.2892	179.29	1317.71	179.96	1261.97	1441.94	0.7097	5.3298	0
2	4.6270	1.5727	0.2695	188.53	1319.34	189.20	1254.77	1444.03	0.7435	5.3038	2
4	4.9773	1.5796	0.2514	197.80	1320.92	198.59	1247.48	1446.07	0.7770	5.2781	4
6	5.3479	1.5866	0.2348	207.10	1322.47	207.95	1240.09	1448.04	0.8105	5.2529	6
8	5.7395	1.5936	0.2195	216.42	1323.96	217.34	1232.61	1449.94	0.8438	5.2279	8
10	6.1529	1.6008	0.2054	225.77	1325.42	226.75	1225.03	1451.78	0.8769	5.2033	10
12	6.5890	1.6081	0.1923	235.14	1326.82	236.20	1217.35	1453.55	0.9099	5.1791	12
16	7.5324	1.6231	0.1691	253.95	1329.48	255.18	1201.70	1456.87	0.9755	5.1314	16
20	8.5762	1.6386	0.1492	272.86	1331.94	274.26	1185.64	1459.90	1.0404	5.0849	20
24	9.7274	1.6547	0.1320	291.84	1334.19	293.45	1169.16	1462.61	1.1048	5.0394	24
28	10.993	1.6714	0.1172	310.92	1336.20	312.75	1152.24	1465.00	1.1686	4.9948	28
32	12.380	1.6887	0.1043	330.07	1337.97	332.17	1134.87	1467.03	1.2319	4.9509	32
36	13.896	1.7068	0.0930	349.32	1339.47	351.69	1117.00	1468.70	1.2946	4.9078	36
40	15.549	1.7256	0.0831	368.67	1340.70	371.35	1098.62	1469.97	1.3569	4.8652	40
45	17.819	1.7503	0.0725	393.01	1341.81	396.31	1074.84	1470.96	1.4341	4.8125	45
50	20.331	1.7765	0.0634	417.56	1342.42	421.17	1050.09	1471.26	1.5109	4.7604	50

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Table B.10 *Properties of* ***Saturated Ammonia*** *(Liquid–Vapour): Pressure Entry*

Press. bar	*Temp. °C*	*Specific Volume m^3/kg*		*Internal Energy kJ/kg*		*Enthalpy kJ/kg*			*Entropy kJ/kg · K*		*Press. bar*
		Sat. Liquid $v_f \times 10^3$	*Sat. Vapour v_g*	*Sat. Liquid u_f*	*Sat. Vapour u_g*	*Sat. Liquid h_f*	*Evap. h_{fg}*	*Sat. Vapour h_g*	*Sat. Liquid s_f*	*Sat. Vapour s_g*	
0.40	–50.36	1.4236	2.6795	–45.52	1264.54	–45.46	1417.18	1371.72	–0.1992	6.1618	0.40
0.50	–46.53	1.4330	2.1752	–28.73	1269.31	–28.66	1406.73	1378.07	–0.1245	6.0829	0.50
0.60	–43.28	1.4410	1.8345	–14.51	1273.27	–14.42	1397.76	1383.34	–0.0622	6.0186	0.60
0.70	–40.46	1.4482	1.5884	–2.11	1276.66	–2.01	1389.85	1387.84	–0.0086	5.9643	0.70
0.80	–37.94	1.4546	1.4020	8.93	1279.61	9.04	1382.73	1391.78	0.0386	5.9174	0.80
0.90	–35.67	1.4605	1.2559	18.91	1282.24	19.04	1376.23	1395.27	0.0808	5.8760	0.90
1.00	–33.60	1.4660	1.1381	28.03	1284.61	28.18	1370.23	1398.41	0.1191	5.8391	1.00
1.25	–29.07	1.4782	0.9237	48.03	1289.65	48.22	1356.89	1405.11	0.2018	5.7610	1.25
1.50	–25.22	1.4889	0.7787	65.10	1293.80	65.32	1345.28	1410.61	0.2712	5.6973	1.50
1.75	–21.86	1.4984	0.6740	80.08	1297.33	80.35	1334.92	1415.27	0.3312	5.6435	1.75
2.00	–18.86	1.5071	0.5946	93.50	1300.39	93.80	1325.51	1419.31	0.3843	5.5969	2.00
2.25	–16.15	1.5151	0.5323	105.68	1303.08	106.03	1316.83	1422.86	0.4319	5.5558	2.25
2.50	–13.67	1.5225	0.4821	116.88	1305.49	117.26	1308.76	1426.03	0.4753	5.5190	2.50
2.75	–11.37	1.5295	0.4408	127.26	1307.67	127.68	1301.20	1428.88	0.5152	5.4858	2.75
3.00	–9.24	1.5361	0.4061	136.96	1309.65	137.42	1294.05	1431.47	0.5520	5.4554	3.00
3.25	–7.24	1.5424	0.3765	146.06	1311.46	146.57	1287.27	1433.84	0.5864	5.4275	3.25
3.50	–5.36	1.5484	0.3511	154.66	1313.14	155.20	1280.81	1436.01	0.6186	5.4016	3.50
3.75	–3.58	1.5542	0.3289	162.80	1314.68	166.38	1274.64	1438.03	0.6489	5.3774	3.75
4.00	–1.90	1.5597	0.3094	170.55	1316.12	171.18	1268.71	1439.89	0.6776	5.3548	4.00
4.25	–0.29	1.5650	0.2921	177.96	1317.47	178.62	1263.01	1441.63	0.7048	5.3336	4.25
4.50	1.25	1.5702	0.2767	185.04	1318.73	185.75	1257.50	1443.25	0.7308	5.3135	4.50
4.75	2.72	1.5752	0.2629	191.84	1319.91	192.59	1252.18	1444.77	0.7555	5.2946	4.75
5.00	4.13	1.5800	0.2503	198.39	1321.02	199.18	1247.02	1446.19	0.7791	5.2765	5.00
5.25	5.48	1.5847	0.2390	204.69	1322.07	205.52	1242.01	1447.53	0.8018	5.2594	5.22
5.50	6.79	1.5893	0.2286	210.78	1323.06	211.65	1237.15	1448.80	0.8236	5.2430	5.50
5.75	8.05	1.5938	0.2191	216.66	1324.00	217.58	1232.41	1449.99	0.8446	5.2273	5.75
6.00	9.27	1.5982	0.2104	222.37	1324.89	223.32	1227.79	1451.12	0.8649	5.2122	6.00
7.00	13.79	1.6148	0.1815	243.56	1328.04	244.69	1210.38	1455.07	0.9394	5.1576	7.00
8.00	17.84	1.6302	0.1596	262.64	1330.64	263.95	1194.36	1458.30	1.0054	5.1099	8.00
9.00	21.52	1.6446	0.1424	280.05	1332.82	281.53	1179.44	1460.97	1.0649	5.0675	9.00
10.00	24.89	1.6584	0.1285	296.10	1334.66	297.76	1165.42	1463.18	1.1191	5.0294	10.00
12.00	30.94	1.6841	0.1075	324.99	1337.52	327.10	1139.52	1466.53	1.2152	4.9625	12.00
14.00	36.26	1.7080	0.0923	350.58	1339.56	352.97	1115.82	1468.79	1.2987	4.9050	14.00
16.00	41.03	1.7306	0.0808	373.69	1340.97	376.46	1093.77	1470.23	1.3729	4.8542	16.00
18.00	45.38	1.7522	0.0717	394.85	1341.88	398.00	1073.01	1471.01	1.4399	4.8086	18.00
20.00	49.37	1.7731	0.0644	414.44	1342.37	417.99	1053.27	1471.26	1.5012	4.7670	20.00

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Table B.11 *Properties of* ***Superheated Refrigerant Ammonia Vapour***

T °C	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m^3/kg	u kJ/kg	h kJkg	s kJ/kg·K
	p = 0.4 bar = 0.04 MPa (T_{sat} = – 50.30°C)				*p = 0.6 bar = 0.06 MPa* (T_{sat} = – 43.28°C)			
Sat.	2.6795	1264.54	1371.72	6.1618	1.8345	1273.27	1383.34	6.0186
–50	2.6841	1265.11	1372.48	6.1652				
–45	2.7481	1273.05	1382.98	6.2118				
–40	2.8118	1281.01	1393.48	6.2573	1.8630	1278.62	1390.40	6.0490
–35	2.8753	1288.96	1403.98	6.3018	1.9061	1286.75	1401.12	6.0946
–30	2.9385	1296.93	1414.47	6.3455	1.9491	1294.88	1411.83	6.1390
–25	3.0015	1304.90	1424.96	6.3882	1.9918	1303.01	1422.52	6.1826
–20	3.0644	1312.88	1435.46	6.4300	20343	1311.13	1433.19	6.2251
–15	3.1271	1320.86	1445.95	6.4711	2.0766	1319.25	1443.85	6.2668
–10	3.1896	1328.87	1456.45	6.5114	2.1188	1327.37	1454.50	6.3077
–5	3.2520	1336.88	1466.95	6.5509	2.1609	1335.49	1465.14	6.3478
0	3.3142	1344.90	1477.17	6.5898	2.2028	1343.61	1475.78	6.3871
5	3.3764	1352.95	1488.00	6.6280	2.2446	1351.75	1486.43	6.4257
	p = 0.8 bar = 0.08 MPa (T_{sat} = – 37.94°)				*p = 1.0 bar = 0.10 MPa* (T_{sat} = – 33.60°C)			
Sat.	1.4021	1279.61	1391.78	5.9174	1.1381	1284.61	1398.41	5.8391
–35	1.4215	1284.51	1398.23	5.9446				
–30	1.4543	1292.81	1409.15	5.9900	1.1573	1290.71	1406.71	5.8723
–25	1.4868	1301.09	1420.04	6.0343	1.1838	1299.15	1417.53	5.9175
–20	1.5192	1309.36	1430.90	6.0777	1.2101	1307.57	1428.58	5.9616
–15	1.5514	1317.61	1441.72	6.1200	1.2362	1315.96	1439.58	6.0046
–10	1.5834	1325.85	1452.53	6.1615	1.2621	1324.33	1450.54	6.0467
–5	1.6153	1334.09	1463.31	6.2021	1.2880	1332.67	1461.46	6.0878
0	1.6471	1342.31	1474.08	6.2419	1.3136	1341.00	1472.37	6.1281
5	1.6788	1350.54	1484.84	6.2809	1.3392	1349.33	1483.25	6.1676
10	1.7103	1358.77	1495.60	6.3192	1.3647	1357.64	1494.11	6.2063
15	1.7418	1367.01	1506.35	6.3568	1.3900	1365.95	1504.96	6.2442
20	1.7732	1375.25	1517.10	6.3539	1.4153	1374.27	1515.80	6.2816
	p = 1.5 bar = 0.15 MPa (T_{sat} = – 25.22°C)				*p = 2.0 bar = 0.20 MPa* (T_{sat} = – 18.86°C)			
Sat.	0.7787	1293.80	1410.61	5.6973	0.59460	1300.39	1419.31	5.5969
–25	0.7795	1294.20	1411.13	5.6994				
–20	0.7978	1303.00	1422.67	5.7454				
–15	0.8158	1311.75	1434.12	5.7902	0.60542	1307.43	1428.51	5.6328
–10	0.8336	1320.44	1445.49	5.8338	0.61926	1316.46	1440.31	5.6781
–5	0.8514	1329.08	1456.79	5.8764	0.63294	1325.41	1452.00	5.7221
0	0.8689	1337.68	1468.02	5.9179	0.64648	1334.29	1463.59	5.7645
5	0.8864	1346.25	1479.20	5.9585	0.65989	1343.11	1475.09	5.8066
10	0.9037	1354.78	1490.34	5.9981	0.67320	1351.87	1486.51	5.8473
15	0.9210	1363.29	1501.44	6.0370	0.68640	1360.59	1497.87	5.8871
20	0.9382	1371.79	1512.51	6.0751	0.69952	1369.28	1509.10	5.9260
25	0.9553	1380.28	1523.56	6.1125	0.71256	1377.93	1520.44	5.9641
30	0.9723	1388.76	1534.60	6.1492	0.72553	1386.56	1531.67	6.0014

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Table B.11 *Contd.*

T °C	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K
	p = 2.5 bar = 0.25 MPa (T_{sat} = – 13.67°C)				*p = 3.0 bar = 0.30 MPa* (T_{sat} = – 9.24°C)			
Sat.	0.48213	1305.49	1426.03	5.5190	0.40607	1309.65	1431.47	5.4554
–10	0.49051	1312.37	1435.00	5.5534				
–5	0.50180	1321.65	1447.10	5.5989	0.41428	1317.80	1442.08	5.4953
0	0.51293	1330.83	1459.06	5.6431	0.42382	1327.28	1454.43	5.5409
5	0.52393	1339.91	1470.89	5.6860	0.43323	1336.64	1466.61	5.5851
10	0.53482	1348.91	1482.61	5.7278	0.44251	1345.89	1478.65	5.6280
15	0.54560	1357.84	1494.25	5.7685	0.45169	1355.05	1490.56	5.6697
20	0.55630	1366.72	1505.80	5.8083	0.46078	1364.13	1502.36	5.7103
25	0.56691	1375.55	1517.28	5.8471	0.46978	1373.14	1514.07	5.7499
30	0.57745	1384.34	1528.70	5.8851	0.47870	1382.09	1525.70	5.7886
35	0.58793	1393.10	1540.08	5.9223	0.48756	1391.00	1357.26	5.8264
40	0.59835	1401.84	1551.42	5.9589	0.49637	1399.86	1548.77	5.8635
45	0.60872	1410.56	1562.74	5.9947	0.50512	1408.70	1560.24	5.8998
	p = 3.5 bar = 0.35 MPa (T_{sat} = – 5.36°C)				*p = 4.0 bar = 0.40 MPa* (T_{sat} = – 1.90°C)			
Sat.	0.35108	1313.14	1436.01	5.4016	0.30942	1316.12	1439.89	5.3548
0	0.36011	1323.66	1449.70	5.4522	0.31227	1319.95	1444.86	5.3731
10	0.37654	1342.82	1474.61	5.5417	0.32701	1339.68	1470.49	5.4652
20	0.39251	1361.49	1498.87	5.6259	0.34129	1358.81	1495.33	5.5515
30	0.40814	1379.81	1522.66	5.7057	0.35520	1377.49	1519.57	5.6328
40	0.42350	1397.87	1546.09	5.7818	0.36884	1395.85	1543.38	5.7101
60	0.45363	1433.55	1592.32	5.9249	0.39550	1431.97	1590.17	5.8549
80	0.48320	1469.06	1638.18	6.0586	0.42160	1467.77	1636.41	5.9867
100	0.51240	1504.73	1684.07	6.1850	0.44733	1503.64	1682.58	6.1169
120	0.54136	1540.79	1730.26	6.3056	0.47280	1539.85	1728.97	6.2380
140	0.57013	1577.38	1776.92	6.4213	0.49808	1576.55	1775.79	6.3541
160	0.59876	1616.60	1824.16	6.5330	0.52323	1613.86	1823.16	6.4661
180	0.62728	1652.51	1872.06	6.6411	0.54827	1651.85	1871.16	6.5744
200	0.65572	1691.15	1920.65	6.7460	0.57322	1690.56	1919.85	6.6796
	p = 4.5 bar = 0.45 MPa (T_{sat} = 1.25°C)				*p = 5.0 bar 0.50 MPa* (T_{sat} = 4.13°C)			
Sat.	0.27671	1318.73	1443.25	5.3135	0.25034	1321.02	1446.19	5.2765
10	0.28846	1336.48	1466.29	5.3962	0.25757	1333.22	1462.00	5.3330
20	0.30142	1356.09	1491.72	5.4845	0.26949	1353.32	1488.06	5.4234
30	0.31401	1375.15	1516.45	5.5674	0.28103	1372.76	1513.28	5.5080
40	0.32631	1393.80	1540.64	5.6460	0.29227	1391.74	1537.87	5.5878
60	0.35029	1430.37	1588.00	5.7926	0.31410	1428.76	1585.81	5.7362
80	0.37369	1466.47	1634.63	5.9285	0.33535	1465.16	1632.84	5.8733
100	0.39671	1502.55	1681.07	6.0564	0.35621	1501.46	1679.56	6.0020
120	0.41947	1538.91	1727.67	6.1781	0.37681	1537.97	1726.37	6.1242
140	0.44205	1575.73	1774.65	6.2946	0.39722	1574.90	1773.51	6.2412
160	0.46448	1613.13	1822.15	6.4069	0.41749	1612.40	1821.14	6.3537
180	0.48681	1651.20	1870.26	6.5155	0.43765	1650.54	1869.36	6.4626
200	0.50905	1689.97	1919.04	6.6208	0.45771	1689.38	1918.24	6.5681

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Table B.11 *Contd.*

T °C	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m^3/kg	u kJ/kg	h kJkg	s kJ/kg·K
	p = 5.5 bar = 0.55 MPa (T_{sat} = 6.79°C)				*p = 6.0 bar = 0.60 MPa* (T_{sat} = 9.27°C)			
Sat.	0.22861	1323.06	1448.80	5.2430	0.21038	1324.89	1451.12	5.2122
10	0.23227	1329.88	1457.63	5.2743	0.21115	1326.47	1453.16	5.2195
20	0.24335	1350.50	1484.34	5.3671	0.22155	1347.62	1480.55	5.3145
30	0.25403	1370.35	1510.07	5.4534	0.23152	1367.90	1506.81	5.4026
40	0.26441	1389.64	1535.07	5.5345	0.24118	1387.52	1532.23	5.4851
50	0.27454	1408.53	1559.53	5.6114	0.25059	1406.67	1557.03	5.5631
60	0.28449	1427.13	1583.60	5.6848	0.25981	1425.49	1581.38	5.6373
80	0.30398	1463.85	1631.04	5.8230	0.27783	1462.52	1629.22	5.7768
100	0.32307	1500.36	1678.05	5.9525	0.29546	1499.25	1676.52	5.9071
120	0.34190	1537.02	1725.07	6.0753	0.31281	1536.07	1723.76	6.0304
140	0.36054	1574.07	1772.37	6.1926	0.32997	1573.24	1771.22	6.1481
160	0.37903	1611.66	1820.13	6.3055	0.34699	1610.92	1819.12	6.2613
180	0.39742	1649.88	1868.46	6.4146	0.36390	1649.22	1867.56	6.3707
200	0.41571	1688.79	1917.43	6.5203	0.38071	1688.20	1916.63	6.4766
	p = 7.0 bar = 0.70 MPa (T_{sat} = 13.79°C)				*p = 8.0 bar = 0.80 MPa* (T_{sat} = 17.84°C)			
Sat.	0.18148	1328.04	1455.07	5.1576	0.15958	1330.64	1458.30	5.1099
20	0.18721	1341.72	1472.77	5.2186	0.16138	1335.59	1464.70	5.1318
30	0.19610	1362.88	1500.15	5.3104	0.16948	1357.71	1493.29	5.2277
40	0.20464	1383.20	1526.45	5.3958	0.17720	1378.77	1520.53	5.3161
50	0.21293	1402.90	1551.95	5.4760	0.18465	1399.05	1546.77	5.3986
60	0.22101	1422.16	1576.87	5.5519	0.19189	1418.77	1572.28	5.4763
80	0.23674	1459.85	1625.56	5.6939	0.20590	1457.14	1621.86	5.6209
100	0.25205	1497.02	1673.46	5.8258	0.21949	1494.77	1670.37	5.7545
120	0.26709	1534.12	1721.12	5.9502	0.23280	1532.24	1718.48	5.8801
140	0.28193	1571.57	1768.92	6.0688	0.24590	1569.89	1766.61	5.9995
160	0.29663	1609.44	1817.08	6.1826	0.25886	1607.96	1815.04	6.1140
180	0.31121	1647.90	1865.75	6.2925	0.27170	1646.57	1683.94	6.2243
200	0.32571	1687.02	1915.01	6.3988	0.28445	1685.83	1913.39	6.3311
	p = 9.0 bar = 0.90 MPa (T_{sat} = 21.52°C)				*p = 10.0 bar = 1.00 MPa* (T_{sat} = 24.89°C)			
Sat.	0.14239	1332.82	1460.97	5.0675	0.12852	1334.66	1463.18	5.0294
30	0.14872	1352.36	1486.20	5.1520	0.13206	1346.82	1478.88	5.0816
40	0.15582	1374.21	1514.45	5.2436	0.13868	1369.52	1508.20	5.1768
50	0.16263	1395.11	1541.47	5.3286	0.14499	1391.07	1536.06	5.2644
60	0.16922	1415.32	1567.61	5.4083	0.15106	1411.79	1562.86	5.3460
80	0.18191	1454.39	1618.11	5.5555	0.16270	1451.60	1614.31	5.4960
100	0.19416	1492.50	1667.24	5.6908	0.17389	1490.20	1664.10	5.6332
120	0.20612	1530.30	1715.81	5.8176	0.18478	1528.35	1713.13	5.7612
140	0.21788	1568.20	1764.29	5.9379	0.19545	1566.51	1761.96	5.8803
160	0.22948	1606.46	1813.00	6.0530	0.20598	1604.97	1810.94	5.9981
180	0.24097	1645.24	1862.12	6.1639	0.21638	1643.91	1860.29	6.1095
200	0.25237	1684.64	1911.77	6.2711	0.22670	1683.44	1910.14	6.2171

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Table B.11 *Contd.*

T °C	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K
	p = 12.0 bar = 1.20 MPa (T_{sat} = 30.94°C)				*p = 14.0 bar = 1.40 MPa* (T_{sat} = 36.26°C)			
Sat.	0.10751	1337.52	1466.53	4.9625	0.09231	1339.56	1468.79	4.9050
40	0.11287	1359.73	1495.18	5.0553	0.09432	1349.29	1481.33	4.9453
60	0.12378	1404.54	1553.07	5.2347	0.10423	1396.97	1542.98	5.1360
80	0.13387	1445.91	1606.56	5.3906	0.11324	1440.06	1598.59	5.2984
100	0.14347	1485.55	1657.71	5.5315	0.12172	1480.79	1651.20	5.4433
120	0.15275	1524.41	1707.71	5.6620	0.12986	1520.41	1702.21	5.5765
140	0.16181	1563.09	1757.26	5.7850	0.13777	1559.63	1752.52	5.7013
160	0.17072	1601.95	1806.81	5.9021	0.14552	1598.92	1802.65	5.8198
180	0.17950	1641.23	1856.63	6.0145	0.15315	1638.53	1852.94	5.9333
200	0.18819	1681.05	1906.87	6.1230	0.16068	1678.64	1903.59	6.0427
220	0.19680	1721.50	1957.66	6.2282	0.16813	1719.35	1954.73	6.1485
240	0.20534	1762.63	2009.04	6.3303	0.17551	1760.72	2006.43	6.2513
260	0.21382	1804.48	2061.06	6.4297	0.18283	1802.78	2058.75	6.3513
280	0.22225	1847.04	2113.74	6.5267	0.19010	1845.55	2111.69	6.4488
	p = 16.0 bar = 1.60 MPa (T_{sat} = 41.03°C)				*p = 18.0 bar = 1.80 MPa* (T_{sat} = 45.38°C)			
Sat.	0.08079	1340.97	1470.23	4.8542	0.07174	1341.88	1471.01	4.8086
60	0.08951	1389.06	1532.28	5.0461	0.07801	1380.77	1521.10	4.9627
80	0.09774	1434.02	1590.40	5.2156	0.08565	1427.79	1581.97	5.1399
100	0.10539	1475.93	1644.56	5.3648	0.09267	1470.96	1637.78	5.2937
120	0.11268	1516.34	1696.64	5.5008	0.09931	1512.22	1690.98	5.4326
140	0.11974	1556.14	1747.72	5.6276	0.10570	1552.61	1742.88	5.5614
160	0.12663	1595.85	1798.45	5.7475	0.11192	1592.76	1794.23	5.6828
180	0.13339	1635.81	1849.23	5.8621	0.11801	1633.08	1845.50	5.7985
200	0.14005	1676.21	1900.29	5.9723	0.12400	1673.78	1896.98	5.9096
220	0.14663	1717.18	1951.79	6.0789	0.12991	1715.00	1948.83	6.0170
240	0.15314	1758.79	2003.81	6.1823	0.13574	1756.85	2001.18	6.1210
260	0.15959	1801.07	2056.42	6.2829	0.14152	1799.35	2054.08	6.2222
280	0.16599	1844.05	2109.64	6.3809	0.14724	1842.55	2107.58	6.3207
	p = 20.0 bar = 2.00 MPa (T_{sat} = 49.37°C)							
Sat.	0.06445	1342.37	1471.26	4.7670				
60	0.06445	1372.05	1509.54	4.8838				
80	0.07596	1421.36	1573.27	5.0696				
100	0.08248	1465.89	1630.86	5.2283				
120	0.08861	1508.03	1685.24	5.3703				
140	0.09447	1549.03	1737.98	5.5012				
160	0.10016	1589.65	1789.97	5.6241				
180	0.10571	1630.82	1841.74	5.7409				
200	0.11116	1671.33	1893.64	5.8530				
220	0.11652	1712.82	1945.87	5.9611				
240	0.12182	1754.90	1998.54	6.0658				
260	0.12706	1797.63	2051.74	6.1675				
280	0.13224	1841.03	2105.50	6.2665				

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Table B.12 *Properties of Saturated* ***Propane*** *(Liquid–Vapour): Temperature Entry*

Temp. °C	*Press.* bar	*Specific Volume* m^3/kg		*Internal Energy* kJ/kg		*Enthalpy* kJ/kg			*Entropy* kJ/kg · K		*Temp.* °C
		Sat. Liquid $v_f \times 10^3$	*Sat. Vapour* v_g	*Sat. Liquid* u_f	*Sat. Vapour* u_g	*Sat. Liquid* h_f	*Evap.* h_{fg}	*Sat. Vapour* h_g	*Sat. Liquid* s_f	*Sat. Vapour* s_g	
–100	0.02888	1.553	11.27	–128.4	319.5	–128.4	480.4	352.0	–0.634	2.140	–100
–90	0.06426	1.578	5.345	–107.8	329.3	–107.8	471.4	363.6	–0.519	2.055	–90
–80	0.1301	1.605	2.7704	–87.0	339.3	–87.0	462.4	375.4	–0.408	1.986	–80
–70	0.2434	1.633	1.551	–65.8	349.5	–65.8	453.1	387.3	–0.301	1.929	–70
–60	0.4261	1.663	0.9234	–44.4	359.9	–44.3	443.5	399.2	–0.198	1.883	–60
–50	0.7046	1.694	0.5793	–22.5	370.4	–22.4	433.6	411.2	–0.098	1.845	–50
–40	1.110	1.728	0.3798	–0.2	381.0	0.0	423.2	423.2	0.000	1.815	–40
–30	1.677	1.763	0.2585	22.6	391.6	22.9	412.1	435.0	0.096	1.791	–30
–20	2.444	1.802	0.1815	45.9	402.4	46.3	400.5	446.8	0.190	1.772	–20
–10	3.451	1.844	0.1309	69.8	413.2	70.4	388.0	458.4	0.282	1.757	–10
0	4.743	1.890	0.09653	94.2	423.8	95.1	374.5	469.6	0.374	1.745	0
4	5.349	1.910	0.08591	104.2	428.1	105.3	368.8	474.1	0.410	1.741	4
8	6.011	1.931	0.07666	114.3	432.3	115.5	362.9	478.4	0.446	1.737	8
12	6.732	1.952	0.06858	124.6	436.5	125.9	356.8	482.1	0.482	1.734	12
16	7.515	1.975	0.06149	135.0	440.7	136.4	350.5	486.9	0.519	1.731	16
20	8.362	1.999	0.05525	145.4	444.8	147.1	343.9	491.0	0.555	1.728	20
24	9.278	2.024	0.04973	156.1	448.9	158.0	337.0	495.0	0.591	1.725	24
28	10.27	2.050	0.04483	166.9	452.9	169.0	329.9	498.9	0.627	1.722	28
32	11.33	2.078	0.04048	177.8	456.7	180.2	322.4	502.6	0.663	1.720	32
36	12.47	2.108	0.03659	188.9	460.6	191.6	314.6	506.2	0.699	1.717	36
40	13.69	2.140	0.03310	200.2	464.3	203.1	306.5	509.6	0.736	1.715	40
44	15.00	2.174	0.02997	211.7	467.9	214.9	298.0	512.9	0.772	1.712	44
48	16.40	2.211	0.02714	223.4	471.4	227.0	288.9	515.9	0.809	1.709	48
52	17.89	2.250	0.02459	235.3	474.6	239.3	279.3	518.6	0.846	1.705	52
56	19.47	2.293	0.02227	247.4	477.7	251.9	269.2	521.1	0.884	1.701	56
60	21.16	2.340	0.02015	259.8	480.6	264.8	258.4	523.2	0.921	1.697	60
65	23.42	2.406	0.01776	275.7	483.6	281.4	243.8	525.2	0.969	1.690	65
70	25.86	2.483	0.01560	292.3	486.1	298.7	227.2	526.4	1.018	1.682	70
75	28.49	2.573	0.01363	309.5	487.8	316.8	209.8	526.6	1.069	1.671	75
80	31.31	2.683	0.01182	327.6	488.2	336.0	189.2	525.2	1.122	1.657	80
85	34.36	2.827	0.01011	347.2	486.9	356.9	164.7	521.6	1.178	1.638	85
90	37.64	3.038	0.008415	369.4	482.2	380.8	133.1	513.9	1.242	1.608	90
95	41.19	3.488	0.006395	399.8	467.4	414.2	79.5	493.7	1.330	1.546	95
96.7	42.48	4.535	0.004535	434.9	434.9	454.2	0.0	457.2	1.437	1.437	96.7

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Table B.13 *Properties of Saturated* ***Propane*** *(Liquid–Vapour): Pressure Entry*

		Specific Volume m^3/kg		*Internal Energy* kJ/kg		*Enthalpy* kJ/kg			*Entropy* $kJ/kg \cdot K$		
Press. *bar*	*Temp.* *°C*	*Sat. Liquid* $v_f \times 10^3$	*Sat. Vapour* v_g	*Sat. Liquid* u_f	*Sat. Vapour* u_g	*Sat. Liquid* h_f	*Evap.* h_{fg}	*Sat. Vapour* h_g	*Sat. Liquid* s_f	*Sat. Vapour* s_g	*Press.* *bar*
0.05	–93.28	1.570	6.752	–114.6	326.0	–114.6	474.4	359.8	–0.556	2.081	0.05
0.10	–83.87	1.594	3.542	–95.1	335.4	–95.1	465.9	370.8	–0.450	2.011	0.10
0.25	–69.55	1.634	1.513	–64.9	350.0	–64.9	452.7	387.8	–0.297	1.927	0.25
0.50	–56.93	1.672	0.7962	–37.7	363.1	–37.6	440.5	402.9	–0.167	1.871	0.50
0.75	–48.68	1.698	0.5467	–19.6	371.8	–19.5	432.3	412.8	–0.085	1.841	0.75
1.00	–42.38	1.719	0.4185	–5.6	378.5	–5.4	425.7	420.3	–0.023	1.822	1.00
2.00	–25.43	1.781	0.2192	33.1	396.6	33.5	406.9	440.4	0.139	1.782	2.00
3.00	–14.16	1.826	0.1496	59.8	408.7	60.3	393.3	453.6	0.244	1.762	3.00
4.00	–5.46	1.865	0.1137	80.8	418.0	81.5	382.0	463.5	0.324	1.751	4.00
5.00	1.74	1.899	0.09172	98.6	425.7	99.5	372.1	471.6	0.389	1.743	5.00
6.00	7.93	1.931	0.07680	114.2	432.2	115.3	363.0	478.3	0.446	1.737	6.00
7.00	13.41	1.960	0.06598	128.2	438.0	129.6	354.6	484.2	0.495	1.733	7.00
8.00	18.33	1.989	0.05776	141.0	443.1	142.6	346.7	489.3	0.540	1.729	8.00
9.00	22.82	2.016	0.05129	152.9	447.6	154.7	339.1	493.8	0.580	1.726	9.00
10.00	26.95	2.043	0.04606	164.0	451.8	166.1	331.8	497.9	0.618	1.723	10.00
11.00	30.80	2.070	0.04174	174.5	455.6	176.8	324.7	501.5	0.652	1.721	11.00
12.00	34.39	2.096	0.03810	184.4	459.1	187.0	317.8	504.8	0.685	1.718	12.00
13.00	37.77	2.122	0.03499	193.9	462.2	196.7	311.0	507.7	0.716	1.716	13.00
14.00	40.97	2.148	0.03231	203.0	465.2	206.0	304.4	510.4	0.745	1.714	14.00
15.00	44.01	2.174	0.02997	211.7	467.9	215.0	297.9	512.9	0.772	1.712	15.00
16.00	46.89	2.200	0.02790	220.1	470.4	223.6	291.4	515.0	0.799	1.710	16.00
17.00	49.65	2.227	0.02606	228.3	472.7	232.0	285.0	517.0	0.824	1.707	17.00
18.00	52.30	2.253	0.02441	236.2	474.9	240.2	278.6	518.8	0.849	1.705	18.00
19.00	54.83	2.280	0.02292	243.8	476.9	248.2	272.2	520.4	0.873	1.703	19.00
20.00	57.27	2.308	0.02157	251.3	478.7	255.9	265.9	521.8	0.896	1.700	20.00
22.00	61.90	2.364	0.01921	265.8	481.7	271.0	253.0	524.0	0.939	1.695	22.00
24.00	66.21	2.424	0.01721	279.7	484.3	285.5	240.1	525.6	0.981	1.688	24.00
26.00	70.27	2.487	0.01549	293.1	486.2	299.6	226.9	526.5	1.021	1.681	26.00
28.00	74.10	2.555	0.01398	306.2	487.5	313.4	213.2	526.6	1.060	1.673	28.00
30.00	77.72	2.630	0.01263	319.2	488.1	327.1	198.9	526.0	1.097	1.664	30.00
35.00	86.01	2.862	0.009771	351.4	486.3	361.4	159.1	520.5	1.190	1.633	35.00
40.00	93.38	3.279	0.007151	387.9	474.7	401.0	102.3	503.3	1.295	1.574	40.00
42.48	96.70	4.535	0.004535	434.9	434.9	454.2	0.0	454.2	1.437	1.437	42.48

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Table B.14 *Properties of Superheated* ***Propane***

T °C	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m^3/kg	u kJ/kg	h kJkg	s kJ/kg·K
	$p = 0.05$ bar $= 0.005$ MPa ($T_{sat} = -93.28$°C)				$p = 0.1$ bar $= 0.01$ MPa ($T_{sat} = -83.87$°C)			
Sat.	6.752	326.0	359.8	2.081	3.542	367.3	370.8	2.011
–90	6.877	329.4	363.8	2.103				
–80	7.258	339.8	376.1	2.169	3.617	339.5	375.7	2.037
–70	7.639	350.6	388.8	2.233	3.808	350.3	388.4	2.101
–60	8.018	361.8	401.9	2.296	3.999	361.5	401.5	2.164
–50	8.397	373.3	415.3	2.357	4.190	373.1	415.0	2.226
–40	8.776	385.1	429.0	2.418	4.380	385.0	428.8	2.286
–30	9.155	397.4	443.2	2.477	4.570	397.3	433.0	2.346
–20	9.533	410.1	457.8	2.536	4.760	410.0	457.6	2.405
–10	9.911	423.2	472.8	2.594	4.950	423.1	472.6	2.463
0	10.29	436.8	488.2	2.652	5.139	436.7	488.1	2.520
10	10.67	450.8	504.1	2.709	5.329	450.6	503.9	2.578
20	11.05	570.6	520.4	2.765	5.518	465.1	520.3	2.634
	$p = 0.5$ bar $= 0.05$ MPa ($T_{sat} = -56.93$°C)				$p = 1.0$ bar $= 0.1$ MPa ($T_{sat} = -42.38$°C)			
Sat.	0.796	363.1	402.9	1.871	0.4185	378.5	420.3	1.822
–50	0.824	371.3	412.5	1.914				
–40	0.863	383.4	426.6	1.976	0.4234	381.5	423.8	1.837
–30	0.903	396.0	441.1	2.037	0.4439	394.2	438.6	1.899
–20	0.942	408.8	455.9	2.096	0.4641	407.3	453.7	1.960
–10	0.981	422.1	471.1	2.155	0.4842	420.7	469.1	2.019
0	1.019	435.8	486.7	2.213	0.5040	434.4	484.8	2.078
10	1.058	449.8	502.7	2.271	0.5238	448.6	501.0	2.136
20	1.096	464.3	519.1	2.328	0.5434	463.3	517.6	2.194
30	1.135	479.2	535.9	2.384	0.5629	478.2	534.5	2.251
40	1.173	494.6	553.2	2.440	0.5824	493.7	551.9	2.307
50	1.211	510.4	570.9	2.496	0.6018	509.5	569.7	2.363
60	1.249	526.7	589.1	2.551	0.6211	525.8	587.9	2.419
	$p = 2.0$ bar $= 0.2$ MPa ($T_{sat} = -25.43$°C)				$p = 3.0$ bar $= 0.3$ MPa ($T_{sat} = -14.16$°C)			
Sat.	0.2192	396.6	440.4	1.782	0.1496	408.7	453.6	1.762
–20	0.2251	404.0	449.0	1.816				
–10	0.2358	417.7	464.9	1.877	0.1527	414.7	460.5	1.789
0	0.2463	431.8	481.1	1.938	0.1602	429.0	477.1	1.851
10	0.2566	446.3	497.6	1.997	0.1674	443.8	494.0	1.912
20	0.2669	461.1	514.5	2.056	0.1746	458.8	511.2	1.971
30	0.2770	476.3	531.7	2.113	0.1816	474.2	528.7	2.030
40	0.2871	491.9	549.3	2.170	0.1885	490.1	546.6	2.088
50	0.2970	507.9	567.3	2.227	0.1954	506.2	564.8	2.145
60	0.3070	524.3	585.7	2.283	0.2022	522.7	583.4	2.202
70	0.3169	541.1	604.5	2.339	0.2090	539.6	602.3	2.258
80	0.3267	558.4	623.7	2.394	0.2157	557.0	621.7	2.314
90	0.3365	576.1	643.4	2.449	0.2223	574.8	641.5	2.369

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Table B.14 *Properties of Superheated* ***Propane***

T °C	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m^3/kg	u kJ/kg	h kJkg	s kJ/kg·K
	$p = 4.0$ bar $= 0.4$ MPa ($T_{sat} = -5.46$°C)				$p = 5.0$ bar $= 0.5$ MPa ($T_{sat} = 1.74$°C)			
Sat.	0.1137	418.0	463.5	1.751	0.09172	425.7	471.6	1.743
0	0.1169	426.1	472.9	1.786				
10	0.1227	441.2	490.3	1.848	0.09577	438.4	486.3	1.796
20	0.1283	456.6	507.9	1.909	0.1005	454.1	504.3	1.858
30	0.1338	472.2	525.7	1.969	0.1051	470.0	522.5	1.919
40	0.1392	488.1	543.8	2.027	0.1096	486.1	540.9	1.979
50	0.1445	504.4	562.2	2.085	0.1140	502.5	559.5	2.038
60	0.1498	521.1	581.0	2.143	0.1183	519.4	578.5	2.095
70	0.1550	538.1	600.1	2.199	0.1226	536.6	597.9	2.153
80	0.1601	555.7	619.7	2.255	0.1268	554.1	617.5	2.209
90	0.1652	573.5	639.6	2.311	0.1310	572.1	637.6	2.265
100	0.1703	591.8	659.9	2.366	0.1351	590.5	658.0	2.321
110	0.1754	610.4	680.6	2.421	0.1392	609.3	678.9	2.376
	$p = 6.0$ bar $= 0.6$ MPa ($T_{sat} = 7.93$°C)				$p = 7.0$ bar $= 0.7$ MPa ($T_{sat} = 13.41$°C)			
Sat.	0.07680	432.2	478.3	1.737	0.06598	438.0	484.2	1.733
10	0.07769	435.6	482.2	1.751				
20	0.08187	451.5	500.6	1.815	0.06847	448.8	496.7	1.776
30	0.08588	467.7	519.2	1.877	0.07210	465.2	515.7	1.840
40	0.08978	484.0	537.9	1.938	0.07558	481.9	534.8	1.901
50	0.09357	500.7	556.8	1.997	0.07896	498.7	554.0	1.962
60	0.09729	517.6	576.0	2.056	0.08225	515.9	573.5	2.021
70	0.1009	535.0	595.5	2.113	0.08547	533.4	593.2	2.079
80	0.1045	552.7	615.4	2.170	0.08863	551.2	613.2	2.137
90	0.1081	570.7	635.6	2.227	0.09175	569.4	633.6	2.194
100	0.1116	589.2	656.2	2.283	0.09482	587.9	654.3	2.250
110	0.1151	608.0	677.1	2.338	0.09786	606.8	675.3	2.306
120	0.1185	627.3	698.4	2.393	0.1009	626.2	696.8	2.361
	$p = 8.0$ bar $= 0.8$ MPa ($T_{sat} = 18.33$°C)				$p = 9.0$ bar $= 0.9$ MPa ($T_{sat} = 22.82$°C)			
Sat.	0.05776	443.1	489.3	1.729	0.05129	447.2	493.8	1.726
20	0.05834	445.9	492.6	1.740				
30	0.06170	462.7	512.1	1.806	0.05355	460.0	508.2	1.774
40	0.06489	479.6	531.5	1.869	0.05633	477.2	528.1	1.839
50	0.06796	496.7	551.1	1.930	0.05938	494.7	548.1	1.901
60	0.07094	514.0	570.8	1.990	0.06213	512.2	568.1	1.962
70	0.07385	531.6	590.7	2.049	0.06479	530.0	588.3	2.022
80	0.07669	549.6	611.0	2.107	0.06738	548.1	608.7	2.081
90	0.07948	567.9	631.5	2.165	0.06992	566.5	629.4	2.138
100	0.08222	586.5	652.3	2.221	0.07241	585.2	650.4	2.195
110	0.08493	605.6	673.5	2.277	0.07487	604.3	671.7	2.252
120	0.08761	625.0	695.1	2.333	0.07729	623.7	693.3	2.307
130	0.09026	644.8	717.0	2.388	0.07969	643.6	715.3	2.363
140	0.09289	665.0	739.3	2.442	0.08206	663.8	737.7	2.418

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Table B.14 *Contd.*

T °C	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K
	p = 10.0 bar = 1.0 MPa (T_{sat} = 26.95°C)				*p = 12.0 bar = 1.2 MPa* (T_{sat} = 34.39°C)			
Sat.	0.04606	451.8	497.9	1.723	0.03810	459.1	504.8	1.718
30	0.04696	457.1	504.1	1.744				
40	0.04980	474.8	524.6	1.810	0.03957	469.4	516.9	1.757
50	0.05248	492.4	544.9	1.874	0.04204	487.8	538.2	1.824
60	0.05505	510.2	565.2	1.936	0.04436	506.1	559.3	1.889
70	0.05752	528.2	585.7	1.997	0.04657	524.4	580.3	1.951
80	0.05995	546.4	606.3	2.056	0.04869	543.1	601.5	2.012
90	0.06226	564.9	627.2	2.114	0.05075	561.8	622.7	2.071
100	0.06456	583.7	648.3	2.172	0.05275	580.9	644.2	2.129
110	0.06681	603.0	669.8	2.228	0.05470	600.4	666.0	2.187
120	0.06903	622.6	691.6	2.284	0.05662	620.1	688.0	2.244
130	0.07122	642.5	713.7	2.340	0.05851	640.1	710.3	2.300
140	0.07338	662.8	736.2	2.395	0.06037	660.16	733.0	2.355
	p = 14.0 bar = 1.4 MPa (T_{sat} = 40.97°C)				*p = 16.0 bar = 1.6 MPa* (T_{sat} = 46.89°C)			
Sat.	0.03231	465.2	510.4	1.714	0.02790	470.4	515.0	1.710
50	0.03446	482.6	530.8	1.778	0.02861	476.7	522.5	1.733
60	0.03664	501.6	552.9	1.845	0.03075	496.6	545.8	1.804
70	0.03869	520.4	574.6	1.909	0.03270	516.2	568.5	1.871
80	0.04063	539.4	596.3	1.972	0.03453	535.7	590.9	1.935
90	0.04249	558.6	618.1	2.033	0.03626	555.2	613.2	1.997
100	0.04429	577.9	639.9	2.092	0.03792	574.8	635.5	2.058
110	0.04604	597.5	662.0	2.150	0.03952	594.7	657.9	2.117
120	0.04774	617.5	684.3	2.208	0.04107	614.8	680.5	2.176
130	0.04942	637.7	706.9	2.265	0.04259	635.3	703.4	2.233
140	0.05107	658.3	729.8	2.321	0.04407	656.0	726.5	2.290
150	0.05268	679.2	753.0	2.376	0.04553	677.1	749.9	2.346
160	0.05428	700.5	776.5	2.431	0.04696	698.5	773.6	2.401
	p = 18.0 bar = 1.8 MPa (T_{sat} = 52.30°C)				*p = 20.0 bar = 2.0 MPa* (T_{sat} = 57.27°C)			
Sat.	0.02441	474.9	518.8	1.705	0.02157	478.7	521.8	1.700
60	0.02606	491.1	538.0	1.763	0.02216	484.8	529.1	1.722
70	0.02798	511.4	561.8	1.834	0.02412	506.3	554.5	1.797
80	0.02974	531.6	585.1	1.901	0.02585	527.1	578.8	1.867
90	0.03138	551.5	608.0	1.965	0.02744	547.6	602.5	1.933
100	0.03293	571.5	630.8	2.027	0.02892	568.1	625.9	1.997
110	0.03443	591.7	653.7	2.087	0.03033	588.4	649.2	2.059
120	0.03586	612.1	676.6	2.146	0.3169	609.2	672.6	2.119
130	0.03726	632.7	699.8	2.204	0.03299	630.0	696.0	2.178
140	0.03863	653.6	723.1	2.262	0.03426	651.2	719.7	2.236
150	0.03996	674.8	746.7	2.318	0.03550	672.5	743.5	2.293
160	0.04127	696.3	770.6	2.374	0.03671	694.2	767.6	2.349
170	0.04256	718.2	794.8	2.429	0.03790	716.2	792.0	2.404
180	0.04383	740.4	819.3	2.484	0.03907	738.5	816.6	2.459

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Table B.14 *Contd.*

T °C	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K
	p = 22.0 bar = 2.2 MPa (T_{sat} = 61.90°C)				*p = 24.0 bar = 2.4 MPa* (T_{sat} = 66.21°C)			
Sat.	0.01921	481.8	524.0	1.695	0.01721	484.3	525.6	1.688
70	0.02086	500.5	546.4	1.761	0.01802	493.7	536.9	1.722
80	0.02261	522.4	572.1	1.834	0.01984	517.0	564.6	1.801
90	0.02417	543.5	596.7	1.903	0.02141	539.0	590.4	1.873
100	0.02561	564.5	620.8	1.969	0.02283	560.6	615.4	1.941
110	0.02697	585.3	644.6	2.032	0.02414	581.9	639.8	2.006
120	0.02826	606.2	668.4	2.093	0.02538	603.2	664.1	2.068
130	0.02949	627.3	692.2	2.153	0.02656	624.6	688.3	2.129
140	0.03069	648.6	716.1	2.211	0.02770	646.0	712.5	2.188
150	0.03185	670.1	740.2	2.269	0.02880	667.8	736.9	2.247
160	0.03298	691.9	764.5	2.326	0.02986	689.7	761.4	2.304
170	0.03409	714.1	789.1	2.382	0.03091	711.9	786.1	2.360
180	0.03517	736.5	813.9	2.437	0.03193	734.5	811.1	2.416
	p = 26.0 bar = 2.6 MPa (T_{sat} = 70.27°C)				*p = 30.0 bar = 3.0 MPa* (T_{sat} = 77.72°C)			
Sat.	0.01549	486.2	526.5	1.681	0.01263	488.2	526.0	1.664
80	0.01742	511.0	556.3	1.767	0.01318	495.4	534.9	1.689
90	0.01903	534.2	583.7	1.844	0.01506	522.8	568.0	1.782
100	0.02045	556.4	609.6	1.914	0.01654	547.2	596.8	1.860
110	0.02174	578.3	634.8	1.981	0.01783	570.4	623.9	1.932
120	0.02294	600.0	659.6	2.045	0.01899	593.0	650.0	1.999
130	0.02408	621.6	684.2	2.106	0.02007	615.4	675.6	2.063
140	0.02516	643.4	708.8	2.167	0.02109	637.7	701.0	2.126
150	0.02621	665.3	733.4	2.226	0.02206	660.1	726.3	2.186
160	0.02723	687.4	758.2	2.283	0.02300	682.6	751.6	2.245
170	0.02821	709.9	783.2	2.340	0.02390	705.4	777.1	2.303
180	0.02918	732.5	808.4	2.397	0.02478	728.3	802.6	2.360
190	0.03012	755.5	833.8	2.452	0.02563	751.5	828.4	2.417
	p = 35.0 bar = 3.5 MPa (T_{sat} = 86.01°C)				*p = 40.0 bar = 4.0 MPa* (T_{sat} = 93.38°C)			
Sat.	0.00977	486.3	520.5	1.633	0.00715	474.7	503.3	1.574
90	0.01086	502.4	540.5	1.688				
100	0.01270	532.9	577.3	1.788	0.00940	512.1	549.7	1.700
110	0.01408	558.9	608.2	1.870	0.01110	544.7	589.1	1.804
120	0.01526	583.4	636.8	1.944	0.01237	572.1	621.6	1.887
130	0.01631	607.0	664.1	2.012	0.01344	597.4	651.2	1.962
140	0.01728	630.2	690.7	2.077	0.01439	621.9	679.5	2.031
150	0.01819	653.3	717.0	2.140	0.01527	645.9	707.0	2.097
160	0.01906	676.4	743.1	2.201	0.01609	669.7	734.1	2.160
170	0.01989	699.6	769.2	2.261	0.01687	693.4	760.9	2.222
180	0.02068	722.9	795.3	2.319	0.01761	717.3	787.7	2.281
190	0.02146	746.5	821.6	2.376	0.01833	741.2	814.5	2.340
200	0.02221	770.3	848.0	2.433	0.01902	765.3	841.4	2.397

Table B.15 *Properties of* ***saturated Nitrogen*** *(Liquid-Vapour) temperature entry*

Temp.	*Sat. Pressure*	*Specific Volume, m^3/kg*		*Internal energy, kJ/kg*		*Enthalpy, kJ/kg*			*Entropy, kJ/kg*		*Temp.*
K *T*	*kPa* *p*	*Sat. Liquid* v_f	*Sat. Vapour* v_g	*Sat. Liquid* u_f	*Sat. Vapour* u_g	*Sat. Liquid* h_f	*Evap* h_{fg}	*Sat. Vapour* h_g	*Sat. Liquid* s_f	*Sat. Vapour* s_g	*K* *T*
63.1	12.5	0.001150	1.48189	–150.92	45.94	–150.91	215.39	64.48	2.4234	5.8343	63.1
65	17.4	0.001160	1.09347	–147.19	47.17	–147.17	213.38	66.21	2.4816	5.7645	65
70	38.6	0.001191	0.52632	–137.13	50.40	–137.09	207.79	70.70	2.6307	5.5991	70
75	76.1	0.001223	0.28174	–127.04	53.43	–126.95	201.82	74.87	2.7700	5.4609	75
77.3	101.3	0.001240	0.21639	–122.27	54.76	–122.15	198.84	76.69	2.8326	5.4033	77.3
80	137.0	0.001259	0.16375	–116.86	56.20	–116.69	195.32	78.63	2.9014	5.3429	80
85	229.1	0.001299	0.10148	–106.55	58.65	–106.25	188.15	81.90	3.0266	5.2401	85
90	360.8	0.001343	0.06611	–96.06	60.70	–95.58	180.13	84.55	3.1466	5.1480	90
95	541.1	0.001393	0.04476	–85.35	62.25	–84.59	171.07	86.47	3.2627	5.0634	95
100	779.2	0.001452	0.03120	–74.33	63.17	–73.20	160.68	87.48	3.3761	4.9829	100
105	1084.6	0.001522	0.02218	–62.89	63.29	–61.24	148.59	87.35	3.4883	4.9034	105
110	1467.6	0.001610	0.01595	–50.81	62.31	–48.45	134.15	85.71	3.6017	4.8213	110
115	1939.3	0.001729	0.01144	–37.66	59.70	–34.31	116.19	81.88	3.7204	4.7307	115
120	2513.0	0.001915	0.00799	–22.42	54.21	–17.61	91.91	74.30	3.8536	4.6195	120
125	3208.0	0.002355	0.00490	–0.83	39.90	6.73	48.88	55.60	4.0399	4.4309	125
126.2	3397.8	0.003194	0.00319	18.94	18.94	29.79	0	29.79	4.2193	4.2193	126.2

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Table B.16 *Properties of* ***superheated Nitrogen***

Temp. *K*	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg·K*	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg K*	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg·K*
	100 kPa (77.24)			*200 kPa (83.62)*			*500 kPa (93.98)*		
Sat.	0.21903	76.61	5.4059	0.11520	81.05	5.2673	0.04834	86.15	5.0802
100	0.29103	101.94	5.6944	0.14252	100.24	5.4775	0.05306	94.46	5.1660
120	0.35208	123.15	5.8878	0.17397	121.93	5.6753	0.06701	118.12	5.3821
140	0.41253	144.20	6.0501	0.20476	143.28	5.8399	0.08007	140.44	5.5541
160	0.47263	165.17	6.1901	0.23519	164.44	5.9812	0.09272	162.22	5.6996
180	0.53254	186.09	6.3132	0.26542	185.49	6.1052	0.10515	183.70	5.8261
200	0.59231	206.97	6.4232	0.29551	206.48	6.2157	0.11744	205.00	5.9383
220	0.65199	227.83	6.5227	0.32552	227.41	6.3155	0.12964	226.18	6.0392
240	0.71161	248.67	6.6133	0.35546	248.32	6.4064	0.14177	247.27	6.1310
260	0.77118	269.51	6.6967	0.38535	269.21	6.4900	0.15385	268.31	6.2152
280	0.83072	290.33	6.7739	0.41520	290.08	6.5674	0.16590	289.31	6.2930
300	0.89023	311.16	6.8457	0.44503	310.94	6.6393	0.17792	310.28	6.3653
350	1.03891	365.24	7.0063	0.51952	363.09	6.8001	0.20788	362.63	6.5267
400	1.18752	415.41	7.1456	0.59392	415.31	6.9396	0.23777	414.99	6.6666
450	1.33607	467.77	7.2690	0.66827	467.70	7.0630	0.26759	467.49	6.7902
500	1.48458	520.41	7.3799	0.74258	520.37	7.1740	0.29739	520.24	6.9014
600	1.78154	626.94	7.5741	0.89114	626.94	7.3682	0.35691	626.93	7.0959
700	2.07845	735.58	7.7415	1.03965	735.61	7.5357	0.41637	735.68	7.2635
800	2.37532	846.60	7.8897	1.18812	846.64	7.6839	0.47581	846.78	7.4118
900	2.67217	960.01	8.0232	1.33657	960.07	7.8175	0.53522	960.24	7.5454
1000	2.96900	1075.68	8.1451	1.48501	1075.75	7.9393	0.59462	1075.96	7.6673

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Table B.16 *(Contd.) Superheated Nitrogen*

Temp. *K*	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg·K*	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg·K*	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg·K*
	600 kPa (96.37)			*800 kPa (100.38)*			*1000 kPa (103.73)*		
Sat.	0.04046	86.85	5.0411	0.03038	87.52	4.9762	0.02416	87.51	4.9237
120	0.05510	116.79	5.3204	0.04017	114.02	5.2191	0.03117	111.08	5.1357
140	0.06620	139.47	5.4953	0.04886	137.50	5.4002	0.03845	135.47	5.3239
160	0.07689	161.47	5.6422	0.05710	159.95	5.5501	0.04522	158.42	5.4772
180	0.08734	183.10	5.7696	0.06059	181.89	5.6793	0.05173	180.67	5.6082
200	0.09766	204.50	5.8823	0.07293	203.51	5.7933	0.05809	202.52	5.7234
220	0.10788	225.76	5.9837	0.08067	224.94	5.8954	0.06436	224.11	5.8263
240	0.11803	246.92	6.0757	0.08835	246.23	5.9880	0.07055	245.53	5.9194
260	0.12813	268.01	6.1601	0.09599	267.42	6.0728	0.07670	266.83	6.0047
280	0.13820	289.05	6.2381	0.10358	288.54	6.1511	0.08281	288.04	6.0833
300	0.14824	310.06	6.3105	0.11115	309.62	6.2238	0.08889	309.18	6.1562
350	0.17326	362.48	6.4722	0.12998	362.17	6.3858	0.10401	361.87	6.3187
400	0.19819	414.89	6.6121	0.14873	414.68	6.5260	0.11905	414.47	6.4591
450	0.22308	467.42	6.7359	0.16743	467.28	6.6500	0.13404	467.15	6.5832
500	0.24792	520.20	6.8471	0.18609	520.12	6.7613	0.14899	520.04	6.6947
600	0.29755	626.93	7.0416	0.22335	626.93	6.9560	0.17883	626.92	6.8895
700	0.34712	735.70	7.2093	0.26056	735.76	7.1237	0.20862	735.81	7.0573
800	0.39666	846.82	7.3576	0.29773	846.91	7.2721	0.23837	847.00	7.2057
900	044618	960.60	7.4912	0.33488	960.42	7.4058	0.26810	960.54	7.3394
1000	0.49568	1076.02	7.6131	0.37202	1076.16	7.5277	0.29782	1076.30	7.4614
	1500 kPa (110.38)			*2000 kPa (115.58)*			*3000 kPa (123.61)*		
Sat.	0.01555	85.51	4.8148	0.01100	81.25	4.7193	0.00582	63.47	4.5032
120	0.01899	102.75	4.9650	0.01260	92.10	4.8116	—	—	—
140	0.02452	130.15	5.1767	0.01752	124.40	5.0618	0.01038	111.13	4.8706
160	0.02937	154.50	5.3394	0.02144	150.43	5.2358	0.01350	141.85	5.0763
180	0.03393	177.60	5.4755	0.02503	174.48	5.3775	0.01614	168.09	5.2310
200	0.03832	200.03	5.5937	0.02844	197.53	5.4989	0.01857	192.49	5.3596
240	0.04682	243.80	5.7933	0.03496	242.08	5.7021	0.02312	238.66	5.5702
260	0.05099	265.36	5.8796	0.03814	263.90	5.7894	0.02531	261.02	5.6597
280	0.05512	286.78	5.9590	0.04128	285.53	5.8696	0.02746	283.09	5.7414
300	0.05922	308.10	6.0325	0.04440	307.03	5.9438	0.02958	304.94	5.8168
350	0.06940	361.13	6.1960	0.05209	360.39	6.1083	0.03480	358.96	5.9834
400	0.07949	413.96	6.3371	0.05791	413.47	6.2500	0.03993	412.50	6.1264
450	0.08953	466.82	6.4616	0.06727	466.49	6.3750	0.04502	465.87	6.2521
500	0.09953	519.84	6.5733	0.07480	519.65	6.4870	0.05008	519.29	6.3647
600	0.11948	626.92	6.7685	0.08980	626.93	6.6825	0.06013	626.95	6.5609
700	0.13937	735.94	6.9365	0.10474	736.07	6.8507	0.07012	736.35	6.7295
800	0.15923	847.22	7.0851	0.11965	847.45	6.9994	0.08008	847.92	6.8785
900	0.17906	960.83	7.2189	0.13454	961.13	7.1333	0.09003	961.73	7.0125
1000	0.19889	1076.65	7.3409	1.14942	1077.01	7.2553	0.09996	1077.72	7.1347

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Table B.16 *(Contd.) Superheated Nitrogen*

Temp. *K*	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg·K*	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg·K*	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg·K*
		6000 kPa			*8000 kPa*			*10000 kPa*	
140	0.002941	47.44	4.2926	0.002224	27.78	4.1167	0.002003	20.87	4.0373
160	0.005556	112.16	4.7292	0.003748	91.80	4.5453	0.002908	76.52	4.4088
180	0.007309	148.02	4.9411	0.005193	134.69	4.7988	0.004021	122.65	4.6813
200	0.008771	177.29	5.0955	0.006387	167.47	4.9717	0.005014	158.35	4.8697
220	0.010095	203.77	5.2217	0.007449	196.07	5.1082	0.005902	188.88	5.0153
240	0.011337	228.73	5.3303	0.008433	222.48	5.2231	0.006721	216.64	5.1362
260	0.012526	252.73	5.4264	0.009367	247.55	5.3235	0.007495	242.72	5.2406
280	0.013678	276.09	5.5130	0.010264	271.74	5.4131	0.008235	267.69	5.3331
300	0.014803	298.99	5.5920	0.011135	295.32	5.4945	0.008982	291.90	5.4167
350	0.017532	354.95	5.7646	0.013236	352.51	5.6709	0.010670	350.26	5.5967
400	0.020187	409.18	5.9111	0.015264	408.24	5.8197	0.012320	406.79	5.7477
450	0.022794	464.19	6.0392	0.017248	463.22	5.9492	0.013927	462.36	5.8786
500	0.025370	518.37	6.1534	0.019202	517.88	6.0644	0.015507	517.48	5.9948
600	0.030463	627.12	6.3516	0.023053	627.32	6.2639	0.018611	627.58	6.1955
700	0.035506	737.27	6.5214	0.026856	737.94	6.4344	0.021669	738.65	6.3667
800	0.040519	849.37	6.6710	0.030631	850.38	6.5845	0.024700	851.43	6.5172
900	0.045514	963.59	6.8055	0.034388	964.86	6.7194	0.027714	966.15	6.6523
1000	0.050495	1079.88	6.9281	0.038132	1081.35	6.8421	0.030715	1082.84	6.7753
		15000 kPa			*20000 kPa*			*50000 kPa*	
140	0.001770	14.81	3.9272	0.001655	13.75	3.8587	0.001391	28.05	3.6405
160	0.002183	59.14	4.2232	0.001929	53.63	4.1250	0.001497	61.62	3.8647
180	0.002749	102.34	4.4778	0.002281	93.02	4.2570	0.001612	94.31	4.0573
200	0.003365	140.60	4.6796	0.002687	130.17	4.5529	0.001736	126.15	4.2250
220	0.003964	174.10	4.8394	0.003108	164.26	4.7154	0.001867	157.12	4.3726
240	0.004531	204.33	4.9710	0.003525	195.59	4.8518	0.002003	187.24	4.5037
260	0.005071	232.41	5.0834	0.003930	224.82	4.9689	0.002143	216.53	4.6209
280	0.005589	259.01	5.1820	0.004323	252.50	5.0714	0.002285	245.02	4.7266
300	0.006088	284.56	5.2702	0.004704	279.01	5.1629	0.002428	272.78	4.8223
350	0.007280	345.47	5.4581	0.005617	341.86	5.3568	0.002786	339.44	5.0280
400	0.008416	403.79	5.6139	0.006487	401.65	5.5166	0.003138	403.08	5.1980
450	0.009517	460.71	5.7480	0.007329	459.70	5.6534	0.003484	464.64	5.3431
500	0.010593	516.88	5.8664	0.008149	516.78	5.7737	0.003823	524.82	5.4699
600	0.012697	628.50	6.0699	0.009748	629.76	5.9797	0.004484	642.94	5.6853
700	0.014759	740.63	6.2427	0.011310	742.85	6.1540	0.005129	760.04	5.8658
800	0.016797	854.18	6.3943	0.012849	857.11	6.3065	0.005762	877.47	6.0226
900	0.018818	969.50	6.5301	0.014373	972.98	6.4430	0.006385	995.87	6.1621
1000	0.020828	1086.64	6.6535	0.015887	1090.55	6.5668	0.007001	1115.51	6.2881

C

Appendix

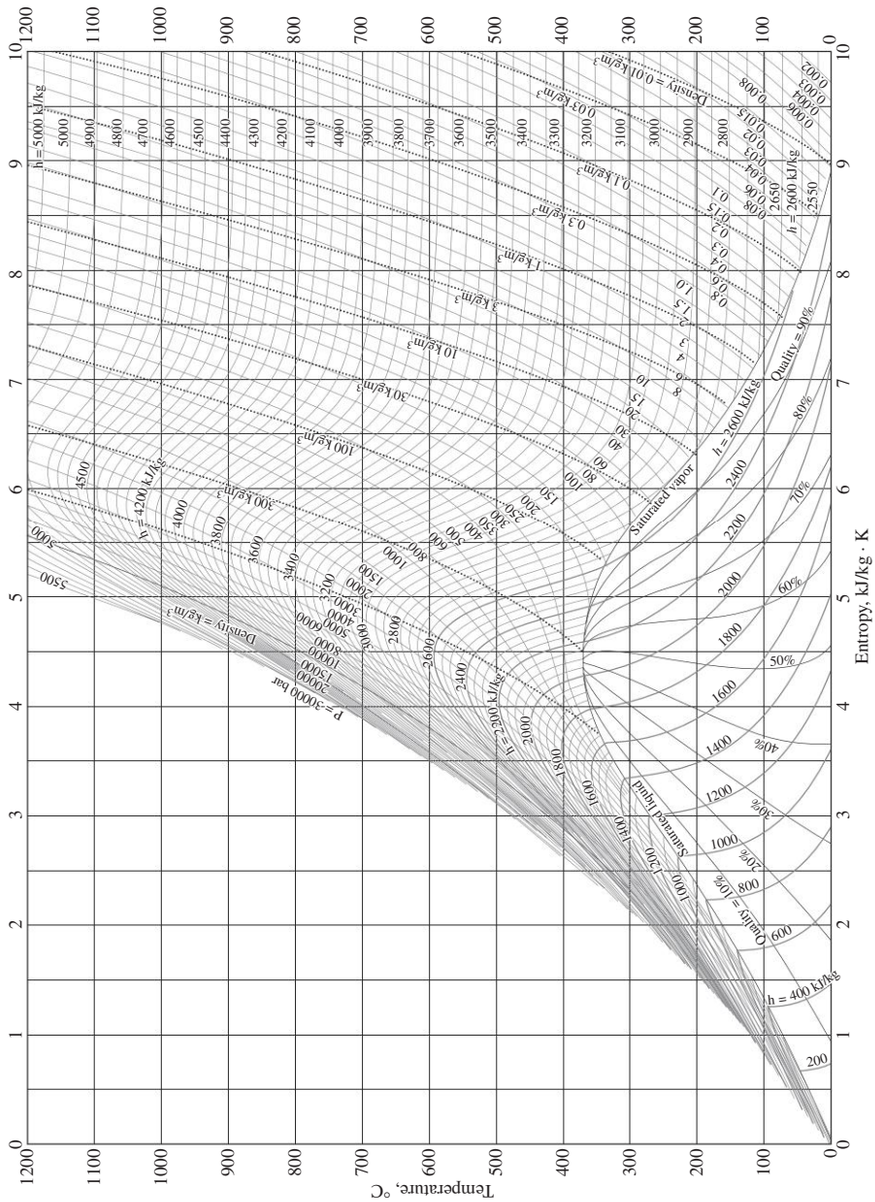

Fig. C.1 *Temperature entropy diagram for water*

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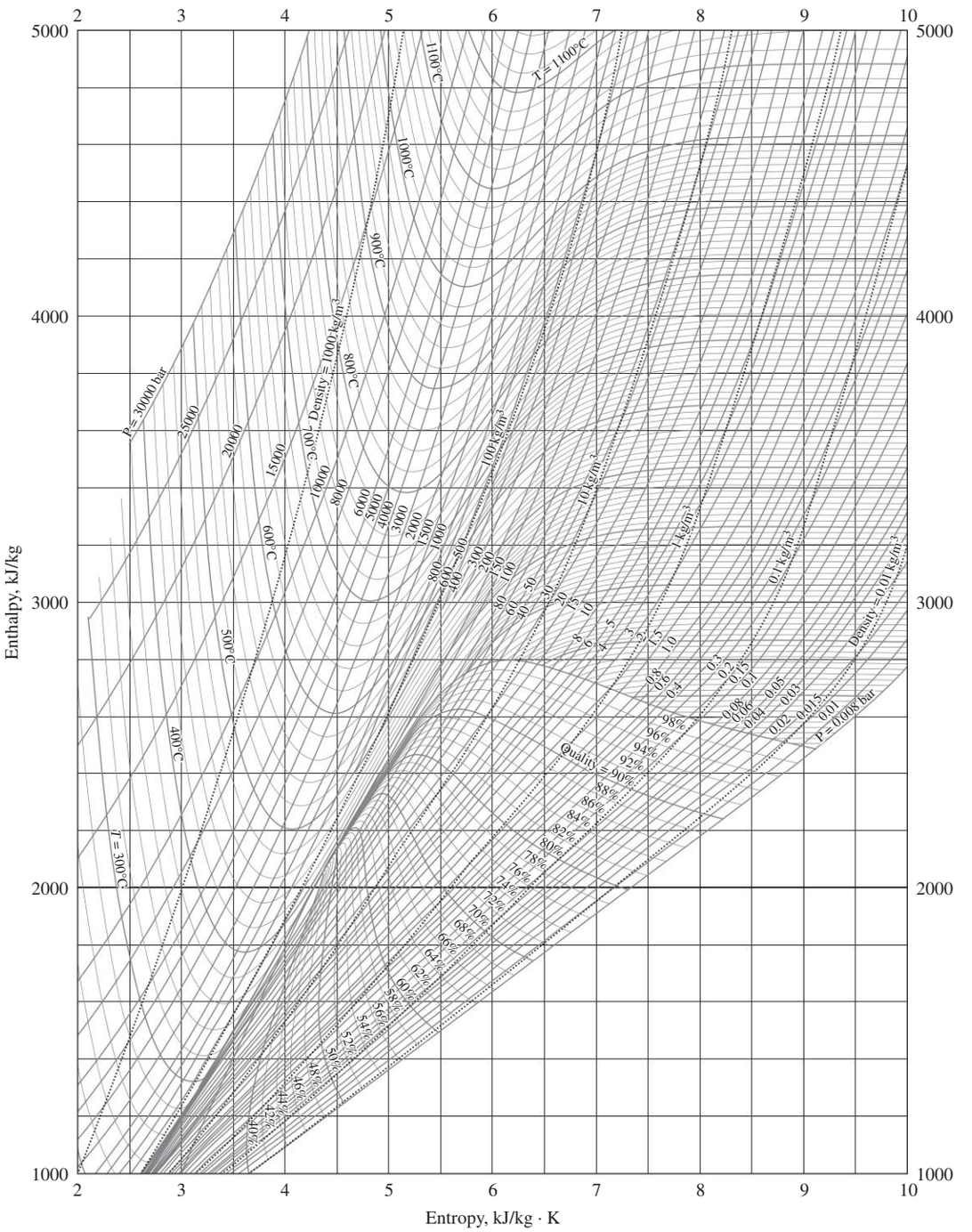

Fig. C.2 *Mollier diagram for water*

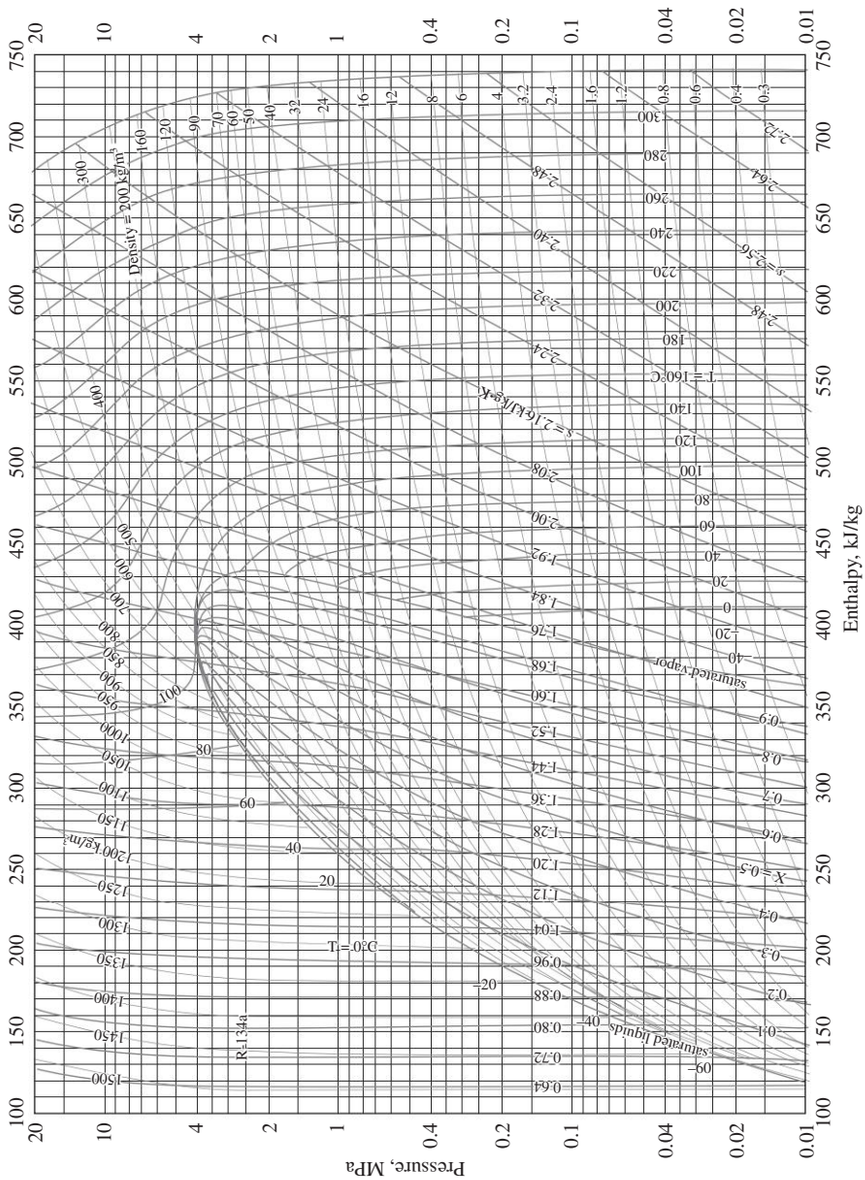

Fig. C.3 *Pressure-enthalpy diagram for refrigerant R134a*

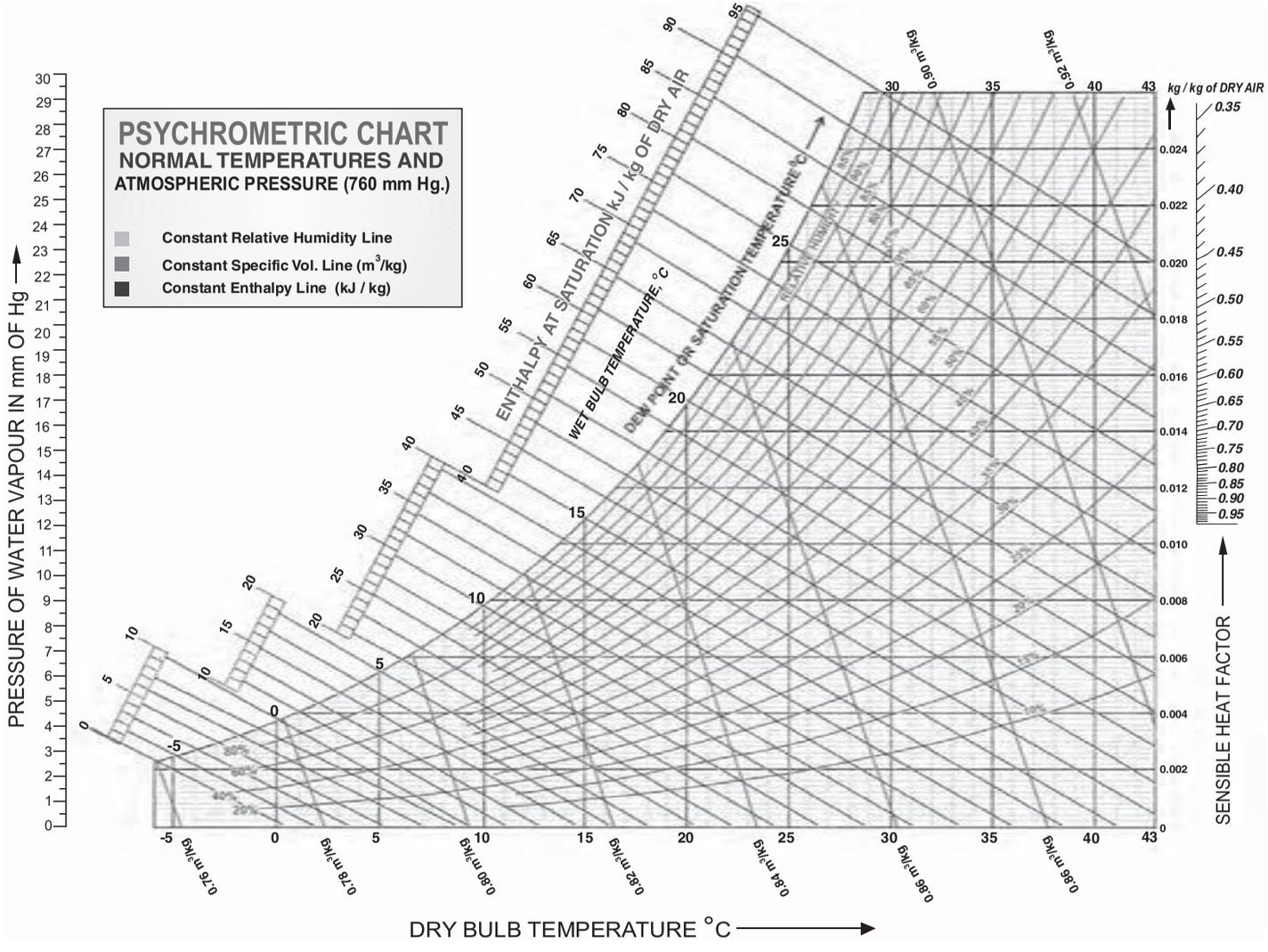

Fig. C.4 *Psychrometric chat at 1-atm total pressure. (Reprinted by permission of the American Society of Heating, Refrigerating and Air-Conditioning Engineers,)*

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